UNIVERSITI TEKNOLOGI MARA

FINANCIAL BASED COMPANY VALUATION USING INTEGRATED ANALYTICAL HIERARCHY PROCESS AND DATA ENVELOPEMENT ANALYSIS

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BSc (HONS.) MATHEMATICAL MODELLING AND ANALYTICS

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I certify that this report and the project to which it refers is the product of my own work and that any idea or quotation from the work of the other people, published or otherwise are fully acknowledged in accordance with standard referring practices of the discipline.

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ABSTRACT

In today's complex financial landscape, accurately evaluating a company's performance requires more than conventional financial analysis. This study proposes a hybrid company valuation framework by integrating the Analytic Hierarchy Process (AHP) and Data Envelopment Analysis (DEA) to evaluate and rank financial efficiency in Malaysian banking firms. Traditional financial analysis methods often fail to capture the multidimensional nature of company performance and lack a systematic integration of expert judgment. To address these limitations, this research applies AHP to derive expert-informed weights for key financial ratios, including profitability, valuation, liquidity, and operational efficiency. These weights are then embedded into a Variable Returns to Scale (VRS) DEA model as upper bounds, ensuring that efficiency scores reflect both empirical data and stakeholder priorities. Furthermore, a cross-efficiency DEA approach is used to enhance the discriminatory power and generate a complete firm ranking. The model is tested using financial data from Bursa Malaysia-listed banks between 2019 and 2024, and its outputs are validated against actual stock price performance using Spearman's rank correlation. The results confirm a weak alignment between DEA-derived financial efficiency and market behavior, demonstrating that market prices are also influenced by external and behavioral factors beyond financial data. This research contributes to the field of multi-criteria decision-making (MCDM) by offering a reproducible, expert-driven, and data-supported framework for corporate financial performance evaluation.

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Abbreviations

AHP	Analytic Hierarchy Process
DEA	Data Envelopment Analysis
DMU	Decision-Making Unit
CR	Consistency Ratio
VRS	Variable Returns to Scale
CE	Cross Efficiency
MCDM	Multi-Criteria Decision Making

CHAPTER 1
INTRODUCTION

1.1 Research Background

The assessment of the company performance is a pillar of effective financial decision-making. In the past, such assessments have been conducted with the help of financial ratio analysis and other quantitative methods, such as trend analysis and regression models Gibson (2004); Penman (2013). Nevertheless, these traditional methods tend to fail in describing the complexity of contemporary business environments, which encompass several, and sometimes contradictory, performance dimensions. This shortcoming has opened the door to the use of Multi-Criteria Decision-Making (MCDM) techniques, which enable more detailed and complex analyses due to the inclusion of both qualitative and quantitative criteria Asadabadi et al. (2019).

The most well-known MCDM methods include the Analytic Hierarchy Process (AHP) and Data Envelopment Analysis (DEA). AHP, as proposed by Saaty (1985), offers a systematic methodology to give weighting to evaluation criteria by using expert-based pairwise comparisons. The approach has been adopted in financial analysis to add subjective judgment to decision models Aljuhaishi and Kadhim (2023); Eghbali Amooghin et al. (2023). It aids decision-makers to justify investment decisions, particularly in the face of uncertainty.

On the other hand, DEA, suggested by Charnes et al. (1978), is a non-parametric method of measuring the relative efficiency of decision-making units (DMUs) in terms of their input-output ratios. It has been generalized by Banker et al. (1984; 1989) to variable returns to scale and alternative inefficiency structure. DEA has proved to be applicable in gauging the efficiency of companies in various sectors including the banking and manufacturing sectors Nikoomaram et al. (2010); Halkos and Salamouris (2004).

After the recognition of the strengths of both methods, researchers have resorted to hybrid AHP–DEA models that combine the subjective weighting capacity of AHP and the objective efficiency measurement of DEA Tavana et al. (2023). It is a mixed approach that allows more accurate and situation-specific performance assessment models. To give an example, the effectiveness of the AHP–DEA method to measure the national R&D organizations was demonstrated by Jyoti et al. (2008), and the model was applied to measure the technical efficiency in the telecommunications sector in Saudi Arabia by Abdel-Halim et al. (2023).

Besides this, recent studies have emphasized the strategic value of integrated models. To illustrate, Kasemi (2022) applied DEA–AHP to assess the financial efficiency of food industry companies in Kosovo, whereas Yang (2022) applied a hybrid AHP–DEA model to optimize the accounting business processes. The article by Hong and Qu (2024) also illustrates the potential of AHP–DEA in cross-sector analysis of the operational risk in the banking sector.

AHP combined with DEA thus provides a powerful analytical tool to the valuation of firms in the more dynamic and intricate financial world. It enables stakeholders to gain an entire image of organizational strengths and inefficiencies and coordinate the outcomes of the assessment with the strategic objectives Zhu et al. (2024); Aljuhaishi and Kadhim (2023).

1.2 Motivation for This Work

The rationale behind the study is that there is an emerging need to have an advanced methodology that is data-driven to assess corporate performance and value from a multi-dimensional perspective. Although essential, the traditional approaches to financial analysis are usually unable to capture the complexity and interdependence of factors that determine organizational efficiency and decisions made by stakeholders Gibson (2004); Penman (2013). Considering these shortcomings, there is an increased focus on implementing Multi-Criteria Decision-Making (MCDM) models that have the potential to incorporate quantitative information and qualitative expert opinion in a systematic manner Asadabadi et al. (2019).

Being a student who is specialising in mathematical modelling and analytics, I am especially interested in the possibility of using the Analytic Hierarchy Process (AHP) and Data Envelopment Analysis (DEA) in combination. AHP, proposed by Saaty

(1985), allows structured elicitation of expert preferences to calculate weights of criteria, whereas DEA, proposed by Charnes et al. (1978), allows the objective assessment of efficiency using input-output relationships. The combination of the two approaches has proven to be rather promising in the creation of balanced and holistic performance ratings, as it is applied in practice to telecommunications Abdel-Halim et al. (2023) and food industry ratings Kasemi (2022).

The peculiar feature of the AHP–DEA hybrid method is its capability to bridge the gap between subjective and objective valuation and data-driven performance evaluation Tavana et al. (2023). A combination of these methods gives a more comprehensive way of valuing companies, not only in respect to internal financial metrics, but also in the environment of the decision-making process. In addition, the hybrid models have gained popularity in contemporary research because of their flexibility, adaptability, and applicability in various industries Yang (2022); Zhu et al. (2024).

Through the application of this holistic approach, the study will not only add value to the scholarly debate on the measurement of financial performance but will also help in the creation of viable tools that can be applied by analysts and investors in making sound strategic decisions. The relevance of this study to modern trends in the fields of decision science and financial modeling proves its timeliness and possible contribution.

1.3 Problem Statement

Historically, the valuation of companies has been based mainly on financial ratios and market-based indicators. Although they are convenient, such methods tend to fail to reflect the multidimensional and complex nature of corporate performance, especially in decision environments that contain more than one and sometimes conflicting financial criteria Gibson (2004); Penman (2013); Subramanyam (2014). In addition, they seldom incorporate expert opinion systematically, which can lead to biased or incomplete assessments Aljuhaishi and Kadhim (2023); Eghbali Amooghin et al. (2023). Such shortcomings have led to the consideration of Multi-Criteria Decision-Making (MCDM) methods like the Analytic Hierarchy Process (AHP) and Data

Envelopment Analysis (DEA) which are more comprehensive in nature. AHP allows integrating expert opinions in the form of pairwise comparisons and consistency tests Saaty (1980); Franek and Kresta (2014), whereas DEA gives an objective efficiency measure of the decision-making units (DMUs) based on multiple inputs and outputs Banker et al. (1989); Nikoomaram et al. (2010).

However, AHP or DEA when used alone has limitations. In large matrices, AHP can be affected by rank reversal and inconsistency Cheng and Li (2003); Wang and Elhag (2006), whereas DEA does not have the ability to deal with qualitative or subjective variables directly Zhu et al. (2024); Hong and Qu (2024). In order to overcome these deficiencies, recent literature has suggested combined AHP–DEA models that combine the advantages of the two approaches. It has been shown that this hybrid method has a lot of promise in other fields like telecommunications Abdel-Halim et al. (2023), banking Ghazali et al. (2022), and portfolio selection Jana et al. (2024); Vásquez et al. (2021), where it integrates the weight assignment of experts and objective efficiency benchmarking.

Nevertheless, the application of this integrated AHP–DEA methodology to company valuation—especially in connection with real-world market outcomes—remains relatively underexplored. In particular, it is unclear whether the financial efficiency of firms, as measured by expert-informed DEA models, aligns with actual investor behavior as reflected in stock price performance.

Therefore, this study aims to address the following research problems:

i. How can expert judgment be systematically translated into quantitative priority weights for financial performance metrics using the AHP method?
ii. How can AHP-derived expert weightings be integrated into DEA to enable robust ranking of company financial efficiency?
iii. To what extent does the financial efficiency measured through AHP–DEA correlate with actual company stock performance?

1.4 Research Objectives

In the field of financial performance evaluation, the demand for models that combine analytical rigor with expert insight is growing. Traditional assessment methods often fail to incorporate subjective expert knowledge in a structured and reproducible way, leading to oversights in the multidimensional evaluation of companies. To address this, the integration of the Analytic Hierarchy Process (AHP) with Data Envelopment Analysis (DEA) presents a robust methodological framework. AHP captures expert judgment through pairwise comparisons and translates it into quantifiable weights, while DEA objectively benchmarks financial efficiency across firms. This integrated approach enhances both the interpretability and validity of performance assessments. Furthermore, in a capital market context, it is essential to determine whether operational efficiency correlates with investor behavior, as reflected in stock market performance. Therefore, this study is guided by the following three research objectives:

i. To enhance DEA evaluation by integrating AHP-derived expert weightage into the efficiency analysis framework.

ii. To rank companies based on their financial efficiency using a bounded, expert-informed DEA model.

iii. To examine the correlation of the company financial efficiency and real-world company stock performance.

1.5 Significance of Study

The significance of this study lies in that it bridges the gap between subjective financial analysis and objective performance analysis by integrating the Analytic Hierarchy Process (AHP) and Data Envelopment Analysis (DEA). Traditional methods of financial analysis are more likely to lack the multidimensionality of corporate performance, especially when quantitative and qualitative aspects are to be considered in decision-making Gibson (2004); Penman (2013); Subramanyam (2014). The study offers a hybrid model of expert judgment and data-driven models since it concentrates on financial measures through AHP and then measures technical efficiency via DEA Saaty (1985); Banker et al. (1984); Tavana et al. (2023).

By combining the benefits of both methods, the AHP–DEA combination can contribute to the greater transparency of the decision-making process and its analytical richness. The structure of expert judgment in AHP is consistent Franek and Kresta (2014); Cheng and Li (2003), but in DEA, efficiency can be measured objectively using various financial inputs and outputs Banker et al. (1989); Nikoomaram et al. (2010). This combination eliminates the shortcomings of each of the methods, e.g., the inability of DEA to handle qualitative criteria and the vulnerability of AHP to rank reversal and subjectivity Wang and Elhag (2006); Zhu et al. (2024).

The practicality of the AHP–DEA method in the financial environment is proved by the empirical verification of the model in the research. Such efficacy has been exhibited in previous sectoral applications, including the Saudi telecommunications sector Abdel-Halim et al. (2023) and the food sector in Kosovo Kasemi (2022). By developing efficiency scores that reflect not only internal financial performance but also external professional views, the model enables more reasonable, transparent, and potent company valuations, especially in a dynamic market environment Hong and Qu (2024); Zhao et al. (2021).

The study can be said to contribute to the increasing body of literature on Multi-Criteria Decision-Making (MCDM) academically. It demonstrates how hybrid techniques can make financial analysis and performance benchmarking more rigorous Asadabadi et al. (2019); Zopounidis et al. (2015); Vaidya and Kumar (2006). Besides, the methodological design of the study introduces a replicable and flexible tool that can be used by investors, analysts, policymakers, and corporate strategists Aruldoss et al. (2013); Jana et al. (2024).

Lastly, this paper points out the significance of integrating mathematical modeling with expertise in financial analysis. It shows that hybrid MCDM approaches, in case of their proper application, can offer deeper insights into company performance and improve the investment decision-making process in more data-driven financial settings.

1.6 Thesis Scope

The primary purpose of the study is the evaluation of the performance of Malaysian banking companies in terms of their financials using hybrid multi-criteria decision-making (MCDM) approach that integrates Analytical Hierarchy Process (AHP) and Data Envelopment Analysis (DEA). The relative importance (weights) of the selected financial metrics is obtained through ten expert judgments using AHP method from Portfolio Manager, Fund Manager, Managing Director, and Investment Analyst. These weights are then applied in the DEA model to establish relative efficiency of the companies. The research study will be limited to the companies listed at Bursa Malaysia in the banking sector namely RHB Bank Berhad, Hong Leong Bank Berhad, CIMB Group Holdings Berhad, Public Bank Berhad, and Malayan Banking Berhad, between the year 2018 and 2024. Metric that we use are explained on Table 1.1 Appendix. The data is analyzed using computational tools such as RStudio and Excel to make the model accurate, efficient and reproducible (Gibson, 2004; Penman, 2013).

1.7 Limitation

Although the proposed AHP-DEA integrated model of financial performance assessment is associated with a range of strengths, this research recognizes a number of limitations that are inherent to the proposed framework. To begin with, the study is limited to Malaysian banking institutions which are listed in Bursa Malaysia. Such sector-specific scope restricts the generalizability of findings to businesses that are not in the same industry or geographical location, especially those that are subject to different regulatory or macroeconomic conditions. Furthermore, the AHP technique implies the subjective expert opinion in order to define the relative significance of financial criteria. In as much as calculation of consistency ratio (CR) is used to confirm reliability of responses, the process remains susceptible to personal biases, which can influence objectivity of the weightings obtained. Moreover, the analysis is done on the historical financial data of 2018-2024. Although this range allows analyzing trends, it might not capture market disruptions and emerging factors as well as post-pandemic recovery, inflationary pressure, or sustainability-driven market shifts as well. The other limitation is the assumption of the model that the environment of decision making is stable. Both AHP and DEA models are fixed and fail to consider the swift structural changes or shocks, which may skew the performance measures, like sudden policy changes or financial crisis. Lastly, the research only considers financial metrics, leaving out qualitative aspects like corporate governance, environmental responsibility, or stakeholder engagement that may also be important in comprehensive company valuation.

1.8 Assumption

In order to make the research viable within the stipulated time and resource limits, a number of assumptions have been made. It is presumed that the chosen financial ratios, such as profitability, valuation, liquidity, and operational efficiency, are sufficient to reflect the entire financial health and performance of the firms under investigation. The expert judgments that are applied in the AHP pairwise comparisons are assumed to be rational, consistent, and reflective of industry knowledge, which allows prioritizing financial indicators reliably. Also, it is presumed that the financial information obtained through official sources, such as annual reports and regulatory filings, are correct, complete, and not materially misstated. These assumptions support the validity and the relevance of the model results and are critical in ensuring analytical consistency in the study.

1.9 Ethical Committee

This research complies with the ethical standards set by Universiti Teknologi MARA (UiTM). The data used is all publicly available and there is no confidential or sensitive data accessed or revealed. The AHP survey is voluntary and informed consent is obtained before data collection by experts. Possible conflict of interest is declared and there is integrity in data handling, analysis and reporting during the study. Ethical approval, where required, has been duly sought in alignment with UiTM’s academic and research regulations.

1.10 Thesis Outline

This thesis is introduced by a chapter that gives reasons why company valuation is This thesis begins with a chapter that explains the importance of company valuation, particularly in Malaysian banking firms. It outlines the study's rationale, main objectives, and the methodological framework combining the Analytic Hierarchy Process (AHP) and Data Envelopment Analysis (DEA). The chapter also presents the scope, limitations, assumptions, and ethical considerations to provide a clear roadmap for the study.

Chapter Two reviews related literature on financial performance analysis, multi-criteria decision-making techniques, and the specific applications of AHP and DEA in company evaluation. It justifies the chosen methodology by highlighting gaps and limitations in existing approaches.

Chapter Three describes the research methodology, detailing the development of the AHP–DEA framework, financial data collection, and data processing using tools such as Excel and R. Each step is methodologically justified to ensure relevance and reliability.

Chapter Four addresses the implementation of the AHP–DEA model, including AHP survey design, pairwise comparisons, consistency checks, and weight derivation. It also covers variable selection, the VRS DEA model formulation, efficiency evaluation, and cross-efficiency analysis.

Chapter Five presents the results and their interpretation. It analyzes the financial performance of the companies using the integrated model and assesses the model's effectiveness. Visual aids such as graphs and tables support the discussion.

The final chapter concludes the thesis by summarizing key findings and discussing their implications. It outlines study limitations and provides future research recommendations, including potential model enhancements and broader applications. The chapter reinforces the study's academic and practical contributions.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter reviews the theoretical and empirical literature related to company financial performance evaluation, focusing on traditional and modern decision-making frameworks. It specifically explores the foundations and applications of fundamental financial analysis and advances in multi-criteria decision-making (MCDM) methods—namely, the Analytic Hierarchy Process (AHP) and Data Envelopment Analysis (DEA). By synthesizing these insights, the chapter establishes a conceptual foundation for integrating AHP and DEA in a hybrid evaluation model to rank company efficiency and relate it to stock market performance.

2.2 Traditional Company Financial focused fundamental analysis

Fundamental analysis is considered to be one of the pillars of investment decision-making and valuation of a company. It dates back to the seminal work of Benjamin Graham and David Dodd (1934), in their book Security Analysis, which established the foundation of assessing the intrinsic value of a firm using its financial statements and economic fundamentals. Fundamental analysis has over time been developed to be more quantitative and systematic to include financial ratios, trend analysis and valuation models. It is the major instrument of analysts, investors, and corporate decision-makers who want to evaluate the financial health of a firm, its growth opportunities, and its investment potential.

2.2.1 Financial-Focused Fundamental Analysis

Financial-oriented fundamental analysis The quantitative evaluation of the performance of a company based on standardized financial measures based on the income statement, balance sheet and cash flow statement. This method allows analysts to measure many factors of financial health profitability, liquidity, leverage, and market valuation in ratios that summarize complicated financial information into meaningful measures.

These financial ratios are classified into categories like valuation, profitability, liquidity, operational efficiency and growth each providing a different perspective in which performance may be viewed. These categories have been extended by scholars and practitioners over time to incorporate cash flow measures, stock volatility indicators and dividend policies to reflect more accurately the realities of modern finance.

2.2.2 Core Methodology and Metrics Used

The traditional approach to the fundamental analysis that is financial-oriented has a systematic approach that involves four major steps. First, the data is collected based on audited financial statements, including annual reports and quarterly filings that give standardized and reliable information on the financial status of a firm. Second, a set of financial ratios are calculated in a variety of categories, including valuation, profitability, liquidity, operational efficiency, growth, and debt ratios, in order to measure performance indicators. These ratios are brief, understandable measures to summarize complicated financial information into similar forms.

The third step is benchmarking and trend analysis in which ratios calculated are compared to historical data, average industry or peer companies to determine the strengths and weaknesses of performance and changing trends. Lastly, modeling of valuation is done. This can be in the form of conventional instruments such as Discounted Cash Flow (DCF) analysis and relative valuation with similar companies. In more recent times, multi-criteria decision-making models like the AHP-DEA hybrid have been proposed to address the same problem, with the advantage that they allow combining expert judgment with data-driven benchmarking, providing a more structured and comprehensive view of financial performance.

2.2.3 Contribution of Literature to This Study

Several classic and recent publications have been of great help to the methodology used in this study, especially in the choice of financial indicators, the organization of the hierarchies of analysis, and the rationale of combining expert judgment with quantitative models.

Osadchy et al. (2018) also highlight the importance of financial statements as the source of reliable information in decision-making, especially in emerging or changing economies. Their results prove the thesis that the interpretation of financial data should be standardized and strategic to evaluate the company properly. This observation forms the basis of the current research, whereby financial statement-based measures are used to form the basis of selection of criteria in the AHP-DEA model.

Penman (2013) brings a comprehensive and methodological procedure of security valuation by means of financial statements analysis. His model provides methodological advice on how to interpret valuation ratios including Price-to-Earnings (P/E) and Price-to-Book (P/B) which is part of the variable selection process in this study. The work of Penman enhances the connection between financial ratio analysis and intrinsic value estimation, which supports the combination of AHP and DEA in the company evaluation.

Subramanyam (2014) explores how ratio analysis can detect trends, risks, and operational inefficiencies within a firm. His insights assist in determining which financial indicators are best suited for conversion into DEA inputs and outputs. Furthermore, his work provides a conceptual structure for organizing financial metrics into distinct analytical categories—such as profitability and valuation—which is directly applicable in designing the AHP hierarchy used in this study.

Gibson (2004) provides critical insights into financial reporting and the importance of comparability across firms. His discussion on benchmarking validates the use of standardized ratios across companies in the same industry, thereby justifying the cross-sectional application of DEA in this research. The emphasis on interpretability and consistency enhances the reliability of comparative efficiency analysis.

Graham and Dodd’s seminal work Security Analysis (1951) offers the foundational philosophy for fundamental financial evaluation. Their core argument—that a company’s intrinsic value can be determined through rigorous analysis of its financial data—provides the underlying justification for the entire AHP–DEA methodology applied in this study. This classical perspective supports the use of financial ratios as reliable indicators of company performance.

Tracy (2012) offers a practical breakdown of 17 key financial ratios widely used in business analysis. His explanation of each ratio’s relevance to financial decision-making enables the precise selection of meaningful indicators for inclusion in the AHP–DEA model. Moreover, his categorization aids in structuring the hierarchy of criteria and subcriteria for expert comparison.

Lastly, Palepu et al. (2020) present an integrated approach to business analysis that combines strategic, financial, and accounting perspectives. Their framework supports the multi-dimensional evaluation of company performance and strengthens the rationale for treating DEA outputs as indicators of financial value creation. This theoretical perspective reinforces the appropriateness of combining AHP-derived expert priorities with DEA’s benchmarking capabilities.

Collectively, these works form a comprehensive scholarly foundation for this study’s integrated methodology, bridging theoretical rigor with applied financial analysis to deliver a robust and multidimensional evaluation model.

2.3 Weight assignment using Analytic Hierarchy Process (AHP)

Analytic Hierarchy Process (AHP) is an early multi-criteria decision-making (MCDM) technique invented by Thomas L. Saaty in the late 1970s to assist a decision-maker in resolving complex problems with multiple and often conflicting criteria. AHP was first designed to be used in military and strategic purposes and has been evolved to various fields of study including economics, healthcare, supply chain management, and especially within the study of financial analysis and evaluation of performances Saaty (1980).

2.3.1 Core Methodology of AHP

The Analytic Hierarchy Process (AHP) is grounded in a systematic methodology that facilitates decision-making through structured evaluation of multiple criteria. At the foundation of the method lies the construction of a hierarchical framework, which begins by clearly defining the decision problem and breaking it down into several levels: a goal at the top, followed by layers of criteria, sub-criteria, and potential alternatives.

Once the hierarchy is established, experts perform pairwise comparisons among elements at each level to assess their relative importance with respect to the element above. These comparisons utilize Saaty's 1–9 fundamental scale, where each numerical value corresponds to a specific level of importance: 1 for equal importance, 3 for moderate importance, 5 for strong importance, 7 for very strong importance, and 9 for extreme importance. Intermediate values such as 2, 4, 6, and 8 provide finer granularity for nuanced judgments, while reciprocal values (e.g., 1/3, 1/5, 1/7) are used to express inverse preferences.the problem and construct a hierarchy.

Table 2.1
Scale of Relative Importance

SCALE	IMPORTANCE
1	Equal Importance
3	Moderate Importance
5	Strong Importance
7	Very Strong Importance
9	Extreme Importance
2, 4,6,8	Intermediate Value
$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$	Inverse value

The resulting judgments are compiled into a square comparison matrix, where each element p_{ij} represents the relative importance of criterion iii over criterion j. To maintain logical consistency, the matrix is constructed such that $p_{ij} = {}^{1}/_{p_{ij}}$, and all diagonal elements equal 1, indicating self-comparison.

$$P = \begin{pmatrix} \frac{\omega_1}{\omega_1} & \frac{\omega_1}{\omega_2} & \cdots & \frac{\omega_1}{\omega_m} \\ \frac{\omega_2}{\omega_1} & \frac{\omega_2}{\omega_2} & \cdots & \frac{\omega_2}{\omega_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\omega_m}{\omega_1} & \frac{\omega_m}{\omega_2} & \cdots & \frac{\omega_m}{\omega_m} \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1m} \\ P_{21} & P_{22} & \cdots & P_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ P_{m1} & P_{m2} & \cdots & P_{mm} \end{pmatrix} \tag{2.1}$$

The next step involves deriving the priority weights from this matrix. This is achieved by calculating the principal eigenvector ω, which corresponds to the maximum eigenvalue λ_{max} of the matrix. The eigenvector is then normalized so that the sum of its components equals one, and these normalized values represent the final weights or priorities assigned to each criterion in the decision structure.

$$P\omega = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1m} \\ P_{21} & P_{22} & \cdots & P_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ P_{m1} & P_{m2} & \cdots & P_{mm} \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_1 \\ \vdots \\ \omega_{\mathrm{m}} \end{pmatrix} = \begin{pmatrix} m\omega_1 \\ m\omega_1 \\ \vdots \\ m\omega_{\mathrm{m}} \end{pmatrix} = m\omega \tag{2.2}$$

In other word, the well-known mathematical problem of matrix P eigenvalues with eigenvector $\boldsymbol{\omega}$:

$$P\omega = \lambda\omega \tag{2.3}$$

Where $\boldsymbol{\lambda} = m$ is an eigenvalue, m is the order of matrix P, to put it another way, the number of criteria compared. The weights in Saaty ‘s approach are the vector $\boldsymbol{\omega}$ are normalized components of eigenvector corresponding to the largest eigenvalue λmax:

$$P\omega = \lambda_{max}\omega \tag{2.4}$$

To ensure the reliability of expert judgments, AHP includes a consistency verification step. The Consistency Index (CI) is computed using the formula

$$CI = \frac{\lambda_{max} - n}{n-1} \tag{2.5}$$

Where,

n is the number of criteria.

This index is then used to calculate the Consistency Ratio

$$CR = \frac{CI}{RI} \tag{2.6}$$

Where,

RI is the Random Index, a value based on average CI scores from randomly generated matrices of the same size.

A CR value of less than 0.10 is typically regarded as indicating acceptable consistency in the judgments. In cases involving multiple experts, the aggregation of individual judgments is performed using methods such as Aggregated Individual Judgments (AIJ) or Aggregated Individual Priorities (AIP), ensuring a consolidated and representative set of priority weights for further analysis.

2.3.2 Contribution of Literature to This Study

The use of AHP in the current research is based on a significant amount of academic literature that confirms the theoretical strength and practical adaptability of the tool. The initial principles of AHP such as the pairwise comparison matrix, consistency index, and the derivation of weights based on eigenvalues are all founded on the seminal work of Saaty (1985)whose approach lays the foundation of the present study in terms of providing meaningful weights to the financial performance indicators. To supplement this, Wang and Elhag (2006)discussed the famous problem of rank reversal in the context of AHP and suggested the ways to preserve ranking stability, in case new alternatives are added. They are used here to maintain consistency in prioritization of financial metrics.

The relevance of AHP in complex decision-making, such as in finance, has been confirmed by Vaidya and Kumar (2006)who conducted an extensive survey of the AHP applications in different fields. Their review supports the application of AHP in assessing multidimensional financial criteria. Likewise, Kułakowski (2015) also pointed out the necessity of order maintenance and consistency in the development of comparison matrices, which is the methodological basis of the present study to make the results reliable prior to incorporating the AHP outputs into the DEA framework. The influence of judgment scales on the AHP performance is discussed by Franek and Kresta (2014), whose findings contribute to the current study and its attempts to enhance the accuracy and reliability of expert judgments by conducting enhanced consistency checks.

Jana et al. (2024) used fuzzy AHP to evaluate financial indexes, and the study demonstrated the flexibility of the technique in dealing with uncertainty, which can be used to guide future versions of the present research. The critical review of AHP and its generalization, ANP, provided by Asadabadi et al. (2019) provided important information about the advantages and drawbacks of hierarchical models, which justifies the use of a hybrid AHP-DEA approach. Zhao et al. (2021) also proved the practical utility of AHP in portfolio selection, which once again confirmed the relevance of the model in financial decision-making where the expert knowledge is crucial.

AHP implementation was enhanced technologically, including the introduction of AHP-OS software by Goepel (2018), which allowed an efficient analysis of consistency and the calculation of a priority vector. The given tool was utilized in the present study to facilitate the processing of expert input. Also, Eghbali Amooghin et al. (2023) applied fuzzy AHP in analyzing the factors influencing the quality of financial reporting, which proves the applicability of the method in financial analysis and contributes to the implementation of the DEA integration with variables.

AHP integration with other models can also be seen in the studies of Xu (2021), who integrated AHP with entropy measures in the assessment of FinTech, and Lam et al. (2023), who suggested a fuzzy-TOPSIS hybrid model to combine expert opinion and quantitative data. These hybrid solutions reflect the methodological trend of the current study, where the weights of AHP are developed by experts and incorporated into DEA to enhance the analysis of financial efficiency. Moreover, Aljuhaishi and Kadhim

(2023) showed how AHP can be used to make rational investment decisions, which is why subjective expert weighting should be used in capital allocation. Lastly, Vásquez et al. (2021) demonstrated the combination of AHP and TOPSIS to optimize the portfolio, which confirms the usefulness of AHP as a pre-processing method in the general context of MCDM applications, which it plays in this study prior to the application of DEA.

2.4 Efficiency measurements using Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA), introduced by Charnes, Cooper, and Rhodes in 1978, is a non-parametric linear programming methodology used to assess the relative efficiency of peer entities, commonly referred to as decision-making units (DMUs). These units convert multiple inputs into multiple outputs, and DEA is capable of handling such multi-dimensional data without requiring a predefined functional form. Based on the earlier work of Farrell (1957) on efficiency measurement, DEA has evolved into a dominant performance evaluation framework in fields including banking, healthcare, education, and corporate financial management.

2.4.1 Core Methodology of DEA

The original DEA model developed by Charnes, Cooper, and Rhodes—commonly referred to as the CCR model—assumes constant returns to scale (CRS). Under this formulation, the DEA model can be input-oriented or output-oriented, depending on whether the focus is minimizing inputs or maximizing outputs.

In the output-oriented CCR model, the linear programming objective seeks to:

$$\max_{\theta,u,,v}\left(\frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}\right)$$

$$s.t. \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \qquad (2.7)$$

$$u_r, v_i \geq 0$$

In the input-oriented version, the objective is to:

$$\min_{\theta,u,,v}\left(\frac{\sum_{i=1}^{m} v_i x_{io}}{\sum_{r=1}^{s} u_r y_{ro}}\right)$$

$$s.t.\frac{\sum_{i=1}^{m} v_i x_{ij}}{\sum_{r=1}^{s} u_r y_{rj}} \geq 1 \tag{2.8}$$

$$u_r\,, v_i \;\geq\; 0$$

Where:

x_{ij} = input i of DMU j,

y_{rj} = output r of DMU j,

v_i and u_r = weights assigned to inputs and outputs, respectively.

The dual (envelopment) form of the model constructs a convex hull from observed data and benchmarks each DMU relative to this hull. DEA evaluates each unit's performance in a manner that is both comparative and constructive, identifying the source and magnitude of inefficiencies.

Two major notable extensions of the original DEA model are BCC Model (Banker, Charnes, and Cooper, 1984) introduces Variable Returns to Scale (VRS), accommodating situations where firms operate under increasing or decreasing returns to scale—more realistic in economic and financial settings. Cross-Efficiency DEA (Doyle and Green, 1994) refines traditional DEA by incorporating peer evaluations, thus reducing bias from self-assigned optimal weights and enhancing ranking discrimination among DMUs.

2.4.2 Contribution of Literature to This Study

The foundational work by Charnes et al. (1978) established the original DEA model and forms the theoretical basis for this study's performance measurement framework. Although the CCR model assumes constant returns to scale, this research use the BCC (VRS) model for greater flexibility in analyzing companies of varying sizes.

Halkos and Salamouris (2004) applied DEA to the financial sector by evaluating Greek commercial banks, using financial ratios as inputs and outputs. Their approach mirrors the structure used in this study and confirms the applicability of DEA in benchmarking firm-level financial efficiency.

Nikoomaram et al. (2010) demonstrated the use of DEA for evaluating financial performance by extracting efficiency insights from corporate financial statements. Their research supports the premise that DEA is suitable for handling standardized financial ratios, an approach similarly adopted in this study.

Ismail (2013) applied DEA to the Malaysian stock market context by modeling stock selection. His work affirms the practical relevance and contextual applicability of DEA in Malaysian financial markets. This study builds on his methodological insights to ensure localized and market-relevant application of the DEA model for company efficiency evaluation.

2.5 Variable Returns to Scale (VRS) Model of DEA

The Variable Returns to Scale (VRS) model, introduced by Banker, Charnes, and Cooper in 1984, extends the original Constant Returns to Scale (CRS) DEA model developed by Charnes et al. (1978). While the CRS model assumes all decision-making units (DMUs) operate at an optimal scale, this assumption is often unrealistic in practical scenarios, especially where external conditions or organizational limitations affect scale. The VRS model relaxes this assumption, making it possible to evaluate DMUs that operate under increasing, constant, or decreasing returns to scale. By accounting for scale inefficiencies, the VRS model provides a more accurate reflection of real-world operational environments Banker et al. (1984).

The VRS DEA model assesses the technical efficiency of DMUs by recognizing that proportional changes in input may not always lead to proportional changes in output. This is critical in many industries, including finance, where firms operate under differing scales due to market size, competition, capital constraints, or managerial effectiveness.

The key distinction between CRS and VRS models lies in their treatment of returns to scale. While the CRS model assumes proportional output change for any given input change, the VRS model allows for:

- Increasing Returns to Scale (IRS): Output increases more than proportionally with inputs.
- Decreasing Returns to Scale (DRS): Output increases less than proportionally with inputs.
- Constant Returns to Scale (CRS): Output increases proportionally with inputs.

This flexibility makes the VRS model suitable for assessing firms that are not operating at optimal scale, which is common in the financial and corporate sectors.

2.5.1 Core Methodology of the VRS Model

The VRS model modifies the basic envelopment form of DEA by introducing a convexity constraint through a free variable (μ). This constraint ensures that DMUs are only compared against others operating at a similar scale, thereby isolating pure technical efficiency from scale efficiency.

For an output-oriented model, the VRS DEA formulation is as follows:

$$\max_{\theta,u,,v} \sum_{r=1}^{s} \mathrm{u}_r y_{ro} + \mu$$

$$s.t. \sum_{i=1}^{m} v_i x_{io} = 1,$$

$$\sum_{i=1}^{m} \mathrm{u}_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \mu \leq 0, \tag{2.9}$$

$$u_r , v_i \geq 0$$

For the input-oriented model, the structure is:

$$\min_{\theta,u,,v} \sum_{r=1}^{s} v_i x_{io} - \mu$$

$$s.t. \sum_{i=1}^{m} u_r y_{ro} = 1,$$

$$\sum_{i=1}^{m} \mathrm{u}_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \mu \leq 0, \tag{2.10}$$

$$u_r , v_i \geq 0$$

Where:

x_{ij} = input i of DMU j,

y_{rj} = output r of DMU j,

μ is a free variable,

v_i and u_r = weights assigned to inputs and outputs, respectively.

The introduction of the convexity constraint via μ ensures that only convex combinations of observed data are used to define the efficient frontier. This adjustment distinguishes the VRS model from the CRS model by allowing for comparisons across DMUs operating under different scales.

2.5.2 Contribution of Literature to This Study

The development of the VRS model by Banker et al. (1984) marked a significant advancement in the DEA methodology, particularly in its applicability to real-world operational conditions where scale inefficiencies are prevalent. Their work forms the foundational model adopted in this study to assess firm-level financial efficiency.

Further clarification and expansion of the VRS model’s practical applications were provided by Banker et al. (1989), who detailed the implementation and interpretation of DEA results within managerial and financial contexts. Their guidance is crucial to this study’s justification for employing the VRS model, ensuring that the evaluation of efficiency among companies of varying sizes is both methodologically sound and practically relevant.

2.6 Company Ranking Using Cross -Efficiency DEA

Cross-Efficiency DEA, originally proposed by Sexton, Silkman, and Hogan (1986), is an extension of the classical Data Envelopment Analysis (DEA) model that aims to overcome one of its key limitations—multiple optimal solutions. In traditional DEA models, efficient Decision Making Units (DMUs) can achieve a score of 1 using self-selected input and output weights, leading to subjectivity and a lack of discrimination among efficient units. Cross-efficiency evaluation introduces a peer-appraisal mechanism, where each DMU is evaluated not only by its own optimal weights but also by the weights derived from other DMUs, resulting in a more objective and comparative performance assessment.

2.6.1 Core Methodology of Cross-Efficiency DEA

Let $u_r^{(j)}$ and $v_i^{(j)}$ be the optimal weights for output r and input i, respectively, determined from the DEA model for DMU j. y_{rk} and x_{ik} be the outputs and inputs of DMU k. Then the cross-efficiency score of DMU k using the weights of DMU j is calculated as:

$$E_{jk} = \frac{\sum_{r=1}^{s} u_r^{(j)} y_{rk}}{\sum_{i=1}^{m} v_i^{(j)} x_{ik}} \tag{2.11}$$

The average cross-efficiency score for DMU k is then calculated as:

$$CE_k = \frac{1}{\mathrm{n}} \sum_{j=1}^{n} E_{jk} \tag{2.12}$$

Where:

n is the total number of DMUs,

CE_k is the final cross-efficiency score used for ranking.

2.6.2 Contribution of Literature to This Study

The idea of cross-efficiency analysis was first presented by Sexton et al. (1986) who developed the model as a post-evaluation method to increase discrimination between efficient units. The theoretical basis of their methodology is the ranking mechanism used in this study that not only aims at measuring financial efficiency but also distinguishing between high-performing companies.

Lim et al. (2014)used cross-efficiency DEA to rank the stocks in the Korean stock market according to the financial performance. Their effort showed how cross-efficiency can be practically useful in financial decision-making especially in portfolio selection. This is quite consistent with the current research aim of ranking companies based on valuation with the aid of DEA outputs.

The next step is in Rasoulzadeh et al. (2022), where a multi-objective cross-efficiency DEA model was developed and combined with the Markowitz portfolio theory and intuitionistic fuzzy sets. Their input demonstrates the importance of combining cross-efficiency with other decision-making models to consider uncertainty and multidimensional goals. This strategy supports the decision to use a hybrid AHP-DEA model in this study, which increases the strength and validity of the final ranking of the companies.

2.7 Integration of AHP and DEA

The integration of the Analytic Hierarchy Process (AHP) and Data Envelopment Analysis (DEA) represents a widely adopted hybrid strategy within the multi-criteria decision-making (MCDM) literature. This combined approach is designed to capitalize on the strengths of both methods while mitigating their individual limitations. AHP is recognized for its ability to capture expert judgment and establish priority rankings through pairwise comparisons. However, it lacks an objective performance evaluation mechanism. Conversely, DEA provides an empirical assessment of relative efficiency without requiring predefined preferences but suffers from issues such as weight flexibility, rank indeterminacy, and insufficient discrimination among efficient units.

To address these challenges, researchers have proposed integrating AHP-derived weights into DEA formulations. This synergy was first explored in depth by Sinuany-Stern et al. (1994), who demonstrated that combining subjective weight prioritization with objective efficiency benchmarking leads to more balanced and decision-relevant evaluation frameworks. The hybrid AHP–DEA method has since gained traction in applications ranging from public sector performance measurement to financial efficiency assessments, including the current study on company valuation.

2.7.1 Core Methodology of the AHP–DEA Integration

The integration of AHP and DEA generally involves several key steps. First, an AHP hierarchy is developed, beginning with the overarching goal (e.g., financial efficiency evaluation), followed by criteria (e.g., profitability, liquidity) and sub-criteria (e.g., ROE, P/E, etc.). Experts then conduct pairwise comparisons among these criteria to derive normalized weights ω for each indicator.

Next, these AHP-derived weights are embedded into the DEA model as bounds. A commonly used approach involves applying them as upper and lower limits on the input and output multipliers in the DEA formulation. The output-oriented DEA model with VRS and bounded weights is expressed as:

$$\max_{\theta,u,,v} \sum_{r=1}^{s} \mathrm{u}_r y_{ro}$$

$$s.t. \sum_{i=1}^{m} v_i x_{io} = 1,$$

$$\sum_{i=1}^{m} \mathrm{u}_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \mu \leq 0, \tag{2.13}$$

$$\omega_{il} \leq v_i \leq \omega_{iu}$$

$$\omega_{rl} \leq u_r \leq \omega_{ru}$$

Where:

x_{ij} = input i of DMU j,

y_{rj} = output r of DMU j,

μ is a free variable,

v_i and u_r = weights assigned to inputs and outputs, respectively.

ω_{il}, ω_{iu}, ω_{rl}, and ω_{ru} = upper and lower bound for v_i and u_r, respectively

This bounded DEA model ensures that the decision-maker’s preferences, derived from expert insight via AHP, are embedded into the efficiency evaluation while preserving the objectivity and benchmarking strength of DEA.

2.7.2 Contribution of Literature to This Study

Tavana et al. (2023) present a detailed meta-analysis of AHP-DEA integration methods and divide diverse methods into weight restriction, hybrid scoring, and input/output importance bounds. Their results support the application of AHP-based weight limits in the present study and provide the best practices in organizing the hybrid model.

Jyoti et al. (2008) illustrate the possibility of employing AHP-derived weights in DEA in measuring national R&D performance and how expert-informed criteria may be used to increase the objectivity of DEA efficiency ratings. Their organization is a methodological guide to the use of this hybrid framework at the firm level financial data.

By integrating entropy and AHP weighting with cross-efficiency DEA, Zhu et al. (2024) use them to assess the complex integration indices. Their analysis confirms the strength of the subjective and objective approach, especially in the ranking and assessment of multidimensional performance, which is the main issue of this study.

Abdel-Halim et al. (2023) use a mixed window DEA-AHP model to measure telecom firm efficiency. Their construction proves the feasibility of applying AHP-based bounds in dynamic DEA models, which is also the case here with regard to bounded VRS DEA using AHP-derived weights.

Kasemi (2022) applies the DEAHP model to assess financial performance in the food sector, validating the relevance of the AHP–DEA hybrid in financial data environments. This supports the use of AHP for financial ratio prioritization and DEA for performance benchmarking in this study.

Hong and Qu (2024) use the AHP–DEA model for risk-based evaluation of financial institutions, highlighting the model's suitability in high-stakes environments requiring multidimensional analysis. This aligns with the complexity and sensitivity of corporate valuation addressed in this research.

Yang (2022) explores AHP-optimized DEA for process reengineering, focusing on structured preference transformation into quantitative scoring. This directly supports the methodology used in this thesis to translate expert priority on financial indicators into DEA-based efficiency rankings.

De Vicente Oliva and Romero-Ania (2022) contribute to AHP–DEA literature by applying it in operational process transformation, reinforcing the credibility of AHP-based constraints in hybrid evaluation environments.

Premachandra (2001) and Wang et al. (2008) also support the structured integration of AHP into DEA, providing early empirical and theoretical validation for using expert-informed weight controls in performance evaluation.

Together, these studies offer strong empirical and theoretical support for the integration of AHP and DEA as a robust MCDM tool, justifying its application in this research to rank and evaluate company financial efficiency with improved discrimination, consistency, and relevance.

CHAPTER 3
RESEARCH METHODOLOGY

3.1 Introduction

This research uses a quantitative research design that is structured to assess the efficiency of corporate finances using a composite analytical framework. The methodology supports the objective analysis of financial statements with the expert-based decision modeling with the help of Multi-Criteria Decision-Making (MCDM) tools. In particular, the methodology uses data normalization, variable transformation and dimensionality refinement, and then integrates the Analytic Hierarchy Process (AHP) and Data Envelopment Analysis (DEA) in both Variable Returns to Scale (VRS) and Cross-Efficiency (CE) environments.

The analysis starts with strict preprocessing of the data so that the financial variables involved could be compared between firms, removing scale-related distortions and data inconsistencies. Considering the various units and scales that are usually present in financial figures (e.g., earnings, ratios, market capitalization), normalization is necessary to provide a consistent assessment. After transformation, the financial data is organized into a consistent form, which allows a strong application of MCDM tools.

3.2 Data Pre-Processing for Classification

Data preprocessing is an essential part of the modeling process because it guarantees reliability and comparability of financial inputs among decision-making units (DMUs). Financial information which is usually obtained through annual reports and standardized databases is often heterogeneous in scale, unit and distribution. Such inconsistencies without preprocessing might result in biased efficiency evaluations and skewed results. The preprocessing stage of the study involves row scaling, column scaling, and individual transformation, which have different purposes in the preparation of the data to be used in further AHP-DEA modeling.

3.2.1 Row Scaling

Preprocessing of data is an initial step in the modeling process since it guarantees reliability and comparability of financial data among decision-making units (DMUs). Financial information, which is usually obtained through annual reports and standardized databases, is usually heterogeneous in scale, unit, and distribution. Such inconsistencies may result in skewed results and biased efficiency measurements without preprocessing. The preprocessing step in the study involves row scaling, column scaling, and individual transformation, which have different functions in the preparation of the data to be used in the later AHP-DEA modeling. The formula applied is:

$$Normalize\ value, x'_{ij} = \frac{x_{ij}}{\sum_{j=1}^{n} x_{ij}} \tag{3.1}$$

Where:

x_{ij} = value of variable j for company i

n = number of variables

This approach ensures that each firm's metrics are expressed as a fraction of its own internal totals, highlighting strategic emphasis and resource allocation within that firm.

3.2.2 Column Scaling

Column scaling, in contrast, normalizes financial variables across all firms to ensure consistency of comparison. Since financial indicators may exist in different units (e.g., millions of currency, percentages), column scaling ensures that no single variable dominates the model due to numerical magnitude. Standard normalization techniques such as min-max scaling or z-score standardization may be used depending on the distribution characteristics. This step is essential prior to AHP-based weight assignment, as it ensures that all indicators contribute proportionately to the priority matrix.

3.3 Research step

The methodology of this research was structured into ten sequential stages, each contributing to the development, implementation, and validation of an integrated AHP–DEA framework for company performance evaluation. These stages are detailed as follows:

Stage 1: Project Definition

The research began by clearly defining the study's objectives, outlining the problem statement, and specifying the scope of analysis. The core aim was to evaluate and rank the financial efficiency of selected companies using a multi-criteria decision-making (MCDM) framework that integrates expert opinion with financial data, as demonstrated by Tavana et al. (2023).

Stage 2: Research Study

An in-depth review of academic literature was conducted to identify relevant financial performance indicators, valuation techniques, and the application of AHP and DEA in financial efficiency analysis. Studies such as Asadabadi et al. (2019), Jyoti et al. (2008), and Vaidya and Kumar (2006) provided a strong theoretical basis for integrating AHP and DEA into a cohesive methodology.

Stage 3: Data Collection and Preprocessing

Primary data from expert surveys and secondary data from financial statements were collected. The data were then preprocessed through normalization, rounding, and transformation into AHP-compatible formats, following procedures discussed by Eghbali Amooghin et al. (2023) and Gibson (2004).

Stage 4: AHP Weightage

Pairwise comparisons from experts were analyzed using AHP to generate priority weights for each financial criterion. The Consistency Ratio (CR) was calculated, with responses requiring a $CR < 0.1$ for acceptance. The approach adheres to the methodology introduced by Saaty (1985) and improved upon by Franek and Kresta (2014).

Stage 5: Variable Selection and DEA Input/Output Configuration

Based on the AHP-derived weights, the most significant financial subcriteria were selected and categorized as DEA inputs or outputs. This ensured alignment with expert judgment and avoided redundancy, following the guidance of Premachandra (2001) and Wang et al. (2008).

Stage 6: Bounded DEA Modeling (VRS)

A Variable Returns to Scale (VRS) DEA model was constructed to address firm size differences. The model incorporated AHP-derived weights as assurance region constraints, ensuring expert-informed efficiency scores. This stage was aligned with the methodologies proposed by Banker et al. (1984) and De Vicente Oliva and Romero-Ania (2022).

Stage 7: Programming Code for Model Application

The VRS DEA model with bounded weights was executed over a six-year period (2019–2024) for each company. Custom R and Python code ensured reproducibility and allowed for iterative testing, in line with practical implementations such as those by Zhu et al. (2024).

Stage 8: Cross-Efficiency DEA Application

To reduce the bias of self-appraisal and improve ranking accuracy, the Cross-Efficiency DEA (CE-DEA) approach was used. Each firm's performance was evaluated with its own weights and also peer-derived weights, based on the methodology of Sexton et al. (1986), Lim et al. (2014), and Rasoulzadeh et al. (2022).

Stage 9: Model Result and Real-Life Ranking Comparison

The results from the DEA model were validated against actual stock price performance. Spearman's Rank Correlation Coefficient (ρ) was used to measure the alignment between technical efficiency and market valuation, following procedures used by Hong and Qu (2024) and Kasemi (2022).

Stage 10: Discussion and Conclusion

In the final stage, results were interpreted in relation to the original research questions. The study concluded with practical insights and recommendations for future research, emphasizing the advantages of combining expert judgment with data-driven models, as highlighted by Aljuhaishi and Kadhim (2023) and Yang (2022).

3.4 Research step flowchart

Figure 3.1 : Research Flowchart

CHAPTER 4
IMPLEMENTATION

4.1 Introduction

This chapter presents the practical implementation of the integrated Analytical Hierarchy Process (AHP) and Data Envelopment Analysis (DEA) methodology for evaluating the financial performance of companies. The focus is on applying the theoretical framework introduced earlier to real-world data derived from financial statements and expert surveys. The implementation comprises two main phases: deriving the priority weights of financial metrics using AHP and applying these weights within a DEA model to assess company efficiency. The process is supported by Python programming and Microsoft Excel for computational accuracy, scalability, and reproducibility. Consistency Ratio (CR) is also calculated to ensure the reliability of expert judgments before applying the results in DEA.

4.2 AHP Survey

To derive the relative importance of financial criteria and subcriteria, an Analytic Hierarchy Process (AHP) survey was conducted involving expert respondents. The survey included a series of pairwise comparisons between decision criteria and subcriteria using a 9-point Likert scale, where values were later converted into reciprocal ratios based on Saaty's fundamental scale. This enabled the transformation of subjective judgments into a quantitative priority model.

4.2.1 Questionnaire Design

The survey utilized a 9-point Likert scale structured according to Saaty's fundamental comparison scale. Experts were asked to compare each pair of financial performance criteria in terms of relative importance. The design of each question followed a left-to-right comparative format, where:

- **1** indicated that the left-hand side (LHS) criterion was extremely more important than the right-hand side (RHS),
- **5** meant equal importance,
- **9** indicated the RHS criterion was extremely more important than the LHS.

An example of the survey interface is shown in Figure 4.1, illustrating the comparison between the P/E Ratio and the P/B Ratio.

Figure 4.1 : Questionnaire Example Comparison of P/E Ratio and P/B Ratio

Survey responses were translated into AHP matrix values based on Saaty's fundamental scale using a direct conversion table. This table maps the 1–9 Likert scale used in the survey into corresponding pairwise comparison values that quantify the relative importance of criteria. The mapping is shown below:

Table 4.1
Conversion Table

SURVEY SCORE	AHP MATRIX VALUE
1	9
1.5	8
2	7
2.5	6
3	5
3.5	4
4	3
4.5	2
5	1
5.5	1/2
6	1/3
6.5	1/4
7	1/5
7.5	1/6
8	1/7
8.5	1/8
9	1/9

In cases where expert responses included decimal values not ending in .0 or .5 (e.g., 4.6 or 3.2), a rounding rule was applied to align with the nearest defined conversion in Table 4.1. The rounding decision was based on proximity to 0.0, 0.5, or 1.0, using the table shown below:

Table 4.2
Rounding Table

0	0
0.1	
0.2	
0.3	0.5
0.4	
0.5	
0.6	
0.7	
0.8	1
0.9	
1.0	

These rounded values were then used to find the corresponding AHP matrix value from Table 4.1, ensuring consistent and valid transformation across all participant inputs.

4.2.2 Criteria-Level Pairwise Comparisons

A total of 28 pairwise comparisons between the eight main criteria (C1 to C8) were collected and converted into AHP values. Full questionnaire and response link are provided in Reference. The raw responses were aggregated and averaged to produce a mean score for each pairwise comparison. Table 4.3 presents selected entries from the averaged results:

Table 4.3
Questionnaire Mean Result for Criteria

LEFT CRITERION (LHS)	MEAN SCORE (LHS COMPARED TO RHS)	MEAN SCORE (RHS COMPARED TO LHS)	RIGHT CRITERION (RHS)
C1	5.4	4.6	C2
C1	3.8	6.2	C3
C1	6.1	3.9	C4
C1	5.4	4.6	C5
C1	4.8	5.2	C6
C1	5.6	4.4	C7
C1	4.5	5.5	C8
C2	4.1	5.9	C8
C3	5.6	4.4	C8
C4	3.8	6.2	C8
C5	4.9	5.1	C8
C6	4.8	5.2	C8
C7	4.2	5.8	C8
C7	4.9	5.1	C2
C7	3.9	6.1	C3
C7	5.6	4.4	C4
C7	3.9	6.1	C5
C7	4.9	5.1	C6
C6	5.5	4.5	C2
C6	4	6	C3
C6	5.6	4.4	C4
C6	4.8	5.2	C5
C2	3.7	6.3	C5
C3	5.8	4.2	C5
C4	3.6	6.4	C5
C4	4.7	5.3	C2
C4	3.6	6.4	C3
C2	3.7	6.3	C3

Using the rounded and converted values, a full 8×8 pairwise comparison matrix was constructed. Process of conversion are shown in Table 4.1 & 4.2 Appendix. Reciprocity was ensured such that if element $a_{ij} = x$, then $a_{ji} = \frac{1}{x}$. Diagonal elements were all set to 1, representing self-comparison.

Table 4.4
Pairwise Comparison Matrix with Converted Value

	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
C1	1	1/2	3	1/3	1/2	1	1/2	2
C2	2	1	4	1/2	4	2	1	3
C3	1/3	1/4	1	1/4	1/3	1/3	1/3	1/2
C4	3	2	4	1	4	2	2	3
C5	2	1/4	3	1/4	1	1	1/3	1
C6	1	1/2	3	1/2	1	1	1	1
C7	2	1	3	1/2	3	1	1	3
C8	1/2	1/3	2	1/3	1	1	1/3	1

The rounded matrix values were then transformed into AHP-compatible reciprocal format, as presented in the Pairwise Comparison Matrix with Converted Value (Appendix X). The matrix was further normalized to ensure each column sums to 1, yielding the Normalized Pairwise Comparison Matrix with the derived criteria weights, as shown in Table 4.5.

Table 4.5
Normalized Pairwise Comparison Matrix with Criteria Weight

	C1	C2	C3	C4	C5	C6	C7	C8	weight
C1	0.0845	0.0857	0.1304	0.0909	0.0337	0.1071	0.0769	0.1379	0.09341
C2	0.1690	0.1714	0.1739	0.1364	0.2697	0.2143	0.1538	0.2069	0.18693
C3	0.0282	0.0429	0.0435	0.0682	0.0225	0.0357	0.0513	0.0345	0.04083
C4	0.2535	0.3429	0.1739	0.2727	0.2697	0.2143	0.3077	0.2069	0.25519
C5	0.1690	0.0429	0.1304	0.0682	0.0674	0.1071	0.0513	0.0690	0.08816
C6	0.0845	0.0857	0.1304	0.1364	0.0674	0.1071	0.1538	0.0690	0.10430
C7	0.1690	0.1714	0.1304	0.1364	0.2022	0.1071	0.1538	0.2069	0.15967
C8	0.0423	0.0571	0.0870	0.0909	0.0674	0.1071	0.0513	0.0690	0.07151
SUM	1	1	1	1	1	1	1	1	1

The final AHP weights for each criterion are presented below and are also visualized in Figure 4.2

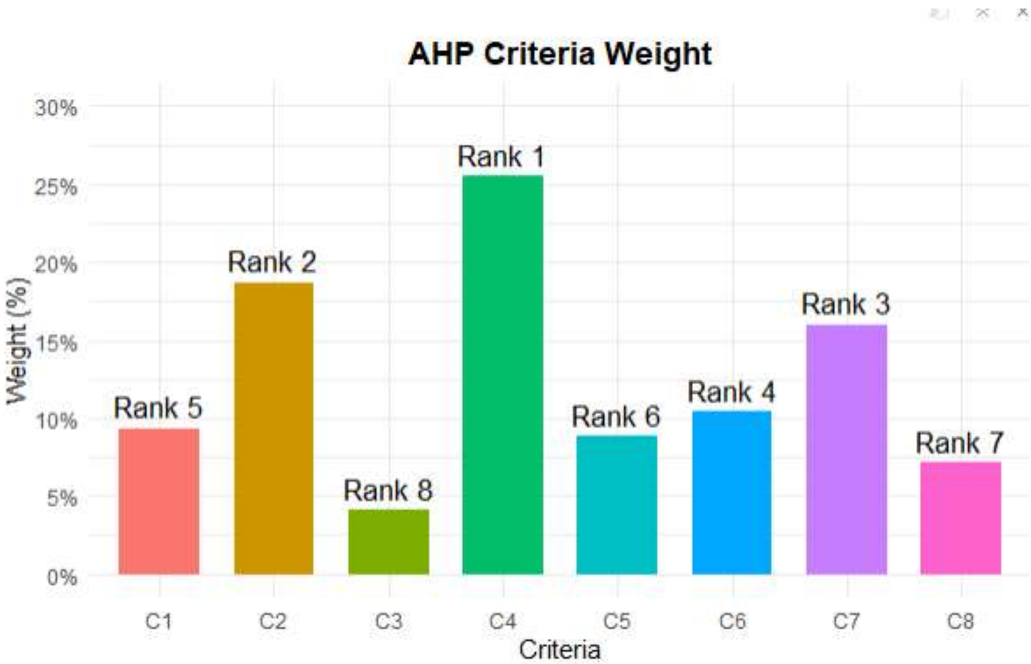

Figure 4.2 : AHP Criteria Weight Ranking

These weights represent the relative importance of each main criterion in the context of evaluating company financial performance.

4.2.3 Consistency Check

To assess the logical consistency of expert pairwise comparisons in the Analytic Hierarchy Process (AHP), a Weight Sum Value Table and a corresponding Ratio Table were computed.

The Weight Sum Value was calculated shown in Table 4.3 Appendix using the formula:

$$Weight\ Sum\ Value = \sum \mathrm{x}_j \cdot Weight$$

Using these values, the Ratio for each criterion shown in Table 4.4 Appendix was determined as:

$$Ratio = \frac{Weight\ Sum\ Value}{Weight},$$

The individual ratios are listed in the Ratio Table, and the average ratio across all criteria is:

$$Average\ Ratio = 8.963279$$

This average ratio corresponds to the principal eigenvalue λ_{max} of the comparison matrix:

$$\lambda_{max} = 8.963279$$

Using this value, the Consistency Index (CI) was calculated as:

$$CI = \frac{\lambda_{max} - n}{n - 1}, \tag{4.1}$$

Where:

λ_{max} = 8.963279
N = 8 (number of criteria)

$$CI = \frac{8.963279-8}{8-1} = 0.10903985$$

$$CR = \frac{CI}{RI} \tag{4.2}$$

The Random Index (RI) for n = 8 is approximately 1.41, giving:

$$CR = \frac{0.10903985}{1.41} = 0.097357009$$

$$CR < 0.1 \tag{4.3}$$

Since CR < 0.10, the pairwise comparison matrix is considered acceptably consistent. Minor adjustments were made during matrix construction to refine responses, ensuring reliable consistency for subsequent DEA integration.

4.2.4 Subcriteria Weight Determination

For each main criterion that contained multiple subcriteria, additional AHP pairwise comparisons were performed to derive the relative weights of those subcomponents. Expert judgments were collected and averaged to construct reciprocal matrices, using the same Saaty scale and conversion method described previously.

Tables 4.6 through 4.10 present the summarized mean survey scores for various subcriteria groups. These were converted to AHP matrix values, from which priority weights were calculated.

Table 4.6
Questionnaire Mean Result for Subcriteria C1 Valuation Ratios

LEFT CRITERION (LHS)	MEAN SCORE (LHS COMPARED TO RHS)	MEAN SCORE (RHS COMPARED TO LHS)	RIGHT CRITERION (RHS)
P/E Ratio	4.5	5.5	P/B Ratio
P/E Ratio	4.7	5.3	EV/EBITDA
P/B Ratio	5.3	4.7	EV/EBITDA

Table 4.7
Questionnaire Mean Result for Subcriteria C2 Profitability Ratios

LEFT CRITERION (LHS)	MEAN SCORE (LHS COMPARED TO RHS)	MEAN SCORE (RHS COMPARED TO LHS)	RIGHT CRITERION (RHS)
Net Profit Margin	4.7	5.3	Return on Asset (ROA)
Net Profit Margin	5.5	4.5	Return on Equity (ROE)
Return on Asset (ROA)	5.8	4.2	Return on Equity (ROE)

Table 4.8
Questionnaire Mean Result for Subcriteria C5 Operational Efficiency

LEFT CRITERION (LHS)	MEAN SCORE (LHS COMPARED TO RHS)	MEAN SCORE (RHS COMPARED TO LHS)	RIGHT CRITERION (RHS)
Gross Margin	5.5	4.5	Operating Margin

Table 4.9
Questionnaire Mean Result for Subcriteria C6 Debt Metrics

LEFT CRITERION (LHS)	MEAN SCORE (LHS COMPARED TO RHS)	MEAN SCORE (RHS COMPARED TO LHS)	RIGHT CRITERION (RHS)
Debt-to-Equity (D/E) Ratio	4.2	5.8	Interest Coverage Ratio

Table 4.10
Questionnaire Mean Result for Subcriteria C7 Revenue and Earnings Growth

LEFT CRITERION (LHS)	MEAN SCORE (LHS COMPARED TO RHS)	MEAN SCORE (RHS COMPARED TO LHS)	RIGHT CRITERION (RHS)
Revenue Growth Rate	4.5	5.5	Earnings Per Share (EPS) Growth

Table 4.11
Questionnaire Mean Result for Subcriteria C8 Dividend Metrics

LEFT CRITERION (LHS)	MEAN SCORE (LHS COMPARED TO RHS)	MEAN SCORE (RHS COMPARED TO LHS)	RIGHT CRITERION (RHS)
Dividend Yield	5	5	Dividend Payout Ratio

The derived weights from each subcriteria group are shown below, calculated using the eigenvector method and normalized so that the sum of weights in each group equals 1.

Table 4.12
Subcriteria Weight

CRITERIA	SUBCRITERIA	WEIGHT
C1 Valuation Ratios	P/E Ratio	0.49047619
	P/B Ratio	0.19761905
	EV/EBITDA	0.31190476
C2 Profitability Ratios	Net Profit Margin	0.2972583
	Return on Asset (ROA)	0.16378066
	Return on Equity (ROE)	0.53896104
C5 Operational Efficiency	Gross Margin	0.33333333
	Operating Margin	0.66666667

C6 Debt Metrics	Debt-to-Equity (D/E) Ratio	0.75
	Interest Coverage Ratio	0.25
C7 Revenue and Earnings Growth	Revenue Growth Rate	0.66666667
	Earnings Per Share (EPS) Growth	0.33333333
C8 Dividend Metrics	Dividend Yield	0.5
	Dividend Payout Ratio	0.5

For single-indicator categories such as Liquidity (C3) and Cash Flow (C4), the sole subcriteria (e.g., Current Ratio, Free Cash Flow) were assigned full weight (1.0) by default.

4.3 Variable Selection

To reduce model complexity while preserving analytical accuracy, a structured variable selection process was undertaken based on the initial AHP results. The primary objectives were to avoid redundancy, retain only the most influential financial indicators, and enhance the interpretability of the subsequent DEA (Data Envelopment Analysis) model.

From the normalized AHP weights at the criteria level, the five most influential financial domains were identified: Cash Flow (C4), Profitability (C2), Revenue and Earnings Growth (C7), Debt Metrics (C6), and Valuation Ratios (C1). These criteria exhibited the highest relative importance in the overall evaluation of company financial performance. Pairwise matrix for new AHP are shown in Table 4.5 Appendix.

To ensure simplicity and focus within the model, only the top-weighted subcriterion from each of these five criteria was retained. This ensured that each major financial domain was represented without overloading the model with correlated or overlapping variables. The selected subcriteria an respective weight were as follows:

Table 4.13
Highest Weight Variable with New AHP Weight

CRITERIA	SUBCRITERIA	WEIGHT
C1	P/E Ratio	0.112505
C2	Return on Equity (ROE)	0.213245
C4	Free Cash Flow (FCF)	0.347125
C6	Debt-to-Equity (D/E) Ratio	0.142451
C7	Revenue Growth Rate	0.184674

This selection yielded a set of five high-impact variables, each representing a key dimension of firm performance. These variables were later used as inputs or outputs in the DEA model to evaluate financial efficiency across firms, with AHP-derived weights guiding their influence within the optimization process.

The Weight Sum Value was calculated shown in Table 4.6 Appendix using the formula:

$$Weight\ Sum\ Value = \sum \mathrm{x}_j \cdot Weight$$

Using these values, the Ratio for each criterion shown in Table 4.7 was determined as:

$$Ratio = \frac{Weight\ Sum\ Value}{Weight},$$

The individual ratios are listed in the Ratio Table (Table 4.4 in Appendix), and the average ratio across all criteria is:

$$Average\ Ratio = 5.36327895$$

This average ratio corresponds to the principal eigenvalue λ_{max} of the comparison matrix:

$$\lambda_{max} = 5.36327895$$

The consistency of the new matrix was verified with the following calculations:

$$CI = \frac{\lambda_{max} - n}{n - 1}, \tag{4.4}$$

Where:

λ_{max} = 5.36327895
N = 5 (number of criteria)

$$CI = \frac{5.36327895 - 5}{5 - 1} = 0.090819738$$

$$CR = \frac{CI}{RI} \tag{4.5}$$

The Random Index (RI) for n = 8 is approximately 1.12, giving:

$$CR = \frac{0.090819738}{1.12} = 0.081089051 \tag{4.6}$$

$$CR < 0.1$$

Since CR < 0.10, the consistency of expert judgment is acceptable. Minor trade-offs in consistency were accepted for the sake of model parsimony and clarity.

These five variables with their derived AHP weights served as the foundation for the DEA input/output configuration. Upper and lower multiplier bounds were applied in the next modeling phase to guide the DEA's optimization in line with expert-defined importance.

4.4 Variable Return to Scale DEA Linear Programming

To evaluate the relative efficiency of firms, this study adopts the Data Envelopment Analysis (DEA) approach under the Variable Returns to Scale (VRS) assumption. The VRS model is well-suited to real-world conditions where firms operate at differing scales, particularly within the financial sector where firm size, capital structure, and market dynamics vary substantially. This approach ensures a more flexible and realistic efficiency evaluation compared to constant returns models.

4.4.1 Model Formulation

The DEA-VRS model used in this study incorporates **assurance region-type weight restrictions**, derived from the AHP-generated expert priority weights. This integration ensures that the DEA optimization respects expert judgment on the importance of each financial indicator. The output-oriented DEA-VRS model is formulated as the following linear programming problem:

$$\max_{\theta,u,,v} \sum_{r=1}^{s} \mathrm{u}_r y_{ro} + \mu$$

$$s.t. \sum_{i=1}^{m} v_i x_{io} = 1,$$

$$\sum_{i=1}^{m} \mathrm{u}_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \mu \leq 0, \tag{4.7}$$

$$\omega_{il} \leq v_i \leq \omega_{iu}$$

$$\omega_{rl} \leq u_r \leq \omega_{ru}$$

Where:

x_{ij} = input i of DMU j,

y_{rj} = output r of DMU j,

μ is a free variable,

v_i and u_r = weights assigned to inputs and outputs, respectively.

ω_{il}, ω_{iu}, ω_{rl}, and ω_{ru} = upper and lower bound for v_i and u_r, respectively

4.4.2 Input and Output Variable Specification

The DEA model uses the five most important subcriteria selected via AHP (see Section 4.5). Two of these were designated as inputs, while the remaining three were treated as outputs to reflect their role in financial performance. Table 4.13 summarizes the variable roles, labels, and their associated AHP weights.

Table 4.14
DEA Model Variable Configuration

ROLE	VARIABLE	LABEL	AHP WEIGHT
Input	PE	x_1	0.112505
Output	ROE	y_1	0.213245
Output	FCF	y_2	0.347125
Input	DE	x_2	0.142451
Output	RG	y_3	0.184674

Table 4.15
Company Data 2019

	DMU	PE	ROE	FCF	DE	RG
RHB BANK BERHAD .	DMU 1	0.146417934	0.193889098	0.194210819	0.244080146	0.196673869
HONG LEONG BANK BHD	DMU 2	0.229189528	0.20350811	0.192305077	0.091074681	0.196619708
CIMB GROUP HOLDINGS BERHAD	DMU 3	0.171813764	0.159751037	0.202529507	0.276867031	0.201844729
PUBLIC BANK BHD .	DMU 4	0.26822386	0.245756318	0.202930177	0.118397086	0.194361619
MALAYAN BANKING BHD	DMU 5	0.184354915	0.197095436	0.208024419	0.269581056	0.210500075

4.4.3 Weight Restrictions

To embed expert preference from the AHP into the DEA model, weight bounds were applied to each variable through upper and lower limits on their multipliers. This assurance region constraint prevents the DEA from assigning disproportionately high or low importance to any variable, thus preserving the integrity of expert judgments.

The bounds for each variable were determined using dynamic multiplier factors applied to the AHP weights. These bounds were adjusted annually to account for structural financial changes and evolving priority considerations.

The general form of the assurance region constraints is:

$$\omega_{il} \le v_i \le \omega_{iu}$$
$$\omega_{rl} \le u_r \le \omega_{ru} \tag{4.8}$$

Where:

ω_{il}, ω_{iu} : lower and upper bounds for input weights

ω_{rl}, ω_{ru} : lower and upper bounds for output weights

Tables 4.14 to 4.19 present the year-by-year upper and lower bounds for each variable based on applied multipliers and adjusted AHP weights.

Table 4.16
2019 Upper and Lower Bound Tightening

VARIABLE	VARIABLE WEIGHT LOWER BOUND	VARIABLE WEIGHT UPPER BOUND
P/E Ratio	0.05625244	0.29251269
Debt-to-Equity (D/E) Ratio	0.071225702	0.370373652
Return on Equity (ROE)	0.106622528	0.554437145
Free Cash Flow (FCF)	0.173562516	0.902525083
Revenue Growth Rate	0.092336814	0.48015143
Multiplier	0.5	2.6

Using bounded DEA weights, also known as multiplier constraints or assurance regions, enhances the robustness and interpretability of efficiency analysis. In traditional DEA, each decision-making unit (DMU) can freely choose weights that maximize its own efficiency, which may lead to unrealistic or biased weight allocations, overemphasizing less relevant variables. By bounding the weights, such distortions are avoided, and logical consistency is maintained across all evaluations. These bounds are often derived from expert input via methods like AHP, ensuring the model reflects domain knowledge and decision-maker preferences. Furthermore, restricting weight flexibility helps improve discrimination between DMUs, reducing the likelihood of identical efficiency scores. The DEA multiplier model inherently relies on these weights to assess performance, and bounding them within a defined range (e.g., 0.5× to 2.6× of AHP weight) ensures that evaluations remain aligned with real-world financial relevance. Additionally, yearly adjustments to bounds account for shifting priorities or market dynamics, making the model temporally adaptive and practically relevant.

4.4.1 Efficiency Results

The Variable Returns to Scale (DEA-VRS) model was implemented to evaluate the efficiency of five decision-making units (DMUs) over the period 2019 to 2024. For each year, the model produced efficiency scores and optimal multiplier weights (input and output weights), constrained by the assurance region bounds derived from AHP. Figure 4.2 provides a sample output from the RMarkdown implementation, showing the 2019 results.

Description: df [5 × 8]

DMU <int>	Efficiency <dbl>	mu <dbl>	v1 <dbl>	v2 <dbl>	u1 <dbl>	u2 <dbl>	u3 <dbl>
1	0.8783	0	0.2925	0.2342	0.1066	0.2523	0.0923
2	1.0000	0	0.2891	0.3704	0.2382	0.1736	0.0923
3	0.8154	0	0.2925	0.1797	0.1066	0.2265	0.0923
4	0.9191	0	0.2925	0.1819	0.1576	0.1736	0.0923
5	0.8670	0	0.2925	0.1709	0.1066	0.2223	0.0923

5 rows

Figure 4.3 :Example of RMarkdown Output 2019

Table 4.17
Dea Result 2019

DMU	EFFICIENCY	μ	v1	u1	u2	u3	u4
1	0.8783	0	0.2925	0.2342	0.1066	0.2523	0.0923
2	1	0	0.2891	0.3704	0.2382	0.1736	0.0923
3	0.8154	0	0.2925	0.1797	0.1066	0.2265	0.0923
4	0.9191	0	0.2925	0.1819	0.1576	0.1736	0.0923
5	0.867	0	0.2925	0.1709	0.1066	0.2223	0.0923

Notably, DMU 1 and DMU 2 consistently achieved efficiency scores near or equal to 1, indicating optimal resource usage under the VRS assumptions and weight bounds. Conversely, DMU 3 exhibited persistent inefficiencies, especially in earlier years, with scores as low as 0.2259 in 2020. These variations suggest differences in resource allocation strategies and financial performance structures.

4.5 Cross Efficiency DEA

While standard DEA allows each DMU to self-select the most favorable weight configuration, this can result in biased or overly optimistic assessments. To address this limitation and enable a more objective comparison, Cross-Efficiency DEA (CE-DEA) was employed. This approach incorporates peer evaluation, where each DMU is also assessed using the optimal weights derived from other DMUs.

4.5.1 Methodology

Let $u_r^{(j)}$ and $v_i^{(j)}$ denote the optimal weights for output r and input i, respectively, derived from the DEA model for DMU j. Let y_{rk} and x_{ik} represent the outputs and inputs of DMU k, respectively. Then, the cross-efficiency score, E_{jk} of DMU k, evaluated using the weights of DMU j, is calculated as:

$$E_{jk} = \frac{\sum_{r=1}^{s} u_r^{(j)} y_{rk}}{\sum_{i=1}^{m} v_i^{(j)} x_{ik}} \qquad (4.9)$$

The average cross-efficiency score CE_k of DMU k is then given by:

$$CE_k = \frac{1}{\mathrm{n}}\sum_{j=1}^{n} E_{jk} \tag{4.10}$$

Where n is the total number of DMUs. These scores serve as the final basis for performance ranking, as they incorporate both self-appraisal and peer appraisal, ensuring a more robust comparative assessment.

4.5.2 Results Across Years

The CE-DEA was applied to five DMUs over the period from 2019 to 2024. The results highlight significant trends in inter-DMU consistency and comparative efficiency. The following tables summarize the annual cross-evaluation matrices and the corresponding mean cross-efficiency scores.

Table 4.18
Cross Efficiency Dea Result 2019

	DMU1	DMU2	DMU3	DMU4	DMU5
EVALUATED BY DMU1	0.878290321	0.999919735	0.753775791	0.897841635	0.79381798
EVALUATED BY DMU2	0.738698278	1.000150388	0.603339468	0.920176584	0.669216084
EVALUATED BY DMU3	0.95526369	0.999940126	0.815257071	0.903435171	0.855320655
EVALUATED BY DMU4	0.944964373	1.000007381	0.784814226	0.919068884	0.841141477
EVALUATED BY DMU5	0.969885438	0.999864195	0.826896594	0.904336758	0.866875068
MEAN CE_k	0.930213	1.010391	0.735643	0.912352	0.912777

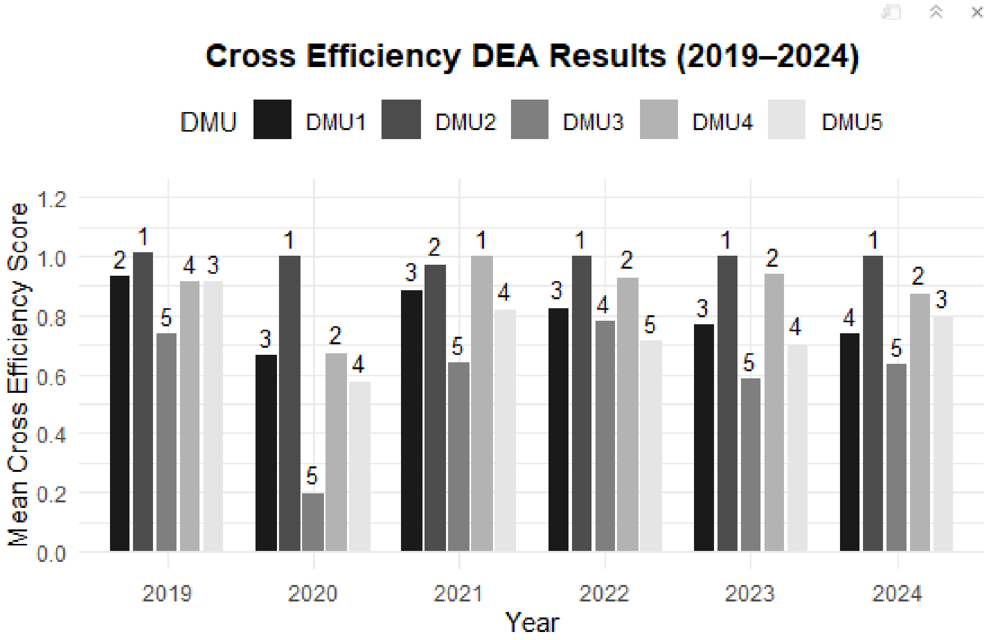

Figure 4.4 : Cross Efficiency DEA Result

From the cross-efficiency analysis, DMU2 consistently ranked the highest, often exceeding a CE score of 1.0 due to favorable peer evaluations. DMU4 also exhibited strong performance, particularly in later years. In contrast, DMU3 repeatedly showed the lowest cross-efficiency scores, reinforcing its inefficient status across both self- and peer-evaluation frameworks.

CHAPTER 5
RESULTS AND DISCUSSIONS

5.1 Introduction

This chapter presents and discusses the findings from the Data Envelopment Analysis (DEA) and compares them against real-world stock market performance data. The goal is to determine whether financial efficiency, as computed using the AHP-weighted DEA model, aligns with how investors respond in the market. The discussion is organized into financial efficiency rankings derived from DEA over a six-year span, and comparative analysis against actual stock returns, serving as a validation of the model's predictive and practical relevance.

5.2 DEA result

The DEA-VRS model was applied annually from 2019 to 2024 to assess the financial efficiency of five firms (DMUs). Rankings were determined using bounded weights derived from AHP-based priorities, ensuring integration of expert knowledge. Table 5.1 summarizes the efficiency rankings over the six-year period.

Table 5.1
Financial Efficiency Ranking Across Year

	2019	**2020**	**2021**	**2022**	**2023**	**2024**
DMU1	3	3	3	3	3	4
DMU2	1	1	2	1	1	1
DMU3	5	5	5	4	5	5
DMU4	2	2	1	2	2	2
DMU5	4	4	4	5	4	3

DMU2 consistently outperformed others, ranking first in most years, while DMU3 lagged significantly across the board. DMU1 showed steady but non-dominant performance, and DMU4 maintained second place consistently, except in 2021 when it ranked first. DMU5 showed fluctuating performance but improved in 2024.

These patterns suggest structural financial strength for DMU2, while DMU3 may require strategic financial reform. The results reflect how well firms transform financial inputs into value-generating outputs over time.

These efficiency scores reflect a company's ability to convert financial inputs (e.g., valuation metrics) into desirable outputs (e.g., profitability, growth, and cash flow measures), as perceived through a multi-criteria lens. Importantly, the consistency of ranks over time suggests structural strength or weakness in the firms' financial management and strategy.

5.3 Real Life Stock Comparison

To assess the practical relevance of the DEA model, the computed efficiency rankings were compared against actual stock price growth for each company (DMU) over the six-year period from 2019 to 2024. This comparison aims to identify whether companies deemed financially efficient by the DEA model also deliver superior returns in real market conditions.

Table 5.2 presents positively scaled annual percentage increase in stock prices for each DMU. These values represent real investor responses to firm performance.

Table 5.2
Stock Growth Across Year

YEAR	RHB BANK BERHAD (DMU1)	HONG LEONG BANK BHD (DMU2)	CIMB GROUP HOLDINGS BERHAD (DMU3)	PUBLIC BANK BHD (DMU4)	MALAYAN BANKING BHD (DMU5)
2019	2.076779026	1.850877193	1.877224199	1.807692308	1.935828877
2020	1.947826087	2.042382589	1.872210953	2.032581454	1.966857143
2021	1.985321101	2.023076923	2.26744186	2.009708738	1.98108747
2022	2.078212291	2.104189044	2.064220183	2.038461538	2.048192771
2023	1.941278066	1.9192607	2.00862069	1.993055556	2.02183908
2024	2.179816514	2.084656085	2.386324786	2.048951049	2.140607424

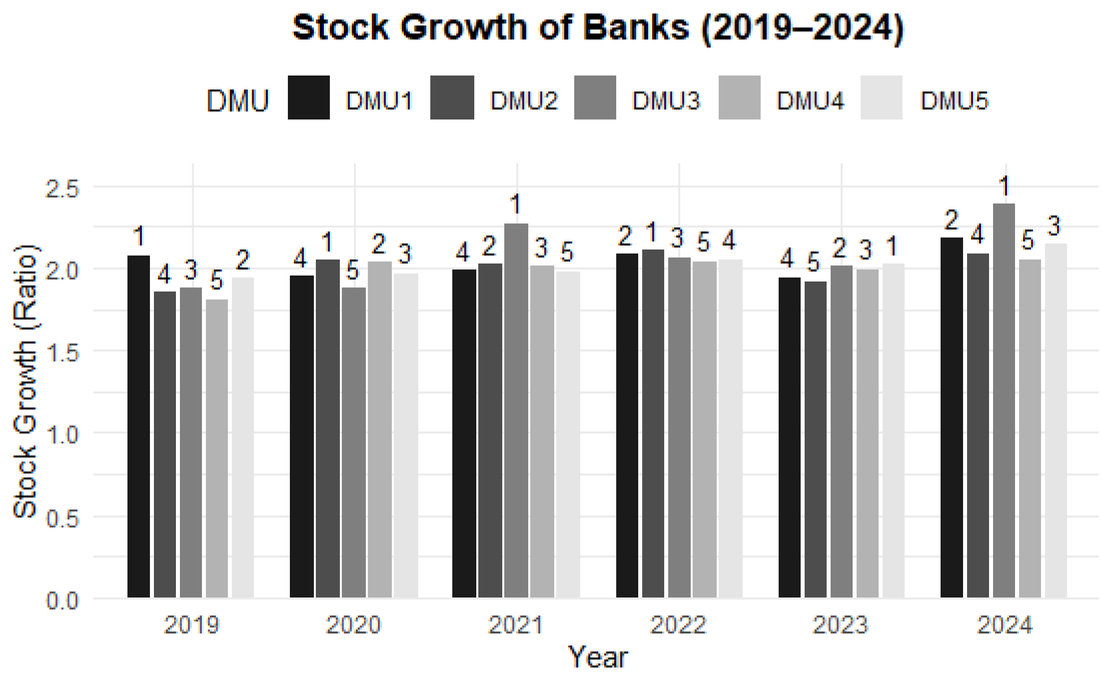

Figure 5.1 : Stock Growth (2019-2024)

To align stock performance with DEA outcomes, both sets of rankings were compared annually. The Spearman's Rank Correlation Coefficient (ρ) was calculated to measure the monotonic relationship between the two rankings.

$$\text{Spearman's Rank Correlation}, \rho = 1 - \frac{6\sum d_i^2}{n(n^2 - 1)} \qquad (5.1)$$

Where:

d_i is the difference between DEA rank and stock rank for each DMU

n is the number of DMUs

Table 5.3
Rank Comparison

YEAR	DMU	COMPANY FINANCIAL EFFICIENCY	STOCK PRICE INCREASE (END OF YEAR)	SPEARMAN'S RANK CORRELATION, ρ
2019	DMU1	3	1	-0.50 Moderate inverse correlation
	DMU2	1	4	
	DMU3	5	3	
	DMU4	2	5	
	DMU5	4	2	
2020	DMU1	3	4	0.90 Strong positive correlation
	DMU2	1	1	
	DMU3	5	5	
	DMU4	2	2	
	DMU5	4	3	
2021	DMU1	3	4	-0.10 Very weak inverse correlation
	DMU2	2	2	
	DMU3	5	1	
	DMU4	1	3	
	DMU5	4	5	
2022	DMU1	3	2	0.40 Weak positive correlation
	DMU2	1	1	
	DMU3	4	3	
	DMU4	2	5	
	DMU5	5	4	
2023	DMU1	3	4	-0.80 Strong inverse correlation
	DMU2	1	5	
	DMU3	5	2	
	DMU4	2	3	
	DMU5	4	1	
2024	DMU1	4	2	-0.90 Very strong inverse correlation
	DMU2	1	4	
	DMU3	5	1	
	DMU4	2	5	
	DMU5	3	3	

The comparison reveals mixed levels of alignment between financial efficiency (DEA) and actual market outcomes. In 2020, the model aligned strongly with stock performance ($\rho = 0.90$), demonstrating the practical accuracy of the DEA evaluation. However, in 2023 and 2024, the relationship turned strongly inverse, indicating that stock prices did not reflect fundamental efficiency.

A notable case is DMU3, which ranked last in DEA efficiency across most years but showed high stock returns in 2021 and 2024. This suggests that investor sentiment, speculative dynamics, or external factors (e.g., news, hype, or industry trends) may drive stock prices in ways that are not always aligned with financial fundamentals.

Thus, while DEA provides a strong framework for evaluating intrinsic financial quality, market behavior is influenced by broader and often irrational factors, highlighting the importance of combining quantitative analysis with qualitative insights for investment decisions.

CHAPTER 6
CONCLUSION AND RECOMENDATION

6.1 Introduction

This chapter presents the concluding insights drawn from the implementation of the integrated AHP–DEA model for evaluating financial performance and its relationship with actual stock market behavior. The discussion revisits the research objectives, summarizes the main findings, and offers both practical and methodological recommendations. The study aimed to evaluate company financial performance by integrating subjective expert input (via AHP) with objective data-driven evaluation (via DEA), providing a multi-criteria and empirically grounded model for decision-making in financial analysis.

6.2 Conclusion

This research addressed three revised core objectives: enhancing DEA evaluation using AHP expert weights, ranking companies by financial efficiency, and examining the relationship between DEA scores and stock performance.

Integrating AHP-derived weights into the DEA model added expert judgment to the traditional quantitative framework. Through pairwise comparisons and consistency validation, expert preferences were quantified and applied as assurance region constraints in the DEA-VRS model. This ensured weight assignments reflected both data and domain insight, producing more credible efficiency evaluations.

Using the bounded DEA-VRS model, the study generated efficiency scores for five companies (DMUs) across six years. Year-by-year rankings revealed that DMU2 consistently performed best, while DMU3 underperformed. These rankings provide a valuable benchmarking tool for internal assessment and peer comparison.

A comparison between DEA efficiency scores and stock price growth tested practical relevance. Some years (e.g., 2020) showed strong positive correlations, while others reflected weak or inverse links. This suggests DEA captures intrinsic financial quality, though stock prices are influenced by external and behavioral factors.

In summary, the AHP–DEA integration offers a comprehensive, flexible, and data-informed approach to evaluating corporate financial performance, effectively balancing expert insight with quantitative rigor.

6.3 Recommendation

Based on these findings, several practical recommendations are proposed. Investors and analysts may adopt the AHP–DEA model as a supporting tool to identify firms with strong intrinsic performance. It helps detect gaps between actual financial health and market valuation—useful for long-term strategies. Corporate managers can use DEA scores to benchmark performance and pinpoint inefficiencies. Divergences between market valuation and DEA results may warrant further review of internal operations or strategy. Financial institutions may integrate this model into credit risk or performance evaluations, enriching decisions with both data and expert-derived weights.

Methodologically, future research could improve AHP consistency by refining the survey and providing better instructions or training. Automated feedback on inconsistencies could also enhance reliability. DEA could be expanded with more financial variables for deeper analysis. Applying the AHP–DEA model across sectors or with larger datasets would help test its robustness. Finally, integrating qualitative factors such as ESG scores, innovation metrics, or governance quality could make the model more holistic for modern capital markets.

Overall, the AHP–DEA framework developed in this study offers a valuable, stakeholder-relevant approach to assessing financial performance—combining expert input with analytical precision.

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APPENDICES

Metric Commonly Used in Financial Analysis

CRITERIA	DEFINITION	SUBCRITERIA	DEFINITION & FORMULA
C1 Valuation Ratios	Financial metrics that assess a company's stock price relative to its earnings, revenue, book value, or other financial fundamentals.	P/E Ratio	measures how much investors are willing to pay for each dollar of a company's earnings, indicating whether a stock is overvalued or undervalued relative to its earnings.
		P/B Ratio	compares a stock's market price to its book value per share, indicating whether a stock is undervalued (P/B < 1) or overvalued based on its net asset value.
		EV/EBITDA	measures a company's total value (including debt) relative to its operating earnings, to compare profitability and valuation across businesses.
C2 Profitability Ratios	Indicators of a company's ability to generate profit relative to its revenue, assets, or equity.	Net Profit Margin	indicates how much of each dollar of revenue remains as profit after all expenses, reflecting a company's overall profitability.
		Return on Asset (ROA)	measures how effectively a company utilizes its assets to generate profit, indicating overall operational efficiency and profitability.
		Return on Equity (ROE)	measures a company's profitability by showing how efficiently it generates returns on shareholders' equity.

C3	Liquidity Ratios	Measures of a company’s ability to meet its short-term obligations.	Current Ratio	assesses a company's ability to cover short-term liabilities with its current assets, with higher values indicating stronger liquidity.
C4	Cash Flow Analysis	The process of evaluating a company’s cash inflows and outflows to assess its liquidity, financial health, and ability to generate free cash flow.	Free Cash Flow (FCF)	measures the cash a company generates after covering capital expenditures, indicating its ability to reinvest, pay dividends, or reduce debt.
C 5	Operational Efficiency	A measure of how effectively a company utilizes its resources to generate revenue and profit.	Gross Margin	indicates the percentage of revenue retained after covering production costs, with higher values reflecting better cost efficiency and profitability.
			Operating Margin	measures the percentage of revenue remaining after covering operating expenses, indicating the profitability of a company’s core business activities.
C6	Debt Metrics	Financial ratios that evaluate a company's leverage and ability to manage debt.	Debt-to-Equity (D/E) Ratio	measures a company's financial leverage by comparing its total debt to shareholder equity, with higher values indicating greater reliance on debt and increased financial risk.
			Interest Coverage Ratio	measures a company's ability to pay interest expenses using its earnings before interest and taxes (EBIT), with higher values indicating stronger debt repayment capacity.

C7	Revenue and Earnings Growth	Measures of a company's financial expansion over time.	Revenue Growth Rate	measures the percentage increase in a company's sales over time, reflecting the rate of business expansion and market performance.
			Earnings Per Share (EPS) Growth	measures the percentage increase in earnings per share over time, indicating a company's improving profitability on a per-share basis.
C8	Dividend Metrics	Ratios that evaluate a company's dividend payments relative to earnings and share price.	Dividend Yield	indicates the percentage of a stock's price returned to investors as dividends, reflecting its income-generating potential.
			Dividend Payout Ratio	measures the percentage of net income distributed to shareholders as dividends, reflecting a company's dividend policy and sustainability.

Pairwise Comparison Matrix with Mean Score Value

	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
C1	5	5.4	3.8	6.1	5.4	4.8	5.6	4.5
C2	4.6	5	3.7	5.3	3.7	4.5	5.1	4.1
C3	6.2	6.3	5	6.4	5.8	6	6.1	5.6
C4	3.9	4.7	3.6	5	3.6	4.4	4.4	3.8
C5	4.6	6.3	4.2	6.4	5	5.2	6.1	4.9
C6	5.2	5.5	4	5.6	4.8	5	5.1	4.8
C7	4.4	4.9	3.9	5.6	3.9	4.9	5	4.2
C8	5.5	5.9	4.4	6.2	5.1	5.2	5.8	5

Pairwise Comparison Matrix with Mean Score Rounded Value

	C1	C2	C3	C4	C5	C6	C7	C8
C1	5	5.5	4	6	5.5	5	5.5	4.5
C2	4.5	5	3.5	5.5	3.5	4.5	5	4
C3	6	6.5	5	6.5	6	6	6	5.5
C4	4	4.5	3.5	5	3.5	4.5	4.5	4
C5	4.5	6.5	4	6.5	5	5	6	5
C6	5	5.5	4	5.5	5	5	5	5
C7	4.5	5	4	5.5	4	5	5	4
C8	5.5	6	4.5	6	5	5	6	5

Weight Sum Value Table

WEIGHT	WEIGHT SUM VALUE
0.093409	0.702889
0.186926	1.848971
0.040830	0.331701
0.255195	2.737796
0.088162	0.755574
0.104299	0.905802
0.159672	1.552037
0.071509	0.605468

Ratio Table

	RATIO
C1	7.526379477
C2	9.890168739
C3	8.120462600
C4	10.725759859
C5	8.570461104
C6	8.685220178
C7	9.719645719
C8	8.468134325
AVERAGE	8.963279

Pairwise Matrix of New AHP Weight

	C1	C2	C4	C6	C7
C1	1	1/2	1/3	1	1/2
C2	2	1	1/2	2	1
C4	3	2	1	2	2
C6	1	1/2	1/2	1	1
C7	2	1	1/2	1	1

Weight Sum Value Table of New AHP Weight

WEIGHT	WEIGHT SUM VALUE
0.112504881	0.520980768
0.213245056	1.173420808
0.347125032	2.259345351
0.142451405	0.685479110
0.184673627	0.990633717

Ratio Table of New AHP Weight

	RATIO
C1	4.631352525
C2	5.502605851
C4	6.505836171
C6	4.813763950
C7	5.362836317
AVERAGE	5.36327895

Company Data 2020

	DMU	PE	ROE	FCF	DE	RG
RHB BANK BERHAD	DMU 1	0.118181818	0.200742509	0.198863526	0.220779221	0.20098933
HONG LEONG BANK BHD	DMU 2	0.123529412	0.251127022	0.195954463	0.090909091	0.203001803
CIMB GROUP HOLDINGS BERHAD	DMU 3	0.381925134	0.056483691	0.18575335	0.330241187	0.195102591
PUBLIC BANK BHD	DMU 4	0.219465241	0.28453991	0.202412583	0.115027829	0.197976434
MALAYAN BANKING BHD	DMU 5	0.156898396	0.207106868	0.217016077	0.243042672	0.202929842

Company Data 2021

	DMU	PE	ROE	FCF	DE	RG
RHB BANK BERHAD	DMU 1	0.137060861	0.196450681	0.189762933	0.245173745	0.201127472
HONG LEONG BANK BHD	DMU 2	0.2210127	0.208212959	0.192277556	0.108108108	0.19378403
CIMB GROUP HOLDINGS BERHAD	DMU 3	0.20979713	0.154354106	0.205761767	0.305019305	0.198197684
PUBLIC BANK BHD	DMU 4	0.235526967	0.244737928	0.196341041	0.106177606	0.204029987
MALAYAN BANKING BHD	DMU 5	0.196602342	0.196244325	0.215856703	0.235521236	0.202860826

Company Data 2022

	DMU	PE	ROE	FCF	DE	RG
RHB BANK BERHAD	DMU 1	0.151733378	0.18502548	0.190707351	0.262541806	0.203948609
HONG LEONG BANK BHD	DMU 2	0.213532072	0.213249706	0.180714108	0.08361204	0.198134949
CIMB GROUP HOLDINGS BERHAD	DMU 3	0.186233462	0.175813407	0.228991074	0.287625418	0.216451696
PUBLIC BANK BHD	DMU 4	0.229442304	0.244021952	0.19115189	0.128762542	0.206420704
MALAYAN BANKING BHD	DMU 5	0.219058784	0.181889455	0.208435577	0.237458194	0.175044043

Company Data 2023

	DMU	PE	ROE	FCF	DE	RG
RHB BANK BERHAD	DMU 1	0.161741835	0.171678513	0.204877363	0.23255814	0.194637497
HONG LEONG BANK BHD	DMU 2	0.197900467	0.214142519	0.202989092	0.11461794	0.202565068
CIMB GROUP HOLDINGS BERHAD	DMU 3	0.174183515	0.194459632	0.152407545	0.313953488	0.203576973
PUBLIC BANK BHD	DMU 4	0.243390358	0.231091671	0.215262851	0.099667774	0.189410031
MALAYAN BANKING BHD	DMU 5	0.222783826	0.188627665	0.224463149	0.239202658	0.209810431

Company Data 2024

	DMU	PE	ROE	FCF	DE	RG
RHB BANK BERHAD	DMU 1	0.165870683	0.174893466	0.182959694	0.26348228	0.198941062
HONG LEONG BANK BHD	DMU 2	0.17229978	0.208984375	0.179273778	0.117103236	0.204965785
CIMB GROUP HOLDINGS BERHAD	DMU 3	0.209404849	0.199573864	0.188836991	0.303543914	0.193609912
PUBLIC BANK BHD	DMU 4	0.227406319	0.2265625	0.190861195	0.103235747	0.198276785
MALAYAN BANKING BHD	DMU 5	0.225018369	0.189985795	0.258068342	0.212634823	0.204206456

2020 Upper and Lower Bound Tightening

VARIABLE	VARIABLE WEIGHT LOWER BOUND	VARIABLE WEIGHT UPPER BOUND
P/E Ratio	0.022500976	0.427518547
Debt-to-Equity (D/E) Ratio	0.028490281	0.541315338
Return on Equity (ROE)	0.042649011	0.810331211
Free Cash Flow (FCF)	0.069425006	1.319075121
Revenue Growth Rate	0.036934725	0.701759783
Multiplier	0.2	3.8

2021 Upper and Lower Bound Tightening

VARIABLE	VARIABLE WEIGHT LOWER BOUND	VARIABLE WEIGHT UPPER BOUND
P/E Ratio	0.045001952	0.303763178
Debt-to-Equity (D/E) Ratio	0.056980562	0.384618793
Return on Equity (ROE)	0.085298022	0.57576165
Free Cash Flow (FCF)	0.138850013	0.937237586
Revenue Growth Rate	0.073869451	0.498618793
Multiplier	0.4	2.7

2022 Upper and Lower Bound Tightening

VARIABLE	VARIABLE WEIGHT LOWER BOUND	VARIABLE WEIGHT UPPER BOUND
P/E Ratio	0.05625244	0.315013666
Debt-to-Equity (D/E) Ratio	0.071225702	0.398863933
Return on Equity (ROE)	0.106622528	0.597086156
Free Cash Flow (FCF)	0.173562516	0.971950089
Revenue Growth Rate	0.092336814	0.517086156
Multiplier	0.5	2.8

2023 Upper and Lower Bound Tightening

VARIABLE	VARIABLE WEIGHT LOWER BOUND	VARIABLE WEIGHT UPPER BOUND
P/E Ratio	0.045001952	0.29251269
Debt-to-Equity (D/E) Ratio	0.056980562	0.370373652
Return on Equity (ROE)	0.085298022	0.554437145
Free Cash Flow (FCF)	0.138850013	0.902525083
Revenue Growth Rate	0.073869451	0.48015143
Multiplier	0.4	2.6

2024 Upper and Lower Bound Tightening

VARIABLE	VARIABLE WEIGHT LOWER BOUND	VARIABLE WEIGHT UPPER BOUND
P/E Ratio	0.05625244	0.371266106
Debt-to-Equity (D/E) Ratio	0.071225702	0.470089636
Return on Equity (ROE)	0.106622528	0.703708683
Free Cash Flow (FCF)	0.173562516	1.145512605
Revenue Growth Rate	0.092336814	0.609422969
Multiplier	0.5	3.3

DEA result 2020

DMU	EFFICIENCY	μ	v1	u1	u2	u3	u4
1	0.7178	0	0.4275	0.2241	0.0426	0.2806	0.0369
2	1	0	0.4112	0.5413	0.3142	0.0694	0.0369
3	0.2259	0	0.2372	0.0285	0.0426	0.0694	0.0373
4	0.7728	0	0.1719	0.5413	0.1965	0.0694	0.0369
5	0.6829	0	0.4275	0.1355	0.0426	0.2394	0.0369

DEA result 2021

DMU	EFFICIENCY	μ	v1	u1	u2	u3	u4
1	1	0	0.3438	0.1503	0.155	0.0562	0.1007
2	1	0	0.2819	0.1503	0.1662	0.0618	0.0658
3	0.6239	0	0.1997	0.0265	0.0293	0.2175	0.0658
4	1	0	0.2674	0.1503	0.0293	0.1304	0.2041
5	0.9079	0	0.1104	0.0265	0.0293	0.0562	0.2502

DEA result 2022

DMU	EFFICIENCY	μ	v1	u1	u2	u3	u4
1	1	0	0.3438	0.0321	0.0293	0.3183	0.0866
2	1	0	0.3318	0.0265	0.0293	0.2834	0.2655
3	0.8161	0	0.3066	0.0265	0.0293	0.2818	0.0658
4	0.9782	0	0.265	0.1503	0.0293	0.0562	0.2714
5	0.9275	0	0.1493	0.0265	0.0293	0.0562	0.2708

DEA result 2023

DMU	EFFICIENCY	μ	v1	u1	u2	u3	u4
1	1	0	0.3438	0.1503	0.0293	0.1591	0.1458
2	1	0	0.3034	0.1503	0.0293	0.1633	0.2283
3	0.8561	0	0.3438	0.1503	0.0293	0.1506	0.0965
4	1	0	0.2681	0.1503	0.0293	0.0562	0.3017
5	0.8552	0	0.1871	0.0265	0.0293	0.1432	0.1886

DEA result 2024

DMU	EFFICIENCY	μ	v1	u1	u2	u3	u4
1	1	0	0.3438	0.0265	0.0293	0.1587	0.2275
2	1	0	0.3434	0.1503	0.0293	0.2472	0.0658
3	0.7272	0	0.3438	0.1285	0.0293	0.0562	0.1654
4	1	0	0.2814	0.0675	0.0293	0.0562	0.3727
5	0.8946	0	0.2382	0.0265	0.0293	0.0562	0.2947

Cross Efficiency DEA result 2020

	DMU1	DMU2	DMU3	DMU4	DMU5
EVALUATED BY DMU1	0.717697086	0.999891357	0.260147797	0.637326569	0.635228086
EVALUATED BY DMU2	0.501420865	0.99989732	0.112676998	0.726222336	0.446878636
EVALUATED BY DMU3	0.869620337	0.999282976	0.225737951	0.606362747	0.712523851
EVALUATED BY DMU4	0.433858742	0.999895936	0.127610557	0.772720895	0.398950315
EVALUATED BY DMU5	0.790370459	0.999589585	0.259949367	0.6204712	0.682601808
MEAN CE_k	0.662593498	0.999711435	0.197224534	0.672620749	0.575236539

Cross Efficiency DEA result 2021

	DMU1	DMU2	DMU3	DMU4	DMU5
EVALUATED BY DMU1	0.910907038	0.976536253	0.652946068	0.999953428	0.821418794
EVALUATED BY DMU2	0.836877097	0.985431574	0.63238938	0.999980492	0.809835966
EVALUATED BY DMU3	1.00008065	0.984057878	0.716381926	0.999991547	0.899740066
EVALUATED BY DMU4	0.679791684	0.927764814	0.468162229	1.000095611	0.651642447
EVALUATED BY DMU5	1.000329867	0.989706068	0.726228588	1.000292442	0.908487355
MEAN CE_k	0.885597267	0.972699317	0.639221638	1.000062704	0.818224926

Cross Efficiency DEA result 2022

	DMU1	DMU2	DMU3	DMU4	DMU5
EVALUATED BY DMU1	0.837929012	1.000011739	0.803125808	0.923719803	0.731320449
EVALUATED BY DMU2	0.628704024	1.000118786	0.585690745	0.893291929	0.584247316
EVALUATED BY DMU3	0.922800514	0.999997457	0.872016981	0.942218317	0.772565525
EVALUATED BY DMU4	0.793994474	1.00009677	0.738806727	0.929911468	0.688568336
EVALUATED BY DMU5	0.947596452	1.000014343	0.891912295	0.947106508	0.784067562
MEAN CE_k	0.826204895	1.000047819	0.778310511	0.927249605	0.712153838

Cross Efficiency DEA result 2023

	DMU1	DMU2	DMU3	DMU4	DMU5
EVALUATED BY DMU1	0.80118565	1.000036768	0.57041428	0.931962397	0.73366681
EVALUATED BY DMU2	0.663956791	1.000156408	0.526190757	0.977319163	0.630849233
EVALUATED BY DMU3	0.852568399	1.000316818	0.672630464	0.888172351	0.757545368
EVALUATED BY DMU4	0.651310661	1.000174236	0.508958232	0.985007048	0.623559884
EVALUATED BY DMU5	0.871768497	1.000141267	0.6484271	0.904965816	0.773241239
MEAN CE_k	0.768158	1.000165099	0.585324167	0.937485355	0.703772507

Cross Efficiency DEA result 2024

	DMU1	DMU2	DMU3	DMU4	DMU5
EVALUATED BY DMU1	0.776568287	0.999779759	0.664742208	0.852988207	0.841923032
EVALUATED BY DMU2	0.670431685	1.000013079	0.573077105	0.880996492	0.748368249
EVALUATED BY DMU3	0.852962553	0.999896425	0.721710669	0.824255862	0.849715135
EVALUATED BY DMU4	0.53876621	1.000113863	0.487586542	0.991552809	0.687029721
EVALUATED BY DMU5	0.847525449	0.99970527	0.717688652	0.825850203	0.849017122
MEAN CE_k	0.737250837	0.999901679	0.632961035	0.875128715	0.795210652

C1, Valuation Ratio

Input

$$P/E\ Ratio\ = \frac{Stock\ Price}{Earning\ Per\ Share}$$

C2, Profitability Ratio

Output

$$ROE\ = \frac{Net\ Income}{Shareholder\ Equity} \times 100$$

C4, Cash Flow Analysis

Output

$$FCF\ = Cash\ Flow\ Operation\ - Capital\ Expenditure$$

C6, Debt Metric

Input

$$D/E\ Ratio\ = \frac{Total\ Debt}{Shareholder\ Equity}$$

C7, Revenue and Earning Growth

Output

$$\text{Revenue Growth} = \frac{Current - Previous\ Revenue}{Previous\ Revenue} \times 100$$

DEA Programming

```{r}
library(readxl)
library(lpSolveAPI)
library(dplyr)
```

```{r}
# Load Excel file (update filename if needed)
data <- read_excel("2019.xlsx", skip = 1)

head(data)

# Define input/output columns by name or index (update as needed)
inputs <- as.matrix(data[, c("PE", "DE")])     # Inputs
outputs <- as.matrix(data[, c("ROE", "FCF", "RG")]) # Outputs

# Optional: scale
inputs <- inputs * 10
outputs <- outputs * 10
head(inputs)
head(outputs)
```

```{r}
n <- nrow(inputs)
m <- ncol(inputs)
s <- ncol(outputs)

# Storage
eff_scores <- numeric(n)
v_weights <- matrix(0, n, m)
u_weights <- matrix(0, n, s)
mu_vals <- numeric(n)
```

```
    # mu is unrestricted
  set.type(lpmodel, columns = total_vars, type = "real")

   # Normalization: sum of weighted inputs = 1
  add.constraint(lpmodel, xt = c(x0, rep(0, s), 0), type = "=", rhs =
1)

  # Solve LP
  status <- solve(lpmodel)
  if (status == 0) {
    vars <- get.variables(lpmodel)
    v <- vars[1:m]
    u <- vars[(m + 1):(m + s)]
    mu <- vars[total_vars]
    eff <- sum(u * y0)  # Objective value = efficiency

    eff_scores[dmu_index] <- eff
    v_weights[dmu_index, ] <- v
    u_weights[dmu_index, ] <- u
    mu_vals[dmu_index] <- mu
  } else {
    eff_scores[dmu_index] <- NA
    warning(paste("Infeasible model for DMU", dmu_index))
  }
}
```

```
# === CUSTOM BOUNDS ===
# Inputs
set.bounds(lpmodel, lower = 0.05625244, upper = 0.29251269, columns =
1)  # v1
set.bounds(lpmodel, lower = 0.071225702, upper = 0.370373652, columns =
2)  # u4

# Outputs
set.bounds(lpmodel, lower = 0.106622528, upper = 0.554437145, columns =
3)  # u1
set.bounds(lpmodel, lower = 0.173562516, upper = 0.902525083, columns =
4) # u2
set.bounds(lpmodel, lower = 0.092336814, upper = 0.48015143, columns =
5) # u3
```

```{r}
for (dmu_index in 1:n) {
  x0 <- inputs[dmu_index, ]
  y0 <- outputs[dmu_index, ]
  total_vars <- m + s + 1  # v_i + u_r + mu

  lpmodel <- make.lp(0, total_vars)
  lp.control(lpmodel, sense = "max")  # ✅ output-oriented

  # Objective: maximize weighted outputs
  set.objfn(lpmodel, c(rep(0, m), y0, 0))

   # Constraints for all DMUs: u*y_j - v*x_j + mu <= 0
  for (j in 1:n) {
    row <- c(-inputs[j, ], outputs[j, ], 1)  # +mu
    add.constraint(lpmodel, xt = row, type = "<=", rhs = 0)
  }


  # Set bounds
  for (i in 1:(m + s)) {
    set.bounds(lpmodel, lower = 0.0001, columns = i)
  }
```

```{r}
results <- data.frame(
  DMU = 1:n,
  Efficiency = round(eff_scores, 4),
  mu = round(mu_vals, 4)
)

results <- cbind(
  results,
  setNames(as.data.frame(round(v_weights, 4)), paste0("v", 1:m)),
  setNames(as.data.frame(round(u_weights, 4)), paste0("u", 1:s))
)

print(results)
```

```{r}
library(ggplot2)
library(dplyr)
```

```{r}
# Create data frame
ahp_weights <- data.frame(
  Criteria = c("C1", "C2", "C3", "C4", "C5", "C6", "C7", "C8"),
  Weight = c(0.093409, 0.186926, 0.04083, 0.255195, 0.088162, 0.104299,
0.159672, 0.071509)
)

# Add ranking (1 = highest weight)
ahp_weights <- ahp_weights %>%
  arrange(desc(Weight)) %>%
  mutate(Rank = paste0("Rank ", row_number())) %>%
  arrange(Criteria)  # Sort back by Criteria for original bar order
```

~~~~
```{r}
library(ggplot2)
library(tidyr)
library(dplyr)
library(ggpattern)
```
~~~~

~~~~
# Plot bar chart
ggplot(ahp_weights, aes(x = Criteria, y = Weight, fill = Criteria)) +
  geom_bar(stat = "identity", width = 0.7) +
  geom_text(aes(label = Rank), vjust = -0.5, size = 4.5) +
  scale_y_continuous(
    limits = c(0, 0.3),  # Slightly above max value (0.255195)
    breaks = seq(0, 0.3, by = 0.05),
    labels = scales::percent_format(accuracy = 1)
  ) +
  labs(title = "AHP Criteria Weight", x = "Criteria", y = "Weight (%)")
+
  theme_minimal() +
  theme(
    legend.position = "none",
    plot.title = element_text(hjust = 0.5, face = "bold")
  )
```
~~~~

~~~~
```{r}
# Grayscale color palette (visually distinct in B/W)
gray_palette <- c("gray10", "gray30", "gray50", "gray70", "gray90")
names(gray_palette) <- c("DMU1", "DMU2", "DMU3", "DMU4", "DMU5")

# Plot
ggplot(dea_ce, aes(x = factor(Year), y = Efficiency, fill = DMU)) +
  geom_bar(stat = "identity", position = position_dodge(width = 0.8),
width = 0.7) +
  geom_text(aes(label = Rank),
            position = position_dodge(width = 0.8),
            vjust = -0.5, size = 3.5) +
  scale_fill_manual(values = gray_palette) +
  scale_y_continuous(limits = c(0, 1.2), breaks = seq(0, 1.2, 0.2)) +
  labs(title = "Cross Efficiency DEA Results (2019-2024)",
       x = "Year", y = "Mean Cross Efficiency Score", fill = "DMU") +
  theme_minimal() +
  theme(
    plot.title = element_text(hjust = 0.5, face = "bold"),
    legend.position = "top"
  )


```
~~~~

```{r}
# DEA data
dea_ce <- data.frame(
  Year = rep(2019:2024, each = 5),
  DMU = rep(c("DMU1", "DMU2", "DMU3", "DMU4", "DMU5"), times = 6),
  Efficiency = c(
    0.930213, 1.010391, 0.735643, 0.912352, 0.912777,
    0.662593498, 0.999711435, 0.197224534, 0.672620749, 0.575236539,
    0.885597267, 0.972699317, 0.639221638, 1.000062704, 0.818224926,
    0.826204895, 1.000047819, 0.778310511, 0.927249605, 0.712153838,
    0.768158, 1.000165099, 0.585324167, 0.937485355, 0.703772507,
    0.737250837, 0.999901679, 0.632961035, 0.875128715, 0.795210652
  )
)

# Rank each DMU within its year
dea_ce <- dea_ce %>%
  group_by(Year) %>%
  arrange(desc(Efficiency), .by_group = TRUE) %>%
  mutate(Rank = row_number())
```

```{r}
# Create stock growth data
stock_growth <- data.frame(
  Year = 2019:2024,
  DMU1 = c(2.076779026, 1.947826087, 1.985321101, 2.078212291,
1.941278066, 2.179816514),
  DMU2 = c(1.850877193, 2.042382589, 2.023076923, 2.104189044,
1.9192607,   2.084656085),
  DMU3 = c(1.877224199, 1.872210953, 2.26744186,   2.064220183,
2.00862069,   2.386324786),
  DMU4 = c(1.807692308, 2.032581454, 2.009708738, 2.038461538,
1.993055556, 2.048951049),
  DMU5 = c(1.935828877, 1.966857143, 1.98108747,   2.048192771,
2.02183908,   2.140607424)
)

# Convert to long format for ggplot
stock_long <- pivot_longer(stock_growth, cols = starts_with("DMU"),
                           names_to = "DMU", values_to = "StockGrowth")

# Add rank within each year
stock_long <- stock_long %>%
  group_by(Year) %>%
  arrange(desc(StockGrowth), .by_group = TRUE) %>%
  mutate(Rank = row_number())
```

```{r}
# Grayscale palette
gray_palette <- c("gray10", "gray30", "gray50", "gray70", "gray90")
names(gray_palette) <- c("DMU1", "DMU2", "DMU3", "DMU4", "DMU5")

# Final plot without y-axis limits
ggplot(stock_long, aes(x = factor(Year), y = StockGrowth, fill = DMU))
+
  geom_bar(stat = "identity", position = position_dodge(width = 0.8),
width = 0.7) +
  geom_text(aes(label = Rank),
            position = position_dodge(width = 0.8),
            vjust = -0.5, size = 3.5) +
  scale_fill_manual(values = gray_palette) +
  scale_y_continuous(expand = expansion(mult = c(0, 0.1))) +
  labs(title = "Stock Growth of Banks (2019-2024)",
       x = "Year", y = "Stock Growth (Ratio)", fill = "DMU") +
  theme_minimal() +
  theme(
    plot.title = element_text(hjust = 0.5, face = "bold"),
    legend.position = "top"
  )
```