Fundamentals of Probability

Outlines:

1. Experiment
2. Outcomes
3. Random experiment
4. Sample space
5. Event
6. Mutually exclusive event
7. Non mutually exclusive event
8. Conditional probability
9. Independent event
10. Dependent event
11. Classical definition of probability, axiomatic Probability
12. Additive law of probability

Experiment: Experiment is an act that can be repeated under given conditions.

Example: Tossing coin, throwing dice etc.

Outcomes: The result of an experiment are called outcomes.

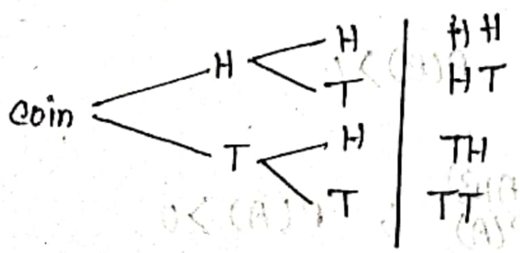
Random experiment (चिप प्रयोग): Experiment are called random experiments if the outcomes depend on chance and cannot be predicted with certainty.

Examples: Tossing of a fair coin, throwing of dice etc.

Sample space: (नमुना क्षेत्र) The collection or totality of all possible outcomes of a random experiment is called sample space. It is deonoted by S or Ω.

Example: 1. If we toss a coin the sample space is $S = \{H, T\}$

2. If we toss a coin two times, the sample space is $S = \{HH, HT, TH, TT\}$



Each element of a sample space is called sample Point.

Event: Any subset of a sample space is called event. It is denoted by A, B, C. There are two types of event. They are:

① Simple event

② Compound event

Mutually exclusive events: Two events are said to be mutually exclusive if they have no common points. If A and B are mutually exclusive events, then

$AB = \phi$ / $P(AB) = P(A \cap B)$

Non mutually exclusive events: Two events are said to be not mutually exclusive event if they have common points. If A and B are two not mutually exclusive events, then $AB \neq \phi$

Conditional Probability: If A and B are two events in S. Then the conditional probability of A for given value of B denoted by $P[A|B]$ is defined by

$$P(A|B) = \frac{P(AB)}{P(B)} \quad ; \quad P(B) > 0$$

Similarly, $$P(B|A) = \frac{P(AB)}{P(A)} \quad ; \quad P(A) > 0$$

⊛ $P[A|B]$ ⇒ It is written by A conditional B or A given B.

↓

B ঘটনা আগে ঘটেছে। B কে Base করে A নির্ণয়,

Independent event: Two events A and B are said to be independent if and only if one of the following conditions holds:

(i) $P[AB] = P[A]P[B]$

(ii) $P[A|B] = P[A]$

(iii) $P[B|A] = P[B]$

Dependent event: Two events A and B are said to be dependent if and only if one of the following conditions holds:

(i) $P[AB] \neq P[A]P[B]$

(ii) $P[A|B] \neq P[A]$

(iii) $P[B|A] \neq P[B]$

Definition of Probability:

Classical or mathematical probability (Priori Probability):

If A is the event in S then the probability of the event A is denoted by P(A) is defined as

$$P(A) = \frac{\text{favorable number of outcomes}}{\text{total number of outcomes}}$$

$$P(A) = \frac{N(A)}{N(S)} = \frac{m}{n} \quad ; \quad 0 \leq P(A) \leq 1$$

Example: $S = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2\}$

$\therefore N(S) = 6 \quad , \quad N(A) = 2$

$\therefore P(A) = \frac{N(A)}{N(S)} = \frac{m}{n} = \frac{2}{6} = \frac{1}{3}$ (Ans)

Axiomatic probability: Suppose S is a sample space and A is an event of this sample space. Then the the probability of the event A denoted by $P(A)$ must satisfy the following ~~fo~~ axioms:

(i) $P(A) \geq 0$

(ii) $P(S) = 1$

(iii) If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$

(iv) $P(A_1 \cup A_2 \cup \cdots \cup A_k) = P(A_1) + P(A_2) + \cdots + P(A_k)$

Additive laws of probability:

~~① Non mutually~~

① Mutually exclusive event

② Non mutually exclusive event

If A and B are two events then

$P[A \cup B] = P[A] + P[B] \rightarrow$ Mutually exclusive

$P[A \cup B] = P[A] + P[B] - P[AB] \Rightarrow$ Non mutually exclusive

Theorem 1: The values of probability lies between 0 to 1.

i.e, $0 \leq P(A) \leq 1$

Theorem 2: $P(A) + P(\bar{A}) = 1$.

Problem - 1 : If $P(A) = 0.6$, $P(B) = 0.8$ and $P(AB) = 0.50$ Find
(i) $P(\bar{A})$; (ii) $P(A \cup B)$; (iii) $P(A|B)$; (iv) $P(B|A)$; (v) $P(A\bar{B})$
(vi) $P(\bar{A}B)$; (vii) $P(\bar{A}\bar{B})$; (viii) $P(\overline{A \cup B})$; (xi) Are the events A and B independent? (x) Are A and B mutually exclusive?

Solution: Given that,

$$P(A) = 0.6$$
$$P(B) = 0.8$$
$$P(AB) = 0.50$$

(i) We know that, $P(A) + P(\bar{A}) = 1$

$$\Rightarrow P(\bar{A}) = 1 - P(A)$$
$$\Rightarrow P(\bar{A}) = 1 - 0.6$$
$$\therefore P(\bar{A}) = 0.4$$

(ii) By Additive law of probability,

$$P(A \cup B) = P(A) + P(B) \pm P(A \cap B)$$
$$= P(A) + P(B) \pm P(AB) \qquad [P(AB) - P(A \cap B)]$$
$$= 0.6 + 0.8 - 0.50$$
$$= 0.9$$

(iii) We have $\quad P(A|B) = \dfrac{P(AB)}{P(B)} \qquad [P(B) > 0]$

$$= \frac{0.50}{0.8}$$
$$= 0.625$$

(iv) $P(B|A) = \frac{P(AB)}{P(A)}$; $P(A) > 0$

$= \frac{0.50}{0.6}$

$= 0.833$

(v) $P(A\ \overline{B}) = P(A \cap \overline{B})$

$= P(A) - P(AB)$

$= 0.6 - 0.50$

$= 0.10$

(vii) $P(\overline{A}\ B) = P(\overline{A} \cap B)$

$= P(B) - P(AB)$

$= 0.8 - 0.50$

$= 0.30$

(viii) $P(\overline{A}\ \overline{B}) = P(\overline{A} \cap \overline{B})$

$= P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= 1 - 0.9$

$= 0.1$

(Ans)

(Viii) ~~$P(\bar{A}\bar{B})$~~ $P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= 1 - 0.9$

$= 0.1$

(Ans)

(xi) Are the events A and B independent?

Solution: The events A and B are independent if

$P[AB] = P[A]P[B]$

Here, $P[AB] = 0.5$

and $P[A]\,P[B] = 0.6 \times 0.8$

$= 0.48$

As, $P[AB] \neq P[A]P[B]$ Hence the events A and B are not independent.

(x) Are A and B mutually exclusive?

Solution: A and B mutually exclusive if $P[AB] = 0$

But here $P[AB] = 0.5 \neq 0$.

Hence A and B are not mutually exclusive.

Problem 2: If $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{4}$, $P(A \cup B) = \frac{11}{12}$.

Find (i) $P(A|B)$

Solution: Given that, $P(A) = \frac{1}{3}$

$P(B) = \frac{3}{4}$

$P(A \cup B) = \frac{11}{12}$

We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

or, $P(A \cup B) = P(A) + P(B) - P(AB)$ $[P(A \cap B) = P(AB)]$

~~or, P(AUB)~~

or, $P(AB) = P(A) + P(B) - P(A \cup B)$

$$= \frac{1}{3} + \frac{3}{4} - \frac{11}{12}$$

$$= \frac{1}{6}$$

$$P(A|B) = \frac{P(AB)}{P(B)}$$

$$= \frac{1/6}{\cancel{3/6}\ 3/4}$$

$$= \frac{2}{9}$$

(ii) $P(\bar{A}|B)$

Soln: $P(\bar{A}|B) = \frac{P(\bar{A}B)}{P(B)}$

$$= \frac{P(\bar{A} \cap B)}{P(B)}$$

$$= \frac{P(B)-P(AB)}{P(B)}$$

$$= \frac{3/4 - 1/6}{3/4}$$

$$= \frac{7}{9}.$$

Problem: 03 – A and B are two mutually exclusive events with $P[A] = 0.35$ and $P[B] = 0.15$, Find

(I) $P[A \cup B]$

Soln: Given that,

$$P[A] = 0.35$$

$$P[B] = 0.15$$

$$P[A \cup B] = P[A] + P[B]$$

$$= 0.35 + 0.15$$

$$= 0.5$$

(II) $P(\bar{A})$

Soln: $P(\bar{A}) = 1 - P(A)$

$$= (1 - 0.35)$$

$$= 0.65$$

(III) $P[A \cap B]$

Soln: $P[A \cap B] = 0$ since A and B are mutually exclusive.

(IV) $P[\bar{A} \cup \bar{B})$

Soln: $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B})$

$$= 1 - P(A \cap B)$$

$$= 1 - 0 = 1$$

(Ans).