charge and electric potential

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~~⑬~~

columb's law: The force of attraction or repulsion between two electric charges is directly proportional to the product of the charges and inversely to the square of the distance between them.

or,

The force between two point charges is directly Proportional to the product of the charges, and inversely Proportional to the square of the distance between them.

$$F \propto q_1 q_2$$

$$F \propto \frac{1}{r^2}$$

$$F = K\frac{q_1 q_2}{r^2} \text{ ; Here, } K = \frac{1}{4\pi\epsilon_0}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \ Nm^2C^{-2}$$

$$\text{So, } F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

Electric field: The space around a charged body where its influence is experienced is called electric field of that charged body.

Electric field intensity: A measure of the force exerted by one charged body on another. Imaginary lines of force or electric field lines originate on positive charges and terminate on negative charges. It is denoted by $\vec{E}$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q q_0}{q_0 r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

Vector form, $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^3} \cdot \vec{r}$

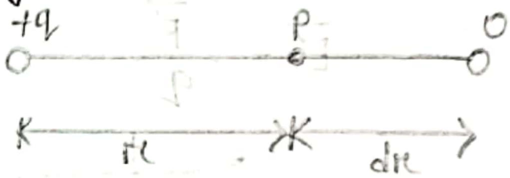
Electric potential: The amount of work done in bringing a unit positive charge from infinity to a point in an electric field is called electric potential at that point. It is denoted by V. The unit of electric potential is JC^{-1}.

$$V = \frac{W}{q}$$

Electric potential difference: The amount of work done is transforming a unit charge from one point to another point in an electric field is called a potential difference of two points.

Potential due to a point charge:

Let P be a point at a distance r from a point charge q placed at O. Let the permitivitively constant of the medium be ϵ_0. The intensity at P is given by



$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \quad \text{—— (1)}$$

The work done to displace a unit charge through dr

$$-dV = -\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}\, dr \quad \text{—— (2)}$$

The negative sign indicates decrease of potential

$$V = \int_{\infty}^{r} -\frac{1}{4\pi\epsilon_0} \frac{q}{r^2}\, dr$$

$$= -\frac{q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_{\infty}^{r}$$

$$= -\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$$

$$E = -\frac{dV}{dr} = -\frac{d}{dr}\left[-\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}\right]$$

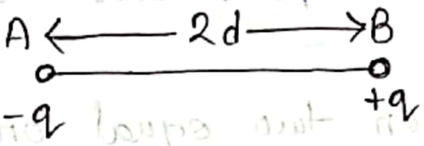
$$= -\frac{q}{4\pi\epsilon_0} \cdot \frac{d}{dr}\left(\frac{1}{r}\right)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

$$= E$$

i.e. intensity at a point in an electric field is equal to the negative gradiant of potential at the same point.

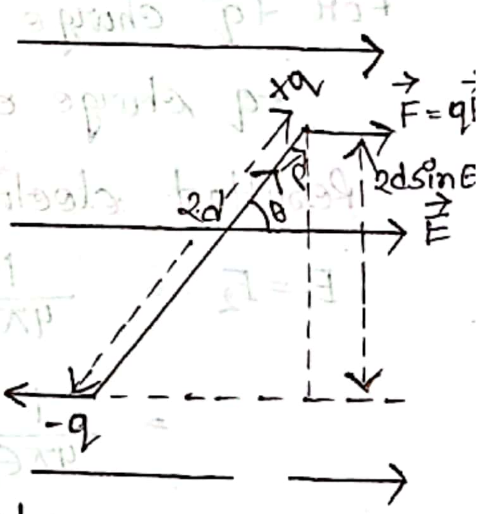
Dipole: When two equal and opposite ~~charg~~ charge are placed in such way that distance between them is very small then both charge are all together called Electric dipole.



Dipole moment, p = charge × distance between them

$= q \times 2d$.

Dipole in an external electric field:



Let us consider when two equal and opposite charge $+q$ and $-q$ are placed in such way that distance them is very small $2a$. then they form a dipole.

Dipole moment, $p = 2aq$ —— (1)

The arrangement is placed in an external electric field $\vec{E}$ and the dipole moment $\vec{p}$ has created θ angle with the $\vec{E}$.

Here also for external electric field, two equal and opposite force are constructive couple

$$\text{Torque}, \vec{\tau} = \vec{F} \times 2\vec{a}$$

$$= 2aF\sin\theta$$

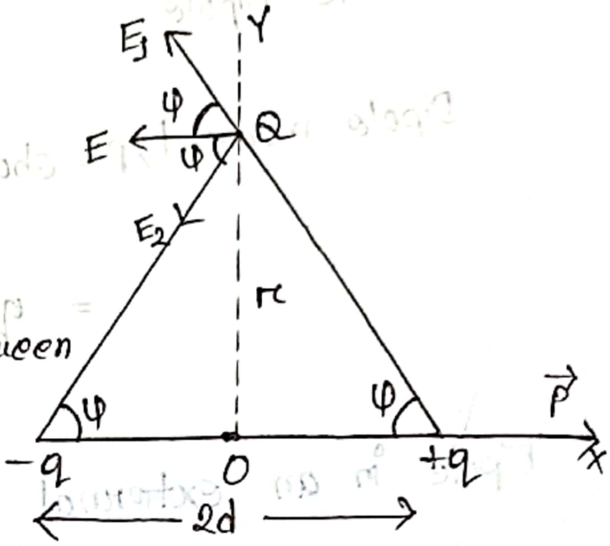
$$= 2aqE\sin\theta \quad [F = qE]$$

$$= pE\sin\theta, \quad \vec{\tau} = \vec{p} \times \vec{E}$$

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

Electric field due to dipole at a point along perpendicular bisector :

or, dipole at a point P, distance r along with Perpendicular bisector.



When two equal or opposite charge +q and -q are placed in such way that distance between them is very small and then they form a dipole.

Dipole moment, $p = 2aq$

For +q charge electric field $= E_1$

-q charge electric field $= E_2$

Resultant electric field, $E = E_1 + E_2$

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(\sqrt{a^2+r^2})^2}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a^3+r^2}$$

From the figure,

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos 2\theta}$$

$$= \sqrt{2E_1^2 + 2E_1^2\cos 2\theta}$$

$$= \sqrt{2E_1^2(1+\cos 2\theta)}$$

$$= \sqrt{2E_1^2 \cdot 2\cos^2\theta}$$

$$= \sqrt{2E_1E_1^2}$$

$$= \sqrt{4E_1^2\cos^2\theta}$$

$$= 2E_1\cos\theta$$

$$= \frac{2q}{4\pi\epsilon_0} \times \frac{1}{a^2+r^2} \times \frac{a}{\sqrt{a^2+r^2}} \quad \left[\cos\theta = \frac{a}{\sqrt{a^2+r^2}}\right]$$

$$= \frac{2aq}{4\pi\epsilon_0} \cdot \frac{1}{(a^2+r^2)^{3/2}}$$

$$= \frac{p}{4\pi\epsilon_0} \times \frac{1}{r^3}$$ [For a is very small we can ignore it

$$\therefore E = \frac{p}{4\pi\epsilon_0} \times \frac{1}{r^3}$$

Electric field for point charge on a wire of a charge ring at a distance from the center of the ring:

Let us consider a differential element of the ring of length ds located at the top of the ring. Suppose it contains an element of charge dq. The electric field dE which is the charge produced at the point p and its magnitude is,

$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{(a^2+x^2)}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q\,ds}{2\pi a} \cdot \frac{1}{(a^2+x^2)}$$

dE can be restored into two components one along the axis the other perpendicular to it.

The perpendicular components are found on another differential element.

The resulting field intensity,

$$E = \int dE \cos\theta$$

$$= \int \frac{1}{4\pi\epsilon_0} \cdot \frac{q\,ds}{2\pi a} \cdot \frac{1}{(a^2+x^2)} \cos\theta$$

$$= \int \frac{1}{4\pi\epsilon_0} \cdot \frac{q\,ds}{2\pi a} \cdot \frac{1}{(a^2+x^2)} \cdot \frac{x}{\sqrt{a^2+x^2}}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{qx}{2\pi a} \cdot \frac{1}{(a^2+x^2)^{3/2}} \int ds$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{qx}{2\pi a} \cdot \frac{1}{(a^2+x^2)^{3/2}} \times 2\pi a$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{qx}{(a^2+x^2)^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{qx}{x^3}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{x^2}$$

Gauss's law: Gauss's law state that the flux of electric field through any closed surface is equal to $\frac{1}{\epsilon_0}$ times the total charge enclosed by the surface. If the surface does not enclosed any charge the flux of $\vec{E}$ is zero.

Let us consider,

The flux of electric field $\vec{E}$ on a small area of surface $= ds$

Amount of ~~surface~~ charge $= q$

When q charge is inside the surface that means,

$$\oint \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$$

when q charge is outside the surface that means surf that means surface does not close.

$$\oint \vec{E}.\vec{ds} = 0$$

~~Columb's law from gauss's law~~:

Columb's law from gauss's law:

Let us consider a sphere of radius r and q amount of charge is enclosed by the surface area of the surface, $S = 4\pi r^2$

Again, let small area $\vec{ds}$ and flux of electric field $\vec{E}$ passing parallely with $\vec{ds}$ through the enclosed surface that means the angle between ds and $\vec{E}$ is zero.

$$\vec{E}.\vec{ds} = Eds\cos\theta$$
$$= Eds$$

According to Gauss's law:

$$\oint \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$$

$$\Rightarrow E\oint dS = \frac{q}{\epsilon_0}$$

$$\Rightarrow ES = \frac{q}{\epsilon_0}$$

$$\Rightarrow E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

Let us consider a test charge q_0 in the enclosed surface and force between q and q_0 is,

$$E = \frac{F}{q_0}$$

$$\therefore F = qE = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{r^2}$$

$\therefore$ This is the form of columb's law.

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Electric Energy:

Again let when the dipole placed in external field initiality with angle 90° after a short time the angle become θ for the change of angle and it stores a potential energy.

$$\text{Energy} = \text{Work done} = U = \int_{90^\circ}^{\theta} \tau \, d\theta$$

$$U = \int_{90}^{\theta} PE \sin\theta \, d\theta$$

$$= PE \int_{90}^{\theta} \sin\theta \, d\theta$$

$$= -PE [\cos\theta]_{90}^{\theta}$$

$$= -PE (\cos\theta - \cos 90^\circ)$$

$$= -PE \cos\theta$$

$$\therefore U = -\vec{P} \cdot \vec{E}$$

This is the stored energy.

① When a 5×10^{-9} test charge is placed at a point it experiences a force of 2×10^{-4} N in the x-direction that is electric field at that point.

Here, $F = 2\times10^{-4} N$

$q = 5\times10^{-9} C$

We know that, $F = qE$

$$\Rightarrow E = \frac{F}{q}$$

$$= \frac{2\times10^{-4}}{5\times10^{-9}}$$

$$= 4\times10^{4} NC^{-1}$$

(Ans)

② What is the force on an electron placed at the point where the electric field intensity is $4\times10^{-4} NC^{-1}$.

Here, $E = 4\times10^{-4} NC^{-1}$

$q = 1.6\times10^{-19} C$ (electron)

We know that,

$$F = qE$$

$$= (1.6\times10^{-19})\times(4\times10^{-4})$$

$$= 6.4\times10^{-23} N$$

For electron, $F = -6.4\times10^{-23}$ N.

(Ans)

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③ Calculate the field E due to a dipole moment 4.5×10^{-10} c/m at a distance 1m from it on the perpendicular bisector.

Given that,

dipole moment, $p = 4.5 \times 10^{-10}$ c/m

$r = 1m$

We know that,

$$E = \frac{p}{4\pi\epsilon_0} \cdot \frac{1}{r^3}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3}$$

$$= (9 \times 10^9) \times \frac{(4.5 \times 10^{-10})}{1^3}$$

$$= 4.05 \text{ V/m}$$

(Ans)

(4) What is the potential at the surface of an Aluminium nucleas $r = 4.5 \times 10^{-13}$ cm and it's change $q = 13e$.

Given that,

$r = 4.5 \times 10^{-15}$ cm

Charge, $q = 13 \times (-1.6 \times 10^{-19})$

$= -2.08 \times 10^{-18}$ C

Potential, $V = ?$

charge density, $\sigma = \frac{\phi}{A}$

$= \frac{\phi}{4\pi r V}$

We know that,

$$E = \frac{V}{r} \quad \text{——} (I)$$

$$E = \frac{\sigma}{E_0} \quad \text{——} (II)$$

Equating (I) and (II) $\Rightarrow$

$$\frac{V}{r} = \frac{\sigma}{E_0}$$

or, $\frac{V}{r} = \frac{\phi}{E_0 4\pi r V}$

or, $V = \frac{1}{4\pi E_0} \cdot \frac{\phi}{r}$

or, $V = (9 \times 10^9) \times \frac{2.08 \times 10^{-18}}{4.5 \times 10^{-15}}$

$\therefore V = 4.16 \times 10^6$ V

(Ans)

(5) Two equal charges of magnitude 2×10^{-6} C are placed at a distance 8 cm from each other. Find the magnitude of electric intensity at point 3 cm form the midpoint of the line to the two charges along the perpendicular bisector of the line.

Given that, $q = 2\times10^{-6}$ C

$2a = 8$ cm $= 0.8$ m

$a = 4$ cm $= 0.4$ m

$r = 3$ cm

$= 0.3$ m

$\frac{1}{4\pi\epsilon_0} = (9\times10^9)$

$E = ?$

We know that,

$$E = \frac{2aq}{4\pi\epsilon_0} \times \frac{1}{(a^2+h^2)^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_0} \times \frac{2aq}{(a^2+h^2)^{3/2}}$$

$$= (9\times10^9) \times \frac{0.8\times(2\times10^{-6})}{\{(0.4)^2+(0.3)^2\}^{3/2}}$$

$$= 9\times10^9 \times 1.28\times10^{-5}$$

$$= 1.152\times10^5 \text{ N/C}$$

(Ans)

(6) An electric dipole consists of two opposite charges of magnitude $q = 2\times10^{-6}$ C separate by 1 cm. The dipole is placed in an external field 2×10^5 N/C. Caculate (1) The maximum torque on the dipole (2) Work done to tur the dipole through 180° starting from a position of $\theta = 0°$

Given that,

$$P = qr = 2\times10^{-6} \times 0.01$$
$$= 2\times10^{-8} \text{ m}$$

$$E = 2\times10^5 \text{ N/C}$$

Again, $P = 2\times10^{-7}$

$E = 2\times10^5$ N/C

$W = ?$

(i) We know that,

$$\tau = PE\sin\theta$$

$$= 2\times10^{-12} \times 2\times10^{5}\times \sin 90^\circ$$

$$= 4\times10^{-4}\ Nm$$

(ii) We know that,

$$W = \int_0^{180} PE\sin\theta .\ d\theta$$

$$= -PE\ [\cos\theta]_0^{180}$$

$$= -PE\ (\cos180 - \cos0)$$

$$= -PE\ (-2)$$

$$= 2PE$$

$$= 2\times 2\times10^{-7}\times 2\times10^{5}$$

$$= 8\times10^{-2}\ J.$$

(Ans)

⑦ A 5 columbs charge is placed on the circumference of a circle of radius 0.5 m. Find the potential and intensity at a distance 25 cm from the centre.

Given that,

$q = 5C$

$r = 0.5\ m$

Potential $V = ?$

(i) We know that,

$$V = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r}$$

$$= (9\times10^9) \times \frac{5}{0.5}$$

$$= 9\times10^{9}\ V$$

$q = 5C$

$r = 25\times10^{-2}$

$E = ?$

(ii) Electric field intensity

$$E = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r^2}$$

$$= (9\times10^9) \times \frac{5}{(25\times10^{-2})^2}$$

$$= 7.2\times10^{11}\ N/C$$

⑧ What is the electric potential at the surface of a told nucleas. The radius is 6.6×10^{-15} m and the atomic number is 79.

We know that,

$$V = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r}$$

$$= (9\times10^9) \times \frac{1.264\times10^{-27}}{6.6\times10^{-15}}$$

$$= 1.723 \times 10^{-3} \text{ V}$$

Given that,

$r = 6.6\times10^{-15}$

Atomic number $Z = 79$

$V = ?$

Charge, $q = Ze$

$= 79\times1.6\times10^{-19}$

$= 1.264\times10^{-17}$ C

⑨ At one time the positive charge in the atom was thought to be distributed uniformly throughtout a sphere with a radius of about 1×10^{-10}m. That is throughtout the entire atom. Calculate the electric field strength at the surface of a told atom ($Z = 79$) on this ssuption.

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

$$= (9\times10^9) \times \frac{(1.264\times10^{-17})}{(1\times10^{-10})^2}$$

$$= 1.1376 \times 10^{13} \text{ N/C}$$

Given that,

$r = 1\times10^{-10}$m

$Z = 79$

$q = Ze = 1.264\times10^{-}$ C

$E = ?$

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(10) A negative point charge of $10^{-6}c$ is distributed in air of the origin of a rectangular co-ordinate system. A second negative point charge of 10^{-4} c. is situated on the positive x-axis at a distance of 50 cm from the origin. What is the force on the second charge.

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2} \times \hat{r}$$

$$= (9\times10^9) \times \frac{(-10^{-6})(-10^{-4})}{(50\times10^{-2})^2}$$

$$F = 3.6\ N$$

$$\therefore \vec{F} = 3.6N\ (+\hat{i})$$

Given that,

$q_1 = -10^{-6}c$

$q_2 = -10^{-4}c$

$r = 50\times10^{-2}\ m$

$F = ?$