The Map Method

KARNAUGH MAP Basics and key points:

- ⇒ Developed by karanaugh in 1953
- ⇒ It is used to minimize boolean equations.
- ⇒ It is build based on gray code.

Two Variable k-map:

B \ A	$\bar{A}$ 0	A 1
$\bar{B}$ 0	1 (0)	(2)
B 1	(1)	1 (3)

Here, $0 \rightarrow \bar{A}$, $1 \rightarrow A$

$\rightarrow y(A,B) = \Sigma_m(0,3)$

Three Variable k-map

Total cells $= 2^n = 2^3 = 8$

C \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$ 0	1 (0)	0 (2)	1 (6)	1 (4)
C 1	1 (1)	0 (3)	0 (7)	0 (5)

It is two bit Gray code

$y(A,B,C) = \Sigma_m(0,1,4,6)$

Four Variable k-map

Total cells $= 2^n = 2^4 = 16$

CD \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}\bar{D}$ ~~$\bar{A}\bar{B}$~~ 00	1 (0)	1 (4)	1 (12)	0 (8)
$\bar{C}D$ ~~$\bar{A}B$~~ 01	1 (1)	0 (5)	0 (13)	0 (9)
CD ~~AB~~ 11	0 (3)	0 (7)	0 (15)	0 (11)
$C\bar{D}$ ~~$A\bar{B}$~~ 10	0 (2)	1 (6)	0 (14)	1 (10)

$y(A,B,C,D) = \Sigma_m (0, 1, 4, 6, 10, 12)$

K-Map rules for grouping:-

1. Group should not contain zero and cells containing '1' must be grouped.
2. We can group 1, 2, 4, 8, ... 2^n cells
3. Each groups should be as large as possible.
4. Group may overlap.
5. Opposite grouping and corner grouping is allowed.
6. There should be as few groups as possible.

Examples:-

CD \ AB	$\bar A\bar B$ 00	$\bar A B$ 01	AB 11	$A\bar B$ 10
$\bar C\bar D$ 00	1 $_0$	1 $_4$	0 $_{12}$	1 $_8$
$\bar C D$ 01	0 $_1$	0 $_5$	0 $_{13}$	0 $_9$
CD 11	0 $_3$	0 $_7$	0 $_{15}$	0 $_{11}$
$C\bar D$ 10	1 $_2$	1 $_6$	0 $_{14}$	1 $_{10}$

2nd

$\rightarrow y = \bar B\bar D + \bar A\bar D$

1st (corner) $\bar B\bar D$

2nd $(\bar A\bar D)$.

$y = \bar B\bar D + \bar A\bar D$.

Some terms in k- Map:-

Implicants:- Any group of '1' can be considered to implicants.

Prime implicants:- It is the largest possible group of 1's.

Essential prime implicants:- In the group, at least, there is single '1'. which cannot combine in other way.

Example:

CD \ AB	00	01	11	10
00	1			
01	1	1	1	
11		1	1	
10			1	

II

I

III

- group I, II, III are Prime implicants
- group II and III are essential prime implicants.

Example-① In the sum of product function is $f(x,y,z) = \Sigma m(2,3,4,5)$, The prime implicants are.

$\bar{x}y, x\bar{y}$.

solution:

z \ xy	00	$\bar{x}y$ 01	11	$x\bar{y}$ 10
0	0	1 2	6	1 4
1	1	1 3	7	1 5

1st group $(\bar{x}y)$

2nd group $(x\bar{y})$

□ Solve the given boolean expression using k-map.

$$y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C}.$$

Solution:

C \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$ 0	1 (0)	1 (2)	1 (6)	0 (4)
C 1	0 (1)	1 (3)	0 (7)	0 (5)

1st group, 2nd, 3rd group

$$= \bar{A}\bar{C} + \bar{A}B + B\bar{C}$$

□ Find the boolean expression for k-map given below.

CD \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}\bar{D}$ 00	1	0	0	1
$\bar{C}D$ 01	0	1	1	1
CD 11	0	1	1	1
$C\bar{D}$ 10	1	0	0	1

OR (3rd group) ($A\bar{B}$) we can choose one.

3rd group (AD)

2nd group (BD)

1st group (corner) ($\bar{B}\bar{D}$)

Solution:

$$y = \bar{B}\bar{D} + BD + AD$$

or, $y = \bar{B}\bar{D} + BD + A\bar{B}$

□ $y = AB + \bar{A}BC + A\bar{B} + C$, Solve boolean expression by K-Map.

Solution:

C \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$ 0	0 (0)	(2)	1 (6)	1 (4)
C 1	1 (1)	1 (3)	1 (7)	1 (5)

1st group (A)

2nd group (C)

$\therefore y = A + C$

□ Find the boolean expression for k-map below.

CD \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}\bar{D}$ 00	0	1	1	0
$\bar{C}D$ 01	1	1	0	1
CD 11	1	1	0	1
$C\bar{D}$ 10	0	1	1	0

2nd grp ($\bar{B}D$)

1st grp ($B\bar{D}$)

3rd grp ($\bar{A}D$)

$$y = \bar{B}D + B\bar{D} + \bar{A}D$$

K-Map with don't care:

Don't care: Don't care is used to increase the size of group but it is not compulsory to use all don't care.

Examples:

① Solve given boolean k-map. [x is don't care]

CD \ AB	00	01	11	10
00	1	x	0	1
01	0	0	0	0
11	0	0	0	1
10	x	1	0	1

2nd

1st group (corner) $(\bar{B}\bar{D})$

3rd group $(A\bar{B}C)$

2nd group $(\bar{A}\bar{D})$

$\therefore y = \bar{B}\bar{D} + \bar{A}\bar{D} + A\bar{B}C$

② Solve the given boolean k-map

CD \ AB	00	01	11	10
00	x	0	1	1
01	1	0	0	1
11	x	0	0	0
10	x	0	0	x

1st

1st group $(\bar{B}\bar{C})$

2nd group $(A\bar{C}\bar{D})$

$y = \bar{B}\bar{C} + A\bar{C}\bar{D}$

③ Solve k map

F: CD \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}\bar{D}$ 00	1	0	0	0
$\bar{C}D$ 01	1	X	1	0
CD 11	X	1	1	X
$C\bar{D}$ 10	1	0	0	0

1st group (BD)

2nd group $(\bar{A}\bar{B})$

$F = BD + \bar{A}\bar{B}$

K-Map for POS expression:

Steps for POS expression:-

- take grouping of 0
- find function (f_d)
- Put Compliment of all Variables (f).

Examples:

① If boolean function is given by $y = \Sigma m(3,6)$ then find the POS expression.

Solution: Given that, $y(A,B,C) = \Sigma m(3,6)$

k-Map:-

1st, 2nd $(\bar{A}+\bar{C})$, 1st group $(\bar{B})$, 3rd $(A+C)$

C \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$ 0	0 $_{0}$	0 $_{2}$	1 $_{6}$	0 $_{4}$
C 1	0 $_{1}$	1 $_{3}$	0 $_{7}$	0 $_{5}$

$y_d = \bar{B}.(\bar{A}+\bar{C}).(A+C)$

$\therefore y = B.(A+C).(\bar{A}+\bar{C})$ (Ans)

② If a boolean function is given by the following k-map, find the POS expression.

Solution: Given k-Map,

2nd $(\bar{A}+B+\bar{C})$, 4th $(A+\bar{B}+\bar{C})$, 3rd $(A+B+C)$, 1st $(\bar{A}+\bar{B}+C)$

C \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$ 0	1	0	1	0
C 1	0	1	0	1

Here, $y_d = (\bar{A}+\bar{B}+C).(\bar{A}+B+\bar{C}).(A+B+C)(A+\bar{B}+\bar{C})$

$\therefore y = (A+B+\bar{C}).(A+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})(\bar{A}+B+C)$

5 Variable k-Map:-

① $f(A,B,C,D,E) = \Sigma m\,(0,1,6,7,8,9,21,22,23,29,31)$

Total cell $= 2^n = 2^5 = 32$

A = 0

DE \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{D}\bar{E}$ 00	1 (0)	0 (4)	0 (12)	1 (8)
$\bar{D}E$ 01	1 (1)	0 (5)	0 (13)	1 (9)
DE 11	0 (3)	1 (7)	0 (15)	0 (11)
$D\bar{E}$ 10	0 (2)	1 (6)	0 (14)	0 (10)

2nd

2nd $(\bar{A}\bar{C}\bar{D})$

A = 1

DE \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{D}\bar{E}$ 00	0 (16)	0 (20)	0 (28)	0 (24)
$\bar{D}E$ 01	0 (17)	1 (21)	1 (29)	0 (25)
DE 11	0 (19)	1 (23)	1 (31)	0 (27)
$D\bar{E}$ 10	0 (18)	1 (22)	0 (30)	0 (26)

1st grp $(\bar{B}CD)$

3rd (ACE)

$y = \bar{B}CD + ACE + \bar{A}\bar{C}\bar{D}$

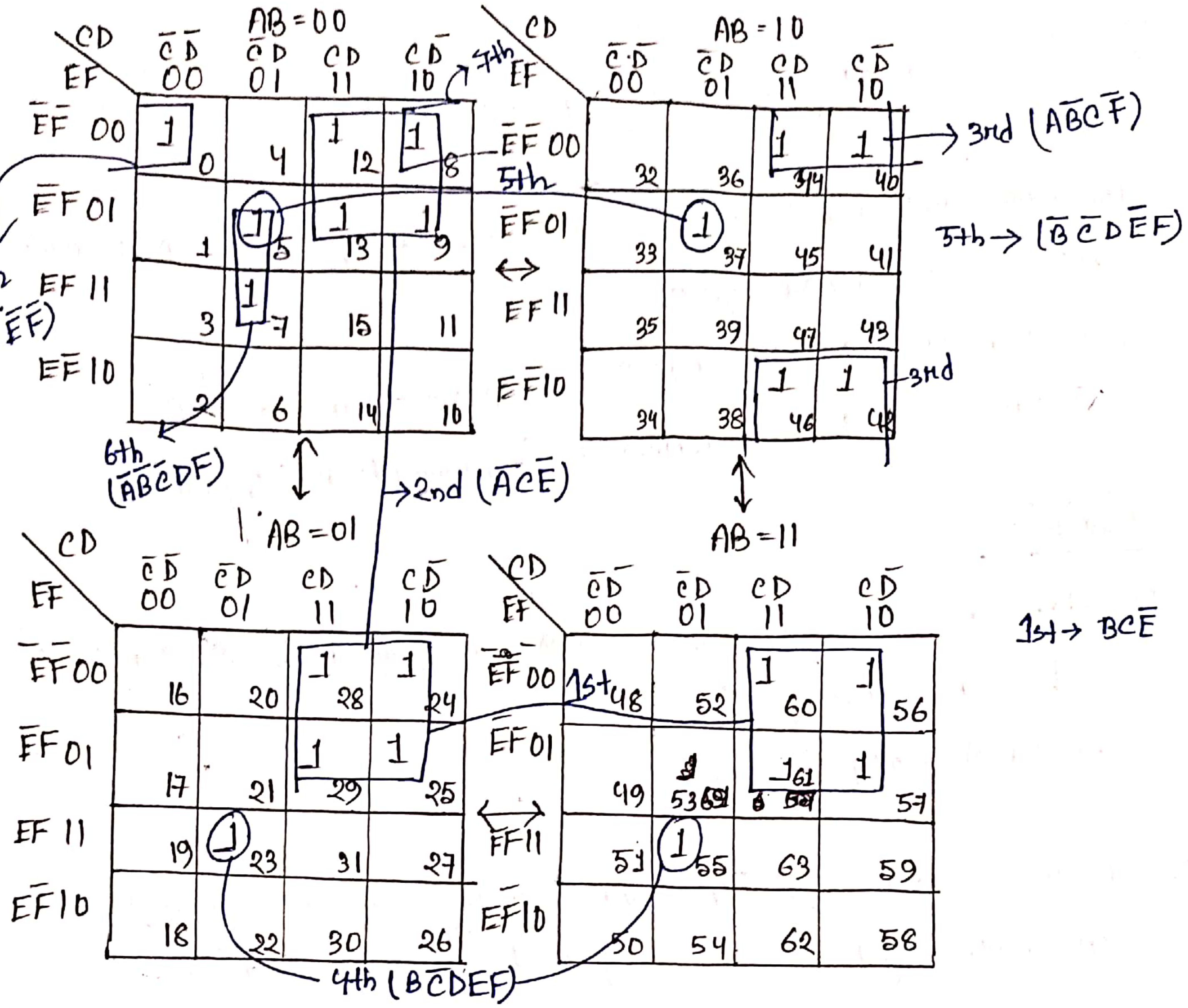
(Ans)

6 Variable k-Map:

① Solve the following 6 variable function by boolean expression.

$f(A,B,C,D,E,F) = \Sigma m\,(0,5,7,8,9,12,13,23,24,25,28,29,37,37,40,42,44,46,55,56,57,60,61)$

Solution: Total cells $= 2^n = 2^6 = 64$



$$y = BC\bar{E} + B\bar{C}DEF + \bar{A}C\bar{E} + A\bar{B}C\bar{F} + \bar{B}\bar{C}D\bar{E}F + \bar{A}\bar{B}\bar{C}DF + \bar{A}\bar{B}\bar{D}\bar{E}\bar{F}$$

$$= BC\bar{E} + \bar{A}C\bar{E} + A\bar{B}C\bar{F} + B\bar{C}DEF + \bar{B}\bar{C}D\bar{E}F + \bar{A}\bar{B}\bar{C}DF + \bar{A}\bar{B}\bar{D}\bar{E}\bar{F}.$$

NAND and NOR Implementations

Minimum two input NAND gates for Multi input AND and multi input NAND gate:-

$\Rightarrow$ $2(n-1)$ [two i/p NAND to implement n i/p AND)

$\Rightarrow$ $2n-3$ [two i/p NAND to implement n i/p NAND)

Examples:-

① How many two input NAND requirement to implement 4 i/p AND gate?

$\Rightarrow$ $2(n-1) = 2(4-1)$ 6 (Ans)

② If we have 4 i/p NAND gate, then how many 2 i/p NAND gates are required to implement it?

$\Rightarrow$ $2n-3 = 2 \cdot 4 - 3$

$= 5$

(Ans)

③ Find two i/p NAND gate for given between function,

(I) $F = A.B.C.\bar{D}$

$\Rightarrow$ We need 1 NAND gate for $\bar{D}$

For 4 i/p AND gate, 2 input NAND gates

$= 2(n-1)$

$= 2(4-1)$

$= 6$

$\therefore$ Total 2 i/p NAND gate $= 6+1 = 7$

(II) $F = \overline{\bar{A}.B.\bar{C}}$

$\Rightarrow$ For $\bar{A}$ and $\bar{C}$, we need two NAND gates.
For 3 i/p NAND gate, 2 i/p NAND gates

$= 2n-3$

$= 2.3-3$

$= 3$

$\therefore$ Total 2 i/p ~~inpw~~ NAND gate $3+2=5$

(III) $F = (\bar{a}+\bar{b})(c+b)$

$\Rightarrow$ Given that,

$F = (\bar{a}+\bar{b})(c+b)$

$= \overline{a.b}(c+b)$ [using De Morgans theorem]

$= A(c+b)$ [Let, $A = \overline{a.b}$] [1 NAND for A]

Or, $F = \overline{\overline{A.c + A.b}}$

~~Ac~~ $= \overline{\overline{A.c}.\overline{A.b}}$

$\therefore$ Required NAND gates 4.

(IV) $F = A + A\bar{B} + A\bar{B}C$

$\Rightarrow F = A(1+\bar{B}+\bar{B}C)$

$= A.1$ [$1.A = 1$]

$= A$

$\therefore$ Required NAND gates $= 0$.

(Ans)

(v) $F = AB + BC + CA$

$$= \overline{\overline{AB + BC + CA}}$$

$$= \overline{\overline{AB} \cdot \overline{BC} \cdot \overline{CA}}$$

For, $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$, we need three two i/p, NAND gates.

For three terminal NAND, 2-terminal NAND gates

$= 2n - 3$

$= 2 \cdot 3 - 3$

$= 3$

$\therefore$ Total 2 i/p NAND gate $= 3 + 3$

$= 6$

(Ans)

Boolean expression to NAND gate implementation:

Steps: ① Implement given expression in terms of basic gates by AOI [AND, OR and Inverter]

② Apply bubble to o/p of AND gate and to i/p of OR gate.

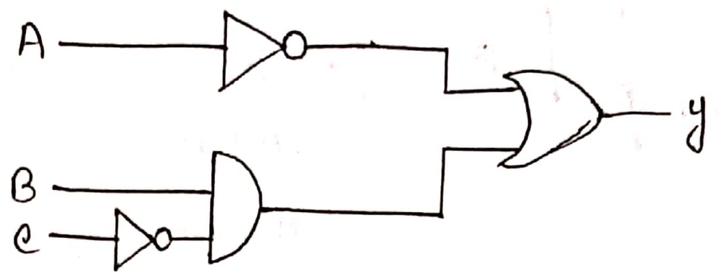
③ Apply NOT gate at places where bubbles have been inserted.

④ Look out for double inversions and cancel extra NOT gates.

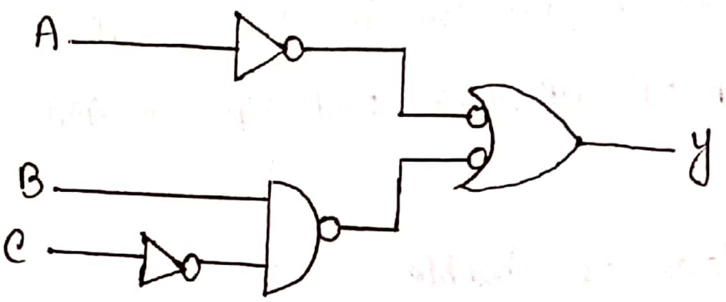
⑤ Place NAND equivalent.

Example: ① $y = \overline{A} + B\overline{C}$

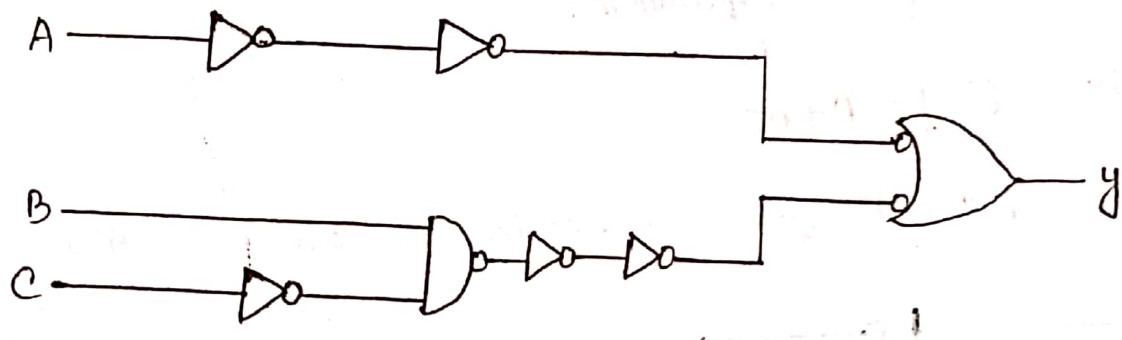
Step-1: Implement given expression by AOI



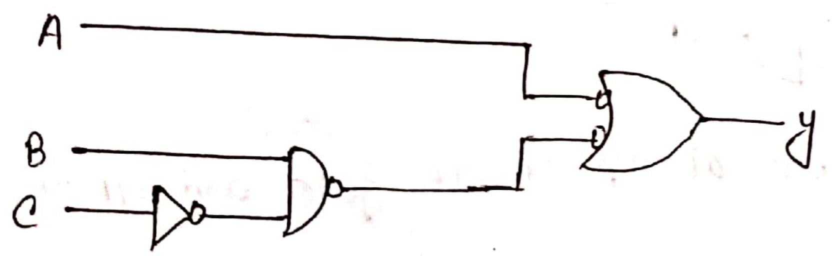
Step-2: Apply bubble to o/p of AND and i/p of OR



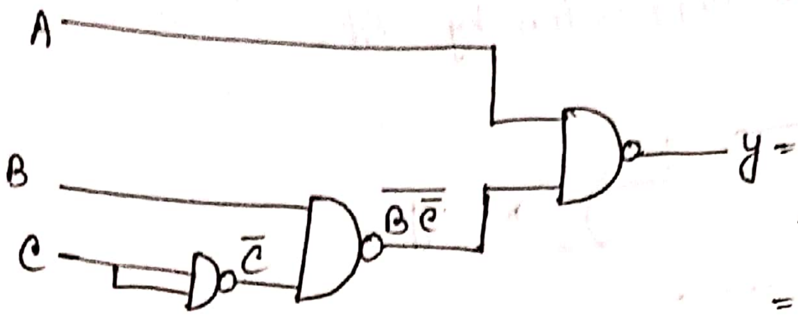
Step-3: Apply NOT gates at places of bubbles



Step-4: Look out the double inversions and cancel extra NOT gates



Step-5: Place NAND equivalent



$y = \overline{A \cdot \overline{B\bar{C}}}$

$= \bar{A} + \overline{\overline{B\bar{C}}}$

$= \bar{A} + B\bar{C}$

Boolean expression to NOR gate Implementation:

Steps: ① Implement given expression in terms of AOI.

② Apply bubble at O/P of OR gate and I/P of AND gate.

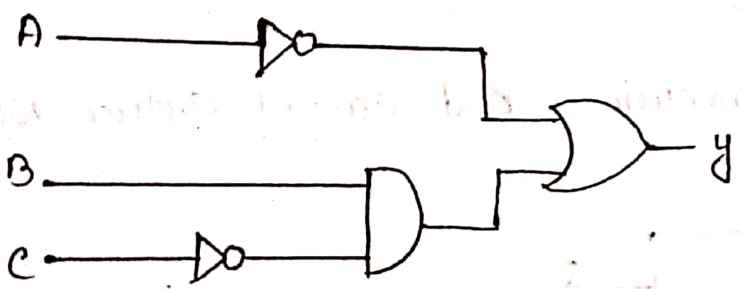
③ Apply NOT gate in place of bubble.

④ Cancel NOT gates connected in series.

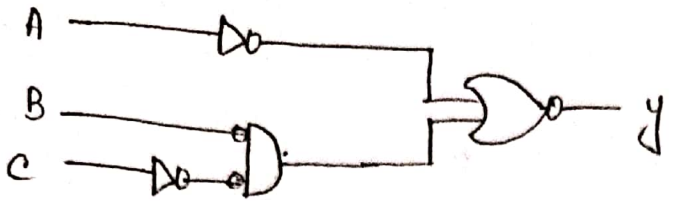
⑤ Place NOR gate equivalent.

Example:- ① $y = \bar{A} + B\bar{C}$

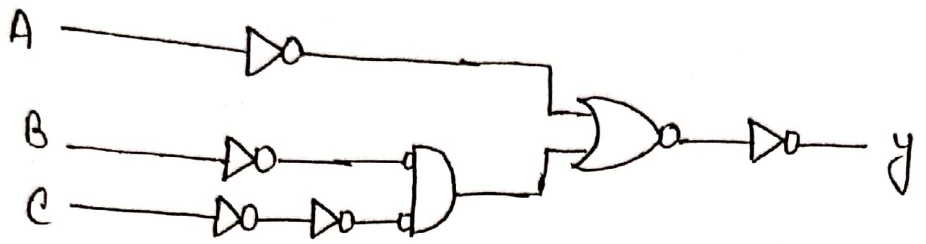
Step 1:- Implement given expression in terms of AOI.



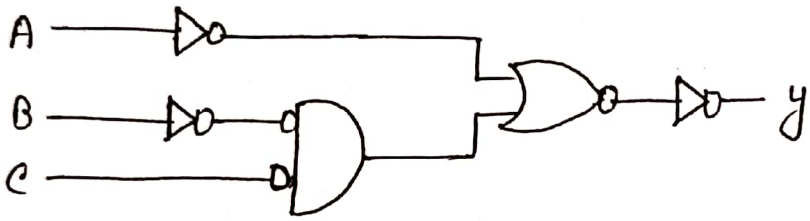
Step-2:- Apply bubble at O/P of OR gate and I/P of AND gate.



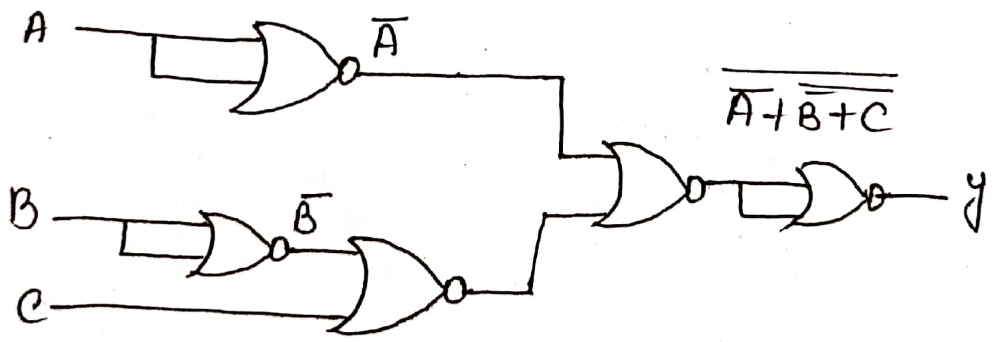
Step-3: Apply NOT gate in place of bubble:



Step-4: Cancel NOT gates connected in series.



Step-5: Place NOR equivalent



$$y = \overline{\overline{\bar{A}+\overline{\bar{B}+C}}}$$
$$= \bar{A}+\overline{\bar{B}+C}$$
$$= \bar{A}+\bar{\bar{B}}\cdot\bar{C}$$
$$= \bar{A}+\overline{BC}$$