Magnetic field

(Zahid sir)

① Definition of magnetic field, Magnetic Induction, Magnetic flux, lorentz force.

② Magnetic Induction at a point due to a straight conduction carrying current.

③ Magnetic Induction at a point due on the circular coil carrying current.

④ Faradays laws of electromagnetic Induction.

⑤ Definition of self Induction and self Induction a long solenoid wire (Proof)

⑥ Definition of Mutual Induction and Mutual Induction between two coaxial solenoids. (Proof)

Magnetic field: The space around the current carrying conductor is defined as the site of a magnetic field. It is a vector field whose magnitude and direction at any point specified by a vector it called magnetic induction.

Magnetic induction: It is the process of generating electric current with a magnetic field.

Magnetic flux: The magnetic flux ϕ through a surface S is defined as

$$\phi = \int_S B.dS$$

It represents the lines of induction crossing surface S. SI unit of magnetic flux is a weber (wb).
If B is uniform and normal to area A

$$\phi = BA$$

If $A = 1m^2$, $\phi = B$
i.e, magnetic induction is numerically to normal flux per unit area. It is also called magnetic flux density.

Lorentz force: ~~suppose~~ a The force exerted on a charged particl q moving with velocity v through an electric field E and magnetic field B. The entire electromagnetic force F on the charged Particle is called the lorentz force.

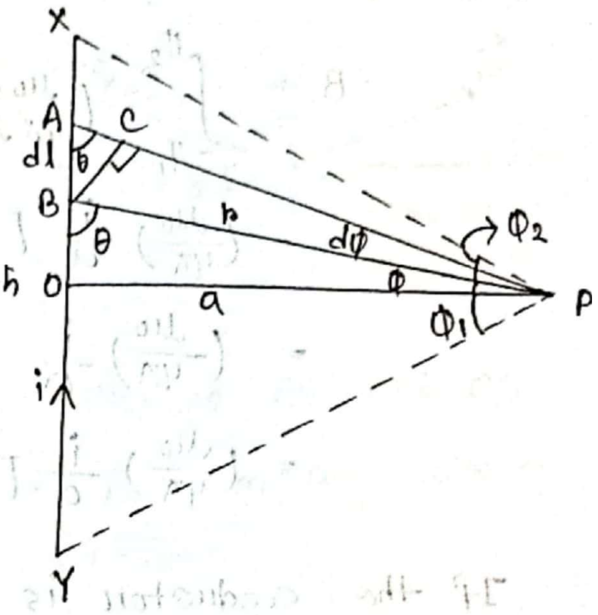
Suppose a charge q moves with a velocity v through a region where both electric field E and magnetic field B are present. Then the resultant force F on the moving charge is

$$F = qE + q(v \times B)$$
$$= q(E + v \times B)$$

This equation is called the lorentz force equation

Magnetic Induction at a point due to a Straight Conduction carrying current:

Let us consider a straight conductor carrying a current i in the direction Y to X. P is a perpendicular distance a from the conductor.



Consider an element AB of length dl. Let $BP = r$ and $\angle OBP = \theta$

Magnetic induction at P due to the element AB

$$= dB$$

$$= \left(\frac{\mu_0}{4\pi}\right) \frac{i\,dl\sin\theta}{r^2} \quad \text{——} (1)$$

From B, draw BC perpendicular to PA.

Let $\angle OPB = \phi$, $\angle BPA = d\phi$

Then $BC = dl\sin\theta = r\,d\phi$

We can write, $dB = \left(\frac{\mu_0}{4\pi}\right) \frac{i r\,d\phi}{r^2}$

$$= \left(\frac{\mu_0}{4\pi}\right) \frac{i\,d\phi}{r}$$

In ΔOPB, $\cos\phi = \frac{a}{r}$

$$\Rightarrow r = \frac{a}{\cos\phi}$$

$$\therefore dB = \left(\frac{\mu_0}{4\pi}\right) \frac{i\cos\phi\,d\phi}{a} \quad \text{——} (2)$$

The direction of dB will be perpendicular to the plane containing dl and r.

Let, ϕ_1 and ϕ_2 be the angles made by the ends of the wire at P. Then magnetic induction at P due to the whole conductor is,

$$B = \int_{-\phi_1}^{\phi_2} \left(\frac{\mu_0}{4\pi}\right) \frac{i\cos\phi \, d\phi}{a}$$

$$= \left(\frac{\mu_0}{4\pi}\right) \frac{i}{a} \left[\sin\phi\right]_{-\phi_1}^{\phi_2}$$

$$= \left(\frac{\mu_0}{4\pi}\right) \frac{i}{a} \left[\sin\phi_2 - \sin(-\phi_1)\right]$$

$$= \left(\frac{\mu_0}{4\pi}\right) \frac{i}{a} \left[\sin\phi_2 + \sin\phi_1\right] \quad \text{—— (3)}$$

If the conductor is infinitely long $\phi_1 = \phi_2 = 90°$

$$\therefore B = \left(\frac{\mu_0}{4\pi}\right) \frac{i}{a} [1+1]$$

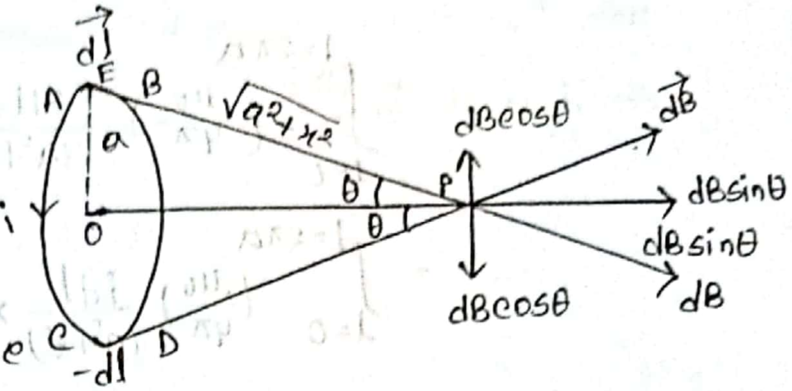
$$\Rightarrow B = \frac{\mu_0 i}{2\pi a}$$

Magnitude of B depends on i and a

$$\therefore B \propto \frac{1}{a}$$

Magnetic Induction at a point due on the circular coil carrying current:

Let us consider a circular coil of radius a, carrying current i. P is a point on its axis at a distance x from the centre O. Consider two opposite current elements AB and CD each of length dl. The distance of P from any point on the circumference of the coil is $\sqrt{a^2+x^2}$.



The field at P due to AB $= dB = \left(\frac{\mu_0}{4\pi}\right)\frac{i\,dl\sin 90^\circ}{(a^2+x^2)}$

$$= \left(\frac{\mu_0}{4\pi}\right)\frac{i\,dl}{(a^2+x^2)}$$

(∵ The direction of the current is at right angles to the line joining P to AB)

This is in the direction PL, perpendicular to the line joining the midpoint of AB with P. Considering the element CD, the magnitude of dB at P due to this element is the same as that given in equation ①

$$\frac{\mu_0}{4\pi}\,\frac{i\,dl\sin\theta}{r^2}.$$

But it is directed along PM.

E is the midpoint of AB.

Let $\angle EPO = \theta$.

$$B = \int_{l=0}^{l=2\pi a} dB \sin\theta$$

$$= \int_{l=0}^{l=2\pi a} \left(\frac{\mu_0}{4\pi}\right) \frac{i\,dl \sin\theta}{(a^2+x^2)}$$

$$= \int_{l=0}^{l=2\pi a} \left(\frac{\mu_0}{4\pi}\right) \frac{i\,dl}{(a^2+x^2)} \times \frac{a}{(a^2+x^2)^{1/2}} \quad \left[\because \sin\theta = \frac{a}{(a^2+x^2)^{1/2}}\right]$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{ia}{(a^2+x^2)^{3/2}} \times 2\pi a$$

$$\therefore B = \frac{\mu_0 i a^2}{2(a^2+x^2)^{3/2}}$$

This is the required magnetic field due to a circular current coil loop.

If the coil has N turns, then

$$B = \frac{\mu_0 N i a^2}{2(a^2+x^2)^{3/2}}$$

At the centre of the coil, $x = 0$

$$B = \frac{\mu_0 N i}{2a}$$

when, $x \gg a$, $$B = \frac{\mu_0 N i a^2}{2x^3}$$

$$\therefore \oint \vec{B}\, d\vec{l} = \frac{\mu_0 i}{2\pi r}(2\pi r)$$

$$= \mu_0 i$$

Thus the integral $\oint \vec{B} \cdot d\vec{l}$ is ~~the~~ μ_0 times current thought the area bounded by the circle. This is Ampere's law.

Faradays law of electromagnetic induction:

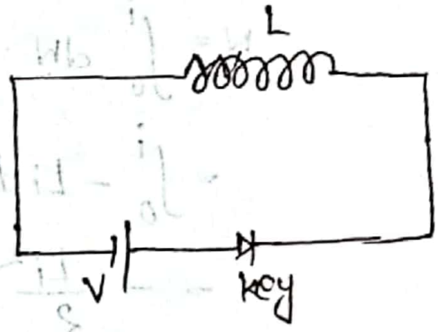
Faradays law state that the electromagnetic force E induced in a circuit equals the negative of the rate of charge of magnetic flux ϕB through the circuit.

$$E = -\frac{d\phi B}{dt}$$

For N number of turns,

$$E = -\frac{N d\phi B}{dt}$$

Self inductance: If N be the Number of lines of force linked with the coild at the same instant it is Proportional to the current 'i' of the circuit,



i.e, $N \alpha i$

$$N = Li$$

where L is the self inductance of the coil. The self inductance of a coil is equate to the number of lines of force linked through it where a current i flows through it.

The work done is equal to,

$$dW = E\,i\,dt$$

$$= \frac{-d\phi_B}{dt}\,i\,dt$$

$$= \frac{-dN}{dt}\,i\,dt$$

$$= \frac{-d}{dt}(Li)\,i\,dt$$

$$= -L\frac{di}{dt}\,i\,dt$$

$$= Li\,di$$

The total work done when the current varies from 0 to i is,

$$W = \int_0^i dW$$

$$= \int_0^i -Li\,di$$

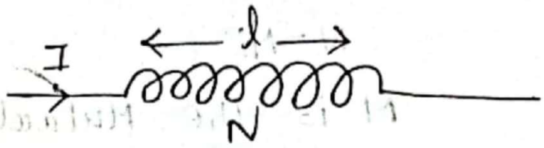
$$= -\frac{Li^2}{2}$$

If $i = 1$, $2W = L$

The inductance of coil is twice the amount of work done in establishing a unit current in the circuit.

Self induction due to a long ~~to~~ solenoid wire:

Considering a long air cored soleniod of length l, total number of turns N and ~~other~~ area of cross section A. The number of turns per unit length is $\frac{N}{l}$.



A current I flows through the solenoid and the magnetic field inside the solenoid is

$$\vec{B} = \mu_0 \left(\frac{N}{l}\right) I$$

The flux through each turn is $\phi_B = BA = \frac{\mu_0 N I A}{l}$

Total flux through N turns, $\phi = \phi_B N = \frac{\mu_0 N^2 I A}{l}$

Self Inductance of solenoid is,

$$L = \frac{\phi}{I} = \frac{\mu_0 N^2 A}{l}$$

If the ~~ind~~ solenoid is wound on a core of constant permeability μ, then

$$L = \frac{\mu_0 N^2 A}{l} \quad \text{where } \mu = \mu_0 \mu_r$$

[μ_r = Relative permeability]

Mutual Inductance: If two coil A and B are connected side by side or one round over the other. The current in coil A produces magnetic lines of force and linked with the coil B.

Then $N \propto i$

$N = Mi$

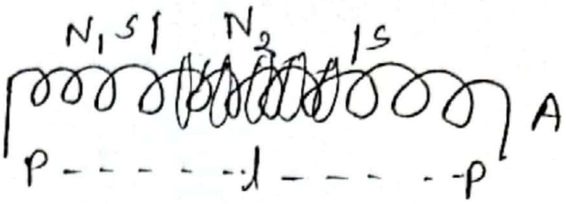
M is the Mutual inductance.

Definition of Mutual inductance: The mutual inductance is between two coils is the flux linked with the one when the unit current flows through the other.

Mutual Inductance between two coaxial solenoid :

Considering a long air-cored solenoid with primary -PP and secondary -SS

Let, N_1 = number of turns in the primary.



N_2 = number of turns in the secondary

A = Area of cross section

l = length of the primary

I = current in the primary

Magnetic field at any point inside the primary,

$$B = \frac{\mu_0 N I}{l}$$

Magnetic flux through each turn of the primary is

$$BA = \frac{\mu_0 N_1 I A}{l}$$

Since the secondary is wound closely over the central portion of the primary, the same flux is also linked with each turn of the secondary.

Magnetic flux through each ~~other~~ turn of the secondary, $= \dfrac{\mu_0 N_1 I A}{l}$

Total magnetic flux through N_2 turns of the secondary, $\phi = \dfrac{\mu_0 N_1 I A N_2}{l}$

Example-1 A solenoid having an air core and 10 cm long has 100 turns and its area of cross section is 5 sq.cm Find the co-efficient of self-inductance of the solenoid.

Solution: Here, $l = 10\,cm = 0.1\,m$

$N = 100$

$A = 5\,sq.cm = 5\times10^{-4}\,m^2$

$$\therefore L = \frac{\mu_0 N^2 A}{l}$$

$$= \frac{(4\pi\times10^{-7})\times(100\times100)\times(5\times10^{-4})}{0.1}$$

$= 62.8\times10^{-6}$ Henry

(Ans)

Example-2 Calculate the self inductance of a solenoid having 1000 turns and length 1 m. The area of cross-section is 7 cm^2 and the relative permeability of the core is 1000.

Solution: Here, $l = 1\,m$,

$N = 1000$

$A = 7\times10^{-4}\,m^2$

$\mu_r = 1000$

$$L = \frac{\mu_r \mu_0 N^2 A}{l}$$

$$= \frac{1000\times(4\pi\times10^{-7})\times(1000)^2\times(7\times10^{-4})}{1}$$

$= 0.88$ henry (Ans)

Example-3 A solenoid having a core of cross-section 4 cm² half air and half iron (relative permeability 500) is 22 cm long. If the number of turns on is 1000, what will be its self inductance?

Solution: Here, $N = 1000$

$l = 0.22 m$

$A_1 = A_2 = 2 \times 10^{-4} m^2$

$\mu_r = 500$

$$L = \frac{\mu_0 N^2 A_1}{l} + \frac{\mu_r \mu_0 N^2 A_2}{l}$$

$$= \frac{(4\pi \times 10^{-7}) \times (1000)^2 \times (2 \times 10^{-4})}{0.22} + \frac{500 (4\pi \times 10^{-7})(1000)^2 \times (2 \times 1}{0.22}$$

$= 0.57$ henry

(Ans)

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Example-1 A solenoid of length 30 cm and area of cross section 10 sq cm has 1000 turns wound over a core of constant permeability 600. Another coil of 500 turns is wound over the same coil at its middle. Calculate the mutual inductance between the

Solution: Here, $N_1 = 1000$

$N_2 = 500$

$\mu_r = 600$

$A = 10 \times 10^{-4}\ m^2$

$l = 30 \times 10^{-2}\ m$

$$M = \frac{\mu_0 \mu_r N_1 N_2 A}{l}$$

$$= \frac{(4\pi \times 10^{-7}) \times 600 \times 1000 \times 500 \times 10 \times 10^{-4}}{30 \times 10^{-2}}$$

$$= 1.257 \text{ henry}$$

(Ans)

Example: 2 Two coils, a primary of 600 turns and a secondary of 30 turns are wound on an iron ring of mean radius 0.1m and cross-section 4×10^{-2} m diameter. Find their mutual inductance. (μ_r for iron = 800)

Solution: $l = 2\pi \times 0.1$

$= 0.2\pi\ m$

$N_1 = 600$

$N_2 = 30$

$A = \pi\,(2 \times 10^{-2})^2$

$= 4\pi \times 10^{-4}\ m^2$

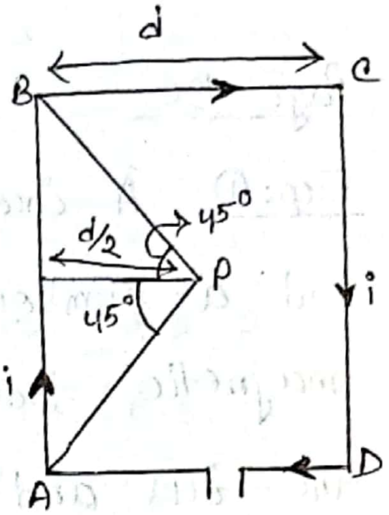
$$M = \frac{\mu_r \mu_0 N_1 N_2 A}{l} = \frac{800 \times (4\pi \times 10^{-7}) \times 600 \times 30 \times (4\pi \times 10^{-4})}{0.2\pi}$$

$$= 3.45 \times 10^{-2} \text{ henry}$$ (Ans)

P-134

Exp-1 A square coil of side "d" carries a current "i". calculate the magnetic induction at the centre of the coil.

Solution: ABCD is a square coil carrying a current i



Magnetic induction at the centre due to AB (one side) is,

$$B' = \left(\frac{\mu_0}{4\pi}\right)\frac{i}{a}\left[\sin\phi_2 + \sin\phi_1\right]$$

$$= \left(\frac{\mu_0}{4\pi}\right)\frac{i}{d/2}\left(\sin 45^\circ + \sin 45^\circ\right)$$

$$= \left(\frac{\mu_0}{4\pi}\right)\frac{2i}{d} \times \frac{2}{\sqrt{2}}$$

$$= \frac{\mu_0 i}{\sqrt{2}\pi d}$$

The magnetic induction due to current in the remaining three arms of the square loop will also have the same magnitude and direction. Thus total magnetic induction at the centre due to the square coil is

$$B = 4B' = \frac{2\sqrt{2}\mu_0 i}{\pi d}.$$

② Find the magnetic induction at the centre of a square current loop of side 1 metre carrying a current of 1 ampere.

Solution:

$$B = \frac{2\sqrt{2}\mu_0 i}{\pi d}$$

$$= \frac{2\sqrt{2} \times (4\pi \times 10^{-7}) \times 1}{\pi \times 1}$$

$$= 8\sqrt{2} \times 10^{-7}\ wbm^{-2}$$

(Ans)

Page–136

Exp: ① A circular coil has a radius of 0.1 m and a number of turns of 50. Calculate the magnetic induction at a point (i) on the axis of the coil and distance 0.2 m from the centre; (ii) at the centre of the coil, when a current of 0.1 A flows in it.

Solution: Here, $a = 0.1\,m$

$N = 50$

$x = 0.2\,m$

$i = 0.1\,A$

$B = ?$

(i) $$B = \frac{\mu_0 N i a^2}{2(a^2+x^2)^{3/2}} = \frac{(4\pi \times 10^{-7}) \times 50 \times 0.1 \times (0.1)^2}{2\left[(0.1)^2 + (0.2)^2\right]^{3/2}}$$

$$= 2.81 \times 10^{-6}\ T.$$

(ii) At the centre,

$$B = \frac{\mu_0 N i}{2a}$$

$$= \frac{(4\pi \times 10^{-7}) \times 50 \times 0.1}{2 \times 0.1}$$

$$= 3.14 \times 10^{-5}\ T.$$

(Ans)

Pg-137

Exp-2. In the Bohr model of the hydrogen atom, the electron circulates around the nucleus in a path of radius 5.29×10^{-11} m at a frequency of 6.58×10^{15} Hz. Find the magnitude of the magnetic induction at the centre of the orbit. What is its dipole moment?

Solution: current = charge/time = $e\nu$

where e is the electronic charge and ν is the frequency of revolution.

$$\therefore i = e\nu = (1.602 \times 10^{-19})(6.58 \times 10^{15})$$

$$= 1.054 \times 10^{-3} A$$

Magnetic induction at the centre of the orbit is

$$B = \frac{\mu_0 i}{2a} = \frac{(4\pi \times 10^{-7}) \times (1.054 \times 10^{-3})}{2 \times (5.29 \times 10^{-11})}$$

$$= 12.52\ T$$

Let, $A = \pi a^2$ be the area of the current loop.

Then, $M = Ai$ is called the magnetic dipole moment of the current loop.

$$M = Ai = \pi a^2 i$$
$$= \pi (5.29 \times 10^{-11})^2 \times (1.054 \times 10^{-3})$$
$$= 9.266 \times 10^{-24}\ Am^2.$$

(Ans)

Pg-140

Exp-1 : A solenoid of 1200 turns is wound uniformly in a single layer on a glass tube 2m long and 0.2 m in diameter. Find the magnetic induction at the centre of the solenoid, when a current of 2A flows through it.

Solution: The length of the solenoid is ten times the diameter so that the formula for a long solenoid can be used. Here,

$N = 1200,$

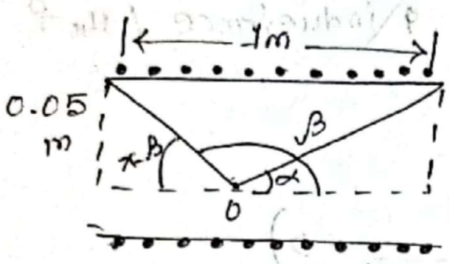
$L = 2m,$

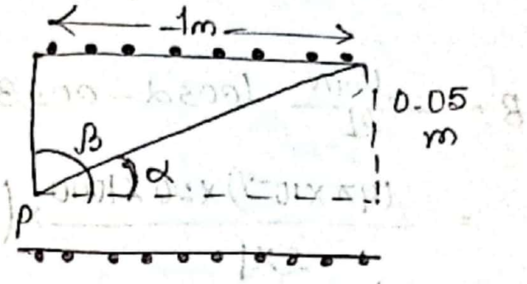
$i = 2A$

$\mu_0 = 4\pi \times 10^{-7}\ wb A^{-1} m^{-1}$

$$\therefore B = \frac{\mu_0 i N}{l} = \frac{(4\pi \times 10^{-7}) \times 2 \times 1200}{2}$$
$$= 1.51 \times 10^{-3}\ wb m^{-2}$$ (Ans)

Exp-2 A long solenoid of length 1m and mean diameter 0.1m consists of 100 turns of wire. A current of 20A flows through it. calculate the magnetic induction on its axis (i) at its centre, (ii) at one of its ends.





Solution: (i) At the centre O,

$$\cos\alpha = \frac{0.5}{\sqrt{(0.5)^2+(0.05)^2}} = \frac{0.5}{\sqrt{.2525}}$$

$$B = \frac{\mu_0 i N}{2L}(\cos\alpha - \cos\beta)$$

$$= \frac{(4\pi\times10^{-7})\times20\times1000}{2\times1}\left[\frac{0.5}{\sqrt{(0.2525)}} - \left(-\frac{0.5}{\sqrt{(0.2525)}}\right)\right]$$

$$= 2.501\times10^{-2}\ \text{wbm}^{-2}$$

$i = 20A$
$N = 1000$
$L = 1m$
$B = ?$

(ii) At and end P.

$$\cos\alpha = \frac{1}{\sqrt{[(1)^2+(.05)^2]}}$$

$$= \frac{1}{\sqrt{1.0025}}$$

$\cdot \cos\beta$

$$= \cos\pi/2$$

$$= 0$$

$$B = \frac{\mu_0 i N}{2L}(\cos\alpha - \cos\beta)$$

$$= \frac{(4\pi \times 10^{-7}) \times 20 \times 1000}{2 \times 1} \times \left(\frac{1}{\sqrt{(1.0025)}} - 0 \right)$$

$$= 1.255 \times 10^{-2} \ wbm^{-2}.$$

(Ans)