Probability Distribution

① Binomial distribution (discrete distribution)
② Poisson distribution (discrete)
③ Normal distribution (continuous)

Binomial distribution: A discrete random variable x is said to have binomial distribution if its probability function is as follows:

$$f(x) = {}^{n}c_{x}\, p^{x} q^{n-x}; \quad x = 0, 1, 2, \ldots, n$$

where n and p are the parameters of the distribution and $p+q=1$

[parameters means unknown value]

Here, n = number of trial
x = number of success
p = probability of success $= \frac{x}{n}$

Assumption of binomial distribution: Four properties of a Binomial.

① The sample consists of a fixed number of observation.

② Each observation is classified into one of two mutually exclusive and collectively exhaustive categories, called success and failure.

③ Probability of an observation being classified as a success p, or a failure 1-p, is constant over all observation.

④ The outcome of any observation is independent of the outcome of any other observation.

Poison distribution: A discrete random variable x is said to have poisson distribution if its probability is as follows:

$$f(x) = \frac{e^{-m} m^x}{x!} \; ; \; x = 0, 1, 2, \dots \infty$$

where m is the parameter of the distribution and

$e = 2.718$

Practical ~~physical~~ application of poisson distribution:

① The number of suicides reported in a particular day.

② The number of printing mistakes per page of a book.

③ The number of faculty blades in a packet of 100.

④ The number of letters lost in a mail per day.

Normal Distribution: A continuous random variable is said to have normal distribution if its Probability density function is as follows:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} ; \quad -\infty \le x \le \infty$$

$$-\infty \le \mu \le \infty$$

$$\sigma > 0$$

where, μ and σ^2 are the parameter of the distribution.

Importance of normal distribution.

① In practice under certain condition most of the probability and sampling distributions can be approximated by normal distribution.

② According to central limit theorem, if mean and variance of distribution exist then the distribution converted to normal distribution.

③ Normal distribution is the basis of all ~~the test~~ sampling distribution.

④ Assumption of normality is the basis of all the test of significance in applied statistics.

⑤ Normal distributions find its application in industrial statistics such as quality control.

Problem: A fair coin is tossed 5 times. Find the probability that,

① Exactly two head

② No head

③ At least 3 heads

④ At ~~least~~ most 2 heads

Solution: Here, $n = 5$

$p = q = \frac{1}{2}$ (since the coin is fair)

We know, $f(x) = {}^{n}c_{x}\, p^{x} q^{n-x}, \quad x = 0, 1, 2 \ldots, n$

$f(x) = {}^{5}c_{x} \left(\frac{1}{2}\right)^{x} \left(\frac{1}{2}\right)^{5-x}; \quad x = 0, 1, 2, 3, \ldots, n$

Let, x be the number of head.

① Here, $x = 2$

$\therefore f(2) = {}^{5}c_{2} \left(\frac{1}{2}\right)^{½} \left(\frac{1}{2}\right)^{5-2}$

$= {}^{5}c_{2} \left(\frac{1}{2}\right)^{½} \left(\frac{1}{2}\right)^{3}$

$= \frac{5}{54}$

$= 0.0926$.

② $x = 0$,

$$\therefore f(0) = {}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{5-0}$$

$$= {}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5$$

$$= 1 \cdot 1 \cdot \frac{1}{32}$$

$$= \frac{1}{32}$$ (Ans)

③ At least 3 heads.

$$\therefore x \geq 3$$

$$\therefore f(x \geq 3) = f(x=3) + f(x=4) + f(x=5)$$

$$= f(3) + f(4) + f(5)$$

$$= {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3} + {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{5-1} + {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{5-0}$$

$$= 10 \cdot \frac{1}{8} \cdot \frac{1}{4} + 5 \cdot \frac{1}{16} \cdot \frac{1}{2} + 1 \cdot \frac{1}{32} \cdot 1$$

$$= \frac{5}{16} + \frac{5}{32} + \frac{1}{32}$$

$$= \frac{16}{32}$$

$$= \frac{1}{2}$$ (Ans)

④ A most 2 heads.

$x \leq 2$

$\therefore f(x \leq 2) = f(0) + f(1) + f(2)$

$= {}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5 + {}^5C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4 + {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3$

$= 1 \cdot 1 \cdot \frac{1}{32} + 5 \cdot \frac{1}{2} \cdot \frac{1}{16} + 10 \cdot \frac{1}{4} \cdot \frac{1}{8}$

$= \frac{1}{32} + \frac{5}{32} + \frac{5}{32}$

$= \frac{11}{32}$

(Ans)