Current and resistance

(Zahid sir)

1. Current
2. current density
3. Ohm's law
4. potential difference

} Definition

5. RC circuit

⇒ Growth of charge

⇒ Decay of charge

} Prove

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EMF included in a coil rotating in a magnetic field

Current: It is defined as the net charge flowing across the area per unit time.

Current density: The current density at any point is defined as the quantity of charge passing per second through a unit area taken perpendicular to the direction of the flow of charge at that point.

$$J = \frac{q/t}{A} = \frac{i}{A}$$

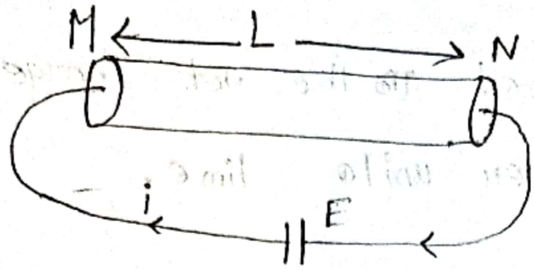
∴ The unit of current density is A/m^2.

Ohm's law: In a metalic conductor at constant temperature, the current density J is linearly proportional to the electric field E. Thus,

$$J = \sigma E \quad \text{——} (1)$$

The equation $J = \sigma E$ is general vector.

Statement of ohm's law: Here σ is a constant called the conductivity of the conductor. The unit of conductivity of the conductor. The unit of conductivity is called the resistivity ρ. In term of ρ eq. (1) can be written as, $E = \rho J$ —— (2)



In the fig. the electric field along the rod is in the direction MN and its value is.

$E = V/L$ everywhere, Here V is the total potential drop from M to N. Thus $J = \sigma (V/L)$

The total amount $i = JA = \sigma\ VA/L$

Here A is the cross-sectional area of the rod.

This leads to,

$$\frac{V}{i} = \frac{L}{\sigma A} = \frac{\rho L}{A} \quad \text{③}$$

The ratio V/i is called the resistance R of the rod

$R = \frac{\rho L}{A}$

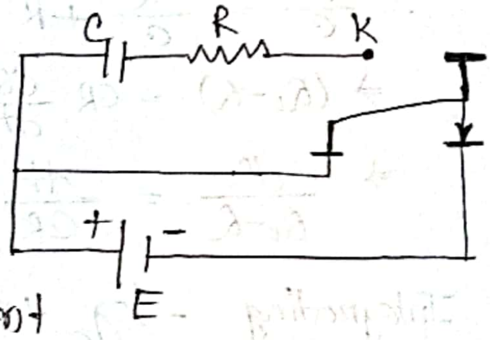
Eq ③ becomes $V = iR$

This is known as ohm's law.

Potential difference: The difference in potential between two points that represents the work involved or the energy released in the transfer of a unit quantity of electricity from one point to the other.

Charge and discharge of a capacitor through a Resistor:

(a) Growth of charge: A capacitor C and a resistance R are connected to a cell of emf E through a Morse key K. When the key is pressed, a momentary current I flows through R. At any instant t, let Q be the charge on the capacitor of capacitance C.



Potential Drop across capacitor $= Q/C$

Potential Drop across resistor $= IR$

The emf equation of the circuit is

$$E = (Q/C) + IR \quad \text{——} \ (1)$$

$$= (Q/C) + R\left(\frac{dQ}{dt}\right) \qquad \left[I = \frac{dQ}{dt}\right]$$

The capacitor continues getting charged till it attains the maximum charge Q_0. At that instant $I = \frac{dQ}{dt} = 0$ The P.D across the capacitor is,

$$E = \frac{Q_0}{C}$$

i.e when,

$$Q = Q_0, \frac{dQ}{dt} = 0$$

$$\text{and } E = \frac{Q_0}{C}$$

$$\therefore \frac{Q_0}{C} = \frac{Q}{C} + R\frac{dQ}{dt}$$

$$\Rightarrow (Q_0 - Q) = CR\frac{dQ}{dt}$$

$$\Rightarrow \frac{dQ}{Q_0 - Q} = \frac{dt}{CR} \quad \text{--- (2)}$$

Integrating $-\log_e (Q_0 - Q) = \frac{t}{CR} + k$

where k is a constant,

when, $t = 0$, $Q = 0$

$$\therefore -\log_e Q_0 = k$$

$$\therefore -\log_e (Q_0 - Q) = \frac{t}{CR} - \log_e Q_0$$

$$\Rightarrow \log_e (Q_0 - Q) = -\frac{t}{CR} + \log_e Q_0$$

$$\Rightarrow \log_e (Q_0 - Q) - \log_e Q_0 = -\frac{t}{CR}$$

$$\Rightarrow \log_e \left(\frac{Q_0 - Q}{Q_0}\right) = -\frac{t}{CR}$$

$$\Rightarrow \frac{Q_0 - Q}{Q_0} = e^{-t/CR}$$

$$\Rightarrow 1 - \frac{Q}{Q_0} = e^{-t/CR}$$

$$\Rightarrow Q = Q_0 (1 - e^{-t/CR}) \quad \text{——} (3)$$

The term CR is called time constant of the circuit.
At the end of time $t = CR$

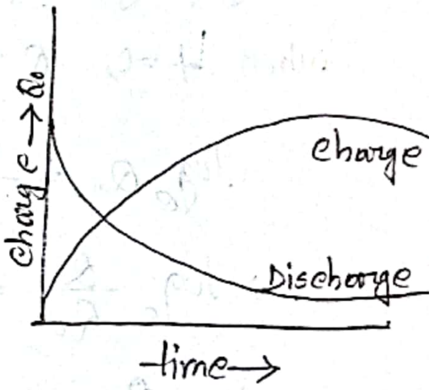
$$Q = Q_0 (1 - e^{-1}) = 0.632 Q_0$$

Thus the time constant maybe defined as the time taken by the capacitor to get charged to 0.632 times its maximum value.

The rate of growth of charge ~~of~~ is,

$$\frac{dQ}{dt} = \frac{Q_0}{CR} e^{-t/CR} = \frac{1}{CR}(Q_0 - Q)$$

Thus it is seen that smaller the Product CR, the more rapidly does the charge grow on the capacitor.



The rate of growth of the charge is rapid in the beginning and it becomes less and less as the charge approaches nearer and nearer the steady value.

Decay of charge: Let the capacitor having charge Q_0 be now discharged by ~~realisi~~ releasing the Morse key k. The charge flows out of the capacitor and this constitute a current. In this case $E=0$,

$$R\frac{dQ}{dt}+\frac{Q}{C}=0 \quad \text{——} (1)$$

or, $$\frac{dQ}{Q} = -\frac{1}{CR}dt$$

Integrating, $\log_e Q = -\frac{t}{CR}+K$

when $t=0$, $Q=Q_0$ $\therefore \log_e Q_0 = K$

$$\log_e Q = -\frac{t}{CR}+\log_e Q_0$$

$$\Rightarrow \log_e \frac{Q}{Q_0} = -\frac{t}{CR}$$

$$\Rightarrow \frac{Q}{Q_0} = e^{-t/CR}$$

$$\therefore Q = Q_0 e^{-t/CR} \quad \text{——} (2)$$

This shows that the charge in the capacitor decays exponentially and becomes zero after infinite interval of time.

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The rate of discharge is,

$$I = \frac{dQ}{dt} = -\frac{Q_0}{CR} e^{-t/CR}$$

$$= -\frac{Q}{CR} \quad \text{——— (3)}$$

Thus, smaller the time constant CR, the quicker is the discharge of the capacitor.

In eq. (2) if we put $t = CR$ then $Q = Q_0 e^{-1} = 0.368 Q_0$

Hence time constant may also be defined as the time taken by the current to fall from maximum to 0.368 of its maximum Value.

Exp: 1 A copper wire of diameter 0.5 mm and length 20 m is connected across a battery of emf 1.5 V and the internal resistance 1.25Ω. Calculate the current density in the wire and the drift velocity V_d, assuming one conduction electron per atom of copper. What is the heat dissipated per metre of the wire?

Solution: Total resistance of the wire,

$$R = \frac{\rho L}{A} = \frac{(1.7\times10^{-8})\times20}{\pi\,(0.25\times10^{-3})^2}$$

$$= 1.732\Omega$$

$$\text{Current, } i = \frac{V}{R_{total}} = \frac{1.5}{(1.732+1.25)} = 0.5A$$

$$\text{Resistance of unit length of wire} = \frac{1.7320}{20}$$

$$= 0.0866\,\Omega/m$$

$$\text{Power dissipated per metre} = i^2R$$

$$= (0.5)^2\times(0.0866)$$

$$= 0.0216\ W/m$$

The number of copper atoms per m^3 is

$$n = \frac{\text{Avogadro No}\times\text{density}}{\text{Atomic weight}}$$

$$= \frac{(6.025\times10^{26})\times(8.89\times10^3)}{63.54} = 8.43\times10^{28}$$

current density, $j = \frac{i}{A}$

$$= \frac{0.5}{\pi (0.25\times10^{-3})^2}$$

$$= 2.546\times10^{6}\ A m^{-2}$$

$$V_d = \frac{J}{ne} = \frac{2.546\times10^{6}}{(8.43\times10^{28})(1.6\times10^{-19})}$$

$$= 0.1888\times10^{-3}\ ms^{-1}$$

(Ans)

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Example-1 A capacitor is charged by DC supply through a resistance of 2 mega ohms. If it takes 0.5 seconds for the charge to reach three quarters of its final value. What is the capacitance of the capacitor?

Solution: Here, $R = 2\times10^{6}\Omega$

$t = 0.5\,s$

$Q/Q_0 = 3/4$

$$Q = Q_0(1-e^{-t/CR})$$

$$\Rightarrow \frac{Q}{Q_0} = (1-e^{-t/CR})$$

$$\Rightarrow \frac{3}{4} = \left(1-e^{-\frac{0.5}{C\times(2\times10^{6})}}\right)$$

$$\Rightarrow e^{\frac{0.5}{C\times(2\times10^{6})}} = 4$$

$$\Rightarrow \frac{0.5}{C\times(2\times10^{6})} = \log_e 4$$

$$\Rightarrow C = \frac{0.5}{(2.3026 \log_{10} 4)(2\times10^{6})}$$

$$= 0.18 \times 10^{-6}\ F$$

$$= 0.18\ \mu F$$

(Ans)

Example-2 A capacitor of capacitance 0.1 μF is first charged and then discharged through a resistance of 10 megaohm. Find the time, the potential will take to fall its original value.

Solution: $Q = Q_0\, e^{-t/CR}$

$$\Rightarrow \frac{Q}{Q_0} = e^{-t/CR}$$

$$\Rightarrow \ln\left(\frac{Q_0}{Q}\right) = \frac{t}{CR}$$

$$\Rightarrow t = CR\ln\left(\frac{Q_0}{Q}\right)$$

$\therefore$ $Q = CV$ and $Q_0 = CV_0$ $\quad \therefore \frac{Q_0}{Q} = \frac{V_0}{V}$

$$\therefore t = CR\ln\left(\frac{V_0}{V}\right)$$

Here, $C = 10^{-7} F$

$R = 10^{7}\ \Omega$

$V_0/V = 2$

$\therefore t = 10^{-7} \times 10^{7} \times \ln 2 = 0.6931$ s. (Ans)

Example-3 A resistance R and a 2μF capacitor in series are connected to a 200 Volt direct supply. Across the capacitor is a neon lamp that strikes at 120 Volts. Calculate the value of R to make the lamp strike 5 seconds after switch has been closed.

Solution: The Resistance R must be such that the P.d across the capacitor should rise to 120 Volt in 5 seconds after the switch is closed. The lamp would then strike. The equation of charging is

$$Q = Q_0(1 - e^{-t/RC})$$

$$\Rightarrow CV = CV_0(1 - e^{-t/RC})$$

$$\Rightarrow V = V_0(1 - e^{-t/RC})$$

Here, $V = 120$ Volt, $V_0 = 200$ Volt, $t = 5$ sec and

$$C = 2 \times 10^{-6} \text{ farad}$$

$$\therefore 120 = 200\left(1 - e^{-5/2 \times 10^{-6} R}\right)$$

$$\Rightarrow e^{5/2 \times 10^{-6} R} = 5/2$$

$$\Rightarrow \frac{5}{2 \times 10^{-6} R} = \log_e 5 - \log_e 2$$

$$\Rightarrow \cancel{1.6094} \quad \frac{5}{2 \times 10^{-6} R} = 1.6094 - \cancel{0.639}\, 0.6931 = 0.9163$$

$$\Rightarrow R = \frac{5}{2 \times 10^{-6} \times 0.9163} = 2.73 \times 10^{6} \text{ ohm}$$

$$= 2.73 \text{ megaohm}$$

(Ans)