Surface Tension

Cohesive Force

Cohesive force is the force of attraction between molecules of the same substance. This force is different from the ordinary gravitational force and does not obey the ordinary inverse square law. It is the greatest in the case of solids, less in the case of liquids and the least in the case of gases, almost negligible at ordinary temperature and pressure. That's why a solid has a definite shape, a liquid has a definite free surface and a gas has neither.

Adhesive Force

Adhesive force is the force of attraction between molecules of different substances, and is different for different pairs of substances. For example, gum has a greater adhesive force than water or alcohol.

Molecular Range

The maximum distance up to which the force of cohesion between two molecules can act is called their molecular range. It is generally of the order of 10^{-7} cm in the case of solids and liquids, being differents for different substances.

Sphere of Influence

A sphere drawn around a molecule as center, with a radius equal to its molecular range is called the sphere of influence of the molecule. The molecule is affected only by the molecules inside this sphere.

Surface Tension

Surface tension is a one kind of force acting per unit length of a line drawn in the liquid surface. The force acts perpendicularly to the line, tangentially along the surface of the liquid and tends to pull the surface apart along the line.

Dimensions of surface Tension

We know that, surface tension,

$$T = \frac{F}{L}$$

$$\text{So, } [T] = \left[\frac{MLT^{-2}}{L}\right] = [MT^{-2}]$$

[SI unit of surface tension is Nm^{-1}]

Surface Energy

If a plane be drawn parallel to the free surface layer and at a distance equal to the molecular range from it, the layer of the liquid, lying in-between the free surface and this plane is called the surface film.

The potential energy of a system tends towards a minimum. The potential energy per unit area of the surface film is called its surface energy.

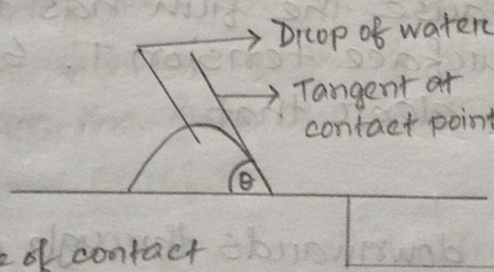
Capillarity

The action by which the surface of a liquid where it is in contact with a solid (as in a capillary tube) is elevated or depressed depending on the relative attraction of the molecules of the liquid for each other and for those of the solid.

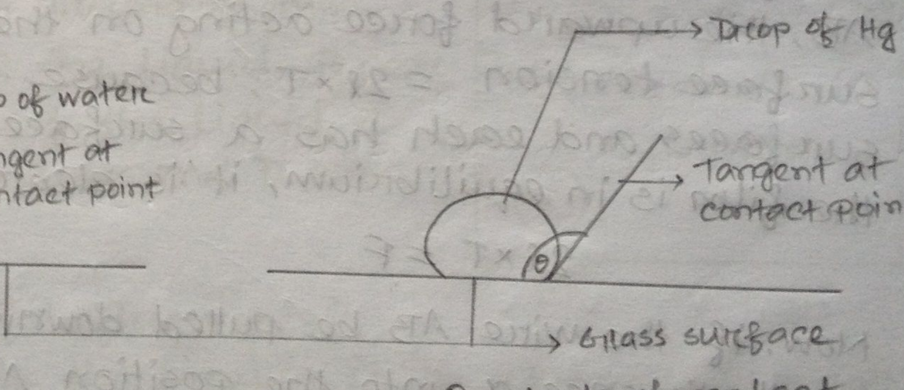
Angle of contact

When a liquid meets a solid, its surface near its plane of contact with the solid is, in general curved. The angle between the tangent to the liquid surface at the point of contact and the solid surface, inside the liquid, is called the angle of contact for that pair of solid and liquid.

The angle may have any value between 0° and 180°. For most liquids and glass, it is less than 90°, for mercury and glass, it is about 140°.



θ = Angle of contact
θ < 90°
Fig-I



θ = Angle of contact
θ > 90°
Fig-II

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For fig-I, Adhesive force is greater than cohesive force and for fig-II, cohesive force is greater than adhesive force. That is why θ is acute in fig-I and obtuse in fig-II.

Relation between Surface Tension and surface Energy

Let PQRS be a rectangular wire framework with a horizontal wire AB placed across it, free to move up and down. A soap film is formed across ABQR by dipping it in a soap solution. The wire AB is pulled upwards by the surface tension of the film, acting perpendicularly to the wire and in the plane of the film. To keep the wire in position, therefore a force has to be applied downwards,

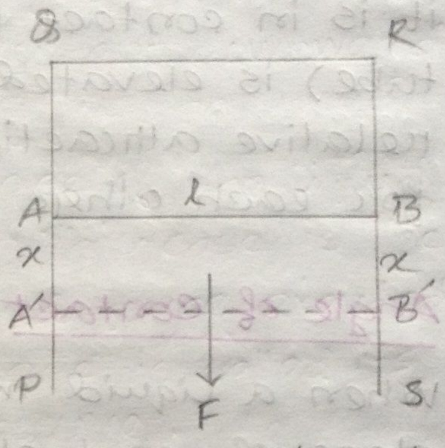

Fig: Rectangular wire framework

equal and opposite to the upward force due to surface tension. Let this downward force be equal to F including the weight of the wire AB which is also acting downwards. If T be the surface tension of the film i.e. the force per unit length and l the length of the wire AB, the upward force acting on the wire due to surface tension $= 2l \times T$, because the film has two surfaces and each has a surface tension T. Since, the film is in equilibrium, it is clear that,

$$2l \times T = F$$

Now, if the wire AB be pulled downwards through a small distance x into the position A'B', i.e. if the film be extended by an area lx on each side, we have

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Work done $= Fx = 2l \times Tx$

The film takes up heat from the atmosphere to come to its original temperature. This heat absorbed together with the mechanical work done, forms the energy of the new surface area $2lx$ of the film formed.

If E be the surface energy of the film and Q heat be absorbed per unit area of the new surface formed, we have,

$$E \times 2lx = 2lTx + Q \times 2lx$$

$$\Rightarrow E = T + Q \quad [\text{Dividing throughout by } 2lx]$$

$$\Rightarrow T = E - Q$$

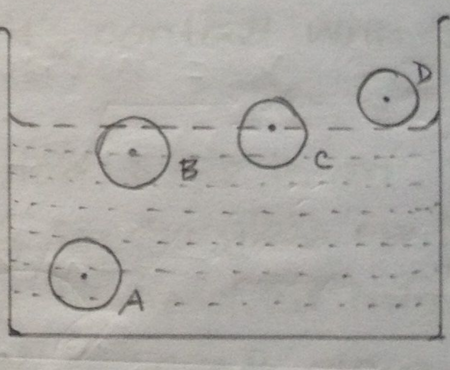
Therefore, T = surface energy − heat energy per unit area

So, surface tension = potential energy per unit area.

We can say that, the surface energy of a liquid is numerically equal to its surface tension.

Molecular Theory of Surface Tension

Let us consider four molecules A, B, C, D of a liquid with their sphere of influence drawn around them. A being well inside the liquid, B near to the free surface of the liquid, C just on the free surface and D above the free surface.



Since the sphere of influence of A lies wholly inside the liquid, it is attracted equally in all directions by the other molecules lying within its sphere of influence. So, there is no resultant cohesive force on it one way or the other, and it, therefore, possesses its thermal velocity.

The sphere of influence of molecule B lies partly outside the liquid and this part contains only a comparatively few molecules of the gas or vapour above the liquid. So, there is a resultant downward force acting on B.

The molecule C lies on the surface of the liquid. The resultant downward force in this case is the maximum. This downward or inward force exerted per unit area of a liquid surface is called its internal, intrinsic or cohession pressure.

In the case of molecule D, which has passed out of the liquid surface, there is no downward forces on the molecule at all and it is free to wander about as a molecule of the vapour or gas.

⊞ Why does a liquid rise in a capillary tube?

A liquid rises in a capillary tube because of its surface tension.

⊞ Derive an expression for surface tension using capillary tube method.

or, Show that the height to which a liquid rises in a capillary tube of radius r is given by $h = \frac{2T}{\rho g r} - \frac{r}{3}$.

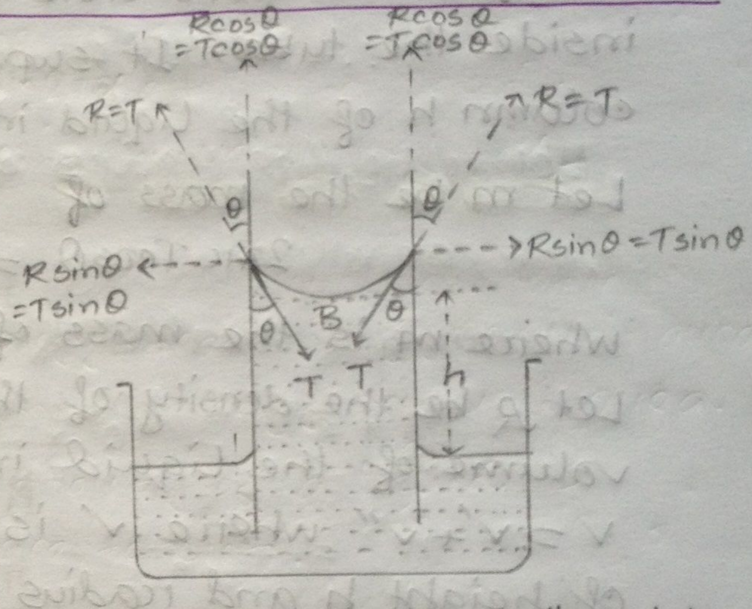


Fig : Water in capillary tube

Let r be the radius of the capillary tube at B, the point up to which the liquid rises into it. Then, it will be practically the same as the radius of the concave meniscus.

Let θ be the angle of contact between the liquid and the glass. The surface tension T of the liquid acts inwards along the tangent to the liquid meniscus at every point of its contact with the inner surface of the tube.

According to Newton's third law of motion, there is an equal and opposite reaction R exerted by the glass on the liquid. Thus $R = T$. This reaction R may be resolved into two rectangular components, (i) $R\cos\theta = T\cos\theta$, along the vertical and in the upward direction, (ii) $R\sin\theta = T\sin\theta$, at right angles to it, in the outward direction as shown in figure. Taking the whole meniscus into consideration, the horizontal or outward components all cancel each other out, and only the vertical components are effective, which are thus added up.

the meniscus touches along a length $2\pi r$, the circumference of the circle of radius r. So, total upward force $= 2\pi r \cdot T\cos\theta$.

This force is responsible for pulling up the liquid inside the tube. It supports the weight of the column h of the liquid in the tube. So,

$$2\pi r \cdot T\cos\theta = mg, \quad \text{—— (1)}$$

where m is the mass of the liquid inside the tube. Let ρ be the density of the liquid and V be the volume of the liquid inside the tube. So,

$V = V' + V''$ where V' is the volume of the cylinder of height h and radius r and V'' is the volume of the liquid in the meniscus of radius r.

So, $V' = \pi r^2 h$ and

$$V'' = \pi r^2 \cdot r - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$= \pi r^3 - \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^3$$

Now, $V = V' + V'' = \pi r^2\left(h + \frac{r}{3}\right)$

We know that,

$$\rho = m/V$$

$$\Rightarrow m = \rho V = \pi r^2 \rho\left(h + \frac{r}{3}\right) \quad \text{—— (2)}$$

Putting this in eq^n (1),

$$2\pi r \cdot T\cos\theta = \pi r^2 \rho\left(h+\frac{r}{3}\right)g \quad \text{———— (3)}$$

$$\Rightarrow T = \frac{r\rho g(h+r/3)}{2\cos\theta} \quad \text{———— (expression i)}$$

* When the liquid is pure water, $\theta = 0^\circ$. In that case, from eq^n (3)

$$2\pi r \cdot T\cos 0^\circ = \pi r^2 \rho\left(h+\frac{r}{3}\right)g$$

$$\Rightarrow T = \frac{r\rho g(h+r/3)}{2} \quad \text{———— (expression ii)}$$

$$\text{or, } 2T = r\rho g\left(h+\frac{r}{3}\right)$$

$$\Rightarrow h+\frac{r}{3} = \frac{2T}{r\rho g}$$

$$\therefore h = \frac{2T}{r\rho g} - \frac{r}{3} \quad \text{———— (expression iii)}$$

* When the radius of tube r is too small with compare to height h, $r/3$ can be negligible. So, in that case, from eq^n (3),

$$T = \frac{hr\rho g}{2\cos\theta} \quad \text{———— (expression iv)}$$

Calculate the amount of energy needed to break a drop of water of diameter 2×10^{-3} m into 10^9 droplets of equal size. Surface tension of water = 72×10^{-3} Nm^{-1}.

Sol^n: Given,

Surface tension, $T = 72 \times 10^{-3}\ Nm^{-1}$

Diameter, $D = 2 \times 10^{-3}$ m

$\therefore$ Radius, $R = \frac{2 \times 10^{-3}}{2} = 10^{-3}$ m

Let r be the radius of the small droplets.

So, we can write,

volume of the drop of water = volume of 10^9 droplets

$$\Rightarrow \frac{4}{3}\pi R^3 = 10^9 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow r^3 = \frac{R^3}{10^9} = \frac{(10^{-3})^3}{10^9}$$

$$\therefore r = 10^{-6} \text{ m}$$

change in surface area,

$$\Delta A = 10^9 \times 4\pi r^2 - 4\pi R^2$$

$$= 4\pi(10^9 r^2 - R^2)$$

$$= 4 \times 3.1416 \times \{10^9(10^{-6})^2 - (10^{-3})^2\}$$

$$= 12.55 \times 10^{-3} \text{ m}^2$$

So, energy needed, $W = \Delta A \times T$

$$= 12.55 \times 10^{-3} \times 72 \times 10^{-3}$$

$$= 9.036 \times 10^{-4} \text{ J} \quad \text{(Ans)}$$

☐ Calculate the work done in spraying a spherical drop of mercury of radius 10^{-3} m into a million drops of equal size. Surface tension of mercury is 550×10^{-3} Nm^{-1}.

Sol^n: Given,

Surface tension, $T = 550 \times 10^{-3} Nm^{-1}$

Radius, $R = 10^{-3}$ m

Let r be the radius of small equal drops.

So, we can write,

volume of the drop of mercury = volume of 10^6 drops

$$\Rightarrow \frac{4}{3}\pi R^3 = 10^6 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow r^3 = \frac{R^3}{10^6} = \frac{(10^{-3})^3}{10^6}$$

$$\therefore r = 10^{-5} m$$

Change in surface area,

$$\Delta A = 10^6 \times 4\pi r^2 - 4\pi R^2$$

$$= 4\pi (10^6 r^2 - R^2)$$

$$= 4 \times 3.1416 \times \{10^6 (10^{-5})^2 - (10^{-3})^2\}$$

$$= 1.24 \times 10^{-3} m^2$$

So, work done, $W = \Delta A \times T$

$$= 1.24 \times 10^{-3} \times 550 \times 10^{-3}$$

$$= 6.82 \times 10^{-4} J$$ (Ans)

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⊞ Find the work done in blowing a bubble of soap of radius 10 cm. Surface tension for soap = $0.05\ Nm^{-1}$.

We know that, work done in blowing a soap bubble is equal to its surface area (inner and outer) × its free surface energy (surface tension)

Here, the surface area of the bubble,

$$A = 2 \times 4\pi r^2$$
$$= 2 \times 4 \times 3.1416 \times (0.1)^2$$
$$= 0.251\ m^2$$

Work done in blowing the bubble,

$$W = A \times T$$
$$= 0.251328 \times 0.05$$
$$= 12.57 \times 10^{-3}\ J$$

(Ans)

⊞ A liquid of density 1.05 gm/cc and angle of contact 20° has a vertical capillary tube of 2mm diameter dipping into it. If the surface tension of the liquid be 235 dynes/cm, find the rise of the liquid in the capillary tube.

Solⁿ: Given,

density, $\rho = 1.05\ gm/cc$

$$= \frac{1.05/1000}{1/100^3}\ kgm^{-3}$$

$$= 1050\ kgm^{-3}$$

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angle of contact, $\theta = 20^\circ$

diameter, $d = 2mm = 2 \times 10^{-3} m$

$\therefore$ radius, $r = 10^{-3} m$

surface tension, $T = 235$ dynes/cm

$$= \frac{235/10^5}{1/100} Nm^{-1}$$

$$= 0.235 \ Nm^{-1}$$

We know that, rise of the liquid,

$$h = \frac{2T\cos\theta}{r\rho g} - \frac{r}{3}$$

$$= \frac{2 \times 0.235 \times \cos 20^\circ}{10^{-3} \times 1050 \times 9.81} - \frac{10^{-3}}{3}$$

$$= 42.54 \times 10^{-3} m$$

$$= 4.25 \ cm \ \text{(Ans)}$$

囸 A capillary tube of 0.5 mm bore stands vertically in a wide vessel containing a liquid of surface tension $0.072 \ Nm^{-1}$. The liquid wets the tube and density of the liquid is $0.8 \times 10^3 \ kgm^{-3}$. Calculate the rise of the liquid inside the capillary tube.

Soln: Given,

diameter, $d = 0.5mm = 0.5 \times 10^{-3} m$

$\therefore$ radius, $r = d/2 = 2.5 \times 10^{-4} m$

surface tension, $T = 0.072 \ Nm^{-1}$

angle of contact, $\theta = 0°$ (because the liquid wets the tube)

hence, $\cos 0° = 1$

density, $\rho = 0.8 \times 10^3 \ kgm^{-3}$

We know that,

rise of the liquid,

$$h = \frac{2T\cos\theta}{\rho r g}$$

$$= \frac{2 \times 0.072 \times 1}{0.8 \times 10^3 \times 2.5 \times 10^{-4} \times 9.8}$$

$$= 0.0735 m$$

$$= 7.35 \ cm \ \text{(Ans)}$$

Ⅲ Calculate the depth of water at which an air bubble of radius 4×10^{-4}m may remain in equilibrium. Surface tension of water $= 70 \times 10^{-3} \ Nm^{-1}$.

Soln: We've to calculate the length of water column.

Clearly,

$$T = \frac{rh\rho g}{2\cos\theta}$$

where h is the height of the water column.

Neglecting angle of contact, $\theta = 0°$, hence $\cos 0° = 1$

Given,

radius, $r = 4 \times 10^{-4}$ m

surface tension, $T = 70 \times 10^{-3} \ Nm^{-1}$

$\therefore h = \dfrac{2T}{r\rho g}$

$= \dfrac{2 \times 70 \times 10^{-3}}{4 \times 10^{-4} \times 1000 \times 9.8}$ [assuming, the density of water $= 10^3\ kgm^{-3}$]

$= 0.0357\ m$

$= 3.57\ cm$

(Ans)

HEAT & THERMODYNAMICS

⊞ Define thermodynamics.

Thermodynamics is the branch of physics that deals with the relationships between heat and other forms of energy. In particular, it decribes how thermal energy is converted to and from other forms of energy and how it affects matter.

⊞ Define system

A system is a portion of the universe that has been chosen for studying the changes that make place within it in response to varying conditions. A system may be complex, such as a planet, or relatively simple, as the liquid within a glass. Those portions of a system that are physically distinct and mechanically separable from other portions of the system are called phases.

⊞ Define thermodynamic system

A thermodynamic system is a definite macroscopic region or space in the universe, in which one or more thermodynic processes take place.

Everything external to a thermodynic system is called ~~sou~~ surroundings.

⊞ Define Entropy

In therodynamical process we must search for a quantity which tells about the direction of flow of heat and which could efficiently define the thermodynamical state of any working substance. The required quantity was supplied by Clasius who called it entropy.

It is denoted by the symbol S. For infinitesimal small change in entropy is given by,

$$S = \frac{dQ}{T}$$

Total change in entropy is given by

$$S = \int \frac{dQ}{T}$$

⊞ ~~Def~~ Define Heat Engine

Any practical machine which converts heat into mechanical work is called a heat engine.

Heat engine in their operation absorbs heat at a higher temperature, converts part of it into mechanical work, and rejects the remaining heat at a low temperature. In this process, a working substance is used. In steam engines, the working substance is water vapour, and in all gas engines the working substance is a combustible mixture of gases.

⊞ Define Efficiency

The efficiency, η of a heat engine is defined as the ratio of mechanical work done by the engine in one cycle to the heat absorbed from the high temperature source. Thus, if W be the amount of work obtainable from a heat engine in one cycle at the expence of Q amount of heat, then its efficiency, η is given by

$$\eta = \frac{W}{Q}$$

$$= \frac{Q_1 - Q_2}{Q_1} \times 100\%$$

⊞ Classification of Thermodynamic system

1) Open system

A system which can exchange matter and heat energy with the surroundings is called an open system. For example, Air compressor. Air at low pressure enters and air at high pressure leaves the system i.e. there is an exchange of matter and heat energy with surroundings.

2) Closed system

A system which can exchange only heat energy, not matter with the surroundings is called a closed system. For example, gas enclosed in a cylinder expands when heated and drives the piston outwards. The boundary of the system moves but the matter (here gas) in the system remains constant.

3) Isolated System

A system which is thermally insulated and has no communication of heat or work with the surroundings is called isolated system.

⊞ State and explain the zeroth law of thermodynamics.

The zeroth law of thermodynamics states that if two bodies A and B are each ~~sp~~ separately in thermal equilibrium with a third body C, then A and B are also in thermal equilibrium with each other.

⊞ State and explain the first law of thermodynamics

The heat supplied to the system is equal to the sum of the change in internal energy of the system and the external work done by the system.

Let the quantity of heat supplied to the system is dQ, the change in internal energy of the molecules be dU and the amount of external work done be dW.

We can write according to the first law of thermodynamics,

$$dQ = dU + dW \quad \text{------} \quad (1)$$

All the quantities in eqn (1) are measured in unit of energy.

4) State and explain the second law of thermodynamics.

The second law of thermodynamics is a general principle which places constraints upon the direction of heat transfer and attainable efficiency of heat engine.

→ **Clasius's statement**

It is impossible to make heat flow from a body at a lower temperature to a body at a higher temperature without doing external work on the working substance.

→ **Kelvins's statement**

It is impossible to get a continuous supply of work from a body by cooling it to a temperature lower than that of its surroundings.

→ **Planck's statement**

It is impossible to construct an engine which, working in a complete cycle, will produce no effect other than the raising of a weight and the cooling of a heat reservoir.

→ **Kelvin-Planck's statement**

It is impossible to extract an amount of heat from a hot reservoir and use it all to do work.

→ **Edser's statement**

Heat flows of itself from higher to lower temperature.

⊞ Reversible Process

A reversible process is a process whose direction can be reversed by inducing infinitesimal changes to some property of the system via its surroundings, while not increasing entropy. Throughout the entire reversible process, the system is in thermodynamic equilibrium with its surroundings.

Example: A given mass of ice changes to water when a certain amount of heat is absorbed by it and the same mass of water changes to ice when the same quantity of heat is removed from it.

⊞ Irreversible Process

The process is said to be an irreversible process if it cannot return the system and the surroundings to their original conditions when the process is reversed. The irreversible process is not a equilibrium throughout the process.

Example: Friction.

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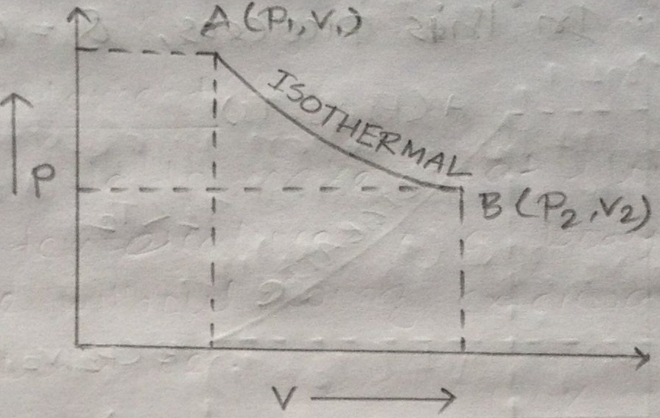
Isothermal Process

If a system is perfectly conducting to the surroundings and the temperature remains constant throughout the process is called an isothermal process.



Let us consider a working substance at a certain pressure and temperature and having a volume represented by the point A.

Pressure is decreased and work is done by the working substance at the cost of its internal energy and there should be fall in temperature. But, the system is perfectly conducting to the surroundings. It absorbs heat from the surroundings and maintains a constant temperature. Thus from A to B, the temperature remains constant. The curve AB is called the isothermal curve or isothermal.

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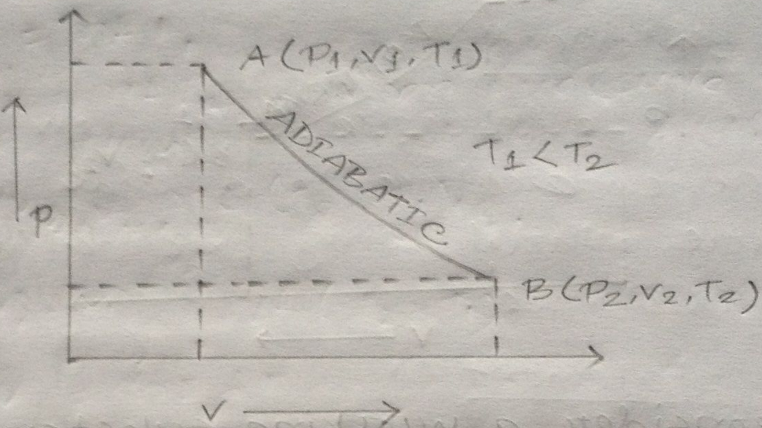
⊞ Adiabatic Process

When a system undergoes from an initial ~~to a~~ state to a final state in such a way that no heat leaves or enters the system, the process is called adiabatic. In this process, $Q = 0$.



During the adiabatic process the working substance is perfectly insulated from the surroundings. All along the process, there is change in temperature. ~~A curve to~~ The curve AB is called an adiabatic curve or an adiabatic.

⊞ Deduce the expression $C_p - C_v = R$ where the symbols have their usual meaning.

Let us consider n moles of an ideal gas is contained in a cylinder fitted with a frictionless piston. If the piston is fixed and the gas is heated, its volume remains constant and all heat supplied goes to increase the internal energy of the molecules due to which the temperature of the gas increases.

If dQ_v is the amount of heat supplied and dT is the increase in temperature, then,

$$dQ_v = nC_v dT \quad \text{———} (1)$$

The pressure of the gas increases during this process, but no work is done because the volume remains constant. Hence, $dW = 0$.
According to the first law of thermodynamics,

$$\cancel{dQ_v = dU}$$

Heat supplied = Increase in internal energy + work done

$$\therefore dQ_v = dU + dW$$

$$\Rightarrow dQ_v = dU + 0$$

$$\Rightarrow dU = nC_v dT \quad \text{———} (2)$$

If the piston is free to move, the gas ~~ma~~ might be allowed to expand at a constant pressure. Let the amout of heat supplied be dQ_p now. The addition of heat causes two changes now in the system.

1) Increase in internal energy
2) work done due to expanding volume

In this case, work done against external pressure, $dW = PdV$.
According to the first law of thermodynamics,

$$dQ_p = dU + dW$$

$$\Rightarrow dQ_p = dU + PdV$$

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Since, $dQ_p = nC_p dT$ and $dU = nC_v dT$, we can write,

$$nC_p dT = nC_v dT + PdV \text{ —— (3)}$$

For an ideal gas, $PV = nRT$

At T_1 K, $PV_1 = nRT_1$

At T_2 K, $PV_2 = nRT_2$, when pressure is contant

$$\therefore PV_2 - PV_1 = nR(T_2 - T_1)$$

$$\Rightarrow P(V_2 - V_1) = nR(T_2 - T_1)$$

$$\Rightarrow PdV = nRdT$$

Putting this value in eq^n (3),

$$nC_p dT = nC_v dT + nRdT$$

$$\Rightarrow C_p = C_v + R$$

$$\therefore C_p - C_v = R$$

[Deduced]

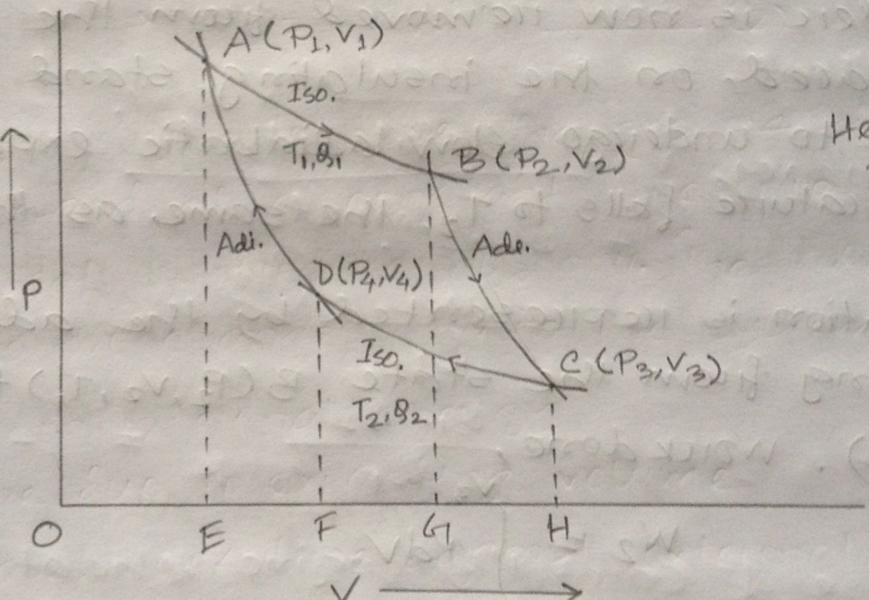
Ⅲ Show that the efficiency of a Carnot engine using an ideal gas as the working substance is

$$\eta = \cancel{1}\,\frac{T_1 - T_2}{T_1}.$$

The Carnot Cycle

A cycle in which the working substance starting from a given condition of temperature, pressure and volume is made to undergo two successive expansion (one isothermal and another adiabatic) and then brought back finally to its initial condition is called Carnot's cycle.

In order to obtain a continuous supply of work, the working substance is subjected to the following cycle of quasi-static operations known as Carnot's cycle.



Here,
Iso. = Isothermal
Adi. = Adiabatic

Fig: Carnot's cycle

1) Isothermal Expansion

The cylinder is first placed on the source, so that the gas acquires the temperature T_1 of the source. The gas therefore, undergoes slow isothermal expansion at the constant temperature T_1.

Let the working substance during isothermal expansion goes from its initial state $A(P_1, V_1, T_1)$ to the state $B(P_2, V_2, T_1)$ along AB.

Workdone, $W_1 = \int_{V_1}^{V_2} PdV$

$= \text{area } ABGEA$

2) Adiabatic Expansion

The cylinder is now removed from the source and is placed on the insulating stand. The gas is allowed to undergo slow adiabatic expansion until its temperature falls to T_2, the same as that of the sink.

The operation is represented by the adiabatic BC, starting from the state $B(P_2, V_2, T_1)$ to the state $C(P_3, V_3, T_2)$. Workdone,

$$W_2 = \int_{V_2}^{V_3} PdV$$

$= \text{area } BCHGB$

3) Isothermal compression:

The cylinder is now removed from the insulating stand and is placed on the sink which is at temperature T_2. The gas undergoes isothermal compression at a constant temperature T_2.

The operation is represented by the isothermal CD, starting from the state C (P_3, V_3, T_2) to the state D (P_4, V_4, T_2). Work done,

$$W_3 = \int_{V_3}^{V_4} P\,dV$$

$$= \text{area CHFDC}$$

4) Adiabatic Compression

The cylinder is now removed from the sink and again placed on the insulating stand. The gas is adiabatically compressed and the temperature rises.
This operation is represented by adiabatic DA, starting from D (P_4, V_4, T_2) to the final state A(P_1, V_1, T_1). Work done,

$$W_4 = \int_{V_4}^{V_1} P\,dV$$

$$= \text{area DFEAD}$$

work done by the gas is positive and work done on the gas is negative. From graph, net work done,

$$W = W_1 + W_2 - W_3 - W_4$$

$$= \text{area ABGEA} + \text{area BCHGB} - \text{area CHFDC} - \text{area DFEAD}$$

$$= \text{area ABCDA}$$

Thus, the area enclosed by the carnot's cycle consisting of two isothermals and two adiabatics gives the net amount of work done per cycle.

Efficiency

We know that,

$$\text{efficiency, } \eta = \frac{\text{Useful output}}{\text{Input}}$$

$$= \frac{W}{Q_1}$$

$$= \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

In the case of carnot's engine, heat is proportional to temperature, i.e. $Q \propto T$.

So, $\frac{Q_2}{T_2} = \frac{Q_1}{T_1} \Rightarrow \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

$$\therefore \eta = 1 - \frac{T_2}{T_1}$$

$$= \frac{T_1 - T_2}{T_1}$$

[showed]

[Network done, $W = Q_1 - Q_2$

$$= RT_1 \log_e \frac{V_2}{V_1} - RT_2 \log_e \frac{V_2}{V_1}$$

$$= R(T_1 - T_2) \log_e \frac{V_2}{V_1} \quad]$$

⊞ Is 100% efficiency possible for a Carnot heat engine. Explain your answer mathematically.

No. It is impossible to gain 100% efficiency for a carnot heat engine.

We know that, the efficiency for carnot engine,

$$\eta = 1 - \frac{T_2}{T_1} \quad \text{--- (1)}$$

When the efficiency is 100%, we get from eqn (1), $\eta = 1$ only when $T_2 = 0K$ i.e. the temperature of the sink is at absolute zero degree. In practice, it is never possible to reach absolute zero i.e. T_2 will never be equal 0K. Hence, 100% efficiency i.e. 100% conversion of heat energy into mechanical work is impossible.

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MATHEMATICAL PROBLEMS

Calculate the efficiency of carnot heat engine working between the temperature

(1) 120°c and 30°c

(2) 127°c and 27°c

(3) 430°c and 20°c

Solution

(1) Here,

$T_1 = (120 + 273)\,K = 393\,K$

$T_2 = (30 + 273)\,K = 303\,K$

Efficiency, $\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$

$= \left(1 - \frac{303}{393}\right) \times 100\%$

$= 22.9\%$ (Ans)

(2) Here,

$T_1 = (127 + 273)\,K = 400\,K$

$T_2 = (27 + 273)\,K = 300\,K$

$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$

$= \left(1 - \frac{300}{400}\right) \times 100\%$

$= 25\%$ (Ans)

(3) Here, $T_1 = (430 + 273)\,K = 703\,K$

$T_2 = (20 + 273)\,K = 293\,K$

$$\therefore \eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

$$= \left(1 - \frac{293}{703}\right) \times 100\%$$

$$= 58.32\%$$ (Ans)

(4) A carnot engine whose temperature of the source is 400K takes 200 calories of heat at this temperature and rejects 150 calories of heat to the sink. What is the temperature of the sink? Also calculate the efficiency of the engine.

> Here, $Q_1 = 200\text{ cal} = 200 \times 4.2\text{ J} = 840\text{ J}$

$Q_2 = 150\text{ cal} = 150 \times 4.2\text{ J} = 630\text{ J}$

$T_1 = 400\,K$

$T_2 = ?$

$\eta = ?$

We know that,

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \Rightarrow \frac{T_2}{Q_2} = \frac{T_1}{Q_1}$$

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$$\Rightarrow T_2 = \frac{Q_2 \times T_1}{Q_1}$$

$$= \frac{630 \times 400}{840}$$

$$= 300K \quad \text{(Ans)}$$

$$\therefore \eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

$$= \left(1 - \frac{300}{400}\right) \times 100\%$$

$$= 25\% \quad \text{(Ans)}$$

SOUND

⊞ Define simple harmonic motion.

If the acceleration of a body is proportional to its displacement from its position of equilibrium or any other fixed point in its path and be always directed towards it, the body is said to execute a simple harmonic motion.

⊞ Write down the characteristics of SHM.

~~1) It is periodic motion.~~
~~2) The acceleration is proportional to the displacement but is in the opposite direction.~~

~~3)~~

A body executing simple harmonic motion shows the following characteristics.

1) Its motion is vibratory and periodic.
2) Some restoring force acts on the vibrating system.
3) Acceleration of the body is directly proportional to its displacement ~~and alwaly direc~~ but is in the opposite direction.
4) Energy of the system oscillates between kinetic energy and potential energy but the total energy remains constant.

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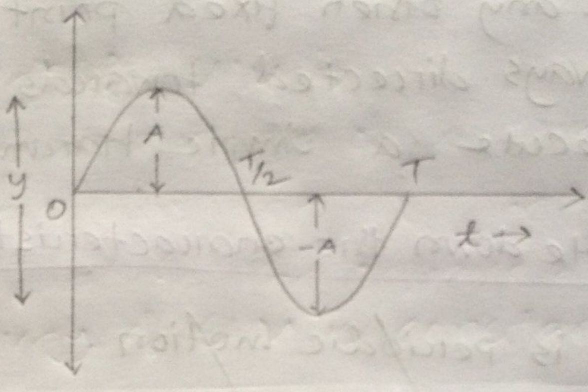
⊞ Give the graphical representation of SHM

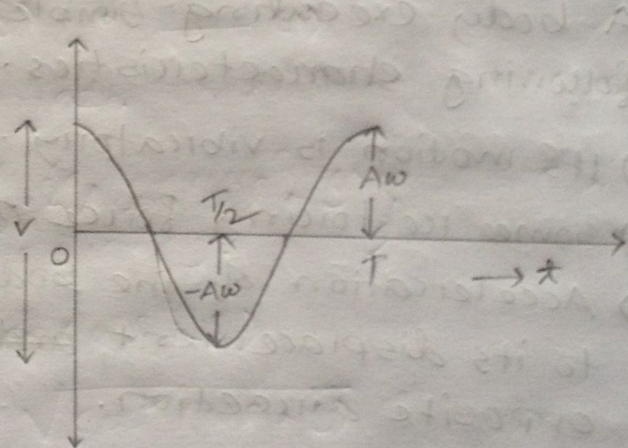
Graphical representation of displacement, velocity and acceleration of a particle vibrating harmonically with respect to time is shown below.

(1) $y = A\sin\omega t$.
Displacement graph is sine curve. Maximum displacement of the particle is, $y = \pm A$



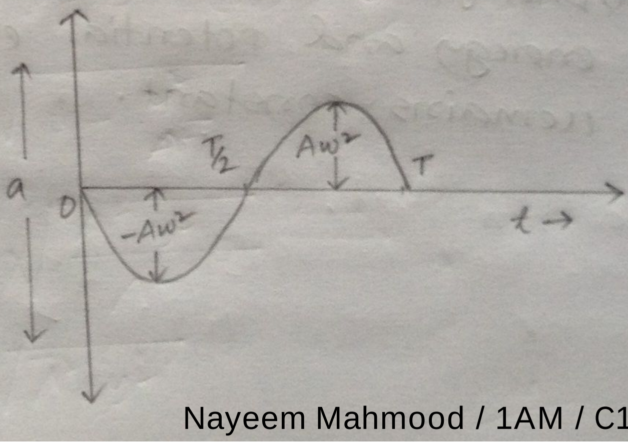
(2) $v = \frac{d}{dt}(y) = \frac{d}{dt}(A\sin\omega t) = A\omega\cos\omega t$

The velocity of the vibrating particle is maximum at the mean position, i.e. $v = \pm A\omega$ and it is zero at extreme position. Velocity graph is cosine curve.



(3) acceleration, $a = \frac{d}{dt}(v)$
$\Rightarrow a = \frac{d}{dt}(A\omega\cos\omega t)$
$= -A\omega^2\sin\omega t$

Acceleration graph is sine curve. Acceleration is zero at mean position ~~at~~ and



at the extreme position, $a = \mp A\omega^2$.

⊞ Obtain differential equation for SHM

Let us consider a physical system that consists of a block of mass m attached to the end of a spring, with the block free to move on a horizontal, frictionless surface.

When the spring is neither stretched nor compressed, the block is at the position $x=0$, called the equilibrium position of the system.

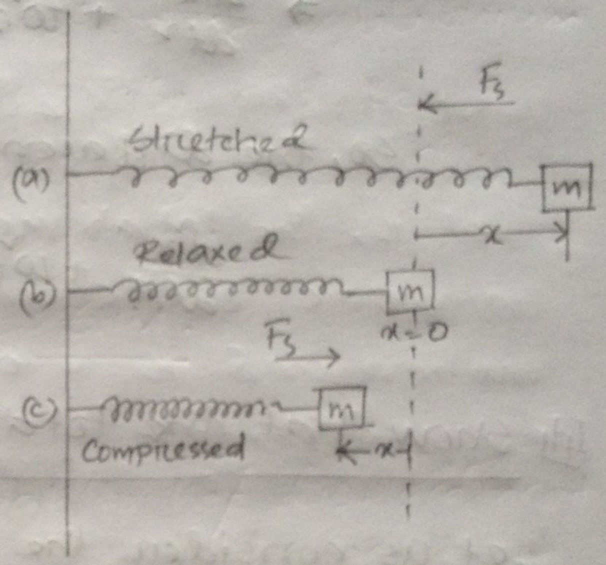


Fig: Mass-block with spring

When the block is displaced a small distance x from equilibrium, the spring exerts on the block a force that is proportional to the displacement and given by Hooke's law,

$$F_s = -kx \quad \text{— (1)}$$

Here, k is stiffness or spring constant and F_s is the restoring force.

Applying Newton's law to the motion of the block, together with eqn (1), we obtain,

$$F_s = -kx = ma$$

$$\Rightarrow -kx = m\frac{d}{dt}(v)$$

$$\Rightarrow -kx = m\frac{d}{dt}\left(\frac{dx}{dt}\right) = m\frac{d^2x}{dt^2}$$

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$$\Rightarrow m\frac{d^2x}{dt^2} + Kx = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{K}{m}x = 0 \quad [\text{Dividing both sides by } m]$$

$$\Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0, \text{ where } \omega^2 = \frac{K}{m}$$

So, the differential eqn of SHM is

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

⊞ Show that the total energy of SHM is constant.

Let us consider the displacement of a particle executing simple harmonic motion,

$$y = A\sin(\omega t + \delta), \quad \text{—— (1)}$$

where δ is phase difference.

So, velocity, $v = \frac{dy}{dt} = \frac{d}{dt}\{A\sin(\omega t + \delta)\}$

$$= A\omega\cos(\omega t + \delta)$$

$$= A\omega\sqrt{\cos^2(\omega t + \delta)}$$

$$= A\omega\sqrt{1 - \sin^2(\omega t + \delta)}$$

$$= A\omega\sqrt{1 - \frac{y^2}{A^2}}$$

$$= A\omega\sqrt{\frac{A^2 - y^2}{A^2}}$$

$$\Rightarrow v = \omega\sqrt{A^2 - y^2}$$

∴ Kinetic energy of the particle, ~~at~~

$$K.E. = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m\omega^2(A^2-y^2)$$

Potential energy of the vibrating particle is the amount of work done in overcoming the force through a distance y.

$$v = A\omega\cos(\omega t+\delta)$$

$$\Rightarrow \frac{d}{dt}(v) = \frac{d}{dt}\{A\omega\cos(\omega t+\delta)\}$$

$$\Rightarrow a = -A\omega^2\sin(\omega t+\delta)$$

$$= -\omega^2 y \quad [\because y = A\sin(\omega t+\delta)]$$

∴ Acceleration, $a = -\omega^2 y$

and Force $= -m\omega^2 y \quad [\because F=ma]$

The minus sign shows that the directions of the acceleration and force are opposite to the direction of the motion of the vibrating particle. So potential energy,

$$P.E. = \int_0^y m\omega^2 y\,dy$$

$$= \frac{1}{2}m\omega^2[y^2]_0^y$$

$$= \frac{1}{2}m\omega^2 y^2$$

Total energy, $E = K.E + P.E$

$$= \frac{1}{2} m\omega^2 (A^2 - y^2) + \frac{1}{2} m\omega^2 y^2$$

$$= \frac{1}{2} m\omega^2 (A^2 - y^2 + y^2)$$

$$= \frac{1}{2} m\omega^2 A^2$$

That is, the total mechanical energy of a simple harmonic oscillator is a constant of the motion and is proportional to the square of the amplitude.

MATHEMATICAL PROBLEMS

The equation of a particle executing SHM is $y = 10 \sin(\omega t + \delta)$. If time period is 30 sec, find out the angular frequency.

Here,

$T = 30s$

$\omega = ?$

We know that,

$$\omega = 2\pi f$$

$$\Rightarrow \omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{30}$$

$$= 0.209 \text{ rad s}^{-1}$$

(Ans)

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WAVES

Define stationary wave/standing wave

Stationary wave is the combination of two waves moving in opposite direction, each having the same amplitude and frequency. The phenomenon is the result of interference, i.e. when waves are superimposed, their energies are either added together or cancelled out. The locations at which the amplitude is minimum are called nodes. and where amplitude is maximum are called anti-nodes.

Derive an expression for stationary waves. Explain nodes and antinodes from stationary waves.

Let us consider wave functions for ~~tow~~ two transverse sinusoidal waves having same amplitude A, frequency f and wavelength λ but traveling in opposite direction in the same medium.

$$y_1 = A \sin(kx + \omega t)$$

$$y_2 = A \sin(kx - \omega t)$$

where y_1 represents a wave traveling to the ~~righ~~ left and y_2 represents one traveling to the right. Adding these two functions gives the resultant wave y.

$$y = y_1 + y_2$$

$= A\sin(kx+\omega t) + A\sin(kx-\omega t)$

$= 2A\sin\left(\frac{kx+\omega t+kx-\omega t}{2}\right)\cos\left(\frac{kx+\omega t-kx+\omega t}{2}\right)$

$\Rightarrow y = 2A\sin kx\cos\omega t$

Wave number, $k = \frac{2\pi}{\lambda}$

$\Rightarrow kx = \frac{2\pi x}{\lambda}$

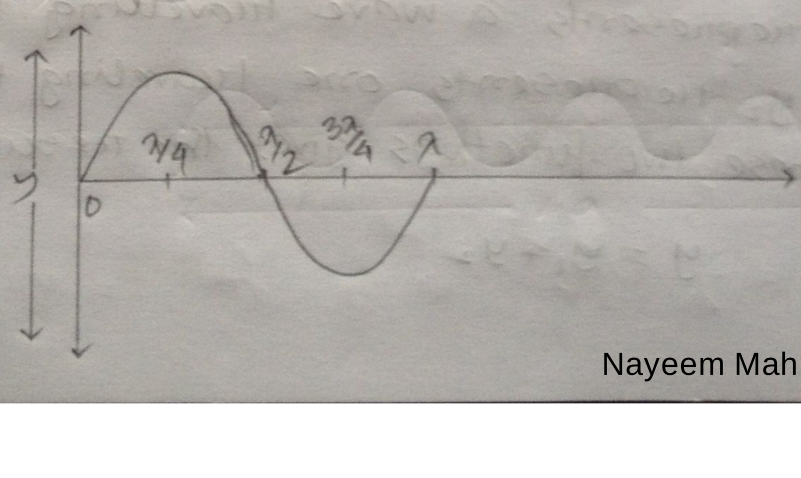
$\therefore y = 2A\sin\frac{2\pi x}{\lambda}\cos\omega t$

When phase angles are $0, \pi, 2\pi, 3\pi, \ldots n\pi$

$kx = 0$	$kx = \pi$	$kx = 2\pi$
$\Rightarrow \frac{2\pi x}{\lambda} = 0$	$\Rightarrow \frac{2\pi x}{\lambda} = \pi$	$\Rightarrow \frac{2\pi x}{\lambda} = 2\pi$
$\Rightarrow x = 0$	$\Rightarrow x = \frac{\lambda}{2}$	$\Rightarrow x = \lambda$

$\therefore x = 0, \frac{\lambda}{2}, \lambda, 3\frac{\lambda}{2}, 2\lambda \ldots \frac{n\lambda}{2}$ where $n = 0, 1, 2, \ldots$

these points of zero displacement are called nodes. Amplitude of the wave is minimum at nodes.



When phase angles are $\pi/2, 3\pi/2, 5\pi/2, \ldots (2n+1)\pi/2$

$Kx = \frac{\pi}{2}$ $\quad Kx = \frac{3\pi}{2}$ $\quad Kx = \frac{5\pi}{2}$

$\Rightarrow \frac{2\pi x}{\lambda} = \frac{\pi}{2}$ $\quad \Rightarrow \frac{2\pi x}{\lambda} = \frac{3\pi}{2}$ $\quad \Rightarrow \frac{2\pi x}{\lambda} = \frac{5\pi}{2}$

$\Rightarrow x = \frac{\lambda}{4}$ $\quad \Rightarrow x = 3\frac{\lambda}{4}$ $\quad \Rightarrow x = 5\frac{\lambda}{4}$

$\therefore x = \frac{\lambda}{4}, 3\frac{\lambda}{4}, 5\frac{\lambda}{4} \ldots (2n+1)\frac{\lambda}{4}$ where $n = 1, 2, 3, \ldots$

These points of maximum displacement are called antinodes. Amplitude of the wave is maximum at antinodes.

[4]

⊞ What is Doppler effect?

the doppler effect is the apparent change in frequency or wavelength of a wave for an observer moving relative to its source.

⊞ Find the apparent frequency -

i) When the source and the observer move towards each other

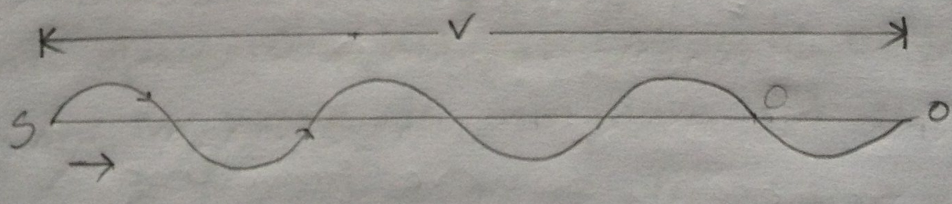
ii) When the source and the observer move away from each other

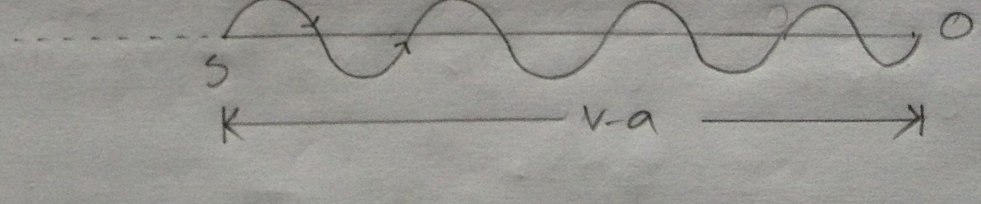
iii) When the source moves towards the stationary observer

iv) When the source moves away from the stationary observer

(iii) When source moves towards the stationary observer

Let us assume a source S is producing sound of ~~frequency~~ pitch n and wavelength λ. The velocity of sound is v.





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Let the source move with a velocity a towards the observer O. In one second, n waves will be contained in a length (v−a) and the apparent wavelength,

$$\lambda' = \frac{v-a}{n} \quad \text{——— (1)}$$

Note: 1) If source moves towards observer, the apparent frequency will be greater than real one.
2) If source moves towards observer at a velocity of sound, apparent frequency will be infinity.

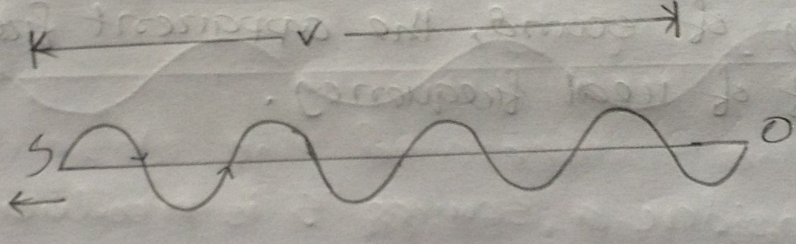
The apparent pitch,

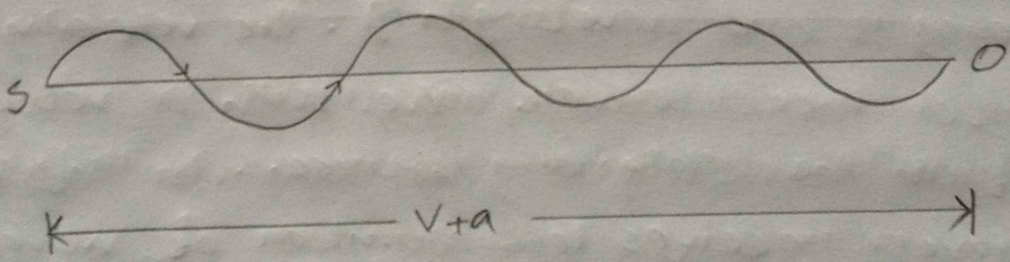
$$n' = \frac{v}{\lambda'}$$

$$\Rightarrow n' = \left(\frac{v}{v-a}\right) n \quad [\text{from eq}^n\ (1)]$$

——— (2)

(iv) When the source moves away from stationary observer





Let us assume a source S is producing sound of pitch n and wavelength λ. The velocity of sound is v. Let the source move with a velocity a away from the stationary observer O. In one second, n waves

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will be contained in a length $(v+a)$ and the apparent wavelength,

$$\lambda' = \frac{v+a}{n} \quad \text{——} \quad (3)$$

The apparent pitch,

$$n' = \frac{v}{\lambda'}$$

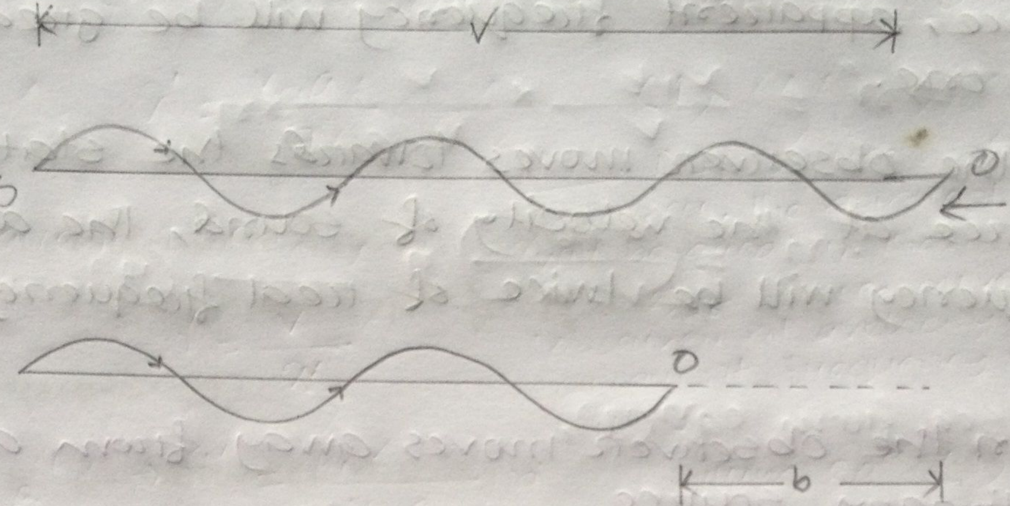
$$\therefore n' = \left(\frac{v}{v+a}\right) n \quad \text{——} \quad (4)$$

We can see from eqn (4),

i) If the source moves away from observer, the apparent frequency will be less than real one.

ii) If the source moves away from observer at the velocity of sound, the apparent frequency will be half of real frequency.

(v) When observer moves towards a stationary source



Let us consider a source S is producing sound of frequency n and wavelength λ. The velocity of sound is v. Let the observer move with a velocity b towards a stationary source. In this case, observer receives more number of waves in one second. The apparent wavelength remains same. The apparent frequency,

$$n' = n + \frac{b}{\lambda}$$

$$\Rightarrow n' = \frac{v}{\lambda} + \frac{b}{\lambda}$$

$$\Rightarrow n' = \frac{v+b}{\lambda}$$

But, $\lambda = \frac{v}{n}$, $\therefore n' = \left(\frac{v+b}{v}\right) n$ —— (5)

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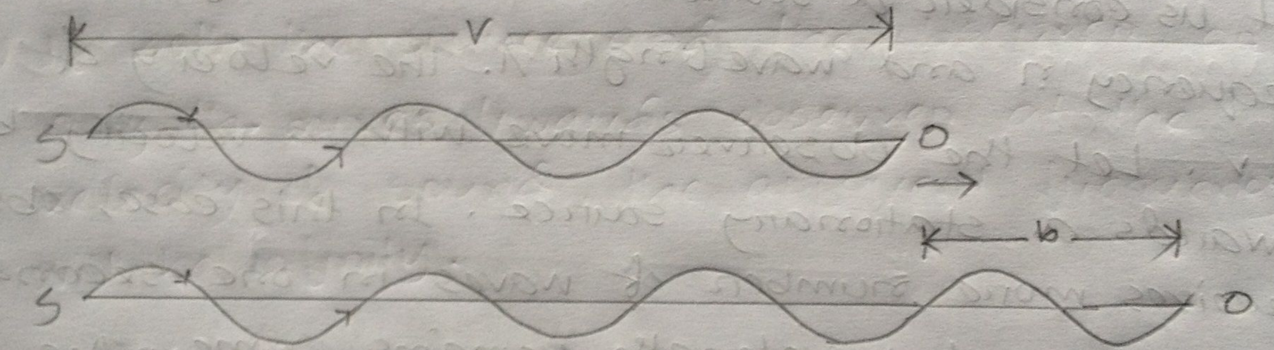
We can see from eqn (5),

i) If the observer moves towards the stationary source, apparent frequency will be greater than real one.

ii) If the observer moves towards the stationary source at the velocity of sound, the apparent frequency will be twice of real frequency.

(VI) When the observer moves away from a stationary source



Let us consider a source S is producing sound of frequency n and wavelength λ. The velocity of sound v. Let the observer move with a velocity b away from the stationary source. In this case, the observer receives less number of waves in one second. The apparent wavelength remains the same. The apparent frequency,

$$n' = n - \frac{b}{\lambda}$$

$$\Rightarrow n' = \frac{v}{\lambda} - \frac{b}{\lambda} = \frac{v-b}{\lambda}$$

But, $\lambda = \frac{v}{n}$,

$$\therefore n' = \frac{v-b}{\lambda} = \left(\frac{v-b}{v}\right)n \quad \text{——— (6)}$$

We can see from eq^n (6),

i) If the observer moves away from stationary source, the apparent frequency will be less than real frequency.

ii) If the observer moves away from stationary source at the velocity of sound, the apparent frequency will be zero.

① When the source and the observer move towards each other

When the source moves towards stationary observer with a velocity a, the apparent frequency,

$$n' = \left(\frac{v}{v-a}\right)n \quad \text{[from eq}^n \text{ (2)]}$$

where n is the actual frequency and v is the velocity of sound.

Now, let us assume that the observer is also moving with velocity b towards the source which appears to be emitting waves of frequency n'. Due to the motion of the observer the frequency will change from n' to n''.

According to eq^n (5),

$$n'' = \left(\frac{v+b}{v}\right) n'$$

$$\Rightarrow n'' = \frac{v+b}{v} \times \frac{v}{v-a} \times n$$

$$\therefore n'' = \left(\frac{v+b}{v-a}\right) n \quad \text{———} \quad (7)$$

we can see from eq^n (7),

i) The apparent frequency will be greater than the actual frequency.

ii) If ~~both~~ the source ~~and the observer or only th~~ moves towards observer at the velocity of sound, the apparent frequency will be infinity.

2) When the source and the observer move away from each other

When the source moves away from the stationary observer with velocity a, the apparent frequency,

$$n' = \left(\frac{v}{v+a}\right) n \quad [\text{from eq}^n (4)]$$

where n is the actual frequency and v is the velocity of sound.

Now let us assume that the observer is also

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moving with velocity b away from the source which appears to be emitting waves of frequency n'. Due to the motion of observer, the frequency will change from n' to n''.

According to eq^n (6),

$$n'' = \left(\frac{v-b}{v}\right) n'$$

$$\Rightarrow n'' = \frac{v-b}{v} \times \frac{v}{v+a} n$$

$$\therefore n'' = \left(\frac{v-b}{v+a}\right) n \quad \text{—— (8)}$$

We can see from eq^n (8),

i) the apparent frequency will be less than actual frequency.

ii) If the observer moves with the velocity of sound, the apparent frequency will be zero.

FLUID DYNAMICS & VISCOSITY

What is liquid?

A liquid is a substance that is completely mobile and practically incompressible. Therefore, the same amount of it flows accross every section of a tube in a given time.

What is fluid?

A fluid is a substance that continually deforms or flows under an applied shear stress. Fluids are a subset of the phases of matter and include liquids, gases, plasmas and to some extent plastic solids. Fluids are substances that have zero shear modulus or in simpler term, a fluid is a substance which can't resist any shear force applied to it.

Define Viscosity

The property of a liquid by virtue of which it opposes relative motion between its different layers is known as viscosity or internal friction of the liquid.

Explain Co-efficient of viscosity

The co-efficient of viscosity of a liquid may be defined as the tangential force required per unit area to maintain unit relative velocity between two layers unit distance apart.

Let us assume that PQ is a stationary surface. A fluid is flowing over the surface as a streamline motion. If any two layers are considered, the upper layer DE tends to acclerate the motion of lower layer and the lower layer BC tends to retard the motion of the upper layer.

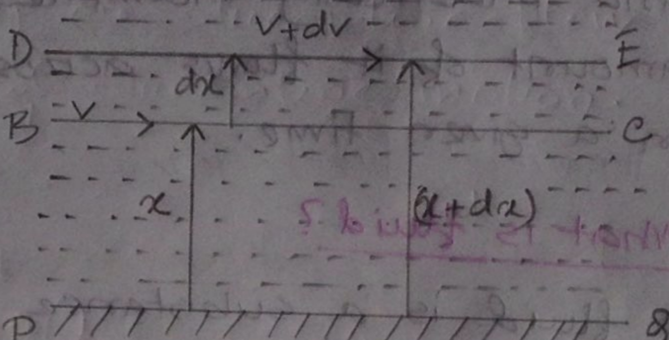


Fig: Flow of fluid

Let the layer BC be at a distance x from ~~the~~ PQ. The other layer DE at a distance $(x+dx)$ from PQ. The velocity of layer BC and DE are v and $(v+dv)$ respectively.

The difference of velocity between two layers

$$= v+dv-v \quad = dv$$

and distance between these layers.

$$= x+dx-x \quad = dx$$

Therefore, for the difference of distance dx, the velocity difference is dv. So, the rate of change of velocity with respect to distance is $\frac{dv}{dx}$. This is known as velocity gradient.

According to Newton's law of viscous flow, viscous ~~flow~~ force is directly proportional to the surface area A i.e. $F \propto A$,

and is directly proportional to the velocity gradient i.e. $F \propto \frac{dv}{dx}$

Therefore,

$$F \propto A \frac{dv}{dx}$$

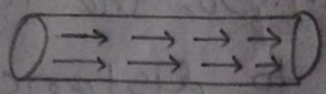
$$\Rightarrow F = -\eta A \frac{dv}{dx} \quad \text{——} \quad (1)$$

Here η is a constant, depending upon the nature of the liquid, and is called its co-efficient of viscosity. The -ve sign in eqn (1) indicates that the direction of the force is opposite to that of velocity.

Q) Define the rate of flow of liquid

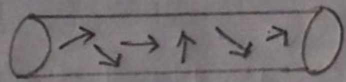
The ~~flow~~ rate of flow of liquid is defined as the volume of it that flows across any section in unit time.

Streamline motion



If fluid flows in such a way that each particle flows exactly in same path and has the same velocity as its predecessor and the fluid is said to have an orderly or streamline flow and the motion is called streamline motion.

Turbulant motion



If fluid flows in such a way that each particle flows with randomly changing the direction of path as well as the velocity, then the motion is called turbulant motion.

Write down the various forms of energy possessed by liquid in motion.

There are three types of energy possessed by liquid in motion. They are,

i) Kinetic energy,

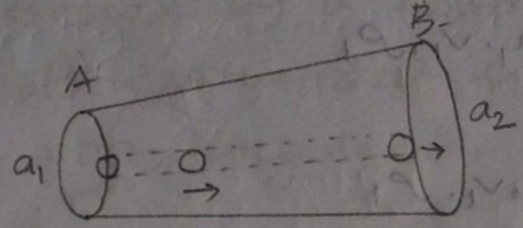
ii) Potential energy,

iii) Pressure energy.

Define Terminal velocity

When a body rises ~~is~~ in the same velocity and there is no acceleration, then the velocity of the body is called terminal velocity.

Explain the equation of continuity of flow for ~~tio~~ fluid.



Let us imagine a fluid ~~be~~ to be flowing through a pipe AB with a_1 and a_2 as its areas of cross-section at sections A and B respectively and consider an infinitesimally small tube of flow (shown dotted) of cross-sectional areas da_1 and da_2 at its two ends and with velocities of the fluid v_1 and v_2 at section A and B respectively.

If the fluid covers distances ds_1 and ds_2 in time dt at the two ends and ρ_1 and ρ_2 be the densities of the fluid at A and B, we have

mass of fluid entering flow-tube at end A per unit time $= \frac{da_1 . ds_1 . \rho_1}{dt}$

$= da_1 v_1 \rho_1 \quad [\because \frac{d}{dt}(s_1) = v_1]$

mass of fluid leaving flow-tube at end B per unit time $= \frac{da_2 \cdot s_2 \cdot \rho_2}{dt}$

$$= da_2 \cdot v_2 \cdot \rho_2$$

$\therefore$ mass of fluid entering the whole section A per ~~se~~ unit time i.e. mass rate of flow at A

$$= \int_0^{a_1} da_1 \cdot v_1 \cdot \rho_1$$

$$= a_1 \cdot v_1 \cdot \rho_1$$

and mass rate of flow at B

$$= \int_0^{a_2} da_2 \cdot v_2 \, \rho_2$$

$$= a_2 \cdot v_2 \cdot \rho_2$$

Since the fluid is incompressible, $\rho_1 = \rho_2$ and since we have no source or sink in between sections A and B, we have, from the law of conservation of matter,

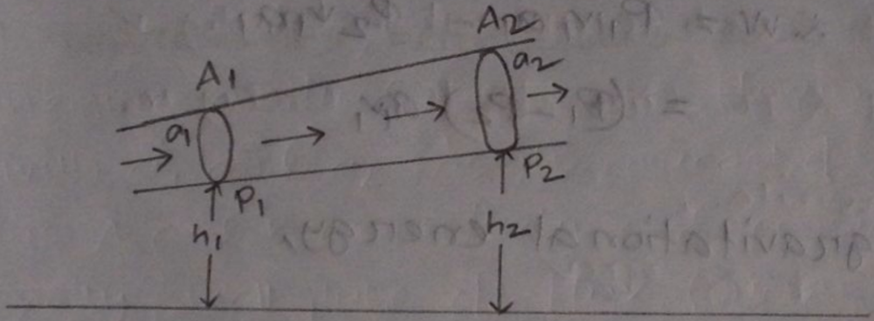
$$a_1 v_1 = a_2 v_2$$

$$\text{or, } av = \text{constant}$$

This is called the equation of continuity and states that the quantity of fluid entering one end of the pipe per second is the same as leaving the pipe at the other end per second.

State and explain Bernoulli's theorem

Bernoulli's theorem states that the sum of the energies possessed by a flowing liquid at any point is constant provided the flow is ~~sted~~ steady and non-turbulent.



Let us consider an incompressible liquid in a pipe line. The motion is stream line. Let the pressure of the liquid at the cross-sections A_1 and A_2 be P_1 and P_2 respectively. The area of cross-section at $A_1 = a_1$ and at $A_2 = a_2$. The velocity of flow of the liquid at $A_1 = v_1$ and at $A_2 = v_2$.

Work done per second on the liquid entering A_1,

$$W_1 = P_1 v_1 a_1$$

Work done per second by the liquid leaving A_2,

$$W_2 = P_2 v_2 a_2$$

Net work done, $W = W_1 - W_2$

$$= P_1 v_1 a_1 - P_2 v_2 a_2$$

~~Change in gravitational energy~~

But, $a_1 v_1 = a_2 v_2$ [from eq^n of continuity]

$$\therefore W = P_1 v_1 a_1 - P_2 v_1 a_1$$

$$= (P_1 - P_2) a_1 v_1$$

Change in gravitational energy,

$$E_1 = P_2 a_2 v_2 - P_1 a_1 v_1$$

$$= h_2 \rho g a_2 v_2 - h_1 \rho g a_1 v_1 \quad [\because P = h\rho g]$$

$$= a_1 v_1 \rho g (h_2 - h_1) \quad [\because a_1 v_1 = a_2 v_2]$$

Change in kinetic energy,

$$E_2 = \frac{1}{2} a_2 v_2 \rho v_2^2 - \frac{1}{2} a_1 v_1 \rho v_1^2$$

$$= \frac{1}{2} a_1 v_1 \rho (v_2^2 - v_1^2) \quad [\because a_1 v_1 = a_2 v_2]$$

$$\therefore \quad W = E_1 + E_2$$

$$\Rightarrow (P_1 - P_2) a_1 v_1 = a_1 v_1 \rho g (h_2 - h_1) + \frac{1}{2} a_1 v_1 \rho (v_2^2 - v_1^2)$$

$$\Rightarrow P_1 - P_2 = \rho g (h_2 - h_1) + \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$\Rightarrow P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

$$\Rightarrow \frac{P_1}{\rho} + \frac{1}{2} v_1^2 + g h_1 = \frac{P_2}{\rho} + \frac{1}{2} v_2^2 + g h_2$$

We can write,

$$\frac{P}{\rho} + \frac{1}{2} v^2 + gh = \text{constant} \quad \text{—— (1)}$$

This eq^n represents Bernoulli's equation.

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□ Prove the relation $F = 6\pi v r \eta$, where the symbols have their usual meaning.

Stokes proved that if a small spherical body of radius r moves downwards in a medium whose co-efficient of viscosity is η and the terminal velocity of the body is v, a viscous force will act on the body.

Let F be the viscous force. According to Stokes' law,

$F \propto$ co-efficient of viscosity η of the medium

$F \propto$ ~~vel~~ terminal velocity v

$F \propto$ radius of the body r

We can write,

$$F \propto \eta r v$$

$$\Rightarrow F = K \cdot \eta r v$$

where K is a constant. Stokes proved that $K = 6\pi$.

$$\therefore F = 6\pi v r \eta$$

(Proved)

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MATHEMATICAL PROBLEMS

[5] Find the limiting velocity of a rain drop of diameter 10^{-3} m. Density of air relative to water is 1.3×10^{-3}, coefficient of viscosity of air is 1.81×10^{-5} S.I. units and density of water is 10^3 kg/m³.

Here, $g = 9.8 ms^{-2}$

radius, $r = \frac{10^{-3}}{2} = 5 \times 10^{-4}$ m

density of water, $\rho = 10^3 \ kgm^{-3}$

~~density of air, σ = 1.3 × 10⁻³~~

density of air, $\sigma = 1.3 \times 10^{-3} \times 10^3 \ kgm^{-3}$

$= 1.3 \ kgm^{-3}$

coefficient of viscosity, $\eta = 1.81 \times 10^{-5} \ Nsm^{-2}$

Limiting velocity, $v = ?$

We know that,

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

$$= \frac{2 \times (5 \times 10^{-4})^2 \times (10^3 - 1.3) \times 9.8}{9 \times 1.81 \times 10^{-5}}$$

$$= 30.04 \ ms^{-1}$$ (Ans)

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[14] A plate of metal $10^{-2} m^2$ area rests on a layer of castor oil $2 \times 10^{-3} m$ thick, whose coefficient of viscosity is $1.55\ Nsm^{-2}$. Calculate the horizontal force required to move the plate with a uniform speed of $3 \times 10^{-2} ms^{-1}$.

Here,

area, $A = 10^{-2} m^2$

coefficient of viscosity, $\eta = 1.55\ Nsm^{-2}$

thickness, $dx = 2 \times 10^{-3} m$

speed, $dv = 3 \times 10^{-2}\ ms^{-1}$

force, $F = ?$

We know that,

$$F = \eta A \frac{dv}{dx}$$

$$= \frac{1.55 \times 10^{-2} \times 3 \times 10^{-2}}{2 \times 10^{-3}}$$

$$= 0.2325\ N$$ (Am)

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田 Water flows through a horizontal pipe line of varying cross section at the rate of $0.2 m^3/s$. Calculate the velocity of water at a point where the area cross section of the pipe is $0.02 m^2$.

Here,

flow of rate of water, $av = 0.2 m^3/s$

$\therefore$ velocity at $0.02 m^2 = \frac{0.2}{0.02} = 10 ms^{-1}$

(Ans)

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11) Water flows through a horizontal pipe line of varying cross-section. At a point where the pressure of water is $5.78 \times 10^3 \ Nm^{-2}$. The velocity of flow is $0.3 ms^{-1}$. Calculate the pressure at another point where the velocity of flow is $0.4 ms^{-1}$. Density of water is $10^3 \ kgm^{-3}$.

From Bernoulli's theorem, we can write,

$$P + \frac{1}{2}\rho v^2 = \text{constant}$$

so, $$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \quad \text{---} (1)$$

Here,

$P_1 = 5.78 \times 10^3 \ Nm^{-2}$

$v_1 = 0.3 ms^{-1}$

$v_2 = 0.4 ms^{-1}$

$\rho = 10^3 \ kgm^{-3}$

$P_2 = ?$

From eqn (1),

$$P_2 = P_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2$$

$$= (5.78 \times 10^3) + \frac{1}{2} \times 10^3 \{(0.3)^2 - (0.4)^2\}$$

$$= 5745 \ Nm^{-2}$$

$$= 5.745 \times 10^3 \ Nm^{-2}$$

AM

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[5] A rigid ball of area 6.5 m^2 falls vertically downward through a viscous liquid with a terminal velocity of 25 m/s. If the coefficient of viscosity is 1.7×10^{-6} SI unit, then find the upward force due to viscosity.

Here,

area, $A = 6.5 m^2$

$\therefore$ radius, $r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{6.5}{3.14}} = 1.44 m$

terminal velocity, $v = 25 ms^{-1}$

coefficient of viscosity, $\eta = 1.7\times10^{-6} Nsm^{-2}$

upward force, $F = ?$

We know that,

$$F = 6\pi\eta r v$$

$$= 6\times3.14\times1.7\times10^{-6}\times1.44\times25$$

$$= 1.15\times10^{-3} N \text{ (Ans)}$$