Vectors and scalars

Definition:

Vector: A vector is a quantity having both magnitude and direction. Such as displacement, velocity, force and acceleration etc.

scalar: A scalar is a quantity having magnitude but no direction. Such as mass, length, time, temperature and any real number.

Unit vector: A unit vector is a vector having unit magnitude. If $\vec{A}$ is a vector with magnitude $A \neq 0$, then $\vec{A}/A$ is a unit vector having the same direction as $\vec{A}$.

$$\text{Unit vector } \hat{a} = \frac{\vec{A}}{|\vec{A}|}$$

Vector field: If to each point (x,y,z) of a region R in space there corresponds a vector $V(x,y,z)$ then V is called a vector function and we say that a vector field V has been defined in R.

Scalar field: If to each point (x,y,z) of a region R in space there corresponds a number or scalar $\phi(x,y,z)$ then ϕ is called a scalar function of position or scalar point function and we say that a scalar field ϕ has been defined in R.

~~22. If $\vec{r_1} = 3\hat{i} - \hat{j}$~~

22. Given $\vec{r_1} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{r_2} = 2\hat{i} - 4\hat{j} - 3\hat{k}$, $\vec{r_3} = -\hat{i} + 2\hat{j} + 2\hat{k}$ find the magnitudes of (a) $\vec{r_3}$, (b) $\vec{r_1} + \vec{r_2} + \vec{r_3}$, (c) $2\vec{r_1} - 3\vec{r_2} - 5\vec{r_3}$.

Solution: Given that, $\vec{r_1} = 3\hat{i} - 2\hat{j} + \hat{k}$

$\vec{r_2} = 2\hat{i} - 4\hat{j} - 3\hat{k}$

$\vec{r_3} = -\hat{i} + 2\hat{j} + 2\hat{k}$

(a) $|\vec{r_3}| = \sqrt{(-1)^2 + (2)^2 + (2)^2}$

$= 3$

(b) $\vec{r_1} + \vec{r_2} + \vec{r_3} = (3\hat{i} - 2\hat{j} + \hat{k}) + (2\hat{i} - 4\hat{j} - 3\hat{k}) + (-\hat{i} + 2\hat{j} + 2\hat{k})$

$= 3\hat{i} - 2\hat{j} + \hat{k} + 2\hat{i} - 4\hat{j} - 3\hat{k} + -\hat{i} + 2\hat{j} + 2\hat{k}$

$= 4\hat{i} - 4\hat{j} + 0\hat{k}$

$= 4\hat{i} - 4\hat{j}$

$\therefore |\vec{r_1} + \vec{r_2} + \vec{r_3}| = \sqrt{(4)^2 + (-4)^2 + (0)^2}$

$= \sqrt{32}$

$= 4\sqrt{2}$

(c) $2\vec{r_1} - 3\vec{r_2} - 5\vec{r_3} = 2(3\hat{i} - 2\hat{j} + \hat{k}) - 3(2\hat{i} - 4\hat{j} - 3\hat{k}) - 5(-\hat{i} + 2\hat{j} + 2\hat{k})$

$= 6\hat{i} - 4\hat{j} + 2\hat{k} - 6\hat{i} + 12\hat{j} + 9\hat{k} + 5\hat{i} - 10\hat{j} - 10\hat{k}$

$= 5\hat{i} - 2\hat{j} + \hat{k}$

$\therefore |2\vec{r_1} - 3\vec{r_2} - 5\vec{r_3}| = \sqrt{(5)^2 + (-2)^2 + (1)^2}$

$= \sqrt{30}$

(Ans)

23. If $\vec{r_1} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{r_2} = \hat{i} + 3\hat{j} - 2\hat{k}$, $\vec{r_3} = -2\hat{i} + \hat{j} - 3\hat{k}$ and $\vec{r_4} = 3\hat{i} + 2\hat{j} + 5\hat{k}$, find scalars a, b, c such that $\vec{r_4} = a\vec{r_1} + b\vec{r_2} + c\vec{r_3}$

Solution: Given that,

$$\vec{r_1} = 2\hat{i} - \hat{j} + \hat{k}$$
$$\vec{r_2} = \hat{i} + 3\hat{j} - 2\hat{k}$$
$$\vec{r_3} = -2\hat{i} + \hat{j} - 3\hat{k}$$
and $$\vec{r_4} = 3\hat{i} + 2\hat{j} + 5\hat{k}$$

Again, Given that, $\vec{r_4} = a\vec{r_1} + b\vec{r_2} + c\vec{r_3}$

or, $3\hat{i} + 2\hat{j} + 5\hat{k} = a(2\hat{i} - \hat{j} + \hat{k}) + b(\hat{i} + 3\hat{j} - 2\hat{k}) + c(-2\hat{i} + \hat{j} - 3\hat{k})$

or, $3\hat{i} + 2\hat{j} + 5\hat{k} = (2a + b - 2c)\hat{i} + (-a + 3b + c)\hat{j} + (a - 2b - 3c)\hat{k}$

Now, equating the coefficient of i, j, k we get,

$$2a + b - 2c = 3 \quad \text{---} (1)$$
$$-a + 3b + c = 2 \quad \text{---} (2)$$
$$a - 2b - 3c = 5 \quad \text{---} (3)$$

From (1) + 2×(2) we get,

$(2a + b - 2c) + 2(-a + 3b + c) = 3 + 2 \times 2$

or, $2a + b - 2c - 2a + 6b + 2c = 7$

or, $7b = 7$

or, $b = 1$

Again from (2)+(3) we get,

$\cancel{(2a+b-2c)+}$

$(-a + 3b + c) + (a - 2b - 3c) = 2 + 5$

or, $-a + 3b + c + a - 2b - 3c = 7$

or, $b - 2c = 7$

or, $1 - 2c = 7$

or, $2c = -6$

or, $c = -3$

Putting the values of b and c in eqn ③ we get.

$$a - 2\cdot 1 - 3(-3) = 5$$

$$\text{or, } a - 2 + 9 = 5$$

$$\text{or, } a + 7 = 5$$

$$\text{or, } a = -2$$

$$\therefore (a, b, c) = (-2, 1, -3)$$

(Ans)

24. Find a unit vector parallel to the resultant of vectors $\vec{r_1} = 2\hat{i} + 4\hat{j} - 5\hat{k}$, $\vec{r_2} = \hat{i} + 2\hat{j} + 3\hat{k}$ with justification.

Solution: Given that, $\vec{r_1} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

$\vec{r_2} = \hat{i} + 2\hat{j} + 3\hat{k}$

Let the resultant vectors of the given two vectors be $\vec{R}$

$$\therefore \vec{R} = \vec{r_1} + \vec{r_2}$$

$$= (2\hat{i} + 4\hat{j} - 5\hat{k}) + (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= 3\hat{i} + 6\hat{j} - 2\hat{k}$$

$$\therefore |\vec{R}| = \sqrt{(3)^2 + (6)^2 + (-2)^2}$$

$$= \sqrt{49}$$

$$= 7$$

Hence the required unit vector $= \dfrac{\vec{R}}{|\vec{R}|}$

$$= \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$$

$= \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$

(Ans)

Justification: Here, unit vector of resultant vector

$= \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$

$\therefore \left|\frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}\right| = \sqrt{\left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2 + \left(-\frac{2}{7}\right)^2}$

$= 1$

As, $\left|\frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}\right| = 1$, Hence justified.

29. Given the scalar field by $\phi(x,y,z) = 3x^2z - xy^3 + 5$, find ϕ at the points (a) $\phi(0,0,0)$, (b) $\phi(1,-2,2)$, (c) $\phi(-1,-2,-3)$.

Solution: Given that, $\phi(x,y,z) = 3x^2z - xy^3 + 5$

(a) $\phi(0,0,0) = 3(0)^2.0 - 0.(0)^3 + 5$

$= 5$

(b) $\phi(1,-2,2) = 3(1)^2.2 - 1.(-2)^3 + 5$

$= 6 + 8 + 5$

$= 19$

(c) $\phi(-1,-2,-3) = 3(-1)^2(-3) - (-1)(-2)^3 + 5$

$= -9 - 8 + 5$

$= -12$

(Ans)