The sphere (গোলক)

1. General equation of a sphere,

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$ whose centre is at $(-u,-v,-w)$ and radius is $\sqrt{u^2+v^2+w^2-d}$

2. Equation of the sphere through the origin,

$$x^2+y^2+z^2+2ux+2vy+2wz=0$$

whose centre is at $(-u,-v,-w)$ and radius $\sqrt{u^2+v^2+w^2}$

3. Equation of a sphere whose centre is at (a,b,c) and radius r is, $(x-a)^2+(y-b)^2+(z-c)^2=r^2$

4. Equation of a sphere whose centre is at $(0,0,0)$. i.e, origin and radius r is $x^2+y^2+z^2=r^2$

5. The equation of a sphere through the two end points of a diameter is,

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+(z-z_1)(z-z_2)=0$$

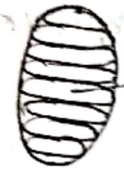
6. The equation of a tangent plane to a sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ at the point (x_1,y_1,z_1) is, $xx_1+yy_1+zz_1+u(x+x_1)+v(y+y_1)+w(z+z_1)+d=0$

7. The equation of a circle

$$x^2+y^2+z^2+2ux+2vy+\cancel{2z}\,2wz+d=0 = ax+by+cz+d$$

or,
$$\left.\begin{aligned} x^2+y^2+z^2+2ux+2vy+2wz+d&=0 \\ ax+by+cz+d&=0 \end{aligned}\right\}$$

8. The equation of the sphere through the above circle, $x^2+y^2+z^2+2ux+2vy+2wz+k(ax+by+cz+d)=0$

9.  → Great circle with respect to the sphere.

(Centre এর circle হলো Great circle.)

Centre of the great circle = centre of the sphere

Radius of the great circle = Radius of the sphere

① Find the equation of the sphere with centre $(2,1,-3)$ that is tangent to the plane $x-3y+2z-4=0$

Solution: Given the plane,

$$x-3y+2z-4=0 \quad \text{—}$$

Given the centre of the required sphere is $(2,1,-3)$

Let the radius be r has the equation of sphere

$$(x-2)^2+(y-1)^2+(z+3)^2=r^2 \quad \text{——} ①$$

Again, the sphere ① is the tangent of the plane $x-3y+2z-4=0$

so, the radius of the sphere is the perpendicular distance from the origin to the plane.

That is $r = \left| \frac{1.2+3.1+2(-3)-4}{\sqrt{1^2+3^2+2^2}} \right|$

$$= \frac{11}{\sqrt{14}}$$

Now, putting the values of r in eq. ① we get

$$(x-2)^2+(y-1)^2+(z+3)^2 = \left(\frac{11}{\sqrt{14}}\right)^2$$

$$\Rightarrow x^2-4x+4+y^2-2y+1+z^2+6z+9 = \frac{121}{14}$$

$$\Rightarrow 14(x^2+y^2+z^2)-56x-28y+84z+75=0$$

(Ans)

② Find the equation of the sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z=8$ is a great circle.

Solution: Given that,
$$\left.\begin{array}{l} x^2+y^2+z^2+7y-2z+2=0 \\ 2x+3y+4z=8 \end{array}\right\} \quad \text{--- ①}$$

The equation of the sphere through the circle is ① is,
$$x^2+y^2+z^2+7y-2z+2+k(2x+3y+4z-8)=0 \quad \text{--- ②}$$
$$\Rightarrow x^2+y^2+z^2+2kx+(3k+7)y+(4k-2)z+2-8k=0$$

Here the centre is at $\left(-k, -\frac{3k+7}{2}, 1-2k\right)$

Since the given circle is a great circle, so the centre, $\left(-k, \frac{-3k+7}{2}, 1-2k\right)$ must lies on the plane
$$2x+3y+4z-8=0$$

i.e, $2(-k)+3\left\{\frac{-(3k+7)}{2}\right\}+4(1-2k)-8=0$
$$\Rightarrow -4k-3(3k+7)+8(1-2k)-16=0$$
$$\Rightarrow -4k-9k-21+8-16k-16=0$$
$$\Rightarrow -29k-29=0$$
$$\therefore k=-1$$

Putting the value of k in eq-② we have,
$$x^2+y^2+z^2+7y-2z+2-1(2x+3y+4z-8)=0$$
$$\Rightarrow x^2+y^2+z^2-2x+4y-6z+10=0$$ which is the required sphere.

③ Find the equation of the sphere with center $(2,-3,2)$ and tangent to the plane $6x-3y+2z-8=0$

<u>Solution:</u> Given the centre of the required sphere is $(2,-3,2)$,

Let the radius be r has the equation of the Sphere,

$$(x-2)^2+(y+3)^2+(z-2)^2=r^2 \quad \text{--- ①}$$

Again, the sphere ① is the tangent of the plane

$$6x-3y+2z-8=0$$

So the radius of the sphere is perpendicular distance from the origin to the plane.

That is $r = \left| \dfrac{6\cdot 2-3(-3)+2\cdot 2-8}{\sqrt{6^2+(-3)^2+(2)^2}} \right|$

$$= \frac{17}{7}$$

Now, putting the value of r in eq ①,

$$(x-2)^2+(y+3)^2+(z-2)^2=\left(\frac{17}{7}\right)^2$$

$$\Rightarrow x^2-4x+4+y^2+6y+9+z^2-4z+4=\frac{289}{49}$$

$$\Rightarrow 49(x^2-4x+4+y^2+6y+9+z^2-4z+4)-289=0$$

$$\Rightarrow 49(x^2+y^2+z^2)-196x+196+294y+441-194z+196-289=$$

$$\Rightarrow 49(x^2+y^2+z^2)-196x+294y-194z+544=0.$$

<u>(Ans)</u>

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④ Find the equation of the sphere with its centre at $(-4, 2, 3)$ and tangent to the plane $2x - y - 2z + 7 = 0$.

Solution: Given the centre of the sphere is $(-4, 2, 3)$

Let the radius be r has the equation of the sphere,

$$(x+4)^2 + (y-2)^2 + (z-3)^2 = r^2 \quad \text{①}$$

Again, the sphere ① is the tangent of the plane, $2x - y - 2z + 7 = 0$

So, the radius of the sphere is perpendicular distance from the origin to the plane.

That is, $$r = \left| \frac{2(-4) - 1(2) - 2(3) + 7}{\sqrt{2^2 + (-1)^2 + (-2)^2}} \right|$$

$$= \left| \frac{-9}{3} \right|$$

$$= \frac{9}{3}$$

$$= 3$$

Now, putting the value of r in eq ①

$$(x+4)^2 + (y-2)^2 + (z-3)^2 = 3^2$$

$$\Rightarrow x^2 + 8x + 16 + y^2 - 4y + 4 + z^2 - 6z + 9 - 9 = 0$$

$$\Rightarrow x^2 + y^2 + z^2 + 8x - 4y - 6z + 20 = 0$$

(Ans)