Loss Versus Rebalancing (LVR) Computation

Assumptions

Let's say that the following is true:

- There is a DEX where asset prices are determined by the $xy = k$ bonding curve.
- The asset X is volatile (e.g. ETH), and its price is defined in terms of Y (e.g. USDC), i.e. $P_{DEX} = \frac{y}{x}$
- A LP provides full-range liquidity on the DEX.
- DEX traders are required to pay swap fee $f_{swap} = \gamma \cdot V$ that depends on the trade volume; the fee is paid in the assets sold by the trader.
- The swap fees are not compounding.
- There is a CEX which trades X and Y for the price P_{CEX}, where the traders are not required to pay any trading fees, and its liquidity is infinitely deep.
- There is a CEX/DEX arbitrager that has unlimited amount of Y, fast connections to both CEX and DEX, and will take all profitable trades at their maximum volume.
- Arbitrage trades can only take place in discrete time intervals (blocks).
- Each arbitrage trade is required to pay a fixed transaction fee f_{tx}.

Metrics

There are several distinct but related metrics that can be computed for a distinct time interval:

- LVR of the LP *vs.* someone who has the same portfolio rebalanced on the CEX;
- PnL of the LP, defined as *fee income* $-$ *LVR*.
- Profits of the arbitrager, defined as *LVR* $-$ *fee income* $-Kf_{tx}$, where K is the number of arbitrage transactions that took place.

Example Mechanism

If the price of X in the DEX is below the price on the CEX ($P_{DEX} < P_{CEX}$), the arbitrager might buy some X on the DEX to sell it on the CEX.

If $P_{DEX} > P_{CEX}$, the arbitrager might buy some X in the CEX and sell it on the DEX.

After each trade, the prices are made equal: $P_{DEX} = P_{CEX}$.

Empirical LVR Estimation Algorithm

To estimate the empirical LVR in a situation where these assumptions are true, let fix a specific time period with duration τ, and let's say that we have the history of CEX prices at the discrete time intervals $P_{CEX} = \{p_1, p_2, \ldots, p_n\}$, where N is the total number of blocks in the interval τ. The blocks can uniformly distributed over time, Poisson-distributed, or otherwise, the result is not affected.

Algorithm 1 LVR and related metric estimation

```
function ESTIMATELVR(x, y, P_CEX)
    R_x ← x                                  ▷ X reserves in the pool
    R_y ← y                                  ▷ Y reserves in the pool
    L ← √(xy)                                ▷ Liquidity in the pool
    γ ← 0.003                                ▷ Fee tier, 0.3 % here

    LVR ← 0
    PnL_arb ← 0
    F_swap ← 0

    for p_i ∈ P_CEX do
        Δx ← L/√p_i − R_x
        Δy ← L · √p_i − R_y

        if Δx > 0 then
                                             ▷ Arber sells X to the DEX
            f_swap ← γ · Δx · p_i
        else
                                             ▷ Arber buys X from the DEX
            f_swap ← γ · Δy
        end if

        lvr_swap ← −(Δx · p_i + Δy)
        pnl_arb ← lvr_swap − f_swap − f_tx

        if pnl_arb > 0 then
                                             ▷ The trade is profitable to the arbitrager
            LVR += lvr_swap
            PnL_arb += pnl_arb
                                             ▷ Update the pool reserves
            R_x += Δx
            R_y += Δy
                                             ▷ Update the collected fees
            F_swap += f_swap
        end if
    end for

    PnL_LP ← F_swap − LVR
    return LVR, PnL_arb, PnL_LP
end function
```