Adding assets to a pool in a specific proportion

Let's look at Uniswap v3 position that has assets of types X and Y, in amounts x and y, respectively. The values V_x and V_y of the tokens in the pool are:

$$V_x = x \cdot P \tag{1}$$

$$V_y = y \tag{2}$$

Balanced positions

If assets are added in 50:50 proportion, then value V_x of the X tokens in the position is equal to the value V_y of the Y tokens in the position, at the current price P: $V_x = V_y$.

Since the values are equal, the price range is symmetrical on both sides of the central (current) price, and we can define a single range factor r so that:

$$P_a = \frac{P}{r^2} \tag{3}$$

$$P_b = P \cdot r^2 \tag{4}$$

It's more convenient to use the square of r, because a lot the equations involve square roots of prices.

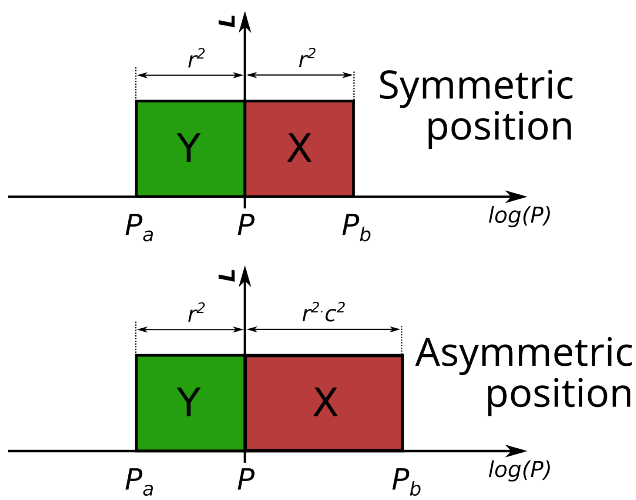

Figure 1: Symmetric and asymmetric concentrated liquidity positions.

Unbalanced positions with specific token value ratio

Let's say that an user wants to create a position with a specific value ratio, for example 80 % in X and 20 % in Y. The value ratio R in this example is 4:1.

Such as position is going to be asymmetric with respect to the current price, and we can introduce a new skew factor c to account for the size of this assymetry:

$$P_a = \frac{P}{r^2} \tag{5}$$

$$P_b = P \cdot r^2 \cdot c^2 \tag{6}$$

subject to $c > \frac{1}{r}$, because we want to exclude single-sided / out-of-range positions.

Question: ***how to find the value of c from a given R, and vice versa?***

By definition and Eq. 1:

$$R = \frac{V_x}{V_y} = \frac{x \cdot P}{y} \tag{7}$$

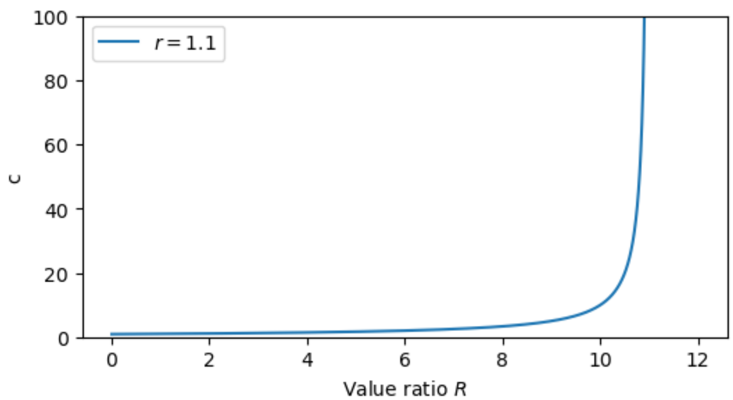

Figure 2: The skew factor c depending on the desired value ratio.

The amounts x and y are given by liquidity math formulas:

$$x = L\frac{\sqrt{P_b} - \sqrt{P}}{\sqrt{P} \cdot \sqrt{P_b}} \tag{8}$$

$$y = L(\sqrt{P} - \sqrt{P_a}) \tag{9}$$

Putting the right-side equation from above in Eq. 7, replace P_a, P_b using Eq. 5 and simplify. The liquidity amounts L cancel out, as they're are equal on both sides of the current price. After simplification:

$$R = \frac{r \cdot c - 1}{r \cdot c - c} \tag{10}$$

Now introduce $q := r - 1$, and simplify it further:

$$R = 1 + \frac{1}{q} - \frac{1}{q \cdot c} \tag{11}$$

The last term is negative and its magnitude grows inversely with c. As a result, when c grows R also grows, and when c gets smaller R also gets smaller.

Solve in the opposite direction:

$$c = \frac{1}{1 + q - R \cdot q} = \frac{1}{r - R \cdot r + R} \tag{12}$$

Question: ***can we get an arbitrary value ratio*** R***?***

The answer is no. If we fix a specific r then the minimum possible ratio R can be arbitrary close to zero, when c approaches the minimum allowed value $\frac{1}{q}$.

However, the maximum possible ration R is finite, and limited by Eq. 11. For example, if we select $r = 1.1$, it follows that $q = 0.1$, and the limit value of R is given by $1 + \frac{1}{q} = 1 + 10 = 11$, when c goes to infinity. See the figure for a graphical demonstration of this.