

## Empirical Loss Versus Rebalancing (LVR) Estimation

### Assumptions

Let's say that the following is true:

- There is a DEX where asset prices are determined by the  $xy = k$  bonding curve.
- The asset  $X$  is volatile (e.g. ETH), and its price is defined in terms of  $Y$  (e.g. USDC), i.e.  $P_{DEX} = \frac{y}{x}$ .
- A LP provides full-range liquidity on the DEX.
- DEX traders are required to pay swap fee  $f_{swap} = \gamma \cdot V$  that depends on the trade volume; the fee is paid in the assets sold by the trader.
- The swap fees are not compounding.
- There is a CEX which trades  $X$  and  $Y$  for the price  $P_{CEX}$ , where the traders are not required to pay any trading fees, and its liquidity is infinitely deep.
- There is a CEX/DEX arbitrageur that has unlimited amount of  $Y$ , fast connections to both CEX and DEX, and will take all profitable trades at their maximum volume.
- Arbitrage trades can only take place in discrete time intervals (blocks).
- Each arbitrage trade is required to pay a fixed transaction fee  $f_{tx}$ .

### Metrics

There are several distinct but related metrics that can be computed for a distinct time interval:

- LVR of the LP vs. someone who has the same portfolio rebalanced on the CEX;
- PnL of the LP, defined as  $fee\ income - LVR$ .
- Profits of the arbitrageur, defined as  $LVR - fee\ income - K f_{tx}$ , where  $K$  is the number of arbitrage transactions that took place.

### Example Mechanism

If the price of  $X$  in the DEX is below the price on the CEX ( $P_{DEX} < P_{CEX}$ ), the arbitrageur might buy some  $X$  on the DEX to sell it on the CEX.

If  $P_{DEX} > P_{CEX}$ , the arbitrageur might buy some  $X$  in the CEX and sell it on the DEX.

After each trade, the prices are made equal:  $P_{DEX} = P_{CEX}$ .

## Empirical LVR Estimation Algorithm

To estimate the empirical LVR in a situation where these assumptions are true, let fix a specific time period with duration  $\tau$ , and let's say that we have the history of CEX prices at the discrete time intervals  $P_{CEX} = \{p_1, p_2, \dots, p_n\}$ , where  $N$  is the total number of blocks in the interval  $\tau$ . The blocks can uniformly distributed over time, Poisson-distributed, or otherwise, the result is not affected.

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### Algorithm 1 LVR and related metric estimation

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function ESTIMATELVR( $x, y, P_{CEX}$ )
   $R_x \leftarrow x$                                  $\triangleright$   $X$  reserves in the pool
   $R_y \leftarrow y$                                  $\triangleright$   $Y$  reserves in the pool
   $L \leftarrow \sqrt{xy}$                              $\triangleright$  Liquidity in the pool
   $\gamma \leftarrow 0.003$                              $\triangleright$  Fee tier, 0.3 % here

   $LVR \leftarrow 0$ 
   $PnL_{arb} \leftarrow 0$ 
   $F_{swap} \leftarrow 0$ 

  for  $p_i \in P_{CEX}$  do
     $\Delta x \leftarrow L/\sqrt{p_i} - R_x$ 
     $\Delta y \leftarrow L \cdot \sqrt{p_i} - R_y$ 

    if  $\Delta x > 0$  then                             $\triangleright$  Arber sells  $X$  to the DEX
       $f_{swap} \leftarrow \gamma \cdot \Delta x \cdot p_i$ 

    else                                             $\triangleright$  Arber buys  $X$  from the DEX
       $f_{swap} \leftarrow \gamma \cdot \Delta y$ 

    end if

     $lvr_{swap} \leftarrow -(\Delta x \cdot p_i + \Delta y)$ 
     $pnl_{arb} \leftarrow lvr_{swap} - f_{swap} - f_{tx}$ 

    if  $pnl_{arb} > 0$  then                             $\triangleright$  The trade is profitable to the arbitrage
       $LVR += lvr_{swap}$ 
       $PnL_{arb} += pnl_{arb}$                              $\triangleright$  Update the pool reserves
       $R_x += \Delta x$ 
       $R_y += \Delta y$                              $\triangleright$  Update the collected fees
       $F_{swap} += f_{swap}$ 

    end if
  end for

   $PnL_{LP} \leftarrow F_{swap} - LVR$ 
  return  $LVR, PnL_{arb}, PnL_{LP}$ 
end function

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