Cumulant Function

$$S_x(t) = n \log [pe^t + (1-p)]$$

$$E(x) = \frac{dS_x(t)}{dt} = n \cdot \frac{p \cdot e^t}{pe^t + (1-p)} \Big|_{t=0} = np$$

$$Var(x) = \frac{d^2 S_x(t)}{dt^2} = n \frac{pe^t \cdot [pe^t + (1-p)] - pe^t \cdot pe^t}{(pe^t + (1-p))^2}$$

$$= \frac{npe^t (1-p)}{(1)} \Big|_{t=0}$$

$$= np(1-p)$$

Characteristic Function

$$\varphi_x(t) = [pe^{it} + (1-p)]^n$$

$$\frac{d\varphi_x(t)}{dt} = n[pe^{it} + (1-p)]^{n-1} \cdot pie^{it} \Big|_{t=0}$$

$$= npi \Rightarrow E(x) = (-i) \cdot \varphi^{(1)}(0)$$

$$= npi(-i) = np.$$

$$\frac{d^2 \varphi_x(t)}{dt^2} = npi \cdot \{(n-1)[pe^{it} + (1-p)]^{n-2} \cdot pi \cdot e^{it} \cdot e^{it}$$

$$[pe^{it} + (1-p)]^{n-1} \cdot i \cdot e^{it}\} \Big|_{t=0}$$

$$= npi\{(n-1)pi + i\} \qquad E(x^2) = (-i)^2 \varphi^{(2)}(0)$$

$$= -n(n-1)p^2 - np \qquad = n(n-1)p^2 + np.$$

$$Var = E(x^2) - E(x)^2$$