Department of Economics
University of California, Riverside

CORE CUMULATIVE EXAMINATION FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

MICROECONOMIC THEORY

September 19, 2008

The examination contains four equally weighted parts. Please allocate your time carefully and write concisely and legibly. The exam will be monitored by the Graduate Assistant. There will be no communication with faculty members; if you believe a question contains an error or an ambiguity, say so on your written examination, make an assumption to correct the alleged error or to resolve the ambiguity, and answer the question as well as you can.

Part I. Answer all three questions.

1. Explain, in general and in the specific context of income-constrained utility maximization, the difference between a local and a global maximum. Under what assumption is a local maximum also a global maximum, again in general and in the specific context of income-constrained utility maximization? Explain your answer.

2. The expected utility function exists under several axioms. Describe at least two of them and briefly explain their roles in establishing the existence of the expected utility function.

3. Comment on each of the following statements:

   (a) If a competitive equilibrium exists, preferences must be convex.

   (b) Whenever both commodity prices increase (in a two-commodity economy), the real income of workers declines.

Part II. Answer both questions.

1. The demand share functions are defined by $s_i(p,y) := p_i \cdot \phi_i(p,y)/y,\ i = 1,\ldots,n$, where p is the price vector, y is the income endowment, and $\phi_i,\ i = 1,\ldots,n$, are the Marshallian demand functions.

   (a) Prove that

   $$s_i(p/y) = -\frac{\partial V(p,y)/\partial \ln p_i}{\partial V(p,y)/\partial \ln y} \quad \forall\, i,$$

   where V is the indirect utility function.

   (b) Without doing any calculations, explain why it is also true that

   $$s_i(p/y) = -\frac{\partial \ln V(p,y)/\partial \ln p_i}{\partial \ln V(p,y)/\partial \ln y} \quad \forall\, i.$$

   (c) Derive the share functions from the translog indirect utility function,

   $$\ln \hat{V}(p/y) := \alpha_0 + \sum_i \alpha_i \ln(p_i/y) + \frac{1}{2}\sum_i \sum_j \beta_{ij} \ln(p_i/y)\ln(p_j/y),$$

   where α_0, α_i $(i = 1,\ldots,n)$, and β_{ij} $(i,j = 1,\ldots,n)$ are parameters satisfying $\beta_{ij} = \beta_{ji}$ for all i,j. (If you are unable to do the calculations for this n-commodity translog, answer this part for the special case where $n = 2$.)

2. The distance function is given by $D(u,x) = \Gamma(x)/\phi(u)$, where u is a scalar output quantity, x is an input quantity vector, and Γ and ϕ are functions.

   (a) Write the expression for the technology (set) in terms of this distance function.

   (b) What properties of the technology guarantee that the distance function is well defined? Explain why each is needed.

   (c) Derive the production function from this distance function.

   (d) Explain why this production function is homothetic.

   (e) Under what condition is this production function homogeneous (of degree α)?

Part III. Signaling with useful education.

A worker with education level e knows his productivity $\theta(1+e)$ while his potential employer does not know θ (but the employer can observe e). The value of the worker to the employer is his expected productivity: we assume that the employer pays the worker a wage w that is equal to his expected productivity (the job market is competitive).

The worker chooses the level of education e, which the employer observes, and receives a payoff of $w - \frac{e^2}{\theta}$. Assume that θ takes the value of either θ^L or θ^H, $\theta^H > \theta^L$, and that the probability that a worker is of type θ^H is p^H.

Assume that $\theta^H = 3, \theta^L = 2$, and $p^H = 0.5$.

(a) What would be the equilibrium under symmetric information, i.e., when the employer knows the worker's type?

(b) Define the concept of perfect Bayesian equilibrium (PBE) *in the context of this question.*

(c) Under asymmetric information, what is the minimum level of education $\underline{e}$ that type θ^H worker acquires in a separating equilibrium in pure strategies? What is the maximum? What is the level of education that type θ^L worker acquires in separating equilibria? Specify a separating equilibrium where type θ^H worker acquires the level of education $\underline{e}$. Show your work.

(d) What is the maximum level of education that can be supported by a pooling equilibrium? Show your work.

Part IV.

"Since competitive markets ensure the efficient allocation of resources, government intervention to remove a barrier to competition is warranted, irrespective of the presence of other distortions in the economy." Evaluate this statement.

Part III

1. Prove or disprove the following claims.
(a) Assignment of property rights necessarily resolves the problem of inefficiency in the presence of externality.
(b) The welfare loss cannot be avoided in a few sellers industry.

2. Write down the conditions or assumptions for a proof of existence of a competitive equilibrium (CE) in an exchange economy. Outline the role of those assumptions in establishing the existence of a CE.

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CORE CUMULATIVE EXAMINATION FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

MICROECONOMIC THEORY

July 7, 2007

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Part I. Answer all of the following questions.

1 Answer both parts of the following question.

(a) Using the requisite mathematical theorem, sketch a proof of the fact that continuity of preferences (along with positive prices and a complete preordering of the consumption set) suffices for the existence of an optimal consumption bundle in a non-empty budget set.

(b) Provide an example showing that continuity of preferences is not necessary for the existence of an optimal bundle at *all* positive prices.

2. What is the full dimensionality condition in the Perfect Folk Theorem? Explain briefly why sufficient dimensionality is important in establishing the result.

3. Evaluate each of the following statements.

(a) Pareto efficiency is not attainable in the absence of perfect competition.

(b) An aggregation procedure that maps a profile of individual preference orderings to a social preference relation is necessarily dictatorial if it is required to satisfy the following conditions: unrestricted domain, weak Pareto principle and independence of irrelevant alternatives.

Part II. Answer the following question.

Consider the utility function with image $U(x_g, m)$, where x_g is consumption of gasoline and m is a Hicks composite commodity (Hicks-Marshall money).

(a) What assumptions are needed to express the consumer's utility as a function of these two variables?

(b) Find, illustrate, and interpret the first-order (necessary) conditions for (x_g^*, m^*) to be an interior solution to the problem of maximizing this utility function subject to a budget constraint.

(c) Write the second-order necessary conditions for (x_g^*, m^*) to be an interior solution to the problem of maximizing this utility function subject to a budget constraint.

(d) Now suppose the government decides to ration gasoline, limiting each consumer to $\overline{x}_g$ units per time period, and find, illustrate, and interpret the first-order (necessary) conditions for (x_g^*, m^*) to be a (not necessarily interior) solution to the problem of maximizing this utility function subject to a budget constraint. (Be sure to consider all interesting cases.)

(e) Illustrate the loss of consumer surplus—both the compensating and the equivalent variation—from the imposition of rationing.

(f) What structure for the function U would guarantee that these compensating and equivalent variations are equal? Illustrate your answer.

Part III. Answer **ONE** and **ONLY ONE** of the following two questions.

1. Pareto efficiency is not a useful concept in resource allocation in a real world since it is attainable only in a private good economy without any externality. Evaluate.

2. Consider a private ownership economy with two produced final goods (A and B), two non-produced factors of production (L and K) available in fixed quantities, one consumer, and two producers one of whom produces only A and the other produces only B. The production functions of the two producers are given by

$$A = G(L_A, K_A)$$

and

$$B = H(L_B, K_B, K_A),$$

where L_A is the amount of L employed in industry A, K_A is the amount of K employed in industry A, and so on. Assume that $\frac{\partial B}{\partial K_A} < 0$ (thus, there is an external diseconomy generated by the use of K in industry A).

(a) Derive the first-order conditions for Pareto optimality of an allocation in this economy (ignore the possibility of some of the quantities being zero in a Pareto optimal allocation).

(b) Interpret the conditions that you derive in answering (a).

(c) Show that a competitive equilibrium allocation will not be Pareto optimal in this economy (assume that all the quantities figuring in a competitive equilibrium are positive).

(d) Show that, in the absence of any government action relating to the external diseconomy, it is not possible to achieve a given Pareto optimal allocation for this economy through a competitive equilibrium, no matter what the government may do about the distribution of income. (Assume that all the quantities figuring in the given Pareto optimal allocation are positive.)

(e) Specify a Pigovian tax scheme that will correct the "distortion" caused by the external diseconomy.

(f) What information will be necessary to devise such a Pigovian tax scheme?

(g) Give a "real life" example of the type of externality referred to in this question.

Part IV. Answer the following question.

Feints. Army A has to decide whether to march through a narrow mountain pass (p), or through a broad road (r); Army B has to decide whether to defend the pass or the road. If one of the armies chooses the pass and the other chooses the road, A gets a payoff of 1 and B gets a payoff of -1 because it fails to stop Army A; if they choose the same location, then A gets a payoff of -1 and B gets a payoff of 1, because it succeeds in stopping A.

Before they simultaneously choose between the pass and the road, Army B will make defensive preparations at either the pass, or at the road (but **not** at both locations). Such preparations are observed by A, but have **no** impact on either A or B's payoffs, e.g., the preparations cost nothing. However, with some probability $n \in [0, 1/2)$, Army B is strategically "naive," in that it is committed to an equal mix of the following two strategies: prepare at the pass and defend the road, and prepare at the road and defend the pass. With probability $1 - n$, Army B is "rational" such that these preparations are just "feints": they don't indicate anything about where Army B actually chooses to defend.

(a) Describe this game formally: what are Army A and B's strategies? What is a perfect Bayesian Equilibrium **in this context**?

(b) Suppose that Army B made preparations at the pass. What are its respective expected payoffs if it chooses to defend the pass and if it chooses to defend the road, given A's strategy?

(c) Suppose that Army B prepares at the pass and at the road with equal probability. What must Army A do to support this strategy?

(d) Find a PBE in which a rational Army B prepares at the pass and at the road with equal probability.

(e) Is there any correlation in this equilibrium between the location at which rational B prepares and the location it defends? Show this and explain intuitively.

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CORE CUMULATIVE EXAMINATION FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

MICROECONOMIC THEORY

September ~~14~~ 21, 2007

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Part I. Answer all of the following questions.

1. Define the elasticity of scale and explain how it is simplified by the assumption that the technology is homothetic and further simplified by the assumption that the technology is homogeneous (of degree α).

2. Indicate whether the following statement is true or false. Prove it if it is true; explain and provide a counterexample if it is false.

   "In a finite Prisoners' Dilemma game with incomplete information, the only possible equilibrium is for both players to defect at every stage."

3. Indicate whether the following statement is true or false. Give justification for your answer.

   "If a competitive consumer has intransitive preferences, then there must exist a budget set for which a best commodity bundle defined in terms of the consumer's preferences will not exist."

Part II. Answer the following question.

Consider the following utility function:

$$U^i(x_i, m) = x_i^{1/2} + m, \qquad (*)$$

where x_i is the quantity consumed of commodity i and m is expenditure on all other commodities (Hicks-Marshall money). In answering the following questions (all of which pertain to the particular utility function $(*)$), short cuts are allowable so long as they are carefully justified.

(a) What assumptions rationalize this representation of consumer preferences (as a function of x_i and m)?

(b) Show that U^i is strictly quasi-concave (relative to $\mathbf{R}^2_+$).

(c) Derive the (Marshallian) demand functions for commodity i and Hicks-Marshall money (the Hicks composite commodity) as a function of total expenditure y and the price of the i^{th} commodity, p_i.

(d) Derive the indirect utility function dual to $(*)$ and verify your derivation of the demand function for commodity i using the envelope theorem.

(e) Derive the substitution and income effects of a change in p_i on the demand for commodity i.

(f) Derive the expenditure function and the (Hicksian) compensated demand for commodity i.

(g) Find the expression for the Marshallian consumer surplus owing to a change in the price of commodity i from $\bar{p}_i$ to p_i'.

(h) How does this Marshallian consumer surplus compare to the Hicksian notions of consumer surplus (compensating and equivalent variation)?

Part III. Answer the following question.

With appropriate assumptions, show that a Pareto optimal allocation in a pure exchange economy (i.e., an economy without any production) can be supported by a competitive equilibrium if the government introduces a suitably devised lump sum tax-transfer scheme. Comment on the intuitive plausibility of the assumptions.

Part IV. Answer the following question.

Consider a market for used cars. Only the seller knows whether the car is in good condition or not. Buyers know only that half the cars on sale are good and half are bad. A good used car is worth v dollars and is very unlikely to break down quickly, but if it does, it costs k dollars to fix it. A bad used car is worth $v/2$ dollars as the chance of breaking down quickly is $1/2$, and when it does, it costs $2k$ dollars to fix it.

The sellers compete by offering warranty: a warranty of w percent means the seller must reimburse the buyer w percent of the cost of fixing a breakdown when it happens (w can be greater than one). Suppose that the market is a seller's market, e.g., there are infinitely many buyers, thus the seller gets the expected worth of his car. Note that the warranty does not directly affect the price of the car.

(a) Describe this game formally. What kind of game is this? What is a perfect Bayesian Equilibrium in this context?

(b) If the buyer knows the value of the car, what prices would he pay? If the buyer does not know the value of the car, and no warranty is offered by any seller, what would he pay?

(c) Construct a pooling equilibrium in which no warranty is ever offered. Is this equilibrium plausible? Explain briefly.

(d) Find a separating equilibrium of this game, show your work. How does your answer depend on the cost of repairs k and value of car v?

Department of Economics
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CORE CUMULATIVE EXAMINATION FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

MICROECONOMIC THEORY

July 6, 2007

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Part I. Answer all of the following questions.

1. Explain why theme parks (*e.g.*, Disneyland) charge a flat fee for entry and no charge for specific rides and other attractions (rather than charging separately for each ride or show).

2. Recall the standard Rubinstein infinite-horizon alternating offer bargaining game with two players $i = 1, 2$. The discount rates of the players are respectively δ_1, δ_2. For each of the following claims, prove it if it is true, and give a counterexample if it is false.

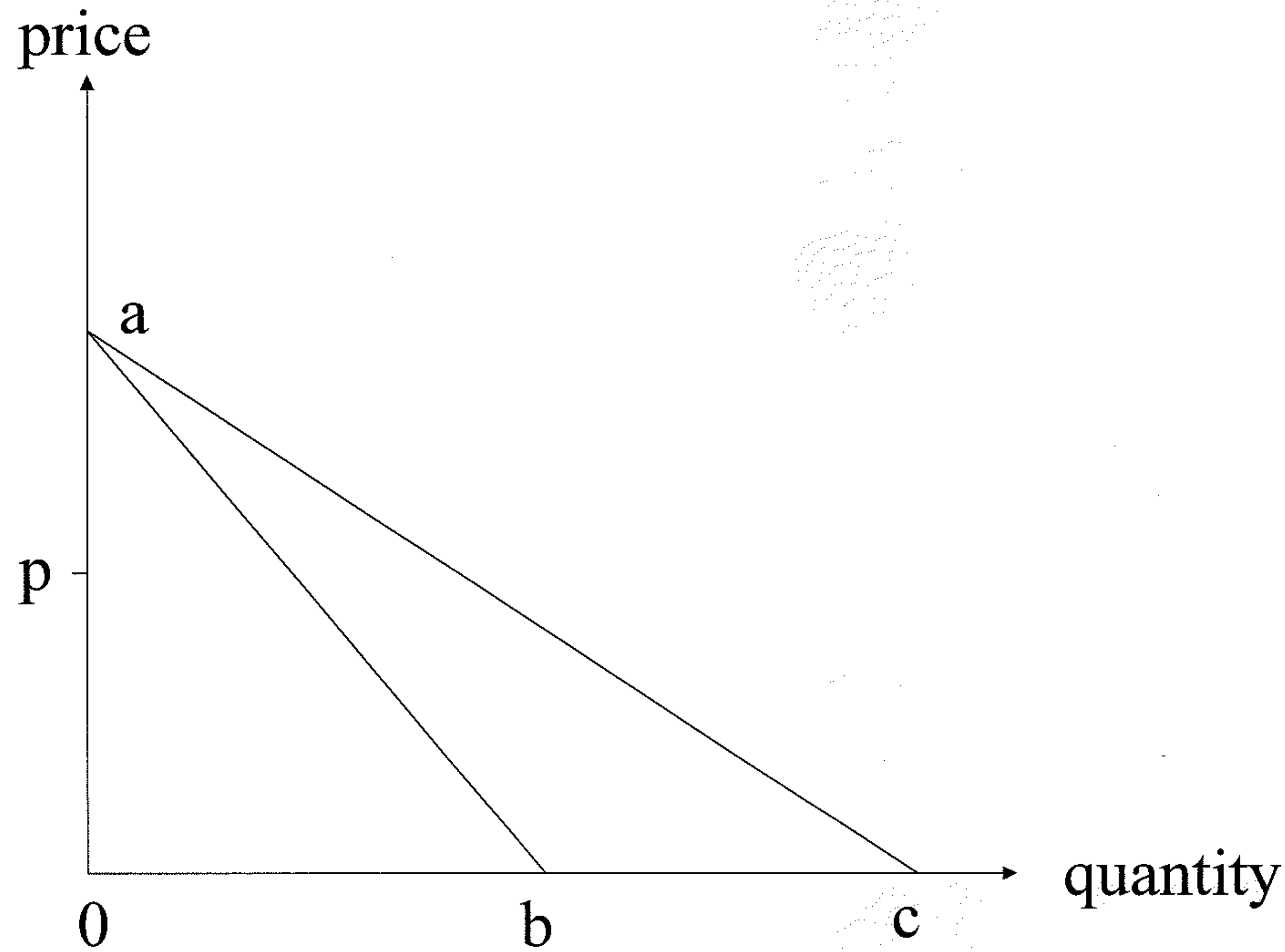
   (i) This game has a unique Nash equilibrium.

   (ii) The player who is more patient (has a higher discount rate) always gets a higher payoff in the SPE.

3. For each of the following statements, indicate whether it is true or false and give justification for your answer.

   (i) A move from a Pareto inoptimal allocation to a Pareto optimal allocation will make somebody better off without making anybody worse off.

   (ii) Since the demand curve ac below is flatter than the demand curve ab, the elasticity of demand at price p is greater for a consumer who has the demand curve ac than for a consumer who has the demand curve ab (see the graph on the next page).


price
a
p
0
b
c
quantity

Part II. Answer both questions.

1. Below are four notions of "nonsatiation" of preferences that can be found in textbooks (where X is the consumption set, $\succeq$ is the weak-preference relation, and $\succ$ is the strict-preference relation):

   (a) There does not exist an $\bar{x} \in X$ such that $\bar{x} \succeq x \ \forall \ x \in X$.

   (b) There does not exist an $\bar{x} \in X$ and an $\epsilon > 0$ such that $\bar{x} \succeq x \ \forall \ x \in N_\epsilon(\bar{x})$.

   (c) For all $\langle \bar{x}, x \rangle \in X \times X$, $\bar{x} > x \implies \bar{x} \succ x$.

   (d) For all $\langle \bar{x}, x \rangle \in X \times X$, $\bar{x} \geq x \implies \bar{x} \succeq x$.

   Which of these assumptions is most appropriate for assuring that, at the optimum, the budget constraint is satisfied as an equality (rather than an inequality). Explain, using illustrations as appropriate.

2. Exploiting the properties of the profit function π (and assuming it is twice differentiable), prove the following comparative static properties:

$$\frac{\partial d_j(p,w)}{\partial w_j} \leq 0 \quad \forall \ j,$$

   and

$$\frac{\partial s_i(p,w)}{\partial p_i} \geq 0 \quad \forall \ i,$$

   where d_j is the demand function for the j^{th} input, s_i is the supply function of the i^{th} output, and p and w are the output and input price vectors.

Part III. Answer the following question.

Consider a private ownership economy with two (produced) consumption goods, A and B, and two (non-produced) factors of production, L and K. The aggregate initial endowment of the economy consists of 0 units of each of A and B, 100 units of L, and 150 units of K. There are exactly two firms; one of the firms produces A and the other produces B. The production functions are given by

$$A \leq \min(L_A, K_A),$$

and

$$B \leq L_B + K_B$$

(the notation has the usual meaning). Every consumer $i \in \{1, 2, ..., n\}$ in the economy has the following utility function:

$$U^i(A_i, B_i) = A_i B_i,$$

where A_i and B_i denote i's consumption of A and B, respectively.

First, assuming the economy to be closed, answer the following questions.

(a) What are the production-wise efficient allocations of L and K between the two industries? (If you like, you can show these allocations in a production box diagram.)

(b) Identify the production possibility frontier of the economy.

(c) Assuming B to be the numeraire, what will be the prices in a competitive equilibrium for this economy and what quantities of A and B will be produced in a competitive equilibrium? (Give the reasoning underlying your answers.)

Now assume that: (1) the economy is an open economy; (2) A and B are traded in the international market, but L and K are not; (3) the country takes the international prices of A and B as given; and (4) the price of A in terms of B in the international market is 3.

(d) How will this new specification change your answers to the questions in (c) above? (Give the reasoning underlying your answer.)

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CORE CUMULATIVE EXAMINATION FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

MICROECONOMIC THEORY

September 22, 2006

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Part I. Short-Answer Questions.

1. To guarantee that the optimal consumption bundle is on the budget hyperplane, some textbooks assume strict monotonicity of the utility function (globally positive marginal utilities). Is this a sensible approach? Would weak monotonicity (globally non-negative marginal utilities) work as well? Explain your answers carefully and fully.

2. State the (Perfect) Folk Theorem of infinitely repeated games, defining the terms arising in the theorem (there is no need to define the basic concepts of equilibrium).

3. For each of the following statements, give an example to show that the statement is true (diagrammatic examples will suffice):

(1) a private-ownership economy may not have a competitive equilibrium;

(2) a private-ownership economy may have an infinite number of competitive equilibria; and

(3) it may not be possible to support a Pareto optimal allocation of an economy by a competitive equilibrium.

Part II. In each part of this question, assume that the indirect utility function is given by

$$V(p, y) = \left(\sum_{i=1}^{n} \alpha_i \frac{p_i}{y} \right)^{-1},$$

where α_i, $i = 1, \ldots, n$, are parameters.

(a) Derive the (Marshallian) demand functions, $\phi_i(p, y)$, $i = 1, \ldots, n$.

(b) Explain the derivation of the Engel curves.

(c) Derive the expenditure function.

(d) Derive the (Hicksian) demand functions, $\delta_i(p, u)$, $i = 1, \ldots, n$.

(e) Set up the problem for recovering the direct utility function.

(f) For the case where $n = 2$, use your intuition to describe the shape of a typical indirect indifference curve and a typical direct indifference curve.

(g) Derive the direct utility function (for $n = 2$ is good enough).

Part III. Answer the following question.

A seller owns one unit of an indivisible good and receives the payoff p if she trades at the price p and the payoff 0 if she does not trade. There are two buyers, both value the good at 1. A buyer receives the payoff $1-p$ if he obtains the good at the price p and the payoff 0 if he does not trade. Each party discounts payoffs at the rate δ. That is, the payoff u received at $t+1$ is worth $\delta^t u$.

Suppose that in the first period the seller is first randomly matched with one of the buyers (each with probability $\frac{1}{2}$), then one of the matched parties is randomly chosen to make an offer (each with probability $\frac{1}{2}$). The responder either accepts the offer, in which case trade occurs and the game ends, or rejects the offer, in which case the play moves to the second period, in which the seller is again randomly matched with one of the buyers.

(a) First, consider a two-period version of this game, that is, the game ends after the second period and each player gets 0 if they fail to agree to trade. Set up this game properly and use backward induction to find a subgame perfect equilibrium.

(b) Now, consider the infinite version of this game. That is, instead of playing a fixed number of periods, the play continues in the same way until an offer is accepted. Find a SPE of this infinite game with perfect information and show the strategic profile you find is in fact a SPE. [Hint: this game is stationary, so you should look for a pair of offers and the respective accept/reject policies in equilibrium.]

(c) Observe your answer from part (b), how does the equilibrium change as δ converges to 0? And how does the equilibrium change as δ converges to 1? Explain these results intuitively.

Part IV.

(i) With suitable assumptions, show that, in a core allocation of a pure-exchange private-ownership economy, all consumers of the same "type" must consume the same consumption bundle. (If you like, you can assume that there are only two "types" of consumers.)

(ii) Consider a private-ownership pure-exchange economy with two consumers and two commodities. The two consumers are 1 and 2, and the two commodities are A and B. The utility functions of the two consumers are given by

$$u = A_1 + B_1$$

and

$$v = \min(A_2, B_2),$$

respectively, where A_1 denotes 1's consumption of A, B_2 denotes 2's consumption of B, and so on. 1's initial endowment bundle consists of 1 unit of A and 0 units of B. 2's initial endowment bundle consists of 0 units of A and 1 unit of B. Identify the core and the set of competitive equilibria for this economy. (Give full reasoning to justify your answer.)

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Part I. Answer the following three questions.

1. The production function is homogeneous of degree one and the elasticity of substitution between labor and capital is greater than 1. What happens to the relative shares of the two inputs when the wage rate rises relative to the rental rate on capital? Explain your answer.

2. Formally define a Bayesian Nash equilibrium. Then briefly discuss the importance of Harsanyi's purification principle.

3. Examine the validity of the following statements.

   (a) If the society's weak preference relation, R, over social alternatives violates transitivity, then there must exist a non-empty set, A, of social alternatives such that A will not have an R-greatest element.

   (b) When we have exactly two consumers, the set of competitive equilibrium allocations of a private-ownership, pure exchange economy cannot possibly coincide with the core.

Part II. Answer the following question.

If the solution to the consumer's expenditure-constrained utility-maximization problem is unique,

$$\phi\big(p, E(u,p)\big) = \delta(u,p), \qquad (*)$$

where ϕ is the ordinary (Marshallian), vector-valued demand function, δ is the constant-utility (Hicksian), vector-valued demand function, and E is the expenditure function. Assume that each of these functions is differentiable.

(a) What assumption about preferences suffices for the solution to this utility-maximization problem to be unique? State this assumption formally in terms of the preference relation $\succsim$ (and its decomposition, $\succ \cup \sim$).

(b) Explaining each step carefully, derive the Slutsky equations from ($\star$).

(c) Interpret each of the terms in the Slutsky equations.

(d) Explain the occurrence of the Giffen paradox using one of these equations.

(e) Suppose the substitution effect of a price change vanishes at the optimum. What do the direct and indirect indifference curves look like (in two dimensions)?

(f) Suppose the income elasticities of demand are globally equal to 1 for all commodities. What do the direct and indirect indifference curves look like (in two dimensions)? Explain your answers.

(g) What phenomena other than the Giffen paradox, outside the context of neoclassical consumer theory, could explain an upward-sloping demand curve for a commodity?

Part III. In answering the following question, give all the steps involved in your derivations, and give intuitive explanation wherever appropriate.

Consider a private-ownership economy E, which has the following features. E has two producers; one of the producers produces a consumption good, A, and the other produces another consumption good, B. There are fixed total amounts of two non-produced factors of production (L and K), and there are n consumers $(1, 2, ..., n)$. The production functions for A and B are given, respectively, as follows:

$$G(L_A, K_A) = L_A + K_A,$$

and

$$H(L_B, K_B) = min(L_B, K_B),$$

where L_A and K_A denote, respectively, the amounts of L and K used for the production of A, and similarly for L_B and K_B. The economy has 50 units of L and 70 units of K. The utility function of consumer i $(i = 1, 2, ...n)$ is given by

$$u^i(A_i, B_i) = A_i B_i,$$

where A_i and B_i denote, respectively, i's consumption of A and i's consumption of B. (The amount of labor supplied by each consumer is assumed to be fixed and is, therefore, dropped from the utility function.)

(a) Identify the set of efficient allocations of L and K between the two industries.

(b) Identify the production possibility frontier of the economy.

(c) Assuming that E is a closed economy and taking commodity A as the numeraire, compute the competitive equilibrium price vector for E, and specify how much of A and how much of B will be produced in the competitive equilibrium.

(d) Now assume that our economy E becomes open to international trade and that it is a small economy trading A and B (but not L or K) in a competitive international market where the price of B in terms of A is 4. What will be the competitive equilibrium prices of L and K in our open economy? What will be the amounts of A and B produced in the competitive equilibrium in E with international trade?

Part IV. Answer the following question.

Samaritan's Dilemma. Suppose that a parent and a child plays the following game. Let the incomes of child and parent be respectively I_c, I_p, which are fixed exogenously. In the first stage, the child decides how much of income I_c to save (S) for the future, consuming the rest ($I_c - S$) today. In the second stage, the parent observes the child's choice of S and chooses a bequest B. The child's payoff is the sum of the two periods' payoffs:

$$U_1(I_c - S) + U_2(S + B).$$

The parent's payoff is

$$V_p(I_p - B) + k[U_1(I_c - S) + U_2(S + B)].$$

Assume that the utility functions U_1, U_2, V are all increasing and strictly concave.

(a) Find S and B that maximize the **total** family utility. Explain your results intuitively.

(b) Use backward induction to find the SPE of this game. Show that the child saves less in this SPE than what you have found in part (a).

(c) Intuitively explain why the answers differ in part (a) and (b). Can you think of some ways to resolve this problem? What is the implication of this model in public finance, foreign aid programs, etc.?

Department of Economics
University of California, Riverside

CORE CUMULATIVE EXAMINATION FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

MICROECONOMIC THEORY

September 23, 2005

Directions: *The examination contains four equally weighted parts. Please allocate your time carefully and write concisely and legibly. This exam will be monitored by the Graduate Assistant. There will be no communication with faculty members; if you believe a question contains an error or an ambiguity, say so on your written examination, make an assumption to correct the alleged error or to resolve the ambiguity, and answer the question as well as you can.*

Part I. Answer the following three questions.

1. Explain why the assumption of local nonsatiation of preferences (along, of course, with other assumptions) is needed to prove the following two propositions (diagrammatic illustrations will do):

   (a) $x^* = argmax_x\{U(x) \mid p \cdot x \leq y\} \iff x^* = argmin_x\{p \cdot x \mid U(x) \geq u^*\}$, where x is a consumption bundle, p is a price vector, y is total expenditure, u is utility, and $u^* = U(x^*)$.

   (b) A competitive equilibrium is Pareto optimal.

2. Consider the statement: "If a strategy is not a dominant strategy, it must be dominated." Prove it if this statement is true; give a counterexample if it is false.

3. Examine the validity of the following statements.

   (a) A Pareto optimal allocation must be Pareto superior to a Pareto inoptimal allocation.

   (b) If one straight-line demand curve is flatter than another straight-line demand curve (flatness being judged with reference to the quantity axis), then, for any given price, the absolute value of the price elasticity of demand must be higher for the flatter demand curve than for the steeper demand curve.

   (c) Since the number of criminals apprehended in a city has gone down despite an increase in the size of the police force in the city, the police must have become less efficient.

Part II. Answer the following question.

(a) Define homogeneity of a production function F (with a scalar output).

(b) Characterize the isoquant map for a homogeneous production function.

(c) What is the relationship between homogeneity of F and returns to scale?

(d) Define homotheticity of the production function F.

(e) What structure of the cost function is equivalent to homotheticity of F?

(f) What structure of the cost function is equivalent to homogeneity of F?

(g) Under what condition is the unit cost function independent of output?

(h) Is (partial) competitive equilibrium consistent with the condition in part (g)? Explain.

(i) Prove the equivalence in (e) or (f).

Part III. Answer both parts of the following question.

1. (i) Consider a private ownership pure-exchange economy with two consumers and two commodities. The two consumers are 1 and 2, and the two commodities are A and B. The utility functions of the two individuals are given by

$$u = A_1 B_1$$

and

$$v = min(2A_2, 3B_2),$$

respectively, where A_1 denotes 1's consumption of A, B_2 denotes 2's consumption of B, and so on. 1's initial endowment bundle consists of 1 unit of A and 0 amount of B. 2's initial endowment bundle consists of 1 unit of A and 2 units of B. Find the competitive equilibrium prices and allocation for this economy. (Recall that a pure-exchange economy is an economy with exchange but without production.)

(ii) Using a simple model of a pure-exchange economy with two consumers and two commodities, show how the presence of an externality affects the validity of the two fundamental theorems of welfare economics. (Provide intuitive interpretation of the equations involved in your demonstration.)

Part IV. Answer the following question.

Consider the following signaling game. There are two players, a plaintiff (P) and a defendant (D) in a civil suit. The plaintiff knows whether or not he will win the case if it goes to trial, but the defendant does not. The defendant knows that the plaintiff knows who would win, and the defendant has prior beliefs that there is probability $\frac{1}{3}$ that the plaintiff will win, these prior beliefs are common knowledge. If the plaintiff wins, his payoff is 3 and the defendant's payoff is -4; if the plaintiff loses, his payoff is -1 and the defendant's is 0. (This corresponds to the defendant paying cash damages of 3 if the plaintiff wins and the loser of the case paying court fee of 1.)

The plaintiff has two possible actions. He can ask for either a low settlement of $m = 1$ or a high settlement of $m = 2$. If the defendant accepts a settlement offer of m, the plaintiff's payoff is m and the defendant's is $-m$. If the defendant rejects the settlement offer, the case goes to court.

(a) Draw the game tree according to the description above. What type of game is this? What are the strategies of the players? Define perfect Bayesian equilibrium (PBE) in the context of this game.

(b) Find **all** the pure strategy perfect Bayesian equilibria (PBE) of this game. Explain intuitively why there does not exist any other pure strategy PBE.

Department of Economics
University of California, Riverside

CORE CUMULATIVE EXAMINATION FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

MICROECONOMIC THEORY

July 1, 2005

This examination will be monitored by the Graduate Assistant. There will be no communication with faculty members; if you believe a question contains an error or an ambiguity, say so on your written examination, make an assumption to correct the alleged error or to resolve the ambiguity, and answer the question as well as you can.
The examination contains four equally weighted parts; allocate your time carefully.

Part I. Answer **four** out of the following five questions

1. Use Euler's equation to show that, if a function $f : \mathbf{R}^2 \to \mathbf{R}$ is homogeneous of degree one, it is only necessary to check the diagonal elements of the Hessian of f to determine whether it is convex or concave.

2. Explain why the demand system generated by a lexicographic ordering can be rationalized by a utility function, despite the fact that a lexicographic ordering itself cannot be represented by a utility function.

3. Discuss the validity of the following comment: "Whenever the relative supply of labor increases, the production of a labor-intensive commodity will increase."

4. A standard two player, two action normal form game (such as the following) essentially falls into one of three types in terms of its Nash equilibrium behavior. List the three types of games and **briefly** explain their differences.

	S	R
S	a,b	c,d
R	e,f	g,h

5. Discuss the validity of the following statement: "Walras' Law holds for every perfectly competitive private ownership economy."

Part II.

A consumer's expenditure function, $E : \mathbf{R} \times \mathbf{R}^2_{++} \to \mathbf{R}_+$, is given by

$$E(u, p_1, p_2) = up_1^{\beta_1} p_2^{\beta_2} + \gamma p_1 \quad \beta_1 > 0,\ \beta_2 > 0,\ \beta_1 + \beta_2 = 1,\ \gamma > 0.$$

(a) Show that the restrictions on the parameters, β_1, β_2, and γ, imply that E satisfies the required properties of a theoretically plausible expenditure function. (The fact in question 1 of Part I can be used here to simplify some of your calculations.)

(b) Would these restrictions still hold if the direct utility function were not quasi-concave?

(c) What do these restrictions say about the sign of the own substitution effect? Explain.

(d) Derive the indirect utility function and the ordinary (Marshallian) demand functions for the two goods.

(e) Are these goods normal or inferior? Explain.

(f) For the particular preferences represented by this expenditure and indirect utility function, write the Slutsky equation for the effect on the demand for good 1 of a change in its own price.

(g) Can either of these goods be a Giffen good? Explain.

(h) Set up, but do not solve, the optimization problem that recovers the direct utility function from the indirect utility function.

(i) What assumptions about the direct utility function are required in order for this recovery mechanism to be precise?

Part III. Answer **one** of the following two questions.

1. (a) Consider an economy with two produced consumption goods (A and B), two non-produced inputs (L and K) in fixed supplies, two producers, each of whom produces exactly one of the two produced goods, and two consumers (1 and 2) with utility functions

$$u(A_1, B_1) = A_1 \cdot B_1$$

and

$$v(A_2, B_2) = A_2 \cdot B_2,$$

respectively. Assume that A is a public good and B is a private good. The two production functions are given by

$$A \leq L_A^{2/3} \cdot K_A^{1/3}$$

and

$$B \leq 10 L_B^{2/3} \cdot K_B^{1/3}.$$

The economy has 100 units of L and 100 units of K.

Characterize as precisely as you can the set of Pareto optimal allocations for this economy.

(b) With suitable assumptions, show that all consumers of the same type will get identical consumption bundles in a core allocation.

2. Discuss the significance of Arrow's impossibility theorem.

Part IV. Answer **one** of the following three questions.

1. Consider an individual with a von Neumann-Morgenstern utility function and assume that she is risk-averse. How would you measure the degree of risk-aversion of this individual? Provide as many interpretations as you can for the measures that you suggest, giving formal proofs wherever necessary.

2. Consider a Cournot-type duopolistic industry where the two producers produce identical products. Labour is the only input required for production. The producers have identical production functions. To produce each unit of output, a producer needs one unit of labour (thus, to produce x units of output, a producer needs x units of labour). The market demand function is given by $P = 100 - Q$, where P is the price of the product and Q is the total output of the two producers taken together. A single trade union supplies all the labour required in the industry and fixes identical wage rates for both producers. The interaction between the two producers and the trade union takes the following form. First, the trade union demands a single wage rate, w, from both producers. Next, the two producers accept the wage rate demanded by the trade union and determine simultaneously their respective outputs and the respective amounts of labour to be hired. The payoff of each producer is simply her profit. The payoff of the trade union is $(w-2)L$, where L is the total amount of labour hired in the industry and 2 is the wage rate that prevails outside the industry.

   Represent this situation as an extensive form game and predict the wage rate, the total output, and the price that will materialize at the end of the game. (Explain carefully the notion of equilibrium that you use.)

3. Consider two hunters who must decide whether to hunt a stag or a rabbit. With probability p, each player has preferences that always make him hunt the stag (e.g., he does not like rabbit); with probability q, each player always hunts the rabbit (he does not like the stag); with probability $1-p-q$, the player has the following preferences: he gets 1 if he hunts the rabbit, 2 if *both* hunt the stag, and 0 if he hunts the stag alone. Suppose that $2p > 1-q$ and $2q > 1-p$.

   (a) Show that in the one period version of the game there are multiple equilibria if $max(p,q) < 1/2$. What are these equilibria? Show that the equilibrium is unique if $p > 1/2$ or $q > 1/2$.

   (b) Consider the two period version of the above stag-hunt game with incomplete information. Show that for any first period behavior, a player who has no strict preference for either stag or rabbit (the last type) always copies his opponent's first period behavior. Explain your result briefly and intuitively.

Department of Economics
University of California, Riverside

CORE CUMULATIVE EXAMINATION FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

MICROECONOMIC THEORY

September 22, 2004

This examination will be monitored by the Graduate Assistant. There will be no communication with faculty members; if you believe a question contains an error or an ambiguity, say so on your written examination, make an assumption to correct the alleged error or to resolve the ambiguity, and answer the question as well as you can.

The examination contains four equally weighted parts; allocate your time carefully.

Part I. Briefly assess each of the following comments:

1. The scale elasticity in production can be defined locally using the cost function.
2. The Marshall-Depuis consumer surplus always overestimates the "true" (Hicksian) consumer surplus.
3. The price elasticity of a positively sloped, linear supply curve cannot be invariant with respect to changes in price.
4. In the presence of externalities in the economy, a competitive equilibrium allocation must be Pareto inoptimal.
5. If at all a minimum-wage law succeeds in raising the wage rate, it will reduce employment.
6. Convexity of firms' technology (production) sets is a necessary condition for the existence of general equilibrium.

Part II. Consider the following utility function:

$$U(x_1, x_2) = x_1 x_2 + 2x_1.$$

Explain/justify all answers; short cuts are permissable as long as they are explicitly justified.

1. Show that this utility function is quasi-concave.
2. Find the expression for the marginal rate of substitution of commodity 1 for commodity 2, $\mathrm{MRS}_{12}(x_1, x_2)$.
3. Set up the Lagrangean expression for maximizing this utility function subject to a budget constraint (with fixed prices, p_1 and p_2, and income, y).
4. Derive the first order conditions for $\langle \overset{*}{x}_1, \overset{*}{x}_2 \rangle$ to be an interior solution to the budget constrained maximization problem.
5. Find the demand functions for the two commodities.
6. Are the two commodities normal or inferior?
7. Can either commodity be Giffen?
8. Are the two commodities necessities or luxuries?
9. Are the two commodities gross substitutes or gross complements?
10. Are the two commodities net substitutes or net complements?
11. Derive the expression for the value of the Lagrangian multiplier at the optimum. What is the economic interpretation of this concept?
12. Find the expression for the indirect utility function.
13. Set up, but do not solve, the algorithm for recovering the direct utility function from the indirect utility function.

Part III. Consider an exchange economy with two goods, X and Y, and two individuals. The two consumers have identical preferences, represented by a Cobb-Douglas utility function such that the marginal rate of substitution between the two goods is equal to y_i/x_i, where x_i and y_i are the consumptions of X and Y, respectively, by the type-i individual. The endowment vectors are $e_1 = (2, 8)$ and $e_2 = (8, 2)$ for consumers 1 and 2, respectively.

1. Draw an Edgeworth box for this economy.
2. Show that the allocation in which one consumer gets 6 units of each good and the other gets the remaining amount of each good is in the core.
3. Explain what happens to the allocation in (b) when this economy is replicated.

Part IV. *Answer exactly one question.*

1. Consider two firms, 1 and 2, that produce a homogeneous product. The two firms decide their respective outputs, Q_1 and Q_2, simultaneously. The market price is then determined by the following equation:

$$P = a - \left(Q_1 + Q_2\right),$$

where a is positive. The cost per unit of output is constant for each firm. However, each firm has probability r of having the unit cost of $\underline{c}$ and probability $(1 - r)$ of having the unit cost of $\bar{c}$. Assume that $a > \bar{c} > \underline{c}$. Model this market as a Bayesian game and identify the Bayesian Nash equilibrium.

2. Suppose that members of a household in a village can either work in the village or migrate to the city. One can always get a job in the city at a fixed wage w_u. The wage in the village is w_r, which is a random variable. $E(w_r) > w_u$. Each household in the village maximizes expected utility and has a quadratic Bernoulli utility function.

    1. Set up a simple model to analyze a rural household's decision regarding migration.
    2. Who migrate more: the members of poorer rural households or richer rural households?
    3. Show that the adoption of new technology in agriculture, which increases the mean as well as the variability of the rural wage rate, may increase migration to the city.

Department of Economics
University of California, Riverside

CORE CUMULATIVE EXAMINATION FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

MICROECONOMIC THEORY

Spring, 2004

This examination will be monitored by the Graduate Assistant. There will be no communication with faculty members: if you believe a question contains an error or an ambiguity, say so on your written examination, make an assumption to correct the alleged error or to resolve the ambiguity, and answer the question as well as you can.

The examination contains four equally weighted parts; allocate your time carefully.

PART I

Short questions
Check the validity of the following statements.

1. For a given price-income situation, a consumer cannot attain the optimal consumption bundle unless the ratio of the marginal utility of a commodity to its price is the same for all commodities.
2. A trade cannot occur when endowments, preferences and technologies are identical for all agents.
3. There is no relation between the Stolper-Samuelson Theorem and the Rybczynski Theorem.
4. A dominated strategy can never figure in a Nash equilibrium. (Note that this statement is about a dominated strategy and not about a strictly dominated strategy.)
5. Let a be a Pareto optimal allocation and let a' be a Pareto inoptimal allocation for a given economy. Then a must yield a higher level of social welfare as compared to a'.
6. If the price effect is negative, then the good is necessarily normal.

PART II

1. A consumer-worker with a fixed time endowment and a positive endowment of non-labor income faces a fixed wage rate.
   a. Taking account of corner solutions and illustrating your answer graphically, find the first-order conditions for maximizing utility in leisure-income space. Identify graphically the optimal choice of leisure time, the optimal earned income, and the labor supply under different assumptions about the preference ordering.

   b. Answer (a) under the modified condition where the consumer-worker faces a straight-time wage rate for the first θ hours of work and a higher overtime wage rate for hours worked beyond that.

   c. In the situation of part (b), would the consumer-worker ever choose to work exactly θ hours? Explain.

PART III Answer exactly one of the two questions.

1. "Since a competitive equilibrium allocation is Pareto efficient, there is no role of the government in a competitive economy." Evaluate.

2. Show that the core approaches the set of competitive equilibrium allocations as the economy is replicated.

Part II

The utility function is given by $U(x_1, x_2) = x_1 + \ln x_2$. In answering the following questions justify all answers; short cuts are allowable as long as they are explicitly justified.

(a) Find the expression for the marginal rate of substitution.

(b) Does the utility function satisfy quasi-concavity? How about strict quasi-concavity?

(c) Find the first-order (necessary) conditions for $\langle \bar{x}_1, \bar{x}_2 \rangle$ to be an interior solution to the problem of maximizing the utility function subject to a budget constraint with fixed positive prices and non-negative "income" (total expenditure).

(d) Are second-order (sufficient) conditions satisfied?

(e) Derive the ordinary (Marshallian) and compensated (Hicksian) demand functions (continuing to assume an interior solution).

(f) Write the Slutsky equations for the differential effects on $\bar{x}_1$ and $\bar{x}_2$ of changes in their own prices. Identify clearly the income and substitution effects and their signs (positive, negative, or zero).

(g) Explain diagrammatically, in $\langle x_1, x_2 \rangle$ space, the calculated income effect for commodity 1 in (f).

(h) Draw a typical indifference curve, being careful to take account of whether it intersects either or both axis.

(i) Taking account of the information in (h), re-state first order conditions for $\langle \bar{x}_1, \bar{x}_2 \rangle$ to be a (possibly non-interior) solution to the budget-constrained utility maximization problem.

(j) Illustrate graphically the possible corner solution in (i).

Part III

Consider an economy with n individuals with strictly convex preferences, who are divided equally among t types.

(a) For n = 2 and t= 1, where both preferences and endowments are identical, describe the core of this economy and illustrate it in an Edgeworth box.

(b) Show that, in the presence of different types, all individuals of a particular type receive the same bundle at any core allocation.

Part IV

Answer exactly one of the following two questions.

1. (a) Consider the case where all the outcomes in risky situations are sums of money. Suppose the agent under consideration has a von Neumann-Morgenstern utility function. How would you measure the degree of risk aversion of the agent?
   (b) Carefully interpret the measure(s) of risk aversion that you specify in your answer to part (a) above.
   (c) Suppose each tax payer has exactly two options – not to file the tax return at all or to file the tax return and declare the entire true income (for simplicity, we rule out the possibility of filing but under-reporting the income). If someone does not file the tax return, the probability that this will be detected is p $(0 < p < 1)$, and, if detected, the person will have to pay the tax she was trying to evade and a fine. All individuals have von Neumann-Morgenstern utility functions and are risk-averse. Will a t % increase in the probability of detection or a t % increase in the fine be more effective in deterring tax evasion, where t is a 'very small' positive number? Explain your answer.

2. (2.1) Explain clearly the notion of a sequential equilibrium.
   (2.2) Consider the two strategic games, **G** and **G′**, given below.

G

Player 2 across the top; player 1 down the side.

1		c	d
	a	10, 6	4, 2
	b	4,2	6, 10

G′

Player 2 across the top; player 1 down the side.

1		c'	d'
	a'	12, 12	1, 2
	b'	1,2	3, 3

Consider the extensive game Z, where 1 first (publicly) chooses the game to be played, $\mathbf{G}$ or $\mathbf{G'}$, and then 1 and 2 play the game chosen by 1.

(i) Draw a diagram representing the extensive game Z.

(ii) Does there exist a subgame perfect Nash equilibrium of Z, where 1 first chooses $\mathbf{G}$ and then (a, c) is played? Explain your answer.

(iii) Does there exist a subgame perfect Nash equilibrium of the game Z, where 1 first chooses $\mathbf{G'}$ and then (b', d') is played? Explain your answer.

Department of Economics
University of California, Riverside

CORE CUMULATIVE EXAMINATION FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

MICROECONOMIC THEORY

January 2, 2004

This examination will be monitored by the Graduate Assistant. There will be no communication with faculty members: if you believe a question contains an error or an ambiguity, say so on your written examination, make an assumption to correct the alleged error or to resolve the ambiguity, and answer the question as well as you can.

The examination contains four equally weighted parts; allocate your time carefully

PART I

1. Using the properties of the profit function, sketch a proof of the proposition that input demand curves are "downward sloping" and output supply curves are "upward sloping."

2. Characterize the isoquant map for different values of ρ in the CES technology:

$$f(x_1, x_2) = (\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho})^{1/\rho}, \ 1 \geq \rho \neq 0.$$

What happens in the limiting case as $\rho \to 0$?

3. Consider a competitive consumer in a two-commodity world. The wealth (W) of the consumer and the two prices (p_1 and p_2) are always positive. If $p_1 \leq p_2$, then the consumer spends her entire wealth on commodity 2, and, if $p_1 > p_2$, then the consumer spends her entire wealth on commodity 1. Does this consumer satisfy the weak axiom of revealed preference? Justify your answer.

4. Assess the validity of the following statements.
   (i) A competitive equilibrium can exist only if the consumers' preferences are convex.
   (ii) In the presence of a monopoly it is not possible to attain Pareto efficiency.
   (iii) In less developed countries, which export labor-intensive commodities and import capital-intensive commodities, a lowering of tariffs on imported commodities will always benefit the workers.

Part III

Answer **one** of the following two questions.

1. Consider the following strategic game (with complete information) that is played twice.

		Player 2		
		D	E	F
	A	6, 2	0, 0	10, 0
Player 1	B	4, 2	2, 4	6, 2
	C	2,4	0, 2	8, 8

The way in which this game is played twice is as follows. The two players play the game for the first time. Then, after observing the two strategies that are adopted in this first round, they play the game for the second time. There is no discounting so that the final payoff of each player after both the rounds of the game is simply the sum of the payoffs of that player in those two separate rounds.

(i) Model this situation, where the given strategic game is played twice, as an extensive game, clearly identifying the strategies in this extensive game.

(ii) Is it possible that, in a subgame perfect Nash equilibrium of the extensive game, players 1 and 2 will adopt, respectively and simultaneously, C and F in the first round? Give full justification for your answer.

Part IV

Answer **one** of the following two questions.

1. Consider an economy with two consumers (1 and 2), two produced final goods (A and B) and two factors of production (L and K) available in fixed supplies. A is a public good while B is a private good. The amount of L in the economy is 27 and the amount of K in the economy is 64. The production functions are given by:

$$A = L_A^{1/3}.K_A^{2/3}$$

and

$$B = 5L_B^{1/3}.K_B^{2/3},$$

where L_A, K_A, etc. have the obvious interpretations. The utility functions of 1 and 2 are given, respectively, by:

$$U = A_1.B_1$$

and

$$V = A_2.B_2,$$

Where A_1, B_1, etc. have the obvious interpretations.

(i) Identify the class of Pareto optimal allocations for this economy as precisely and as explicitly as you can (provide explicit reasoning for each of the steps involved).

(ii) Derive the equation for the utility possibility frontier for this economy.

2. Let E be a pure exchange economy. State a set of assumptions, which, together, are sufficient to ensure that every Pareto optimal allocation in this economy can be sustained by a competitive equilibrium after a suitable specification of the property rights. Give a proof of such sufficiency.

PART IV

Answer *exactly one* of the following two questions.

1. Consider the following story about a father and his son (the original story is due to the Nobel Laureate J. Buchanan). There are two periods in the story: period 1 and period 2. Both the father and the son know that the father will die at the end of the first period and that the son will die at the end of the second period without leaving an heir. The father has a fixed income, y, in the first period (at the end of which he dies). The son has a fixed income, x, in period 1 and a fixed income, 0, in period 2. The son has to decide the amount (call it s) that he would save from his income in period 1 to consume in period 2. The father observes how much the son has decided to save in the first period and then decides the amount (call it b) that he would leave as a bequest for his son to consume in the second period. Thus, the son's consumption in the first period is $x - s$ and his consumption in the second period is $s+b$. The father's consumption in the first period is $y-b$ (given our story, the father cannot consume anything in the second period). The son's utility is given by

$$u(x-s)+v(s+b),$$

where u and v are both strictly increasing and strictly concave functions. The father cares about both himself and the son, and the father's utility is given by

$$w(y-b) + a[u(x-s)+v(s+b)],$$

where w is a strictly increasing and strictly concave function, and $1>a>0$. Each of the two persons (the father and the son) knows the other person's utility function as well as his own utility function.

   (i) Derive the conditions for Pareto optimality in the simple two-person economy consisting of the father and son.
   (ii) Represent the story in terms of an extensive game of perfect information.
   (iii) Show that, in general, the outcome of the subgame perfect Nash equilibrium of this extensive game will not satisfy the conditions referred to in Part (i) of the question and will not, therefore, be Pareto optimal.

2. In some less developed countries, farmers often like to have several plots of land at considerable distances from each other, rather than having one consolidated plot of land (though a consolidated plot has some advantage insofar as it saves the time lost in going from one plot to another). It has been suggested that this may be due to the farmers' desire to avoid certain types of risk. For example, with one consolidated plot, there is some risk that the entire crop may be destroyed if the area where the plot is located happens to be attacked by pests, but having several plots located in different areas reduces the risk of the entire crop being destroyed in this fashion. With the help of a simple model, provide an analytical basis for this suggestion (if you like, you can assume that farmers have quadratic Bernoulli utility functions).

2. Consider a situation where we have two individuals, 1 and 2. There is no uncertainty about the personality or 'type' of individual 1 and it is common knowledge. However, so far as 2 is concerned, there are two possible personalities or types, Types A and B. There is probability 2/3 that nature will assign Type A to 2 and probability 1/3 that nature will assign Type B to 2. After nature has assigned one of these two types to 2, 2 knows the type assigned to him, but all that 1 knows is that the type of 2 is A with probability 2/3 and B with probability 1/3. 1 has two possible actions, a and b. 2 has two possible actions c and e. The payoffs are as follows.

Player 2 (Type A)

		c	e
Player 1	a	10, 8	5, 7
	b	8, 10	16, 12

Player 2 (Type B)

		c	e
Player 1	a	10, 12	5,10
	b	8, 7	16, 8

(i) Model this situation as a Bayesian game, identifying all the constituent features of the game precisely and explicitly.

(ii) Identify the class of all (pure-strategy) Bayesian Nash equilibria of this Bayesian game.

Department of Economics

University of California, Riverside

CORE CUMULATIVE EXAMINATION FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

MICROECONOMIC THEORY

September 24, 2003

This examination will be monitored by the Graduate Assistant. There will be no communication with faculty members: if you believe a question contains an error or an ambiguity, say so on your written examination, make an assumption to correct the alleged error or to resolve the ambiguity, and answer the question as well as you can.

The examination contains four equally weighted parts; allocate your time carefully.

Part I. Briefly assess each of the following comments:

1. It is not possible for all pairs of commodities to be (net) complements in the consumer demand system.
2. Continuity of preferences is a necessary condition for the existence of a solution to the consumer's budget-constrained optimization problem.
3. The imposition of a minimum wage in an industry must reduce employment in that industry.
4. A rise in the price of leisure (i.e., a rise in the wage rate) can increase the consumption of leisure only if leisure is a normal good.
5. An aggregation procedure that maps a profile of individual preference orderings to a social preference ordering is dictatorial if all other Arrowian conditions, namely, unrestricted domain, weak Pareto principle, and independence of irrelevant of alternatives, are satisfied; however, this procedure is not necessarily dictatorial if any one of these three conditions is no longer required.
6. It is impossible to attain Pareto optimality in the presence of an externality.

Part II. The variable cost function of a (cost minimizing) firm has the following structure:

$$VC(u, p, k) = \theta(u)\ \Pi(p)\ \gamma(k), \qquad (*)$$

where u is the (scalar) output quantity, $p = \langle p_1, \ldots\ldots, p_n \rangle$ is the variable-input price vector, k is the scalar amount of the fixed input ("capital"), and θ, Π, and γ are functions. Assume that this variable cost is well defined for all positive p and all non-negative u and k.

a) Identify the restrictions on the functions, θ, Π, and γ, for VC to be a proper variable cost function.

b) What properties of the technology are implied by the form of this variable cost function?

c) What additional restriction is implied by the assumption of free disposability of all (variable and fixed) inputs?

d) Derive the expressions for the marginal cost (MC), average variable cost (AVC), and average fixed cost (AFC) functions.

e) Find the expression for the differential (local) returns-to-scale coefficient.

f) Under what restrictions on (*) would the coefficient in (e) be globally constant, and what does this restriction imply about the technology?

g) Derive the expressions for the cost-minimizing values of the variable inputs (i.e., the variable input demands conditional on u and k as well as p).

h) In terms of the information in the variable cost functions, under what conditions would the amount of capital, k, be (i) larger than, (ii) smaller than, or (iii) equal to the (long-run) cost-minimizing amount (given the values of u and p)?

i) Discuss the dilemma posed to a regulator implementing marginal-cost pricing for this firm when θ is globally concave, and suggest a resolution of this dilemma. What further impediments might there be to your proposed resolution?

j) Reinterpret k in (*) as an externality: the output of another firm that affects the cost of only the firm with variable cost function (*). Design a Pigouvian tax to internalize this externality.

Part III. Answer the following question.

a) Describe an exchange economy with n individuals. Make all assumptions that you need to prove the existence of a competitive equilibrium.

b) Sketch a proof of the existence of a competitive equilibrium of such an economy.

Part IV. Answer exactly one of the following two questions.

1. a) State a set of assumptions, which, together, ensure the existence of a von Neumann-Morgenstern utility functions, and comment briefly on their intuitive contents.

   b) Consider a risk averse individual who has a von Neumann-Morgenstern utility function and whose initial wealth is W. He has to decide whether he should insure his house, and, if so, how much insurance coverage he should buy. Assume that fire is the only source of hazard for a house. The probability of a house catching fire is p ($1>p>0$). If the individual's house catches fire, then there will be a damage of D dollars. The price of insurance prevailing in the competitive house insurance market is such that the expected profit of each insurance company is zero (we assume that the insurance companies have zero costs). Construct a simple model to determine how much insurance coverage our individual will buy.

2. a) Explain clearly the notion of a Bayesian Nash equilibrium.

   b) Identify the set of all mixed-strategy Nash equilibria of the following strategic game.

Player 2

Player 1	d	e	f
a	4, 6	8, 4	4, 3
b	5, 2	4, 4	2, 7
c	8, 4	2, 3	2, 5

Department of Economics

University of California, Riverside

CORE CUMULATIVE EXAMINATION FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

MICROECONOMIC THEORY

July 1, 2003

This examination will be monitored by the Graduate Assistant. There will be no communication with faculty members; if you believe a question contains an error or an ambiguity, say so on your written examination, make an assumption to correct the alleged error or to resolve the ambiguity, and answer the question as well as you can.

The examination contains four equally weighted parts; allocate your time carefully.

Part I. Briefly assess each of the following comments:

(a) Strict monotonicity of preferences is a reasonable assumption to make in order to guarantee that a utility-maximizing consumer spends his or her entire endowment income.

(b) Convexity of lower level sets in price space ("convexity toward the origin" of indirect indifference curves) follows directly from convexity of upper level sets in consumption space ("convexity toward the origin" of direct indifference curves).

(c) A competitive equilibrium of an economy with production externalities can not be Pareto efficient.

(d) If every consumer has a bliss point in his or her consumption set, then a Pareto efficient allocation might not be attainable as a competitive equilibrium allocation with a suitable tax-transfer policy.

(e) If a person willingly plays a game entailing an "unfair" gamble—one in which the expected monetary payoff to the player is negative—he or she must get utility out of playing the game itself.

(f) The only explanation for charging a fixed fee for entry to Disneyland (irrespective of how many rides or shows one participates in) is to save on transactions costs.

Part II. A regulated public utility produces a single commodity with a fixed amount of capital and two variable inputs, labor and energy. The firm's price is controlled and it must satisfy demand for the one commodity it produces.

(a) Would you expect the variable production function, expressing output as a function of labor and energy, to exhibit (i) increasing, (ii) decreasing, or (iii) constant returns to scale? Explain.

(b) Find the first-order conditions for cost mimimization subject to the technological and regulatory constraints.

(c) Now suppose, because of environmental concerns, the public utility is restricted to using no more than a stipulated amount of the energy input. Find the first-order conditions for cost mimimization and illustrate these conditions graphically.

(d) Suppose next that the public utility is allowed to use more than the stipulated amount of the energy input but that it must pay a per-unit tax for energy usage above that amount. Find the first-order conditions for cost mimimization and illustrate these conditions graphically. (If you're running short on time, skip the algebra and just illustrate these conditions graphically.)

(e) Briefly, under what conditions would the regulatory regimes in (c) and (d) be socially optimal.

Part III. Consider a small open economy with two production sectors producing two goods with constant-returns-to-scale technologies in a perfectly competitive environment, where each factor is homogeneous, inelastically supplied, and perfectly mobile between the two sectors.

(a) Derive the first-order conditions for an efficient allocation of resources.

(b) Discuss the effect of an increase in the price of one of the two goods.

(c) If one of the sectors were unionized, what policy would you prescribe for an efficient allocation of resources.

Part IV. Answer <u>exactly one</u> of the following two questions:

1. (i) Define the notion of a Bayesian Nash equilibrium and explain the intuition underlying the formal definition.

(ii) Consider two duopolists producing identical products. The market demand curve for the product is given by

$$p = 12 - q_1 - q_2$$

where q_1 and q_2 are the outputs of the two producers and p is the market price of the product. Each producer has to choose the level of his own output. Neither producer has any fixed cost. The (constant) marginal cost of producer 1 is 5, and this is common knowledge. However, with probability 1/3, 2's (constant) marginal cost is 4, and, with probability 2/3, it is 6 (producer 2 knows his own marginal cost at the time of choosing his output, but producer 1 does not know 2's marginal cost for sure at the time of taking his own output decision, though 1 knows the probabilities involved). Model this situation as a Bayesian game and identify the set of Bayesian Nash equilibria.

2. Let G denote the following strategic game.

		Player 2		
		d	e	f
Player 1	a	2, 4	18, 0	2, 0
	b	0, 0	14, 14	0, 0
	c	0, 0	0, 0	8, 2

(i) Identify the set of all pure-strategy Nash equilibria for G.

(ii) Identify the set of all mixed-strategy Nash equilibria for G.

(iii) Suppose G is played twice. We assume that when the second round starts, each player knows what action each player took in the first round. The final payoff for each player after the game is played twice is simply the sum of her payoff for the first round and her payoff for the second round. Thus, for example, if the two players, 1 and 2, take actions a and d, respectively, in the first round and actions b and e, respectively, in the second round, then, after both the rounds are complete, the payoff of player 1 is

16 and the payoff of player 2 is 18. Consider this situation where G is played twice as an extensive game. Identify the set of (pure) strategies of each player in this extensive game. Does there exists a subgame perfect Nash equilibrium of this extensive game such that players 1 and 2 take actions b and e, respectively, in the first round? Justify your answer.