# ECON200A (FALL 2016): MICROECONOMIC THEORY
# FINAL

**Problem 1.** Let $X = \mathbb{R}^2$, and suppose that there is a consumer whose preference relation on $X$ is represented by a utility function $u(x_1, x_2) = x_1 x_2$. The unit price of good 1 is \$1 for the first $M > 0$ units, but it is discounted to \$.50 per unit for any amount exceeding $M$. (For example, if the consumer buys $M + 1$ units of good 1, the total price is $M + 1/2$.) The unit price of good 2 is always \$1. Let $w > 0$ denote the consumer's income.

(1) Draw two graphs of this consumer's budget set, one for the case when $w \leq M$ and the other for the case when $w > M$. (6pts)

(2) Compute the demand function as a function of $w$ and $M$. (20pts)

**Problem 2.** Let $X$ be a finite set, $\mathcal{A}$ the collection of all nonempty subsets of $X$, and $\succcurlyeq_i$ a complete, transitive, and antisymmetric preference relation on $X$ for each $i = 0, 1, 2$. Let $\Gamma = \max(S, \succcurlyeq_1) \cup \max(S, \succcurlyeq_2)$ for every $S \in \mathcal{A}$, and define a singleton-valued choice correspondence $c$ on $\mathcal{A}$ by

$$c(A) = \max(\Gamma(S), \succcurlyeq_0)$$

for any $S \in \mathcal{A}$. Prove or falsify each of the following claims. (Each 9pts)

(1) $c$ is rationalizable.

(2) For any $S \in \mathcal{A}$, $x \notin \Gamma(S)$ implies $\Gamma(S) = \Gamma(S \setminus \{x\})$.

(3) For any $S \in \mathcal{A}$, there exists an $x^* \in S$ such that

$$\big(x^* \in T \in \mathcal{A},\ \ c(T) \in S, \text{ and } c(T) \neq c(T \setminus \{x^*\})\big) \implies c(T) = x^*.$$

**Problem 3.** Answer the following questions.

(1) For any $x \in [0, 100]$, let $\mathbf{p} = \frac{x}{100}\delta_0 \oplus \frac{100-x}{100}\delta_x$ and $\mathbf{q} = \frac{1}{3}\delta_0 \oplus \frac{1}{3}\delta_{10} \oplus \frac{1}{3}\delta_{20}$. Identify a set $I \subseteq [0, 100]$ that satisfies $\mathbf{p}\,\text{FSD}\,\mathbf{q} \Leftrightarrow x \in I$. Also, identify a set $J \subseteq [0, 100]$ that satisfies $\mathbf{q}\,\text{FSD}\,\mathbf{p} \Leftrightarrow x \in J$. (Hint: they are both intervals. Each 9pts))

(2) Let $Z = \mathbb{R}$, and $L(Z)$ be the set of all lotteries with finite support. Let $\succcurlyeq$ be a preference relation on $L(Z)$ admitting a "worst case" representation under a function $u : Z \to \mathbb{R}$, that is,

$$\mathbf{p} \succcurlyeq \mathbf{q} \quad \Longleftrightarrow \quad \min\{u(z) : \mathbf{p}(z) > 0\} \geq \min\{u(z) : \mathbf{q}(z) > 0\}$$

for any $\mathbf{p}$ and $\mathbf{q}$ in $L(Z)$. Choose one or more properties of the function $u$ from the list below to fill the box in the next statement and make it a true proposition. (No proof is needed. 9pts)

$$\succcurlyeq \text{ is risk averse} \quad \text{if and only if} \quad u \text{ is } \boxed{\quad ? \quad}.$$

List of properties: continuous, increasing, decreasing, concave, convex.

**Problem 4.** Consider a firm producing a good under a production function $f(x_1, x_2) = \sqrt{x_1 x_2}$, where $x_1$ and $x_2$ are two inputs. The markets of these input goods are competitive, and their market prices are given by $w_1 > 0$ and $w_2 > 0$, respectively. On the other hand, the firm is a monopolist in the market of the output good, and the inverse market demand function of the good is given by $p = 2q^{-1/2}$. Compute the size of dead weight loss in the market of the output good due to monopoly as a function of $w_1$ and $w_2$. (20pts)