ECON200A (FALL 2017): MICROECONOMIC THEORY
FINAL

Problem 1. Let $X \subseteq \mathbb{R}^2_+$, and consider a consumer whose preference relation on X is represented by a utility function $u(x_1, x_2) = x_1 x_2$. Suppose that the unit price of good 1 is \$1, the unit price of good 2 is \$2, and the consumer has an income of \$10. So, the consumer's budget set is given by

$$B = \{x \in X : x_1 + 2x_2 \leq 10\}.$$

For each of the following cases, compute the consumer's optimal consumption bundles. If there are more than one optimal consumption bundle, identify all of them.

(1) The consumption of good 1 is limited: $X = \{x \in \mathbb{R}^2_+ : x_1 \leq 8\}$.
(2) The consumption of good 1 is tightly limited: $X = \{x \in \mathbb{R}^2_+ : x_1 \leq 3\}$.
(3) Good 1 is indivisible: $X = \{x \in \mathbb{R}^2_+ : x_1 \in \mathbb{N} \cup \{0\}\}$.
(4) Good 2 is indivisible: $X = \{x \in \mathbb{R}^2_+ : x_2 \in \mathbb{N} \cup \{0\}\}$.
(5) Both goods are indivisible: $X = \{x \in \mathbb{R}^2_+ : x_1, x_2 \in \mathbb{N} \cup \{0\}\}$.
(6) Goods are sold as a pair: $X = \{x \in \mathbb{R}^2_+ : x_1 = x_2\}$.
(7) Goods are indivisible and sold as a pair: $X = \{x \in \mathbb{R}^2_+ : x_1 = x_2 \in \mathbb{N}\}$.

Problem 2. Let X be a set, x^* an element of X, and $\mathcal{A}$ the collection of all nonempty finite subsets S of X with $x^* \in S$. In this problem, we call x^* a candy. Consider a household consisting of a mother, a father, and a child who have complete and transitive preference relations $\succsim_m$, $\succsim_f$, and $\succsim_c$, respectively. Assume that

$$z \succ_m x^* \quad \text{and} \quad z \succ_f x^*$$

for any $z \in X \setminus \{x^*\}$, so that the mother and father never choose the candy unless necessary. Facing a choice set $S \in \mathcal{A}$, if the mother and father agree on some alternatives, meaning that $\max(S, \succsim_m) \cap \max(S, \succsim_f) \neq \emptyset$, then the child will choose her best alternatives from them. If the mother and father do not agree on any alternatives, meaning that $\max(S, \succsim_m) \cap \max(S, \succsim_f) = \emptyset$, then the child can choose the candy. Therefore, this household's choice is modeled by a choice correspondence C on $\mathcal{A}$ such that, for any $S \in \mathcal{A}$,

$$C(S) = \max(\max(S, \succsim_m) \cap \max(S, \succsim_f), \succsim_c)$$

if $\max(S, \succsim_m) \cap \max(S, \succsim_f) \neq \emptyset$, and $C(S) = \{x^*\}$ otherwise. Prove or falsify each of the following claims.

(1) If $S, T \in \mathcal{A}$, $S \subseteq T$, and $C(S) = \{x^*\}$, then $C(T) = \{x^*\}$.

(2) If $S, T \in \mathcal{A}$, $S \subseteq T$, and $C(T) = \{x^*\}$, then $C(S) = \{x^*\}$.

(3) If $S, T \in \mathcal{A}$ and $C(S) = C(T) = \{x^*\}$, then $C(S \cup T) = \{x^*\}$.

(4) If $S \in \mathcal{A}$, then $C(C(S) \cup \{x^*\}) = C(S)$.

Problem 3. Let $Z = \mathbb{R}$, and $L(Z)$ be the set of all lotteries with finite support. For any $\mathbf{p} \in L(Z)$ and $a \in \mathbb{R}$, let us denote by $\mathbf{p}+a \in L(Z)$ a lottery such that $\mathbf{p}(z) = (\mathbf{p}+a)(z+a)$ for any $z \in Z$. Also, for any $\mathbf{p}, \mathbf{q} \in L(Z)$, we say that $\mathbf{p}$ second-order stochastically dominates $\mathbf{q}$, denoted $\mathbf{p}\,\mathrm{SSD}\,\mathbf{q}$, if $\mathbf{p} \succcurlyeq \mathbf{q}$ for any preferences $\succcurlyeq$ satisfying the vNM assumptions, monotonicity, and risk aversion. For each of the following statements, identify whether it is true or false. If you answer that some of (1)-(4) are false, discuss why they are false. If you answer that some of (5)-(8) are false, give counterexamples. You do not have to give proofs to true statements.

(1) $.5\delta_{-1} + .5\delta_1 \,\mathrm{SSD}\, \delta_0$.

(2) $\delta_0 \,\mathrm{SSD}\, .5\delta_{-1} + .5\delta_1$.

(3) $.3\delta_0 + .4\delta_1 + .3\delta_2 \,\mathrm{FSD}\, .6\delta_0 + .4\delta_2$.

(4) $.4\delta_0 + .2\delta_1 + .4\delta_2 \,\mathrm{FSD}\, .6\delta_0 + .4\delta_2$.

(5) For any $\mathbf{p}, \mathbf{q} \in L(Z)$, there is an $a \in \mathbb{R}$ such that $\mathbf{p}\, FSD\, \mathbf{q} + a$.

(6) If $\mathbf{p}\,\mathrm{SSD}\,\mathbf{q}$ and $a > 0$, then $\mathbf{p} + a\,\mathrm{FSD}\,\mathbf{q}$.

(7) If $\mathbf{p}\,\mathrm{FSD}\,\mathbf{q}$, then $\mathbb{E}\mathbf{p} \geq \mathbb{E}\mathbf{q}$.

(8) If $\mathbb{E}\mathbf{p} \geq \mathbb{E}\mathbf{q}$, then $\mathbf{p}\,\mathrm{FSD}\,\mathbf{q}$.

Problem 4. Consider a firm producing a good under a production function $f(x_1, x_2) = \sqrt{2ax_1} + x_2$, where x_1 and x_2 are two inputs, and $a > 0$ is a positive constant. The markets of these input goods are competitive, and their market prices happen to be given by $w_1 = w_2 = 2a > 0$. On the other hand, the firm is a monopolist in the market of the output good, and the inverse market demand function of the good is given by $p = b - q$, where $b > 0$ is a positive constant.

(1) Compute the cost function $c(q)$ for the firm to produce q units of the output good. (Hint: the firm will use only input 1 when $q \leq a$.)

(2) Draw the marginal cost curve $c'(q)$ on the graph where the horizontal axis measures q and the vertical axis measures $c'(q)$.

(3) Assuming that $3a < b < 4a$, compute the size of deadweight loss in the market of the output good due to monopoly.