University of California, Riverside

Microeconomics Qualifying Exam

July 10, 2017

INSTRUCTIONS

- *Write solutions on only one side of each page of paper and do not put answers for more than one question on a page.*
- *Assemble your solutions in numerical order, starting with #1, for each part, and assemble parts in order.*
- *Answer all questions.*

Part I

Problem 1. Let X be a nonempty set, and $\succcurlyeq$ be a preference relation on X such that there is a countable subset Z of X with the following property:

$$\text{if } x, y \in X \text{ and } x \succ y, \text{ then there exists a } z \in Z \text{ such that } x \succ z \succ y.$$

(a) Choose one or more properties of the preference relation $\succcurlyeq$ from the list below to fill the box in the next statement and make it a true proposition.

$$\exists \text{ a utility function } v : Z \to [0,1] \text{ that represents } \succcurlyeq \text{ on } Z$$
$$\text{if and only if } \succcurlyeq \text{ is } \boxed{\quad ? \quad} \text{ on } Z.$$

List of properties: complete, transitive, continuous, convex, monotone.

(b) Assume that the preference relation $\succcurlyeq$ satisfies the properties you answered in (a) on the entire X (not just on Z). By (a), there is a utility function $v : Z \to [0,1]$ that represents $\succcurlyeq$ on Z. But, in fact, we can show that there exists a utility function $u : X \to [0,1]$ that represents $\succcurlyeq$ on the entire X. Suggest a construction of such a utility function u.

(c) Prove that the function u you constructed in (b) is indeed a utility function for the preference relation $\succcurlyeq$ on X.

Problem 2. Let $\succcurlyeq$ be a preference relation over monetary lotteries that admits an expected utility representation under a vNM utility function $u(z) = \ln z$. For any infinite sequence $x = (x_0, x_1, x_2, \ldots)$ of real numbers, let $\mathbf{p}_x$ be a lottery such that $\mathbf{p}_x(x_t) = (1-\delta)\delta^t$ for each $t \geq 0$.

(a) Explicitly write an expected utility value of the lottery $\mathbf{p}_x$:

$$\mathbb{E}u(\mathbf{p}_x) = \boxed{\quad ? \quad}.$$

(Your answer can include x, δ, and $\ln(\cdot)$, but neither $\mathbf{p}_x$ nor u.)

(b) Suppose that an economic agent endowed with the preference relation $\succcurlyeq$ must choose a lottery from a set

$$S = \left\{ \mathbf{p}_x : \sum_{t=0}^{\infty} R^{-t} x_t = I \right\}$$

for some $R > 1$ and $I > 0$. Answer the $\succcurlyeq$-maximizing lottery $\mathbf{p}_x$ in S.

Problem 3. Suppose that a monopolistic firm produces a good under a cost function

$$c(q) = \begin{cases} \frac{1}{2}q^2 & \text{if } q < A, \\ q^2 - Aq + \frac{1}{2}A^2 & \text{otherwise} \end{cases}$$

while the inverse demand function for the good is given by $p = B - q$, where $A > 0$ and $B > 0$ are positive parameters.

(a) Draw this firm's marginal cost curve on the graph where the horizontal axis measures q and the vertical axis measures $c'(q)$.

(b) Compute the monopoly price and monopoly quantity as functions of A and B.

(c) Compute the size of a deadweight loss in this market due to monopoly as a function of A and B.

Part II

Problem 1. Jack and Jill contribute to a public good that they both enjoy. If the amount of the good is G, then each receives a gross benefit of $v(G) = G - \frac{1}{2}G^2$. The amount of the public good actually enjoyed is equal to the maximum amount contributed by either Jack or Jill (i.e., $G = \max\{g_{Jack}, g_{Jill}\}$), where g_{Jack} and g_{Jill} are the individual contributions of Jack and Jill. A player contributing g earns net payoff $v(G) - cg$, where $c \in (0, 1)$ is the common marginal cost of a contribution and G is the actual realized level of the public good.

Jack and Jill make their contributions simultaneously.

(a) Determine a *symmetric* equilibrium to this game.

(b) Calculate the probability that none of the good is provided.

Problem 2. A seller has two identical objects for sale. Three players bid on these objects, each wanting at most one of them. Players' values for the objects are independent and identically distributed random variables uniformly distributed over $[0, 1]$. The realizations of the values are private information to each player. Each player submits a bid. The highest two bids win and the winning bidders pay the amounts of their own bids. Winning bidders obtain a final payoff of (private object value) – (amount paid) while the losing bidder receives a payoff of 0. After learning their own values, players simultaneously submit their bids. The foregoing is common knowledge.

(a) Determine a Bayesian equilibrium to this game.

(b) Calculate the bid of a player with object value equal to 1.

Problem 3. Consider the following matrix game played by Row and Column.

	a	b	c
A	3, 3	0, 0	0, 5
B	0, 0	0, 0	0, 1
C	5, 0	1, 0	2, 2

Row's pure strategies are $\{A, B, C\}$ and Column's are $\{a, b, c\}$. In each cell, the first number is the payoff to Row, the second is the payoff to Column.

Over an infinite number of periods players simultaneously choose actions. Payoffs in the repeated game are the discounted sums of per-period payoffs, where the players have a common per-period discount factor $\delta \in (0, 1)$.

Players would like to achieve the cooperative outcome of playing (A, a) in every period.

(a) For what range of discount factors can such cooperation be sustained by a Grim Trigger Strategy? Explain.

(b) For what range of discount factors can such cooperation be sustained by a carrot–stick strategy that uses a 1-period punishment? Explain.

(c) For what range of discount factors can such cooperation be sustained by a carrot–stick strategy that uses a 2-period punishment? Explain.

Part III

Problem 1. There are only two consumers, Amy and Bev, and only two goods, the quantities of which are denoted by x and y. There are 20 units of each good to be allocated between Amy and Bev. Amys and Bevs preferences can be represented by the utility functions

$$u_A(x_A, y_A) = log(x_A) + 4log(y_A)$$

and

$$u_B(x_B, y_B) = y_B + 5log(x_B)$$

.

(a) Determine the set of all Pareto optimal allocations and depict the set carefully in an Edgeworth box diagram.

(b) Verify mathematically that the allocation $((x_A, y_A), (x_B, y_B)) = ((4, 5), (16, 15))$ is Pareto efficient.

(c) Now assume that Amy owns the bundle $(4, 5)$ and Bev owns the bundle $(16, 15)$. Determine a Walrasian equilibrium. Verify that all three conditions in the definition of an Walrasian Equilibrium are satisfied in your proposed solution.

Problem 2. Consider the bilateral cooperative relationship between a Principal (P) and an agent (A) that generates a outcome that depends on the agent's effort level e, and on the type of the agent. Suppose that the agent has type 1 with probability α and he has type 2 with probability $1-\alpha$. The productivity of each of these types is given by $x_1(e) = 10e$ and $x_2(e) = 20e$, respectively. The preferences of both types of agents are represented by the utility functions

$$u_1(w, e) = u_2(w, e) = w - e^2$$

The principal is risk-neutral and the reservation utility of the agent is zero.

(a) Find the symmetric information contracts that will be offered by P. Will the effort levels demanded by P vary when the reservation utility increases?

(b) Find the contracts that P will offer when there is asymmetric information on the agent's type.

(c) Now compute the asymmetric information contracts when there are multiple principals competing for the services of the agent.

Problem 3. An agent has an income $m_0 = 64$ and faces a risk: with probability q his future income will be 36. Depending on his choice $a = 1$ or $a = 2$, the probability q will be either $1/2$ or $1/4$, respectively. The agent has the utility function $U(m) = \sqrt{m} - a$. A monopoly risk-neutral insurance company considers the possibility of offering an insurance to the agent, consisting of a premium p and a compensation C in case of a loss.

(a) Suppose that the agent does not buy the insurance. What is his choice of a?

(b) Suppose there is symmetric infomation. What contract would the insurance company offer to the agent?

(c) Suppose now that there is asymmetric information. Find the optimal contract. Discuss.

University of California, Riverside

Microeconomics Qualifying Exam

September 8, 2017

INSTRUCTIONS

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- *Answer all questions.*

Part I

Problem 1. Let X be a nonempty set, $\mathcal{A}$ a collection of nonempty subsets of X, and C a choice correspondence on $\mathcal{A}$. Recall the following axioms.

- (Axiom α) If $S, T \in \mathcal{A}$ and $T \subseteq S$, then $C(S) \cap T \subseteq C(T)$.
- (Axiom β) If $S, T \in \mathcal{A}$, $T \subseteq S$, and $C(S) \cap T \neq \emptyset$, then $C(T) \subseteq C(S)$.
- (WARP) If $S, T \in \mathcal{A}$ and $C(T) \cap S \neq \emptyset$, then $C(S) \cap T \subseteq C(T)$.

When we proved that Axioms α and β together are equivalent to WARP in class, we assumed that $\mathcal{A}$ is the collection of all nonempty subsets of X. But assuming observability of a decision maker's choice from every choice set is sometimes too strong. In this problem, by weakening this assumption, let us instead assume that $\mathcal{A}$ is just a nonempty collection of nonempty subsets of X. Prove or falsify each of the following claims. (To prove, give a proof. To falsify, give a counterexample.)

(1) If C satisfies Axioms α and β, then C satisfies WARP.

(2) If C satisfies WARP, then C satisfies Axioms α and β.

(3) If C is rationalizable, then C satisfies WARP.

(4) If C satisfies WARP, then C is rationalizable.

Problem 2.

(1) For any $x \in [0, 100]$, let $\mathbf{p} = \frac{x}{200}\delta_0 \oplus \frac{x}{200}\delta_x \oplus \frac{200-2x}{200}\delta_{200}$ and $\mathbf{q} = \frac{1}{2}\delta_0 \oplus \frac{1}{2}\delta_{100}$. Identify a set $S \subseteq [0, 100]$ that satisfies $\mathbf{p}\,\mathrm{FSD}\,\mathbf{q} \Leftrightarrow x \in S$.

(2) Let $Z = \mathbb{R}$, and $L^*(Z)$ be the set of all lotteries $\mathbf{p}$ with finite support such that $\mathbf{p}(z) \neq \mathbf{p}(z')$ for any distinct $z, z' \in \mathrm{supp}(\mathbf{p})$. Let $\succcurlyeq$ be a preference relation on $L^*(Z)$ that compares the most likely prizes by a continuous function $u : Z \to \mathbb{R}$. To recall, this means that, for any $\mathbf{p}$ and $\mathbf{q}$ in $L^*(Z)$, if $z \in Z$ is the most likely prize of $\mathbf{p}$, and $z' \in Z$ is that of $\mathbf{q}$, then $\mathbf{p} \succcurlyeq \mathbf{q}$ if and only if $u(z) \geq u(z')$. Or, succinctly,

$$\mathbf{p} \succcurlyeq \mathbf{q} \iff u(\arg\max_{z \in Z} \mathbf{p}(z)) \geq u(\arg\max_{z \in Z} \mathbf{q}(z))$$

for any $\mathbf{p}$ and $\mathbf{q}$ in $L^*(Z)$. Choose one or more properties of the function u from the list below to fill the box in the next statement and make it a true proposition. Then, prove the proposition.

$$\succcurlyeq \text{ is risk averse} \quad \text{if and only if} \quad u \text{ is } \boxed{\quad ? \quad}.$$

List of properties: increasing, decreasing, concave, convex, constant, polynomial, linear.

Problem 3. Suppose that a monopolistic firm produces a good under a cost function

$$c(q) = \begin{cases} \frac{1}{3}q^3 & \text{if } q < A, \\ Aq^2 - A^2q + \frac{1}{3}A^3 & \text{otherwise} \end{cases}$$

while the inverse demand function for the good is given by $p = B - q$, where $A > 0$ and $B > 0$ are positive parameters.

(1) draw this firm's marginal cost curve on the graph where the horizontal axis measures q and the vertical axis measures $c'(q)$.

(2) compute the monopoly quantity as a function of a and b.

(3) when $A^2 + A < B < A^2 + 2A$, the size of a deadweight loss in this market due to monopoly is given by an equation of the following form:

$$\text{DWL} = \int_{\boxed{\text{(a)}}}^{A} (B - q - q^2)dq + \int_{A}^{\boxed{\text{(b)}}} (B - q + A^2 - 2Aq)dq.$$

Fill the values in (a) and (b).

Part II

Problem 1. Jack and Jill contribute to a public good that they both enjoy. They play a game in which each privately writes on a piece of paper how much he or she is willing to spend on the good. Then the numbers on the papers are revealed. Whoever's number is larger is the (only) one to make the expenditure he or she volunteered to make. If the amount spent on the public good is g, then each receives a *gross* benefit of $v(g) = 2g - \frac{1}{2}g^2$.

Ties are broken with each player having a 1/2 chance of being designated the provider.

The realized payoffs are $v(g) - g$ for the person who is the provider and spends amount g on the good, and the other player's payoff is just $v(g)$.

The foregoing is common knowledge.

Determine a *symmetric* equilibrium to this game.

(Note: this question was *NOT* on the July 2017 exam.)

Problem 2. A seller uses an all-pay auction to sell a single object. Bidders 1 and 2 have values v_1 and v_2 for the good, respectively. These values are private information but they are correlated, with probabilities given in the following table:

	$v_2 = 0$	$v_2 = 1$	$v_2 = 2$
$v_1 = 0$	$\frac{1}{7}$	$\frac{1}{7}$	0
$v_1 = 1$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
$v_1 = 2$	0	$\frac{1}{7}$	$\frac{1}{7}$

Each player submits a bid. The highest bid wins the object and both players pay the amounts of their own bids. The winning bidder obtains a final payoff of (private object value) – (amount paid) while the losing bidder receives a payoff of –(amount paid).

Ties are broken with each player having a 1/2 chance of being designated the winner.

After learning their own values, players simultaneously submit their bids.

The foregoing is common knowledge.

Determine a *symmetric* Bayesian equilibrium to this game. Verify that your proposed strategies indeed constitute an equilibrium.

Problem 3. Consider the following game of incomplete information between Row and Column. Row's set of possible actions is $S_R = \{a, b\}$, and Column's set of possible actions is $S_C = \{c, d\}$. Players' payoffs are determined by the players' actions and the state of Nature. The state of Nature is either A or B, and both of these states are equally likely.

Payoffs, as a function of state, are given in the following matrices, with the first number being the payoff to Row, the second the payoff to Column.

Payoffs when the state is A:

	c	d
a	3, 0	1, 1
b	0, 1	2, 0

Payoffs when the state is B:

	c	d
a	0, 3	4, 0
b	3, 0	1, 3

Before the game is to be played, Row observes the state of Nature. Column does not know the state, but initially believes the states to be equally likely. After learning the state, Row chooses an action, with the knowledge that this action will be observed by Column before Column makes his choice.

The foregoing is common knowledge between the players.

Derive a **Perfect Bayesian equilibrium** to this game. *Be complete in your description of the equilibrium, completely showing strategies and beliefs.*

Part III

Problem 1. There are two goods, x and y, and two people, Andy and Breanne, in an exchange economy. No production is possible. An allocation is a list $(x_A,\ y_A,\ x_B,\ y_B)$ specifying what each person receives of each good. Andy's and Breanne's preferences are described by the utility functions

$$u_A(x_A, y_A) = 2x_A + y_A + \alpha \log x_B$$

and

$$u_B(x_B, y_B) = x_B + y_B$$

The two goods are available in the positive amounts $\overline{x}$ and $\overline{y}$, and α satisfies $0 < \alpha < \overline{x}$. Note that Andy cares directly about how much Breanne receives of the x-good. Determine all the Pareto efficient allocations in which Andy and Breanne both receive a positive amount of each good.

Problem 2. Consider a risk-neutral monopolist Principal who offers an insurance to a risk-averse Agent. Agent has an initial wealth of 64 and faces a risk: with probability π_e he can lose 28. This probability depends on whether or not Agent is cautious. Agent can decide whether to be cautious (e_1) or not (e_2). Being cautious generates a disutility c. When Agent is cautious he loses 28 with probability $\frac{1}{2}$ whereas if he is not, the probability is $\frac{3}{4}$. The elementary utility function of Agent is $u(w) = \sqrt{w}$, where w denotes wealth.

(1) Find the decision of Agent when he has no insurance.

(2) Find the contract that Principal would offer in case of symmetric information, assuming $c \leq 0.5$. An insurance contract consists of a premium P and compensation X.

(3) Discuss what would happen if Principal offers $(P, X) = (15, 29)$ when Agent's action is unobservable.

(4) Find the optimal contract under asymmetric information, assuming $c \leq 0.5$.

(5) What would happen if $c > 0.5$? Discuss both symmetric and asymmetric information cases.

UNIVERSITY OF CALIFORNIA, RIVERSIDE

Microeconomics Qualifying Exam

JULY 9, 2018

INSTRUCTIONS

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- *Assemble your solutions in numerical order, starting with #1, for each part, and assemble parts in order.*
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Part I

Problem 1. Let $X \subseteq \mathbb{R}^2_+$, and consider a consumer whose preference relation on X is represented by a utility function $u(x_1, x_2) = 8\sqrt{x_1} + x_2$. Suppose that the unit prices of good 1 and good 2 are both \$1, and the consumer has an income of \$21. So, the consumer's budget set is given by

$$B = \{x \in X : x_1 + x_2 \leq 21\}.$$

For each of the following cases, compute the consumer's optimal consumption bundles. If there are more than one optimal consumption bundle, identify all of them. Hint: The following information may be found useful.

$$\sqrt{15} \approx 3.87298 \qquad\qquad \sqrt{17} \approx 4.12311$$

(1) The consumption of good 1 is limited: $X = \{x \in \mathbb{R}^2_+ : x_1 \leq 8\}$.

(2) The consumption of good 2 is limited: $X = \{x \in \mathbb{R}^2_+ : x_2 \leq 8\}$.

(3) Good 1 is sold as packs of two: $X = \{x \in \mathbb{R}^2_+ : x_1 = 2k,\ k \in \mathbb{N}\}$.

(4) Good 2 is sold as packs of two: $X = \{x \in \mathbb{R}^2_+ : x_2 = 2k,\ k \in \mathbb{N}\}$.

(5) Goods are sold as a pair: $X = \{x \in \mathbb{R}^2_+ : x_1 = x_2\}$.

(6) Goods are indivisible and sold as a pair: $X = \{x \in \mathbb{R}^2_+ : x_1 = x_2 \in \mathbb{N}\}$.

Problem 2. In both experiments and field studies, decision makers are sometimes observed to make distorted evaluations of probabilities. (For example, some drivers may underestimate a chance of accidents and engage in reckless driving.) To study preferences of such economic agents, consider the following model. Let $Z = \mathbb{R}_+$, and $L(Z)$ be the set of all lotteries with finite support. We say that a preference relation $\succsim$ on $L(Z)$ admits a ψ-distorted expected utility representation if ψ is a continuous and strictly increasing function from $[0,1]$ to $[0,1]$ with $\psi(0) = 0$, and there exists a continuous and strictly increasing function $u : Z \to \mathbb{R}_+$ such that

$$\mathbf{p} \succsim \mathbf{q} \text{ in and only if } \sum_{z \in Z} \psi(\mathbf{p}(z))u(z) \geq \sum_{z \in Z} \psi(\mathbf{q}(z))u(z)$$

for any $\mathbf{p}$ and $\mathbf{q}$ in $L(Z)$. Prove each of the following claims.

(1) Define ψ by $\psi(a) = ta$ for all $a \in [0,1]$, where $t \in (0,1]$ is a constant. Then, any preference relation $\succsim$ on $L(Z)$ that admits a ψ-distorted expected utility representation must satisfy the independence axiom.

(2) Define ψ by $\psi(a) = a^2$ for all $a \in [0,1]$. Then, any preference relation $\succsim$ on $L(Z)$ that admits a ψ-distorted expected utility representation must violate the independence axiom.

Problem 3. Suppose that a monopolistic firm produces a good under a cost function

$$c(q) = \begin{cases} 8q & \text{if } q < 6, \\ \frac{1}{2}q^2 + 2q + 18 & \text{otherwise} \end{cases}$$

while the inverse demand function for the good is given by $p = 16 - q$.

(1) Compute the monopoly price and monopoly quantity of the good.

(2) Compute the competitive price and competitive quantity of the good.

(3) Compute the size of a deadweight loss in this market due to monopoly.

[This is the end of Part I]

Part II

Problem 1. Consider a classic problem of common property. There are N fishing boats on a lake. Assume that the benefit that boat $i \in \{1, 2, \ldots, N\}$ can derive from fishing is $B_i = L_i - \frac{\beta}{2}\bar{L}^2$, where L_i is the amount of labor on boat i, $\beta > 0$ is an exogenous parameter denoting the potential congestion problem, and $\bar{L} \equiv \sum_{i=1}^{N} L_i$ is the total amount of labor on all the boats. Assume each unit of labor costs $w \in (0, 1)$ to the owner of the boat.

(1) Assume all the boats are owned by one person, and she is choosing the amount of labor on each boat to maximize the total net benefit from all the boats. Assume she chooses the same amount of labor across all the boats. Consider only interior solutions. What is the optimal amount of labor that she would like to have on each boat?

(2) Assume each boat is owned by one person, respectively, and each person is choosing the amount of labor on her own boat to maximize the net benefit from her own boat. Assume the choices are simultaneous. Consider only interior solutions. In a symmetric Nash equilibrium, what is the amount of labor that each person will have on her own boat?

(3) Given $N \geq 2$, compare the two scenarios above. Under which scenario is there more labor on each boat? Also discuss the intuition of your answer.

Problem 2. Consider the following two stage games, WLS and BPSD.

	Opera	Boxing	Laze	Work
Opera	1,4	0,0	0,0	5,-1
Boxing	0,0	4,1	0,0	5,-1
Laze	0,0	0,0	3,3	5,-1
Work	-1,5	-1,5	-1,5	4,4

Battle of the Prisoner's Sex Dilemma

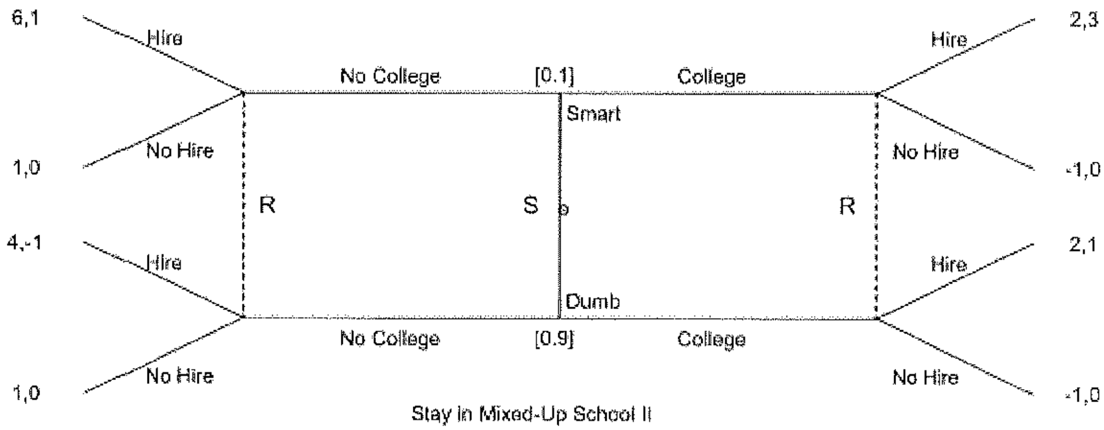
	Work	Laze	Sab
Work	5, 5	-3, 6	-3, -3
Laze	6, -3	-1, -1	-3, -3
Sabotage	-3, -3	-3, -3	-6, -6

Work, Laze, Sabotage

(1) In BPSD ($T = 2, \delta$), is there a range of $\delta \in (0, 1]$ such that a subgame perfect Nash equilibrium induces (Work, Work) in the first period?

(2) In WLS ($T = \infty, \delta$), is there a range of $\delta \in (0, 1]$ such that a subgame perfect Nash equilibrium induces (Work, Work) in the first period?

Problem 3. Consider the following game of signaling.



(1) What are the Bayesian Nash equilibria? Do not forget the ones with mixed strategies!

(2) Are these Bayesian Nash equilibria also perfect Bayesian equilibria?

[This is the end of Part II]

Part III

Problem 1. There are two goods (quantities denoted by x and y) and two people (Al and Bill) in the economy. Al owns 8 units of the x-good and none of the y-good. Bill owns none of the x-good, and 3 units of the y-good. Their preferences are described by the utility functions

$$\text{Al: } u_A(x_A, y_A) = x_A y_A$$
$$\text{Bill: } u_B(x_B, y_B) = y_B + \log(x_B)$$

(a) Determine both consumers' demand functions and the market demand function.

(b) Compute the Walrasian equilibrium prices and allocations.

Problem 2. Assume that a businessman wants to contract a worker, but there are aspects concerning the worker that are unknown to the businessman. He knows that the worker is risk-neutral, but with respect to disutility of effort, the worker can be one of two types. The '*good*' type has disutility e^2, and the '*bad*' type has disutility $2e^2$, where effort is denoted by e. Therefore the worker's utility function is either $u^G(w, e) = w - e^2$ or $u^B(w, e) = w - 2e^2$, where w is wage. Both worker types has reservation utility of zero.
The principal is risk neutral, with profit $\pi(w, e) = e - w$. Effort is observable and contractible. Let the probability of a '*good*' worker be $q = 0.5$

(a) Find the symmetric information contracts that will be offered by the businessman. What effort levels will be demanded and what wages would be paid? Calculate the profits.

(b) Find the contracts that principal will offer when there is asymmetric information on the agent's type. Calculate the profits.

(c) Now compute the asymmetric information contract if the principal wants to only contract the '*good*' type agent. Calculate the profits.

(d) Compare (b) and (c). Which one is the optimal contract?

Problem 3. Consider the regular moral hazard model with a risk-neutral principal and a risk averse agent. The agent can choose between two effort levels, $a \in \{\underline{a}, \overline{a}\}$ with associated cost $C_{\underline{a}} = 0$ and $C_{\overline{a}} = c$, with $c > 0$. Each action generates stochastically one of two possible output levels, $x \in \{\underline{x}, \overline{x}\}$ with $\overline{x} > \underline{x}$ and $p(\underline{x}|\underline{a}) > p(\underline{x}|\overline{a})$. The utility function of the agent is $u(w, C_i) = \log w - C_i$. The value of the outside option is normalized to zero. Risk-neutrality of the principal implies that his payoff function is $x - w$.

(a) Carefully set up the principal's maximization problem under asymmetric information when the principal wishes to implement the high effort level $\overline{a}$.

(b) Solve explicitly for the optimal wage schedule to be offered to the agent which implements the high effort level $\overline{a}$.

[This is the end of Part III]

UNIVERSITY OF CALIFORNIA, RIVERSIDE

Microeconomics Qualifying Exam

SEPTEMBER 7, 2018

INSTRUCTIONS

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- *Assemble your solutions in numerical order, starting with #1, for each part, and assemble parts in order.*
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Part I

Problem 1. Let $X = \mathbb{R}^2_+$. We say that a preference relation $\succsim$ on X satisfies (i) a condition CI if

$$x \sim y \quad \text{implies} \quad x \sim \lambda x + (1-\lambda)y$$

for any $x, y \in X$ and $\lambda \in [0,1]$ and (ii) a condition INT if $(x_1, x_2) \sim (x_2, x_1)$ for any $x \in X$. For each of the following utility functions u, and for each of the two conditions above, prove or falsify that a preference relation represented by u satisfies the condition.

(1) $u(x_1, x_2) = x_1 + x_2$.

(2) $u(x_1, x_2) = \min\{x_1, x_2\}$.

(3) $u(x_1, x_2) = x_1 x_2$.

(4) $u(x_1, x_2) = \sqrt{x_1} + x_2$.

Problem 2. For any $x \in (0, 100)$, let $\mathbf{p}_x = \frac{200-2x}{200}\delta_0 \oplus \frac{x}{200}\delta_x \oplus \frac{x}{200}\delta_{100}$. Answer the following questions.

(1) When $\mathbf{q} = \frac{1}{2}\delta_0 \oplus \frac{1}{2}\delta_{75}$, identify a set $S \subseteq (0,100)$ such that $\mathbf{p}_x \,\mathrm{FSD}\, \mathbf{q} \Leftrightarrow x \in S$.

(2) When $\mathbf{q} = \frac{1}{2}\delta_0 \oplus \frac{1}{2}\delta_{25}$, identify a set $S \subseteq (0,100)$ such that $\mathbf{p}_x \,\mathrm{FSD}\, \mathbf{q} \Leftrightarrow x \in S$.

Problem 3. Consider a firm producing an output good under a production function $f(x)$, where $x = (x_1, \ldots, x_n)$ is a vector of input goods in $\mathbb{R}^n_+$. The unit price of input good x_i is given by w_i. For each of the following cases, compute the cost function $c(q)$ for the firm to produce q units of the output good.

(1) $n = 2$, $f(x_1, x_2) = x_1 x_2$, $w_1 = 1$, $w_2 = 2$.

(2) $n = 2$, $f(x_1, x_2) = \sqrt{x_1} + x_2$, $w_1 = w_2 = 1$.

(3) $n = 3$, $f(x_1, x_2, x_3) = \sqrt{x_1} + \min\{x_2, x_3\}$, $w_1 = w_2 = w_3 = 1$.

[This is the end of Part I.]

Part II

Problem 1. Players A and B are bargaining over a pie of size 1:

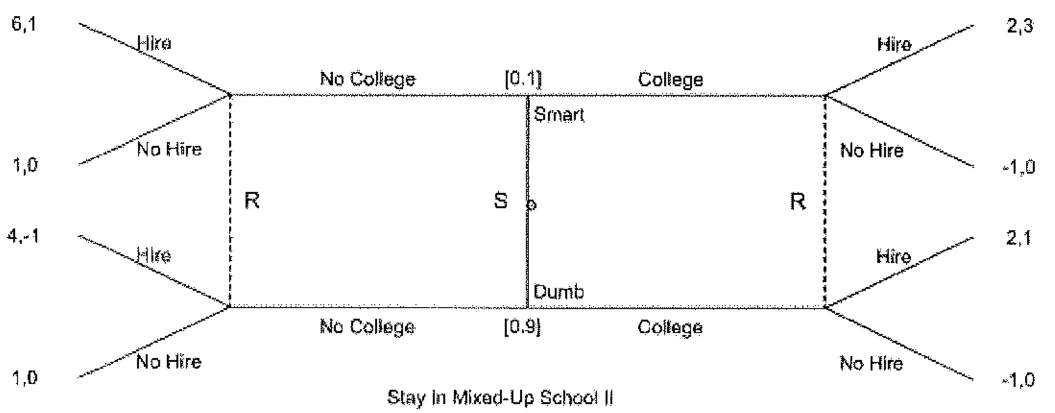
- Period 0: A offers split (x_0, y_0) to B. If B accepts, the game ends with split (x_0, y_0); if B rejects, the game goes on to period 1.
- Period 1: B offers split (x_1, y_1) to A. If A accepts, the game ends with split (x_1, y_1); if A rejects, the game goes on to period 2.
- Period 2: A offers split (x_2, y_2) to B. If B accepts, the game ends with split (x_2, y_2); if B rejects, the game ends with payoffs $(0, 0)$.

Naturally, we assume $x_t + y_t = 1$, $x_t \geq 0$, and $y_t \geq 0$, where $t = 0, 1, 2$. A's payoff is x_t, and B's payoff y_t, where t denotes when they get the payoffs. The time discount factors are δ_A and δ_B, respectively for the players, where $\delta_A \in (0,1)$ and $\delta_B \in (0,1)$. We also assume that players accept an offer if indifferent between accepting and rejecting. What are the reasonable outcomes predicted by backwards induction, and how would they depend on the discount factors?

Problem 2. Prove the Weak Folk Theorem:

> Denote Y_k as player $k \in \{1, 2, \ldots, N\}$'s payoff in the stage-game Nash equilibrium that gives her the lowest payoff among all the stage-game Nash equilibria. For any stage game G, consider any stage-game actions $(a_1, \ldots, a_N)$ with payoffs $(X_1, X_2, \ldots, X_N)$ such that $X_k > Y_k$ for any k. Then there exists a $\delta^* < 1$ such that, for all $\delta > \delta^*$, there exists a subgame perfect Nash equilibrium of $G(\infty, \delta)$ in which the stage-game actions $(a_1, \ldots, a_N)$ are played each period on the equilibrium path.

Problem 3. Consider the following game of signaling.



(1) What are the Bayesian Nash equilibria? Do not forget the ones with mixed strategies!

(2) Are these Bayesian Nash equilibria also perfect Bayesian equilibria?

[This is the end of Part II.]

Part III

Problem 1. Consider the two-person, two-good production economy. Person i's utility function is given by u^1, where $u^1(x_1^1, x_2^1) = (x_1^1)(x_2^1)$ and $u^2(x_1^2, x_2^2) = (x_1^2)(x_2^2)^2$, and where x_i^j denotes person i's consumption of good j, for i and j =1, 2. Each consumer is endowed with one unit of each good.

There is one firm in the economy that produces good 2 from good 1 according to the following production function :

$$y_2 = f(y_1) = 2\sqrt{y_1},$$

where y_1 denotes the amount of good 1 used as input and y_2 denotes the amount of good 2 produced. Suppose that person 1 receives all the profits from the firm. Calculate the Walrasian equilibrium prices in this economy.

Problem 2. The utility of an agent is represented by the function $u(w, e)$ with w as wage, and 'cost' of effort denoted by e. The agent's reservation utility is $\underline{u}$. As a result of his action, two outcomes x_1 and x_2 are possible. The probability for the better outcome x_2 is p^H or p^L, contingent on effort level e^H or e^L. Let $u(w, e) = \sqrt{w} - e^2$, $e^H = 2$, $e^L = 1$, $p^H = 0.5$, $p^L = 0.25$, and $\underline{u} = 2$. The effort chosen by the agent is not observable to the principal. Calculate the optimal contract for the risk neutral principal if she demands

1. Low effort e^L
2. High effort e^H
3. How large would the difference between the good (x_2) and the bad outcome (x_1) have to be so that the principal demands high effort?

Problem 3. Consider a risk-neutral principal (P) hiring an agent (A) whose effort e will be observed. She will pay the agent a wage $w(e)$. From the cooperation between P and A an outcome $x(e) = e + 5$ is obtained.

The agent may have two different types, A or B, each with probability $\frac{1}{2}$. The preferences of the two types of agents are $u_A(w, e) = w - e^2$ and $u_B(w, e) = w - 2e^2$, and their reservation utility is $\underline{u} = 1$.

1. Find the contract that the principal will offer if the principal can observe the agent's type. Discuss if the same contracts will be offered if there is asymmetric information regarding the agent's type.
2. Suppose now that the principal has to compete with many others to secure the services of the agent. Calculate the symmetric information contract. How does it change if there is asymmetric information?

[This is the end of Part III.]