UNIVERSITY OF CALIFORNIA, RIVERSIDE

Microeconomics Cumulative Exam

SEPTEMBER 21, 2012

INSTRUCTIONS

- *Put your identification number on the front of your packet of solutions and each subsequent page (using the first box in the upper right-hand corner).*
- *Write solutions on only one side of each page of paper and do not put answers for more than one question on a page.*
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INSTRUCTIONS: ANSWER PARTS I, II, AND III.

Part I

1. The profit function for a firm is given by

$$\pi(r,p) = r_1^{\alpha_1} + r_2^{\alpha_2} + p_1^{\beta_1} + p_2^{\beta_2},$$

where r_j, $j = 1,2$, and p_i, $i = 1,2$, are output and input prices, respectively, and α_j, $j = 1,2$, and β_i, $i = 1,2$, are parameters.

   (a) Place restrictions on the parameters that suffice for this profit function to satisfy the conditions implied by profit-maximizing behavior.

   (b) Show that the j^{th} output-supply function is nondecreasing in r_j and the i^{th} input-demand function is nonincreasing in p_i.

   (c) To what extent do the properties of π rely on concavity of the production function?

2. Prove the following three statements:

   (a) If the utility function is homothetic, the "cost of living index,"

$$I(u, p^b, p^c) = \frac{E(u, p^c)}{E(u, p^b)},$$

   where E is the expenditure function, p^b and p^c are the price vectors in the base period and the current period, respectively, is independent of the "reference" utility level, u. (Maintain Shephard's decomposition theorem.)

(b) In a consumer's preference ordering, all pairs of goods can be net substitutes for one another, but they can't all be net complements to one another.

(c) If the (aggregate) elasticity of substitution between labor and capital is less than one, factors are competitively priced, and the aggregate technology is homothetic, capital deepening (increasing the capital/labor ratio) increases labor's share of national income.

Part II

1. Two agents jointly own a firm and each has an equal share. The value of the whole firm to agent i is v_i, $i = 1, 2$, and $v_1 > v_2 > 0$. Suppose now the agents must dissolve the firm.

   The parties agree to the following binding arbitration rule. Both parties independently and simultaneously submit bids to the arbitrator, b_1 for player 1 and b_2 for player 2.

   - If $b_1 \geq b_2$ then player 1 gets the firm and pays b_1 to player 2. Agent 1's payoff is $v_1 - b_1$ and agent 2's is b_1.
   - If $b_2 > b_1$ then player 2 gets the firm and pays b_2 to player 1. Agent 1's payoff is b_2 and agent 2's is $v_2 - b_2$.

   (Note the tie-breaking rule implicit in the above description.) The foregoing is common knowledge.

   (a) Calculate the set of of pure strategy equilibria in which player 1 gets the firm.

   (b) Calculate the set of of pure strategy equilibria in which player 2 gets the firm.

2. Gertrude has died and left behind \$1 million. The money will either go to the Save the Whales Foundation or be divided among Gertrude's 4 daughters. The daughters—Andi, Brandi, Candi, and Dorothy—will get the money if they agree upon a division. But Gertrude, who throughout her life embraced a majority-rules decision rule, has left rather unusual instructions, which are as follows.

   > Daughters make proposals in alphabetical order, each making at most one proposal. A proposal has the form (a, b, c, d), where a is the amount for Andi, b the amount for Brandi, c the amount for Candi, and d the amount for Dorothy. All amounts must be nonnegative and satisfy $a+b+c+d \leq \$1$ million. If a daughter's proposal is not accepted by *strictly* more than half of the daughters remaining for consideration, then that daughter is eliminated from the group eligible to split the inheritance. If a proposal is agreed to by a strictly more than half of the remaining daughters, then the game stops and the inheritance is divided as agreed. If all daughters' proposals are rejected, then the Whales get the inheritance.

   Thus, Andi makes the first proposal (a, b, c, d); if this is not agreed to by at least 3 of the 4 sisters (e.g., Andi and two other sisters), then Andi is removed from further consideration. Then in the next period, Brandi makes a proposal, which must be agreed to by at least 2 of the remaining 3 sisters for the game to end, and so on.

   The daughters have a common discount factor of $\delta < 1$ between proposal periods. A daughter's payoff is simply the present value of dollars she will receive. All above is common knowledge.

   Derive the subgame perfect equilibrium outcome to this game.

3. Two agents jointly own a firm and each has an equal share. The value of the whole firm to agent i is a random variable v_i, independently and uniformly distributed on $[0, 1]$. An individual's value for the firm is privately known only to that agent. Suppose now the agents must dissolve the firm.

   The parties have become very disagreeable and can carry out this dissolution only with the help of a middleman, Uncle Fred. Uncle Fred will handle the transaction as follows. Under binding arbitration, agents 1 and 2 simultaneously announce bids b_1 and b_2, respectively.

   - If $b_1 \geq b_2$ then player 1 gets the firm and pays b_1 to Uncle Fred, who then gives player 2 the amount b_2 (Uncle Fred keeps $b_1 - b_2$ for his trouble). Agent 1's payoff is $v_1 - b_1$ and agent 2's is b_2.
   - If $b_2 > b_1$ then player 2 gets the firm and pays b_2 to Uncle Fred, who then gives player 1 the amount b_1 (Uncle Fred keeps $b_2 - b_1$ for his trouble). Agent 1's payoff is b_1 and agent 2's is $v_2 - b_2$.

   The foregoing is common knowledge.

   Determine a symmetric equilibrium bidding strategy under this arbitration rule and determine Uncle Fred's expected revenue.

Part III

1. Assume that, in a world with uncertainty, there are two assets. The first is a riskless asset that pays 1 dollar. The second pays amounts a and b with probabilities of π and $1-\pi$ respectively. Denote the demand for two assets by x_1 and x_2.

   Suppose that a decision maker's preferences satisfy the axioms of expected utility theory and he is risk averse. The decision maker's wealth is 1, and so are the prices of the assets. Therefore, the decision maker's budget constraint is given by

$$x_1 + x_2 = 1$$

   with

$$x_1, x_2 \in [0, 1].$$

   (a) Give a simple necessary condition (involving a and b only) for the demand for the riskless asset to be strictly positive.

   (b) Give a simple necessary condition (involving a, b and π only) for the demand for the risky asset to be strictly positive.

   In the next three parts, assume that the conditions obtained in (*a*) and (*b*) are satisfied.

   (c) Write down the first-order conditions for utility maximization in this asset demand problem.

   (d) Assume that $a < 1$. Show by analyzing the first-order conditions that $\frac{dx_1}{da} \leq 0$.

   (e) What can you say about the sign of $\frac{dx_1}{d\pi}$?

2. Consider an exchange economy with two commodities and two consumers. Both consumers have homothetic preferences of constant elasticity variety. Moreover, the elasticity of substitution is the same for both consumers and is small (i.e. goods are close to perfect complements). Specifically,

$$u_1(x_1, x_2) = (2x_{11}^\rho + x_{21}^\rho)^{\frac{1}{\rho}}$$

   and

$$u_2(x_1, x_2) = (2x_{12}^\rho + x_{22}^\rho)^{\frac{1}{\rho}}$$

   with $\rho = -4$. The endowments are $\omega_1 = (1, 0)$ and $\omega_2 = (0, 1)$.

   Compute the excess demand function of this economy. Verify that $(p_1, p_2) = (1, 1)$ is (part of) an equilibrium and argue that it is *locally* unique.

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Instructions: answer Parts I, II, and III.

Part I

Answer *either* Part I.A *or* Part I.B, but *not* both.

Part I.A

If the consumer's preferences satisfy local non-satiation and the solution to the utility-constrained, expenditure-minimization problem is unique at utility u and price vector p,

$$\phi(p, y) = \phi\big(p, E(u, p)\big) = \delta(u, p), \qquad (\star)$$

where y is income (total expenditure), ϕ and δ are the (vector valued) Marshallian and Hicksian demand functions, and E is the expenditure function. Maintain (twice) differentiability at $\langle u, p\rangle$ (equivalently, at $\langle p, y\rangle$) of each of the functions in $(\star)$.

1. Derive the Slutsky equation for an arbitrary pair of commodities, i and j, and explain each of the components of this equation.
2. Employing the equation for $i = j$ and exploiting the properties of E, prove that the demand curve for good i is "downward sloping" $(\partial\phi_i(p, y)/\partial p_i \leq 0)$ if, but not only if, good i is "normal" $(\partial\phi_i(p, y)/\partial y \geq 0)$.
3. Explain why local non-satiation is necessary for $(\star)$ to hold for all p and u.
4. Is convexity of preferences necessary for any of the above results? Explain.
5. Provide a (graphical or functional) counterexample showing that differentiability of the utility function is not needed for the above derivation of the Slutsky equation.

Part I.B

1. Samantha has von Neumann-Morgenstern preferences over lotteries on money. Samantha is risk-averse. For the lottery awarding \$0 with probability 1/3 and \$200 with probability 2/3, Samantha's certainty equivalent is \$100.

   Now consider the lottery L_0 that awards \$0 with probability 1/2 and \$200 with probability 1/2.

   Derive the *least upper bound* and *the greatest lower bound* on Samantha's certainty equivalent for lottery L_0.

2. Consider a firm with the following production function:

$$f(L,K) = \begin{cases} \sqrt{(L-2)(K-3)} & \text{if } L > 2 \text{ and } K > 3 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Either classify this production function by returns to scale or explain why it does not fit into one of the categories of weakly increasing or weakly decreasing returns to scale.

   (b) Suppose prices of factors L and K are w and r, respectively.
   Calculate this firm's cost function.

3. Consider a monopoly with zero costs of production. The monopolist faces two consumers, Bill and Hillary. Bill's demand function is $q_1(p) = 4 - p$, and Hillary has demand given by $q_2(p) = 3 - p$. You may assume there is no resale of the good.

   If the monopolist can offer only a single two-part tariff, what schedule should it offer to maximize its profit? Calculate the associated deadweight welfare loss.

Part II

1. Two firms are engaged in a symmetric duopoly with differentiated products. The firms choose prices simultaneously at the beginning of an infinite sequence of periods. Firms have no costs of production. The demand function facing firm 1 is $Q_1(p_1, p_2) = 1 - 2p_1 + p_2$; the demand function facing firm 2 is $Q_2(p_1, p_2) = 1 + p_1 - 2p_2$. Firms have a common per period discount factor of $\delta \in (0, 1)$. The foregoing is common knowledge.

   The firms would like to collude to achieve (static) maximum combined profit in every period. Calculate the smallest discount factor for which such collusion can be supported by a "Grim Trigger Strategy."

Answer exactly one of the following questions.

2. Two agents jointly own a firm and each has an equal share. The value of the whole firm to agent i is a random variable v_i, independently and uniformly distributed on $[0, 1]$. An individual's value for the firm is privately known only to that agent. Suppose now the agents must dissolve the firm. Under binding arbitration, agents 1 and 2 simultaneously announce bids b_1 and b_2, respectively. If $b_1 \geq b_2$ then player 1 gets the firm and pays player 2 the amount b_2, and agent 1's payoff is $v_1 - b_2$ and agent 2's is b_2. If $b_2 > b_1$, then player 2 gets the firm and pays player 1 the amount b_1, and agent 2's payoff is $v_2 - b_1$ and agent 1's is b_1. The foregoing is common knowledge.

   Determine a symmetric equilibrium bidding strategy under this binding arbitration rule.

3. Two players are considering contributing to provide a public good. If the good is provided, player i will get a gross benefit of v_i, $i = 1, 2$. Only player i knows his true value v_i. These private values are independently distributed random variables. Player 1's value is uniformly distributed over the interval $[0, 2]$; player 2's value is distributed over the interval $[0, 1]$ with cumulative distribution function $F_2(v_2) = (v_2)^2$, for $v_2 \in [0, 1]$. Only 1 contribution is needed to fund the good. Contributions are not refunded. If a citizen contributes, his contribution is c (other than 0 or c, the level of contribution is not a choice variable). Here $c \in (0, 1)$ is a parameter.

   If the good *is* provided, then payoffs to player i is are v_i if he did not contribute and $v_i - c$ if he did. If the good is *not* provided, then all payoffs are 0.

   Find *all* of the Bayesian equilibria in this game. (You don't need to distinguish among equilibria that differ only on a set of types having probability 0.)

Part III

Answer *either* Part III.A *or* Part III.B, but *not* both.

Part III.A

1. True or false – explain.

   (a) Convexity of preferences is both necessary and sufficient condition for the existence of a competitive equilibrium.

   (b) A loss in welfare is inevitable in any imperfectly competitive market except for a perfectly discriminating monopolist.

2. Show that if a competitive equilibrium exists in an economy where all goods are gross substitutes, then it is indeed unique.

Part III.B

1. Let Z denote a (finite) set of monetary prizes. Let $\mathcal{L}(Z)$ denote the (infinite) set of all lotteries on Z, and $\succcurlyeq$ denote a rational preference relation on $\mathcal{L}(Z)$.

   (a) Write out, clearly and formally, the axioms which ensure that $\succcurlyeq$ admits the von Neuman-Morgenstern utility representation.

   (b) For any $p \in \mathcal{L}(Z)$, define

   $$U(p) = E(p) - \frac{1}{4}var(p)$$

   where $E(p)$ is the expected value of the prize of lottery p, and $var(p)$ is the variance of of the prize p. Show that U defined above induces a preference relation which is not consistent with the assumptions of the von Neuman-Morgenstern theorem.

   (c) Assume now that Z is a set of a finite number of income levels. An income distribution specifies the proportion of individuals at each level. Thus, an income distribution has the same mathematical structure as a lottery. Consider the binary relation 'one distribution is (weakly) more egalitarian than another'. Are the von Neuman-Morgenstern axioms appropriate for this binary relation?

2. An individual has a continuous increasing Bernoulli utility function $u(\cdot)$ and initial wealth ω. Let lottery L offer a payoff of G with probability π and payoff of B with probability $1-\pi$.

   (a) If the individual owns the lottery, what is the minimum price she would sell it for?

   (b) If the individual does not own the lottery, what is the maximum price she would buy it for?

   (c) Are buying and selling prices equal? Give an economic interpretation for your answer. Find an example of preferences for which buying and selling prices are equal.

   (d) Let $G = 20$, $B = 10$, $\omega = 20$, $\pi = \frac{1}{2}$ and $u(x) = \sqrt{x}$. Compute the buying and selling prices for this lottery and this utility function.

3. Consider a pure exchange economy with two consumers, Tom and Jerry, and two goods, cheese and wine. Tom and Jerry have the following utility functions respectively:

   $$u_t = min\{x_{wt}, x_{ct}\}$$

   and

   $$u_j = min\{x_{wj}, \sqrt{x_{cj}}\}$$

   where x_{ct} is Tom's consumption of cheese, x_{cj} is Jerry's consumption of cheese, x_{wt} is Tom's consumption of wine and x_{wj} is Jerry's consumption of wine. Neither of them can consume negative amounts of any good and both are price takers.

   (a) Tom has an endowment of 30 units of wine and none of cheese. Jerry starts with 20 units of cheese and none of wine. Solve for the equilibria in this economy and illustrate your solution in an Edgeworth box diagram.

   (b) Repeat the above supposing that Tom starts with 5 units of wine and none of cheese and Jerry starts with 20 units of cheese and none of wine.

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where y is income (total expenditure), ϕ and δ are the (vector valued) Marshallian and Hicksian demand functions, and E is the expenditure function. Maintain (twice) differentiability at $\langle u,p\rangle$ (equivalently, at $\langle p,y\rangle$) of each of the functions in ($\star$).

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1. Two firms are engaged in a symmetric duopoly with differentiated products. The firms choose prices simultaneously at the beginning of an infinite sequence of periods. Firms have no costs of production. The demand function facing firm 1 is $Q_1(p_1, p_2) = 1 - 2p_1 + p_2$; the demand function facing firm 2 is $Q_2(p_1, p_2) = 1 + p_1 - 2p_2$. Firms have a common per period discount factor of $\delta \in (0, 1)$. The foregoing is common knowledge.

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1. Samantha has von Neumann-Morgenstern preferences over lotteries on money. Samantha is risk-averse. For the lottery awarding \$0 with probability 1/3 and \$200 with probability 2/3, Samantha's certainty equivalent is \$100.

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   Find *all* of the Bayesian equilibria in this game. (You don't need to distinguish among equilibria that differ only on a set of types having probability 0.)

Part III

Answer *either* Part III.A *or* Part III.B, but *not* both.

Part III.A

1. True or false – explain.

   (a) Convexity of preferences is both necessary and sufficient condition for the existence of a competitive equilibrium.

   (b) A loss in welfare is inevitable in any imperfectly competitive market except for a perfectly discriminating monopolist.

2. Show that if a competitive equilibrium exists in an economy where all goods are gross substitutes, then it is indeed unique.

Part III.B

1. Let Z denote a (finite) set of monetary prizes. Let $\mathcal{L}(Z)$ denote the (infinite) set of all lotteries on Z, and $\succcurlyeq$ denote a rational preference relation on $\mathcal{L}(Z)$.

   (a) Write out, clearly and formally, the axioms which ensure that $\succcurlyeq$ admits the von Neuman-Morgenstern utility representation.

   (b) For any $p \in \mathcal{L}(Z)$, define

   $$U(p) = E(p) - \frac{1}{4}var(p)$$

   where $E(p)$ is the expected value of the prize of lottery p, and $var(p)$ is the variance of of the prize p. Show that U defined above induces a preference relation which is not consistent with the assumptions of the von Neuman-Morgenstern theorem.

   (c) Assume now that Z is a set of a finite number of income levels. An income distribution specifies the proportion of individuals at each level. Thus, an income distribution has the same mathematical structure as a lottery. Consider the binary relation 'one distribution is (weakly) more egalitarian than another'. Are the von Neuman-Morgenstern axioms appropriate for this binary relation?

2. An individual has a continuous increasing Bernoulli utility function $u(\cdot)$ and initial wealth ω. Let lottery L offer a payoff of G with probability π and payoff of B with probability $1 - \pi$.

   (a) If the individual owns the lottery, what is the minimum price she would sell it for?

   (b) If the individual does not own the lottery, what is the maximum price she would buy it for?

   (c) Are buying and selling prices equal? Give an economic interpretation for your answer. Find an example of preferences for which buying and selling prices are equal.

   (d) Let $G = 20$, $B = 10$, $\omega = 20$, $\pi = \frac{1}{2}$ and $u(x) = \sqrt{x}$. Compute the buying and selling prices for this lottery and this utility function.

3. Consider a pure exchange economy with two consumers, Tom and Jerry, and two goods, cheese and wine. Tom and Jerry have the following utility functions respectively:

   $$u_t = min\{x_{wt}, x_{ct}\}$$

   and

   $$u_j = min\{x_{wj}, \sqrt{x_{cj}}\}$$

   where x_{ct} is Tom's consumption of cheese, x_{cj} is Jerry's consumption of cheese, x_{wt} is Tom's consumption of wine and x_{wj} is Jerry's consumption of wine. Neither of them can consume negative amounts of any good and both are price takers.

   (a) Tom has an endowment of 30 units of wine and none of cheese. Jerry starts with 20 units of cheese and none of wine. Solve for the equilibria in this economy and illustrate your solution in an Edgeworth box diagram.

   (b) Repeat the above supposing that Tom starts with 5 units of wine and none of cheese and Jerry starts with 20 units of cheese and none of wine.

Department of Economics
University of California, Riverside

CORE CUMULATIVE EXAMINATION FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY
MICROECONOMIC THEORY
SEPTEMBER 16, 2011

INSTRUCTIONS

- *The exam will be monitored by the Graduate Assistant. There will be no communication with faculty members; if you believe a question contains an error or an ambiguity, say so on your written examination, make an assumption to correct the alleged error or to resolve the ambiguity, and answer the question as well as you can.*
- *Put your identification number on the front of your packet of solutions and each subsequent page (using the first box in the upper right-hand corner).*
- *Write solutions on only one side of each page of paper and do not put answers for more than one question on a page.*
- *Assemble your solutions in numerical order, starting with #1.*
- *After compiling your solutions in order, number your pages sequentially in the second box in the upper right-hand corner of each page.*

Part I

1. Consider the function $v(p_1, p_2, m) = \sqrt{m}\left(\frac{1}{\sqrt{p_1}} + \frac{2}{\sqrt{p_2}}\right)$, where p_1 and p_2 denote prices of goods 1 and 2, respectively, and m denotes income.

   Verify that v has the properties of an indirect utility function, and then recover the direct utility function that yields v.

2. In the perfectly competitive wonder-drug market has been around for years, with annual demand for the drug given by $Q_0(p) = 60 - p$. The commonly available technology has cost function $C(q) = q^2$, where q denotes an individual firm's level of output. Anyone wishing to enter the market for the year can do so by paying the *annual* license fee of $F = 16$ on January 2nd.
   Entry later in the year is not possible.

   On January 3rd it was announced that the wonder drug, previously thought effective only for women, has been discovered to be effective for men, too, so annual demand for the drug has been doubled: the new demand is $Q_N(p) = 2Q_0(p)$.

   Determine the *short*-run competitive equilibrium price of the drug after the announcement. You should ignore considerations of discounting over the year.

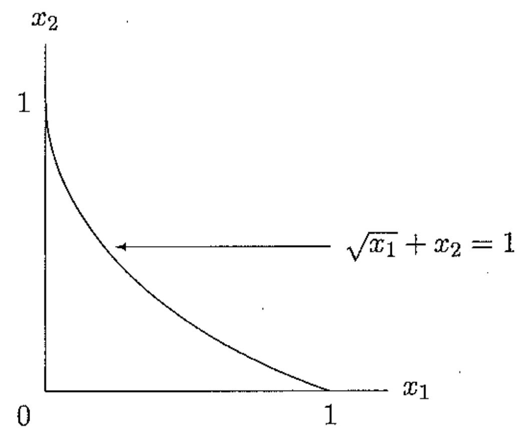
   Determine the *long*-run competitive equilibrium price of the drug after the announcement.

3. A competitive firm uses factors 1 and 2 in production (let (x_1, x_2) denote an input vector), with respective prices w_1 and w_2. The isoquant corresponding to 1 unit of output is shown in the figure below. The firm's production function is homogeneous of degree 1/3.

   (a) Graph the output-expansion path when $(w_1, w_2) = (2, 5)$.

   (b) Derive the firm's cost function.

   (c) Calculate the firm's elasticity of supply with respect to output price when $(w_1, w_2) = (2, 1)$ and the output price is $p = 10$.



Part II

1. Prove or disprove the following propositions:

   (a) If the price elasticity of a downward sloping demand curve is constant everywhere then it must be unitary elastic.

   (b) In the absence of any government intervention, welfare loss arises in a monopoly can be avoided only under perfect price discrimination.

2. Construct an economy to show that an increase in the relative price of a capital intensive good will not necessarily decrease the price of labor services.

Part III

1. Gertrude has died and left behind \$1 million. The money will either go to the Save the Whales Foundation or be divided among Gertrude's 4 daughters. The daughters—Andi, Brandi, Candi, and Dorothy—will get the money if they agree upon a division. But Gertrude, who throughout her life embraced a majority-rules decision rule, has left rather unusual instructions, which are as follows.

   > Daughters make proposals in alphabetical order, each making at most one proposal. A proposal has the form (a, b, c, d), where a is the amount for Andi, b the amount for Brandi, c the amount for Candi, and d the amount for Dorothy. All amounts must be nonnegative and satisfy $a+b+c+d \leq \$1$ million. If a daughter's proposal is not accepted by *strictly* more than half of the daughters remaining for consideration, then that daughter is eliminated from the group eligible to split the inheritance. If a proposal is agreed to by a strictly more than half of the remaining daughters, then the game stops and the inheritance is divided as agreed. If all daughters' proposals are rejected, then the Whales get the inheritance.

   Thus, Andi makes the first proposal (a, b, c, d); if this is not agreed to by at least 3 of the 4 sisters (e.g., Andi and two other sisters), then Andi is removed from further consideration. Then in the next period, Brandi makes a proposal, which must be agreed to by at least 2 of the remaining 3 sisters for the game to end, and so on.

   The daughters have a common discount factor of $\delta < 1$ between proposal periods. A daughter's payoff is simply the present value of dollars she will receive. All above is common knowledge.

   Derive the subgame perfect equilibrium outcome to this game.

INSTRUCTIONS: *Answer* ***one and only one*** *of the following two problems.*

2. Two players may contribute effort toward the production of a public good. If the total amount of effort contributed is G, then the amount of the public good produced equals G. Players may differ in their costs of effort. Let c_i denote the marginal cost of effort for player i. If player i contributes effort g_i, then the total amount of the public good is $G = g_1 + g_2$ and the realized payoff to player i is $v(G) - c_i g_i$, $i = 1, 2$.

   Now suppose c_1 and c_2 are independent random variables, uniformly distributed over the interval $[0, 1]$ and suppose $v(G) = G - \frac{1}{2}G^2$. Players' costs are private information.

   The foregoing description is common knowledge. Derive a Bayesian equilibrium to this game and calculate each player's expected contribution.

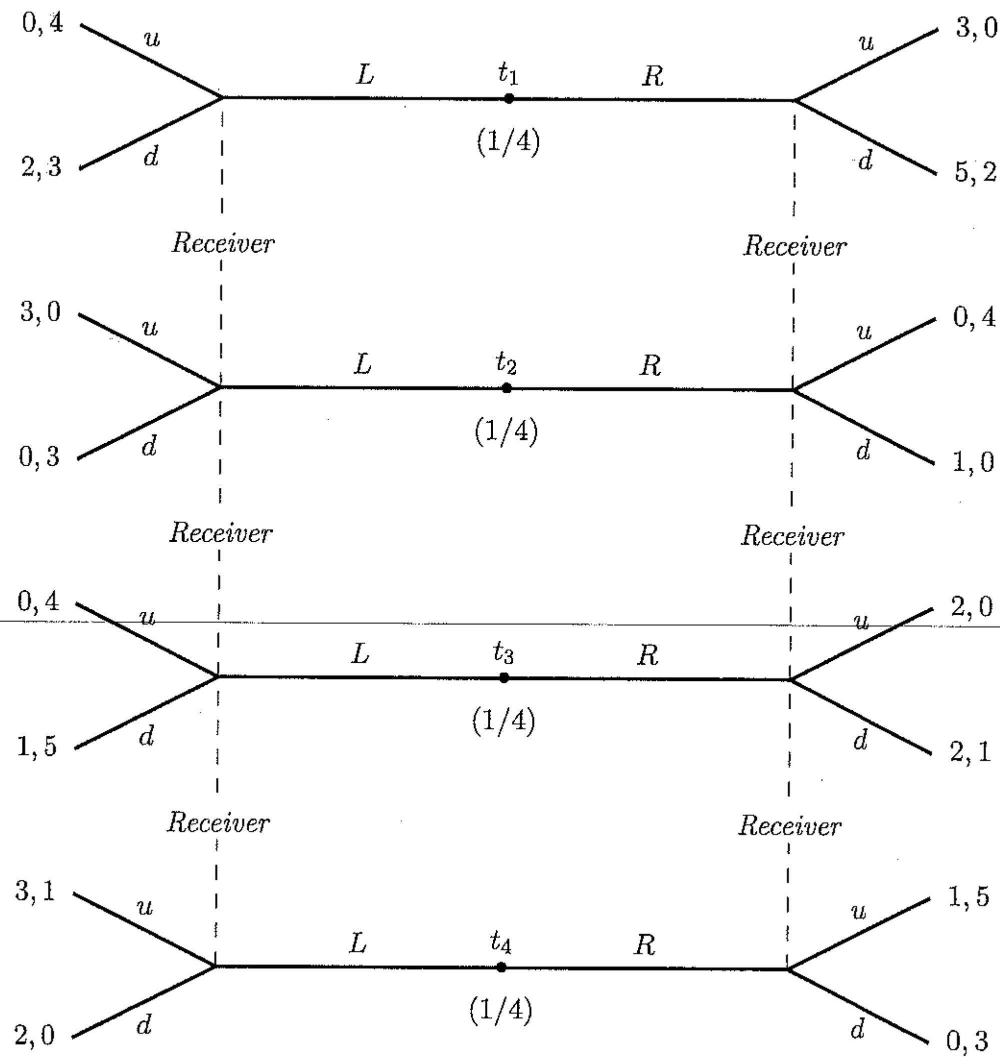
3. Consider the Sender-Receiver game shown below. The Sender's type is either t_1, t_2, t_3, or t_4. This type is chosen by Nature, and *ex ante* both the Sender and Receiver know these possibilities are equally likely.

   Only the Sender observes his type as chosen by Nature; after observing his type, the Sender chooses either L or R. The Receiver observes the Sender's choice but not the Sender's type.

   The game is depicted below. At terminal nodes, the first payoff is the Sender's, the second is the Receiver's.

   This description of the game is common knowledge.

   Derive a perfect Bayesian equilibrium to this game.



Department of Economics
University of California, Riverside

MICROECONOMICS QUALIFYING EXAM

July 1, 2011

INSTRUCTIONS:

- Put your identification number on the front of your packet of solutions and each subsequent page (using the first box in the upper right-hand corner).
- Write solutions on only one side of each page of paper and do not put answers for more than one question on a page.
- Assemble your solutions in numerical order, starting with #1.
- After compiling your solutions in order, number your pages sequentially in the second box in the upper right-hand corner of each page.

INSTRUCTIONS: ANSWER ALL OF THE FOLLOWING QUESTIONS.

Part I

1. State whether each of the following systems of demand functions **could** or **could not** arise from a constrained utility maximization problem of the form $\max U(\cdot)$ s.t. $\mathbf{p}\cdot\mathbf{x}=Y$, where $U(\cdot)$ is strictly quasi-concave and strictly monotonic, and $\mathbf{p}>>0$ and $Y>0$?
Justify your answer.

   (a) $U(x_1,x_2):\quad x_1^*(\mathbf{p},Y)=\frac{Y}{p_1}(1-e^{-p_2})$ and $x_2^*(\mathbf{p},Y)=\frac{Y}{p_2}e^{-p_2}$

   (b) $U(x_1,x_2):\quad x_1^*(\mathbf{p},Y)=\frac{p_1}{p_1^2+p_2^2}Y$ and $x_2^*(\mathbf{p},Y)=\frac{p_2}{p_1^2+p_2^2}Y$

   (c) $U(x_1,x_2):\quad x_1^*(\mathbf{p},Y)=\frac{Y}{2(p_1+\sqrt{p_1}\sqrt{p_2})}$ and $x_2^*(\mathbf{p},Y)=\frac{Y}{2(\sqrt{p_1}\sqrt{p_2}+p_2)}$

2. Oskar has been arrested for terrorism. As part of the enhanced interrogation procedures, the officers place before Oskar a 6-shot revolver (handgun) that is loaded with k rounds of ammunition—Oskar saw the k rounds loaded into the gun (so there are $6-k$ empty chambers). The lead officer spins the cylinder and tells Oskar that if he does not reveal the names of the conspirators in this plot, then he will be required to place the gun at his head and pull the trigger once. If the gun fires, Oskar will be dead and the interrogation is over; if the gun does not fire, Oskar will be allowed to go free.

   Oskar is innocent and has no names to give. But Oskar is also rich, so he offers to pay to have one round removed from the gun before he must pull the trigger (at most one round can be removed).

   Let b_k denote the amount Oskar is just willing to pay to remove one round (i.e., he is indifferent between (i) pulling the trigger with k rounds and (ii) pulling the trigger with $k-1$ rounds after having paid b_k to the officer). (If the revolver has k rounds, then Oskar's chance of dying in this procedure is $k/6$.)

   Oskar has von Neumann-Morgenstern preferences and finds living is preferred to dying, having more money when alive is preferred to having less when alive, and being dead has the same payoff regardless of how much money one has.

   Determine whether b_1 is larger or smaller than b_4, *and carefully explain your answer.*
   (Basically, you are to determine whether Oskar is willing to pay more to remove one round when there are 1 or 4 rounds in the gun.)

3. A firm uses three inputs to produce a single output according to the production function $f(x_1, x_2, x_3)$. This firm takes the prices of its inputs as given, and they are w_1, w_2, and w_3 for inputs 1, 2, and 3, respectively. The total cost function for this firm is given as

$$C(q|w_1, w_2, w_3) = \left(\frac{w_1 w_2 w_3}{w_1 w_2 + 4w_1 w_3 + w_2 w_3}\right) q^2,$$

where q denotes the firm's total output.

(a) Calculate this firm's conditional factor demand for input 1.

(b) Derive the production function $f(x_1, x_2, x_3)$.

Part II

1. At 5 a.m. each day Acme Fruit (AF) delivers q_a pounds of fruit to the farmers' market; at 8 a.m. each day Best Fruit (BF) delivers q_b to the market. At 8:30 a.m. it is common knowlege how much fruit each firm has delivered. Buyers show up starting at 9 a.m. to purchase fruit, and the price is given by $P(q_a + q_b) = 1 - (q_a + q_b)$. AF and BF have the same cost function, $c(q) = \frac{1}{2}q^2$, where q is the amount of fruit that a firm brings to market. Everyone knows that BF has spies who are able to observe AF's delivery decision—so each day the outputs can be modeled as sequentially chosen, with AF choosing first, followed by BF. Firms' common discount factor per day is δ. The foregoing is common knowledge.

   In the past, realized outputs *in each period* have simply been the Stackelberg equilibrium outputs. Bill, the manager at BF, realizes that the companies are not as profitable as they could be. Bill suggests to Al, the manager at AF, that they each produce q^*, which is determined so that combined profit is maximized each day. Bill hopes to sustain this plan with the Grim Trigger Strategy that would threaten eternal reversion to the daily noncooperative sequential choice of outputs (which they have already been doing) if either player ever deviates from producing the joint-profit-maximizing outputs.

   Calculate the minimum value of δ required for Bill's collusive plan to be successful.

2. The University recently hired a Hot Shot empirical economist (HS) fresh out of graduate school. When arriving at the University, the Graybeard theorist (GB) suggests they collaborate on a paper. He would write the theoretical portion of the paper, she the empirical portion. The quality of the paper depends on the quality of each component: if GB exerts effort e_1 and HS exerts effort e_2 on the paper, then the quality of the paper is $q \equiv \min\{e_1, e_2\}$; thus, the paper quality equals the lesser of the efforts by the two authors. HS and GB each get benefit $v(q) = q - \frac{1}{2}q^2$ from a paper of quality q. The marginal cost of effort is c_1 for GB and c_2 for HS. Final net payoffs to players are $v(q) - c_i e_i$, $i = 1, 2$.

   Because GB has been around so long, it is common knowledge that his marginal cost of effort is c_1, where $c_1 \in (0, 1)$. However, HS's marginal cost is known only to her—but *ex ante* it is known to be distributed according to a uniform distribution on $[0, 1]$.

   GB and HS choose their efforts simultaneously, given the information described above. Find the (pure strategy) Bayesian equilibrium that gives the highest possible realized paper quality. For that equilibrium, calculate the *ex ante* probability that they actually achieve this highest quality. (*Hint:* first figure out what an equilibrium looks like, and then find the one for which the possible paper quality is greatest.)

Part III

1. Prove or disprove the following propositions:
(a) Two countries with identical endowment, identical taste and preferences, identical technology, identical population and identical demography will never engage in trade.
(b) An equilibrium outcome of an exchange economy cannot be Pareto efficient in the presence of a monopoly.

2. A monopolistic firm produces a product without any fixed factor, and its marginal cost is constant. It is selling in a market where every consumer is willing to buy at the initial asking price and the demand curve for its product is downward sloping linear. The monopolist chooses a point of the demand curve DD that would maximize its profit. An enthusiastic employee of the firm, who is a UCR graduate with a major in Economics, observes that at a profit maximizing price the firm is making huge profit and there are potential customers who are left out because they are not willing to pay the profit maximizing price. This enthusiastic employee suggests that after selling the product to its first round customers the firm can indeed increase its profit further by choosing another point of DD that sets the lower price for the customers who didn't buy in the first round.
(a) Demonstrate two prices the monopolist will choose.
(b) Since the price elasticity of demand decreases as one moves along a linear downward sloping demand curve, this led the employee to conclude that a monopolist charges higher price when price elasticity is higher, and lower price when price elasticity is lower. Explain in detail whether you agree or disagree.

3. Answer either (a) or (b).
(a) Show that a non-imposed, non-manipulable, social choice function is dictatorial if and only if it is required to be single-valued.
(b) "Local non-satiation is both necessary and sufficient for a competitive equilibrium allocation to be Pareto efficient" – Evaluate.

Department of Economics
University of California, Riverside

MICROECONOMICS QUALIFYING EXAM

September 17, 2010

INSTRUCTIONS:

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- Write solutions on only one side of each page of paper and do not put answers for more than one question on a page.
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Part I

1. Many businesses offer so-called "flexible spending plans" to employees, under the direction of the federal government. These work as follows. The income tax rate is t—so an individual gets to keep fraction $1-t$ of his or her income. However, flexible spending plans allow a person to shelter some income for use on allowable expenses, which here are called "medical expenses."

   If an individual puts amount s into the flexible spending plan, then these dollars are "sheltered," not taxed, and they can be used for medical expenses. The drawback of these plans is that any money in the plan not used for medical expenses is lost—it can not be recovered. (These plans operate on an annual basis—you can think of this problem as applying to a single year.) Thus, if an individual with income m puts s into the flexible spending plan and has total medical expenses L, then that individual will have after-tax income, net of medical expenses, equal to

$$\text{net after-tax income} = \begin{cases} (m-s)(1-t)-(L-s) & \text{if } L > s \\ (m-s)(1-t) & \text{if } L \leq s. \end{cases}$$

   The cumulative probability distribution for an employee's total medical expenses, L, has a probability density function that is strictly positive on the interval $[0, \bar{L}]$.

   In answering the following questions, you should assume that the tax rates, incomes, and possible medical expenses considered never force a person into bankruptcy; that is,

$$(1-t)m - \bar{L} > 0.$$

   Tiffany's income is m. Tiffany prefers more money to less and her preferences are representable by von Neumann-Morgenstern utility function. Her Bernoulli utility function u is a function of realized net after-tax income (i.e., her payoff is determined as $\mathrm{E}[u(\text{net after-tax income})]$). Tiffany is strictly risk averse. (You may assume her utility function is as differentiable as you like.)

   (a) Show that for all $t \in (0,1)$, Tiffany will put amount $s(t)$ into her flexible spending plan, where $s(t)$ lies strictly between 0 and $\bar{L}$; i.e., $0 < s(t) < \bar{L}$.

   (b) Fix $t \in (0,1)$. Consider two cumulative distribution functions for L, namely, F and G. Let s_f denote the amount of income Tiffany puts into the plan if the distribution of her medical expenses is given by F; let s_g denote the amount she puts into the plan if the distribution of her medical expenses is given by G. Suppose F strictly dominates G in the sense of first-order stochastic dominance—this implies, for example, that the expected expense under distribution F is greater than under G.

       Show that $s_f \geq s_g$.

   (c) Keke is a risk-neutral employee with random medical expenses that are distributed according to a 37-times differentiable cumulative distribution function. Show that Keke increases her contribution to the flexible spending plan as the tax rate t increases.

2. Suppose there are two consumers, A and B in the market served by the widget monopolist. Their inverse demands are $P_A(q_A) = 9 - q_A$ and $P_B(q_B) = 10 - q_B$ for consumers A and B, respectively. The total cost function for the monopolist is $C(q) = \frac{1}{2}q^2$, where $q = q_A + q_B$ denotes the total output sold to the two consumers. Assume resale among consumers is impossible.

   (a) Suppose the monopolist must sell its good at a single (i.e., uniform) per-unit price. Calculate its profit-maximizing price. Calculate the associated deadweight welfare loss.

   (b) Suppose the monopolist can determine the individual consumers' types. Calculate the monopolist's profit-maximizing prices under third-degree price discrimination.

   (c) Calculate the profit-maximizing (single) two-part tariff for the monopolist to offer.

Part I - Alternative Question

A standard functional representation of a (closed) multiple-output technology T is the "distance function" (technically, a gauge function), defined by $D(u, x) = \max\{\lambda \mid x/\lambda \in L(u)\}$, where $u \in \mathbf{R}^m_+$ is the output-quantity vector, $x \in \mathbf{R}^n_+$ is the input-quantity vector, and $L(u)$ is the input-requirement set for producing u.

(a) What restrictions on T guarantee that D provides a functional representation of the technology?

(b) Maintaining the assumptions in (a), state and explain the properties of D.

(c) Show that the technical rate of substitution between any two inputs, say i and k, at a point where D is differentiable is given by $D_i(u, x)/D_k(u, x)$ (where subscripts indicate partial differentiation). (Use the implicit function theorem if you can; otherwise, use cookbook calculus.)

(d) Assuming differentiability of D (in x) at $\overset{*}{x}$, derive and interpret the first-order conditions for $\overset{*}{x}$ to be a cost-minimizing input-quantity vector.

(e) Describe the symmetric duality between the cost function and the distance function. (Derivation of this duality is not necessary: a formulaic description will suffice.)

(f) Assuming differentiability of D at $\langle \overset{*}{u}, \overset{*}{x} \rangle$, derive and interpret the first-order conditions for $\langle \overset{*}{u}, \overset{*}{x} \rangle$ to be a profit-maximizing production vector.

Part II

1. Consider the following game of incomplete information between Row and Column. Row's set of possible actions is $S_R = \{a, b\}$, and Column's set of possible actions is $S_C = \{c, d\}$. Players' payoffs are determined by the players' actions and the state of Nature. The state of Nature is either A or B, and both of these states are equally likely.

   Payoffs, as a function of state, are given in the following matrices, with the first number being the payoff to Row, the second the payoff to Column.

   Payoffs when the state is A:

	c	d
a	3, 0	1, 1
b	0, 1	2, 0

   Payoffs when the state is B:

	c	d
a	0, 3	3, 1
b	3, 0	1, 3

   Before the game is to be played, Row observes the state of Nature. Column does not know the state, but believes the states to be equally likely.
   The foregoing is common knowledge between the players.

   (a) Suppose players choose actions simultaneously. Derive a Bayesian equilibrium in which Column (strictly) randomizes between c and d.

   (b) Suppose after learning the state, Row chooses an action, with the knowledge that this action will be observed by Column.
   Derive a Perfect Bayesian equilibrium to this game (you need only construct one equilibrium). Be complete in your description of the equilibrium.

Answer one *and only one* of the following questions.

2. Two bidders, A and B, are in an auction to purchase up to 2 units of the good. The two units of the good are identical.

   Player i is characterized by a pair of numbers (v_1^i, v_2^i), where $\max\{v_1^i, v_2^i\}$ is player i's gross value of obtaining one unit, and $v_1^i + v_2^i$ is her gross value of obtaining two units, $i = A, B$. Assume that v_1^i and v_2^i are independently and uniformly distributed over $[0, 1]$, $i = A, B$; furthermore, player A's values are independent of player B's. A player's net utility is her total gross value of goods consumed minus the amount she pays.

   The two-unit auction works as follows. Player i submits a bid (b_1^i, b_2^i), where b_1^i is the bid for one unit and b_2^i is the bid for a second unit, $i = A, B$. (You may assume $b_1^i \geq b_2^i$, $i = A, B$.)

   The first unit goes to the player submitting the highest bid; the second unit goes to the player submitting the second-highest bid (so a player could receive 0, 1, or 2 units). In the case of ties, the player to receive the good is determined by the flip of a fair coin. Both units are sold at a price equal to the *third*-highest bid. (For example, if $b_1^A > b_2^A > b_1^B > b_2^B$, then player A would get both units at price b_1^B.)

   Now consider the strategy $b^*(v_1^i, v_2^i) = (\max\{v_1^i, v_2^i\}, \min\{v_1^i, v_2^i\})$, according to which a player bids her true values for the first unit and the second unit.

   Is b^* a weakly dominant strategy for each player? Explain carefully.

3. Consider the following version of Rubinstein's alternating-offers bargaining model.

   There are *three* players who can split \$1 if they come to an agreeable division. At the beginning of the first period, a player is chosen at random (all are equally likely to be chosen), and that player proposes to the other players a division of the dollar. If both players agree to this division, then the dollar is divided as proposed; if not, then at the beginning of the next period a player is chosen at random to make a proposal. This process continues until there is agreement.

   Players have a common per-period discount factor of $\delta < 1$. A player gets payoff x in the period where she receives amount x; amount x is worth x if it is received today and has present value $\delta^k x$ if it is received k periods from now.

   Determine a subgame perfect equilibrium in which players' payoffs are strictly positive.

Part III

1. Prove or disprove the following claims.

(a) Assignment of property rights necessarily resolves the problem of inefficiency in the presence of externality.

(b) The welfare loss cannot be avoided in a few sellers industry.

2. Write down the conditions or assumptions for a proof of existence of a competitive equilibrium (CE) in an exchange economy. Outline the role of those assumptions in establishing the existence of a CE.

UNIVERSITY OF CALIFORNIA, RIVERSIDE

Microeconomics Qualifying Exam

JULY 7, 2010

INSTRUCTIONS

- *Put your identification number on the front of your packet of solutions and each subsequent page (using the first box in the upper right-hand corner).*
- *Write solutions on only one side of each page of paper and do not put answers for more than one question on a page.*
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Part I

1. Suppose that a perfectly competitive firm has profit function

$$\pi(p_1, p_2, p_3) = \frac{(p_1 - p_2)^2}{p_3}, \quad \text{for } p_1 > 0,\ p_2 > 0, \text{ and } p_3 > 0.$$

   (a) Does this firm's production function exhibit constant returns to scale? Explain.

   (b) Discuss whether each of the following production plans is efficient:

$$(2, -3, -1/2), \quad (1, -1, -1/4), \quad \text{and} \quad (4, -4, -3).$$

2. Nikita's preferences for goods x and y are described by the utility function $u : \mathbf{R}^2_+ \to \mathbf{R}$. Nikita's preferences are homothetic, and her indifference curve for utility level 1 (i.e., (x, y) such that $u(x, y) = 1$) is described as follows:

$$0 \le x \le 4, \quad 0 \le y \le 1, \quad \text{and} \quad 2 = \sqrt{x} + 2y.$$

   (a) Suppose the unit prices of goods x and y are given by p_x and p_y. For utility level $u = 1$ and prices $p_x = 1$ and $p_y = 4$, calculate Nikita's Hicksian demands for goods x and y.

   (b) Suppose Nikita's income is 12. Calculate Nikita's Marshallian demands for goods x and y when the unit prices of goods x and y are $p_x = 1$ and $p_y = 4$.

   (c) Derive a utility function that represents Nikita's preferences.

3. Consider a perfectly competitive firm that faces an uncertain price, p, for its product. Before learning the market price of its output, this firm makes an investment. By making a monetary investment of amount I, the firm obtains the total cost function $TC(q) = \frac{1}{2}c(I)q^2$, where q denotes the firm's output level. The firm is risk neutral, caring only about its expected profit. For a given distribution of price, the firm chooses its investment I to maximize its expected profit.

   Timing is as follows:

   - Knowing the distribution of p, the firm chooses an investment level, I.
   - The market price is realized and the firm observes this price.
   - The firm chooses its output to maximize profit, given this price realization and its original investment level.

   Higher initial investments lead to lower variable costs. In particular, assume that c is continuous and differentiable, with $c(I) > 0$ and $c'(I) < 0$ for any $I > 0$.

   For all distributions considered here, *assume that the firm's optimal investment is positive, finite, and unique.*

   Consider two probability distributions of p: F and G. These cumulative distribution functions may be related as follows:

   A1. F first-order stochastically dominates G.

   A2. F is a mean-preserving spread of G.

   (a) This question concerns the optimal investment level, I.

      i. Given A1, determine and explain whether the optimal I is higher with F or with G, or explain why no ranking is possible.

      ii. Given A2, determine and explain whether the optimal I is higher with F or with G, or explain why no ranking is possible.

   (b) Consider the firm's *ex ante* profit, allowing for the optimal level of investment.

      i. Given A1, determine and explain whether expected profit is higher with F or with G, or explain why no ranking is possible.

      ii. Given A2, determine and explain whether expected profit is higher with F or with G, or explain why no ranking is possible.

Part II

1. The town's mayor wishes to fund a public good for his community. There are n people in the town, and if $G > 0$ is spent on the public good, then person i receives (gross) benefit $\beta_i G > 0$ from having the public good. Assume $\beta_1 + \cdots + \beta_n > 1$ so it is socially worthwhile to provide the public good; and assume for each i that $\beta_i < 1$, so no individual, even if sufficiently wealthy, would fully fund the public good. The mayor wishes to fund the public good at level G_0.

   The mayor's daughter is home for the summer, studying for her economics qualifying exams at UCR. She knows her father hates taxes and so proposes funding the public good with a lottery to be played by the city's residents.

   Person i has income w_i and can choose any amount of money, x_i, to spend on the lottery. If expenditures on the lottery are $x_1, \ldots, x_n$, then player i's chance of winning the prize is just

$$p_i(x_1, \ldots, x_n) \equiv \frac{x_i}{x_1 + \cdots + x_n}.$$

   Residents are risk neutral, and player i's payoff, given expenditures $x_1, \ldots, x_n$, level of the lottery prize R, and level of the good G, is just

$$w_i - x_i + p_i R + \beta_i G.$$

   All money raised but not spent on the prize is to be spent on the public good (so $G = \sum x_i - R$).

   Assume that all of the information above about personal characteristics is common knowledge.

   As a former political science major, the mayor is sure that using a lottery rather than taxes to fund the good will improve his chances for reelection. But he is clueless when it comes to figuring out what level the lottery's prize should be. Ever resourceful, his daughter saves the day by telling her father exactly what the level of the prize should be so that the lottery yields funding exactly G_0 for the public good.

   Your task: determine the level of prize R that the mayor's daughter proposes.

   (*You should assume that parameters are such that every individual participates in the lottery but no one spends all of his or her money on the lottery—i.e., focus on an "interior" solution.*)

2. Consider the following independent private-values sealed-bid auction with two risk-neutral bidders. Player i's value for the object, v_i, is uniformly distributed over the interval $[0, 1]$, $i = 1, 2$.

   As usual, in this auction the bidder with the higher bid wins (in the case of ties, each is equally likely to be designated the winner); but, somewhat unusually, *the winner must pay the **SUM** of both players' bids.*

   Realized payoffs are v_i − (amount paid) for the winner and 0 for the loser.

   The foregoing is common knowledge.

   Calculate a Bayesian-Nash equilibrium to this game.

Part III

1. Prove or disprove the following propositions:

   (a) A monopolistically competitive firm will never choose a point in an inelastic portion of a demand curve.

   (b) Pareto efficiency is not attainable in the presence of a monopoly.

2. Show that there is no aggregation procedure which maps a profile of individual preference orderings to a social preference relation which is reflexive, complete and acyclic that satisfies the following conditions: (a) unrestricted domain, (b) weak Pareto (or unanimity) principle, and (c) minimum liberalism.

Department of Economics
University of California, Riverside

CORE CUMULATIVE EXAMINATION FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY
MICROECONOMIC THEORY
SEPTEMBER 18, 2009

INSTRUCTIONS

- *The exam will be monitored by the Graduate Assistant. There will be no communication with faculty members; if you believe a question contains an error or an ambiguity, say so on your written examination, make an assumption to correct the alleged error or to resolve the ambiguity, and answer the question as well as you can.*
- *Put your identification number on the front of your packet of solutions and each subsequent page (using the first box in the upper right-hand corner).*
- *Write solutions on only one side of each page of paper and do not put answers for more than one question on a page.*
- *Assemble your solutions in numerical order, starting with #1.*
- *After compiling your solutions in order, number your pages sequentially in the second box in the upper right-hand corner of each page.*

Part I

1. Say whether each of the following statements is True, False, or Uncertain. Explain your answer.
   (a) In a two-good, two-factor economy, an increase in the supply of one factor will necessarily increase the production of one commodity and decrease the production of the other.
   (b) Whenever the price of a commodity is less than its average cost, a competitive firm will cease to produce and sell the output.
2. State and prove (by utilizing the quasi-neutrality lemma) the Arrow impossibility theorem. Write down the significance of this result.

Part II

1. The variable profit function of a firm is given by $\pi(p, w, K) = (p^2K)/w$, where p is the output price, w is the price (wage rate) of the variable input (labor), and K is the amount of the fixed input (capital).
   (a) Derive the supply function and the labor demand function.
   (b) Find the expression for the shadow price of capital.
   (c) Over what range of the price of (rental rate on) capital, r, is the firm (i) overcapitalized (i.e., employing more capital than is optimal) and (ii) undercapitalized?

2. A preference relation, $\succeq$, completely preorders the consumer's consumption space, X.

   (a) What properties does $\succeq$ satisfy? (Provide rigorous definitions of these properties.)

   (b) What does it mean to say that a function $U : X \rightarrow \mathbf{R}$ represents $\succeq$?

   (c) What additional property of $\succeq$ suffices to guarantee that it can be represented by a (real-valued) utility function? (Provide a rigorous definition of this property.)

   (d) Is this property necessary as well as sufficient? Justify your answer.

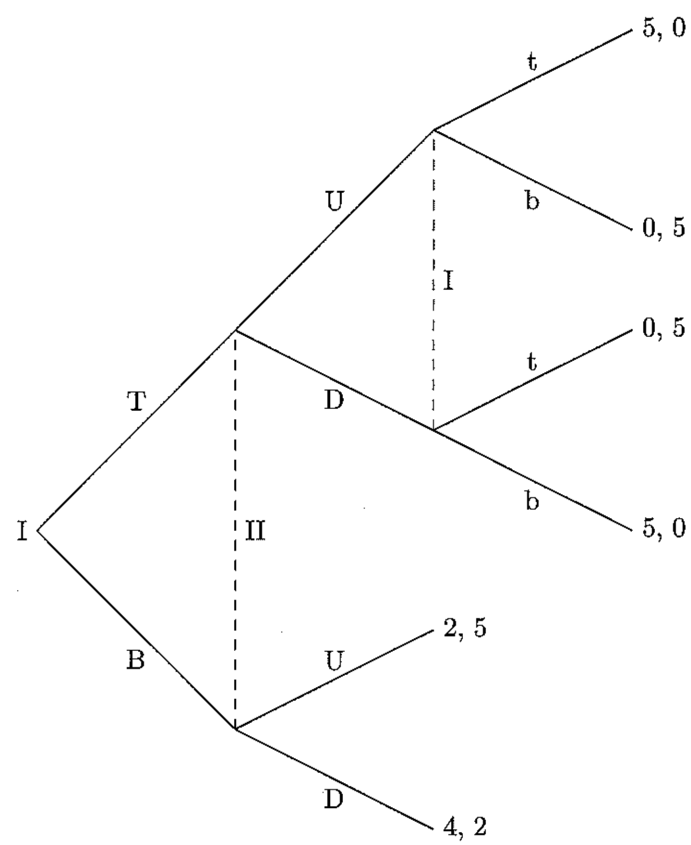
   (e) Provide a two-dimensional example of an ordering that is not representable by a real-valued utility function. Can the choices of a budget-constrained consumer with this utility function be rationalized (in the sense of Afriat) by a real-valued utility function?

3. Exploiting the appropriate duality theorems (and assuming local nonsatiation and a unique solution to the budget-constrained optimization problem), derive the Slutsky equation and characterize its components.

Part III

1. The figure below depicts an extensive-form game between players I and II. Dashed lines represent information sets. At terminal nodes the first number is the payoff to player I, the second the payoff to player II.

   Determine a perfect Bayesian equilibrium to this game.



Instructions: *Answer* ***one and only one*** *of the following two questions.*

2. Consider the following common-value sealed-bid auction with two risk-neutral bidders. The common value, v, for the object is either \$1 or \$2, and these values are equally likely. Before players submit bids, player 1 learns the value of the object. Both players then submit bids simultaneously. Player 2 knows nothing about v other than that the values are equally likely.

   This is an all-pay auction: the player with the higher bid wins the object; each player pays the amount of her bid. In the case of tying bids, each bidder receives the good with probability 1/2. Realized payoffs are

   $$v - (\text{amount paid})$$

   for the winner and

   $$-(\text{amount paid})$$

   for the loser.

   The foregoing is common knowledge.

   Calculate a Bayesian-Nash equilibrium to this game.

3. The cost, c, of a discrete public good is random: it is uniformly distributed over $[0, 2]$. There are two potential contributors to fund the good. Player i's value v_i for the good is uniformly distributed over $[0, 1]$, $i = 1, 2$. The random variables c, v_1, and v_2 are independently distributed.

   Players will submit contributions and if these contributions cover the realized cost of the good, then the good will be provided. In this case, each player's payoff is simply that player's value for the good minus his contribution. If contributions fall short of realized cost, then contributions are refunded and the good is not provided: players' realized payoffs are 0.

   As a professional fundraiser, you have been asked to design a scheme to raise money for the public good. You have decided on a binary scheme, asking players to contribute b if they are going to contribute anything—otherwise they should just contribute 0.

   You recognize that after you choose the value b, players will observe their own values and decide whether or not to contribute—playing a Bayesian equilibrium in this subsequent donation game.

   Determine the level of b that maximizes the *ex ante* probability that the good will actually be provided.

Part IV. Answer the following question.

A seller (S) has a good which he values at 0 that a buyer (B) is interested in. The buyer's value of the good is $v \in \{1, 2\}$, where $v = 2$ with probability $\pi < \frac{1}{2}$. B's value is private information: only he knows how much he cares about this good.

First, consider the one period version of this game. That is, S sets a price p_0. B can buy at this price, and if he rejects this price, the game ends and both players get 0.

(a) Find a BNE of this game. Show your work. Is the equilibrium you find socially efficient?

Next, consider the two period version of this game. At $t = 0$, S sets a price p_0, and B either accepts or rejects. If B rejects, S can set another price p_1 at $t = 1$, and B can accept or reject. The second period payoffs are discounted by a common discount factor $\delta \in (0, 1)$. If B still rejects, the game ends and both get 0.

(b) **In the context of this game:** briefly define the concept of PBE.

(c) Find a PBE of this game. Show your work. Is the equilibrium you find socially efficient?

(d) Without doing any calculation, how do you think your results may change if $\pi > \frac{1}{2}$? Explain your reasoning.

Department of Economics
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CORE CUMULATIVE EXAMINATION FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY
MICROECONOMIC THEORY
July 6, 2009

Directions: *The examination contains four equally weighted parts. Please allocate your time carefully and write concisely and legibly. This exam will be monitored by the Graduate Assistant. There will be no communication with faculty members; if you believe a question contains an error or an ambiguity, say so on your written examination, make an assumption to correct the alleged error or to resolve the ambiguity, and answer the question as well as you can.*

Part I. Answer all of the following questions.

1. Answer all parts of the following question.

The probability of a house burning down is p. The owner is risk averse and has wealth w (including the value of the house).

(i) Illustrate diagrammatically (a) an "actuarially fair" insurance contract that fully insures the homeowner and (b) an insurance contract that fully insures the owner and maximizes the insurer's expected revenue (from this homeowner). Also identify the risk premium associated with the latter contract.

(ii) How would you measure the degree of risk aversion of the homeowner?

(iii) Provide some intuition about the relationship between the risk premium and the degree of risk aversion?

2. Consider the following private-values sealed-bid auction with two risk-neutral bidders. Player i's value, v_i, for the object is either \$1 or \$2, and these values are equally likely, $i = 1, 2$. Values v_1 and v_2 are independently distributed. Players learn their own values for the object before submitting their sealed bids; they do not learn the other player's value.

Suppose this is a standard first-price auction; that is, the player with the higher bid wins the object and pays the amount of her bid, and the loser pays nothing. In the case of tying bids, each bidder receives the good with probability 1/2, and the winner pays the amount of her bid. Realized payoffs are

$$v_i - (\text{amount paid})$$

for the winner, and 0 for the loser. The foregoing is common knowledge. Calculate a Bayesian-Nash equilibrium to this game.

3. Prove or disprove the following propositions:

(i) For any country free trade is an optimum policy.

(ii) A change in relative price in the international market will necessarily change the relative factor rewards in the home country.

Part II. Answer the following question.

The cost function of a firm is given by $C(u, p)= u^{\alpha}p^{\beta}\ p^{\gamma}$, where u is the scalar output quantity, p_1 and p_2 are the input prices, and α, β, and γ are parameters. Justify your answers to the following questions.

(a) What restrictions on the parameters are necessary to guarantee that this cost function is derived from output-constrained cost minimization?

(b) What additional restrictions on the parameters imply homotheticity of the technology? What additional restrictions imply homogeneity of the production function?

(c) What additional restrictions on the parameters are necessary to guarantee a unique solution to the firm's profit maximization problem (when facing a fixed output price r)?

(d) Set up, but do not execute, the optimization problem that would recover the technology of the firm.

(e) Derive the constant-output demand function for input 1.

(f) Derive the supply function and the profit function.

(g) Explain precisely how you would derive the demand function for input 1from the profit function.

Part III. Answer the following question.

1. The significance of the concept of Pareto efficiency is its usefulness as a reference point to the policy maker – Evaluate.

Part IV. Answer all of the following questions.

1. Consider the following somewhat unusual price negotiations between a potential buyer and the seller of a car. The car is worth 0 to the seller but has value v to the buyer.

The players negotiate over a series of periods on the price of the car. At the beginning of each period a fair coin is flipped.

If the coin turns up Heads, then the *seller* proposes a transaction price, and the buyer either accepts the offer or rejects it; if the buyer rejects the offer, then the coin is again flipped.

If the coin turns up Tails, then the *buyer* proposes a transaction price, and the seller either accepts the offer or rejects it; if the seller rejects the offer, then the coin is again flipped.

The negotiations end when an offer is accepted. If a price p is accepted, then the net payoff to the seller is p and the net payoff to the buyer is $v - p$. Delay in the negotiations is costly. The players discount their payoffs by the common per-period discount factor δ, where $0 < \delta < 1$.

The foregoing description of the game is common knowledge.

Derive the subgame perfect equilibrium strategies for the buyer and seller in this game.

2. Consider the Sender–Receiver game shown below. The Sender's type is either t_1, t_2, t_3, or t_4. This type is chosen by Nature, and ex ante both the Sender and Receiver know these possibilities are equally likely.

Only the Sender observes his type as chosen by Nature, and then the Sender chooses either L or R. The Receiver observes the Sender's choice but not the Sender's type. The game is depicted below. At terminal nodes, the first payoff is the Sender's, the second is the Receiver's.

This description of the game is common knowledge.

Derive a perfect Bayesian equilibrium to this game.