# What are we after in 200C?

In part I of the micro sequence, we learnt about economic agents, their preferences and maximizing behavior.

In part II, we learnt tools of game theory to analyze strategic behavior by agents in interactive situations.

Part III is about economic transactions/exchanges between agents; we will discuss various institutions/mechanisms for trade and resource allocation and the outcomes they generate.

We will cover the competitive market mechanism for multilateral trade (General Equilibrium Theory), contractual mechanisms for bilateral trade (Contract Theory), and some general mechanisms for multilateral allocations (Mechanism Design).

# Model Components

The basic components in all our models will be the following:

- objects to be allocated
- agents, their preferences and endowments
- strategies available to agents
- rules of allocation
- information available to agents

In our discussions, we will highlight the role of various assumptions made about preferences, rules of transaction, and information available to the agents.

# Value of Information

What is the ‘value’ of ‘information’ to an economic agent? (Note the quotation marks!)

Why do we need to discuss this?

Because one of the most important insights of economic theory is that how agents behave in economic transactions, and what outcomes are obtained from these transactions depend crucially on what information they have, and how they view/interpret that information.

This is the subject matter of a vast field ‘Information Economics’, but we will only discuss a handful of concepts that will be relevant for our discussion of various exchange mechanisms.

# Playing against Nature

We will start by looking at the ‘value’ of ‘information’ in a single person decision problem under uncertainty.

The model used here was pioneered by David Blackwell in the ’50s. It combines the ideas of Bayesian probability with Expected Utility Theory.

It is now the standard framework in ‘Decision Theory’, i.e the theory of single person decision making under uncertainty.

We will assume that individuals are expected utility maximizers.

## What is information and what is its value?

What is ‘information’?

Imagine you are asked to make a decision. Assume that you do not have perfect knowledge of some important aspect regarding the situation that will impact the outcome of your decision.

Formally, we will consider this as an unknown 'state' of the world, which could be one of multiple possible states. Your uncertainty over the possible states is modeled as a probability distribution.

Now, imagine that there is a way for you to get more information about the unknown state.

Formally, such information will be represented by a signal/message, which is a random variable correlated with the state.

Your information structure, as described above, will be represented by a joint distribution over states and signal values.

Given a joint distribution, we can readily recognize two important components:

-prior distribution over states: this is what you believe about the probabilities of various underlying states <u>before</u> you see the signal (technically, this is the marginal distribution for the states)

-posterior distribution: your updated (using Bayes' Rule) beliefs after you observe the signal (this is the conditional distribution for the states, conditional on the signal)

Let's write down some notations (taken from BHR):

$\pi_s$ = the unconditional or marginal (prior) probability of state $s$
$q_m$ = the unconditional probability of receiving message $m$
$j_{sm}$ = the joint probability of state $s$ and message $m$
$q_{m \cdot s}$ = the conditional probability (or "likelihood") of message $m$, given state $s$
$\pi_{s \cdot m}$ = the conditional (posterior) probability of state $s$, given message $m$

Table 5.2. *Joint probability matrix* ($J = [j_{sm}]$)

| | | | | Messages ($m$) | $M$ | Probabilities for states |
|---|---|---|---|---|---|---|
| | $J$ | 1 | 2 | ... | | |
| | 1 | $j_{11}$ | $j_{12}$ | ... | $j_{1M}$ | $\pi_1$ |
| States | 2 | $j_{21}$ | $j_{22}$ | ... | $j_{2M}$ | $\pi_2$ |
| ($s$) | ... | ... | ... | ... | ... | |
| | $S$ | $j_{S1}$ | $j_{S2}$ | ... | $j_{SM}$ | $\pi_S$ |
| Probabilities for messages | | $q_1$ | $q_2$ | ... | $q_M$ | 1.0 |

Table 5.4. *Potential posterior matrix* ($\Pi \equiv [\pi_{s \cdot m}]$)

| | | Messages ($m$) | | | |
|---|---|---|---|---|---|
| | $\Pi$ | 1 | 2 | ... | $M$ |
| | 1 | $\pi_{1 \cdot 1}$ | $\pi_{1 \cdot 2}$ | ... | $\pi_{1 \cdot M}$ |
| | 2 | $\pi_{1 \cdot 2}$ | $\pi_{2 \cdot 2}$ | ... | $\pi_{2 \cdot M}$ |
| States ($s$) | ... | ... | ... | ... | ... |
| | $S$ | $\pi_{S \cdot 1}$ | $\pi_{S \cdot 2}$ | ... | $\pi_{S \cdot M}$ |
| | | 1.0 | 1.0 | ... | 1.0 |

The potential posterior matrix $\Pi$ (Table 5.4) shows the conditional probability of each state given any message $m$, which is denoted $\pi_{s \cdot m}$ and defined in:

$$\pi_{s \cdot m} \equiv \frac{j_{sm}}{q_m} \tag{5.2.3}$$

Table 5.3. *Likelihood matrix* ($L \equiv [l_{sm}] \equiv [q_{m \cdot s}]$)

| | Messages ($m$) | | | | | |
|---|---|---|---|---|---|---|
| | $L$ | 1 | 2 | ... | $M$ | |
| | 1 | $q_{1 \cdot 1}$ | $q_{1 \cdot 1}$ | ... | $q_{M \cdot 1}$ | 1.0 |
| | 2 | $q_{1 \cdot 2}$ | $q_{2 \cdot 2}$ | ... | $q_{M \cdot 2}$ | 1.0 |
| States($s$) | ... | ... | ... | ... | ... | ... |
| | $S$ | $q_{1 \cdot S}$ | $q_{2 \cdot S}$ | ... | $q_{M \cdot S}$ | 1.0 |

The likelihood matrix $L = [l_{sm}]$ (Table 5.3) shows the *conditional* probability of any message given any state, which will be denoted $q_{m \cdot s}$. Thus:

$$l_{sm} \equiv q_{m \cdot s} \equiv \frac{j_{sm}}{\pi_s} \qquad (5.2.2)$$

**Example 5.1:** There may be oil under your land. The alternative terminal actions are to undertake a major investment to develop the field or not to do so, and this decision will, of course, depend upon your estimate of the chance of oil really being there. The underlying "states of the world" are three possible geological configurations: state 1 is very favorable, with 90% chance that oil is there; state 2 is much less favorable, with 30% chance; state 3 is hopeless, with 0% chance.

In order to improve your information before taking terminal action, you have decided to drill a test well. The two possible sample outcomes or messages are that the test well is either "wet" or "dry." On the assumption that the test well is a random sample, then, if the true state is really state 1, there is a 90% chance of the message "wet"; if it is state 2, there is a 30% chance; and if it is state 3, no chance. So the given data specify the likelihood matrix $L \equiv [q_{m \cdot s}]$ shown below.

Suppose that, in addition, you initially attach probabilities $(\pi_1, \pi_2, \pi_3) = (0.1, 0.5, 0.4)$ to the three states. Multiplying each likelihood $q_{m.s}$ in the $L$ matrix by the prior probability $\pi_s$ yields the joint probability matrix $J \equiv [j_{sm}]$. The column sums are then the message probabilities $q_1 = 0.24$ and $q_2 = 0.76$. Using these you can easily compute your potential posterior matrix $\Pi = [\pi_{s \cdot m}]$.

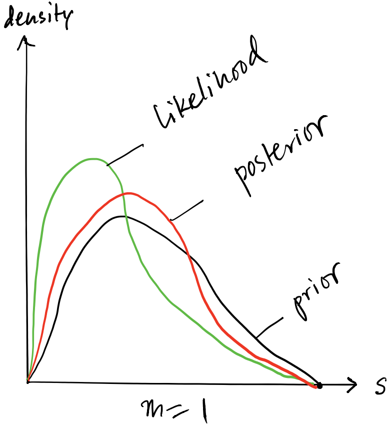
| | | $L \equiv [q_{m\cdot s}]$ Messages ($m$) | | | | $J \equiv [j_{sm}]$ Messages ($m$) | | | | $\Pi \equiv [\pi_{s\cdot m}]$ Messages ($m$) | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Wet | Dry | | | Wet | Dry | | | Wet | Dry |
| | $L$ | 1 | 2 | | $J$ | 1 | 2 | $\pi_s$ | $\Pi$ | 1 | 2 |
| | 1 | 0.9 | 0.1 | 1.0 | 1 | 0.09 | 0.01 | 0.1 | 1 | 0.375 | 0.013 |
| States | 2 | 0.3 | 0.7 | 1.0 | 2 | 0.15 | 0.35 | 0.5 | 2 | 0.625 | 0.461 |
| ($s$) | 3 | 0 | 1.0 | 1.0 | 3 | 0 | 0.40 | 0.4 | 3 | 0 | 0.526 |
| | | | | | $q_m$ | 0.24 | 0.76 | 1.0 | | 1.0 | 1.0 |

# Thinking about Signal Precision

Let's look closely at the relationship between the likelihood and the posterior:

$$\begin{aligned}\pi_{s\cdot m} &= \frac{j_{sm}}{q_m} \\ &= \pi_s \frac{q_{m\cdot s}}{q_m}\end{aligned}$$

posterior= (prior over state)x(likelihood of message given state)x (normalizing factor)



density
likelihood
posterior
prior
s
$n = 1$

posterior is an ‘average’ of likelihood and prior

the more ‘imprecise’ the prior, other things being equal, the greater is the revision

the more ’surprising’ the evidence, other things being equal, the bigger the impact upon the posterior

So now that we have a definition of ‘information’, what about its ‘value’?

Criteria: Are you able to make a better decision after you have observed the signal, compared to the decision you would have made without observing the signal?

If yes, then the information is ‘valuable’ to you, and you will be willing to pay for it.

How much will you be willing to pay for it? That’s the final step in quantifying ‘value of information’.

To answer ‘how much’, we will compare two utility levels:

Your **maximized** utility before observing the signal (here expectation is taken w.r.t the prior distribution)

Your **maximized** utility after observing the signal (expectation taken w.r.t the posterior)

Note: the actions that maximize one, may not maximize the other, this is whole point about making better choice with better information.

The value of the message will be calculated as the expected difference between the two utility levels.

## Revision of Optimal Action and Value of Information

If immediate terminal action is to be taken, the individual will choose whichever act has highest expected utility,

$$\max_x \sum_s \pi_s v(c_{xs})$$

Here $c_{xs}$ denotes consumption/income when action $x$ is chosen and true state is $s$

Denote as $x_0$ the optimal immediate terminal action, which is calculated in terms of the prior probabilities $\pi_s$.

After a particular message $m$ has been received, the decision maker now uses the posterior probabilities $\pi_{s \cdot m}$.

This recalculation could well lead to a choice of a different optimal terminal action $x_m$

$$x_m = \underset{x}{\operatorname{argmax}} \sum_s \pi_{s \cdot m} v(c_{xs})$$

The value of message $m$ is then, in terms of what the agent would know after observing the signal:

$$\omega_m = u(x_m; \pi_{\cdot m}) - u(x_0; \pi_{\cdot m})$$

Here $u(x_m; \pi_{\cdot m})$ is the expected utility of choosing optimal action $x_m$ based on posterior upon realization of $m$, and $u(x_0; \pi_{\cdot m})$ is the expected utility of choosing $x_0$

Note that we are using posteriors here. Why should this value always be non-negative?

However, one cannot purchase a given value of the message but only a message service.

So the value is determined by the expectation of the utility gain from all possible messages weighted by their respective message probabilities $q_m$

Let $c^*_{sm}$ denote the income in state $s$ associated with the best action $x_m$, after receiving message $m$

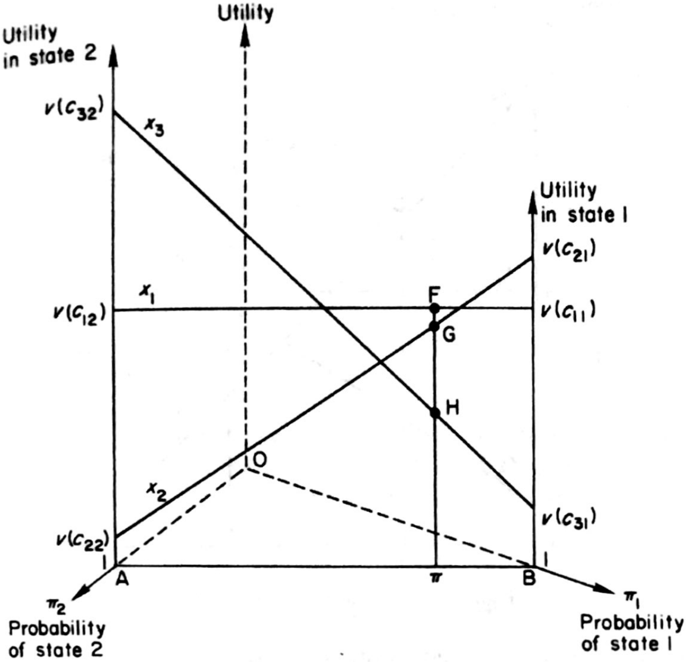
Let $c^*_{s0}$ be the corresponding income for the best uninformed action $x_0$ (that is, the best action in terms of the prior beliefs):

The value of the message service $\mu$ is

$$\Omega(\mu) = \sum_m q_m \omega_m$$

$$\begin{aligned}\Omega(\mu) &= \sum_m q_m \sum_s \pi_{s \cdot m} v\left(c^*_{sm}\right) - \sum_m \sum_s \pi_{s \cdot m} q_m v\left(c^*_{s0}\right) \\ &= \sum_m \sum_s \pi_{s \cdot m} q_m v\left(c^*_{sm}\right) - \sum_s \pi_s v\left(c^*_{s0}\right)\end{aligned}$$

So the value of information is just the difference in maximized expected utility with or without the service.


Utility
Utility in state 2
$v(c_{32})$
$x_3$
Utility in state 1
$v(c_{21})$
$x_1$
F
$v(c_{12})$
$v(c_{11})$
G
H
O
$x_2$
$v(c_{31})$
$v(c_{22})$
1
1
A
$\pi$
B
$\pi_2$
Probability of state 2
$\pi_1$
Probability of state 1

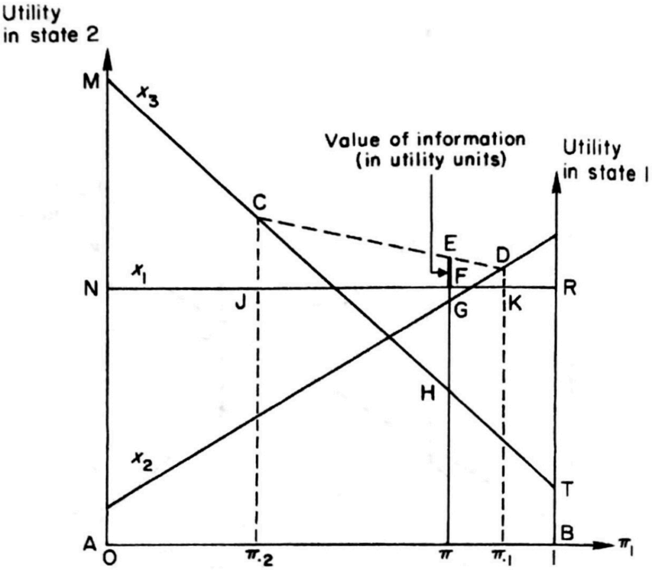



Figure 5.3. The value of information.

**Example 5.2:** In an oil-drilling situation, suppose there are just two possible states: "wet" (with prior probability $\pi_1 = 0.24$) and "dry" (with prior probability $\pi_2 = 0.76$).[5] If you take action $x = 1$ (drill the well) and it is wet, you gain \$1,000,000. If it is dry, you lose \$400,000. Action $x = 2$

(not drilling) involves a \$50,000 cost in relocating your rig. Suppose your utility function is simply linear in income (you are risk neutral), so we can write $v(c) = c$. A message service, taking the form of a geological analysis in advance of drilling, is characterized by the following likelihood matrix $L$. How much should you be willing to pay for it?

| | | Message | | |
|---|---|---|---|---|
| | $L = [q_{m,s}]$ | Wet | Dry | |
| State | Wet | 0.6 | 0.4 | 1.0 |
| | Dry | 0.2 | 0.8 | 1.0 |

**Answer:** In terms of your prior probabilities, action $x = 1$ involves an expected gain of 0.24 (\$1,000,000)−0.76(\$400,000) = −\$64,000, whereas action $x = 2$ leads to a loss of only \$50,000. So the optimal prior action $x_0$ is $x = 2$. As for the value of the message service, straightforward computations lead to the Potential Posterior Matrix shown below.

| | | Message | |
|---|---|---|---|
| | $\Pi = [\pi_{s \cdot m}]$ | Wet | Dry |
| State | Wet | 0.486 | 0.136 |
| | Dry | 0.514 | 0.864 |
| | | 1.0 | 1.0 |

Using the posterior probabilities, if the message is "dry", the best action remains $x = 2$ (not drilling). But if the message is "wet," the expected gain from drilling (action $x = 1$) becomes 0.486(\$1,000,000) − 0.514(\$400,000) = \$280,400. So the expected value of the information is 0.296 (\$280,400) + 0.704 (−\$50,000) − (−\$50,000) = \$97,798. This is the value of the message service, where 0.296 and 0.704 are the message probabilities $q_1$ and $q_2$. □