University of California, Riverside
Department of Economics

Macroeconomic Theory
Cumulative Examination

Fall 2007

This examination will be proctored by the Graduate Student Affairs Assistant, who has been instructed not to answer questions. There also will be no communication with faculty members; if you believe a question contains an error or an ambiguity, say so on your written examination, make an assumption to correct the alleged error or to resolve the ambiguity, and answer the question as well as you can.

Instructions

1. There are 7 pages (including the cover page) of the exam.
2. You have four hours to complete the exam. There are 240 points in total.
3. Read the instructions carefully.
4. Clearly label all the diagrams and underline the keywords in your answers.

Part I. Answer All Questions.

1. [10 points] Two Nobel Laureates, Milton Friedman and Edmund Phelps have argued that permanently low unemployment is unsustainable in the long run since eventually workers and firms will build expectations of price inflation in their wage-setting behavior, and hence the economy has a natural rate of unemployment dubbed as the Non-Accelerating Inflation Rate of Unemployment (NAIRU). Explain and discuss the key features of this particular notion of the natural rate of unemployment.

2. [15 points] Consider an economy in which output y is produced by a production function $y = \left[\beta k^{-\rho} + (1-\beta)n^{-\rho}\right]^{-\frac{1}{\rho}}$, $0 < \beta < 1$, $-1 < \rho < \infty$, where k is capital and n is employment.

(a) [8 points] <u>Derive</u> the expression for the marginal product of per-capita capital k/n.

(b) [7 points] Based on your answers to part (a), <u>find</u> the range of ρ under which <u>per-capita</u> output y/n exhibits sustained growth even though there is no technological progress in the economy. <u>Explain the economic intuition</u>.

3. [15 points] Consider the following two-good two-consumer general equilibrium model. Consumers have preferences represented by a utility function

$$u_h = A_h \left(x_1^h\right)^{\alpha} \left(x_2^h\right)^{1-\alpha},\ A_h > 0,\ 0 < \alpha < 1,$$

where h = 1 and 2 to index the two consumers. Assume that agent h has an endowment $\omega^h = \left(\omega_1^h, \omega_2^h\right)$, where ω_1^h and ω_2^h are both positive. The price of good 1 is normalized to 1 and the (relative) price of good 2 is denoted as p_2. <u>Derive</u> the general-equilibrium price (ratio) and allocations for this economy, and express them as functions of α and agents' endowments.

4. [40 points] Consider an economy with a unit measure of identical infinitely-lived households. The representative household maximizes the expected utility over its lifetime

$$(1)\qquad E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(c_t - A\frac{h_t^{1+\gamma}}{1+\gamma}\right)\right],\ 0 < \beta < 1,\ \gamma \geq 0 \text{ and } A > 0,$$

where c is consumption, h is hours worked, β is the discount factor and A is a preference parameter. On the production side of the economy, output is produced by a unit measure of competitive firms using the following technology:

(2) $$y_t = z_t f(k_t, h_t),\ z_0 \text{ is given,}$$

where y is output, z is a technology shock, $f(\cdot)$ is a concave production function that displays constant returns-to-scale, and k is physical capital. The budget constraint faced by the representative household is

(3) $$c_t + k_{t+1} - (1-\delta)k_t = w_t h_t + r_t k_t,\ 0 < \delta < 1,\ k_0 \text{ is given,}$$

where δ represents the capital depreciation rate, w is the real wage rate and r is the capital rental rate.

(a) [10 points] Based the objective function (1), formulate the Lagrangian for the household's optimization problem, and derive the first-order condition that governs the representative household's choice for its supply of labor in period t. Explain the economic intuition.

(b) [6 points] Based your answers to part (a), explain why in this case the household's labor supply curve cannot be downward-sloping?

(c) [10 points] Using the Lagrangian as in part (a), derive the first-order condition that governs the representative household's choice for consumption across periods t and t+1. Explain the economic intuition.

Now consider the above model with the following expected life-time utility for the representative household:

(4) $$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t - A \frac{h_t^{1+\gamma}}{1+\gamma} \right],\ 0 < \beta < 1,\ \gamma \geq 0 \text{ and } A > 0.$$

(d) [6 points] Based the objective function (4), formulate the Lagrangian for the household's optimization problem, and derive the first-order condition that governs the representative household's choice for its supply of labor in period t.

(e) [8 points] Based your answers to parts (a)-(d), other things being equal, which objective function (equation 1 or equation 4) will generate higher output volatility driven by technology shocks. Explain your answers.

Part II. Answer All Questions.

1. [10 points] Explain the Non-Ponzi Game condition. Give the intuition and the formal definition in the context of growth models.

2. [20 points] Assume a two-period market problem in which a representative agent maximizes the utility function subject to a budget constraint. The agent can transfer resources to the future by investing in capital k_t or buying government issued bonds b_t. Assume that there is a government that levies a lump-sum tax. The government uses the proceeds from tax and from government bonds to finance its expenditure g each period. Assume that: i) the agent lives for two periods and discount the future with the subjective discount factor β where $0<\beta<1$; ii) he is born without debt and dies the same way; iii) he is born with an initial amount of capital, $\bar{k}$;iv) the production function is his only source of income; v) capital totally depreciates from one period to another. In addition, suppose that preferences are given by $U(c_t)$ and his technology is given by $y_t = f(k_{t-1})$.

(a) [5 points] Write down the agent's period-by-period maximization problem and budget constraint.

(b) [5 points] Solve for the agent's two-period intertemporal budget constraint.

(c) [5 points] Write down the government's period-by-period maximization problem and budget constraint.

(d) [5 points] Solve for the government's two-period intertemporal budget constraint.

3. [50 points] Assume that a representative agent has the following preferences:

(1) $$\sum_{t=0}^{\infty} \beta^t U(c_t),\ 0 < \beta < 1, \text{ and } U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma},$$

where c is consumption and $\gamma > 0$ is the coefficient of relative risk aversion. Technology is described by the following homogeneous neoclassical production function with full depreciation ($\delta = 1$):

(2) $$f(k_t) = \varepsilon_t\, k_t^{\theta},\ 0 < \theta < 1,\ k_0 \text{ and } \varepsilon_0 \text{ are given,}$$

where k is capital and $\varepsilon_t \sim iid(\mu,\sigma^2)$ is a random production shock.

(a) [10 points] Note that so far this is essentially a planning problem with uncertain future income stream. Set up the planner's maximization problem as a dynamic programming problem. That is, write down the Bellman's equation, the resource constraint and specify the control and state variables.

(b) [10 points] Derive the first-order conditions, the envelope condition and the Euler equation. Assume that the value of ε_t is a constant μ at steady state. Find the steady state capital (k^*) for the economy.

Assume that there is a government bond market where the agent can lend at the interest rate r. Assume that b_t are government bonds that pay no coupon and, therefore, they have to be sold at a discount price q_t, where $q_t = 1/(1+r)$. In addition, the government levies a marginal tax τ on capital income. The revenue will be dumped into the ocean and does not influence the consumer's utility. This can now be seen as a market problem.

(c) [10 points] Set up the private agent's maximization problem as a dynamic programming problem. Write down the Bellman's equation, the agent's budget constraint and specify the control and state variables.

(d) [10 points] Derive the first-order conditions and the Euler equation (notice that now the Euler equation can be written equivalently in terms of bond returns or capital returns). Assume that the value of ε_t is a constant μ at steady state. Find the steady state capital (k^{**}) for the economy.

(e) [10 points] Compare the steady-state capital before and after tax. Does this income tax affect the amount of capital stock in equilibrium? If yes, how?

Part III. Answer All Questions.

1. [10 points] Let $F(K, L)$ be a neoclassical production function where K denote capital input and L denote labor input. The production function is assumed to satisfy the following conditions:

   (A1) It is twice continuously differentiable in both K and L.
   (A2) It is strictly increasing and strictly concave in both K and L.
   (A3) $F(K, 0) = 0$ for all $K \geq 0$ and $F(0, L) = 0$ for all $L \geq 0$.
   (A4) $F(\lambda K, \lambda L) = \lambda F(K, L)$ for all $\lambda > 0$.
   (A5) The Inada conditions:

   $$\lim_{K\to 0} F_K(K,L) = \infty \quad \text{and} \quad \lim_{K\to\infty} F_K(K,L) = 0, \quad \text{for all } L \geq 0,$$

   $$\lim_{L\to 0} F_L(K,L) = \infty, \quad \text{and} \quad \lim_{L\to\infty} F_L(K,L) = 0, \quad \text{for all } K \geq 0.$$

   Suppose the total labor supply is fixed. Let $\rho > 0$ be the rate of time preferences, $\sigma > 0$ be the inverse of the elasticity of intertemporal substitution, and $\delta > 0$ be the depreciation rate of capital. Using the Euler equation, show that when K grows without bound over time perpetual growth in per-capita consumption can be obtained by relaxing **ONE** of the five conditions stated above, i.e., (A1)-(A5).

2. [15 points] Consider a two-period overlapping-generation model in which the size of generation grows at a constant rate $n > 0$. Let K_t and L_t denote capital and labor at time t, respectively. The production function is given by

   $$F(K_t, L_t) = L_t f(k_t),$$

   where $k_t \equiv K_t / L_t$ and $f(k_t) \equiv F(k_t, 1)$ is the reduced form of the production function. The depreciation rate of capital is $0 < \delta < 1$. Let $c_{y,t}$ denote consumption when young at time t and $c_{o,t+1}$ denote consumption when old at time $t + 1$. The utility function for a typical consumer in generation t is given by $U(c_{y,t}, c_{o,t+1})$.

   Using these notations, define **dynamic efficiency** and **Pareto optimality** in an overlapping-generation model. Explain the relationship between the two.

3. [Total 55 points] Consider an economy inhabited by a continuum of identical, infinitely-lived agents. The size of population is constant and is normalized to one. For each consumer, the felicity function is given by

$$U(c,h) = \frac{(c/h^{\gamma})^{1-\sigma} - 1}{1-\sigma},$$

with $0 < \gamma < 1$ and $\sigma > 1$. The variables c and h denote current consumption and the stock of habit in consumption, respectively. This stock evolves according to

$$\dot{h}(t) = \eta[\bar{c}(t) - h(t)], \tag{1}$$

for all $t \geq 0$ with $h(0) > 0$ and $\eta > 0$. The variable $\bar{c}(t)$ denotes the *economy-wide level* of per-capita consumption. Individual consumers take the value of $\bar{c}(t)$ as exogenously given. The total lifetime utility of a typical consumer is given by

$$\int_0^{\infty} e^{-\rho t} U[c(t), h(t)] dt$$

where $\rho > 0$ is the rate of time preferences.

The only asset available in this economy is physical capital. The agents are not allowed to borrow. Using their capital holdings, the agents can produce their own output using a linear production function: $y(t) = Ak(t)$, where $y(t)$ denote output, $k(t)$ denote capital input and $A > 0$ is a technological factor. Let $\delta > 0$ be the depreciation rate of capital. Assume $A - \delta > \rho$.

(a) [5 points] Explain why the no-Ponzi-game condition in the consumer's problem must be satisfied in this setting. [Hint: You don't need the first-order conditions in order to answer this.]

(b) [15 points] Write down the problem faced by a typical consumer. Derive the first-order condition(s) and the transversality condition(s) for an interior solution.

(c) [20 points] Define $z(t) \equiv c(t)/h(t)$. and $s(t) \equiv k(t)/h(t)$. Derive a pair of differential equations in $z(t)$ and $s(t)$ that characterizes the equilibrium of this economy.

(d) [10 points] Consider a balanced growth path along which $c(t)$, $k(t)$ and $h(t)$ are all growing at the same constant rate. Derive the common growth rate of these variables.

(e) [5 points] Show that the transversality condition(s) that you have stated in part (b) is/are satisfied along a balanced growth path as defined in part (d).

(b) [10 points] Let $z_t^t \equiv x_t^t - 1$ be the excess demand of a young agent. Find an explicit expression for z_t^t as a function of R_t.

(c) [10 points] Write down a difference equation in R_t that describes equilibrium sequences.

(d) [10 points] What are the equilibrium real interest factors at the autarky and at the golden rule steady state, respectively?

(e) [10 points] For what values of α, β and ρ does this economy exhibit a unique perfect foresight equilibrium? Explain your answer.

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Part I. Answer All Questions.

1. [10 points] Is the velocity of circulation of the money supply bigger or smaller than that of the monetary base? Explain your answers. Note: start your answers by precisely defining what are meant by (i) the monetary base, (ii) the money supply and (iii) the velocity of circulation, respectively.

2. [10 points] The first-order condition for the household's labor supply in a standard real business cycle (RBC) model with separable preferences over consumption and leisure implies that the model cannot reproduce the weak correlation between detrended output and real wage observed in the actual data. True, false or uncertain? Explain your answers.

3. [60 points] Consider a simple economy inhabited by N identical households. Each household has one period of planning horizon, is endowed with one unit of time that can be allocated between work at wage w_t or leisure, and is endowed at time t a stock of physical capital k_t that earns a return r_t. Assume the household's preferences over consumption c_t, savings s_t, and hours worked h_t are

(1) $$u(c_t, s_t, h_t) = \log c_t + \beta \log s_t + \gamma \log(1 - h_t),$$

where β and γ are positive constants. The time-t budget constraint faced by the household is given by

(2) $$c_t + s_t = (1-\tau)w_t h_t + r_t k_t + T_t = y_t,$$

where $\tau \in (0,1)$ is a constant proportional wage tax rate, T_t is a lump sum transfer from the government and y_t is total income. Assume that the government balances its budget each period whereby

(3) $$\tau w_t h_t = T_t,$$

but the household take the transfer T_t as given when making its decisions.

(a) [10 points] Formulate the Lagrangian for the household's optimization problem, and derive the first-order conditions that govern the representative household's choices for consumption, savings and labor supply in period t. In addition, show that the optimal household consumption and savings are proportional to total income y_t.

(b) [8 points] Based on your answers to part (a), derive the expression for the optimal household labor supply as a function of β, γ τ, y_t and w_t. In addition, for a given wage and income, what is the impact of an increase in τ on the household's labor supply? Explain your answers.

Now suppose that we append the above household to a closed-economy Solow growth model (with no technical change and no population growth). There are N identical households and the aggregate production function is given by

$$\text{(3)} \qquad Y_t = K_t^{\alpha} H_t^{1-\alpha}, \quad 0 < \alpha < 1,$$

where Y_t is aggregate output, H_t denotes aggregate hours worked, and the aggregate capital stock K_t evolves according to

$$\text{(4)} \qquad K_{t+1} = (1-\delta)K_t + I_t, \quad 0 < \delta < 1,$$

where I_t represents aggregate investment and δ is the capital depreciation rate. Finally, the economy's aggregate resource constraint is

$$\text{(5)} \qquad C_t + I_t = Y_t = K_t^{\alpha} H_t^{1-\alpha},$$

where C_t is aggregate consumption.

(c) [6 points] Under the assumption that factor markets are perfectly competitive, derive expressions for the equilibrium (i) wage rate w_t and (ii) capital rental rate r_t as functions of the capital stock per hour worked K_t/H_t.

(d) [10 points] Based on your answers to part (c), show that in equilibrium, the economy's resource constraint (5) is equivalent to combining the individual household's budget constraint (2) and the government budget constraint (3) in aggregate terms.

(e) [14 points] Based on your answers to parts (b) and (c), derive the expression for the equilibrium household labor supply, which is a constant. Explain the economic intuition of why it is independent of the wage rate?

(f) [12 points] Based on your answers to parts (a), (c), (d), (e) and equation (4), derive the expressions for the steady-state levels of (i) capital stock per household and (ii) output per household as functions of model parameters. In addition, what is the impact of an increase in τ on the steady-state output per household? Explain your answers.

Part II. Answer All Questions.

1. [15 points] Assume a two-period intertemporal consumption model, in which a representative agent can allocate resources to the future only through the purchase of capital and privately issued bonds. Assume that the agent is endowed with $\bar{k}$ amount of capital when he/she is born. Capital totally depreciates from one period to the other. There is no firm and no government.

2. [15 points] What are substitution and income effects? What would be the most probable effect to the agent's consumption today versus tomorrow and labor hours today versus tomorrow if he suddenly wins the lottery? And if a firedestroys part of his stock of capital? Explain.

3. [50 points] Consider an endowment economy consisting of identical agents with preferences given by $E\sum_{t=0}^{\infty}\beta^t \log c_t$. Each period, agents are endowed with y_t units of consumption good, where y_t is an i.i.d. random variable such that $E_t\dfrac{1}{y_{t+1}}=\mu$. In this economy, there is a market for the consumption good and for an annuity that pays one unit of consumption good in each period. (Hint: Think of an annuity as a Lucas tree that provides a certain, as opposed to random, dividend stream.)

(a) [6 points] Define a *recursive competitive equilibrium* for this economy.

(b) [5 points] Write down Bellman's equation and define control and state variables.

(c) [10 points] Derive the Euler equation. Derive a functional equation the solution to which is the equilibrium pricing function for an annuity. Solve for this equilibrium pricing function.

Suppose now that y_t is produced by a tree and that each agent is endowed with one tree.

(d) [6 points] Define a *recursive competitive equilibrium* for this economy.

(e) [5 points] Write down Bellman's equation and define control and state variables.

(f) [10 points] Derive the Euler equation. Derive a functional equation the solution to which is the equilibrium pricing function for an annuity. Solve for this equilibrium pricing function.

(g) [8 points]Compare the price of an annuity with the price of a tree. Is there any risk premium built into the price of the tree?

NOTE: Answer Part III <u>OR</u> Part IV.

Part III. Answer All Questions.

Question 1 [55 points]

Consider an economy that is inhabited by N identical infinitively-lived consumers. Each consumer has access to the production technology

$$y(t) = Ak(t),$$

where $A>0$ and $k(t)$ is the stock of capital hold by an individual consumer at time t. A government exists in this economy which (i) taxes consumption, (ii) imposes a lump sum tax $\tau(t)$ on each consumer and (iii) subsidizes investment. The tax rate on consumption is constant over time and is denoted by $\eta>0$. The subsidy rate on investment is also constant over time and is given by $\varphi \in (0,1)$. The lump-sum tax $\tau(t)$ is a function of time.

The problem faced by a typical consumer is given by

$$\max_{\{c(t),i(t)\}} \int_0^\infty e^{-\rho t} \frac{[c(t)]^{1-\sigma} - 1}{1-\sigma} dt, \qquad \rho > 0, \ \ \sigma > 0,$$

subject to

$$(1+\eta)c(t) + (1-\varphi)i(t) = Ak(t) - \tau(t),$$

$$i(t) = \dot{k}(t) + \delta k(t),$$

$$c(t) \geq 0, \quad k(t) \geq 0, \quad and \quad k(0) = k_0 > 0 \text{ given.}$$

The variable $i(t)$ denote gross investment at time t. The parameter $\delta>0$ is the depreciation rate of capital. **Assume $A > \delta$. Assume that the government's budget is balanced in every time period.**

(a) [10 points] Derive the first-order conditions and the transversality condition that characterize an interior solution of the consumer's problem.

(b) [20 points] Let $\gamma_c(t)$ be the growth rate of $c(t)$, i.e., $\gamma_c(t) \equiv \dot{c}(t)/c(t)$.

(i) Derive an expression for $\gamma_c(t)$. [4 points]
(ii) How is $\gamma_c(t)$ affected by η and φ ? Explain the intuition behind. [8 points]
(iii) How is $\gamma_c(t)$ affected by ρ ? Explain the intuition behind. [8 points]

(c) [10 points] Along a balanced growth path, $k(t)$ and $c(t)$ are growing at the same rate, i.e.,

$$\frac{\dot{c}(t)}{c(t)} = \frac{\dot{k}(t)}{k(t)}.$$

Suppose $\sigma>1$. Determine the values of φ under which the transversality condition in part (a) is satisfied along a balanced growth path.

(d) [15 points] Define $z(t) \equiv c(t)/k(t)$. Assume $\sigma>1$. Derive the value of $z(t)$ along a balanced growth path. Determine the effect of φ on $z(t)$ along a balanced growth path. Explain your answer intuitively.

Question 2 [25 points]

Consider the following problem faced by a typical consumer of generation t in a two-period overlapping-generation model,

$$\max_{c_{y,t}, s_t, c_{o,t+1}} \left[\frac{c_{y,t}^{1-\theta} - 1}{1-\theta} + \beta \frac{c_{o,t+1}^{1-\theta} - 1}{1-\theta} \right], \qquad \theta > 0, \quad \beta \in (0,1),$$

subject to

$$c_{y,t} + s_t = w_t,$$

$$c_{o,t+1} = (1 + r_{t+1}) s_t,$$

$$c_{y,t} \geq 0, \qquad c_{o,t+1} \geq 0, \qquad and \quad s_t \geq 0.$$

where $w_t > 0$ is the wage rate at time t and $r_{t+1} > 0$ is the interest rate.

(a) [10 points] Let s_t be the optimal level of savings when young. Derive an expression for the saving rate $\sigma_t \equiv s_t / w_t$.

(b) [15 points] Determine how an increase in the interest rate r_{t+1} affects the saving rate σ_t when (i) $\theta > 1$, (ii) $\theta = 1$ and (iii) $\theta < 1$. Explain the intuition behind.

NOTE: Answer Part III <u>OR</u> Part IV.

Part IV. Answer All Questions.

1. [10 points] A two-period-life overlapping generations model with endowments admits multiple steady state equilibria. Draw a picture of the savings relation as a function of the interest factor R, and explain what causes multiple steady states.

2. [10 points] A standard two-period-life overlapping generations model with productive capital admits multiple interior steady-state equilibria. State a theorem that causes the model to have a single interior steady state and explain the mechanism through which this occurs.

3. [10 points] Provide two reasons of why some countries have not reached a steady-state level, but instead continued to grow, in per capita income, and discuss why each reason is plausible from a modeling point of view.

4. [50 points] Consider a two-period-life OLG model with production. Population is constant and normalized to 1 and consumers are retired in old age. A consumer who has logarithmic utility solves the following utility maximization problem,

(1) $\quad \text{Max } \alpha \ln(c_{0,t}) + (1-\alpha) \ln(c_{1,t+1}), \; 0 < \alpha < 1,$

subject to

(2) $\quad c_{0,t} = w_t - s_t,$

(3) $\quad c_{1,t+1} = R_{t+1}\, s_t$

where c is consumption, and R is the yield on savings s.

(a) [10 points] Find the optimal savings s*. Derive a condition using a general neoclassical production function f(k), which is strictly increasing, continuous, and concave in capital k, that guarantees that s* is concave in k. <u>Hint</u>: solve the firm's profit maximization problem.

(b) [10 points] Let the wage be $w_t = \beta A k_t$, and the rental rate $r_t = (1-\beta)Ak_t$, for $\beta \in (0,1)$ and $A > 0$. Use these factor prices to construct the capital market clearing condition in terms of current and future state variables. Find all steady states.

(c) [10 points] Derive the phase portrait, including arrows of motion. Draw the phase portrait and identify the stability properties of all steady states.

(d) [10 points] Prove or disprove: this economy exhibits perpetual growth.

(e) [10 points] We now modify the model so that $w_t = \beta k_t$, and $r_t = (1-\beta)k_t$, for $\beta \in (0,1)$. Re-derive the phase portrait and graph it. Then discuss what drives the dynamics of this version of model as compared to the previous version.

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Part I. Answer All Questions.

1. [10 points] Explain in words what is precisely meant by "efficiency wage"? In addition, provide a graphical illustration for the determination of efficiency wage.

2. [20 points] Consider an economy in which output y is produced by a production function $y = \left[(1-\alpha)k^{\rho} + \alpha n^{\rho}\right]^{\frac{1}{\rho}}$, $0 < \alpha < 1$, $-\infty < \rho < 1$, where k is capital and n is employment. Notice that there is no population growth and no technological progress in the economy.

(a) [5 points] Find the expression for the elasticity of substitution between capital and employment (denoted as σ).

(b) [5 points] Find the expression for the marginal product of <u>per-capita</u> capital k/n.

(c) [10 points] Based on your answer to parts (a) and (b), find the range of σ under which <u>per-capita</u> output y/n exhibits sustained/unbounded growth even though there is no technological progress in the economy. <u>Explain the economic intuition.</u>

3. [50 points] This problem concerns a representative agent economy. Technology is given by

(1) $$Y_t = K_t^{\alpha} L_t^{1-\alpha}, \quad 0 < \alpha < 1,$$

where Y is output, K is capital and L is labor hours. The representative agent maximizes the following discounted utility:

(2) $$U = \sum_{t=0}^{\infty} \beta^t \left[C_t - L_t^{\gamma}\right], \quad 0 \le \beta \le 1, \quad \gamma > 1,$$

where β is the discount factor and C is consumption. The capital accumulation equation is given by

(3) $$K_{t+1} = K_t(1-\delta) + Y_t - C_t, \quad K_0 > 0 \text{ given},$$

where $\delta \in (0,1)$ is the capital depreciation rate.

(a) [12 points] Find a set of four equations in the variables C_t, K_t, L_t and Y_t that characterizes a competitive equilibrium for this economy.

A consumer born at time t who has logarithmic utility solves the following lifetime expected utility maximization problem,

$$\text{Max}_{c0,t,\ c1,t+1}\ E_t\{(1-\beta)\ln(c_{0,t}) + \beta\ln(c_{1,t+1})\}$$

s.t.

$$c_{0,t} = w_t(1-\tau) - s_t$$

$$c_{1,t+1} = R_{t+1}\, s_t.$$

(a) [10 points] What is/are the state variable(s) for this problem at time t? Write down the optimal savings function s_t^* that is the consumer's optimum for the problem above.

(b) [10 points] Construct the capital market equilibrium condition in per youngster terms using only the state variable(s) and constants. To do this, let the wage be $w_t = (1-a)e^{At} k_t^a$, where $k_t \equiv K_t/N_t$.

(c) [10 points] Draw a graph of the equilibrium dynamics in this model in k_t, k_{t+1} space when technology is a random variable and follows the autoregressive expression given above. Include a 45 degree line and arrows of motion showing how the economy evolves.

(d) [10 points] What does the persistence of a technology shocks mean for variations in capital, output and consumption?

(e) [10 points] Describe what happens to the economy at time t if before agents make their savings/consumption decisions it is reported in the news that the "experts" expect that next period's value of A_t will be negative and large because it has been positive for so long, and most people believe this is true. Be explicit about causation and effects over time.

(f) [10 points] President Bush has just added you to his Council of Economic Advisors to tell him how to get the economy out of a recession. Using the model above, offer the most effective policy to stimulate economic output and describe to the President how this would produce the desired effect.

(b) [6 points] What is meant by the transversality condition? What is the transversality condition for a competitive equilibrium for this economy?

(c) [14 points] Based on your answer to part (a), derive the expression for the steady-state value of capital K^* as a function of the model parameters.

(d) [10 points] Based on your answer to part (a), derive the non-linear equation that describes how capital evolves over time.

(e) [8 points] Based on your answer to part (d), describe (in words or using a diagram) how capital will adjust over time from its initial value K_0 to its steady state K^*. <u>Explain the economic intuition</u>. <u>Note</u>: assume that $K_0 < K^*$.

Part II. Answer All Questions.

1. [15 points] Provide a derivation of the Euler equation in the context of the Cass-Koopmans model. Discuss the difference between this model and the standard Solow model with respect to the attainability of the Golden-Rule rate of saving.

2. [15 points] Assume a two-period intertemporal consumption model in which the representative agent is faced with a labor-leisure choice. Explain the intratemporal and intertemporal effects on labor and consumption of a pure income shock.

3. [50 points] Assume that the representative agent's preferences over the consumption good are given by:

$$\text{Max} \sum_{t=0}^{\infty} \beta^t \log(c_t), \qquad 0 < \beta < 1,$$

and that the budget constraint is

(i) $k_t^\theta + (1-\delta)k_t + M_{t-1}/p_t + tr_t - c_t - k_{t+1} - M_t/p_t \geq 0$ B.C.

where $k_t^\theta = f(k_t)$ and $0 < \theta < 1$. In addition, previous accumulated cash is required for the purchase of consumption and investment:

(ii) $c_t + k_{t+1} - (1-\delta)k_t \leq M_{t-1}/p_t + tr_t$ CIA

where p_t, $(M_{t-1}/p_t + tr_t \equiv M_t/p_t)$ and tr_t are the price level, the household's beginning of period money balances, and lump sum transfer of new cash in period t in real terms, respectively. The Central Bank allows the money supply to grow according to the following law of motion:

$$M_{t+1} = (1+g)\, M_t.$$

(a) [6 points] Write down Bellman's equation.

(b) [12 points] Derive the F.O.C., the Envelope and Euler equations for this problem.

(c) [10 points] Define a recursive competitive equilibrium for this economy. Make sure you describe completely the problem.

(d) [10 points] Derive a condition involving g that guarantees that the cash-in-advance constraint is binding.

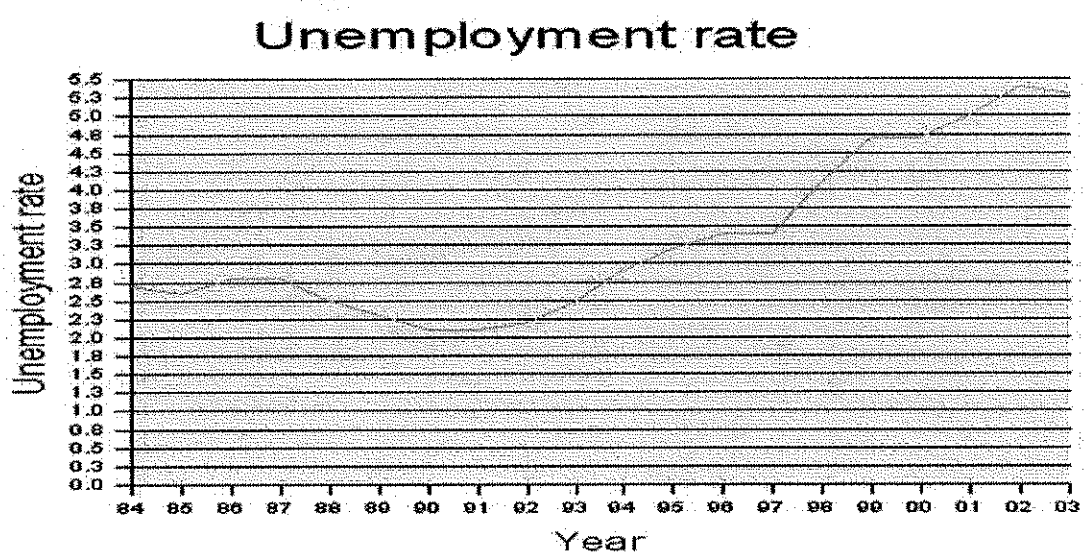
(e) [12 points] Assume that the condition obtained from (d) is satisfied. Solve for the steady state capital stock. How does this depend on the rate of money growth? How does it compare with the Pareto optimal steady state capital stock (that is, the standard Cass-Koopman's model)?

Part III. Answer All Questions.

1. [7 points] The standard neoclassical growth model provides an adequate explanation for Japan's rapid economic growth starting in 1960 [Data on annual average PCGDP growth: 10% in the 1960s, 5% in the 1970s, 4% in the 1980s, 1.5% in 1990s]. Do you agree or disagree with this statement? Explain.

2. [6 points] The standard neoclassical growth model provides an adequate explanation for Japan's growth slowdown starting in 1990. [Data on annual average PCGDP growth: 10% in the 1960s, 5% in the 1970s, 4% in the 1980s, 1.5% in 1990s]. Do you agree or disagree with this statement? Explain.

3. [7 points] The standard neoclassical growth model provides an adequate explanation for Japan's rising unemployment starting in 1990. [Data in graph below] Do you agree or disagree with this statement? Explain.



4. [60 points] Consider a standard two period life overlapping generations "real business cycle" model with production and a random technology shock A_t, for all times t. The technology level A_t is known for certain at time t, but A_{t+1} must be forecasted at using an autoregressive form

$$A_{t+1} = \rho A_t + \varepsilon_t$$

where ε_t is a normally distributed with unconditional expected value 0, and $\rho \in (0,1)$. The population grows geometrically, $N_{t+1} = (1+n) N_t$, where N_t is the number of youngsters at time t, and n$\geq$ -1$. There is also an income tax $\tau \in [0,1]$ on youngsters. The revenue from the tax funds the government but is unproductive from the consumer and firm perspective.

University of California, Riverside
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Macroeconomic Theory
Cumulative Examination

Spring 2006

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Instructions

1. There are nine pages (including the cover page) of the exam.
2. You have four hours to complete the exam. There are 240 points in total.
3. Read the instructions carefully.
4. Clearly label all the diagrams and underline the keywords in your answers.

Part I. Answer All Questions.

1. [10 points] A simple economy produces two commodities: hamburgers and computers. The following table shows total production for this economy in 1999 and 2000:

	1999 Quantity	1999 Price	2000 Quantity	2000 Price
Hamburger	200,000	\$ 3.00	400,000	\$ 4.00
Computer	1,000	600.00	4,000	100.00

(a) [5 points] Use the chain-weighted method to calculate this economy's growth rate of real GDP between 1999 and 2000. Show your work.

(b) [5 points] Based on your answer to part (a), explain the rationale behind the U.S. Commerce Department's switching from the base-year method to the chain-weighted method in calculating real GDP growth since 1996.

2. [20 points] Consider an economy in which output y is produced by a production function $y = \left[\alpha k^{-\rho} + (1-\alpha)n^{-\rho}\right]^{-\frac{1}{\rho}}$, $0 < \alpha < 1$, $-1 < \rho < \infty$, where k is capital and n is employment. Notice that there is no technological progress in the economy.

(a) [10 points] Log-linearize the above production function around an arbitrary point $y_* = \left[\alpha k_*^{-\rho} + (1-\alpha)n_*^{-\rho}\right]^{-\frac{1}{\rho}}$, and then obtain the following approximation form: $\log(y) = \beta_0 + \beta_1 \log(k) + \beta_2 \log(n)$. That is, derive the expressions for β_0, β_1 and β_2 as functions of the production parameters (α, ρ) and the arbitrary point (y_*, k_*, n_*).

(b) [5 points] Use the L'Hospital's rule to prove that $\lim_{\rho \to 0}\left[\alpha k^{-\rho} + (1-\alpha)n^{-\rho}\right]^{-\frac{1}{\rho}} = k^{\alpha} n^{1-\alpha}$. Show your work.

(c) [5 points] Find the expression for the elasticity of substitution between capital and employment.

3. [50 points] Consider the following economic model:

(1) $\quad p_t = \alpha E_t[p_{t+1}] + \beta p_{t-1} + v_t,$

where p_t is the logarithm of the price level, E_t is the conditional expectations operator, α and β are non-zero parameters, and v_t is an i.i.d. exogenous stochastic shock with mean zero and variance σ^2. Let the state vector be X_t where $X_t = \begin{bmatrix} p_t \\ E_t[p_{t+1}] \end{bmatrix}$.

(a) [10 points] Write the model, as in equation (1), as a first-order expectational vector difference equation in the following form:

$$(2) \qquad \Gamma_0 X_t = \Gamma_1 X_{t-1} + \Psi_v v_t + \Psi_w w_t,$$

where w_t is an endogenous expectational error of forecasting p_t. That is, find the elements of the matrices Γ_0, Γ_1, Ψ_v and Ψ_w in terms of the parameters α and β.

(b) [5 points] Rewrite equation (2) in the form $X_t = AX_{t-1} + Bv_t + Cw_t$. That is, find the elements of the matrices A, B and C in terms of the parameters α and β.

(c) [5 points] Let λ and θ be the eigenvalues of the matrix A. Express the matrix A in the form $Q\Lambda Q^{-1}$, where Λ is a diagonal matrix of eigenvalues, and Q is a matrix of eigenvectors. Note: arrange the two eigenvalues such that θ appears in the first row/column of Λ; and normalize the first element of both eigenvectors to be 1.

(d) [5 points] Assume that λ and θ are both positive real numbers, and that $\lambda < \theta$. What is the condition, in terms of the parameters α and β, which will ensure that the model (1) exhibits a unique rational expectations equilibrium? Explain your answer.

(e) [5 points] Define the transformed variables

$$(3) \qquad \begin{bmatrix} z_t^1 \\ z_t^2 \end{bmatrix} = Q^{-1}X_t, \quad \begin{bmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \end{bmatrix} = Q^{-1}Bv_t \quad \text{and} \quad \begin{bmatrix} \eta_t^1 \\ \eta_t^2 \end{bmatrix} = Q^{-1}Cw_t.$$

Using the above definitions, write down two independent scalar difference equations in z_t^1, z_t^2, $\varepsilon_t^1, \varepsilon_t^2$, η_t^1 and η_t^2 that represent the transformed dynamical system.

(f) [10 points] Under the assumption that the condition for equilibrium uniqueness, as in part (d), is satisfied, use the unstable/explosive difference equation in part (e) to express the non-fundamental error w_t as a linear function of the fundamental error v_t and parameters λ and θ.

(g) [10 points] Find a scalar difference equation in terms of the observables p_t, p_{t-1}, v_t, and the parameters λ and θ that characterizes the model's unique rational expectations solution.

Part II. Answer All Questions.

1. [10 points] In Sidrauski's model, do consumption and capital depend on the rate of money growth at the steady state? Why or why not? At the steady state, what is the effect of an increase in the rate of growth of money supply on (i) bond's price; (ii) nominal interest rate; (iii) money demand; (iv) marginal utility of money; and (v) total utility $U(c_t, m_t)$.

2. [10 points] Suppose a model in which the conditions for consumption optimization are: $\frac{dc_t/dt}{c_t} = \frac{1}{\theta}(r_t - \rho)$, where r_t is the interest rate at time t and $\frac{1}{\theta}$ is the elasticity of substitution, which represents the willingness of a household to postpone current consumption for future consumption. Explain the effects of a decrease in interest rate on the household's saving (substitution effect, income effect, and wealth effect). Explain which one would prevail for each of the following cases: (i) $\frac{1}{\theta} > 1$; (ii) $\frac{1}{\theta} < 1$; and (iii) $\frac{1}{\theta} = 1$.

3. [60 points] Lucas' fruit tree model with one-period contingent claim: consider an economy consisting of a large number of identical agents with preferences given by

$$(1) \qquad E\sum_{t=0}^{\infty}\beta^t U(c_t), \quad 0 < \beta < 1.$$

Suppose that each agent is endowed with one tree that pays stochastic dividends (fruit) d_t, as a function of the state of the economy and one-period contingent claim. Denote the price of a tree by p_t and the number of trees owned by the households by s_t. Let $q(x_{t+1}, x_t)$ be the price at t (in state x_t) of a claim to one unit of consumption to be delivered at t+1 if state x_{t+1} is realized; and let $\int_{\Omega} q(x_{t+1},x_t)dx_{t+1} = 1/(1+R_t)$ be the price of a sure claim to a unit of consumption next period, where Ω is the whole state space. In addition, let $f(x_{t+1},x_t)$ be the transition density of the underlying stochastic state, $x_t = d_t$, where $F(x_{t+1},x_t) = \int_{-\infty}^{\infty} f(\mu,x_t)d\mu$, and define $y(x_t)$ as the quantity of assets that pay off in state x_t, that is, deliveries which are due today, measured in consumption goods.

(a) [10 points] Write this optimization problem as a dynamic programming problem. That is, write down Bellman's equation and define control and state variables.

(b) [10 points] Define a recursive competitive equilibrium for this economy. Be complete.

(c) [15 points] Derive the first-order conditions, the envelope equation and Euler equation (for both the fruit tree and contingent claim).

(d) [5 points] Find the *equilibrium* price of the fruit tree.

(e) [6 points] Use repeated substitution and the law of iterated expectations to find the equilibrium price of the fruit tree. Give the interpretation for equilibrium price of the fruit tree.

(f) [6 points] Find the *equilibrium* price of a contingent claim and explain intuitively what it means.

(g) [8 points] Find the *equilibrium* price of a sure claim in terms of utility.

NOTE: Answer Part III <u>OR</u> Part IV.

Part III. Answer All Questions.

1. [10 points] Explain why heterogeneity facilitates trade.

2. [10 points] Explain clearly why the capital stock is the state variable in a standard two-period life overlapping generations model.

3. [10 points] All standard general equilibrium growth models predict that countries converge in per capita income, but this is inconsistent with the data. Offer two reasons to explain why countries do not hit a steady state but instead continue to grow.

4. [50 points] Consider a two-period life OLG model with production. Population is constant and normalized to 1 and consumers are retired in old age. A consumer who has logarithmic utility solves the following utility maximization problem,

(1) Max $(1-\beta)\ln(c_{0,t}) + \beta \ln(c_{1,t+1})$,

subject to

(2) $c_{0,t} = w_t - s_t$,

(3) $c_{1,t+1} = R_{t+1}\, s_t$

where $\beta \in (0,1)$ is the agent's discount factor, c is consumption, and R is the yield on savings, s.

(a) [10 points] Find the optimal savings s*. Derive a condition using a general neoclassical production function f(k), which is strictly increasing, continuous, and concave in capital k, that guarantees that s* is concave in k. <u>Hint</u>: solve the firm's profit maximization problem.

(b) [10 points] Let the wage be $w_t = \alpha A k_t$, and the rental rate $r_t = (1-\alpha)Ak_t$, for $\alpha \in (0,1)$ and $A > 0$. Use these factor prices to construct the capital market clearing condition in terms of current and future state variables. Find all steady states.

(c) [10 points] Derive the phase portrait, including arrows of motion. Draw the phase portrait and identify the stability properties of all steady states.

(d) [10 points] Prove or disprove: this economy exhibits perpetual growth.

(e) [10 points] We now modify the model so that $w_t = \alpha k_t$, and $r_t = (1-\alpha)k_t$, for $\alpha \in (0,1)$. Re-derive the phase portrait and graph it. Then discuss what drives the dynamics of this version of model as compared to the previous version.

NOTE: Answer Part III <u>OR</u> Part IV.

NOTE: Answer Part III OR Part IV.

Part IV. Answer All Questions.

1. [10 points] Lump-sum tax has no effect on the household's steady-state savings rate in a standard Ramsey model. True, false or uncertain? Explain your answers.

2. [10 points] In an overlapping generation model with no altruism, a fully-funded social security system has no real effect on the economy. True, false or uncertain? Explain your answers.

3. [10 points] Consider a standard neoclassical Q-theory model in which the firm is a monopolist whose capital level has reached the steady state at this moment. If there is a demand shock that lowers the price elasticity of demand of the good produced by the firm, then the firm should reduce its production. True, false or uncertain? Explain your answers.

4. [50 points] Consider an economy whose population at time t is N_t, and the population growth rate is a constant n. The utility function of an infinitely-lived representative household takes the form:

(1) $$U_t = \int_0^\infty e^{-\theta t} N_t \log c_t dt, \ \ 0 < \theta < 1,,$$

where θ is the time discount rate, and c_t is per capita consumption at time t. The economy produces a Solow good according to the following production function:

(2) $$Y_t = K_t^\alpha M_t^{1-\alpha}, \ \ 0 < \alpha < 1,$$

where Y_t is the output that can be either consumed or transformed into capital stock. K_t is the capital input that does not depreciate; and M_t is an intermediate good that totally depreciates in production, *i.e.*, it cannot be accumulated after being used in production). The intermediate good is produced according to a CES production function

(3) $$M_t = \left(\int_0^A x_{jt}^\beta dj \right)^{\frac{1}{\beta}}, \ \ A > 0 \text{ and } 0 < \beta < 1,$$

where x_{jt} is the jth raw material and there are a total of A brands. The labor requirement for each unit of raw material is 1. The production sector is perfectly competitive, and the price of this Solow good is normalized to 1.

(a) [10 points] Set up the current value Hamiltonian function for the household and solve its optimization problem.

(b) [5 points] Solve for optimal inputs for the Solow good in terms of the rental rate r_t and intermediate-good price P_{mt}.

(c) [5 points] Solve for optimal raw materials demand for the jth intermediate good in terms of its own price P_{jt} and the intermediate-good price P_{mt}.

(d) [5 points] Given that all raw materials have a unit labor requirement of 1, what is the relationship between the raw-material price P_{jt} and the wage rate w_t? Explain your answer.

(e) [10 points] Solve for the equilibrium rental rate r_t and wage rate w_t in terms of per capita capital using your results in parts (b), (c) and (d).

(f) [5 points] Find out the dynamic trajectories of per capita capital and consumption.

(g) [5 points] Analyze the movement of the dynamic system in a Phase diagram. Explain your answer.

(h) [5 points] Use a Phase diagram to analyze the effects of an unexpected technology shock so that A increases to A' at time t_0.

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Part I. Answer All Questions.

1. [10 points] Real wages are countercyclical in the New-Keynesian model with the labor market equilibrium being characterized by efficiency wages. True, false or uncertain? Explain your answers using the New-Keynesian labor market diagram.

2. [10 points] The Debreu-Sonnenschein-Mantel theorem states that all competitive equilibria in a pure exchange economy are not invariant to changes in the units in which prices are measured. True, false or uncertain? Explain your answers.

3. [60 points] This question is about a representative agent economy whereby choices are made by a large number of identical households, each solves the following problem:

$$(1) \qquad \text{Max}\sum_{t=0}^{\infty}\beta^t \log\left(c_t - Ah_t\ell_t^{\gamma}\right),\ \ t = 0,1,2,\ldots,\ \ 0<\beta<1,\ \ A>0\ \text{ and }\ \gamma>1,$$

subject to the constraints:

$$(2) \qquad c_t + i_t = y_t,$$

$$(3) \qquad k_{t+1} = k_t^{1-\delta} i_t^{\delta},\ \ t = 0,1,2,\ldots,\ \ 0<\delta<1\ \text{ and }\ k_0\ \text{given},$$

$$(4) \qquad y_t = k_t^{\alpha}(h_t\ell_t)^{1-\alpha},\ \ 0<\alpha<1,$$

where k is capital, ℓ is labor supply, y is output, i is investment, c is consumption, and A, α, β, γ and δ are parameters. Moreover, h is an index of knowledge (or human capital) that is outside the household's control. To capture the idea that the mechanism for knowledge accumulation is learning by doing as a by-product of private investment activities, we postulate that in equilibrium, $h_t = k_t$ for all t. Finally, agents are assumed to have perfect foresight.

(a) [5 points] Formulate the Lagrangian for the above optimization problem, and show that the marginal utility of consumption in period t is equal to the Lagrange multiplier associated with the budget constraint λ_t.

(b) [10 points] Using your answers to part (a), derive the equation in terms of ℓ_t, h_t, y_t, and model parameters that describes how a representative agent chooses its supply of labor in period t. Explain the economic intuition.

(c) [10 points] Using your answers to part (a), derive the equation in terms of y_{t+1}, i_t, i_{t+1}, λ_t, λ_{t+1} and model parameters that describes how a representative agent allocates its consumption across periods t and t+1. Explain the economic intuition.

(d) [10 points] Using your answers to parts (a) and (b), equation (4), and the equilibrium condition that $h_t = k_t$, derive the optimal decision rule for household labor hours.

(e) [15 points] Show that the equilibrium savings rate is a constant in this model. Find the expression for this constant savings rate. <u>Hint</u>: guess that $i_t = a_0 y_t$ and $1/\lambda_t = b_0 y_t$. Plug this guess into your answers to part (c) and solve for a_0.

(f) [10 points] Based on your answers to parts (d) and (e), does this model exhibit features that are consistent with a balanced growth path? Explain your answers.

Part II. Answer All Questions.

1. [20 points] In the standard Cass-Koopmans' model the capital stock in which maximum consumption is achieved (golden rule) is smaller, equal or greater than the steady state capital in which utility is maximized? Does your answer depend or not on the functional form of the utility function and/or of the production function? Explain.

2. [10 points] In the Cash-in-Advance-type model of money and growth, differences in inflation and real money balances have no impact on capital intensity, the real interest rate, and consumption in the steady state. Correct, Incorrect or Uncertain?

3. [50 points] Consider the following version of Sidrauski's model below. Assume that the representative agent has preferences given by:

$$\sum_{t=0}^{\infty} \beta^t (c_t^{\alpha} m_t^{1-\alpha}) \qquad\qquad 0<\beta<1,\ 0<\alpha<1$$

where $U(c,m) = (c^{\alpha} m^{1-\alpha})$, the production function is $f(k) = k^{\theta}$, $0<\theta<1$, and the depreciation rate is $\delta=1$. Let $m_t=M_t/P_t$ be the real money balance, tr_t be the real transfers from the government, and b_t be the privately issued bonds in real terms that are sold for a discount price q_t. π_t is defined as p_t/p_{t-1} ($\pi_t = p_t/p_{t-1}$). The Central Bank allows the money supply to grow according to the law of motion: $M_t = (1+g)\, M_{t-1}$.

(a) [10 points] Write down the budget constraint in real terms for this problem - use the specific functional forms given.

(b) [5 points] Write down Bellman's equation, control and state variables.

(c) [10 points] Derive the F.O.Cs. and the envelope equations (Hint: use Lagrangean).

(d) [5 points] Derive the Euler equation in real terms and in nominal terms.

(e) [10 points] Write down the demand of real money balances as a function of consumption and the nominal interest rate. Interpret this expression. How does real money demand depends on the parameter α?

(f) [5 points] Find the steady state capital for this economy (k^*) and show how it depends on the rate of growth of money supply, g. Derive the inflation rate in the steady-state equilibrium. Is money *superneutral*?

(g) [5 points] Let us consider the case in which the government conducts monetary policy so that the nominal interest rate $i = \pi + r$ should be constant. Is money *superneutral* in equilibrium? Show your results.

Part III. Answer All Questions.

1 [10 points] In the context of an overlapping generations model, an effective way in helping the incumbent president get re-elected is to boost domestic consumption through issuing bonds instead of taxing people (to finance the ongoing public projects. True, false or uncertain? Explain your answers.

2. [10 points] According to the Beta convergence theory, if two countries are otherwise exactly the same, but one has a higher income level, then the richer country is to grow faster. True, false or uncertain? Explain your answers.

3. [10 points] What is the intuition for incomplete nominal adjustment in the Phelps-Lucas island model? What is the intuition of Lucas Critique in this context?

4. [50 points] Consider an economy with the following production function:

$$Y_t = AK_t + BK_t^{\beta} L_t^{1-\beta}, \tag{1}$$

where $A \geq 0$, $B > 0$, and $0 < \beta < 1$. Y_t, K_t, and L_t are respectively the aggregate output, capital and labor input at time *t*. A representative agent in the economy has an infinite horizon with an intertemporal utility function

$$U = \int_0^{\infty} e^{-\rho t} \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} dt, \quad 0 < \rho < 1, \ \sigma > 0, \ \sigma \neq 1, \tag{2}$$

and c_t is the per-capita consumption at time t. The population size in this economy is constant over the time. There is no capital depreciation in this economy. In parts (a) to (e), assume that $A > \rho$.

(a) [5 points] Rewrite the production function (1) in per-capita terms. Is the Inada condition violated in (1) or not? From this, can we conjecture that the model can generate a non-zero growth rate? Explain your answers.

(b) [10 points] Write down the consumer's problem, the corresponding Hamiltonian, and derive the first-order conditions.

(c) [10 points] Find out the equilibrium dynamic trajectory for per-capita capital and consumption.

(d) [10 points] Suppose a steady-state balanced growth path exists in which per-capita consumption and capital grow at the same rate, what is the growth rate? In addition, what is the equilibrium per-capita consumption-capital ratio? Show your work.

(e) [5 points] Does a higher rate of intertemporal elasticity of substitution or a lower time discount rate imply a higher long-run steady-state growth rate? Explain your answers.

(f) [5 points] Consider a special case with $A = 0$ (in all the parts above, we assume that $A > \rho$), explain why the steady-state growth rate is zero in this specification.

(g) [5 points] Continue with part (f). Does a lower time discount rate lead to a higher level of capital at the steady state? Show your work.

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Part I. Answer All Questions.

1. [10 points] The Government of Econoland is concerned about employment conditions. You are asked to conduct a research on this matter, and therefore collect the following data:

   Civilian Adult Population = 1,000,000
   Labor Force = 800,000
   Number of People Employed = 760,000

   Compute the (a) labor force participation rate, (b) the employment rate and (c) the unemployment rate. Show your work. Note: start your answers by precisely defining what are meant by (i) the civilian adult population, (ii) the labor force, (iii) the employment rate and (iv) the unemployment rate, respectively.

2. [10 points] In a deterministic optimal growth model with endogenous labor supply, the representative household maximizes $\sum_{t=0}^{\infty}\beta^t U(c_t, \ell_t)$, where $\beta \in (0,1)$ is the discount factor, c_t is consumption and ℓ_t is leisure. In addition, the period utility function $U(\cdot)$ is assumed to be strictly concave and twice continuously differentiable. According to King, Plosser and Rebelo (1988, *JME*, Vol. 21, 195-232), there are only two classes of period utility functions that are compatible with steady-state balanced growth. What are they? Explain your answers. Note: start with your answers by precisely defining what is meant by steady-state balanced growth.

3. [60 points] Consider an economy with a unit measure of identical infinitely-lived households. The representative household maximizes its lifetime utility

$$\text{(1)} \qquad \text{Max} \sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t\left[\frac{c_t^{1-\sigma}}{1-\sigma} - A\frac{h_t^{1+\gamma}}{1+\gamma}\right],\ 0<\rho<1,\ \sigma>1,\ A>0 \text{ and } \gamma \geq 0,$$

   where ρ is the discount rate, c_t is consumption, h_t is hours worked and A is a preference parameter. The budget constraint faced by the representative household is

$$\text{(2)} \qquad c_t + k_{t+1} - (1-\delta)k_t + \tau_{ct}c_t = w_t h_t + r_t k_t,\ 0<\delta<1,\ 0<\tau_{ct}<1,\ k_0 \text{ is given},$$

   where k_t is capital, δ represents the capital depreciation rate, τ_{ct} is the consumption tax rate, w_t is the real wage rate and r_t is the capital rental rate. On the production side of the economy, output y_t is produced by a unit measure of competitive firms using the following technology:

$$\text{(3)} \qquad y_t = \left[\alpha k_t^{\beta} + (1-\alpha)h_t^{\beta}\right]^{\frac{1}{\beta}},\ 0<\alpha<1,\ -\infty<\beta<1.$$

Finally, the government has a stream of constant spending G that is financed by levying taxes on the household's consumption expenditures. It maintains a balanced budget for each period such that

(4) $G = \tau_{ct} c_t$.

(a) [5 points] Based on the firm's production function (3), derive the expression for the elasticity of substitution between capital and labor inputs.

(b) [5 points] Based on your answers to part (a), what is the value of β under which the firm's production function (3) becomes Cobb-Douglas $y_t = k_t^{\alpha} h_t^{1-\alpha}$? Explain your answers. Note: start your answers by deriving the capital and labor shares of national income, respectively, assuming that factor markets are perfectly competitive.

Note: to answer parts (c)-(h) below, replace (3) with $y_t = k_t^{\alpha} h_t^{1-\alpha}$.

(c) [4 points] Under the assumption that factor markets are perfectly competitive, derive the first-order conditions for the firms' profit maximization problem. Explain the economic intuition.

(d) [8 points] Formulate the Lagrangian for the household's optimization problem, and derive the first-order condition that governs the representative household's choice for its supply of labor in period t. Explain the economic intuition.

(e) [8 points] Using the Lagrangian as in part (d), derive the first-order condition that governs the representative household's choice for consumption across periods t and t+1. Explain the economic intuition.

(f) [20 points] Based on equation (2), your answers to parts (c)-(e), and using τ_c and h^* to denote the steady-state consumption tax rate and hours worked, derive the expressions for the steady-state (i) capital-labor ratio $k^*/h^* \equiv \kappa$ (as a function of model parameters), (ii) consumption c^*(as a function of model parameters, τ_c, κ and h^*), and (iii) labor hours h^* (as a function of model parameters, τ_c and κ).

(g) [5 points] Based on your answers to part (f) and the steady-state version of equation (4), derive the expression for $\frac{\partial G}{\partial \tau_c}$ (as a function of model parameters, τ_c and κ).

(h) [5 points] Based on your answers to part (g), discuss the sign of $\frac{\partial G}{\partial \tau_c}$, and explain why the sign of $\frac{\partial G}{\partial \tau_c}$ implies that the model exhibits a unique interior steady state.

Part II. Answer All Questions.

1. [20 points] In a two-period intertemporal consumption model, describe the intratemporal and intertemporal effects of a pure negative income shock on consumption and labor supply (assuming that there is no government, and no capital). Illustrate the intertemporal and intratemporal effects graphically.

2. [10 points] In a two-factor (labor and capital) infinite-horizon model with optimizing agents, a constant proportional tax rate imposed on the total income will have no effect on the economy's steady-state growth rate. True, false or uncertain? Explain your answers.

3. [50 points] Consider a representative household acting to maximize

(1) $\sum_{t=0}^{\infty} \beta^t \log(c_t)$, $0 < \beta < 1$,

subject to

(2) $k_{t+1} = f(k_t) + (1 - \delta)k_t - c_t$, $c_t \geq 0$, $k_t \geq 0$ $(t = 0,1,2,\ldots)$, $0 < \delta < 1$ and k_o given.

(a) [10 points] Write down the Bellman's equation for the associated stationary dynamic programming, the control and state variables.

(b) [15 points] Derive the first-order condition, envelope condition and Euler equation for consumption.

(c) [5 points] Let c* and k* be the steady state to which the optimal path converges. Write down the Euler equation for this optimal steady-state, and derive the expression for the steady-state marginal productivity of capital.

(d) [5 points] Suppose the utility also depends on leisure ℓ_t, so that the period t utility function is now: $\log(c_t + \ell_t)$. Write down the Bellman's equation and derive the first-order conditions for c_t and ℓ_t. Note: there are now two controls.

(e) [15 points] Assume that the representative agent faced no leisure option, but a proportional consumption tax, which was rebated to him lump-sum. What is the correct form of his transition equation for capital now? Set-up the agent's maximization problem, derive his F.O.C.'s and the Euler equation, and then compare the steady-state levels to those derived in part (c).

Part III. Answer All Questions.

1. [10 points] In the Solow growth model without exogenous technological progress, if the production function is not necessarily neoclassical, but exhibits constant return-to-scale and diminishing returns to each of the production factors, then the long-run growth rate of per-capita output is zero. True, false or uncertain? Explain your answers.

2. [10 points] In an overlapping generation model with a fully-funded social security system, the Ricardian equivalence will always hold. True, false or uncertain? Explain your answers.

3. [10 points] Accidental technological discovery, rather than profit-seeking technological discovery, can be compatible with a perfectly competitive economy with neoclassical production technologies. True, false or uncertain? Explain your answers.

4. [50 points] Consider an economy in which the utility function of an infinitely-lived representative agent takes the form:

(1) $$U_t = \int_t^{\infty} e^{-\rho t} \log c_t dt, \quad 0 < \rho < 1,,$$

where ρ is the time discount rate, and c_t is personal consumption at time t. The economy produces a Solow good Y_t that can be either consumed or invested. The common neoclassical production function for the perfectly competitive firms in the economy is given by

(2) $$Y_t = K_{yt}{}^{\alpha} L_{yt}{}^{1-\alpha} G_t{}^{\beta}, \quad 0 < \alpha < 1 \text{ and } \beta > 0,$$

where K_{yt} and L_{yt} are capital and labor inputs, and G_t is an index of the service provided by the infrastructure built by the government. The kind of infrastructure service is made freely available to the private firms. For simplicity, the population is constant at N, and there is no capital depreciation. The initial asset owned by the representative agent is k_0.

The government levies a lump sum tax $\frac{T_t}{N}$ per household. The tax rate in the economy is z (i.e., if the household income is $\frac{Y_t}{N}$, z fraction of the income will be taxed away) at all times. The tax revenue is used to finance the service of infrastructures. In addition, we make the over-simplified assumption that

(3) $G_t = (1-c)T_t$, $0 < c < 1$,

where T_t is the total tax revenue of the government, c is a corruption index of the government.

(a) [10 points] Assuming that the representative household takes the lump sum tax $\frac{T_t}{N}$ as given, set up the current value Hamiltonian function for the household and solve it.

(b) [5 points] Solve for the firms' profit maximization problem.

(c) [5 points] In equilibrium, what would be the relationship between aggregate output and total capital stock and population level?

(d) [10 points] Write down the equilibrium conditions, and find out the dynamic trajectories for per-capita consumption and capital accumulation.

(e) [10 points] If $\beta = 1-\alpha$, find out the long-run per-capita consumption growth rate.

(f) [5 points] With the tax rate unchanged, an increase in the corruption level will increase or decrease the long-run per-capita consumption growth rate? Why?

(g) [5 points] If $\beta < 1-\alpha$, what is the long-run income growth rate? Why?

University of California, Riverside
Department of Economics

Macroeconomic Theory
Cumulative Examination

Fall 2004

This examination will be proctored by the Graduate Student Affairs Assistant, who has been instructed not to answer questions. There also will be no communication with faculty members; if you believe a question contains an error or an ambiguity, say so on your written examination, make an assumption to correct the alleged error or to resolve the ambiguity, and answer the question as well as you can.

Instructions

1. There are 7 pages (including the cover page) of the exam.
2. You have four hours to complete the exam. There are 240 points in total.
3. Read the instructions carefully.
4. Clearly label all the diagrams and underline the keywords in your answers.

Part I. Answer All Questions.

1. [10 points] The sum of a country's employment rate and unemployment rate cannot exceed 100%. True, false or uncertain? Explain your answers. Note: start your answers by precisely defining what are meant by the employment rate and the unemployment rate, respectively.

2. [10 points] Consider the class of linear rational expectations models of the form: $y_t = AE_t y_{t+1} + b$, where y_t is an n vector of endogenous variables, A is an invertible nxn matrix of coefficients and b is an n vector of constants. Suppose, in addition, that k of the variables y_0 are pinned down by initial conditions. Let $\bar{y}$ be the unique interior steady state of the above model.

(a) [5 points] What is meant by the statement that $\bar{y}$ is a hyperbolic steady state?

(b) [5 points] Assuming that that $\bar{y}$ is a hyperbolic steady state, under what condition is $\bar{y}$ a saddle point?

3. [10 points] Lucas Critique is not applicable in an "irregular" macroeconomic model. True, false or uncertain? Explain your answers. Note: start your answers by precisely defining what are meant by Lucas Critique and an irregular model, respectively.

4. [50 points] This question is about a representative-agent economy in which choices are made by a large number of identical households, each of which solves the following problem under perfect foresight:

$$(1)\qquad \text{Max} \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - A\frac{h_t^{1+\gamma}}{1+\gamma} \right],\ 0 < \rho < 1,\ \sigma > 0,\ \sigma \neq 1, A > 0 \text{ and } \gamma \geq 0,$$

where c is consumption, h is hours worked and β is the discount factor. The representative household also has access to the following production technology:

$$(2)\qquad y_t = (u_t k_t)^{\alpha} h_t^{1-\alpha},\ 0 < u_t < 1,\ 0 < \alpha < 1,$$

where y is output, k is capital, and u denotes the rate of capital utilization that is endogenously determined by the representative household. The budget constraint faced by the representative household is

$$(3)\qquad c_t + k_{t+1} - (1-\delta_t)k_t = y_t,\ 0 < \delta_t < 1,\ k_0 \text{ is given},$$

where y_t is given by (2), and δ_t represents the time-varying capital depreciation rate. It is postulated that more intensive capital utilization accelerates its rate of depreciation. In particular,

(4) $$\delta_t = \frac{1}{\theta} u_t^{\theta},\ \theta > 1.$$

(a) [10 points] Given the period utility function in equation (1), what is the value of (intertemporal) elasticity of substitution for consumption between any two points in time. Show your work.

(b) [10 points] Formulate the Lagrangian for the household's optimization problem, and derive the first-order condition that governs the representative household's choice for its supply of labor in period t. Explain the economic intuition.

(c) [10 points] Based on your answer to part (b), what is the economic interpretation of the preference parameter γ? Why is the formulation with $\gamma = 0$ an important special case in studying business cycle fluctuations? Explain your answers.

(d) [10 points] Using the Lagrangian as in part (b), derive the first-order condition that governs the representative household's *choice for the rate of capital utilization in period t*. Explain the economic intuition.

(e) [10 points] Using the Lagrangian as in part (b), derive the first-order condition that governs the representative household's choice *for consumption across periods t and t+1*. Explain the economic intuition.

Part II. Answer All Questions.

1. [15 points] In the simplest version of the Lucas' tree model, what can lead to an increase in the price of a tree (asset)?

2. [15 points] In the standard Cass-Koopman's model, one of the effects of a contractionary fiscal policy -- such as an increase in income tax, is a reduction in consumption. True, False, or Uncertain. Explain.

3. [50 points] Assume that the representative agent's preferences over the consumption good are given by:

(1) $$\text{Max} \sum_{t=0}^{\infty} \beta^t (\log c_t + \log m_t), \qquad 0 < \beta < 1,$$

The technology is described by the production function $y = f(k) = A + \theta \log k$, $0<\theta<1$, $A>0$, with depreciation rate $\delta=1$. Let $m_t=M_t/P_t$ be the real money balance, tr_t be the real transfers from the government, and b_t be the privately issued bonds in real terms that are sold for a discount price q_t. π_t is defined as p_t/p_{t-1} ($\pi_t = p_t/p_{t-1} = 1 + \tilde{\pi}_t$). The Central Bank allows the money supply to grow according to the law of motion:

(2) $$M_t = (1+g)\, M_{t-1}.$$

(a) [7 points] Write down Bellman's equation with the budget constraint expressed in real terms. Define the state and control variables.

(b) [7 points] Define a <u>recursive competitive equilibrium</u> for this economy. Make sure you describe completely the household's problem.

(c) [10 points] Derive the F.O.C., the Envelope equations, and the Euler equations in real and nominal terms.

(d) [7 points] Using your results from (c), derive an expression for real money balances held in period t (m_t) as a function of consumption and inflation. This is the money demand function. How is money demand related to g at steady state? And to β? Give an economic interpretation to this result.

(e) [7 points] Solve for the steady state capital stock, consumption. How do consumption and capital depend on the rate of money growth at steady state? Why? How do these steady state levels compare with the Pareto optimal steady state (that is, the standard Cass-Koopman's model without money)? (Hint: think in terms of a market equilibrium).

(f) [6 points] At steady state, what is the effect of an increase in g on: i. bond's price; ii. nominal and real interest rates; iii. marginal utility of money; iv. total utility $U(c_t, m_t)$; v. Return on bonds. <u>Explain.</u>

(g) [6 points] With a higher rate of inflation, would people hold more money, capital or bonds in equilibrium? Why?

Part III. Answer All Questions.

1. [10 points] Consider a Ramsey model in which the real interest rate is fixed exogenously (this may happen to a small open economy whose capital account is open to the outside world). If the real interest rate increases unexpectedly, then we would expect the savings rate of an individual to increase for sure. True or False? Answer with a Brief Explanation.

2. [10 points] Consider the following question in a Tobin-Hayashi neoclassical investment model: If the price of newly purchased capital good of a manufacturing firm is determined by the world oil price, then the unexpected explosion of a major oil pipeline in the Middle East will plunge the stock price of this firm immediately.

3. [10 points] The AK endogenous growth model implies that a country with a higher population growth rate grows faster.

4. [50 points] A representative household in a closed economy, say, Thailand, has an intertemporal utility function with constant instantaneous elasticity of substitution of 1, and a time discount rate of θ. The typical firms in the economy share the same neo-classical production function for a Solow good Y, which can be either consumed or invested for future production. Its price is normalized to 1. The population growth rate is n. The capital depreciation rate is zero.

(a) [5 points] Write down an intertemporal utility function of the household satisfying the above specification.

(b) [10 points] Write down the household's problem and then solve for the optimal per capita consumption path.

(c) [5 points] Write down the definition of a neoclassical production function.

(d) [5 points] What are the definitions of Hicks-neutral, Solow-neutral and Harrod-neutral technological progresses? Which technological progress is compatible with a balanced growth path of real variables in the Ramsey model (no explanation necessary)?

(e) [10 points] Assume the technological progress compatible with a balanced growth path describes the Thai firms' actual technical development, and the technological growth rate is exogenously given at x. Solve the firms' problem and then solve for the equilibrium path of consumption and capital per efficient capita.

(f) [5 points] Does the capital stock on the balanced growth path satisfies the golden rule?

(g) [10 points] Suppose Thailand now opens up its capital account to another country Indonesia which, like Thailand at this moment, is also on the balanced growth path. The two countries are otherwise exactly the same with the exception that:

(i) Thai technology level is lower than that of Indonesia.

(ii) Thai technological growth rate is higher than that of Indonesia.

For simplicity, we assume that the investors cannot bring his home country's technology with him when he invests in a foreign country. (This can be realistic when you treat "technology" as "human capital" embodied in the workers in different countries.)
Then capital will flow from which country to which country? Why?

University of California, Riverside
Department of Economics

Macroeconomic Theory
Cumulative Examination

Spring 2004

This examination will be proctored by the Graduate Student Affairs Assistant, who has been instructed not to answer questions. There also will be no communication with faculty members; if you believe a question contains an error or an ambiguity, say so on your written examination, make an assumption to correct the alleged error or to resolve the ambiguity, and answer the question as well as you can.

Instructions

1. There are six pages (including the cover page) of the exam.
2. You have four hours to complete the exam. There are 240 points in total.
3. Read the instructions carefully.
4. Clearly label all the diagrams and underline the keywords in your answers.

Part I. Answer All Questions.

1. [10 points] In 1996, the U.S. Commence Department switched from the base-year method to the chain weighted method in calculating real GDP growth. Explain how do these two (base-year and chain weighted) methods work, respectively?

2. [20 points] Consider an infinite-horizon representative-agent model in which the household's preferences are given by

(1) $$E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t), \; 0 < \beta < 1,$$

where E is the expectational operator, β is the discount factor and c_t is consumption. The representative firm's produces output y_t according to

(2) $$y_t = z_t k_t^{\theta}, \; 0 < \theta < 1, \; k_0 \text{ given},$$

where k_t is capital that depreciates at rate $\delta \in (0,1)$, and z_t is an aggregate technology shock that follows

(3) $$z_{t+1} = (1-\rho) + \rho z_t + \varepsilon_{t+1}, \; 0 < \rho < 1, \; z_0 \text{ given},$$

where ε_t is an i.i.d. random variable with zero mean and standard deviation σ_{ε}. Moreover, there are government expenditures on goods g_t, which are thrown away on defense, that satisfy a balanced budget requirement each period, that is, $g_t = \tau_t y_t$, where τ_t is the income tax rate. The law of motion for taxes is given by

(4) $$\tau_{t+1} = (1-\gamma)\bar{\tau} + \gamma \tau_t + u_{t+1}, \; 0 < \gamma < 1, \; \tau_0 \text{ given},$$

where $\bar{\tau}$ denotes the steady-state tax rate, and u_t is drawn from a zero-mean uniform distribution with bounded support such that $\tau_t \in (0,1)$.

(a) [10 points] Write down the aggregate resource constraint for the economy, set up the social planner's problem, and then derive the first-order condition that governs the intertemporal (Pareto efficient) allocations of capital. Explain the economic intuition.

(b) [10 points] Based on your answer to part (a), derive the analytical expression for the optimal steady-state capital $\bar{k}$. How do changes in the steady-state tax rate $\bar{\tau}$ affect the optimal steady-state capital $\bar{k}$? Explain your answers.

3. [50 points] Consider the following growth model with a unit measure of identical infinitely-lived households. Each agent is endowed with one unit of time that (s)he can allocate to producing consumption goods c_t or accumulating human capital h_t. The production function is given by

(1) $$c_t = \alpha h_t n_t,\ \alpha > 0,$$

where n_t is time devoted to production. Human capital is produced using the technology

(2) $$h_{t+1} = \delta h_t(1-n_t),\ \delta > 0, \text{ and } h_0 > 0 \text{ given.}$$

Preferences are given by

(3) $$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}-1}{1-\gamma},\ 0<\beta<1, \text{ and } \gamma > 1.$$

There is no physical capital in the economy. Finally, assume that $(\beta\delta)^{\frac{1}{\gamma}} > 1$, and $(\beta\delta^{1-\gamma})^{\frac{1}{\gamma}} < 1$.

(a) [5 points] Write down the social planner's problem.

(b) [10 points] Derive the first-order conditions that govern the optimal (Pareto efficient) allocations of (i) labor hours n_t, and (ii) human capital accumulation h_{t+1}.

(c) [5 points] Based on your answer to part (b), is it possible for the optimal n_t to be 0 or 1? Explain why or why not?

(d) [10 points] Based on your answer to part (b), derive the optimal growth rate of consumption.

(e) [10 points] Based on your answers to part (d) and equation (1), derive the first-order non-linear difference equation that characterizes the optimal law of motion for labor hours n_t.

(f) [5 points] Based on your answer to part (e), derive the analytical expression for the optimal steady-state labor hours $\bar{n}$.

(g) [5 points] There is empirical evidence that international income levels do not converge. Can this model be used to address this fact? Explain your answers.

Part II. Answer All Questions.

1. [15 points] At the balanced growth path of a one-sector growth model with a representative consumer: changing the intertemporal elasticity of substitution in consumption will change the interest rate in a growing economy, but changing this elasticity will not affect the interest rate if the growth rate is zero along the balanced growth path. True, false or uncertain? Explain your answers.

2. [15 points] Use the framework of a Cash-in-Advance model to explain the role of the nominal interest rate as the opportunity cost of holding money.

3. [50 points] Consider the following variant of the standard Solow-Cass-Koopmans model:

   Preferences: $\sum_{t=0}^{\infty} \beta^t [U(c_t) - V(h_t)], \quad 0 < \beta < 1,$

   Technology: $k_{t+1} = f(k_t, h_t) - c_t,$

   where c is consumption, h is labor hours, k is capital stock, and f is a concave production function with constant returns-to-scale. In addition, U and -V are strictly concave and twice differentiable.

(a) [10 points] Set up the representative agent's maximization problem as a dynamic programming problem. That is, write down Bellman's equation and define control and state variables.

(b) [10 points] Derive the F.O.C. and the envelope equation. Find the Euler equation that relates time t+1 marginal utility to time t marginal utility. Explain in words what the Euler equation means.

(c) [10 points] Discuss the determination of the steady-state levels of his choice variables. Does the steady state depend on the agent's initial capital stock? Explain your answers.

(d) [10 points] Assume that the representative agent faces a proportional consumption tax which is rebated to him as lump-sum transfers. What is the correct form of his transition equation for capital now? Set up the agent's maximization problem, derive his F.O.C.s and Euler equation, and compare the steady-state levels to those derived in part (c).

(e) [10 points] How would your answer in part (d) be changed if the agent's level of labor effort were constrained to be the same as in part (c) in the steady state? Explain your answers.

Part III. Answer All Questions.

1. [10 points] Consider a standard neoclassical Q-theory model in which the firm is a monopolist whose capital level has reached the steady state at this moment. If there is a demand shock that raises the price elasticity of demand of the good produced by the firm, then the firm should expand its production. True, false or uncertain? Explain your answers.

2. [10 points] The user's cost of capital of a firm will increase if the best alternative portfolio for the investors produces a higher certain rate of return. True, false or uncertain? Explain your answers.

3. [10 points] Explain briefly why a pay-as-you-go social security system can be welfare improving?

4. [50 points] Consider an economy with the representative household maximizing the following utility function:

$$(1)\qquad \int_0^\infty u(c_t)e^{-\theta t}\,dt,$$

where $u(c_t)$ is the familiar constant relative risk aversion utility function with the instantaneous elasticity of substitution to be 1. The firms in this economy share the same Cobb-Douglas production function for a Solow good Y as in

$$(2)\qquad Y_t = K_t^{\alpha}(G_t L_t)^{1-\alpha},\ 0 < \alpha < 1,$$

where L_t and K_t are the labor and capital inputs at time t, and G_t is the level of public good provided by the government. For simplicity, we assume that

$$(3)\qquad G_t = I_t,$$

in which I_t is the investment by the government at time t. The government runs a balanced budget at all times whereby it taxes each household an amount of $T_t > 0$ and uses the tax revenue to make public goods. The household number in this economy is normalized to 1. The price of the Solow good is normalized to 1. The population per household is constant at 1. The market structure for this economy is perfectly competitive everywhere. For simplicity, we assume that capital stock for the private firms do not depreciate. The household takes the government tax at any time to be exogenously given.

(a) [5 points] Write out the exact functional form for $u(c_t)$

(b) [5 points] Write out the dynamic budget constraint for the household.

(c) [10 points] Set up the current value Hamiltonian function for the households, and then derive the FOCs.

(d) [10 points] Given G_t at any time, solve for the firm's problem, and derive the equilibrium dynamic trajectories for per capita consumption and capital.

(e) [5 points] Suppose the government tax is constant at $\overline{T}$, would long-run growth be possible? Give a brief reason with less than 3 sentences.

(f) [5 points] If the government tax away $\tau \in (0,1)$ fraction of each family's income at each moment, solve for the equilibrium aggregate output level as a function of capital and labor only.

(g) [10 points] Continuing with part (f), would long run growth be possible? If long-run growth is impossible, solve for the steady-state levels of capital and consumption. If long-run positive growth is possible, solve for the growth rates of per capita output and consumption along a balance growth path. Note: in answering this question, assume that $\frac{(1-\alpha)^2}{\alpha}(\ln\alpha + \ln(1-\tau)) > \ln\theta$.