FALL 2014 ECON200A
MICROECONOMIC THEORY
FINAL

Problem 1. Consider a world with two consumption goods. A consumer's utility function is $u(x_1, x_2) = x_1^2 x_2^3$, where x_1 and x_2 are consumption of good 1 and good 2, respectively. Let $p = (p_1, p_2) \in \mathbb{R}^2_{++}$ be the prices of commodities, and $w > 0$ an income of the consumer.

(1) Compute the demand function $x(p_1, p_2, w)$. (6pts)

(2) Compute the indirect utility function $v(p_1, p_2, w)$. (6pts)

(3) Verify the Roy's identity. (6pts)

Problem 2. Consider a world with three commodities. A firm produces the commodity z_3 using the commodities (z_1, z_2). Its production function is $f(z_1, z_2) = \sqrt{z_1} + \sqrt{z_2}$. Let $p = (p_1, p_2, p_3)$ be the prices of commodities.

(1) Compute the supply function $z(p)$. (6pts)

(2) Compute the profit function $\pi(p)$. (6pts)

(3) Compute the cost function $C(p_1, p_2, y)$. (6pts)

(4) Verify the Hotelling's lemma. (6pts)

(5) Verify the Shepherd's lemma. (6pts)

Problem 3. Consider monetary lotteries prizes of which are always weakly greater than \$1. (So, $Z = [1, \infty)$.) Suppose that person 1 and person 2 have preference relations $\succsim_1$ and $\succsim_2$ over such monetary lotteries that are represented by expected utilities. Person 1's vNM utility function is $u_1(z) = z^\alpha$ with $\alpha \in (0, 1)$, and person 2's vNM utility function is $u_2(z) = \frac{z}{1+z}$. Answer the distinct values of i and j in order to make the following statement true (5pts). Prove the statement (8pts).

$$\text{For any } p \in L(Z) \text{ and } z \in Z,\ p \succsim_i \delta_z \text{ implies } p \succsim_j \delta_z.$$

Problem 4. Consider a market of a certain good that consists of price-taking buyers and a monopoly seller. The inverse demand function of the good is $p(q) = 5 - 2q$, and the cost function is $C(q) = \frac{1}{3}q^3$.

(1) Compute the monopoly price and quantity. (7pts)

(2) Compute the welfare loss due to the monopoly. (8pts)

Problem 5. For any two monetary lotteries $\mathbf{p}$ and $\mathbf{q}$ with finite supports, denote by $\mathbf{p}+\mathbf{q}$ a lottery such that $(\mathbf{p}+\mathbf{q})(z) = \sum_{z'} p(z')q(z-z')$ for every z. For example, if $\mathbf{p} = .6\delta_0 \oplus .4\delta_1$ and $\mathbf{q} = \delta_1$, then

$$(\mathbf{p}+\mathbf{q})(z) = \begin{cases} .6 & \text{if } z = 1, \\ .4 & \text{if } z = 2, \\ 0 & \text{otherwise.} \end{cases} \qquad (*)$$

(1) For each of the following three cases, give the description of a lottery $\mathbf{p}+\mathbf{q}$ in the similar form as $(*)$.

(i) $\mathbf{p} = \delta_2$ and $\mathbf{q} = \delta_3$. (4pts)
(ii) $\mathbf{p} = .6\delta_0 \oplus .4\delta_1$ and $\mathbf{q} = .5\delta_1 \oplus .5\delta_2$. (4pts)
(iii) $\mathbf{p} = .4\delta_{-1} \oplus .2\delta_0 \oplus .4\delta_1$ and $\mathbf{q} = .5\delta_1 \oplus .5\delta_2$. (4pts)

(2) Consider the following two statements. One of the statements is true, while the other is false. Identify the true statement (6pts, no proof). Give a counterexample to the false statement (6pts).

(i) If $\succcurlyeq$ satisfies Independence axiom, then $\mathbf{p} \succcurlyeq \mathbf{q}$ implies $\mathbf{p}+\mathbf{r} \succcurlyeq \mathbf{q}+\mathbf{r}$ for any three lotteries $\mathbf{p}, \mathbf{q}, \mathbf{r}$.
(ii) For any four lotteries $\mathbf{p}, \mathbf{q}, \mathbf{p}', \mathbf{q}'$, if $\mathbf{p}\,\mathrm{FSD}\,\mathbf{p}'$ and $\mathbf{q}\,\mathrm{FSD}\,\mathbf{q}'$, then $\mathbf{p}+\mathbf{q}\,\mathrm{FSD}\mathbf{p}'+\mathbf{q}'$.