University of California, Riverside

Microeconomics Qualifying Exam

September 9, 2016

INSTRUCTIONS

- *Put your identification number on the front of your packet of solutions and each subsequent page (using the first box in the upper right-hand corner).*
- *Write solutions on only one side of each page of paper and do not put answers for more than one question on a page.*
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- *Answer all questions.*

Part I

Problem 1. Let u be a real-valued function on a set X and $\epsilon > 0$. Define a preference relation $\succcurlyeq$ on X by $x \succcurlyeq y$ iff $u(x) \geq u(y) - \epsilon$ for any x and y in X. Let $\mathcal{A}$ be the collection of all finite subsets of X, and c be a choice correspondence on $\mathcal{A}$ such that

$$c(S) = \{x \in S : x \succcurlyeq y \text{ for all } y \in S\}$$

for every S in $\mathcal{A}$. Prove or falsify each of the following statements.

(1) $\succ$ is transitive.

(2) If $x \succ y \succcurlyeq z$, then $x \succ z$.

(3) (Axiom α) If $S, T \in \mathcal{A}$ and $T \subseteq S$, then $c(S) \cap T \subseteq c(T)$.

(4) (Axiom β) If $S, T \in \mathcal{A}$, $T \subseteq S$, and $c(S) \cap T \neq \emptyset$, then $c(T) \subseteq c(S)$.

Problem 2. Suppose that there are three goods in the market, and there is a competitive firm that has a technology Z such that

$$Z = \{(z_1, z_2, z_3) \in \mathbb{R}^3 : z_1 \leq 0,\ z_2 \leq 0,\ z_3 \leq (z_1 z_2)^{1/3}\}.$$

Denote the market price of good i by $p_i > 0$ for $i = 1, 2, 3$.

(1) Compute the firm's supply function $z(p)$.

(2) Compute the firm's profit function $\pi(p)$.

(3) Verify the Hotelling's lemma for this firm.

Problem 3. We sometimes observe people playing gambling games (risk-seeking behavior) while, at the same time, they purchase life/auto/home insurances (risk-averse behavior). To derive such choice patterns, consider a rational economic agent, named John, who maximizes his expected utility when he faces a choice among lotteries. His vNM utility function u is given by

$$u(x) = \begin{cases} \frac{x^2}{225} & \text{if } x \leq 900, \\ 120\sqrt{x} & \text{if } x > 900. \end{cases}$$

(1) Draw a graph of the function u.

(2) At a casino, John can join a certain gambling game by paying s dollars. Once he joins the game, he will win \$200 dollars with probability 9/20 and nothing with probability 11/20. Assuming that the casino is risk neutral, determine a condition on s under which John is willing to join the game and the casino is willing to offer it.

Hint: John compares $\frac{9}{20}\delta_{200} \oplus \frac{11}{20}\delta_0$ with δ_s.

(3) John drives a car that has a market value of \$10,000. During a certain period of time, an accident happens with 2% chance. If it happens, his car loses all the market value. Otherwise, it keeps the original market value. There is an insurance company that offers an auto insurance at a price of t dollars. The insurance will pay the original market value of the car (i.e. \$10,000) to John in the case of an accident. Assuming that the insurance company is risk neutral, determine a condition on t under which John is willing to purchase the insurance and the insurance company is willing to offer it.

Hint: John compares $\frac{98}{100}\delta_{10000} \oplus \frac{2}{100}\delta_0$ with $\delta_{10000-t}$.

Part II

1. Jill Harmony has died and left behind \$1 million. The money will either go to Médecins Sans Frontières (MSF) or be divided among Ms Harmony's 4 daughters. The daughters—Andi, Brandi, Candi, and Dani—will get the money if they agree upon a division. But Ms Harmony, who throughout her life promoted tranquility and consensus, has left rather unusual instructions, which are as follows.

    > Daughters make proposals in alphabetical order, each making at most one proposal. A proposal has the form (a, b, c, d), where a is the amount for Andi, b the amount for Brandi, c the amount for Candi, and d the amount for Dani. All amounts must be nonnegative and satisfy $a+b+c+d \leq \$1$ million. If a daughter's proposal is not accepted by all of the daughters remaining still eligible to share in the inheritance, then that daughter is eliminated from the group eligible to split the inheritance. If a proposal is agreed to by all of the remaining daughters, then the game stops and the inheritance is divided as agreed. If all daughters' proposals are rejected, then MSF gets the inheritance.

    The daughters have a common discount factor of $\delta \in (0,1)$ between proposal periods. A daughter's payoff is simply the present value of dollars she will receive. All the above is common knowledge.

    (a) Derive the subgame perfect equilibrium outcome to this game.

    (b) At the subgame perfect equilibrium outcome, which daughter receives the largest share of the inheritance? Explain.

2. Row and Column play a game in there is an uncertain state of the world, θ. The state is $\theta \in \{\theta_1, \theta_2, \theta_3\}$, and the states are equally likely. In each state, Row's strategy set is $\{A, B\}$ and Column's is $\{a, b\}$. Row learns the true state of the world before choosing her action; Column does not know the state but does know its distribution.

    Payoffs corresponding to each state and action combination are given as follows.

State θ_1:	a	b
A	0, 4	1, 3
B	3, 0	2, 3

State θ_2:	a	b
A	5, 0	0, 3
B	2, 5	1, 0

State θ_3:	a	b
A	2, 1	4, 0
B	1, 2	0, 5

    (a) Suppose players choose their actions simultaneously. Determine a Bayesian equilibrium to this game.

    (b) Suppose Row chooses her action first, knowing this choice will be observed by Column before Column chooses his action. And this sequencing of decisions is common knowledge.

    Determine a perfect Bayesian equilibrium to this game. Describe the solution *completely* (i.e., specify beliefs and actions on and off the equilibrium path).

Part III

1. Consider an Edgeworth box economy with utility functions $u_A(x_1, x_2) = \min(3x_1, x_2)$ and $u_B(x_1, x_2) = \min(x_1, 3x_2)$

   (a) Find the competitive equilibrium when $e_A = (4, 0)$ and $e_B = (0, 4)$ and the price of good 1 is equal to 1. Calculate the utility of person A at the equilibrium.

   (b) Find the competitive equilibrium when $e_A = (6, 0)$ and $e_B = (0, 4)$ and the price of good 1 is equal to 1. Calculate the utility of person A at the equilibrium.

   (c) Compare the utility of person A at the equilibrium found in part (a) to the one found in part (b). Is there something paradoxical about this result? Can you provide an intuitive explanation for how person A's utility changes between (a) and (b)?

2. For several months not a drop of rain has fallen in the city of Oz, and there is a fullblown water crisis. The Mayor is looking for drastic and imaginative solutions. One of her aides tells her about an wizard from India who is able to make it rain. Naturally the Mayor is unconvinced that the wizard is for real, and she worries that he will turn out to be fake. So she wants to offer the wizard a contract that he will accept only if he is real.

   Everyone knows that a phony wizard has no power over the rain. If the Mayor contracted a fake, the probability of rain remains unchanged, which is currently estimated at 2/100. On the other hand, an authentic wizard, in spite of having powers, is not omnipotent, but can only raise the probability to 20/100. Both type of wizards are risk averse, their utility function being of the form $u(w) = \sqrt{w}$. No authentic wizard will work unless the utility received from the contract is at least $\underline{U} = 10$. Fakes, on the other hand, are willing to charge less, since their reservation utility is $\underline{U} = 1$. The Mayor, who is risk neutral, is interested in desigining a contract that would only be accpted by an authentic wizard, because if the wizard turns out to be fake, she would definitely lose the next election.

   (a) Formulate the problem that the Mayor of Oz must solve, when the authencity of the wizard is not known to the Mayor, and the only thing she can observe and verify is whether or not it rains afterwards. Keep in mind that she only wants to hire an authentic wizard.

   (b) Calculate the optimal contract.

   (c) Calculate and compare the cost of this contract to the one that would be offered under symmetric information.

3. Consider an agency relationship in which the principal contracts the agent, whose effort determines the monetary result. Assume that the uncertainty is represented by three states of nature, each with probability 1/3. The agent can choose between two effort (e) levels. The results are shown in the following table:

Table 1: Results of Efforts under Different States

	State 1	State 2	State 3
e=6	60,000	60,000	30,000
e=4	30,000	60,000	30,000

The objective functions of the principal and the agent are, respectively, $B(x, w) = x - w$ and $U(w, e) = \sqrt{w} - e^2$, where x is the monetary result, and w is the wage paid to the agent.

(a) What can you say about the risk attitudes of the two participants looking at their objective functions?

(b) What would be the effort and wage under symmetric information?

(c) What would be the effort and wage under asymmetric information? Discuss the results.

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Microeconomics Qualifying Exam

July 8, 2016

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Part I

Problem 1. Let X be a nonempty finite set of alternatives, $\succcurlyeq$ be a complete and transitive preference relation on X, and c be a choice correspondence on the set of all nonempty subsets of X such that

$$c(S) = \{x \in S : y \succcurlyeq x \text{ for all } y \in S\} \tag{$*$}$$

for every nonempty subset S of X.

(a) The following items (1)–(3) list possible interpretations for choice models. Choose one that best describes the choice correspondence c above.

(1) c chooses anything but the worst alternatives.
(2) c chooses the first-best and the second-best alternatives.
(3) c chooses the worst alternatives.

(b) Prove or falsify each of the following two properties.

(1) (Axiom α) If $T \subseteq S$, then $c(S) \cap T \subseteq c(T)$.
(2) (Axiom β) If $T \subseteq S$ and $c(S) \cap T \neq \emptyset$, then $c(T) \subseteq c(S)$.

(c) Let us consider another choice model instead of ($*$). That is, suppose that c is a choice correspondence such that, for any nonempty $S \subseteq X$,

$$c(S) = \{x \in S : x \succ y \text{ for some } y \in S\} \tag{$**$}$$

if the right hand side of ($**$) is a nonempty set, and $c(S) = S$ otherwise. For this choice correspondence c, answer the same questions (a) and (b) above.

Problem 2. Suppose that a consumer has a preference relation on $X = \mathbb{R}^2$ represented by a utility function $u(x) = \sqrt{x_1} + \sqrt{x_2}$.

(1) Compute the consumer's demand function $x(p, w)$.
(2) Compute the indirect utility function $v(p, w)$.
(3) Compute the value of $\partial v(p, w)/\partial w$ when $p_1 = p_2 = 1$ and $w = 4$.
(4) Compute the value of $\partial v(p, w)/\partial p_1$ when $p_1 = p_2 = 1$ and $w = 4$.

Problem 3. Suppose that there are three goods in the market and a firm's technology is given by

$$Z = \{(z_1, z_2, z_3) \in \mathbb{R}^3 : z_1 \leq \sqrt{-z_3},\ z_2 \leq \ln(1 - z_3)\,,\ z_3 \leq 0\}.$$

Denote the market price of good i by $p_i > 0$ for $i = 1, 2, 3$.

(1) Assuming that the market is perfectly competitive, compute the profit maximizing production vector (z_1^*, z_2^*, z_3^*) when $(p_1, p_2, p_3) = (12, 20, 4)$.

Hint: z_1^* is a natural number.

(2) Instead, suppose that the firm is a monopolist in the market of good 1 and the inverse market demand function of good 1 is given by $p_1 = 4 - z_1$. Continue to assume that the markets of good 2 and good 3 are perfectly competitive, and let $p_2 = 20$ and $p_3 = 4$. Compute the new profit maximizing production vector (z_1^*, z_2^*, z_3^*).

Hint: z_1^* is a natural number.

Part II

1. Consider the following all-pay auction involving players 1 and 2. Player 1's value for the prize is $v_1 = 1$ and player 2's is $v_2 = 2$. Effort costs are $c(x) = \frac{1}{2}x^2$ for effort x, the same for both players. Final payoffs to player i exerting effort x are $-c(x)$ if he loses and $v_i - c(x)$ if he wins, $i = 1, 2$. Players choose efforts simultaneously. Player i wins the auction if his effort is strictly larger than the other player's, $i = 1, 2$; if players' efforts are equal, then each player has a $1/2$ chance of being designated the winner. The foregoing is common knowledge.

   Determine a Nash equilibrium to this game,
   and calculate the probability player 2 wins the auction.

2. Consider the infinitely-repeated Prisoner's Dilemma game between Row and Col depicted below.

	A	B
a	$2, 2$	$0, 5$
b	$5, 0$	$1, 1$

   In the stage game, Row's pure strategies are a and b while Col's are A and B. In each period, the first number in a cell is Row's payoff, the second is Col's. Players evaluate overall payoffs as the present discounted value of the sum of their stage-game payoffs, where players discount future payoffs by the common per-period discount factor $\delta \in (0, 1)$. The foregoing is common knowledge.

   (a) For what values of δ can (a, A) be sustained in every period as a subgame perfect Nash equilibrium outcome (relying on a grim trigger strategy)? Explain.

   (b) Consider the alternating strategy according to which in odd-numbered periods (i.e., 1st, 3rd, 5th, ...) players play (b, A) and in even-numbered periods players play (a, B). For what values of δ can the described play be realized as a subgame perfect Nash equilibrium outcome (relying on a grim-trigger-strategy type "punishment")? Explain.

   (c) For the different possible values of $\delta \in (0, 1)$, what seems like reasonable behavior by the players, in light of your findings in the first two parts of this problem? (Hint: compare players' payoffs under the two scenarios.)

3. In the following scenario, players 1 and 2 contribute to a public good. Each player has a private type that determines his strength of taste for the public good. Player i's parameter is θ_i and his final payoff is

$$2\theta_i G - G^2 - g_i,$$

   where g_1 and g_2 are the contributions, respectively, of players 1 and 2 to the public good, $i = 1, 2$, and $G \equiv g_1 + g_2$. Of course, we require $g_1 \geq 0$ and $g_2 \geq 0$. Players make their contributions simultaneously.

   Assume θ_1 and θ_2 are iid random variables, uniformly distributed over $[0, 1]$.

   The foregoing is common knowledge. Derive a symmetric Bayesian equilibrium to this game.

Part III

1. Robinson and Friday are the only two inhabitants on a tropical island, earning their keep by trading coconuts and bananas between themselves. They both have the same endowment of bananas and coconuts:

$$x_{bf} = x_{br} = 1$$

$$x_{cf} = x_{cr} = c \geq 1$$

but their preferences are different. Friday likes bananas more than coconuts, and Robinson likes coconuts more than bananas (i.e. you can assume that $\alpha \in (0.5, 1)$ in what follows). The utility functions are given by:

$$u_f(x_f) = \alpha \ln(x_{bf}) + (1 - \alpha)\ln(x_{cf}),$$

$$u_r(x_r) = (1 - \alpha)\ln(x_{br}) + \alpha \ln(x_{cr})$$

(a) Normalize the price of bananas to be equal to one, or $p_b = 1$. Compute the demand functions of Robinson and Friday as a function of the price for coconuts p_c.

(b) Find the equilibrium prices in this island economy and determine the net trading quantities.

(c) Consider the island economy again but this time Friday and Robinson have agreed to share their resources and they have also agreed that the weight that Friday receives in the economy is w_f and the weight that Robinson receives is $w_r = 1 - w_f$.

i. For every weight $w_f \in (0, 1)$, find the allocation which would maximize the social surplus given the weights; in other words, we are interested in finding the allocation $(x_{bf}, x_{br}, x_{cf}, x_{cr})$ which maximizes the sum

$$w_f u_f(x_f) + (1 - w_f)u_r(x_r)$$

subject to the resource constraints of the economy.

ii. For every weight $w_f \in (0, 1)$, can you find an initial endowment of bananas and coconuts among Robinson and Friday and a set of prices that the Pareto efficient allocation actually constitutes an equilibrium of the market? (Here we decentralize the Pareto efficient allocation via a market equilibrium.) It is sufficient to discuss the case of $c = 1$.

2. Consider a relationship between a principal and an agent in which the the agent's effort influences the result but is not observable. The principal is risk neutral, and the agent is risk averse, having a utility function $u(w, e) = \sqrt{w} - e^2$, where w represents the wage and e represents the effort. The agent can choose between low effort $e = 0$, or high effort $e = 3$. His reservation utility is 21. The production technology is such that only three results x are possible, where x represents the value of the result to the principal, $x \in \{0, 1000, 2500\}$. The probabilities conditional on effort are : $\text{Prob}(x = 0 \mid e = 0) = 0.4$; $\text{Prob}(x = 1000 \mid e = 0) = 0.4$; $\text{Prob}(x = 2500 \mid e = 0) = 0.2$; $\text{Prob}(x = 0 \mid e = 3) = 0.2$; $\text{Prob}(x = 1000 \mid e = 3) = 0.4$ and $\text{Prob}(x = 2500 \mid e = 3) = 0.4$.

(a) What are the optimal symmetric information contracts for each effort? Which effort will the principal demand from the agent?

(b) What is the optimal contract under which agent will exert an effort $e = 0$ if the only verifiable variable in the relationship is result x?

(c) Given asymmetric information, characterize the optimal contract if the principal wants the agent to exert an effort of $e = 3$.

(d) Which contract will the principal offer the agent under asymmetric information?

(e) Discuss the optimal contract under asymmetric information if the agent were risk neutral.

3. Mr. Jones decides to go to an insurance company in order to insure his car against accidents. From the insurance company's point of view, Mr. Jones could be either a safe driver or a reckless driver. The probability that a safe driver will suffer an accident is $p_s = 1/3$, while the probability that a reckless driver will suffer an accident is $p_r = 1/2$. Let $t \in (0, 1)$ denote the probability that Jones is a safe driver. Assume that there are many insurance companies in the market, and they are all risk neutral. Jones's utility function is $\ln(x)$, where x represents his net wealth. Initially his wealth is $W = 64$, and the accident implies a cost of $C = 63$. All insurance companies offer contracts that include a premium ρ, and a coverage amount q if an accident occurs. Jones will choose the contract (ρ, q) that he most prefers.

   (a) Write Mr. Jones's expected utility function when he signs a contract (ρ, q), contingent on him being a safe or a reckless driver. Write down the participation constraints for each case.

   (b) Write down the profits of an insurance company that insures Mr. Jones under a contract (ρ, q) knowing that he is a safe driver, knowing that he is a reckless driver, and without knowing what type of a driver he is.

   (c) Derive the contracts that will exist in this market, if the information is symmetric, given that there are many insurance companies. Will Jones be fully insured?

   (d) Would the same contracts appear in the market if the insurance companies cannot observe whether Jones is safe or reckless? Why or why not?

   (e) Calculate a set of contracts that could comprise a separating equilibrium in this market under asymmetric information.

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September 11, 2015

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Part I

1. Consider the following two propositions. One of them is true while the other is false. Identify the true proposition, and give a proof. Identify the false proposition, and give a counterexample.

   P1. Let $\succcurlyeq$ be a complete, transitive, and continuous preference relation on $\mathbb{R}^n$, and S be a nonempty compact subset of $\mathbb{R}^n$. Then, there exists a "second-best" alternative in S unless the person is indifferent across all alternatives in S, that is,

   $$\mathrm{MAX}(S, \succcurlyeq) \neq S \quad \text{implies} \quad \mathrm{MAX}(S \setminus \mathrm{MAX}(S, \succcurlyeq), \succcurlyeq) \neq \emptyset.$$

   P2. Let $\succcurlyeq_a$ and $\succcurlyeq_b$ be complete, transitive, and continuous binary relations on $\mathbb{R}^n$, and $\succcurlyeq^L$ be such that

   $$x \succcurlyeq^L y \quad \text{if and only if} \quad x \succ_a y \text{ or } (x \sim_a y \text{ and } x \succcurlyeq_b y)$$

   for every x and y in $\mathbb{R}^n$. (So, $\succcurlyeq^L$ is the lexicographic preference relation induced by $\succcurlyeq_a$ and $\succcurlyeq_b$.) Then, for any nonempty compact subset S of $\mathbb{R}^n$, $\mathrm{MAX}(S, \succcurlyeq^L) \neq \emptyset$.

2. Let $L(\mathbb{R})$ be the set of monetary lotteries with finite support. Suppose that $\succcurlyeq_1$ and $\succcurlyeq_2$ are preference relations on $L(\mathbb{R})$ admitting the expected utility representation under vNM utility functions u_1 and u_2, respectively. Let $\mathbf{p} \in L(\mathbb{R})$ be any lottery, $z, z' \in \mathbb{R}$ be any prizes, and α be any number in $[0, 1]$. Answer questions below regarding the next statement.

   $$\mathbf{p} \succcurlyeq_1 \delta_z \text{ and } \mathbf{p} \succcurlyeq_1 \delta_{z'} \quad \Longrightarrow \quad \mathbf{p} \succcurlyeq_2 \alpha\delta_z \oplus (1-\alpha)\delta_{z'} \qquad (*)$$

   (1) Is the statement $(*)$ true or false when $u_1(x) = \ln x$ and $u_2(x) = x^{1/5}$? If it is true, give a proof. If it is false, give a counterexample.

   (2) Is the statement $(*)$ true or false when $u_1(x) = x^{1/2}$ and $u_2(x) = x^{2/3}$? If it is true, give a proof. If it is false, give a counterexample.

3. In a certain industry, there is a monopolistic firm producing a good under a cost function $c(q) = \frac{1}{3}q^3$. The inverse demand function for the good is given by $p = 2 - q$. Compute the size of welfare loss in this market due to monopoly.

Part II

1. Two agents jointly own a firm and each has an equal share. The value of the whole firm to agent i is v_i, $i = 1, 2$, and $v_1 > v_2 > 0$. Suppose now the agents must dissolve the firm.

   The parties agree to the following binding arbitration rule. Both parties independently and simultaneously submit bids to the arbitrator, b_1 for player 1 and b_2 for player 2.

   - If $b_1 \geq b_2$ then player 1 gets the firm and pays b_2 to player 2. Agent 1's payoff is $v_1 - b_2$ and agent 2's is b_2.
   - If $b_2 > b_1$ then player 2 gets the firm and pays b_1 to player 1. Agent 1's payoff is b_1 and agent 2's is $v_2 - b_1$.

   (Noté the tie-breaking rule implicit in the above description.)
   The foregoing is common knowledge.

   Calculate the set of of pure strategy equilibria.

2. Players A and B engage in a standard first-price all-pay auction. If players A and B exert efforts a and b, respectively, then player A wins with probability 1 if $a > b$ and with probability 0 if $a < b$; if $a = b$, then the player with the higher value is designated the winner (in the case of ties where players have equal values, each has a 50% chance of being designated the winner—this tie breaking rule could be implemented with a second-price auction following a tie in the all-pay auction).

   Players' values for the good are $v_i \in \{0, 1\}$, $i = A, B$. Probabilities for the values are $\Pr(v_A = 1) = p$ and $\Pr(v_B = 1) = q$, where $1 > p \geq q > 0$. Values are independently distributed.

   The final payoff to the winner is (gross value) $-$ (effort), while the final payoff to the loser is $-$(effort).

   The foregoing is common knowledge.

   Calculate a Bayesian equilibrium to this game.

3. Romeo and Juliet play the following infinitely-repeated game. In each period each player chooses an effort level, which benefits both of them. Let x_R be Romeo's effort and x_J, Juliet's. Given these efforts, Romeo's payoff is $x_R + x_J - \frac{1}{2}(x_R)^2$ and Juliet's is $x_R + x_J - \frac{1}{2}(x_J)^2$.

   Romeo and Juliet each value their own payoff streams by the present value of the sum of the current and future payoffs, using the discount factor $\delta \in (0, 1)$ per period.

   (a) Calculate the effort levels for Romeo and Juliet their maximize the *sum* of their payoffs. Call these the efficient effort levels, (x_R^e, x_J^e).

   (b) For what range of discount factors can a Grim Trigger Strategy in the repeated game support the choice of (x_R^e, x_J^e) in every period as a subgame perfect equilibrium outcome? Explain.

   (c) Can a stick-and-carrot punishment strategy be used to increase the range of discount factors for which, in the repeated game, the choice of (x_R^e, x_J^e) in every period is the subgame perfect equilibrium outcome?

   Either construct such a strategy and determine this larger range of discount factors *or* explain why it is not possible in this example.

Part III

1. The Arrow and Debreu families live next door to one another. Each family has an orange grove that yields 30 oranges per week, and the Arrows also have an apple orchard that yields 30 apples per week. The two households' preferences for oranges (x per week) and apples (y per week) are given by the utility functions

$$u_A(x_A, y_A) = x_A y_A^3$$

and

$$u_D(x_D, y_D) = 2x_D + y_D$$

The Arrows and Debreus realize they may be able to make both households better off by trading apples for oranges. Determine all Walrasian equilibrium price lists and allocations.

2. Consider the following asymmetric information problem in the financial market. The principal is a lender who provides a loan of size k to a borrower. Capital costs Rk to the lender since it could be invested elsewhere in the economy to earn the risk-free interest rate R. The lender has thus a utility function $V = t - Rk$. The borrower makes a profit $U = \theta f(k) - t$ where $f(k)$ is the production with k units of capital and t is the borrower's repayment to the lender. We assume that $f' > 0$ and $f'' < 0$. The parameter θ is a productivity shock drawn from $\Theta = \{\underline{\theta}, \overline{\theta}\}$ with respective probabilities $1 - \alpha$ and α.

   (a) Write down incentive and participation constraints.

   (b) Write down and solve for the optimal contract assuming the lender perfectly observes the productivity of the borrower.

   (c) Write down the principal's maximization problem under asymmetric information.

   (d) Which constraints are binding at the optimum? Explain. Is there a capital distortion with respect to the first-best outcome?

3. Consider the regular moral hazard model with a risk-neutral principal and a risk averse agent. The agent can choose between two effort levels, $a_i \in \{\underline{a}, \overline{a}\}$ with associated cost $c_i \in \{0, c\}$ with $c > 0$. Each action generates stochastically one of two possible profit levels, $x_i \in \{\underline{x}, \overline{x}\}$ with probabilities, $p(\overline{x}|\underline{a}) > p(\underline{x}|\overline{a})$. The utility function of the agent is $u(w, c_i) = \ln w - c_i$. The value of the outside option is normalized to 0. Risk-neutrality of the principal implies that his payoff function is $x - w$.

   (a) Carefully describe the principal-agent problem when the principal wishes to implement the high effort level $\overline{a}$.

   (b) Solve for the optimal wage schedule to be offered to the agent which implements the high effort level $\overline{a}$.

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- *After compiling your solutions in order, number your pages sequentially in the second box in the upper right-hand corner of each page.*
- *Answer all questions.*

Part I

1. Let X be a nonempty set of alternatives, $\succcurlyeq$ be a reflexive and transitive (but not necessarily complete) binary relation on X, and c be a choice correspondence on the set of all nonempty subsets of X such that
$$c(S) = \{x \in S : y \succ x \text{ for no } y \in S\}$$
for every nonempty subset S of X. This choice correspondence c satisfies four properties out of the following six properties. Identify these four properties, and give a proof to each of them. For the following properties, let S and T be arbitrary nonempty subsets of X, and x, y, z be any alternatives in X.

   (a) (Axiom α) If $T \subseteq S$, then $c(S) \cap T \subseteq c(T)$.
   (b) (Axiom β) If $T \subseteq S$ and $c(S) \cap T \neq \emptyset$, then $c(T) \subseteq c(S)$.
   (c) If $x \in c(\{x, y\})$ and $y \in c(\{y, z\})$, then $x \in c(\{x, z\})$.
   (d) If $x \notin c(\{x, y\})$ and $y \notin c(\{y, z\})$, then $x \notin c(\{x, z\})$.
   (e) $c(S) \cap c(T) \subseteq c(S \cup T)$.
   (f) $c(c(S)) = c(S)$.

2. For any two monetary lotteries $\mathbf{p}$ and $\mathbf{q}$ with finite supports, denote by $\mathbf{p} + \mathbf{q}$ a lottery such that $(\mathbf{p} + \mathbf{q})(z) = \sum_{z'} p(z')q(z - z')$ for every $z \in \mathbb{R}$. Let $\alpha, \beta \in (0, 1)$, $z_{11}, z_{12}, z_{21}, z_{22} \in \mathbb{R}$ with $z_{11} < z_{12}$ and $z_{21} < z_{22}$, and $\mathbf{p}^1 = \alpha z_{11} \oplus (1 - \alpha) z_{12}$ and $\mathbf{p}^2 = \beta z_{21} \oplus (1 - \beta) z_{22}$ be two monetary lotteries of binary outcomes. Consider the following statement:
$$\mathbf{p}^1 + \delta_w \text{ first-order stochastically dominates } \mathbf{p}^2, \qquad (*)$$
where δ_w is a degenerate lottery of a prize $w \in \mathbb{R}$.

   (1) Draw a graph of cumulative distribution functions of $\mathbf{p}^1$ and $\mathbf{p}^2$ when $\alpha \leq \beta$.
   (2) Give a necessary and sufficient condition for the statement $(*)$ to hold when $\alpha \leq \beta$.
   (3) Give a necessary and sufficient condition for the statement $(*)$ to hold when $\alpha > \beta$.

3. Suppose that there are two towns, indexed by 1 and 2, where a certain good is consumed. The demand function of the good in town 1 is given by $q_1 = 8 - 2p_1$, and that of town 2 is given by $q_2 = 12 - 3p_2$. A monopolistic firm supplies the good to these two towns. The firm has a cost function $c(q) = \frac{1}{5}q^2$, where $c > 0$.

   (1) Assume that the firm must set the same price of the good in two towns. Compute the monopoly price p^*.
   (2) Assume that the firm can set different prices of the good in two towns. Compute the monopoly prices p_A^* and p_B^*.

Part II

1. Heather and Tingting are to split \$1,000,000. Their procedure for this will go through as many rounds as necessary to reach an agreement. (That is, this is an infinite-horizon model.)

   In each period (round) a fair coin is flipped. If the coin comes up Heads, then Heather proposes a split of the money; if it comes up Tails, then Tingting proposes the split. After a proposal is made, the other player either accepts or rejects the proposal. If the proposal is accepted, then the money is immediately divided as proposed (and agreed) and payoffs are realized. If the proposal is rejected, then in the next period the above procedure is repeated, starting with a fresh flip of the coin. If an agreement is never reached, neither Heather nor Tingting receives any money.

   Heather and Tingting care about their present discounted award—so, for a per-period discount factor δ, receiving share s now is worth s but receiving it t periods from now is worth $s\delta^t$.

   Heather's per-period discount factor is $\delta_H = 2/3$, and Tingting's is $\delta_T = 1/3$.

   Determine the equilibrium divisions proposed by Heather and by Tingting.

2. Sisters Andi and Brandi participate in an unusual first-price auction. Each player's value for the good is independently and identically distributed according to a uniform distribution on $[0, 1]$. After learning their own private values (but not their sister's value), each sister submits a bid and the highest bid wins (in the case of ties, the winner is determined by a random choice among the highest bidders). The unusual feature of this auction is that *the winning bidder pays nothing while the losing bidder pays the amount she bid.* Players are risk neutral, with realized payoffs v for a winner with value v and $-$(payment) for a loser.

   Determine a Bayesian equilibrium to this auction.

3. An Incumbent monopolist (I) has marginal cost of zero. A potential entrant (E) has marginal cost of either $MC_E = 0$ or $MC_E = 1$, and the Incumbent knows these are equally likely. The goods of I and E are homogeneous and market inverse demand is given by $P(Q) = 3 - Q$, where Q denotes total output in this market. Of course, E knows his own marginal cost.

   Before entering the market, E conducts an advertising campaign, spending an observable amount k. After all (especially I) have observed this choice of k, I and E simultaneously choose their levels of output for this market. Firms are risk neutral, acting to maximize their individual expected profit, and their is no discounting to consider. The foregoing is common knowledge.

   Derive the equilibrium-path choices of advertising and outputs in the Pareto-dominant separating perfect Bayesian equilibrium. Specify a correspondingly appropriate rule for updating the Incumbent's beliefs, given the observed advertising of the Entrant.

Part III

1. Amy and Bob consume only two goods, quantities of which we will denote by x and y. Amy and Bob have the same preferences, described by the utility function

$$
\begin{aligned}
u(x, y) &= x + y - 1 \text{ if } x \geq 1 \\
&= 3x + y - 3 \text{ if } x < 1
\end{aligned}
$$

   There are 4 units of good x, all owned by Amy and 6 units of good y, all owned by Bob. Draw the Edgeworth box diagram, including each person's indifference curve through the initial endowment point. Determine all Walrasian equilibrium prices and allocations.

2. Ann and Bill work together as water ski instructors in Florida. Each earns 100 dollars per day. Each one also owns orange trees that yield 8 oranges per day. Ann likes oranges "more" than Bill does; specifically, Ann's MRS for oranges is $MRS_A = 12 - x_A$ and Bill's MRS is $MRS_B = 8 - x_B$, where x_i denotes i's daily consumption of oranges and the MRS tells how many dollars one would be willing to give up to get an additional orange.

   (a) Bill has been selling 4 oranges a day to Ann, for which Ann has been paying Bill 3 dollars per day. (Thus Ann ends up with 10 oranges and 97 dollars per day, and Bill ends up with 6 oranges and 103 dollars per day). Is this Pareto efficient? Are they both better off than they would be if they did not trade? Is this a Walrasian equilibrium? Verify your answers.

   (b) In an Edgeworth box diagram, depict clearly all Pareto efficient allocations of oranges and dollars to Ann and Bill.

   (c) A hurricane has destroyed Ann's orange crop but has left Bill's crop undamaged. The Florida legislature has hurriedly passed a law against "price gouging". The law specifies that oranges cannot be sold for more than 4 dollars apiece. At the price of 4 dollars per orange, Bill is willing to sell Ann 4 oranges per day, but not more. Would Ann be willing to buy 4 oranges at 4 dollars apiece? Are there illegal trades (i.e., at a price more than 4 dollars per orange) that would make them both better off than they are at the above-mentioned legal trade? If so, find such a trade; if not, explain why not.

   (d) Determine whether the Walrasian equilibrium (after the hurricane) is a Pareto improvement on the allocation in (a).

3. Consider an economy with two types of jobs, "good" and "bad", and two types of workers "productive" and "unproductive". Sixty percent of the population is productive, the rest is not. In a bad job, either type of worker produces some x units of output where $0 < x < 60$. In a good job, a productive worker produces 100, but an unproductive one produces 0. Assume that there are many jobs of each type, so workers can choose which type of job they want. An employer for a particular job must pay the worker what she expects the worker to produce. A worker's type is only known to himself and not to the employers.

   (a) Explain whether this is an adverse selection problem or a moral hazard problem.

   (b) What are the equilibrium wages for each kind of job and which kind of jobs are filled by which kind of worker?

(c) What is the size of the loss in output from this equilibrium compared to the first-best outcome?

Suppose now that the workers can, if they wish, get educated. Studying s years involves a cost of $s^2/2$ for productive workers, and s^2 for the unproductive workers. Education costs are measured in the same units as output.

(d) Find the minimal years of education, s^*, that achieves separation.

(e) Suppose now that the government imposes a regulation that makes it mandatory for everyone to get s^g years of education, where $s^g > s^*$. Is separation still possible?

(f) Compare each type of worker's payoff under the unregulated separating equilibrium to those in the original equilibrium where signaling was not allowed and discuss how x determines whether each type of worker gains or loses from allowing education. Explain the intuition behind your results.

University of California, Riverside

Microeconomics Cumulative Exam

September 12, 2014

INSTRUCTIONS

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Instructions: answer Parts I, II, and III.

Part I

Answer all questions.

Problem 1. Let $\succcurlyeq$ be a complete and transitive preference relation on $\mathbb{R}^2$. Consider two properties below and answer the following questions.

Mon. If $x_1 \geq y_1$, $x_2 \geq y_2$, and $(x_1, x_2) \neq (y_1, y_2)$, then $(x_1, x_2) \succ (y_1, y_2)$.

Add. $(x_1, x_2) \sim (x_1 + x_2, 0)$ for any $(x_1, x_2) \in \mathbb{R}^2$.

(1) Show that if $\succcurlyeq$ is represented by a utility function $u(x_1, x_2) = x_1 + x_2$, then it satisfies Mon and Add.

(2) Show that if $\succcurlyeq$ satisfies Mon and Add, then it can be represented by a utility function $u(x_1, x_2) = x_1 + x_2$.

Problem 2. Let $\mathbf{p}$ be a monetary lottery with finite support, and δ_x be a certain lottery that takes the value $x \in \mathbb{R}$ with probability 1. Fill the box below to make the statement true, and prove the statement.

$$\delta_x \text{ FSD } \mathbf{p} \quad \text{if and only if} \quad x \geq \boxed{\ ?\ }.$$

Problem 3. Consider a monopolistic firm that can potentially supply its product to three isolated markets indexed by $n = 1, 2, 3$. For any n, Market n's inverse demand function is given by $p_n(q_n) = 2(6 - n) - q_n$. The firm's cost function is $c(q) = q^2$, where q is the total amount of output. Compute the firm's maximum profit for each of the following cases.

(Recall that the firm may sell different amounts and charge different prices to isolated markets.)

(1) The firm can only supply to Market 1.

(2) The firm can only supply to Markets 1 and 2.

(3) The firm can (but is not forced to) supply to all the markets.

Part II

Answer all questions.

1. There are 2 identical objects for sale and there are 3 bidders. Bidders want at most one object. Bidders' values for the good are iid random variables uniformly distributed over $[0, 1]$. A player learns only his own value before bidding. The two highest bidders win the objects and each pays the amount of the *second-highest bid.* After observing their individual values, players independently and simultaneously submit bids.

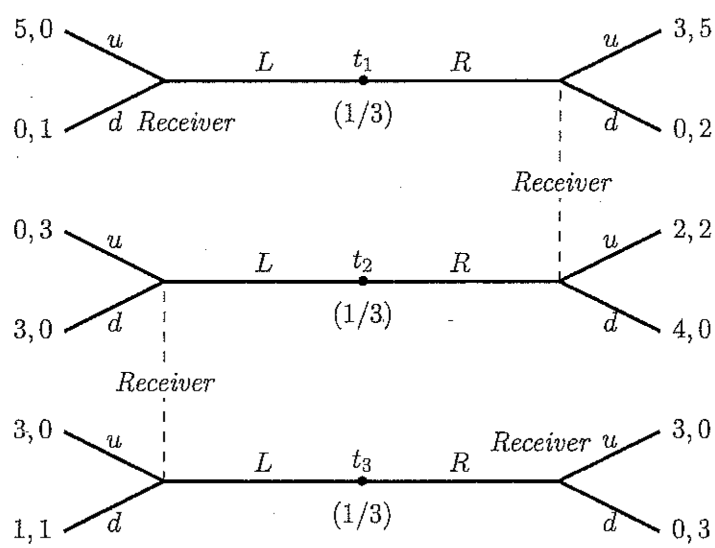
   For a bidder with value v, the final payoff is 0 if he does not get the object, and it is $v - p$ if he obtains the object and pays p for it.

   Find a Bayesian equilibrium to this game.

2. Consider the Sender-Receiver game shown below. The Sender's type is either t_1, t_2, or t_3. This type is chosen by Nature, and *ex ante* both the Sender and Receiver know these possibilities are equally likely.

   Only the Sender observes his type chosen by Nature. The Receiver's information following the choices by Nature and the Sender are shown by the dashed-line information sets in the figure below. At terminal nodes, the first payoff is the Sender's, the second is the Receiver's.

   This description of the game is common knowledge.



   Derive *a* perfect Bayesian equilibrium for this game.
   Use the game tree to ***fully depict*** *your equilibrium—strategies* and *beliefs!*
   (You do *not* need to find them all—just exhibit *one* completely!)

3. Three players are engaged in the following game. Players simultaneously and independently choose a number from the set $\{1, 2, 3\}$. The winner is the player who has the lowest number among those selected by only one player. If all three players choose the same number, then no one wins the prize—*all are losers.* Von Neumann–Morgenstern utilities are as follows: for all players, the payoff of losing is 0; for player k the payoff of winning is $v_k = k$, $k = 1, 2, 3$.

   Derive an equilibrium in which each player has a strictly positive probability of winning.

Part III

Answer All questions.

1. Consider the case of a pure exchange economy with two consumers. Both consumers have Cobb-Douglas utility functions but with different parameters. Consumer 1 has utility function $u(x_1^1, x_2^1) = (x_1^1)^\alpha (x_2^1)^{1-\alpha}$. Consumer 2 has utility function $u(x_1^2, x_2^2) = (x_1^2)^\beta (x_2^2)^{1-\beta}$. The endowment of good j owned by consumer i is ω_j^i. The price of good 1 is p_1, the price of good 2 is normalized to 1 without loss of generality.

   (a) For each consumer, compute the offer curves.

   (b) Assume $\omega_1^1 = 1$, $\omega_2^1 = 3$, $\omega_1^2 = 3$, $\omega_2^2 = 1$ and $\alpha = 1/2$, $\beta = 1/2$. Draw the Pareto set and the contract curve for this economy.

   (c) Solve analytically for the general equilibrium for the numerical values given in (b). Verify Walras's Law.

   (d) What is the comparative statics of p_1^* with respect to the endowments ω_j^i and taste parameters α and β? Discuss why these results make sense.

2. An employer is hiring a worker. Once hired, the agent can choose either a high effort e_H or low effort e_L. With high effort, the output is $y_H = 18$ with probability $\frac{3}{4}$ and $y_L = 1$ with probability $\frac{1}{4}$. With low effort, output is $y_H = 18$ with probability $\frac{1}{4}$ and $y_L = 1$ with probability $\frac{3}{4}$. The utility of the agent is $\sqrt{w} - c(e)$ where $c(e_H) = 0.1$ and $c(e_L) = 0$. The reservation utility for the agent is 0.1. The principal is risk neutral.

   (a) Derive the first best contract. Check carefully that the effort level being implemented by your proposed first best contract is actually optimal.

   (b) Derive the second best contract when effort is unobservable. Again check carefully that your suggested contract is actually maximizing the principal's profit.

   (c) If there is a monitoring system that allows the principal to perfectly observe the actions and hence implement the first best contract, how much would the principal be willing to pay for it?

University of California, Riverside

Microeconomics Cumulative Exam

July 7, 2014

INSTRUCTIONS

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Instructions: answer Parts I, II, and III.

Part I

Answer all questions.

1. Let X be a finite set of alternatives, and $\mathcal{X}$ be the collection of all nonempty subsets of X.

   For a binary relation $\succcurlyeq$, denote

$$\begin{cases} x \sim y & \text{if and only if} \quad x \succcurlyeq y \text{ and } y \succcurlyeq x, \\ x \succ y & \text{if and only if} \quad x \succcurlyeq y \text{ and not } y \succcurlyeq x. \end{cases}$$

   For a binary relation $\trianglerighteq$, denote

$$\begin{cases} x \approx y & \text{if and only if} \quad x \trianglerighteq y \text{ and } y \trianglerighteq x, \\ x \triangleright y & \text{if and only if} \quad x \trianglerighteq y \text{ and not } y \trianglerighteq x. \end{cases}$$

   (These are just notations for indifferences and strict preferences.)

   (1) Recall the concept of indirect preference relations. Let $\succcurlyeq$ be a complete and transitive binary relation on X, and $\trianglerighteq$ be a complete and transitive binary relation on $\mathcal{X}$ such that

$$A \trianglerighteq B \;\Leftrightarrow\; x \succcurlyeq y \text{ for all } x \in \max(A, \succcurlyeq) \text{ and } y \in \max(B, \succcurlyeq) \qquad (*)$$

   for any A and B in $\mathcal{X}$.

   Show that $\trianglerighteq$ satisfies

$$A \trianglerighteq B \text{ implies } A \approx A \cup B \qquad (**)$$

   for any A and B in $\mathcal{X}$.

(2) In this question, you will show that the converse also holds. Let $\trianglerighteq$ be a complete and transitive binary relation on $\mathcal{X}$ that satisfies ($**$), and answer the following questions. (Do not assume ($*$); it is one of claims you need to show.)

Define a binary relation $\succcurlyeq$ on X by $x \succcurlyeq y$ iff $\{x\} \trianglerighteq \{y\}$ for any $x, y \in X$.

(a) Show that $\succcurlyeq$ is complete.

(b) Show that $\succcurlyeq$ is transitive.

(c) Show that $A \approx \{x\}$ for any $A \in \mathcal{X}$ and $x \in \max(A, \succcurlyeq)$.

(d) Show that ($*$) holds for any A and B in $\mathcal{X}$.

2. Let $\mathbf{p}$ and $\mathbf{q}$ be two monetary lotteries such that $\mathbf{p}$ FSD $\mathbf{q}$. For simplicity, we assume finite supports of these lotteries, that is, supp($\mathbf{p}$) and supp($\mathbf{q}$) are finite sets. Answer the following two questions.

(1) Define two lotteries $\mathbf{p}^+$ and $\mathbf{q}^+$ by

$$\mathbf{p}^+(z) = \begin{cases} \mathbf{p}(z) & \text{if } z > 0, \\ F_{\mathbf{p}}(0) & \text{if } z = 0, \\ 0 & \text{if } z < 0, \end{cases}$$

and

$$\mathbf{q}^+(z) = \begin{cases} \mathbf{q}(z) & \text{if } z > 0, \\ F_{\mathbf{q}}(0) & \text{if } z = 0, \\ 0 & \text{if } z < 0, \end{cases}$$

where $F_{\mathbf{p}}$ and $F_{\mathbf{q}}$ are cumulative distribution functions for $\mathbf{p}$ and $\mathbf{q}$. (These lotteries may be interpreted as those under "free default.")

Show that $\mathbf{p}^+$ FSD $\mathbf{q}^+$.

(2) Show that, if expected values of two lotteries are same, that is, $\mathbf{E}(\mathbf{p}) = \mathbf{E}(\mathbf{q})$, then $\mathbf{p} = \mathbf{q}$.

(Hint: By enumerating supp($\mathbf{p}$) $\cup$ supp($\mathbf{q}$) $= \{z_1, \ldots, z_n\}$, where $z_1 < \cdots < z_n$, we can write

$$\mathbf{E}(\mathbf{p}) = z_n - \sum_{k=1}^{n-1} (z_{k+1} - z_k) F_{\mathbf{p}}(z_k)$$

and a similar equation for $\mathbf{E}(\mathbf{q})$. You can use this fact without proof.)

3. Suppose that a monopolistic firm sells its product to two isolated markets, A and B. The market A's inverse demand function is $p_A(q_A) = 120 - 3q_A$, and the market B's inverse demand function is $p_B(q_B) = 200 - 5q_B$. The firm's cost function is $c(q) = \frac{5}{4}q^2$, where q is the total amount of output. Compute (1) the profit maximizing supply to market A, (2) the profit maximizing supply to market B, and (3) the value of maximized profit for the firm. (Note that the firm can supply different amount and charge different prices to two isolated markets.)

Part II

Answer all questions.

1. Three players are engaged in the following game. Players simultaneously and independently choose a number from the set $\{1, 2, 3\}$. The winner is the player who has the lowest number among those selected by only one player. If all three players choose the same number, then each has a $1/3$ chance of being selected the winner. The payoff of winning is 1; the payoff of losing is 0.

   (a) How many pure strategy equilibria are there? Explain.

   (b) Derive a symmetric equilibrium.

2. Ariel and Barak engage in an infinite sequence of ultimatum games. In each period they have the potential to share 1 unit of an infinitely-divisible cake. The cake cannot be stored across periods.

   In the odd-numbered periods, Ariel makes to Barak a take-it-or-leave-it offer for sharing the cake. If Barak accepts, they share that period's cake as Ariel proposed; otherwise neither obtains anything.

   In even-numbered periods, Barak makes the proposal. If Ariel accepts, they share the cake as Barak proposed; otherwise neither obtains anything.

   Payoffs in a period are $\sqrt{x}$ to a player obtaining share x of the cake. A player obtaining a sequence of consumption amounts $(x_t)_{t=1}^{\infty}$ earns a payoff of

   $$\sum_{t=1}^{\infty} \delta^{t-1} \sqrt{x_t},$$

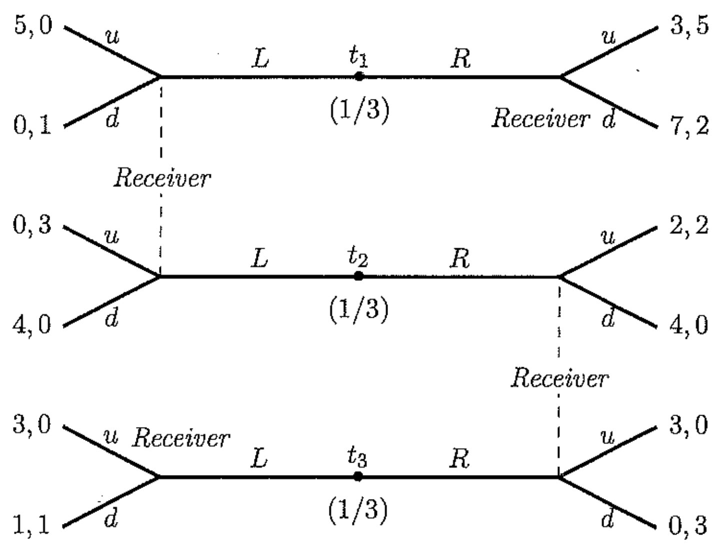
   where δ, $0 < \delta < 1$, is the common per-period discount factor.

   Ariel and Barak's combined payoff is maximized if in every period they share the cake equally. Determine the set of discount factors for which a Grim Trigger Strategy can sustain this outcome as a subgame perfect Nash equilibrium.

3. Consider the Sender-Receiver game shown below. The Sender's type is either t_1, t_2, or t_3. This type is chosen by Nature, and *ex ante* both the Sender and Receiver know these possibilities are equally likely.

   Only the Sender observes his type chosen by Nature. The Receiver's information following the choices by Nature and the Sender are shown by the dashed-line information sets in the figure below. At terminal nodes, the first payoff is the Sender's, the second is the Receiver's.

   This description of the game is common knowledge.



Derive *all* the perfect Bayesian equilibria in this game.
Use the game tree to fully depict the equilibria—strategies and *beliefs!*

Part III

Answer All questions.

1. Consider a two-person, two-good exchange economy with utility functions $U_1(x_{11}, x_{21}) = x_{11}$, $U_2(x_{12}, x_{22}) = x_{22} + \log x_{12}$, and endowments $\omega_1 = (2, 0)$ and $\omega_2 = (0, 2)$. For the purposes of this problem, we will assume that the price p_2 of the second good is always strictly positive.

   (a) What are the offer curves $OC_1$and OC_2? (Also draw a picture.)

   (b) Does there exist an equilibrium? Discuss.

2. A monopolist can produce a good in different qualities. The cost of producing a unit of quality s is s^2. Consumers buy at most one unit and have utility function $u(s|\theta) = \theta s$ if they consume one unit of quality s and 0 if they do not . The monopolist decides on the quality (or qualities) it is going to produce and the price. Consumers observe qualities and prices and decide which quality to buy, if at all.

   (a) Characterize the first-best solution.

   (b) Suppose that the seller cannot observe θ and suppose that θ is uniformly distributed on the interval $[0, 1]$. Characterize the second-best optimal quality-pricing schedule.

3. Suppose that $\geq$ is a linear order on X and consider a profile of preferences $(\succcurlyeq_1, \ldots\ldots, \succcurlyeq_I)$, where for every i, $\succcurlyeq_i$ is single-peaked with respect to $\geq$. Let $h \in I$ be a median agent. Show that the peak x_h of the median agent is a Condorcet winner.

UNIVERSITY OF CALIFORNIA, RIVERSIDE

Microeconomics Cumulative Exam

JULY 8, 2013

FOR STUDENTS ENTERING THE PROGRAM IN **FALL 2011**

INSTRUCTIONS

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INSTRUCTIONS: ANSWER PARTS I, II, AND III.

Part I

Carefully and precisely justify your answers to both questions. Short cuts are allowable as long as they are fully justified.

1. The expenditure function of the consumer is

$$E(u,p) = up_1^{1/2}p_2^{1/2} + \gamma p_1.$$

   (a) Does this expenditure function satisfy the requisite homogeneity condition?

   (b) Derive the Hicksian demand functions.

   (c) Derive the own and cross (Slutsky) substitution effects and show that the own substitution effects have the requisite sign.

   (d) Are the two goods net substitutes or net complements?

   (e) Derive the indirect utility function.

   (f) Explain precisely how you would now find the Marshallian demand functions (but do not carry out the calculations).

   (g) Set up the optimization problem that recovers the direct utility function (but do not carry out the calculations).

   (h) What assumptions about the direct utility function suffice to guarantee that the operation in (g) precisely recovers the direct utility function?

   (i) Assuming either of these assumptions is not satisfied, characterize the "errors" made by the recovery operation in (g).

(j) What is the economic significance of the "errors" in (i).

2. The cost function for a firm producing a single output is given by $C(u, p) = u^{1/\alpha}\Pi(p)$, where α is a parameter and Π is a function.

   (a) What properties must Π satisfy?

   (b) What are the constraints on α?

   (c) Define and derive the scale elasticity.

   (d) Show rigorously that the production function is homogeneous of degree α.

Part II

Answer all questions.

1. The following matrix game is played by Row and Column. Row's pure strategies are $\{t, m, b\}$ and Column's are $\{L, M, R\}$. The first number in a cell is the payoff to Row, the second is the payoff to Column.

	L	M	R
t	5, 3	0, 4	3, 5
m	4, 0	5, 5	4, 0
b	3, 5	0, 4	5, 3

   Derive *all* of the (simultaneous-move) Nash equilibria in this game.

2. Romeo and Juliet contribute effort toward a public good that each will then enjoy. The (constant marginal) cost of effort can vary across players. If a player with cost c contributes effort x and the level of the public good is g, then that player has a realized payoff of $v(g) - cx$, where $v(\cdot)$ is the benefit function common to Romeo and Juliet. A player's cost is private information to that player. A player's objective is to maximize his or her expected payoff.

   After learning their individual costs, Romeo and Juliet simultaneously and independently choose their efforts, x_R and x_J, respectively.

   In this problem, suppose

   - $v(g) = 2\sqrt{g}$, for all $g \geq 0$;
   - Romeo's cost, c_R, and Juliet's cost, c_J, are iid random variables uniformly distributed over the interval $[1, 2]$;
   - the level of the public good is $g = \max\{x_R, x_J\}$; that is, the level of the public good equals the larger of Romeo's and Juliet's efforts.

   Find a symmetric Bayesian equilibrium to this game.

3. A seller has value 0 for the object he has to sell. The single potential buyer has value v for the object, where v is uniformly distributed over $[0, 1]$ and is private information to the buyer (though the distribution is common knowledge). The seller will make up to two take-it-or-leave-it offers. In the first period a price p_1 is offered; if accepted, the payoff to the seller is p_1 and the payoff to the buyer is $v - p_1$. If the offer is rejected, then in the second period the seller makes a final offer p_2, which the buyer can either accept or reject. If the final offer is rejected, all payoffs are 0; if it is accepted, the payoff to the seller is p_2 and the payoff to the buyer is $v - p_2$. From the first period both the seller and buyer discount payoffs realized in period 2 by the factor $\delta \in (0, 1)$.

   Derive a perfect Bayesian equilibrium for this game.

Part III

1. Consider a four-person, two-good pure exchange economy where agents have endowments $\omega_1 = \omega_2 = (10, 10)$ and $\omega_3 = \omega_4 = (10, 30)$ and the same utility function

$$U_i(x_{1i}, x_{2i}) = \log x_{1i} + \log x_{2i}$$

for $i = 1, 2, 3, 4$. For each allocation vector given below show whether it is Pareto optimal and can be supported as a competitive equilibria for some price vector. Explain your reasoning.

   (a) $x_1 = x_2 = (7.5, 15)$ and $x_3 = x_4 = (12.5, 25)$.

   (b) $x_1 = x_2 = (\sqrt{50}, 2\sqrt{50})$ and $x_3 = x_4 = (20 - \sqrt{50}, 40 - 2\sqrt{50})$.

   (c) $x_1 = (8, 12)$, $x_2 = (9, 11)$, $x_2 = (12, 23)$ and $x_4 = (11, 29)$.

2. Adam is an expected utility maximizer and has constant absolute risk aversion over the range $[-\$1000, \$5000]$. He claims that his certainty equivalent for a lottery that pays \$5000 and \$0 each with probability 0.5 is \$2400. Which of the following three lotteries would Adam most prefer?

   (a) \$2000 for sure

   (b) \$5000 with probability 0.6 and $-$\$1000 with probability 0.4

   (c) \$5000 with probability 0.4, \$0 with probability 0.3 and $-$\$1000 with probability 0.3

   You can use the fact that having a constant coefficient of absolute risk aversion λ is equivalent to having vNM utility function

$$u(x) = -\exp(-\lambda x)$$

3. Consider a consumer with a random income y who needs to allocate her income between x_1 and x_2. The problem is that she has to decide BEFORE the uncertainty about income is resolved how much she would allocate to x_1. Her utility function is

$$u(x_1, x_2) = log(x_1 + 1) + log(x_2 + 1)$$

   After she decides her allocation towards x_1, the value of y is realized, which can be y_l probability p and y_h with probability $1 - p$, where $y_l < y_h$. Both goods have price 1. Derive an expression that characterizes the demand for x_1.

UNIVERSITY OF CALIFORNIA, RIVERSIDE

Microeconomics Cumulative Exam

SEPTEMBER 20, 2013

This exam is for students entering the program in Fall 2011.

INSTRUCTIONS

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INSTRUCTIONS: ANSWER PARTS I, II, AND III.

Part I

1. Assume that the consumer's utility function can be written as a function of x_f, the consumption of "food," and x_o, the consumption of "other commodities." The consumer receives each period an allotment, s, of food stamps, denominated in dollars, which can be used only to purchase food (at the given price of food). In addition to food stamps, the consumer is endowed with a positive income, y, which can be spent on either food or other commodities at positive prices, p_f and p_o.

   (a) Write the consumer's optimization problem and the associated Lagrangian function.

   (b) Draw the constraint set for this problem.

   (c) Write, interpret, and illustrate diagrammatically the first-order conditions for $\langle x_f^*, x_o^* \rangle$ to be a (not necessarily interior) solution to this optimization problem (assuming that the utility function is differentiable at $\langle x_f^*, x_o^* \rangle$.

2. Using the implicit function theorem (and the requisite sign conditions), express the following concepts in terms of (partial) derivatives of the (input) distance function, $D : \mathbf{R}_+^{m+n} \to \mathbf{R}_+$, defined by $D(u, x) = \max_{\lambda>0} \{\lambda \mid x/\lambda \in L(u)\}$, where $L(u)$ is the input requirement set for output u:

   (a) the rate of commodity transformation between outputs k and ℓ.

   (b) the technical rate of substitution between inputs i and j.

   (c) the marginal product of input i in producing output k.

Part II

1. A seller has 2 identical objects to sell. She uses a high-bid auction of the following description. All bidders simultaneously submit bids; the highest two bids are designated the winners. Each winner receives an object and pays the amount of the *second highest bid.* Losers get payoff 0; winners get payoff equal to {value} − {amount paid}. Each player wants at most one object.

   There are 3 bidders, and their values are iid random variables, uniformly distributed on $[0, 1]$. Each player's value is private information to that player. The foregoing is common knowledge.

   Derive a Bayesian equilibrium.

2. Romeo and Juliet play the following infinitely-repeated game. In each period each player chooses an effort level, which benefits both of them. Let x_R be Romeo's effort and x_J, Juliet's. Given these efforts, Romeo's payoff is $x_R + x_J - \frac{1}{2}(x_R)^2$ and Juliet's is $x_R + x_J - \frac{1}{2}(x_J)^2$.

   Romeo and Juliet each value their own payoff streams by the present value of the sum of the current and future payoffs, using the discount factor $\delta \in (0, 1)$ per period.

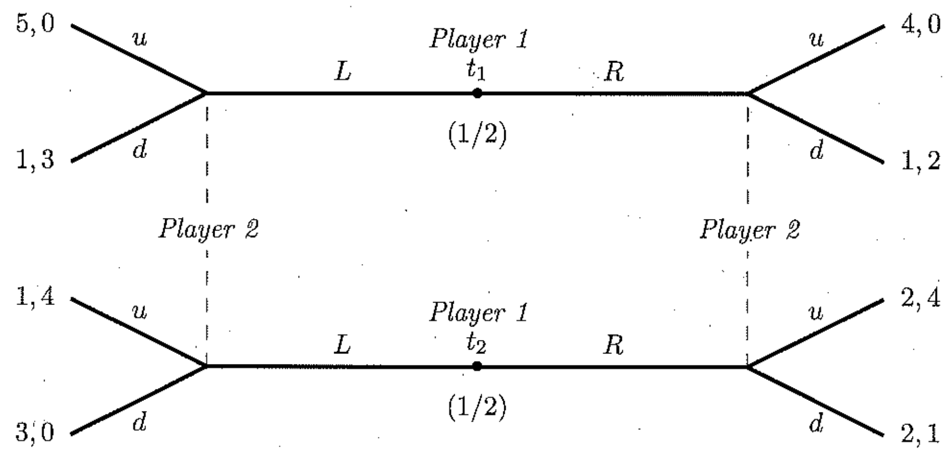
   (a) Calculate the effort levels for Romeo and Juliet their maximize the *sum* of their payoffs. Call these the efficient effort levels, (x_R^e, x_J^e).

   (b) For what range of discount factors can a Grim Trigger Strategy in the repeated game support the choice of (x_R^e, x_J^e) in every period as a subgame perfect equilibrium outcome? Explain.

   (c) Can a stick-and-carrot punishment strategy be used to increase the range of discount factors for which, in the repeated game, the choice of (x_R^e, x_J^e) in every period is the subgame perfect equilibrium outcome?

   Either construct such a strategy and determine this larger range of discount factors *or* explain why it is not possible in this example.

3. Consider the Sender-Receiver game depicted in the figure below, played between Player 1 and Player 2. Player 1's type is either t_1 or t_2, each being equally likely (chosen by Nature with probability 1/2 each). Player 1's pure strategies are L and R; Player 2's pure strategies at each information set are u and d. At the terminal nodes, the first number is the payoff to Player 1, the second number is the payoff to Player 2.



Determine whether this game has Perfect Bayesian Equilibria in which Player 1 uses a pure strategy (i.e., Player 1 uses no randomization). So you must ***EITHER***

- explain why there are no such equilibria ***OR***
- *fully* depict a PBE in which Player 1 uses a pure strategy.

Part III

1. Consider an Edgeworth box economy in which consumers have the Cobb-Douglas utility functions

$$u_1(x_{11}, x_{21}) = x_{11}^{\alpha} x_{21}^{1-\alpha}$$

and

$$u_2(x_{12}, x_{22}) = x_{12}^{\beta} x_{22}^{1-\beta}$$

Consumer i's endowments are $(\omega_{1i}, \omega_{2i})$ for $i = 1, 2$. Solve for the equilibrium price ratio and allocation. How do these change with a differential change in ω_{11}?

2. Argue graphically that in an Edgeworth box economy with locally non-satiated preferences, a Walrasian equilibrium is Pareto optimal.

3. A worker can be *careful* or *careless*, effort levels that generate mistakes with probabilities 0.25 and 0.75. His utility function is

$$U = 100 - 10/w - x$$

where w is his wage and x takes the value 2 if he is careful and 0 otherwise. Whether a mistake is made is contractible, but effort is not. Employer is risk neutral. Output is worth 0 if a mistake is made and 20 otherwise.

   (a) Will the worker be paid anything if he makes a mistake?

   (b) Will the worker be paid more if he does not make a mistake?

   (c) How would the contract be affected if employer is risk-averse?

UNIVERSITY OF CALIFORNIA, RIVERSIDE

Microeconomics Cumulative Exam

JULY 8, 2013

FOR STUDENTS ENTERING THE PROGRAM IN **FALL 2012**

INSTRUCTIONS

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INSTRUCTIONS: ANSWER PARTS I, II, AND III.

Part I

Answer all questions.

1. A consumer in a three-good economy (goods denoted x_1, x_2, and x_3; prices denoted p_1, p_2, and p_3) with wealth level $w > 0$ has demand functions for commodities 1 and 2 given by

$$x_1 = 100 - 5\frac{p_1}{p_3} + \beta\frac{p_2}{p_3} + \delta\frac{w}{p_3}$$

$$x_2 = \alpha + \beta\frac{p_1}{p_3} + \gamma\frac{p_2}{p_3} + \delta\frac{w}{p_3}$$

on an open set of (p_1, p_2, p_3) and all $w > 0$.

Calculate the restrictions on the numerical values of α, β, γ, and δ implied by utility maximization.

2. There are 4 consumers who purchase good 1; they also consume good 2. Good 2 is competitively supplied at per-unit price 1. Good 1 is available in a perfectly competitive market where 4 firms each have production function $f(L,K) = \sqrt{LK}$, where L and K are levels of labor and capital used. The unit prices of L and K are $w = 3$ and $r = 12$, respectively.

The consumer i has indirect utility function $v(p_1, p_2, m_i) = m_i\left(\frac{1}{p_1} + \frac{1}{p_2}\right)$, where m_i is consumer i's income and p_i is the unit price of good i. (Consumers differ only in their incomes.)

Calculate the total equilibrium quantity of good 1 consumed by these 4 consumers.

3. A monopolist has no costs of production. The monopolist sells to two consumers, whose inverse demands are $P_1(q_1) = 1 - \frac{1}{2}q_1$ and $P_2(q_2) = 1 - \frac{1}{3}q_2$. The monopolist is required to offer the same deals to both consumers.

   (a) Under uniform pricing (at a constant per unit price), what price should the monopolist set to maximize its profit?

   (b) Suppose the monopolist can sell according to a single two-part tariff. Calculate the monopolist's profit-maximizing single two-part tariff.

   (c) Now suppose the monopolist can sell according to any nonlinear price schedule, but he must offer the same schedule to both players. Calculate profit-maximizing (nonlinear) price schedule for the monopolist to offer.

   You should assume income effects are not relevant over the ranges considered.

Part II

Answer all questions.

1. The following matrix game is played by Row and Column. Row's pure strategies are $\{t, m, b\}$ and Column's are $\{L, M, R\}$. The first number in a cell is the payoff to Row, the second is the payoff to Column.

	L	M	R
t	5, 3	0, 4	3, 5
m	4, 0	5, 5	4, 0
b	3, 5	0, 4	5, 3

   Derive *all* of the (simultaneous-move) Nash equilibria in this game.

2. Romeo and Juliet contribute effort toward a public good that each will then enjoy. The (constant marginal) cost of effort can vary across players. If a player with cost c contributes effort x and the level of the public good is g, then that player has a realized payoff of $v(g)-cx$, where $v(\cdot)$ is the benefit function common to Romeo and Juliet. A player's cost is private information to that player. A player's objective is to maximize his or her expected payoff.

   After learning their individual costs, Romeo and Juliet simultaneously and independently choose their efforts, x_R and x_J, respectively.

   In this problem, suppose

   - $v(g) = 2\sqrt{g}$, for all $g \geq 0$;
   - Romeo's cost, c_R, and Juliet's cost, c_J, are iid random variables uniformly distributed over the interval $[1, 2]$;
   - the level of the public good is $g = \max\{x_R, x_J\}$; that is, the level of the public good equals the larger of Romeo's and Juliet's efforts.

   Find a symmetric Bayesian equilibrium to this game.

3. A seller has value 0 for the object he has to sell. The single potential buyer has value v for the object, where v is uniformly distributed over $[0, 1]$ and is private information to the buyer (though the distribution is common knowledge). The seller will make up to two take-it-or-leave-it offers. In the first period a price p_1 is offered; if accepted, the payoff to the seller is p_1 and the payoff to the buyer is $v - p_1$. If the offer is rejected, then in the second period the seller makes a final offer p_2, which the buyer can either accept or reject. If the final offer is rejected, all payoffs are 0; if it is accepted, the payoff to the seller is p_2 and the payoff to the buyer is $v - p_2$. From the first period both the seller and buyer discount payoffs realized in period 2 by the factor $\delta \in (0, 1)$.

   Derive a perfect Bayesian equilibrium for this game.

Part III

1. Consider a four-person, two-good pure exchange economy where agents have endowments $\omega_1 = \omega_2 = (10, 10)$ and $\omega_3 = \omega_4 = (10, 30)$ and the same utility function

$$U_i(x_{1i}, x_{2i}) = \log x_{1i} + \log x_{2i}$$

for $i = 1, 2, 3, 4$. For each allocation vector given below show whether it is Pareto optimal and can be supported as a competitive equilibria for some price vector. Explain your reasoning.

   (a) $x_1 = x_2 = (7.5, 15)$ and $x_3 = x_4 = (12.5, 25)$.

   (b) $x_1 = (8, 12)$, $x_2 = (9, 11)$, $x_2 = (12, 23)$ and $x_4 = (11, 29)$.

2. State the Second Welfare Theorem. Discuss why the statement involves "price equilibrium with transfers" instead of "Walrasian equilibrium" (include definitions of both) . Illustrate with an Edgeworth box diagram.

3. I am trying to decide whether or not to hire a manager for my project. Without the manager, I can have a success (with value S) or failure (with value F where $S > F$) with probabilities α and $1 - \alpha$. If I hire the manager, and she puts in effort $e > 0$ then the probability of success is β and of failure is $1 - \beta$. If she puts effort $e = 0$, then the probabilities remain as if she was not hired. Her von Neuman-Morgenstern utility function is $\sqrt{w} - e$, where w is the wage paid to her. I need help in deciding whether or not to hire this person and, if I hire her, how to structure the payments I make to her. I am risk neutral. Write a memo outlining what I should do and why, under the various scenarios involving different parameter values. Please note that you don't have enough information to pin down the exact optimal contract, but be as thorough as you can in your justification for each step in your proposed method of solving this problem.

University of California, Riverside
Microeconomics Cumulative Exam
July 8, 2013

INSTRUCTIONS

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Instructions: answer Parts I, II, and III.

Part I

Carefully and precisely justify your answers to both questions. Short cuts are allowable as long as they are fully justified.

1. The expenditure function of the consumer is

$$E(u,p) = up_1^{1/2}p_2^{1/2} + \gamma p_1.$$

   (a) Does this expenditure function satisfy the requisite homogeneity condition?
   (b) Derive the Hicksian demand functions.
   (c) Derive the own and cross (Slutsky) substitution effects and show that the own substitution effects have the requisite sign.
   (d) Are the two goods net substitutes or net complements?
   (e) Derive the indirect utility function.
   (f) Explain precisely how you would now find the Marshallian demand functions (but do not carry out the calculations).
   (g) Set up the optimization problem that recovers the direct utility function (but do not carry out the calculations).
   (h) What assumptions about the direct utility function suffice to guarantee that the operation in (g) precisely recovers the direct utility function?
   (i) Assuming either of these assumptions is not satisfied, characterize the "errors" made by the recovery operation in (g).

(j) What is the economic significance of the "errors" in (i).

2. The cost function for a firm producing a single output is given by $C(u,p) = u^{1/\alpha}\Pi(p)$, where α is a parameter and Π is a function.

   (a) What properties must Π satisfy?

   (b) What are the constraints on α?

   (c) Define and derive the scale elasticity.

   (d) Show rigorously that the production function is homogeneous of degree α.

Part II

Answer all questions.

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