HW4

February 28, 2019

```
In [1]: import pandas as pd
        from sklearn.linear_model import LinearRegression
        import numpy as np
        import statsmodels.api as sm
```

1 Question #1 (a)

```
In [2]: raw_data = pd.read_csv('Koop-Tobias.csv')
        raw_data.head()

Out[2]:    PERSONID  EDUC  LOGWAGE  POTEXPER  TIMETRND  ABILITY  MOTHERED  FATHERED  \
        0         1    13     1.82         1         0      1.0        12        12
        1         1    18     3.29         3         7      1.0        12        12
        2         1    18     3.21         5         9      1.0        12        12
        3         1    18     3.06         6        10      1.0        12        12
        4         2    15     2.14         4         6      1.5        12        12

           BRKNHOME  SIBLINGS
        0         0         1
        1         0         1
        2         0         1
        3         0         1
        4         0         1
```

```
In [3]: X = raw_data.drop(columns=['PERSONID', 'LOGWAGE', 'TIMETRND'])
        X = sm.add_constant(X)
        y = raw_data['LOGWAGE']

        results = sm.OLS(y, X).fit()

        print(results.summary())

                            OLS Regression Results
==============================================================================
Dep. Variable:                LOGWAGE   R-squared:                       0.176
Model:                            OLS   Adj. R-squared:                  0.176
Method:                 Least Squares   F-statistic:                     546.6
```

```
Date:                Wed, 27 Feb 2019   Prob (F-statistic):                0.00
Time:                        23:24:58   Log-Likelihood:                 -12255.
No. Observations:               17919   AIC:                         2.453e+04
Df Residuals:                   17911   BIC:                         2.459e+04
Df Model:                           7
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.9897      0.034     29.198      0.000       0.923       1.056
EDUC           0.0712      0.002     31.538      0.000       0.067       0.076
POTEXPER       0.0395      0.001     43.970      0.000       0.038       0.041
ABILITY        0.0774      0.005     15.682      0.000       0.068       0.087
MOTHERED    7.099e-05      0.002      0.042      0.967      -0.003       0.003
FATHERED       0.0053      0.001      3.974      0.000       0.003       0.008
BRKNHOME      -0.0529      0.010     -5.292      0.000      -0.072      -0.033
SIBLINGS       0.0049      0.002      2.720      0.007       0.001       0.008
==============================================================================
Omnibus:                     1110.722   Durbin-Watson:                   0.804
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2075.146
Skew:                          -0.458   Prob(JB):                         0.00
Kurtosis:                       4.393   Cond. No.                         218.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

marginal effect

```
In [5]: X_bar = X.mean()
        X_bar['BRKNHOME'] = 0
        X_bar['EDUC'] = 12

In [6]: X_bar = np.asarray(X_bar)
        coef = np.asarray(results.params)

In [7]: np.exp(np.matmul(X_bar, coef))*results.params[1]

Out[7]: 0.6800472008090137
```

2 Question #1 (b)

```
In [8]: X['EDUC_2'] = np.power(X['EDUC'], 2)
        X['EDUC_3'] = np.power(X['EDUC'], 3)

        results_2 = sm.OLS(y, X).fit()
        R = np.array(([0,0,0,0,0,0,0,0,0,1], [0,0,0,0,0,0,0,0,1,0]))
```

```
In [9]: print(results_2.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                LOGWAGE   R-squared:                       0.177
Model:                            OLS   Adj. R-squared:                  0.177
Method:                 Least Squares   F-statistic:                     428.9
Date:                Wed, 27 Feb 2019   Prob (F-statistic):               0.00
Time:                        23:24:58   Log-Likelihood:                -12240.
No. Observations:               17919   AIC:                         2.450e+04
Df Residuals:                   17909   BIC:                         2.458e+04
Df Model:                           9
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.3650      0.440      3.102      0.002       0.502       2.228
EDUC          -0.0601      0.099     -0.606      0.545      -0.254       0.134
POTEXPER       0.0396      0.001     44.060      0.000       0.038       0.041
ABILITY        0.0741      0.005     14.908      0.000       0.064       0.084
MOTHERED       0.0003      0.002      0.182      0.855      -0.003       0.004
FATHERED       0.0052      0.001      3.865      0.000       0.003       0.008
BRKNHOME      -0.0499      0.010     -4.990      0.000      -0.070      -0.030
SIBLINGS       0.0048      0.002      2.707      0.007       0.001       0.008
EDUC_2         0.0131      0.007      1.780      0.075      -0.001       0.028
EDUC_3        -0.0004      0.000     -2.214      0.027      -0.001   -4.55e-05
==============================================================================
Omnibus:                     1114.698   Durbin-Watson:                   0.805
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2107.537
Skew:                          -0.456   Prob(JB):                         0.00
Kurtosis:                       4.411   Cond. No.                     3.08e+05
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 3.08e+05. This might indicate that there are
strong multicollinearity or other numerical problems.
```

```
In [10]: print(results_2.f_test(R))
```

```
<F test: F=array([[14.34648984]]), p=5.948185372443663e-07, df_denom=17909, df_num=2>
```

3 Question #1 (c)

```
In [11]: X = raw_data['EDUC']
         X = sm.add_constant(X)
```

```
y = raw_data['LOGWAGE']
results_3 = sm.OLS(y, X).fit()

y_pred = results_3.predict(X)
lin_params = results_3.params
print(results_3.summary())
```

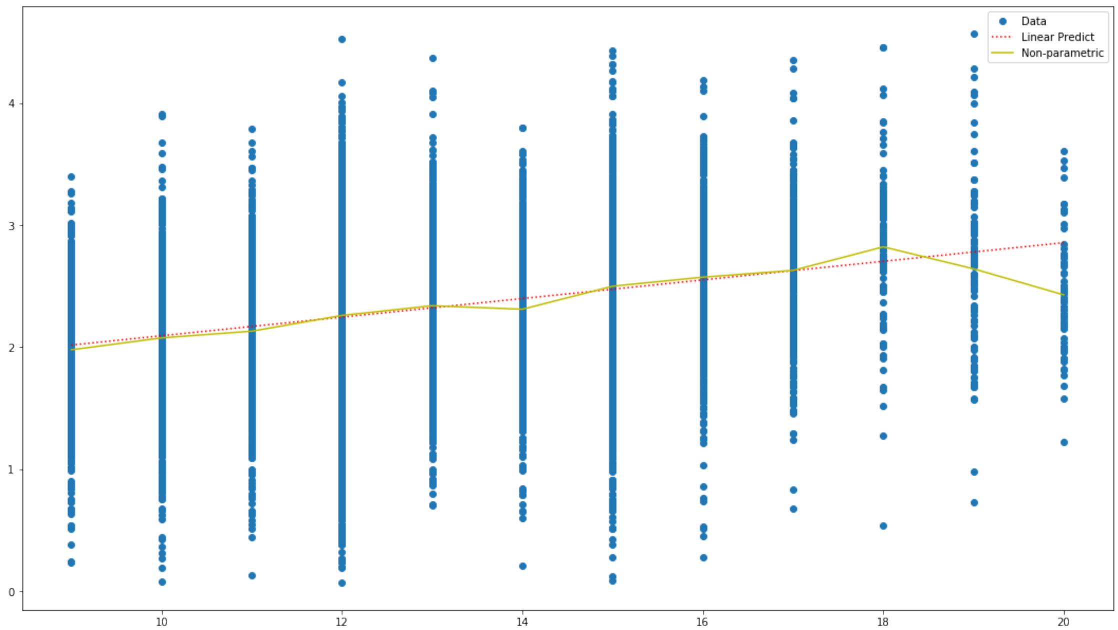
```
                            OLS Regression Results
==============================================================================
Dep. Variable:                LOGWAGE   R-squared:                       0.077
Model:                            OLS   Adj. R-squared:                  0.077
Method:                 Least Squares   F-statistic:                     1497.
Date:                Wed, 27 Feb 2019   Prob (F-statistic):          1.39e-314
Time:                        23:24:58   Log-Likelihood:                -13270.
No. Observations:               17919   AIC:                         2.654e+04
Df Residuals:                   17917   BIC:                         2.656e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.3296      0.025     52.589      0.000       1.280       1.379
EDUC           0.0763      0.002     38.690      0.000       0.072       0.080
==============================================================================
Omnibus:                      578.200   Durbin-Watson:                   0.793
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              878.367
Skew:                          -0.319   Prob(JB):                    1.84e-191
Kurtosis:                       3.878   Cond. No.                         86.0
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

```
In [12]: from statsmodels.nonparametric.kernel_regression import KernelReg
         X = raw_data['EDUC']
         bw = [(np.std(X, ddof=1)* (len(X))**(-1/5))]

         results_4 = KernelReg(y, X, 'c',reg_type='lc', bw=bw).fit()
         y_pred_np = results_4[0]
```

```
In [15]: import matplotlib.pyplot as plt

         fig, ax = plt.subplots()
         fig.set_size_inches(18.5, 10.5)

         new_x_1, new_y_1 = zip(*sorted(zip(X, y_pred)))
```

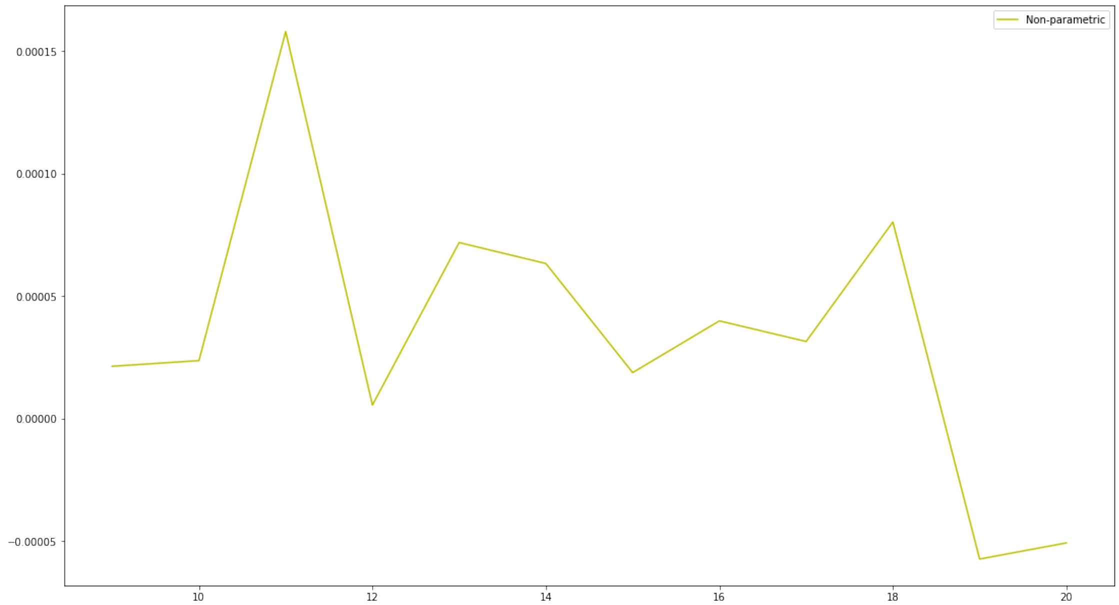
```
new_x_2, new_y_2 = zip(*sorted(zip(X, y_pred_np)))

ax.plot(X, y, 'o', label="Data")
ax.plot(new_x_1, new_y_1, 'r', label="Linear Predict", linestyle='dotted')
ax.plot(new_x_2, new_y_2, 'y', label="Non-parametric")

ax.legend(loc="best")
```

Out[15]: <matplotlib.legend.Legend at 0x20326690748>




```
In [16]: marginal_effect = results_4[1]*100

In [18]: fig, ax_2 = plt.subplots()
         fig.set_size_inches(18.5, 10.5)
         #ax.plot(X, y, 'o', label="Data")
         new_x, new_y = zip(*sorted(zip(X, marginal_effect)))

         ax_2.plot(new_x, new_y, 'y', label="Non-parametric")
         #ax_2.plot(X, np.full(X.shape, lin_params[1]), 'g', label="Linear", linestyle='dashed

         ax_2.legend(loc="best")
```

Out[18]: <matplotlib.legend.Legend at 0x20326717c18>

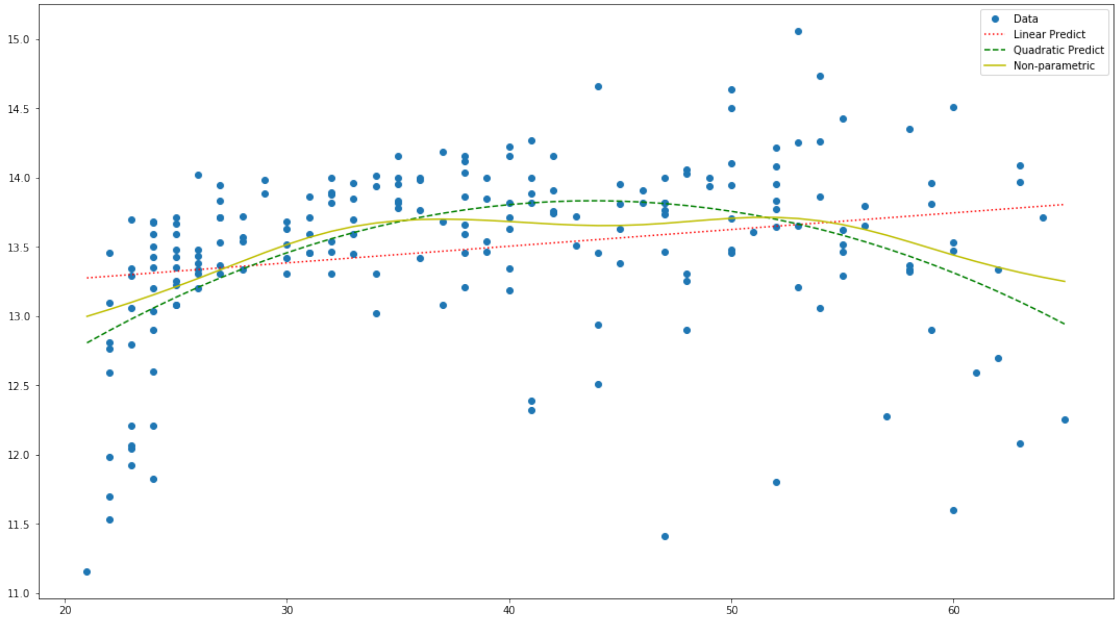


4 Question #2

```
In [19]: raw_data = pd.read_csv('cps71.csv')

In [20]: X = raw_data['age']
         X = sm.add_constant(X)
         y = raw_data['logwage']

         results = sm.OLS(y, X).fit()
         lin_params = results.params

         y_pred_lin = results.predict(X)

         X['age_2'] =np.power(raw_data['age'], 2)

         results_2 = sm.OLS(y, X).fit()
         quad_params = results_2.params

         y_pred_quad = results_2.predict(X)

In [21]: bw = [(np.std(X['age'], ddof=1)* (len(X))**(-1/5))]
         results_3 = KernelReg(y, X['age'], 'c', reg_type='lc', bw=bw).fit()
         y_pred_np = results_3[0]

In [22]: import matplotlib.pyplot as plt

         fig, ax = plt.subplots()
```

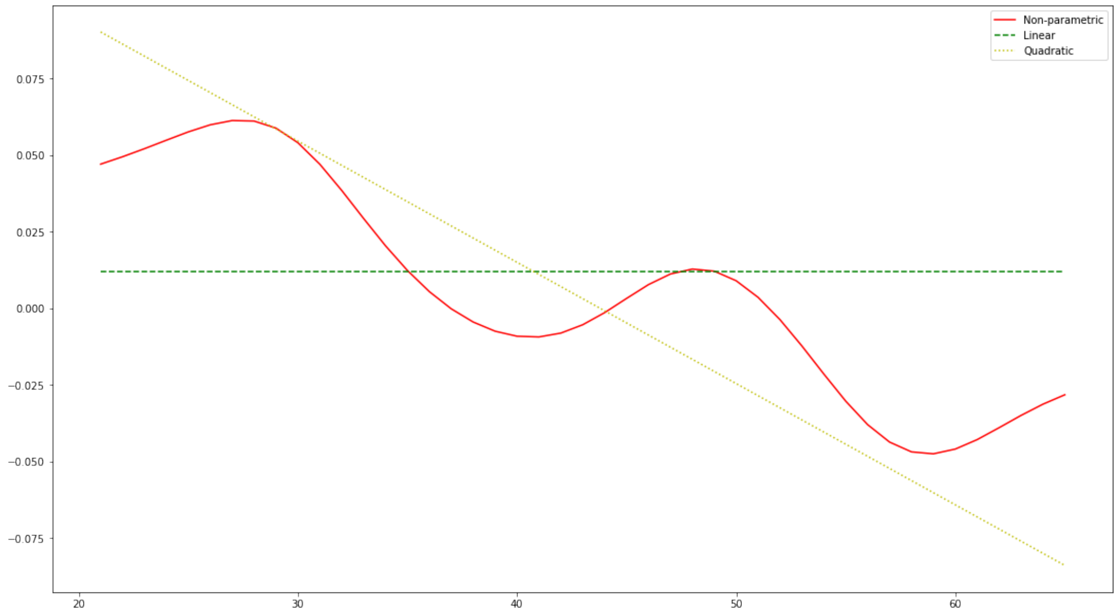
```
fig.set_size_inches(18.5, 10.5)
ax.plot(X['age'], y, 'o', label="Data")
ax.plot(X['age'], y_pred_lin, 'r', label="Linear Predict", linestyle='dotted')
ax.plot(X['age'], y_pred_quad, 'g', label="Quadratic Predict", linestyle='dashed')
ax.plot(X['age'], y_pred_np, 'y', label="Non-parametric")

ax.legend(loc="best")
```

Out[22]: <matplotlib.legend.Legend at 0x20326706a58>



In [23]:
```
np_marginal = results_3[1]

fig, ax_2 = plt.subplots()
fig.set_size_inches(18.5, 10.5)
ax_2.plot(X['age'], np_marginal*100, 'r', label="Non-parametric")
ax_2.plot(X['age'], np.full(X['age'].shape, lin_params[1]), 'g', label="Linear", lines
ax_2.plot(X['age'], X['age']*quad_params[2]*2 + quad_params[1], 'y', label="Quadratic"

ax_2.legend(loc="best")
```

Out[23]: <matplotlib.legend.Legend at 0x2032708be80>



5 Question #3(a)

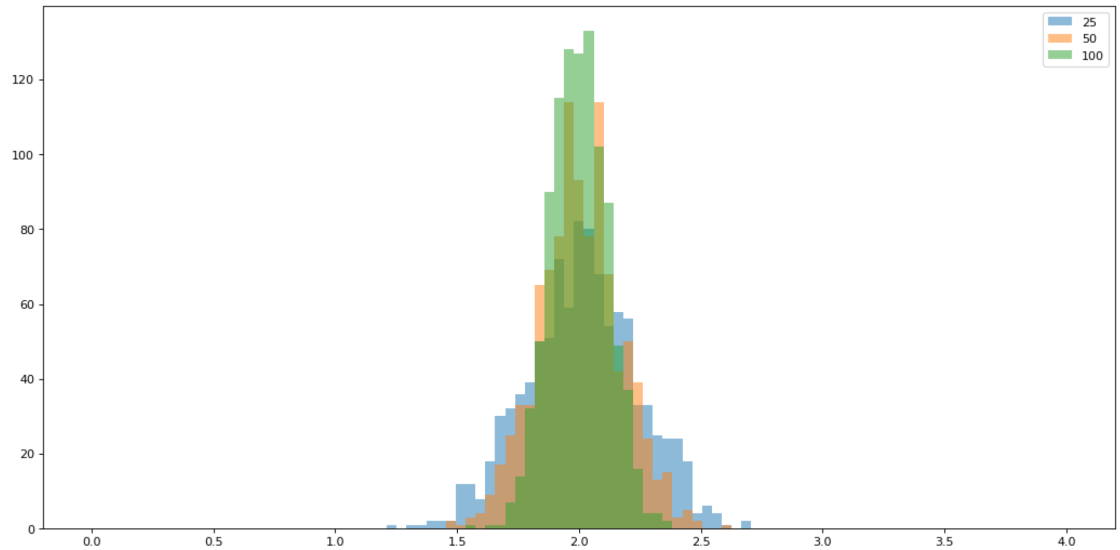
```
In [141]: import math
          sample_sizes = [25, 50, 100] #size of sample
          # y = 4 + 2 * x + error
          # N error N(0,1)
          # T error T(4)
          coef_big_mat = []
          z_big_mat = []
          t_big_mat = []
          for n in sample_sizes:
              coef_n = []
              coef_t = []
              z_n = []
              z_t = []
              t_n = []
              t_t = []
              for t in range(1000):
                  n_error = np.random.normal(0, 1, n)
                  t_error = np.random.standard_t(4, n)
                  x = np.random.uniform(0, 3, n)
                  y_n = 4 + 2*x + n_error
                  y_t = 4 + 2*x + t_error
                  sd = math.sqrt(1/np.sum(np.power(x, 2)))

                  # N Error
```

```
            result_n = sm.OLS(y_n, sm.add_constant(x)).fit()
            coef_n.append(result_n.params[1])
            z = (result_n.params[1]-0)/sd
            se = result_n.bse[1]
            t = (result_n.params[1]-0)/se
            t_n.append(t)
            z_n.append(z)

            # T Error
            result_t = sm.OLS(y_t, sm.add_constant(x)).fit()
            coef_t.append(result_t.params[1])
            z = (result_t.params[1]-0)/sd
            se = result_t.bse[1]
            t = (result_t.params[1]-0)/se
            t_t.append(t)
            z_t.append(z)

        coef_big_mat.append([coef_n, coef_t])
        z_big_mat.append([z_n, z_t])
        t_big_mat.append([t_n, t_t])
```

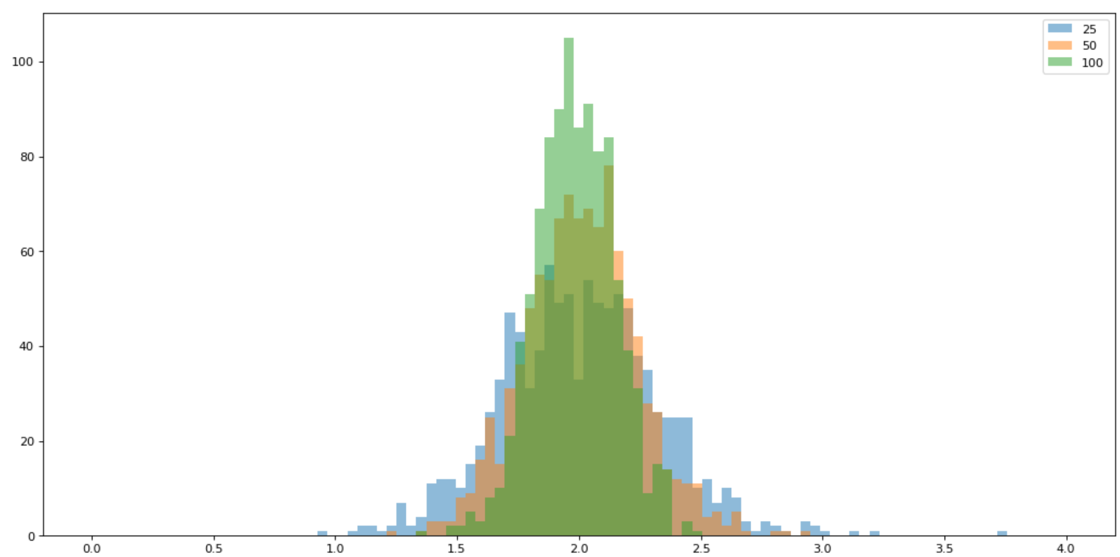
In [142]:
```
from matplotlib import pyplot
from matplotlib.pyplot import figure
```

In [143]:
```
bins = np.linspace(-0, 4, 100)

print("N Error Case:")

figure(num=None, figsize=(16, 8), dpi=80, facecolor='w', edgecolor='k')
for i in range(3):
    print("n="+str(sample_sizes[i])+", mean="+str(np.mean(coef_big_mat[i][0]))+", va
    pyplot.hist(coef_big_mat[i][0], bins, alpha=0.5, label=str(sample_sizes[i]))
pyplot.legend(loc='upper right')
pyplot.show()

print("T Error Case:")

figure(num=None, figsize=(16, 8), dpi=80, facecolor='w', edgecolor='k')
for i in range(3):
    print("n="+str(sample_sizes[i])+", mean="+str(np.mean(coef_big_mat[i][1]))+", va
    pyplot.hist(coef_big_mat[i][1], bins, alpha=0.5, label=str(sample_sizes[i]))
pyplot.legend(loc='upper right')
pyplot.show()
```

```
N Error Case:
n=25, mean=2.012488983333248, var=0.05249862604346628, se=0.2291257865092148
n=50, mean=2.004038638956284, var=0.028643210231471507, se=0.16924305076271673
n=100, mean=1.9998946283499164, var=0.01354834677651397, se=0.1163973658486908
```




```
T Error Case:
n=25, mean=2.0060822548285806, var=0.10567402834654102, se=0.32507541947452906
n=50, mean=2.0113618504889645, var=0.05352231792474029, se=0.23134890949546377
n=100, mean=1.9884132856956598, var=0.02630018530277698, se=0.16217331871419843
```

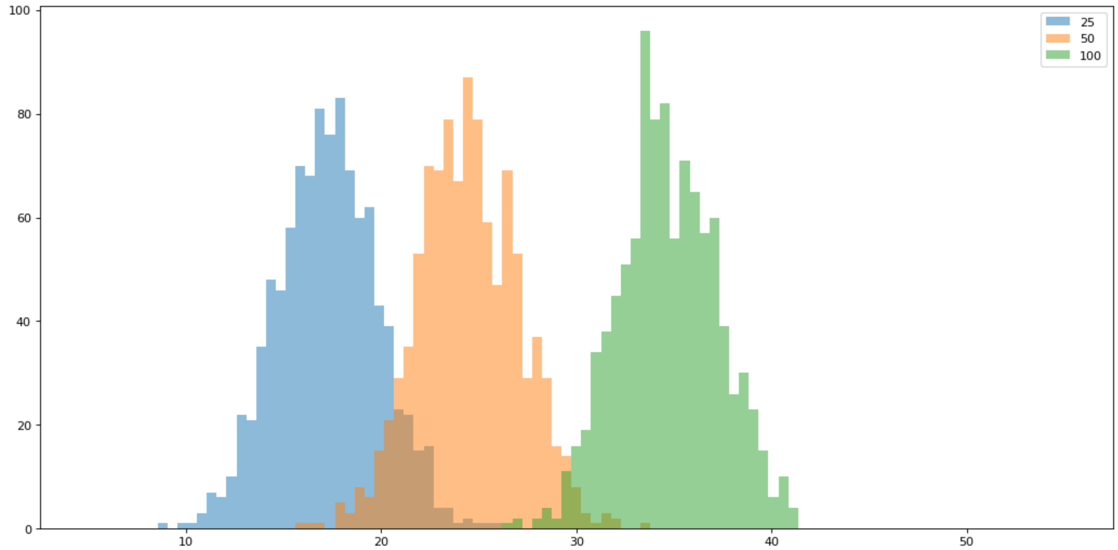


6 Question #3(b)

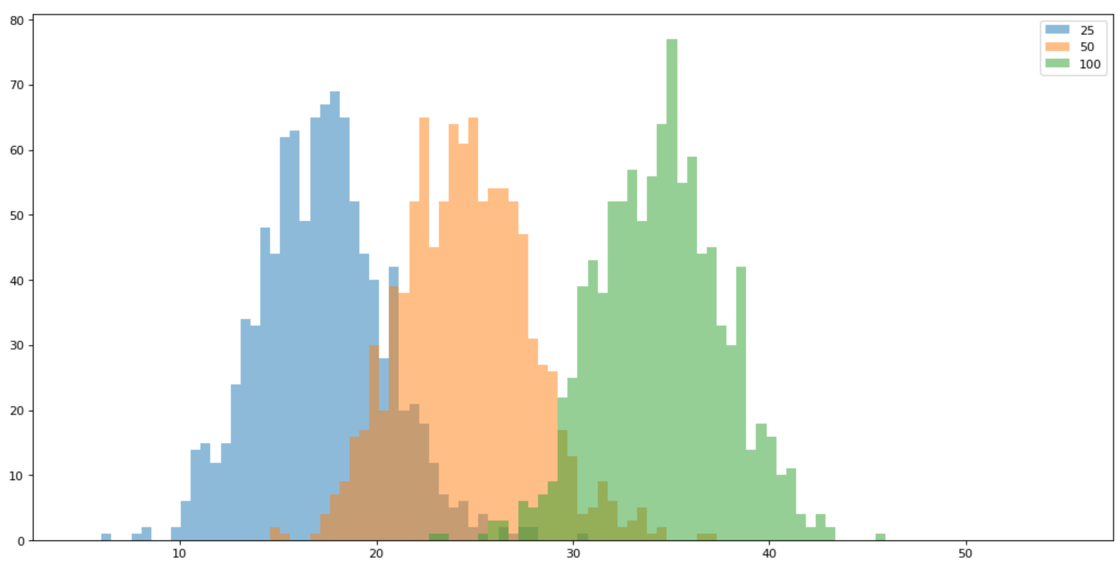
6.0.1 Z Statistics

```
In [144]: bins = np.linspace(5, 55, 100)

          print("N Error Case:")

          figure(num=None, figsize=(16, 8), dpi=80, facecolor='w', edgecolor='k')
          for i in range(3):
              print("n="+str(sample_sizes[i])+", mean="+str(np.mean(z_big_mat[i][0]))+", var="
              count = 0
              for element in z_big_mat[i][0]:
                  if (element > 1.96 or element < -1.96):
                      count += 1
              print("size of test:", 1 - count/1000)
              pyplot.hist(z_big_mat[i][0], bins, alpha=0.5, label=str(sample_sizes[i]))
          pyplot.legend(loc='upper right')
          pyplot.show()

          print("T Error Case:")

          figure(num=None, figsize=(16, 8), dpi=80, facecolor='w', edgecolor='k')
          for i in range(3):
              print("n="+str(sample_sizes[i])+", mean="+str(np.mean(z_big_mat[i][1]))+", var="
              count = 0
              for element in z_big_mat[i][1]:
                  if (element > 1.96 or element < -1.96):
                      count += 1
              print("size of test:", 1 - count/1000)
              pyplot.hist(z_big_mat[i][1], bins, alpha=0.5, label=str(sample_sizes[i]))
          pyplot.legend(loc='upper right')
          pyplot.show()
```

```
N Error Case:
n=25, mean=17.268458433839506, var=6.4753934834130495
size of test: 0.0
n=50, mean=24.478947133064747, var=6.603008365729441
size of test: 0.0
n=100, mean=34.689623939681276, var=6.482227300655119
size of test: 0.0
```



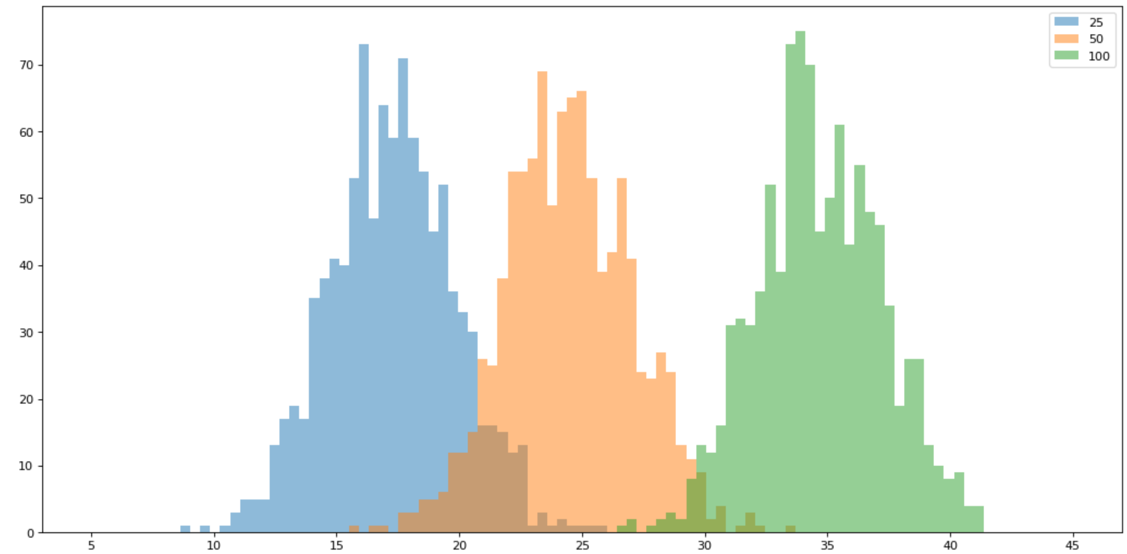

```
T Error Case:
n=25, mean=17.21294748809783, var=10.290138070588648
size of test: 0.0
n=50, mean=24.57051460244579, var=10.509378822227127
size of test: 0.0
n=100, mean=34.48750148998313, var=10.095753493509088
size of test: 0.0
```



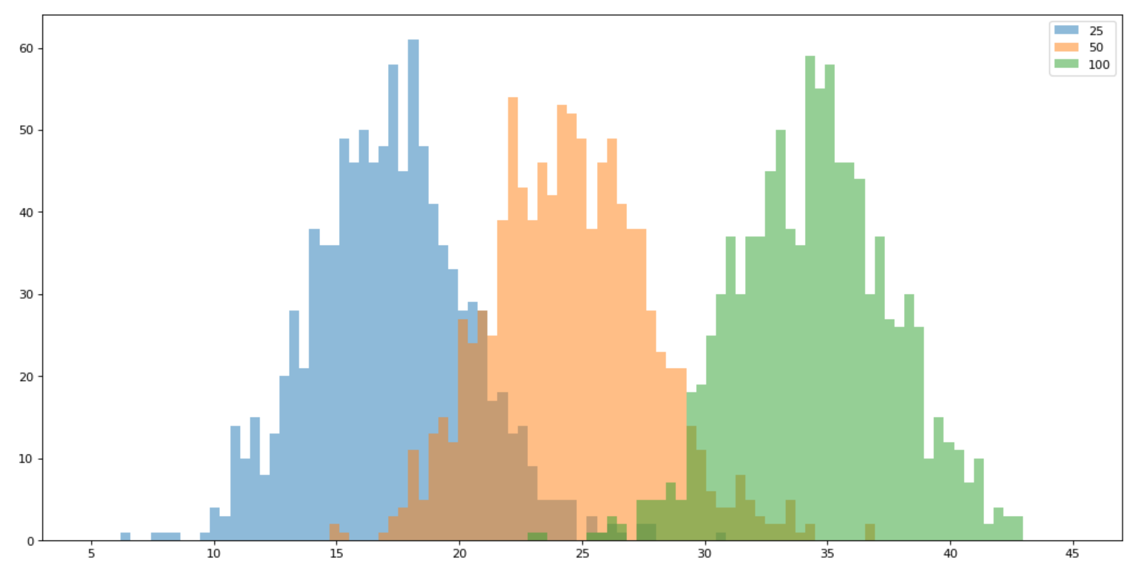
6.0.2 T Statistics

```
In [145]: bins = np.linspace(5, 45, 100)

          print("N Error Case:")

          figure(num=None, figsize=(16, 8), dpi=80, facecolor='w', edgecolor='k')
          for i in range(3):
              print("n="+str(sample_sizes[i])+", mean="+str(np.mean(t_big_mat[i][0]))+", var="
              count = 0
              for element in t_big_mat[i][0]:
                  if (element > 12.706 or element < -12.706):
                      count += 1
              print("size of test:", 1 - count/1000)
              pyplot.hist(z_big_mat[i][0], bins, alpha=0.5, label=str(sample_sizes[i]))
          pyplot.legend(loc='upper right')
          pyplot.show()

          print("T Error Case:")

          figure(num=None, figsize=(16, 8), dpi=80, facecolor='w', edgecolor='k')
          for i in range(3):
              print("n="+str(sample_sizes[i])+", mean="+str(np.mean(t_big_mat[i][1]))+", var="
              count = 0
              for element in t_big_mat[i][1]:
                  if (element > 12.706 or element < -12.706):
                      count += 1
              print("size of test:", 1 - count/1000)
              pyplot.hist(z_big_mat[i][1], bins, alpha=0.5, label=str(sample_sizes[i]))
          pyplot.legend(loc='upper right')
          pyplot.show()
```

```
N Error Case:
n=25, mean=8.735152019547114, var=3.3629344001402646
size of test: 0.968
n=50, mean=12.228941819914041, var=3.263578798541486
size of test: 0.635
n=100, mean=17.351736876535266, var=3.5030257513385066
size of test: 0.0020000000000000018
```



T Error Case:
n=25, mean=6.627049466024269, var=4.000635431527382
size of test: 0.991
n=50, mean=9.186082838319468, var=4.161971560960327
size of test: 0.957
n=100, mean=12.504484595267229, var=4.389229381841637
size of test: 0.5329999999999999



7 Question #3(c)

In [146]:
```
sample_sizes = [25, 50, 100] #size of sample
# y = 4 + 2 * x + error
print("N Error Case:")
sample_X = np.random.uniform(0, 3, 25)
sample_n_error = np.random.normal(0, 1, 25)
sample_y_n = 4 + 2*sample_X + sample_n_error
result_original = sm.OLS(sample_y_n, sm.add_constant(sample_X)).fit()
beta_1 = result_original.params[0]
beta_2 = result_original.params[1]
u_hat = sample_y_n - beta_1 - beta_2 * sample_X

coef = []
for t in range(1000):
    u_star = np.random.choice(u_hat, 25)
    y_star = beta_1 + beta_2 * sample_X + u_star
    result_bootstrap = sm.OLS(y_star, sm.add_constant(sample_X)).fit()
    coef.append(result_bootstrap.params[1])
print("sample beta:", str(beta_2))
print("bootstrap beta:", str(np.mean(coef)))
print("bootstrap se:", str(math.sqrt(np.var(coef))))


print("T Error Case:")
sample_X = np.random.uniform(0, 3, 25)
sample_t_error = np.random.standard_t(4, 25)
sample_y_t = 4 + 2*sample_X + sample_t_error
result_original = sm.OLS(sample_y_t, sm.add_constant(sample_X)).fit()
beta_1 = result_original.params[0]
beta_2 = result_original.params[1]
u_hat = sample_y_n - beta_1 - beta_2 * sample_X

coef = []
for t in range(1000):
    u_star = np.random.choice(u_hat, 25)
    y_star = beta_1 + beta_2 * sample_X + u_star
    result_bootstrap = sm.OLS(y_star, sm.add_constant(sample_X)).fit()
    coef.append(result_bootstrap.params[1])
print("sample beta:", str(beta_2))
print("bootstrap beta:", str(np.mean(coef)))
print("bootstrap se:", str(math.sqrt(np.var(coef))))
```

```
N Error Case:
sample beta: 1.7102102215594508
bootstrap beta: 1.7159064524085028
bootstrap se: 0.1928669790849424
T Error Case:
```

```
sample beta: 1.2898398249410796
bootstrap beta: 1.2824862775758408
bootstrap se: 0.5631121845139818
```