ECON200A (FALL 2015): MICROECONOMIC THEORY
FINAL

Problem 1. Let $L \in \mathbb{N}$, and $\succcurlyeq$ be a preference relation on $X = \mathbb{R}^L$ represented by a utility function $U(x) = \sum_{i=1}^{L} u_i(x_i)$, where u_i is a function from $\mathbb{R}$ to $\mathbb{R}$ for each $i \in [L]$. Choose two from the following three claims, and prove or disprove them. In the following claims, $e^i \in \mathbb{R}^L$ denotes the ith unit vector for each $i \in [L]$.

(1) If $x \succcurlyeq y$, then $x + \epsilon e^i \succcurlyeq y + \epsilon e^i$ for any $\epsilon > 0$ and $i \in [L]$.

(2) If $i \neq j$, then $\alpha e^i \sim \beta e^j$ and $\gamma e^i \sim \delta e^j$ imply $\alpha e^i + \delta e^j \sim \beta e^j + \gamma e^i$.

(3) If u_i is quasi-concave for all $i \in [L]$, then $\succcurlyeq$ is convex.

Problem 2. Let $\mathcal{A}$ be the collection of all compact intervals in $\mathbb{R}$, and C a choice correspondence on $\mathcal{A}$ such that $C([a, b]) = \{x : b \geq x \geq (a + b)/2\}$ for any $a, b \in \mathbb{R}$ with $b \geq a$. Prove or disprove each of the following claims.

(1) $\{S, T\} \subseteq \mathcal{A}$ and $T \subseteq S$ imply $C(S) \cap T \subseteq C(T)$.

(2) $\{S, T\} \subseteq \mathcal{A}$, $T \subseteq S$, and $C(S) \cap T \neq \emptyset$ imply $C(T) \subseteq C(S)$.

(3) $\{S, T\} \subseteq \mathcal{A}$ and $S \cup T \in \mathcal{A}$ imply $C(S) \cap C(T) \subseteq C(S \cup T)$.

(4) $S \in \mathcal{A}$ implies $C(C(S)) = C(S)$.

Problem 3. Suppose that a firm has a production function $f(x_1, \ldots, x_{L-1}) = \sum_{i=1}^{L-1} a_i \sqrt{x_i}$, where $a_i > 0$ for all $i \in [L-1]$. The output is the Lth good.

(1) Compute the firm's supply function.

(2) Compute the firm's profit function.

(3) Verify the Hotelling's lemma.

Problem 4. Consider the following five lotteries. Draw graphs of cumulative distribution functions for lotteries $\mathbf{p}^1$, $\mathbf{p}^2$, and $\mathbf{p}^3$. (Draw one graph for each lottery.) Give the full description of FSD relation for the five lotteries.

$$\mathbf{p}^1 = \delta_0, \qquad \mathbf{p}^2 = \frac{1}{2}\delta_1 \oplus \frac{1}{2}\delta_4, \qquad \mathbf{p}^3 = \frac{1}{3}\delta_0 \oplus \frac{1}{3}\delta_2 \oplus \frac{1}{3}\delta_4,$$
$$\mathbf{p}^4 = \frac{1}{3}\delta_4 \oplus \frac{2}{3}\delta_6, \qquad \mathbf{p}^5 = \frac{1}{3}\delta_0 \oplus \frac{2}{3}\delta_4$$

Problem 5. Suppose that a monopolistic firm produces a good under a cost function $c(q) = \frac{1}{16}q^2$, while the inverse demand function for the good is given by $p = 2q^{-1/2}$. Compute the monopoly price and monopoly quantity. Also, we can write the size of dead weight loss due to monopoly as $a\sqrt[3]{2} - b$. Answer the values of a and b.