The Adverse Selection Problem

Motivating examples: The Market for Lemons (Akerlof, 1970)

Similar problem arises in insurance markets (Rothschild and Stiglitz, 1976)

Key problem: if there are agents of different types in the market, an uniform (average) price creates inefficiency and 'adverse selection'

Adverse Selection

The agent has private information about her type before the contracting stage.

If only one contract is to be offered, then only the ‘wrong type’ might end up entering the market. This is called the ‘adverse selection’ problem.

To mitigate this problem, optimal contract tries to differentiate between the types; in essence, we are looking for a ‘separating’ equilibrium; multiple contracts will be offered in equilibrium.

Two types of problems:

The party with the private information offers the contract: signaling. Eg. If there is uncertainty in the buyer's mind about the quality of the product, the seller offers warranties of different length to signal quality.

The uninformed party offers the contract: screening. A seller who is unsure about the buyer's type offers multiple contracts, buyer then self-selects into one of them.

We will be looking at screening problems. You might have seen 'price discrimination' or 'non-linear pricing' models.

This exposition follows Bolton & Dewatripont, Ch 2 (Based on Maskin and Riley, 1984).

Like with any asymmetric information problem, we start we the analysis of the first best.

Consider a transaction between buyer and seller. Seller does not know buyer's willingness to pay.

Buyer's preferences:

$$u(q, T, \theta) = \int_0^q P(x, \theta) dx - T$$

$q=$ number of units purchased

T =total amount paid to the seller

$P(x, \theta)$ is the inverse demand curve of the buyer with preference characteristics θ

θ is unknown to the seller. He only knows the distribution $F(\theta)$.

We will consider the following functional form:

$$u(q, T, \theta) = \theta v(q) - T$$

$v(0)$=0, $v'(q) >$) and $v''(q) < 0$ for all q

Seller's unit production costs are $c > 0$

Seller's profit from selling q units against a sum of money T:

$$\pi = T - cq$$

Question: What is the profit maximizing (T, q) pair that the buyer will be induced to buy?

First Best Outcome

We start with the example with only two types of buyers, $\theta \in \{\theta_H, \theta_L\}$ with $\theta_H > \theta_L$

$\beta=$ probability of type θ_L, and $1-\beta=$ probability of type θ_H

Seller's profit $= \beta(T_L - cq_L) + (1-\beta)(T_H - cq_H)$

If seller knows θ, he can offer the buyer a type-specific contract (maximize each part of the profit expression separately):

(T_i, q_i) for type θ_i, where $i \in \{H, L\}$

by solving

$$\max_{T_i, q_i} T_i - cq_i$$

subject to

$$\theta_i v(q_i) - T_i \geq \bar{u}$$

The constraint is the Individual Rationality (IR) or Participation Constraint (PC).

Profit maximization under full information will imply binding participation constraint for each type.

We can replace the constraint into the objective function for each type and get the solution.

The solution to the problem will be type-specific contracts $(\tilde{T}_i, \tilde{q}_i)$ such that

$$\theta_i v'(\tilde{q}_i) = c$$

and

$$\theta_i v(\tilde{q}_i) = \tilde{T}_i + \bar{u}$$

These type specific contracts are implementable by a type-specific two-part-tariff , buyer pays per unit price c and a type specific fixed fee $F = \theta_i v(\tilde{q}_i) - c\tilde{q}_i - \bar{u}$

So $\tilde{T}_i = c\tilde{q}_i + F$

We will set $\bar{u} = 0$ for simplicity from now on.

Second Best Outcome:Optimal Non Linear Pricing

Now consider the case where seller does not observe the type of the buyer, so he can only offer a **set of contracts** or a **menu** independent of her type, $[q, T(q)]$

Buyer chooses among the contracts that maximizes her payoff.

Seller's problem:

$$\max_{T(q)} \beta(T(q_L) - cq_L) + (1-\beta)(T(q_H) - cq_H)$$

subject to

$$q_i = \operatorname*{argmax}_q \theta_i v(q) - T(q)$$

for $i = L, H$

and

$$\theta_i v(q_i) - T(q_i) \geq 0$$

for $i = L, H$

Step-by-step Procedure

Step 1: Apply the Revelation Principle

On a general level, a contract is a game designed by the principal and played by the agent or agents. It designates the strategies available to the agents as well as payoffs from the realizations of the strategies. These are 'indirect' mechanisms.

In a thought experiment, imagine the principal using a 'direct' mechanism. In such a mechanism, agents report their 'types' to a neutral intermediary, and the intermediary implements a type-based strategy for the agent. Agent gets the payoff from the strategy.

Revelation Principle

Direct Mechanisms are interesting because of the 'Revelation Principle', proved by several people around the same time, including Gibbard (1973), Meyerson (1979) and Maskin (1981).

It says that for any Bayesian Nash equilibrium for the indirect mechanism, there is a payoff equivalent direct mechanism that implements the same equilibrium.

In contract settings, that hugely simplifies the class of contracts we need to look into.

In particular, for adverse selection problems, we can restrict ourselves to contracts that only specifies one strategy for each 'type'.

Take the example of only two types; write down the problem with appropriate IC's and PCs.

Step 2: Observe that the participation constraint of the 'high' type will not bind at the optimum, given the PC of low type and IC of high type are satisfied.

$$\theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L > \theta_L v(q_L) - T_L \geq 0$$

Step 3: Find the incentive constraint that is satisfied at the first best optimum, and solve a 'relaxed' problem by omitting it. (You need to check afterwards whether the solution you get indeed satisfies the omitted IC)

In our example, if the first best two-part tariff is offered, which type would have the incentive to deviate?

Recall, the FB solution has

$$\theta_i v'(\tilde{q}_i) = c$$

and

$$\theta_i v(\tilde{q}_i) = T_i$$

This outcome is not incentive compatible. The high type buyer would prefer to choose $(\tilde{q}_L, \tilde{T}_L)$, rather than her own FB allocation, and will gain surplus $(\theta_H - \theta_L)v(q_L)$. Although it will restrict her consumption at an inefficient level. So, the FB contract gives high type buyer the incentive to deviate.

On the other hand, low type buyer does not have incentive to deviate; if she chooses (q_H, T_H), she would earn negative surplus. (Check). So ICL will not bind in the optimum.

Hence the IC of the 'low' type is omitted in this case.

Warning: for each problem you solve, you will always have to check which type it is that has incentive to deviate, it will vary from problem to problem.

Note the importance of the 'Spence-Mirrlees single crossing condition'. Here is one version of this:

$$\frac{\delta}{\delta\theta}[-\frac{\frac{\delta u}{\delta q}}{\frac{\delta u}{\delta T}}] > 0$$

Now solve the relaxed problem.

Note that the remaining constraints would bind.

Again, using our familiar variational argument, If ICH didn't bind, seller could increase his profits by raising T_H until it did bind.

Similarly if PCL dis not bind, T_L could be raised till it did bind.

Eliminate T_L and T_H from the maximand using the two binding constraints, perform the unconstrained optimization, and check that the ICL is indeed satisfied.

Substituting for the values of T_H and T_L in the seller's objective function, we obtain

$$\max_{q_L, q_H} \beta[\theta_L v(q_L) - cq_L] + (1-\beta)[\theta_H v(q_H) - cq_H - (\theta_H - \theta_L)v(q_L)]$$

Observe carefully the two terms.

The first term includes the full surplus of the low type (recall her PC is binding).

The second term shows the full surplus of the high type, *minus her information rent*.

High type's information rent is $(\theta_H - \theta_L)v(q_L)$, this is due to the fact that she has incentive to mimic the behavior of low type.

Note that the rent is increasing in q_L.

The interior solution (if it exists) (q_L^*, q_H^*) to the relaxed program are characterized by,

$$\theta_H v'(q_H^*) = c$$

$$\theta_L v'(q_L^*) = \frac{c}{1 - [\frac{1-\beta}{\beta} \frac{\theta_H - \theta_L}{\theta_L}]}$$

If the denominator for the second FOC is negative, then $q_L^* = 0$

At the interior solution, $q_L^* < q_H^*$

The SB consumption of the high type is the same as FB consumption, but for the low type, SB consumption is lower than FB.

So, the high type earns information rent, and low type's consumption is distorted compared to the first best level.