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Taghi Farzad

University of California, Riverside

1. Prove $\nexists A \;\; s.t. \quad A \in A$; i.e. no set is an element of itself.

2. Prove for all A and B

$$\bar{A} \cap \bar{B} = \overline{A \cup B}$$

3. With a counterexample show the intersection of infinitely many open sets is not open.

4. Set up the Hessian matrix for the Cobb-Douglas production function $F(K, L) = K^{\alpha} L^{1-\alpha}$ and argue whether it is positive or negative (semi) definite.

5. A consumer has a total wealth of W and chooses a bundle (x_1, x_2, x_3). Show the Budget Set is convex.

6. Consider the utility maximization problem for an agent with $U = x_1^{\alpha} x_2^{1-\alpha}$ and total wealth of W. Price of good one and two are p_1 and p_2, respectively.

   (a) Derive the demand functions.

   (b) Find the maximum level of attainable utility.

   (c) Set up the indirect utility and derive the first derivate of it with respect to wealth and prices. (*Hint: use Envelope Theorem*)

   (d) Set the Dual problem; the expenditure minimization problem when the utility is greater than or equal to $\bar{U}$.

   (e) Find the expenditure minimizing bundle; (h_1, h_2). Find the minimum expenditure (e) when $U \geq \bar{U}$.

   (f) Show when $\bar{U}$ is the same as utility level in (b), then $e = W$.

7. Log-linearize the following Euler Equation around the steady state:

$$\frac{c_t^{-\sigma}}{\beta c_{t+1}^{-\sigma}} = (1 + r_t)$$

8. Find the eigenvalues and eigenvectors for the following matrix.

$$A = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$$

Show A can be written as $A = PDP^{-1}$; where P is an invertible and D is a diagonal matrix.