Electric Ventricular Assist Device (EVAD) for cardiological illness

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Abstract

This report examines the application of control systems in medical devices, focusing on an Electrical Ventricular Assist Device (EVAD) designed for cardiology patients. Using control theory, we analyse the system's behaviour through transfer functions and Nyquist plots, Thus we are able to explore the system's response to the cardiac cycle's variability. Through this analysis we aim to ensure that EVAD operates in a way that improves cardiac function and patient outcomes.

1 Introduction

With cardiovascular disease as the leading cause of mortality globally, the advancement in supportive devices such as EVADs offers a significant increase in life expectancy and quality of life for patients[1]. We will focus on the mathematical modelling of the device's feedback system, represented by a transfer function that includes a time delay to mimic the cardiac cycle. The main part of the discussion will include conditions for stability; the analysis of system poles; the interpretation of Nyquist plots. This will provide insight into the system's frequency response and its implications for device performance. By integrating control system theory with practical medical applications, the report aims to bridge the gap between mathematical concepts and their effect on patient care within the realm of cardiac assist devices.

2 Model of the feedback control system

The EVAD has a single input, the applied motor voltage, and a single output, the blood flow rate. The control system of the EVAD performs two main tasks. Firstly, it adjusts the motor voltage to drive the pusher plate through its desired stroke. Secondly, it varies the EVAD blood flow to meet the body's cardiac output demand. The blood rate is adjusted by varying the EVAD beat rate[2]. The pump has transfer function $G(s) = e^{-sT}$ where T>0 and $K(s) = \frac{a}{s(s+b)}$ where a,b>0.

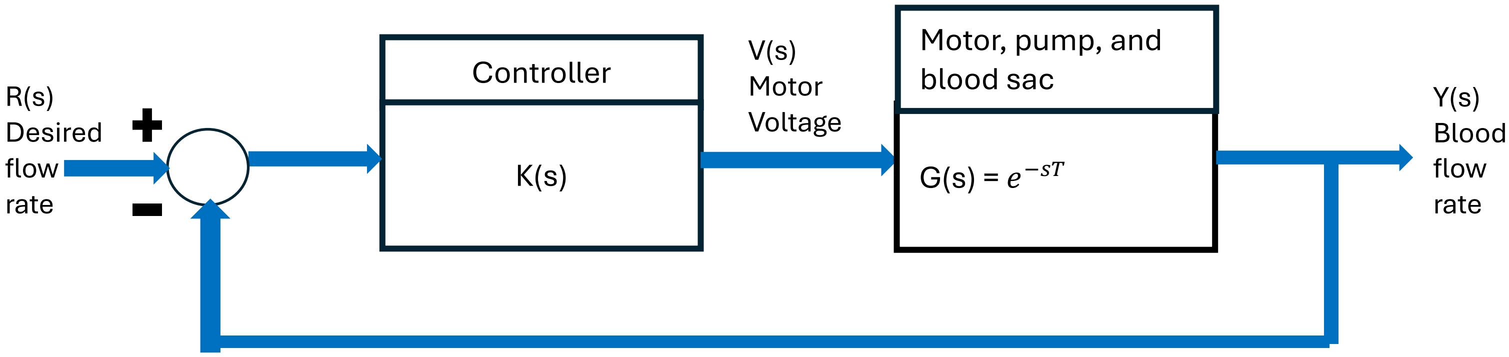

Figure 1: feedback control system of EVAD

3 Discussion

i) Calculating the transfer functions

EVADs are complex systems involving mechanical, electrical, and biological components. Transfer functions simplify this complexity by representing the relationship between input and output in a manageable form. We will consider 4 transfer functions.

We calculate the transfer functions for:

a) Transfer function 1: $\frac{1}{1+K(s)G(s)}$, b) Transfer function 2: $\frac{G(s)}{1+K(s)G(s)}$,
c) Transfer function 3: $\frac{K(s)}{1+K(s)G(s)}$, d) Transfer function 4: $\frac{K(s)G(s)}{1+K(s)G(s)}$.

a) $\frac{1}{1+K(s)G(s)} = 1/(1+\frac{a}{s(s+b)} * e^{-sT}) = \frac{1}{1+\frac{ae^{-sT}}{s(s+b)}} = \frac{1}{\frac{s(s+b)}{s(s+b)}+\frac{ae^{-st}}{s(s+b)}} = \frac{s(s+b)}{s(s+b)+ae^{-sT}}$

For T=1, a=5, b=10 transfer function 1 is : $\frac{1}{1+K(s)G(s)} = \frac{s(s+10)}{s(s+10)+5e^{-s}}$

b) $\frac{G(s)}{1+K(s)G(s)} = \frac{e^{-sT}}{1+\frac{a}{s(s+b)}*e^{-sT}} = \frac{e^{-sT}}{\frac{s(s+b)}{s(s+b)}+\frac{ae^{-sT}}{s(s+b)}} = \frac{s(s+b)e^{-sT}}{s(s+b)+ae^{-sT}}$

For T=1, a=5 and b =10, transfer function 2 is : $\frac{G(s)}{1+K(s)G(s)} = \frac{s(s+10)e^{-s}}{s(s+10)+5e^{-s}}$

c) $\frac{K(s)}{1+K(s)G(s)} = \frac{\frac{a}{s(s+b)}}{1+\frac{a}{s(s+b)}*e^{-sT}} = \frac{\frac{a}{s(s+b)}}{\frac{s(s+b)}{s(s+b)}+\frac{ae^{-sT}}{s(s+b)}} = \frac{a}{s(s+b)+ae^{-sT}}$

For T=1, a=5, b=10, transfer function 3 is : $\frac{K(s)}{1+K(s)G(s)} = \frac{5}{s(s+10)+5e^{-s}}$

d) $\frac{K(s)G(s)}{1+K(s)G(s)} = \frac{\frac{a}{s(s+b)} * e^{-sT}}{1+\frac{a}{s(s+b)} * e^{-sT}} = \frac{\frac{ae^{-sT}}{s(s+b)}}{\frac{s(s+b)+ae^{-sT}}{s(s+b)}} = \frac{ae^{-sT}}{s(s+b)+ae^{-sT}}$

For T=1, a=5, b=10, transfer function 4 is: $\frac{K(s)G(s)}{1+K(s)G(s)} = \frac{5e^{-s}}{s(s+10)+5e-s}$

ii) Conditions that give a pole for the transfer function.

The conditions that produce a pole of a transfer function is when we set the denominator of the transfer function equal to zero. The denominator for the closed-loop transfer function is 1+K(s)G(s) where K(s) is the controller function and G(s) is the plant transfer function. So, we have the equation 1+ K(s)G(s)=0. The equation arises from the feedback loop where the output of K(s)G(s) product is subtracted from the input signal, and the system's stability is determined by the roots of this equation.

Next, the denominator is:

1 + K(s)G(s) = $1 + \frac{a}{s(s+b)} * e^{-sT} = \frac{s(s+b)}{s(s+b)} + \frac{a}{s(s+b)} * e^{-sT} = \frac{s(s+b)+a}{s(s+b)} * e^{-sT} = \frac{s(s+b)+ae^{-sT}}{s(s+b)} = 0$

Then, the equation is : s(s+b) + ae^{-sT} = 0.

For T=1, a=5, b=10: s(s+10) + $5e^{-s}$ = 0. We solve this equation to find the pole.

iii) Determining the poles for a specific function

To determine if there are poles on the real axis or in the right half-plane(RHP) , we plot f(s) along this contour. If the plot crosses the real axis, it indicates a real pole. If the plot encircles the origin, it indicates that poles exist in the RHP by the argument principle. The presence of poles in the RHP would indicate that the system is unstable.

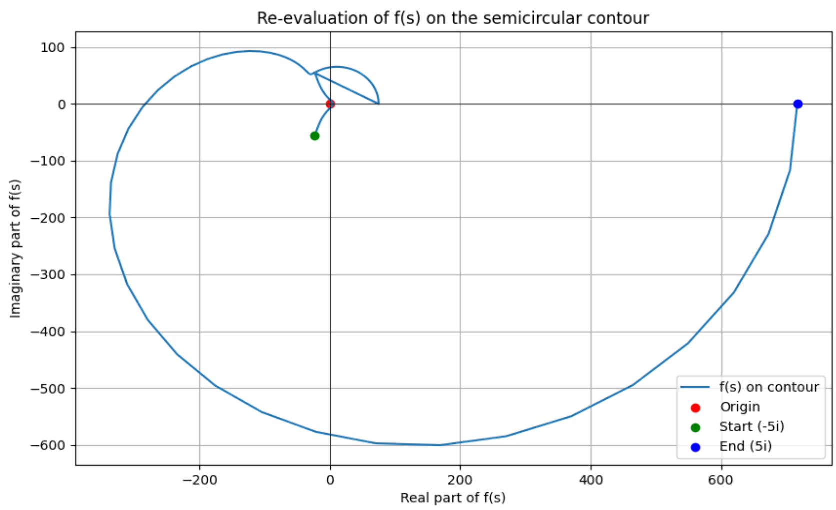

Figure 2: Plot of the function f(s) = s(s+10) + $5e^{-s}$ for s on the semicircular contour made of f [−5i, 5i] ⊕ S_5 using python.

The values of the real axis crossings are the real parts of f(s) where the imaginary part is zero. However, since these parts are positive and not close to zero (75.0 and 717 to 3sf), they do not indicate the presence of poles. Additionally, as the critical point (-1,0) appears not to be encircled by the plot. So, we can infer that there are no poles of $\Re > 0$ within the semicircle of radius 5, centred on the origin in the right half-plane. This is an indication that the system might be stable.

We need to carry out further investigation to definitively conclude that the system is stable i.e. EVAD's are stable.

iv) Considering the solution to an equation in relation to the zeros of the function

The equation $\frac{1}{s(s+10)e^{s}+4}$ = -1 ↔ 1 = $-s(s + 10)e^{s} - 4$ ↔ $5 + s(s + 10)e^{s}$ = 0.
Since this is a transcendental equation as it involves the product of a polynomial and an exponential function of s. We can first plot the graph to see where the function crosses the x-axis.

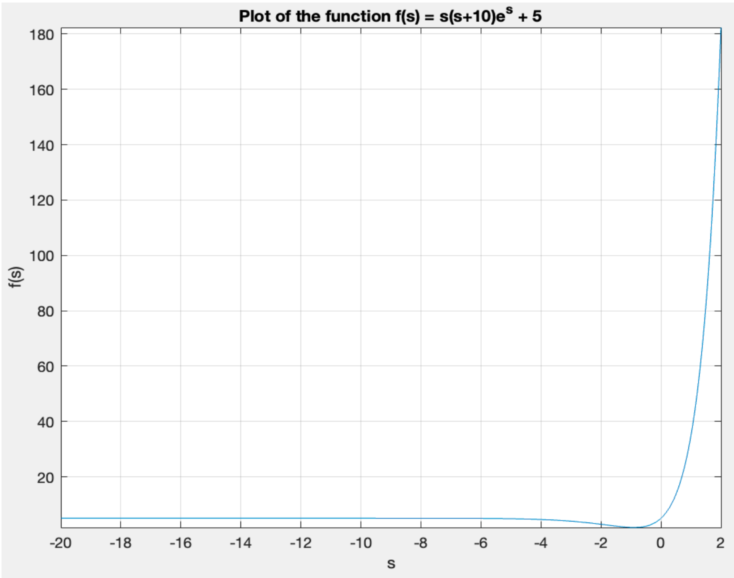

Figure 3: The Nyquist plot for transfer function 1, generated using MATLAB.

Based on the graph we have that the solution crosses the x-axis between 0 and -2. We can use numerical methods in MATLAB to find a solution between 0 and -2. Using numerical methods in MATLAB, we have that the solution is s=-0.90181623. The solution is negative, indicating that the poles would be in the left half plane. The system is stable. This means that the system's response to any bounded input will be bounded, and the system will return to equilibrium after being disturbed.

(v)Producing Nyquist style plots

The four Nyquist plots below shows the four different transfer functions associated with cardiac assist device e.g. ventricle assist device, which is vital in treating patients with cardiovascular diseases. The plots reflect how the system might respond to various frequencies and provide insight into the stability and efficacy of the device. Some of the points that we will discuss include Nyquist criterion, gain margin, and phase margin. Gain margin can be determined by the distance from the plot to the point (-1,0) when the phase is -180 degrees. The phase margin is the additional phase needed to make the plot pass through (-1,0) point when the gain is 1 (0 dB). The Nyquist plots are plotted with their time delays.

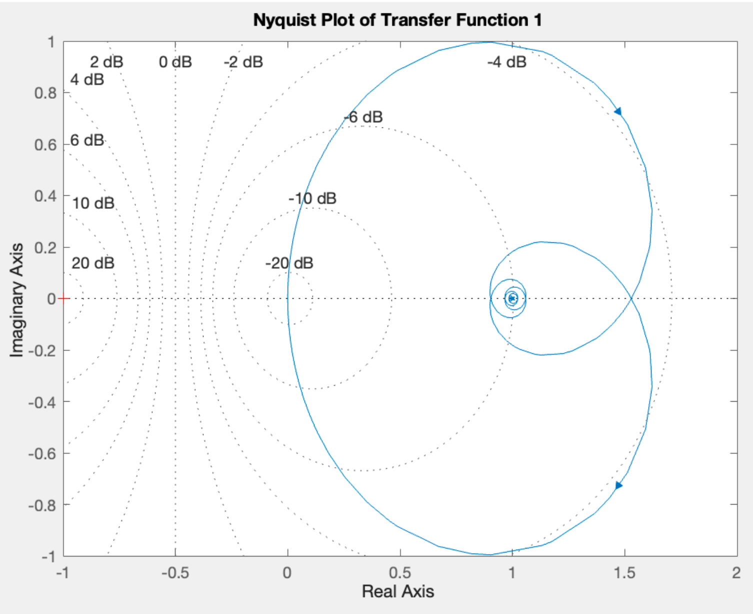

Figure 4: The Nyquist plot for transfer function 1, generated using MATLAB.

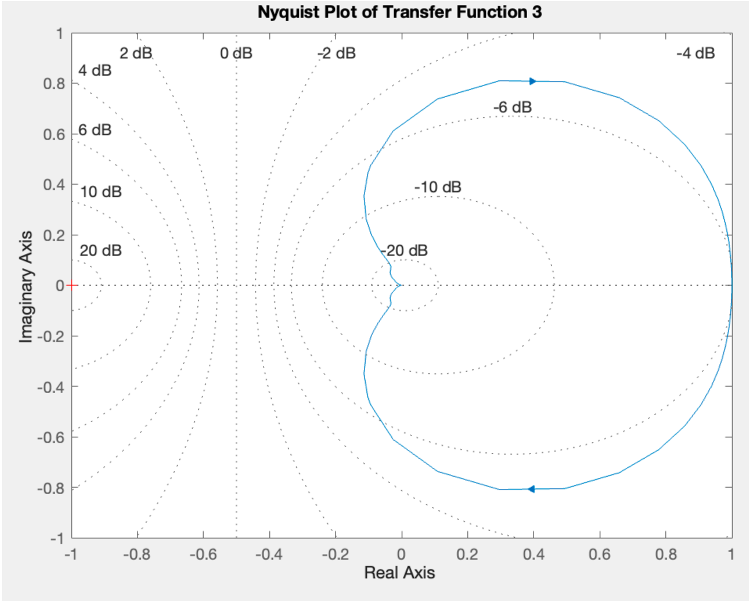

Figure 6: The Nyquist plot for transfer function 3, generated using MATLAB.

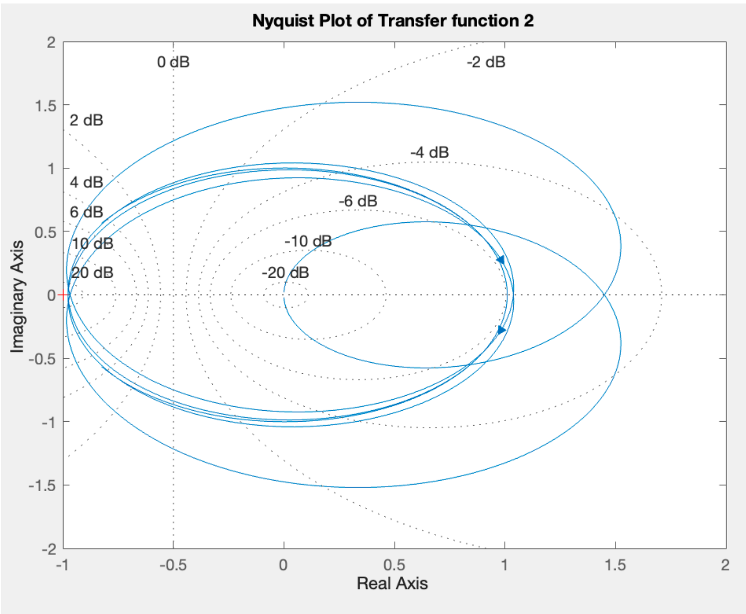

Figure 5: The Nyquist plot for transfer function 2, generated using MATLAB [3].

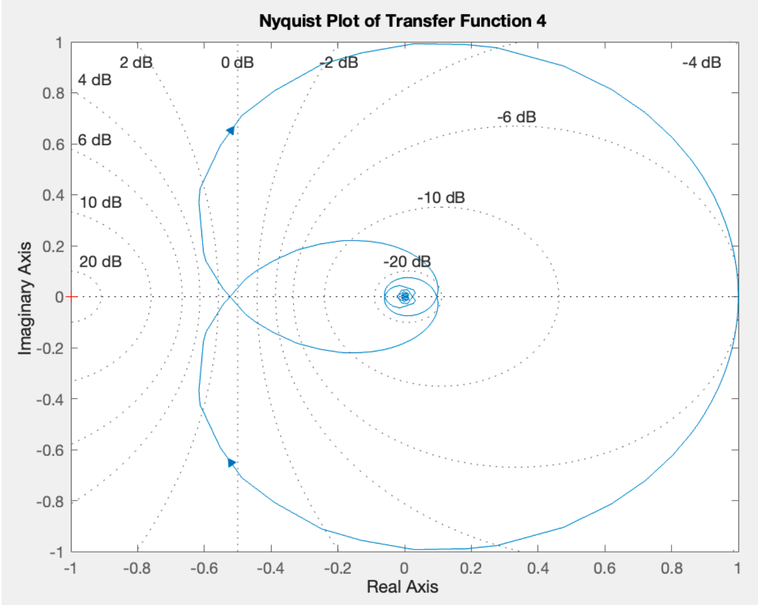

Figure 7: The Nyquist plot for transfer function 4, generated using MATLAB.

Figure 4 indicates that the critical point (-1,0) is not encircled, which suggests that the closed-loop system should be stable if there are no open-loop right -half-plane poles. Since the plot doesn't encircle the critical point, it indicates that there is no excess of poles over zeros in the right half-plane (RHP). The loop extends into the right half plane (RHP) of the complex plane and then loops back, but without encircling the critical point. Also, from part 3)iii, we know there are no solutions in the RHP. This implies stability. The small loop near the origin might represent the system's response to higher frequencies or could be indicative of a minor pole-zero cancellation effect within the system. However, it's limited radius suggests that these effects are relatively contained and may not significantly impact system stability. Additionally, the plot passes close to the -20dB circle, which means that at some frequencies, the system attenuates the signal to the 1/10th of its input. The plot does not cross 0dB circle, indicating the system has infinite phase margin. Also, the transfer function does not appear to cross the negative real axis, so the system has infinite gain margin. Thus, the plot suggests that the transfer function has a large region of stability with both high gain and phase margins.

Figure 5 was plotted over a frequency range to ensure a clearer Nyquist plot [3]. The plot shows no encirclements of the critical point, suggesting stability if there are no poles in RHP (there are none as suggested from part 3)iii)The plot intersects the 0db line at two points, indicating gain crossover frequencies- where the open-loop system has unity again. The points where the Nyquist plot intersects the -20dB circle indicate frequencies where the system attenuates the input signal by a factor of 10. There is an absence of a phase crossover (where the phase angle is -180 degrees), the gain margin is infinite, which indicates a stable system. Since the plot does not intersect the 0dB circle, the system would have an infinite margin at the gain crossover frequency, again suggesting stability. Overall, it is suggested the system is stable.

In figure 6, the plot does encircle the critical pint (-1,0) , indicating that the system is stable. The outer loop of the plot passes through the 20 dB and 10 dB circles. This suggests that at certain frequencies, the system output can be significantly larger than the input, indicative of high gain at these frequencies. As the plot spirals inward, the gain decreases, as shown by the path moving towards the centre, passing the 0 dB circle and into the negative dB region. According to Nyquist criterion, the lack of encirclement indicates that the system should be stable. A significant distance from the point is an indication of a stable system with a healthy gain margin. Since this plot does not cross the negative real axis, the system has an infinite gain margin, which implies high stability. The plot does not cross the unit circle, which indicates infinite phase margin- meaning the system can tolerate any amount of additional phase lag without becoming unstable. The plot indicates a system that is stable with infinite gain and phase margins, which is ideal for most control systems.

Similarly, in figure 7, the plot does not encircle the critical point (-1,0), on the complex plane. Using the Nyquist criterion, we can infer that the closed-loop system associated with transfer function 4 does not have any unstable poles if the open-loop system is stable. As the plot doesn’t pass through the -180-degree phase line, the system has infinite gain margin, indicating a stable gain characteristic. Also, it appears from the plot that the phase margin is positive since the plot intersects the 0 dB line to the right of the -1 point, indicating stability. Moreover, there seems to be a significant phase margin, implying good stability.

4 Conclusion

From our discussion we can conclude that EVADs modelled in these different ways would operate well as a cardiac assist device. This is further solidified by the Nyquist plots, which all indicate stability. Thus, EVADs would not cause instability in the heart rhythm, which is critical for patient safety [4].

<u>References</u>

[1] Cardiovascular diseases (cvds) (no date) World Health Organization. Available at: https://www.who.int/news-room/fact-sheets/detail/cardiovascular-diseases-(cvds) (Accessed: 05 March 2024).

[2] Dorf, R. and Bishop, R.H. (2011) *Modern Control Systems*. 12th edn. Prentice Hall: Pearson (For the general control systems concepts applied to the EVAD).

[3]*Sys* (no date) *Nyquist plot of frequency response - MATLAB*. Available at: https://www.mathworks.com/help/ident/ref/dynamicsystem.nyquist.html (Accessed: 09 March 2024).

[4]*Ventricular assist device (VAD)* (2023) *Mayo Clinic*. Available at: https://www.mayoclinic.org/tests-procedures/ventricular-assist-device/about/pac-20384529 (Accessed: 09 March 2024).