Runtime Analysis

a) $T(n)=\sum_i \theta(1)$

k	0	1	2	3	4	...	
i	2	4	16	256	...	...	$i=2^{2^k}$

runs until
$i<n$

$2^{2^k}<n \rightarrow \sum_{k=0}^{n-1}\theta(2^{2^k})$

$= \sum_{k=0}^{n-1}\theta(\log\log(2^{2^k}))$

$= \boxed{\theta(\log\log(n))}$

b) $T(n)=\sum_{i=1}^{n}(\theta(1)+\sum_i\sum_{k=0}^{i^3}\theta(1))$

$= \underbrace{\sum_{i=1}^{n}\theta(1)}_{\theta(n)} + \sum_i\sum_{k=0}^{i^3}\theta(1)$

$\sum_i$ is for if statement

k	1	2	3	4	...	
i	$1\sqrt{n}$	$2\sqrt{n}$	$3\sqrt{n}$	$4\sqrt{n}$	...	$i=k\sqrt{n}$

$T(n)=\theta(n)+\sum_{k=1}^{\sqrt{n}}\sum_{k=0}^{i^3}\theta(1)$

$=\theta(n)+\sum_{k=1}^{\sqrt{n}}\theta(i^3)$ $i=k\sqrt{n}$

$=\theta(n)+\sum_{k=1}^{\sqrt{n}}\theta(k\sqrt{n})^3$

$=\theta(n)+(\sqrt{n})^3\sum_{k=1}^{\sqrt{n}}\theta(k^3)$ → use general form of arithmetic series

$=\theta(n)+(n^{3/2})\,\theta(\sqrt{n})^4$

$=\theta(n)+(n^{3/2})\,\theta(n^{4/2})$ $p+1 \rightarrow 3+1=4$

$=\boxed{\theta(n^{7/2})}$

c) $T(n)=\underbrace{\sum_{i=1}^{n}\sum_{k=1}^{n}}_{\theta(n^2)}(\theta(1)+\sum_i\sum_{m=1,2,4,8}^{n}\theta(1))$

↳ $m *= m$
↳ $m=1,2,4,8,\ldots$

$\sum_i$ is for if statement
if (p[k]==i) → worst case is $\theta(n)$ or n

$=\theta(n^2)+n\sum_{m=1,2,4,8,\ldots}^{n}\theta(1)$

x	0	1	2	3	4	...	
m	1	2	4	8	16	...	$m=2^x$

$2^x<n$ ← upper bound
$\log(2^x)<\log(n)$

$=\theta(n^2)+n\log(n)$

$=\boxed{\theta(n^2)}$

d) $T(n)=\sum_{i=0}^{n-1}(\theta(1)+\sum_i\sum_{j=0}^{size-1}\theta(1))$

$=\underbrace{\sum_{i=0}^{n-1}\theta(1)}_{\theta(n)}+\sum_i\sum_{j=0}^{size-1}\theta(1)$

$\sum_i$ for if statement
if (i==size)

$i=size=10 \rightarrow size=10\cdot(\frac{3}{2})$

$i=size=10\cdot\frac{3}{2}\rightarrow size=10\cdot(\frac{3}{2})^2$

$i=size=10\cdot(\frac{3}{2})^2\rightarrow size=10\cdot(\frac{3}{2})^3$

$i=size=10\cdot(\frac{3}{2})^3\rightarrow size=10\,(\frac{3}{2})^4$

$i=10\cdot(\frac{3}{2})^k$
↳ $i<n$ ← upper bound

$10\cdot(\frac{3}{2})^k<n$

$k<\log_{\frac{3}{2}}(n/10)$

$=\theta(n)+\sum_{i=0}^{\log_{3/2}(n/10)}\sum_{j=0}^{size-1}\theta(1)$

$=\theta(n)+\sum_{i=0}^{\log_{3/2}(n/10)}\theta(size)$ size $=i$

$=\theta(n)+\sum_{i=0}^{\log_{3/2}(n/10)}(10\cdot\frac{3}{2}^k)$

$=\theta(n)+10\sum_{i=0}^{\log_{3/2}(n/10)}(\frac{3}{2})^k$

use geometric series

$\theta(n)+\dfrac{(\frac{3}{2})^{\log_{3/2}(n/10)+1}-1}{\frac{3}{2}-1}$

$=\theta(n)+\theta(\frac{3}{2})^{\log_{\frac{3}{2}}(n/10)}$

$=\theta(n)+\theta(n)=\boxed{\theta(n)}$