$$
\begin{aligned}
\frac{dS}{dt} &= -\beta SI, \\
\frac{dI}{dt} &= \beta SI - \gamma I, \\
\frac{dR}{dt} &= \gamma I.
\end{aligned}
$$

$$
\begin{aligned}
\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} &= 0 \\
S + I + R &= 1
\end{aligned}
$$

$$\frac{dS}{dt} = -\beta SI + \gamma I$$
$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dS}{dt} + \frac{dI}{dt} = 0$$
$$S + I = 1$$

$$
\begin{aligned}
\frac{dS}{dt} &= -\beta SI, \\
\frac{dE}{dt} &= \beta SI - \sigma E, \\
\frac{dI}{dt} &= \sigma E - \gamma I, \\
\frac{dR}{dt} &= \gamma I.
\end{aligned}
$$

$$
\begin{aligned}
\frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dR}{dt} &= 0 \\
S + E + I + R &= 1
\end{aligned}
$$

$$
\begin{aligned}
\frac{dS}{dt} &= -\beta SI - \epsilon\nu S, \\
\frac{dI}{dt} &= \beta(S+V)I - \gamma I, \\
\frac{dR}{dt} &= \gamma I + \delta V, \\
\frac{dV}{dt} &= \epsilon\nu S - (\beta I + \delta)V.
\end{aligned}
$$

$$
\begin{aligned}
\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} + \frac{dV}{dt} &= 0 \\
S + I + R + V &= 1
\end{aligned}
$$

$$x_{k+1} = x_k + \frac{dx}{dt} \mid_k \Delta t, \quad \text{for steps } k = 0, 1, 2, ...$$

$$
\begin{aligned}
\frac{dS_1}{dt} &= -\frac{\beta S_1}{N_1}(I_1 + I_2\rho_{12}), \\
\frac{dI_1}{dt} &= \frac{\beta S_1}{N_1}(I_1 + I_2\rho_{12}) - \gamma I_1, \\
\frac{dR_1}{dt} &= \gamma I_1, \\
N_1 &= S_1 + I_1 + R_1,
\end{aligned}
$$

$$
\begin{aligned}
\frac{dS_2}{dt} &= -\frac{\beta S_2}{N_2}(I_2 + I_1\rho_{21}), \\
\frac{dI_2}{dt} &= \frac{\beta S_2}{N_2}(I_2 + I_1\rho_{21}) - \gamma I_2, \\
\frac{dR_2}{dt} &= \gamma I_2, \\
N_2 &= S_2 + I_2 + R_2.
\end{aligned}
$$

$$
\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0.
$$

$$
\begin{aligned}
\frac{dS_1}{dt} &= -\frac{\beta S_1 I_1}{N_1} + m_{12}S_2 - m_{21}S_1, \\
\frac{dI_1}{dt} &= \frac{\beta S_1 I_1}{N_1} - \gamma I_1 + m_{12}I_2 - m_{21}I_1, \\
\frac{dR_1}{dt} &= \gamma I_1 + m_{12}R_2 - m_{21}R_1, \\
N_1 &= S_1 + I_1 + R_1,
\end{aligned}
$$

$$
\begin{aligned}
\frac{dS_2}{dt} &= -\frac{\beta S_2 I_2}{N_2} - m_{12}S_2 + m_{21}S_1, \\
\frac{dI_2}{dt} &= \frac{\beta S_2 I_2}{N_2} - \gamma I_2 - m_{12}I_2 + m_{21}I_1, \\
\frac{dR_2}{dt} &= \gamma I_2 - m_{12}R_2 + m_{21}R_1, \\
N_2 &= S_2 + I_2 + R_2.
\end{aligned}
$$

$$I_{reported} = \mathrm{Bin}(p_r, I_{true}) + \mathrm{Bin}(p_m, N - I_{true}),$$

p_r : probability that a case is reported,
p_m : probability that a healthy individual is mis-diagnosed.

$$
\begin{aligned}
\frac{dS}{dt} &= -\frac{\beta(1+\epsilon)SI}{N}, \\
\frac{dI}{dt} &= \frac{\beta(1+\epsilon)SI}{N} - \gamma(1+\xi)I, \\
\frac{dR}{dt} &= \gamma(1+\xi)I.
\end{aligned}
$$

$$
\begin{aligned}
\epsilon &\sim \mathrm{N}(0, \sigma_\epsilon), \\
\xi &\sim \mathrm{N}(0, \sigma_\xi).
\end{aligned}
$$

$$
\begin{aligned}
\frac{dS}{dt} &= -\frac{\beta S I}{N}, \\
\frac{dI}{dt} &= \frac{\beta S I}{N} - \gamma I, \\
\frac{dR}{dt} &= \gamma I.
\end{aligned}
$$

$$
S + I + R = N
$$

$$\Longrightarrow \text{Substitute:} \begin{cases} S \to \frac{S}{N} \\ I \to \frac{I}{N} \\ R \to \frac{R}{N} \end{cases} \quad \Longrightarrow$$

$$
\begin{aligned}
\frac{dS_i}{dt} &= -\sum_{j=1}^{n} \frac{\beta S_i I_j}{N_i} \rho_{ij} \\
\frac{dI_i}{dt} &= \sum_{j=1}^{n} \frac{\beta S_i I_j}{N_i} \rho_{ij} - \gamma I_i \\
\frac{dR_i}{dt} &= \gamma I_i
\end{aligned}
$$

where

$$
\begin{aligned}
&i, j = \{1, 2, ..., n\} \\
&\rho_{ii} = 1 \quad \text{(interaction within patch)}
\end{aligned}
$$

$$\frac{dS_i}{dt} = -\frac{\beta S_i I_i}{N_i} + \sum_{j=1}^{n} m_{ij} S_j - \sum_{j=1}^{n} m_{ji} S_i$$

$$\frac{dI_i}{dt} = \frac{\beta S_i I_i}{N_i} - \gamma I_i + \sum_{j=1}^{n} m_{ij} I_j - \sum_{j=1}^{n} m_{ji} I_i$$

$$\frac{dR_i}{dt} = \gamma I_i + \sum_{j=1}^{n} m_{ij} R_j - \sum_{j=1}^{n} m_{ji} R_i$$

where

where

$$i, j = \{1, 2, ..., n\}$$

$$m_{ii} = 1 - \sum_{j \neq i} m_{ji}$$

(remain in patch)

$$\beta^* = \beta(1+\epsilon)$$
$$\gamma^* = \gamma(1+\xi)$$

1. E_1: transmission, E_2: recovery.
2. $R_1 = \frac{\beta SI}{N}$, $R_2 = \gamma I$.
3. $R_{total} = R_1 + R_2$.
4. $\delta t = \frac{-1}{R_{total}} log(RAND_1)$.
5. $P = RAND_2 \cdot R_{total}$.
6. If $P < R_1$,
   then transmission occurs: $S \to S - 1$, $I \to I + 1$,
   otherwise,
   recovery occurs: $I \to I - 1$, $R \to R + 1$.
7. $t \to t + \delta t$.
8. Return to step 2.