CTA200H project: Polarzied radiative transfer as a probe of cosmic magnetic fields

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The goals of this project are to (i) reinforce your understanding of polarized radiative transfer and its utility in studying cosmic magnetic fields, and (ii) write a code to visualize a real data cube acquired from the Green Bank Telescope (GBT).

Unless otherwise stated, this project uses c.g.s. Gaussian units.

1 Preliminaries

1.1 Software

Writing the code in python 3 in a jupyter notebook is highly recommended for this project but you can use a different language if you prefer.

If you are using python, the first step is to import the necessary packages. This will typically include the numpy and matplotlib packages, which are used for numerical and plotting operations, respectively. The data cube you are working with are in FITS files, so having astropy for working with FITS and getting WCS (World Coordinate System) information would be convenient. Using Anaconda (https://www.anaconda.com/download) for installing packages is usually simple and straight-forward.

1.2 Science

Polarization of radiation is a powerful tool for studying the magnetic fields in the cosmos. As light travels through magnetized regions of space, its polarization properties are altered in ways that depend on the magneto-ionic properties of the medium it passes through. By analyzing these changes in polarization, astronomers can infer the properties of magnetic fields in a variety of astrophysical environments, from the surfaces of stars to the large-scale structure of galaxies. By combining polarimetric observations with numerical simulations and theoretical models, astronomers can build a detailed picture of how magnetic fields shape the structure and evolution of the cosmos.

In the absence of scattering, the polarized radiative transfer (PRT) equation in the Stokes representation reads:

$$\frac{\mathrm{d}}{\mathrm{d}s}\begin{bmatrix} I_\nu \\ Q_\nu \\ U_\nu \\ V_\nu \end{bmatrix} = -\begin{bmatrix} \kappa_\nu & q_\nu & u_\nu & v_\nu \\ q_\nu & \kappa_\nu & f_\nu & -g_\nu \\ u_\nu & -f_\nu & \kappa_\nu & h_\nu \\ v_\nu & g_\nu & -h_\nu & \kappa_\nu \end{bmatrix}\begin{bmatrix} I_\nu \\ Q_\nu \\ U_\nu \\ V_\nu \end{bmatrix} + \begin{bmatrix} \epsilon_{\mathrm{I},\nu} \\ \epsilon_{\mathrm{Q},\nu} \\ \epsilon_{\mathrm{U},\nu} \\ \epsilon_{\mathrm{V},\nu} \end{bmatrix} \tag{1}$$

[see e.g. Unno, 1956, Sazonov, 1969, Jones and Odell, 1977, Pacholczyk, 1977, On et al., 2019], where $[I_\nu, Q_\nu, U_\nu, V_\nu]$ are the specific 4-Stokes parameters at radiatiion frequency ν travelling along the ray path s; κ_ν, q_ν, u_ν, v_ν are the absorption coefficients, ϵ_ν the emission coefficients , f_ν the Faraday rotation coefficient, and g_ν, h_ν the Faraday conversion coefficients. The expression of the transfer coefficients appropriate for the context of cosmic plasmas can be found in the Appendix C of [Chan et al., 2019, and references therein].

Here we will focus on the effects of Faraday rotation and Faraday conversion in a magnetized thermal plasma. With only thermal electrons present, the coefficients of Faraday rotation and Faraday conversion are

$$f_{\mathrm{th}} = \frac{\left(\omega_{\mathrm{p}}^2/c\,\omega_{\mathrm{B}}\right)\cos\theta}{(\omega^2/\omega_{\mathrm{B}}^2)-1} \tag{2}$$

$$h_{\mathrm{th}} = \frac{\left(\omega_{\mathrm{p}}^2/c\,\omega_{\mathrm{B}}\right)\sin^2\theta}{2\ (\omega^3/\omega_{\mathrm{B}}^3-\omega/\omega_{\mathrm{B}})} \tag{3}$$

[Pacholczyk, 1977], respectively, where $\omega = 2\pi\nu$ is the radiation angular frequency, $\omega_{\mathrm{p}} = (4\pi n_{\mathrm{e,th}} e^2/m_{\mathrm{e}})^{1/2}$ is the plasma frequency, $\omega_{\mathrm{B}} = (eB/m_{\mathrm{e}}c)$ is the electron gyrofrequency, $n_{\mathrm{e,th}}$ is the thermal electron number density, B is the magnetic field strength, and θ is the angle between the radiation propagation and the magnetic field vector.

2 Exercise

1. Show that in the high frequency limit, i.e., $\omega \gg \omega_{\mathrm{B}}$, the Faraday rotation due to only thermal electrons (Eqn. 2) can be expressed as

$$f_{\mathrm{th}} = \frac{1}{\pi}\left(\frac{e^3}{m_{\mathrm{e}}^2 c^4}\right) n_{\mathrm{e,th}}\, B_{\parallel}\, \lambda^2\ , \tag{4}$$

where $B_{\parallel} = |\boldsymbol{B}| \cos\theta$ is the magnetic field along the line-of-sight and $\lambda = 2\pi c/\omega$ is the wavelength of radiation. (10 pt)

2. Derive from Eqn. 3 the expression of the coefficient of Faraday conversion due to only thermal electrons in the high frequency limit $\omega \gg \omega_{\rm B}$. What is the wavelength dependence? (10 pt)

3. In the absence of absorption, emission,and Faraday conversion, the PRT equation (Eqn. 1) reduces to

$$\frac{\rm d}{{\rm d}s}\begin{bmatrix} Q_\nu \\ U_\nu \end{bmatrix} = -\begin{bmatrix} 0 & f_\nu \\ -f_\nu & 0 \end{bmatrix}\begin{bmatrix} Q_\nu \\ U_\nu \end{bmatrix} . \tag{5}$$

What is the general solution to the above equation? (10 pt)

4. Make a figure of your answer to Question 3, showing $Q_\nu(s)$ and $U_\nu(s)$ for a constant f_ν. Label the axes, with units (in c.g.s units). (10 pt)

5. Write a function to evaluate $f_{\rm th}$ in the high frequency limit, i.e., Eqn. 4 (10 pt)

6. Using this function, evaluate $f_{\rm th}$ at radio frequencies $\nu = 700$ MHz and $\nu = 1.4$ GHz for the following cases:

   i Warm-ionized Interstellar Medium (ISM): $n_{\rm e,th} = 10^{-1}\ {\rm cm}^{-3}$ and $B = 10\ \mu$G
   ii Intracluster Medium (ICM): $n_{\rm e,th} = 10^{-3}\ {\rm cm}^{-3}$ and $B = 1\ \mu$G.
   iii Intergalactic Medium (IGM): $n_{\rm e,th} = 10^{-7}\ {\rm cm}^{-3}$ and $B = 1$ nG.

   Take $\cos\theta = 1$ here for simplicity. (10 pt)

7. Write a function to evaluate $h_{\rm th}$ in the high frequency limit and repeat the same as in Question 6. (20 pt)

8. The electron number density and magnetic field strength in cosmic diffuse media can vary widely, so the values listed in the Question 6 should be taken with a pinch of salt. Why cosmic diffuse media can vary so widely in their magneto-ionic properties? (15 pt)

9. (Bonus) Rotation measure (RM) is defined as

$$\mathcal{R} = (\Delta\varphi)\,\lambda^{-2} = (\varphi - \varphi_0)\,\lambda^{-2} , \tag{6}$$

where $\varphi \equiv 0.5 \arctan(U/Q)$ is the (linear) polarisation angle. Derive the expression of rotation measure

$$\mathcal{R}(s) = \frac{e^3}{2\pi m_{\rm e}^2 c^4} \int_{s_0}^{s} {\rm d}s'\ n_{\rm e}(s')\, B_{\parallel}(s') , \tag{7}$$

from the restrictive form of PRT equation shown in Eqn. 5. List all the assumptions in the derivation of the rotaion measure. (20 pt)

10. (Bonus) Download one of the FITS files from this link `https://utoronto-my.sharepoint.com/:f:/r/personal/jenniferyh_chan_utoronto_ca/Documents/GBT_Pol_Datasets_AbsCal/FITS_FlippedSignU_CombinedMaps_Com20arcmin?csf=1&web=1&e=umCA4t` (*Data not yet public, please do not circulate*). These data cubes stored $I(\nu, {\rm DEC}, {\rm RA})$, $Q(\nu, {\rm DEC}, {\rm RA})$, $U(\nu, {\rm DEC}, {\rm RA})$, $V(\nu, {\rm DEC}, {\rm RA})$ of the GBT 1-hr observational field and 11-hr field respectively. Visualize the data and make a map at the highest frequency and the lowest frequency, respectively. (20 pt)

References

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