

Assignment 3 Writeup

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1 Question 1

In this question, the goal was to iterate the equation $z_{i+1} = z_i^2 + c$ over the complex plane $-2 < x < 2$ and $-2 < y < 2$. This is what is called the Mandelbrot set. To do this, a function in a separate file called `complexplane.py` was defined which iterated the equation over each point in the complex plane.

To visualize the results of the iteration, points which diverged (went to infinity) were coloured to be blue while points that converged were coloured to be red. An image of the plane showing which points converged vs diverged was created using matplotlib:

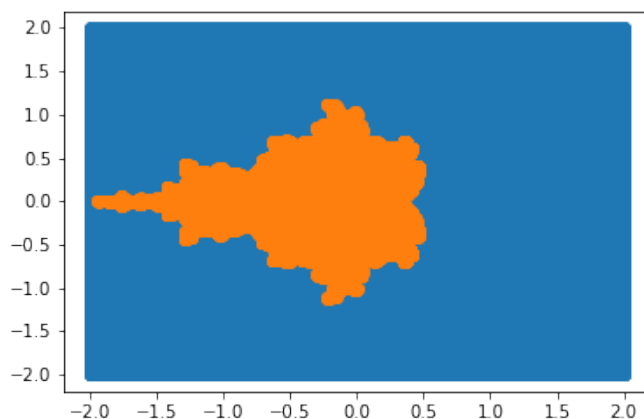


Figure 1: Mandelbrot set. Orange points converge while blue points diverge.

A second image was created with colour as a third variable, indicating the index at which a given point diverged:

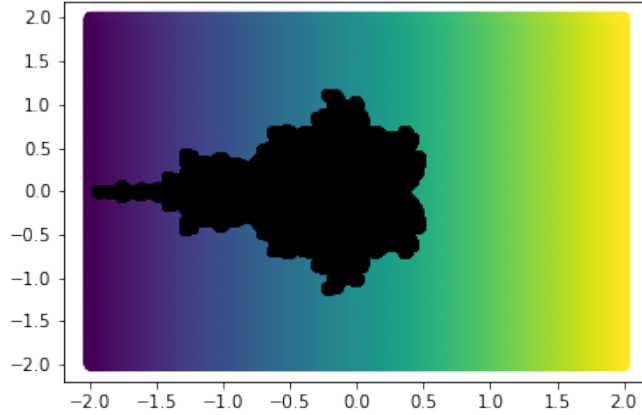


Figure 2: Mandelbrot set where black points converge and the points that diverge are coloured by a colourscale which indicates when they diverged .

2 Question 2

In this question, the goal was to solve the differential equations defined by Edward Lorenz, and recreate his numerical analysis:

$$\dot{X} = -\sigma(X - Y) \quad (1)$$

$$\dot{Y} = rX - Y - XZ \quad (2)$$

$$\dot{Z} = -bZ + XY \quad (3)$$

First, using the same initial conditions $W_0 = [0., 1., 0.]$ and parameters $[\sigma, r, b] = [10., 28, 8./3.]$ used by Lorenz, the numpy function `solve_ivp` was used to solve this system. The timespan of integration used was 0 to 60 seconds. The solution to the Lorenz equations were plotted to recreate the original plots in Lorenz's paper:

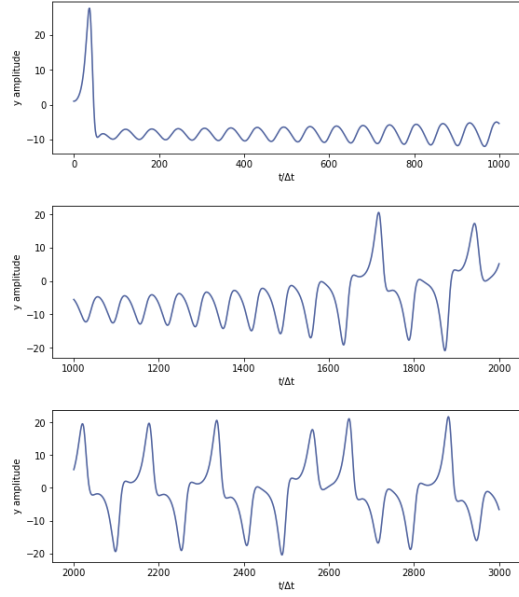


Figure 3: Recreation of Lorenz's Figure 1 Using solve.

This method was also used to recreate Figure 2 in Lorenz's paper:

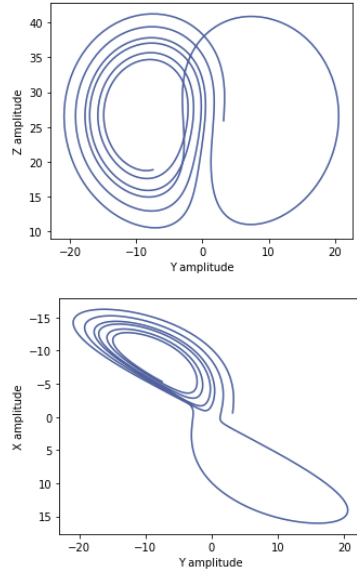


Figure 3: Recreation of Lorenz's Figure 2 Using solve_{ivp}.

Next, the initial conditions were changed to be W_0 , say $W'_0 = W_0 + [0., 1.e -$

$8, 0] = [0., 1.00000001, 0.]$. `solve_ivp` was once again used to solve the equation. Then, the difference between each point in the two solutions was calculated in a loop. Using this, in combination with `np.linalg.norm`, the distance between the two solutions was computed. The following is a plot of this distance vs time:

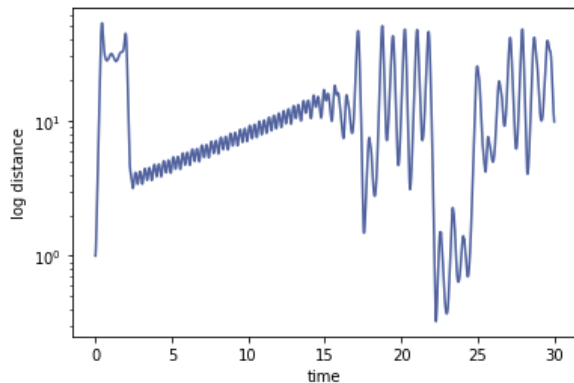


Figure 5: Distance vs time of the two solutions (with distance log-scaled). .

3 References

Lorenz, E. N., 1963: Deterministic Nonperiodic Flow. J. Atmos. Sci., 20, 130–141, [https://doi.org/10.1175/1520-0469\(1963\)020<0130:DNF;2.0.CO;2](https://doi.org/10.1175/1520-0469(1963)020<0130:DNF;2.0.CO;2).