Math 301: 2017 **Midterm** **Name:**___________________________

Counting

1. How many different committees of 2 faculty members and 5 students can be formed from 7 faculty and 8 students?

2. You've now selected your committee of 7 total members. How many different subcommittees are possible? A subcommittee must have at least 1 member and no more than 6 members, and subcommittees are considered different if they differ in at least one member.

Which would be correct solutions for this combinatorics problem? Select as many as you think are valid, and for each one you pick, explain why it is valid.

A 2^7
B $\sum_{k=1}^{6} \binom{7}{k}$
C $2^7 - \binom{7}{0} - \binom{7}{7}$
D $\prod_{k=1}^{6} \binom{7}{k}$
E $7!$
F $\frac{7!}{6!1!}$

3. How many distinct anagrams of the letters in 'MESSES' are there?

4. How many letters do you have to randomly select from 'MESSES' to guarantee you have 2 distinct letters?

Logic and Sets

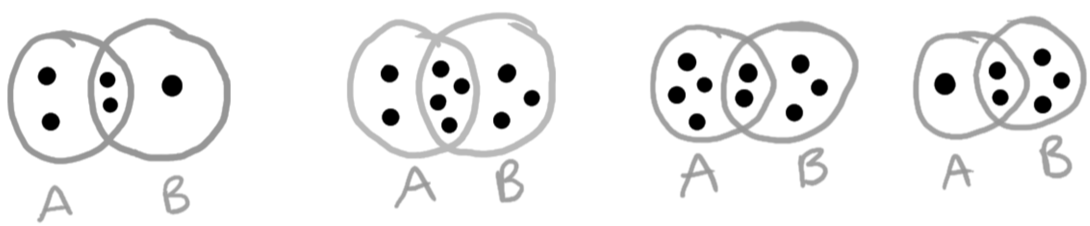
Let A and B be two sets. Given the statement " If $|A \Delta B| > |A|$, then $|B| > |A|$",

1. Give the contrapositive, and the converse of the statement.

Contrapositive:

Converse:

2. Which of the following is a counterexample that disproves the original statement? [Circle one]



3. In situations where a statement is true, which of the following is also definitely true?

A Negation

B Converse

C Contrapositive

4. Give the truth table for $p \wedge \neg q$

Recursion and Induction

Consider the recurrence relation $T(n) = 3T(n-2) - 2T(n-1)$ with $T(0) = 6$ and $T(1) = -2$,

1. Find an explicit formula for $T(n)$. Show your work.

2. Prove your explicit formula from the previous problem is valid by using induction. Pay attention to the form of your proof.

3. Consider the following (inefficient) recursive algorithm.

ge(m,n):

· **If** $n == 0$ **then return** $TRUE$

· **elseif** $m == 0$ **then return** $FALSE$

· **else return** $ge(m-1, n-1)$

a) Trace the call $ge(3, 5)$. Write down the input each time it calls $ge()$ again, as well as the final output.

b) When does this algorithm return TRUE/what is this algorithm computing?

Number Theory

1. Prove that if $a \equiv b \pmod{m}$, then $m|(a-b)$

2. True or False for each:

T or F $72 \equiv -2 \pmod{7}$

T or F 3 is a solution to $x^2 = x$ in $\mathbb{Z}_6$

T or F $34103 \cdot 212 \equiv 4 \pmod{5}$

T or F $2x \equiv 1 \pmod{6}$ has no (integer) solutions.

3. The number 102 in ternary (base 3 representation) corresponds to what number in decimal?

BONUS

1. How many distinct 3 word phrases can be built out of the letters in 'MESSES' by including 2 spaces?

2. Prove that $8^n - 3^n$ is always divisible by 5.

3. If you had to use the recursive implementation from #7 instead of it's constant run time equivalent, what would be the run-time complexity of Binary Search?