

## Section 6.4

Penn State University

Math 141 - Section 001 - Summer 2016

### 6.4\*: General Logarithmic and Exponential Functions

We discussed the exponential function with base  $e$  in the previous section. There are plenty of cases where we wish to take the exponent of numbers other than  $e$ . The definition of the general exponential function is  $a^x = e^{x \ln a}$ . I should emphasize the meaning of the word *definition*. When we say “the definition of  $a^x$  is  $a^x = e^{x \ln a}$ ,” what we mean is that  $a^x$  is just a shorthand for  $e^{x \ln a}$ . Whenever you deal with an expression involving a general exponential, it is almost always the case that rewriting it using the definition is a good first step.

Note that if  $a = e$ , then  $e^{x \ln e} = e^x$ . All the usual properties of exp function apply to general exponential functions (3).  $\frac{d}{dx}(a^x) = a^x \ln a$  (4) and  $\int a^x dx = \frac{a^x}{\ln a} + C$  (p440) are formulas that you should know how to derive, and eventually memorize.

**Exercise 1.** Differentiate  $5^x$ .

Solution: By the formula,  $(5^x)' = \ln 5 \cdot 5^x$ .

**Exercise 2.** Differentiate  $f(x) = 3^{\cos 2x}$ .

Solution: By the chain rule,  $f'(x) = \ln 3 \cdot 3^{\cos 2x} \cdot (\cos 2x)' = -2 \ln 3 \cdot 3^{\cos 2x} \cdot \sin 2x$ .

There is a nice discussion of how know when to use the Power Rule or Exponential Rule on p441, which you should read carefully.

**Exercise 3.** Differentiate  $y = x^{\sqrt{x}}$ .

Now that we know what general exponential functions are, we can also define general log functions. The textbook's definition is that the log function with base  $a$  is the inverse of  $a^x$  ( $\boxed{5}$ ). But it's easier to think of it simply as  $\log_a x = \frac{\ln x}{\ln a}$  ( $\boxed{6}$ ). A consequence of this formula is that  $\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$ .

## Problems

1. Find  $(x^4 + 5^x)'$ .
2. Evaluate  $\int x^4 + 5^x dx$ .
3. Evaluate  $\int x 2^{x^2} dx$ .
4. Find  $(x^{\cos x})'$ .
5.  $\int \frac{\log_{10} x}{x} dx$ .
6.  $\int \frac{2^x}{2^x + 1} dx$ .
7. (Exam 1 Sample B) If  $f(x) = 3^x$ , find the second derivative  $f''(x)$ .
8. (Exam 1 Sample B) If  $f(x) = x^{2x}$ , find  $f'(e)$ .
9. (Exam 1 Sample C) Differentiate  $y = 3^{x^2}$ .