Quiz 1	(5 POINTS TOTAL)
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MATH 220, MATRICES, SPRING 2015

NAME:

Problem 1. What are the three elementary row operations?

Problem 2. Is the following statement true?:

Elementary row operations on an augmented matrix do not change its solution set.



Problem 1 (2 points) Show that $\begin{pmatrix} 2\\3\\1 \end{pmatrix}$ is a linear combination of $\begin{pmatrix} 2\\0\\1 \end{pmatrix}$, $\begin{pmatrix} 0\\1\\1 \end{pmatrix}$, and $\begin{pmatrix} 0\\0\\1 \end{pmatrix}$.

Problem 2 (i)(2 points) What is the span of $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ (pick from: empty, line, plane, \mathbb{R}^3 , or \mathbb{R}^4)? (ii)(1 point) Explain your answer in (i).

Problem 1 (2 points) Let $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$. Do the homogeneous equation $A\mathbf{v} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ and non-homogeneous equation $A\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ have a common solution? In other words, is there a vector \mathbf{v} in \mathbb{R}^3 such that two equations $A\mathbf{v} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ and $A\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ hold simultaneously?

Problem 2 (3 points) (i) Determine whether the set $\left\{ \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 6 \end{pmatrix} \right\}$ is linearly dependent or not. (ii) What about $\left\{ \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 6 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$?

Problem 1 (2 points) Let $A = \begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix}$, and let T be the linear transformation from \mathbb{R}^2 to \mathbb{R}^2 . (i) What is the range of T? (ii) Does the range of T coincide with its codomain?

Problem 2 (3 points) A, B, and C are $n \times n$ matrices. Pick three from the following five statements, and state whether they are true or false (you don't need to explain).

- 1. If AB = AC, then it is always the case that B = C.
- 2. If $A^2 = 0$, then it is always the case that A = 0.
- 3. Regardless of the entries of A and B, AB = BA always holds.
- 4. Regardless of the entries A, B, and C, A(B+C) = AB + AC always holds.
- 5. Regardless of the entries of A, $(A^2)^T = (A^T)^2$ always holds.

Problem 1 (2 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection through the line y = x. (For example, we have $T(\begin{pmatrix} 1 \\ 0 \end{pmatrix}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, and $T(\begin{pmatrix} 2 \\ 1 \end{pmatrix}) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.) Find the *standard* matrix for this linear transformation.

Problem 2 (3 points) Let $A = \begin{pmatrix} 1 & 2 \\ -3 & -6 \end{pmatrix}$. (i) Is A invertible? (ii) How many solutions does $A\mathbf{v} = \mathbf{0}$ have? Explain your anwser without solving the equation explicitly.