Section 7.2

Penn State University

Math 141 - Section 001 - Summer 2016

7.2: Trigonometric Integrals

For one reason or another (one of the major culprits being "Fourier series"), you will end up doing a lot of integration involving sines and cosines in your future courses if you're in engineering, physics, or anything related to them. This section presents some nifty tricks to deal with integrals of that kind. For this section, you need to memorize the following formulas:

$$\sin^2 x + \cos^2 x = 1$$
$$2 \sin x \cos x = \sin 2x$$
$$1 + \tan^2 = \sec^2 x$$
$$\sin^2 x = \frac{1 - \cos 2x}{2}$$
$$\cos^2 x = \frac{1 + \cos 2x}{2}$$
$$\sin x \cos y = \frac{\sin(x - y) + \sin(x + y)}{2}$$
$$\sin x \sin y = \frac{\cos(x - y) - \cos(x + y)}{2}$$
$$\cos x \cos y = \frac{\cos(x - y) + \cos(x + y)}{2}$$

Methods for evaluating $\int \sin^m \cos^n x dx$

There are two methods. Take a look at the box in p497. In my opinion, (a) and (b) are the same thing.

Exercise 1. (Ex1) $\int \cos^3 x dx$

Exercise 2. (Ex2) $\int \sin^5 x \cos^2 x dx$

Exercise 3. (Ex3) $\int \sin^2 x dx$

Methods for evaluating $\int \tan^m \sec^n x dx$

For integrals of this type, knowing $\int \tan x dx = \ln|\sec x| + C$ and $\int \sec x dx = \ln|\sec x + \tan x| + C$ will help. (The latter identity is difficult to derive.) Also $(\sec x)' = \sec x \tan x$.

Exercise 4. (Ex5) $\int \tan^6 x \sec^4 x dx$

Exercise 5. (Ex6) $\int \tan^5 x \sec^7 x dx$

There are integrals of this type which do not fit into the two patterns, as noted in p498.

Exercise 6. (Ex7) $\int \tan^3 x dx$

The next integration is really difficult, and I always need to look it up.

Exercise 7. (Ex8) $\int \sec^3 x dx$

$\int \sin mx \cos nx$

This is discussed in p500. The key idea is to use the fact that you can convert a product of trig functions to a sum of them.

Exercise 8. $\int \sin 2x \cos 3x dx$

Problems

1. $\int \sin^4 x \cos^3 x \, dx$

2. $\int \cos^2 x \sin 2x \, dx$

3. $\int \sin^3 x \, dx$

4. $\int \cos^4 x \, dx$

5. $\int \cos^5 x \, dx$

6. $\int \sin^2 x \cos^2 x \, dx$

7. $\int \tan^5 x \sec^3 x \, dx$

8. $\int \tan^2 x \sec^4 x \, dx$

9. $\int \sin 4\theta \cos 3\theta \, dx$

10. $\int_{-\pi/4}^{0} \sqrt{\sec^2 y - 1} \, dy$

11. $\int \frac{1+\sin x}{\cos^2 x} \, dx$

12. $\int_{0}^{3\pi/4} \sqrt{1 + \cos 2\theta} \, d\theta$