Section 7.1

Penn State University

Math 141 - Section 001 - Summer 2016

7.1: Integration by Parts

Read Examples 1-3. Integration by parts is used in many situations, and, in my opinion, the most important integration technique covered in this course. Example 4 shows a neat trick involving integration by parts. You should remember this trick, because if you didn't know, you wouldn't think about it.

Another way of writing down the formula for integration by parts is this:

$$\int f(x)g(x)dx = F(x)g(x) - \int F(x)g'(x)dx,$$

where $F(x) = \int f(x)dx$ (omitting the integration constant C). I find this way of thinking about integration by parts better than the one in the textbook.

Problems in Chapter 7 can be difficult. As you probably have heard before, doing many problems is the only way to become good at calculus. Here are some problem-solving suggestions specific to 7.1:

1. We learned three methods, exemplified by Example 1, Example 2, and Example 4. When you start a problem, try to guess which method works.

2. If you don't come up with any guess which method is applicable to the problem in hand, just pick a method and try it out. If it works, great. If not, pause

for a moment and analyze why it didn't work. Thinking about why a method doesn't work for a particular problem helps you horn your intuition.

3. Then move on to a different method. Repeat the process until you succeed. When you find a method that works, pause before moving on to the next problem, and analyze why the method is effective. Reflecting on which method applies to the problem and which doesn't helps you develop intuition, or the ability to come up with educated guesses.

4. A lot more can be said about the process of problem solving. For example, the correct solution to Example 1, $\int x \sin x dx$, is to take $f(x) = x$ and $g'(x) = \sin x$. But how do we know that in advance? The answer to that question is—*we don't*. The following is how I imagine a process of solving this problem may go. Let's say you guess that the standard integration by parts might work. Then, you have two options: (a) take $f(x) = x$ and $g'(x) = \sin x$; (b) take $f(x) = \sin x$ and $g'(x) = x$. The point here is that, even though we don't know which of (a) and (b) work, there are only two options, so we try out both of them. If you pick (a) first, then you'll see that it works. If you pick (b) first, then you'll see that it doesn't, and you move on to (a).

   Now, how do we know that the standard integration by parts works here? Again, the answer is: we don't. You may have to try out different integration techniques before thinking of using integration by parts. But eventually, as you become a more experienced integration calculator, you will be able to guess immediately just by seeing the integrand that the standard integration by parts is the way to go.

5. Get used to this process of trying out different methods until you find one that works. Don't be discouraged by failed attempts, because they are what it takes to solve problems. There is science to problem solving, but be aware that the art of problem solving is equally important.

Problems

1. $\int x \cos x dx$.
2. (Ex5) $\int_0^1 \tan^{-1} x dx$
3. $\int e^x \sin 2x dx$.
4. $\int \ln x dx$.
5. $\int (\ln x)^2 dx$.
6. (More Integration Practice #25) $\int \sec^{-1} x dx$ (assume $x > 1$).
7. (Sample A #14) $\int x^2 \ln x dx$.
8. $\int t^3 e^{-t^2} dx$
9. $\int x^2 \sin 3x dx$ (from "More Integration Practice")
10. $\int \frac{\ln x}{\sqrt{x}} dx$
11. $\int \tan^2 x dx$
12. $\int \frac{x e^{\sin^{-1} x}}{\sqrt{1-x^2}} dx$
13. $\int x \tan^2 x dx$