Quiz 13 - Solution

Math 220, Matrices, Spring 2015

Name:

Problem 1 (5 points) Show that $A \begin{pmatrix} 1 & -2 & 3 \\ 0 & -3 & 0 \\ 0 & 0 & 7 \end{pmatrix}$ is diagonalizable, but not orthogonally diagonalizable.

This problem requires almost no computation. Don't get your hands dirty when you don't have to.

Because A is upper triangular, its eigenvalues are its diagonal entries. Therefore, $\lambda = 1$, -3, and 7. Since A is 3×3, and it has three **distinct** eigenvalues, A is diagonalizable. It is easy to check that $A^T \neq A$. So A is not symmetric. Then, by the Spectral Theorem, A is not orthogonally diagonalizable.

Common Mistakes

- Upper triangular doesn't mean it's diagonalizable. In this case, it is diagonalizable because A has three distinct eigenvalues. When a matrix is upper (or lower) triangular, the property helps you to find its eigenvalues, but it doesn't tell you anything about its diagonalizability. For example, $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ is upper triangular, but not diagonalizable.
- The condition for a symmetric matrix is **not** $A^T A = I$. This is for orthogonality. The test for a symmetric matrix is $A^T = A$.
- To say that a matrix is **not** orthogonally diagonalizable, you **need** to use the Spectral Theorem. Just finding one invertible P such that PAP^{-1} is diagonal is **not** enough, because there is still a possibility that there may be some other invertible matrix, say Q, that is orthogonal, and QAQ^{-1} is diagonal.