Section 6.6

Penn State University

Math 141 - Section 001 - Summer 2016

6.6: Inverse Trigonometric Functions

Trig functions are invertible on restricted domains. Let's take sin as an example. Consider its graph. You see that sin is *far* from passing the horizontal line test. However, if we restrict the function to $[-\pi/2, \pi/2]$, then it is indeed one-to-one. (Note that sin is one-to-one on $[\pi/2, 3\pi/2]$, $[3\pi/2, 5\pi/2]$, or any translation of $[-\pi/2, \pi/2]$. But $[-\pi/2, \pi/2]$ is by convention the domain of choice.) Then, the restricted sin function can be inverted, and we call the inverse function $\sin^{-1}$ (also denoted arcsin). The domain of $\sin^{-1}$ is $[-1, 1]$ and the range is $[-\pi/2, \pi/2]$.

The inverse functions of the other trig functions are constructed in a similar manner. In Math 26 and 41, you need to know the domains and ranges of the inverse trig functions, but in this course, that is not the case. Here are important properties that you should know for this course:

1. $\lim_{x\to\infty} \tan^{-1} x = \dfrac{\pi}{2}$
2. $\lim_{x\to-\infty} \tan^{-1} x = -\dfrac{\pi}{2}$
3. The graph of $\tan^{-1}$.
4. $\dfrac{d}{dx} \sin^{-1} x = \dfrac{1}{\sqrt{1-x^2}}$

5. $\dfrac{d}{dx}\cos^{-1}x = \dfrac{-1}{\sqrt{1-x^2}}$
6. $\dfrac{d}{dx}\tan^{-1}x = \dfrac{1}{1+x^2}$
7. $\dfrac{d}{dx}\csc^{-1}x = \dfrac{-1}{x\sqrt{x^2-1}}$
8. $\dfrac{d}{dx}\sec^{-1}x = \dfrac{1}{x\sqrt{x^2-1}}$
9. $\dfrac{d}{dx}\cot^{-1}x = \dfrac{-1}{1+x^2}$

The differentiation rules are derived using implicit differenciation. I will show how to derive them in class.

As a consequence of these differenciation rules, we obtain the following:

1. $$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + C$$
2. $$\int \frac{1}{1+x^2}dx = \tan^{-1}x + C$$

I will prove these identities in a different way, namely, by substitution.

Read Examples 3-9. Example 6 shows a neat method to establish identities. You will find this type of problems in homework, but not in exams.

Problems

1. (Ex3) Simplify $\cos(\tan^{-1}x)$.
2. Simplify $\tan(\sin^{-1}x)$.
3. (Ex4) $\lim_{x\to 2^+}\arctan\dfrac{1}{x-2}$.
4. Show that $(\sec^{-1}x)' = \frac{1}{x\sqrt{x^2-1}}$.

5. (Ex5) $\dfrac{d}{dx}\dfrac{1}{\sin^{-1} x}$
6. (Ex5) $\dfrac{d}{dx} x \arctan \sqrt{x}$
7. $(\arcsin \sqrt{\sin \theta})'$.
8. $(\arctan \sqrt{\frac{1-x}{1+x}})'$.
9. (Ex7) $\int_0^{1/4} \frac{1}{\sqrt{1-4x^2}} dx$
10. (Ex8) $\int \frac{1}{x^2+a^2} dx$
11. (Ex9) $\int \frac{x}{x^4+9} dx$
12. $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx$.
13. $\int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$.