Section 6.2*

Penn State University

Math 141 - Section 001 - Summer 2016

6.2*: The Natural Logarithmic Function

Make sure you're reading 6.2 (note the star) of the textbook, not 6.2 without star.*

In the remaining sections of Chapter 6, we discuss several functions that we will use heavily in the rest of the course. The first is ln, the natural log function. $\boxed{2}$ and $\boxed{3}$ are important properties of ln.

Exercise 1. Work out Example 2 without looking at the solution.

It is useful to have the graph of ln in mind. Input "Plot[Ln(x), (x, 0, 10)]" in WolframAlpha. This will give you the plot of $\ln(x)$ from $x = 0$ to 10. Things that you should note are:

1. ln is **not defined** for $x \leq 0$.
2. ln is one-to-one (for $x > 0$), hence invertible.
3. $\ln(x)$ is zero when $x = 1$.
4. $\ln(x)$ is negative for $0 < x < 1$, and it goes to $-\infty$ as $x \to 0$ ($\boxed{4}$).
5. $\ln(x)$ is positive for $x > 1$, and it goes to $+\infty$ as $x \to \infty$ ($\boxed{4}$).

You should become comfortable with differentiating ln using the chain rule. $\boxed{2}$ says $(\ln(x))' = \frac{1}{x}$, and if we combine $\boxed{2}$ with the chain rule, we get $\boxed{6}$. $\boxed{6}$ gives you two formulas, but personally I like this one better: $\frac{d}{dx}(\ln g(x)) = \frac{g'(x)}{g(x)}$.

Exercise 2. Find $(\ln(2x))'$.

A common mistake is to forget using the chain rule, and do $(\ln(2x))' = \frac{1}{2x}$. The correct solution is the following.

<u>*Solution*</u>: Set $g(x) = 2x$. By the chain rule, $(\ln(2x))' = (\ln g(x))' = \frac{g'(x)}{g(x)} = \frac{2}{2x} = \frac{1}{x}$.

You should read Examples 6, 7, and 8 carefully.

Two more properties of ln that you should know:

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}; \qquad \int \frac{1}{x}dx = \ln|x| + C$$

($\boxed{7}$ and $\boxed{8}$). Note the absolute value!

Read Examples 11, 12, and 13 carefully. The method and result of Example 13 are worth memorizing. This formula $\int \tan x dx = \ln|\sec x| + C$ will come in handy in some of the more difficult integration problems.

The last topic in this section is logarithmic differentiation. Take a look at the expression in Example 14: $y = \frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}$. If you try to compute y' in a straightforward fashion, it would involve lots of product rules and chain rules, so you'd rather not do that. Notice that y is a product of the three expressions: $x^{3/4}$, $\sqrt{x^2+1} = (x^2+1)^{1/2}$, and $(3x+2)^5$. In such case, namely, when you have a product of several polynomial terms, logarithmic differentiation makes computing y' much, much easier. The main idea is that differentiating $\ln y = \frac{3}{4}\ln x + \frac{1}{2}\ln(x^2+1) - 5\ln(3x+2)$ involves much less efforts than y. You should work through this example without looking at the solution.

Problems

1. Expand the quantity $\ln\sqrt{\frac{x^2}{z^3}}$.

2. Expand $\ln \frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}$.

3. Evaluate $\lim_{x\to 1^+} \ln \frac{1}{x-1}$.

4. Differentiate $\ln(x^3 + \sqrt{x^2-1})$.

5. $\int \frac{dt}{8-3t}$.

6. $\int \frac{\cos x}{2+\sin x} dx$.

7. $\int \frac{(\ln x)^2}{x} dx$.

8. (Exam 1 Sample A) Given that $a, b > 0$ and $a \neq b$, evaluate $\lim_{x\to\infty} \ln(3 + ax) - \ln(2 + bx)$.

9. (Exam 1 Sample A) Evaluate the integral $\int_e^{e^2} \frac{2\ln x}{x} dx$.

10. (Exam 1 Sample B) If $f(x) = \ln(x^2 \sin x)$ find $f'(x)$.

11. (Exam 1 Sample C) Differentiate the function $f(x) = \ln(\sin(\ln x))$.

12. Differentiate the function $y = e^{5\cos\sqrt{x}}$.