Section 11.4

Penn State University

Math 141 - Section 001 - Summer 2016

11.4: Comparison Tests

Theorem 1. (Comparison Test) Suppose $a_n \geq b_n \geq 0$.

1. If $\sum_{n=1}^{\infty} a_n$ is convergent, then so is $\sum_{n=1}^{\infty} b_n$.
2. If $\sum_{n=1}^{\infty} b_n$ is divergent, then so is $\sum_{n=1}^{\infty} a_n$.

Theorem 2. (Limit Comparison Test) Suppose $a_n, b_n \geq 0$. If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$, where $0 < c < \infty$, then $\sum a_n$ converges if and only if $\sum b_n$ does.

The Comparison Test for series works in almost exactly the same way as the Comparison Test for improper integrals (p549). Note that both a_n and b_n have to be positive.

Exercise 1. Does $\sum_{n=1}^{\infty} \frac{1}{1+n^2}$ converge? What about $\sum_{n=1}^{\infty} \frac{1}{n-1}$?

Exercise 2. (Ex1) Does $\sum_{n=1}^{\infty} \frac{5}{2n^2+4n+3}$ converge?

Exercise 3. (Ex2) Does $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ converge?

The Limit Comparison Test says when a_n and b_n have the same rate of growth (i.e. $\lim_{n\to\infty} \frac{a_n}{b_n} =$ positive constant), then the corresponding series have the same convergence property. It also requires that the terms are positive, as in the Comparison Test.

Exercise 4. (Ex3) Does $\sum_{n=1}^{\infty} \frac{1}{n^2-1}$ converge?

Solution: Note that the Comparison Test does not work. Use the Limit Comparison Test instead.

Exercise 5. (Ex4) Does $\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{5+n^2}}$ converge?

The Limit Comparison Test works whenever the Comparison Test works, but not the other way around.

Problems

Determine whether the series converges.

1. $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n}$
2. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^4-1}}$
3. $\sum_{n=1}^{\infty} \frac{4^{n+1}}{3^n-2}$
4. $\sum_{n=1}^{\infty} \frac{n+4^n}{n+6^n}$
5. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$
6. $\sum_{n=1}^{\infty} \frac{n+5}{n\sqrt{n}}$
7. $\sum_{n=1}^{\infty} \frac{n^2}{n^3-n+1}$
8. $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$
9. $\sum_{n=1}^{\infty} \frac{1}{n!}$
10. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$
11. $\sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n^{3/2}}$
12. $\sum_{n=1}^{\infty} (1+1/n)^2 e^{-n}$