

Section 7.3

Penn State University

Math 141 - Section 001 - Summer 2016

7.3: Trigonometric Substitution

This section is similar in spirit with 7.2: there are certain patterns that you should recognize, and when you do, there are certain change of variables that you should perform. The table in p502 gives three patterns, and how to deal with them.

0.1 $\sqrt{a^2 - x^2}$

When you substitute in $x = \sin \theta$ for $\frac{1}{\sqrt{1 - x^2}}$, you get $\frac{1}{\sqrt{1 - \sin^2 \theta}} = \frac{1}{\cos \theta}$, and the resulting $\cos \theta$ in the denominator is taken care of by the $\cos \theta$ that we get from $dx = \cos \theta d\theta$.

Exercise 1.

$$\int \frac{\sqrt{9 - x^2}}{x^2} dx$$

0.2 $\sqrt{x^2 + a^2}$

$x = \tan \theta$ is a substitution that you should try when you see $\frac{1}{\sqrt{1 + x^2}}$ (or $\frac{1}{1 + x^2}$) in the integrand. You get $\frac{1}{\sqrt{1 + \tan^2 x}} = \sec x$. Again, this resulting $\sec x$ is taken care of in one way or another by $dx = \sec^2 \theta d\theta$.

Exercise 2.

$$\int \frac{1}{x^2 \sqrt{x^2 + 4}} dx$$

Exercise 3.

$$\int \frac{x}{\sqrt{x^2 + 4}} dx$$

0.3 $\sqrt{x^2 - a^2}$

For integrals of this type, $x = a \sec \theta$ works this a lot of cases.

Exercise 4.

$$\int \frac{\sqrt{x^2 - 4}}{x} dx$$

0.4 Remark

Remember that these substitutions are **not** panaceas for the class of integrals discussed in this section. We saw that, for example, in Exercise 3 that the trig substitution was not the best method. Also, there are integrals that cannot be solved using these substitutions even if the patterns appear. The method described in this section is a great way to *start* solving integrals of particular types, and the great majority of those integrals that you encounter in this course is solved this way. However, if the method fails, you need to seek other ways.

Problems

1. $\int \sqrt{1 - x^2} dx$
2. $\int x^3 \sqrt{1 - x^2} dx$
3. $\int \frac{t}{\sqrt{9 - 4t^2}} dt$ (from “More Integration Practice”)
4. $\int 5x^3 \sqrt{x^2 + 1} dx$ (from “Integration Practice”)
5. $\int \frac{dt}{t^2 - 6t + 13}$

6. $\int \frac{dy}{y^2-2y+5}$
7. $\int \frac{dx}{\sqrt{5-4x-x^2}}$ (from “More Integration Practice”)
8. $\int \frac{dv}{\sqrt{e^{2v}-1}}$ (from “More Integration Practice”)
9. $\int \frac{dt}{t^3\sqrt{t^2-3}}$ (from “More Integration Practice”)
10. (Sample #17) $\int \frac{1}{x^4\sqrt{x^2-9}} dx$.