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# CHAOS, PREDICTION AND LAPLACEAN DETERMINISM

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IN the edenic days before the discovery of quantum indeterminacy, it was a widespread view of the scientific community that the universe was thoroughly deterministic, and that any event was in principle predictable. The only barrier to prediction was our lack of knowledge, either due to a lack of observational data, or due to a lack of knowledge of the relevant laws of nature. This view, which I shall call Scientific Determinism, is eloquently expressed by the French mathematician Pierre-Simon de LaPlace:

> We ought to regard the present state of the universe as the effect of its antecedent state and as the cause of the state that is to follow. An intelligence knowing all the forces acting in nature at a given instant, as well as the momentary positions of all things in the universe, would be able to comprehend in one single formula the motions of the largest bodies as well as of the lightest atoms in the world, provided that its intellect were sufficiently powerful to subject all data to analysis; to it nothing would be uncertain, the future as well as the past would be present to its eyes.[1]

Scientific Determinism, then, advocates two simple theses:

(1) All deterministic systems are predictable.
(2) All systems in nature are deterministic.

From (1) and (2) it would follow that:

(3) All systems in nature are predictable.

We now know (2) to be an empirical hypothesis, one that quantum mechanics has shown to be false. Yet it is all too easy to slip from the belief that:

(4) Because of quantum mechanics, (2) is false;

to the belief that:

(5) *Only* because of quantum mechanics is (2) false.

Although few would readily admit it, (5) is an implicit belief of many contemporary philosophers and scientists alike. My purpose here is to refute (5), the last vestige of scientific determinism. To do so, I will examine work in a new area of scientific inquiry, called *deterministic chaos*, which indicates that not only (2) above, but also (1), is false.

At the outset I want to make two distinctions that will aid the reader in following the discussion. First, the considerations raised here are independent of problems arising from the indeterminacy inherent in quantum mechanics. Indeed we could assume that the "natural world" in this discussion is still the classical one of Newton or Einstein. Second, I am concerned with the sort of predictability that goes hand in hand with determinism. I call this absolute predictability, in contrast to other forms of prediction such as statistical predictability. While it is an interesting question how the limitations I propose for absolute predictability would affect other forms of prediction, that topic is not addressed.

## I. DETERMINISM

Premise (1) is often considered to be a definitional truth. Yet (1) by itself tells us nothing about why it is that all deterministic systems are supposedly predictable. I begin with an examination of what the relationship between determinism and predictability is traditionally taken to be.

According to Scientific Determinism, making a prediction involves applying measurements of the present state of a system to the mathematical apparatus of the relevant scientific theory which generates a desired description of some future state of the system. I will call measurements of the present state of the system *input*, I will call the mathematical apparatus an *algorithm*, and I will call the generated prediction *output*. We should remove the temporal references from this description, leaving open the possibility of retro-prediction. In most general terms, prediction is what enables us to determine an *unknown* state of a system from a *known* state, and we do this by applying the known state as input to a predictive algorithm.

I want now to turn to determinism.[2] What makes determinism a necessary condition for predictability? Bertrand Russell proposed the following definition:

> A system is said to be "deterministic" when, given certain data, $e_1, e_2, \ldots, e_n$, at times $t_1, t_2, \ldots, t_n$ respectively, concerning this system, if $E_t$ is the state of the system at any time $t$, there is a functional relation of the form
>
> $$E_t = f(e_1, t_1, e_2, t_2, \ldots, e_n, t_n, t)$$
>
> The system will be "deterministic throughout a given period" if $t$, in the above formula, may be any time within that period, though outside that period the formula may be no longer true. If the universe, as a whole, is such a system, determinism is true of the universe; if not, not.[3]

Russell's definition looks like not only a definition of determinism, but a passable definition of predictability as well. After all, what is the function in question but the algorithm that, once discovered, scientists will use to generate predictions? This line of thought leads one to view determinism not only as necessary, but sufficient for predictability.

Russell, however, is not happy with this definition of determinism. In particular, Russell is not satisfied with this minimal specification of the function $f$ in terms of which states of the system are said to be related:

> If formulae of any degree of complexity, however great, are admitted, it would seem that any system, whose state at a given moment is a function of certain measurable quantities, *must* be a deterministic system. Let us consider, in illustration, a single material particle, whose co-ordinates at time $t$ are $x_t$, $y_t$, $z_t$. Then, however the particle moves, there must be, theoretically, functions $f_1$, $f_2$, $f_3$, such that
>
> $$x_t = f_1(t),\ y_t = f_2(t),\ z_t = f_3(t).$$
>
> It follows that, theoretically, the whole state of the material universe at time $t$ must be capable of being exhibited as a function of $t$. Hence our universe will be deterministic in the sense defined above. But if this be true, no information is conveyed about the universe in stating that it is deterministic.[4]

Russell's concern is that there is no difficulty in showing that any set of data points are describable by *some* function, indeed by infinitely many functions. This is just the traditional curve plotter's problem. What we want is to say that there exists a function *which is in fact* the function that determines those data points, but no such information is conveyed by a function which simply describes the data.

Suppose we have two possible worlds, $W_1$ and $W_2$. Each is occupied by a single particle, $p_1$ and $p_2$ respectively, whose location at any time can be specified by three spatial coordinates. In $W_1$ there is a law of nature specifiable by a function $f$ which determines the location of $p_1$ at any time. In $W_2$ there are no laws of nature, and $p_2$ moves completely at random. But now suppose that by some incredible chance that for all times, $p_1$'s location in $W_1$ is identical with $p_2$'s location in $W_2$. Russell's concern is that mere knowledge of $f$ and a set of observations could never tell you whether you were observing $W_1$ or $W_2$.

Why then do we believe that there is an important difference between $W_1$ and $W_2$? Suppose for a moment that we treat a world like a tape: you can rewind it and play it through again. $W_1$, because it is a deterministic world, will always play the same. In $W_2$, however, the fact that $f$ describes $p_2$ is coincidental; if we play back $W_2$ again, the location of $p_2$ need not agree with $f$. Thus we should require that a deterministic system be like $W_1$; it will play through the same way every time. This stronger requirement for determinism is spelled out in greater detail by Ernest Nagel, and in a more sophisticated form by Richard Montague.[5]

Distinguishing deterministic from random systems involves a further difficulty. Intuitively we want to say that the output from a predictive algorithm in the case of a deterministic system is always precise, and involves no error. By contrast, attempts to predict the outcome of a system involving some random element will always result in some degree of error. This way of distinguishing deterministic and indeterministic systems is oversimplified.

Suppose the system under study is celestial mechanics. Thus to give a description of the solar system at any given time, we must give the location and angular momentum of each element in the system at that time. Such a description is called a state description, and all of this information can be displayed graphically by locating each element in state space (sometimes called phase space). In general, the dimensionality of state space will correspond to the minimum number of real numbers required to uniquely specify each element in the system, and a state description will be complete when each element is specified by that minimum number of real numbers.

Therein lies the problem. Our measurements do not yield exact values; they do not yield precise real numbers. We do not know from measurements that the angular momentum of Mars is $x$, but only that the angular momentum of Mars is $x \pm e$, where $e$ is some margin of error. The inclination is to respond that this is of no consequence; in actual fact we do not get ideal measurements, but in principle any measured variable takes an exact real number as its value. I am not convinced that in principle perfect measurement is possible. However, even if it were, there is more to the story.

We are limited in our measurements not just by the accuracy of our instruments, but by the numbers we can actually represent and use in calculations. For example, in any calculation in which I must use the value of $\pi$, I must use an approximation because I cannot write down the entire string of digits of $\pi$. Using a more accurate telescope or microscope will not help. The problem is not one of *measurement*, the problem is one of *representation*, and hence it is not just a practical problem but a problem in principle. Thus because our input in making a prediction is restricted to finitely statable numbers, and hence will always involve some error, our output will always involve some error too, even if the system is deterministic.

How then do we distinguish deterministic from indeterministic systems? Our original intuition can be preserved if we refine the distinction as follows: while we lack perfect measuring instruments, and we lack a way of finitely stating every real number, we can nonetheless say that in a deterministic system, our representation of the state space of the system can be made arbitrarily accurate. For deterministic systems, the accuracy of a state description is infinitely refinable, even though any given state description will contain some error. By contrast, a nondeterministic system will have an upper bound of accuracy. Thus in quantum mechanical systems, there is a limit beyond which we cannot extend the accuracy of the description of, say, both the position and momentum of an electron.

To summarize then, a deterministic system is one in which at least the following conditions hold: (a) there exists an algorithm which relates a state of the system at any given time to a state at any other time, and the algorithm is not probabilistic; (b) the system is such that a given state is always followed by the same history of state transitions; (c) any state of the system can be described with arbitrarily small (nonzero) error.

## II. Separating Determinism and Predictability

If determinism and predictability are indeed synonymous, then to say that a system is predictable is just to say that it meets these same three specifications. Certainly one could take that view of predictability. Yet will every system which meets these three conditions satisfy our expectations about predictability? The reasonable answer is no. One problem is that not every algorithm which can "drive" a system can be used as a predictive algorithm.[6] Thus some systems will be deterministic, but not determined by an algorithm of the right sort, and hence not predictable.

To understand how this can occur, let us first consider some simple examples from number theory. Suppose I want to make some "predictions" in number theory. In particular, I want to make "predictions" concerning two problems: (a) I want to predict the sum of the numbers from 1 to $N$; (b) I want to predict the $N$th decimal place in $\pi$. There exist algorithmic procedures for determining the answer to both problems, but I suggest that not all of these algorithms make acceptable models for prediction.

Here are two algorithms we might follow to determine the sum of the numbers from 1 to $N$. First, we might simply add up all the numbers. That is, we might calculate the result of: $1 + 2 + \ldots + N$. Alternatively, we might use the following equation: $N(N + 1)/2$. The difference between the two is that the size of the algorithm in the first method will vary with our choice of $N$; the larger $N$ is, the larger the algorithm. On the other hand, the size of the algorithm in the second method is invariant. Algorithms of the first type are called open form solutions; algorithms of the second type are called closed form solutions. What is peculiar about the open form solution of this problem is that in the system of integers from 1 to $N$ we must examine every element of the system in arriving at our solution. Our method requires complete information about the system under consideration. Prima facie this method seems at odds with our expectation that a prediction generates an unknown state of the system. How could this be if our algorithm must generate *every* state of the system in order to generate *any* state of the system? The puzzle here will be clearer in the context of problem (b).

Not every solvable problem in number theory has a closed form solution. In particular, problem (b) does not have a closed form solution. In order to determine the $N$th decimal place in $\pi$, we must also examine every digit in $\pi$ up to the $N$th decimal place in order to arrive at a solution. What I want to say about such a case is this: in following our algorithm, we have not *predicted* what the $N$th decimal of $\pi$ will be; to do that, we would have to have a closed form solution, and there is none. Rather, we have simply *inspected* $\pi$ to see what the $N$th decimal is; our algorithm tells us not how to make a prediction, but how to carry out such an inspection.

Let me try to motivate this perspective by moving from number theory to scientific theory. Just as there are examples of problems in number theory that lack closed form solutions, so too we should expect that there will be examples of problems in science that lack closed form solutions. Indeed there are apparently such cases.[7] Problems in fluid dynamics, problems in atmospheric turbulence, the three body problem in classical mechanics, are all taken to be problems for which no closed form solution exists. This type of problem forms a growing new area of research in the study of dynamical systems, known as *deterministic chaos* (in the Appendix I discuss two systems that serve as an elementary introduction to chaotic systems).

Suppose that we have some physical system under study, and that the system is a deterministic one: changes in state in the system are driven by a precise algorithm, and we can describe any given state of the system as accurately as we wish. Suppose further that the driving algorithm is not of closed form. Can we say that such a system is predictable? Consider what would be involved in trying to make a prediction. If we try to use some closed form approximation for an algorithm, then that is exactly what we will get: an approximation, and a bad one at that. Whatever error is present in our input state description will be magnified in our output prediction, and this does not accord with what we mean by prediction. In order to get a prediction, we expect that the predicted output will be accurate to any specified degree of accuracy. But if our algorithm magnifies error, then we cannot necessarily get any *arbitrary* degree of accuracy in the output; we will be contrained. I will take up this point in some detail below. I want to emphasize here that this requirement of accuracy in output constitutes an *additional condition* that predictable systems must satisfy. Determinism and predictability part ways if the mere determinism of a system does not guarantee that this additional condition will be satisfied.

Suppose instead that we have actually discovered the (open form) algorithm that drives the system, and we attempt to use this in making our prediction. Crutchfield and others have put the matter this way:

> To find how a system evolves from a given initial state one can employ the dynamic (equations of motion) to move incrementally along an orbit. This method of deducing the system's behavior requires computational effort proportional to the desired length of time to follow the orbit. For simple systems such as a frictionless pendulum the equations of motion may occasionally have a closed-form solution . . . . The unpredictable behavior of chaotic dynamical systems cannot be expressed in a closed-form solution. Consequently there are no possible short cuts to predicting their behavior.[8]

Since the open form algorithm will essentially replicate every state of the system in the transition from the input state to the output prediction, there is no guarantee that the algorithm will produce an output faster than the system itself reaches the end state; the algorithm requires "computational effort proportional to the desired length of time." In other words, we may not be able to predict ahead of time. Our "prediction" may not be produced until after the fact, and hence is not necessarily a prediction at all. But I have already said that I want to characterize the notion of predictability in a time-independent way. So let me restate the point in these terms: we expect that a prediction is accomplished on less than complete information about the system; that is what makes it a prediction and not just an inspection. But an open form algorithm will just replicate all the relevant information about the system under consideration; there is no short cut. Thus it is not a prediction.

One might respond that considerations such as short cuts and requiring less than complete information are not really important to predictability; that all we really care about is getting the desired result. Thus using an open form algorithm to make a prediction *is* still making a prediction, although not one of the usual sort. I believe that this is in part a fair response. It is the response one must make to keep predictability wed to determinism. However, it is only in part a fair response.

One feature of chaotic systems is that points which lie near each other in state space at some given time do not remain close to each other in their evolution: "nearby points in the state space will diverge at an exponential rate, leading to the phrase 'sensitive dependence on initial conditions.'"[9] Thus chaotic systems are error amplifying. Whether we are talking about a closed form algorithm or an open form algorithm in attempting to predict such systems, any error in input will be exponentially amplified in the output. Thus the demand that if a system is predictable we be able to obtain any arbitrary degree of accuracy in our output prediction cannot be satisfied in the case of deterministic chaos. An open form algorithm could get us such accuracy, *if* we had input with no error, but in principle our representations must always contain some error.

Why can we not simply require that the input always be of greater accuracy than the desired accuracy of the output? Would this not remove the problem of amplification of error? No. Between two different states of a system there is always some distance. I do not mean necessarily spatial distance, but rather that there is always some measure which separates the two states. Typically they are either spatial separations or temporal separations, but it could be anything depending on what the dimensions of our state space correspond to. Whatever this measure of separation is, that is what I am calling here "distance." Now suppose that we want to show that, with respect to a certain chaotic system, we can *always* get predictions limited to error measure $e$. Since we know that error is amplified, the problem becomes one of specifying what input error $e'$, where $e' < e$, we require to keep output error from exceeding $e$. It helps that the rate of error amplification in chaotic systems is essentially constant. Indeed this constancy is an important empirical tool in distinguishing noise due to chaos from other noise sources. On a case by case basis, we can limit output error as much as we like: for a given input state and a given output state that are separated by a distance $d$, and where error in the system is amplified at a rate $a$, it is a straightforward calculation to determine what the input error $e'$ must be so as not to be amplified over $d$ to a value greater than $e$. However, there is no general solution for $e'$ to keep error below $e$ for *any* output in the system as a whole. From $a$ it is equally possible to calculate a distance $d'$ such that $e'$ is amplified over $d'$ to a value greater than $e$. Thus for chaotic systems it cannot be shown that we can obtain any arbitrary degree of accuracy in output. The corollary is that for any input there will always be some distance over which error will be sufficiently amplified such that all accuracy is effectively lost. Thus in a strong sense chaotic systems are not predictable even though they are deterministic.

A different way of making the same point has been suggested by G. M. K. Hunt.[10] Hunt suggests that predictable systems (or, as he says, *epistemically* deterministic systems) must obey a condition of continuity. This condition can be stated roughly as follows: Consider two points in a system at a given time $t_1$, and call a region which contains them $R_1$. Now consider a later time $t_2$, and a region $R_2$, which contains the new location of the two points after their evolution within the system from $t_1$ to $t_2$. The system is *continuous* if for any two points, any third point which lies between them in $R_1$ also lies between them in $R_2$. But, as Hunt points out:

> Recently a class of systems have been discovered whose behavior is much more complex and fails the continuity condition in a radical way. These are the chaotic systems. No matter how close two systems are initially, their phase space paths may diverge arbitrarily far

> . . . . Between any two points whose path ends later in, say, area *A* there will be a point whose path ends in *B* . . . and *vice versa*. This phenomenon will occur no matter how close the chosen points are.[11]

Hunt's analysis is another way of illustrating the exponential amplification of error in chaotic systems. Hunt draws the same conclusion that I have drawn here:

> Given the impossibility of perfectly accurate measurement we can restate the thesis of epistemic determinism in the following form. Predictions can be made arbitrarily accurate by making the determination of the initial state of the system arbitrarily accurate . . . . But, as we have seen from our discussion, such a thesis requires continuity across sets of initial and final states of a system. And we have seen that chaotic systems do not exhibit such a continuity.[12]

Thus chaotic systems, even though they are deterministic, are not predictable (they are not *epistemically* deterministic).

Let us state now the requirements for predictability, and indicate where determinism and predictability part ways. Determinism is a necessary condition for predictability. Thus a predictable system must at least meet the conditions we have already stated for determinism: (a) there exists an algorithm which maps a state of the system at any given time to a state at any other time, and the algorithm is not probabilistic; (b) the system is such that a given state is always followed by the same history of state transitions; (c) any state of the system can be described with arbitrarily small (nonzero) error. In addition, we impose the following condition: (d) any state of the system can be generated from the algorithm with arbitrarily small (nonzero) error from any other state of the system. Determinism and predictability part ways because (a), (b), and (c) do not entail (d); in particular, chaotic systems satisfy (a), (b) and (c), but do not satisfy (d).

## III. Explanation and Prediction

One might be tempted to say that chaotic systems must in principle be predictable, that the synonymy between determinism and predictability must be preserved, and that the criteria above are therefore flawed. Doubtless one could give criteria for predictability that would encompass chaotic systems as well, but I believe that first, any such criteria would do unwarranted violence to our notion of predictability, and second, our intuition that chaotic systems are nonetheless intelligible in a way that purely random systems are not can be preserved by carefully distinguishing explanation from prediction. I conclude with a few remarks on this latter point.

The Scientific Determinist is motivated in large part by the faith that nature is thoroughly predictable. The search for predictability thus constrains what will count as a complete explanation. Yet where predictability as conceived of by the Scientific Deterministic fails, we need not abandon the attempt to provide a scientific explanation. Instead, we must redirect our expectations about what counts as a complete scientific explanation. This is precisely what physicists studying chaos have done.

To say that chaotic systems are unpredictable is not to say that science cannot explain them. As Crutchfield and others say:

> The discovery of chaos has created a new paradigm in scientific modeling. On one hand, it implies new fundamental limits on the ability to make predictions. On the other hand, the determinism inherent in chaos implies that many random phenomena are more predictable than had been thought.[13]

Clearly "predictable" is used in two different senses here. I suggest that the last sentence be read as "many random phenomena are more explainable than had been thought."

In a chaotic system we may still discover the driving algorithm, we may still know what quantities will constitute the dimensions of state space, and even this much information provides the basis for a sophisticated scientific explanation. But the task has become a different one. A complete model of an organized system just is the set of equations necessary for making predictions. The knowledge we can obtain about chaotic systems must be captured by some other modeling technique.

Dissipative chaotic systems with state spaces of fairly low dimension are a good example of this modeling technique.[14] Because these systems are dissipative, they converge to an attractor in state space. In other words, if the paths of the elements of the system are traced through state space over time, these paths will converge to a recognizable geometric form, and for a given *type* of chaotic system the attractor will always be the same. In organized systems there are essentially three attractors that the system will converge to: a fixed point (for example, a damped pendulum), a limit cycle (for example, a frictionless pendulum), or a torus of three or more dimensions. Chaotic attractors are not limited to such conventional shapes, and are thus referred to as "strange" attractors.

Yet once a scientist has discovered the attractor of a chaotic system, then he has a *model* of the system, and that model will serve as an explanation. Suppose that one of the dimensions of the state space of some model is temperature. Then the scientist cannot predict exactly what the temperature will be (he cannot determine precisely where the system is on the attractor at any given time), but he can determine, from the attractor, the temperature neighborhoods that the system will visit. This type of information represents neither ignorance in the fact of complete randomness nor predictability of the LaPlacean sort. It seems sensible then to acknowledge the unpredictability of chaotic systems, and yet broaden our notion of scientific explanation. Mere determinism of a system need not send the scientist on a quest for absolute predictability in order to produce useful results.

Independent of what one wants to say about predictability, chaotic systems represent a distinctive subset of classical dynamical systems, and we must have some way of highlighting what is distinctive about them. The methods scientists employ for studying chaotic systems simply do not resemble the methods employed when the system under study is one that can be expected to yield absolute predictions. The most natural way to bring out that distinction is in the context of predictability, and indeed distinguishing them on the basis of predictability is already the working assumption of most scientists. I have tried to argue there that that assumption is reasonable.

## Appendix

To illustrate some of the points I have made concerning chaotic systems, I include here two examples. These two examples differ importantly from most chaotic systems in that they are discrete models: the systems evolve in iterated steps rather than continuously over time. However, discrete models are considerably easier to understand, and thus these models will be more useful than the real physical systems studied in dynamical systems theory.[15]

### *A. The Bit Shift*[16]

Consider a simple rule that allows us to generate a number $x'$ from a number x, as follows: $x' = MOD_1(2x)$. Suppose also that the system evolves by taking each $x'$ from the previous iteration as the new $x$ on the next iteration. Such a system will be deterministic. Consider, however, what happens if we attempt to predict the value of $x'$ after $N$ iterations. Let $x$ and $x'$ each be represented as binary digits. Now our initial value for $x$ must be given some finitely statable representation. Let us suppose that we set $x$ such that:

$$x = .0100110101101$$

We then calculate $x'$, and find that:

$$x' = .100110101101$$

We observe that while $x$ is 13 bits, $x'$ is only 12 bits; we have lost one bit of information. Thus after each iteration we lose one bit of accuracy in our prediction, and after 13 iterations we will not be able to predict anything about the value of $x'$. In general, if we want predictions accurate to $N$ bits, then there will always be some finite number of iterations beyond which we will not have that accuracy in prediction, no matter what (finite) degree of accuracy we required in our initial value of $x$.

The problem is that for any value of $x$ out to $N$ bits, our "knowledge" of $x$ from $N+1$ bit on is random. As Shaw says:

> With each iteration the function shifts these small-scale random variations up to greater significance, and the observer will record "random" numbers in this deterministic system *even on macroscopic length scales*. From the point of view of the observer, the system is acting as an *information source*; after a short time the numbers he records cannot be computed or even estimated from the initial number, no matter how great his analytical capabilities.[17]

One interesting feature of this system is that the *rate* of loss of accuracy is quite steady. It is also independent of our initial accuracy; whether we know $x$ to twenty bits or two bits, we lose one bit of information per iteration. This invariance of rate is a distinctive feature of chaotic systems; the measure of that rate is called the entropy coefficient, or the degree of chaos, of the system. The fact that degree of chaos does not vary with the addition or subtraction of background noise provides scientists with one empirical test by which to distinguish randomness due to chaos from other types of randomness.

### *B. Life: Chaos You Can Do at Home*[18]

Consider an infinite plane, divided by lines into a grid of squares. Each square (called a cell) may be either filled or empty. Suppose we pick some initial state of filled and empty cells, and allow the system to evolve in discrete iterations (called turns) according to the following rules:

1. *Survival*: on a given turn, each filled cell which has exactly two or three filled cells among the eight cells bordering it, remains filled on the next turn.

2. *Birth*: on a given turn, any empty cell which has exactly three cells filled among the eight cells bordering it is a filled cell on the next turn.

3. *Death*: on a given turn, any filled cell which has less than two or more than three filled cells among the eight cells bordering it is an empty cell on the next turn.

4. *Priority*: the above rules are considered to apply simultaneously. Thus births and deaths determined on a particular turn do not affect further births and deaths on that turn; they only determine the state of the next turn.

The "laws" of this system, called Life because of its obvious evolutionary structure, are quite deterministic. Working out a few examples with pencil and graph paper, however, will quickly convince the reader that the system is thoroughly unpredictable. There is simply no way to know what a given initial state will evolve to ultimately, or to have more than a vague idea what it will look like after even a few turns. A few broad features of the system are predictable: an initial state that is symmetrical about some axis will remain symmetrical about that axis; asymmetrical states tend to converge towards symmetry; a configuration that enters a steady state or periodic state will remain in a steady state or periodic state barring interference from another configuration. But nothing like a complete state description on turn $N+i$ can be generated from a complete state description on state $N$ short of working through the state by state transitions from $N$ to $N+i$. Here we have a clear example of a system governed by an open form algorithm, where no closed form algorithm exists. Anyone who feels that the mere absence of a closed form algorithm does not produce unpredictability should have that feeling shaken up quite a bit by working through a few examples.

This system is useful for illustrating two features of chaotic systems. First, the radical divergence of initially close states, what Hunt calls the failure of continuity, is clear. Pick two arbitrary configurations that differ by one or two cells (it is best to select asymmetrical configurations). Work through the evolution of each for about five or ten turns. It becomes quickly clear that initial similarity is no guide to future similarity, even after a very short period of time. Second, and related to the first point, the exponential amplification of error is clear. Give yourself a degree of error in cells, and use this to generate two different configurations. Pick two configurations such that the second differs from the first by as many cells as the selected degree of error. Call the first configuration the *real* system, and the second configuration the *observed* system. Work out the evolution of each over some number of turns. How many cells do they differ by after two turns? After four turns? After eight? A few examples will illustrate that the trend is towards an exponential amplification of error.

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## NOTES

1. Pierre Simon de LaPlace, *Théorie analytique des probabiltés* (Paris: V. Courcier, 1820), preface. Tr. in Nagel, p. 281.

2. Stating determinism precisely is no easy task. John Earman has thought the problem worthy of an entire book. See his *A Primer on Determinism* (Dordrecht: Reidel, 1986). Earman also has a nice discussion of the failure of determinism in classical systems, which I do not discuss here. Earman's argument centers on the irreconcilable demand for boundary conditions on Newton's equations with Newton's concept of absolute space.

3. Bertrand Russell, "On the Notion of Cause, with Applications to the Free-Will Problem," in Herbert Feigl and May Brodbeck (eds.), *Readings in the Philosophy of Science* (New York: Appleton-Century-Crofts, 1953), p. 398.

4. Russell, pp. 400-01.

5. Ernest Nagel, "The Causal Character of Modern Physical Theory," in Herbert Feigl and May Brodbeck (eds.), *Readings in the Philosophy of Science* (New York: Appleton-Century-Crofts, 1953), pp. 419-37; Richard Montague, "Deterministic Theories," in R. H. Thomason (ed.), *Formal Philosophy: Selected Papers of Richard Montague* (New Haven: Yale University Press, 1974), pp. 303-60.

6. In "Determinism, Laws, and Predictability in Principle," *Philosophy of Science*, vol. 39 (1972), pp. 431-56, Richard Boyd pursues a different line of attack against the claim that determinism and predictability are synonymous. Boyd seeks to drive a wedge between the following two claims: "(1) Suppose that the systems in some class C behaved deterministically, then (2) There would be a set L of deterministic laws governing systems in C . . ." (p. 431). Boyd's strategy is to refine the following claim into an acceptable form: "(2) does not follow from (1) because there are only countably many finite sets of linguistic expressions and, hence, only countably many finite sets of scientific laws. But there is at least a continuum of possible deterministic systems. Thus, not every deterministic system can correspond to a finite set of laws which specifies exactly its behavior" (p. 432).

7. The use of the word "apparently" here is required. Showing that a given solution method is the optimal one is, even in mathematics, typically an unsolvable problem. Thus when scientists tell us that a particular physical problem has no closed form solution, this is an empirical matter, not a matter for demonstration. A good brief discussion of this point can be found in Robert Shaw, "Modeling Chaotic Systems," in H. Haken (ed.), *Chaos and Order in Nature: Proceedings of the International Symposium on Synergetics* (New York: Springer-Verlag, 1981), especially pp. 229-30. For a more detailed discussion, see Gregory J. Chaitin, "Information-Theoretic Limitations of Formal Systems," *Journal of the Association for Computing Machinery*, vol. 21 (1974), pp. 403-24.

8. James P. Crutchfield, J. Doyne Farmer, Norman H. Packard and Robert S. Shaw, "Chaos," *Scientific American*, vol. 255 (1986), p. 49.

9. Shaw, p. 220.

10. G. M. K. Hunt, "Determinism, Predictability and Chaos," *Analysis*, vol. 47 (1987), pp. 129-33.

11. Hunt, pp. 130-31.

12. Hunt, p. 132.

13. Crutchfield, p. 46.

14. A dissipative system is one in which the total kinetic energy of the system decreases over time. In dissipative chaotic systems the amplification of error is bound within the system rather than perturbing neighboring systems, hence they are a particularly suitable class of systems for study.

15. A good source for those interested in scientific work in this field is Heinz Georg Schuster, *Deterministic Chaos: An Introduction* (Weinheim, F.D.R.: Physik-Verlag, 1984).

16. This example is discussed in almost every introductory essay on chaos; see, for example, Shaw, pp. 218-19.

17. Shaw, p. 219.

18. This model was introduced in the Mathematical Games section of *Scientific American*. See vol. 223 (1970), pp. 120-23. It is an instance of a cellular automaton, the study of which is a growing field in computer modeling.