11. Chaos, Hyperchaos and the Double-Perspective

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Chaos and Hyperchaos

"Unknotting" is a favorite activity of the mind. When I told Art Winfree of my aim to design a "knotted" limit cycle in 3-space, he immediately sent me his precious collection of reprints and preprints on chaos, including the by then still unpublished "period 3 implies chaos" founding paper [1].

Chaos *is* a locally combable tangle of knots in 3-space, if you so wish [2]. Hairs and noodles and spaghettis and dough and taffy form an irresistible, disentanglable mess. The world of causality is thereby caricatured and, paradoxically, faithfully represented.

An expanding spiral produces chaos if re-injected [3] between two 1-dimensional crossing planes: Fig. 1.

And an expanding screw produces hyperchaos [4] if reinjected between two 2-dimensional crossing planes: Fig. 2.

The principle carries through to arbitrary dimensions. Always, but a single nonlinearity (a quadratic term, say) is needed in a single variable, all other variables being linear, to obtain maximal hyperchaos, as Gerold Baier and Sven Sahle recently found in an explicit example [5].

Waves in a linear chain with a single reflecting variable of nonlinear type on the one end, generate a beautiful carpet [5]. At the same time, they make the connection to wavy media with obstacles [6]. And they contain an uncountable number of periodic solutions of arbitrarily complex shapes in n-space. Chaos control (by Pyragas' feedback method [7]) thus ought to allow

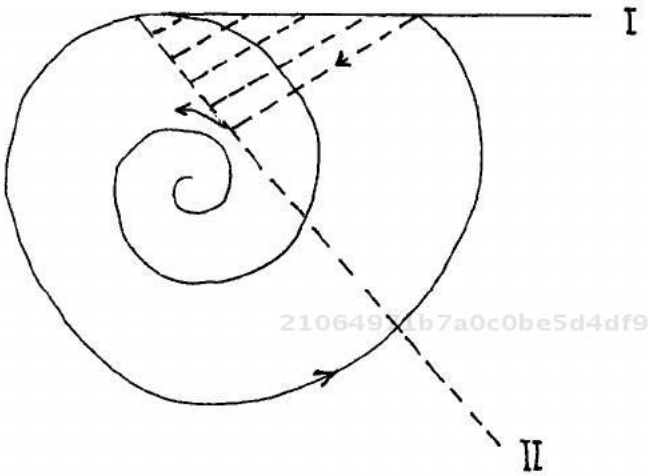

Figure 1

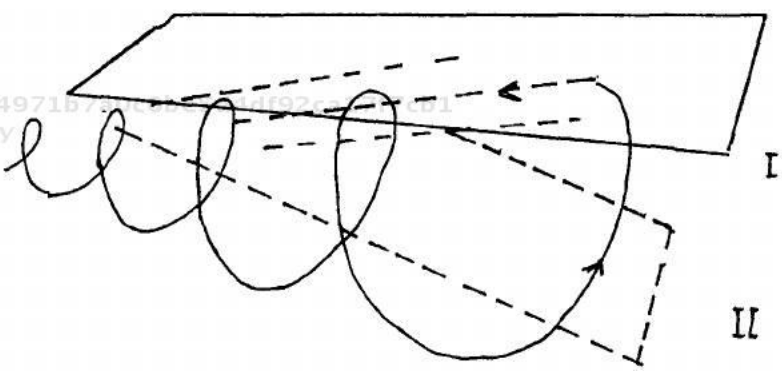

Figure 2

one to extract arbitrary messages (like a speech of Einstein's) from an initial segment, fed into such a system in a custom-made computer. Is this science or fiction?

Most recently, vowels were discovered to represent beautiful tangles embedded in low-dimensional chaotic attractors, by Sebastian Fischer. The hoarse voice and the stuttering car engine [8] and baby cries (H. Herzel) thus are no longer alone. The world of sounds becomes disentanglable.

Superfat Attractors

Let me start all over anew by looking at Georg Cantor's basic map: Fig. 3.

There is a full interval on the left, and there is an empty interval (except for a Cantor set) at the bottom. In between, every submap contains the same structure as the original map (insets). What is so beautiful about this? The sunshine reveals it.

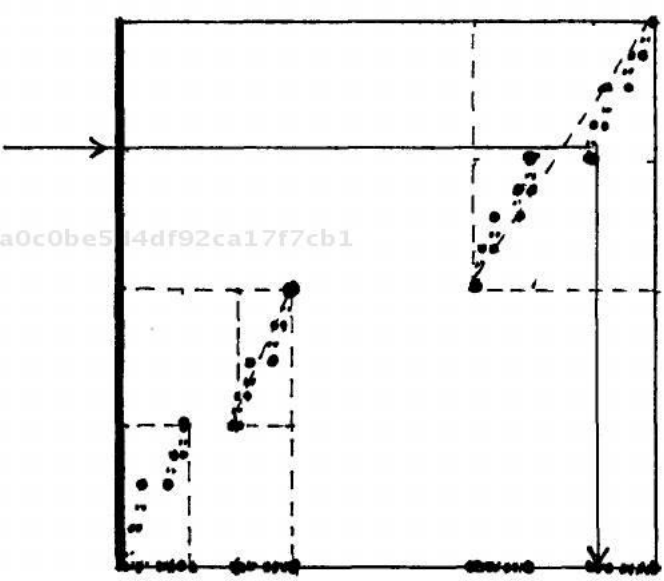

Figure 3

If we elongate everything toward the back, we have the "clotheslines phenomenon," as Jack Hudson calls it: Fig. 4.

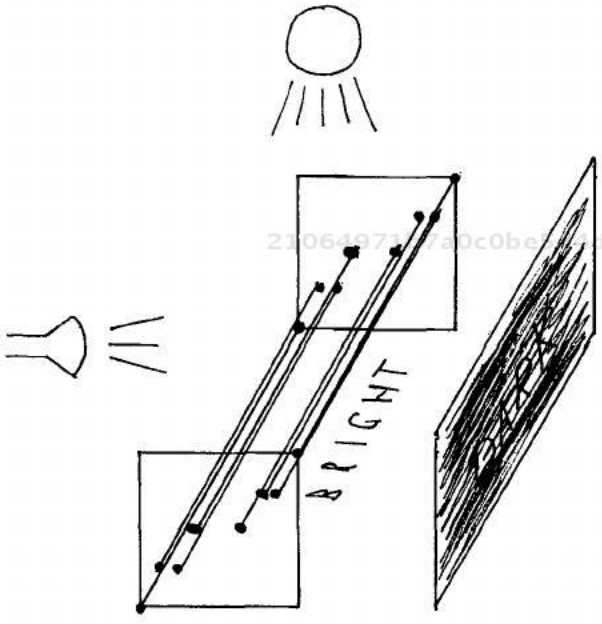

Figure 4

A set of one-dimensional clotheslines is suspended in space in such a manner that, when they are viewed from the top, all light can pass through. But if the same clotheslines are viewed from the side, their shadows make for a continuous dark surface.

This Cantorian phenomenon is robust and occurs in many chaotic attractors, in arbitrary dimensions [9]. Nowhere differentiable attractors (on a Cantor set) are implicit and the paradoxical Kaplan-Yorke phenomenon [10] is thereby explained: "Hyperfat attractors" with topological dimensionality unity but fractal dimensionality maximal (potentially filling almost the whole phase space) become a matter of course.

A special case are the "flare attractors," in which the autocatalytic growth of one variable is multiplicatively dependent on a chaotic input according to the Milnor principle [11]. Economic firms (like in the king of Wienerwald's story "From waiter to millionaire and back" [12]) fit in here [13]. The stock market comes to mind as a testing ground.

Another fractal is that of the galaxies in the sky, as Cantor's time-displaced younger friend Benoit Mandelbrot first saw [14]. Strangely, in such a fractal, gravity is distributed with a bias: Relative to an average point, "most" others have a lower gravitational potential (are redshifted), and apparently the more so the farther they are. Thus, "bang-like events" may stop to exist for a universe that is looked at with a loving (transfinitely exact) eye.

Owls and Cats

We at last come to the two-perspectives problem raised by chaos. It was invented in our time by Ed Lorenz, the second butterfly man of the 20th century after Erwin Schrödinger (as we shall see). His sharp eye saw what no one else had seen so clearly: the most important message of chaos for everybody to take home [15].

Of course, Lorenz had his predecessors, like Maxwell and Poincaré and Julia and Birkhoff and Hopf and Cartwright and Levinson and Ueda and Arnold and Smale and Abraham (cf. [16, 17]). I remember being told by a mathematical friend in the Fall of 1975 to stop running my homespun ODEs on a computer; after, he had found out that their solutions depended on the stepsize chosen for the very first step, all later steps of the Runge-Kutta algorithm having the same step size. This was against the very grain of science and reproducibility: "pathologies should not be looked at."

So what is the meaning of Lorenz's butterfly effect, which to the lay public and the philosophers alike has become the epitome of chaos? Is it a misunderstanding?

Ed Lorenz is like an owl. He can turn his head 180 degrees (or owls can, as he pointed out to my wife and me when he acted as our tour guide through the Boston Science Museum in late 1976). No one else can. With the one position of the head, you look at the universe in which the butterfly did *not* flap its wings; with the other position of the neck, you look at that other

universe in which it did. Both universes diverge exponentially from each other.

The late Joe Ford and I independently invented the "Schrödinger butterfly" in homage to Ed, cf. [18]. In that case, the two universes do *not* diverge (when the butterfly's wings are controlled by a quantum decision between two equiprobable outcomes of a superposition [19]). Everybody sees that something goes awfully awry here. Quantum and chaos become irretrievably entangled, one feels.

The owl pierces deeper with his glance. The self-similar fractal never stops, said Anaxagoras [20] in ancient Greece (if one reads him after having met Mandelbrot). Is the world infinitely exact or transfinitely exact? Anaxagoras, who had invented the technical term "recurrence" (perichoresis [21]), sided with the transfinite camp which came into existence only 2 millennia later with the notion of the long line, cf. [22]. Most recently, Dean Driebe made the distinction clear: The description of the world is qualitatively different dependent on whether you choose the state point infinitely small or transfinitely small [23]. Only in the former case can you replace everything by a time-symmetry breaking distribution.

Is chaos thus metaphysics? It certainly is not. It is the most Platonically idealistic way of looking at the world, the epitome of rationalism. It is Newton's exactness shining back and forth across the centuries and millennia.

But who believes in ODEs, in the song of the spheres, today? Some cohorts of people can be mentioned besides the age-wise and the artists. A 2-year-old once asked me to remove the milk again which I had inadvertently poured into his rosehip tea, and declared in perfect self-confidence that I was "thinking wrongly" when I told him that I was unable to do so.

Everybody has grown accustomed to this impotency in our own world. The demons of Laplace and Maxwell teach us otherwise: They represent Newtonian-chaotic transfinitely-exact myths. Are they perhaps to be revived from the dustbin?

The Interface

We come to a synthesis. Synthesis with the mind is, after all, what chaos is all about. Konton (chaos) means, you need not even open your eyes to see exactly, as Yoshi Kuramoto told me in 1978.

The piercing glance of the owl returns. There is an outside and an inside perspective. Lorenz has invented a demon, too, the demons of Laplace and Maxwell are not alone. Boscovich [24] had struggled with the same problem in the 18th century, and so did Bohr and Einstein. The potentially best-fitting word is "micro relativity." The world is "microscopically relative" to minute changes, as the butterfly effect shows. The sensitive dependence of the world is even more far-reaching, however. The "effective forcing function" that impinges on you and me is dependent on our own internal micro dynamics [25]. This time, the alternatives are not generated by an external butterfly but, so to speak, by the butterfly within.

If this is the ultimate message of chaos, we are all "bubble boys" and "bubble girls," as Peter Weibel used to say. The observer is co-responsible for the world [26]. Microrelativity tries to describe the interface.

Blood-stained cut or tender interface? The "red line"—the fractal attractor in a walking-stick map [27]—was perhaps implicitly referred to already in the Grimm brothers' tales, as a fine scar remaining after a head had been cut off and healed on in the wrong position (much like in that of an owl's). This squinting glance which no one else can follow was called "relativity" by the young Einstein [28]. Microrelativity—the logical extension—was discovered two years after Einstein's passing away by equally young Hugh Everett III in 1957 [29]. His famous "many worlds theory" is indeed no more a many worlds theory than Minkowski's theory (with its many cuts) is. For "world" and "cut" is the same thing.

What we suddenly feel is a tinge of that ominous feeling that is said to occur shortly before the consistency of the world breaks down: Could we possibly have such a transfinitely exact red line cutting right through our neck and brain? Forgive me for positing that this ultimate fear is the penultimate teaching of chaos. The world always tries to tell us to either wake up or emerge smiling serenely. Only we do not listen to it. Chaos is the koan. Ueda's attrator looks like a smile.

If the present memoir stopped here, it could perhaps still have passed as a philosophical essay. But we live between morning and evening in a cold-blooded world that asks for "tangible" implications—applications, that is. Is there such a deeper meaning to the beauty of the chaotic metaphor as everybody knows it to date?

Let me close on a few words on Everett in this context. He not only discovered the cut that presupposes the accuracy of chaos in a quantum context, but also the "world bomb." That would be an application of sorts:

the ultimate icecream bomb. A technology of an unprecedented power, but so at a high cost. Suppose it were true that there is an invisible red interface running right through the universe and your head at this moment, the so-called now-world generator. How could one prove that this is not metaphysics?

"Diós solo basta," said the rabbi's favorite granddaughter Santa Teresa in medieval Spain, with a similar reasoning in her mind. The technology at stake is not levitation. Everett disappeared not in a cloister but in the Pentagon, for the remaining 26 years of his short life. He had only published a single paper in the open literature (titled "relative state formulation" as we saw). If a "world change technology" were implicit in chaos theory, then both relativity and quantum mechanics would have to become explicable as "interface realities" as a first step so to speak. The late Kazuhisa Tomita [30], as well as Joe Ford [31], thought along similar lines. Is the objective world as tight to our skins as the Now is? The argument of "counterfactuality" [32] strongly speaks against the provability of any such dream. Note that the butterfly effect by definition is counterfactual.

Nevertheless, some recent developments in physics (like the "relativistic Bell experiment" [33] and the "gravitation-induced size increase" in proportion to the local clock slow-down [34]) could be taken to speak in favor of this idea. The observer-centered light bubble which unites relativity and quantum mechanics would then no longer be only a prison to fear: Any better-understood bondage provides the seeds for its own undoing. A "technology of liberation" would join the theology of liberation.

Thus, "chaos" oscillates today between discouragement and encouragement in much the same manner as it did through past millennia. Who dares say what the next turn of the mixture will bring to the surface?

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