

CALCULUS I

Practice Problems

Paul Dawkins

Table of Contents

Preface	iii
Outline	iii
Review.....	2
Introduction	2
Review : Functions	3
Review : Inverse Functions	6
Review : Trig Functions	6
Review : Solving Trig Equations.....	7
Review : Solving Trig Equations with Calculators, Part I	9
Review : Solving Trig Equations with Calculators, Part II	10
Review : Exponential Functions	10
Review : Logarithm Functions.....	11
Review : Exponential and Logarithm Equations	12
Review : Common Graphs	14
Limits.....	16
Introduction	16
Rates of Change and Tangent Lines.....	17
The Limit	19
One-Sided Limits	21
Limit Properties.....	22
Computing Limits	24
Infinite Limits	25
Limits At Infinity, Part I.....	26
Limits At Infinity, Part II	27
Continuity.....	28
The Definition of the Limit.....	31
Derivatives.....	31
Introduction	31
The Definition of the Derivative	32
Interpretations of the Derivative	33
Differentiation Formulas	36
Product and Quotient Rule	37
Derivatives of Trig Functions	38
Derivatives of Exponential and Logarithm Functions	39
Derivatives of Inverse Trig Functions.....	40
Derivatives of Hyperbolic Functions.....	40
Chain Rule	41
Implicit Differentiation	43
Related Rates	44
Higher Order Derivatives	46
Logarithmic Differentiation	47
Applications of Derivatives.....	47
Introduction	47
Rates of Change.....	48
Critical Points.....	49
Minimum and Maximum Values.....	50
Finding Absolute Extrema	53
The Shape of a Graph, Part I.....	54
The Shape of a Graph, Part II	56
The Mean Value Theorem	58
Optimization	59
More Optimization Problems	59

Indeterminate Forms and L'Hospital's Rule	60
Linear Approximations	61
Differentials	62
Newton's Method.....	62
Business Applications	63
Integrals.....	64
Introduction	64
Indefinite Integrals.....	65
Computing Indefinite Integrals	65
Substitution Rule for Indefinite Integrals.....	67
More Substitution Rule	69
Area Problem	70
The Definition of the Definite Integral	70
Computing Definite Integrals	71
Substitution Rule for Definite Integrals.....	73
Applications of Integrals.....	74
Introduction	74
Average Function Value	75
Area Between Curves	75
Volumes of Solids of Revolution / Method of Rings.....	76
Volumes of Solids of Revolution / Method of Cylinders	77
More Volume Problems.....	77
Work.....	79

Preface

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Outline

Here is a list of sections for which problems have been written.

[Review](#)

[Review : Functions](#)

[Review : Inverse Functions](#)

[Review : Trig Functions](#)

[Review : Solving Trig Equations](#)

[Review : Solving Trig Equations with Calculators, Part I](#)

[Review : Solving Trig Equations with Calculators, Part II](#)

[Review : Exponential Functions](#)

[Review : Logarithm Functions](#)

[Review : Exponential and Logarithm Equations](#)

[Review : Common Graphs](#)

Limits

[Tangent Lines and Rates of Change](#)

[The Limit](#)

[One-Sided Limits](#)

[Limit Properties](#)

[Computing Limits](#)

[Infinite Limits](#)

[Limits At Infinity, Part I](#)

[Limits At Infinity, Part II](#)

[Continuity](#)

[The Definition of the Limit](#)

Derivatives

[The Definition of the Derivative](#)

[Interpretation of the Derivative](#)

[Differentiation Formulas](#)

[Product and Quotient Rule](#)

[Derivatives of Trig Functions](#)

[Derivatives of Exponential and Logarithm Functions](#)

[Derivatives of Inverse Trig Functions](#)

[Derivatives of Hyperbolic Functions](#)

[Chain Rule](#)

[Implicit Differentiation](#)

[Related Rates](#)

[Higher Order Derivatives](#)

[Logarithmic Differentiation](#)

Applications of Derivatives

[Rates of Change](#)

[Critical Points](#)

[Minimum and Maximum Values](#)

[Finding Absolute Extrema](#)

[The Shape of a Graph, Part I](#)

[The Shape of a Graph, Part II](#)

[The Mean Value Theorem](#)

[Optimization Problems](#)

[More Optimization Problems](#)

[L'Hospital's Rule and Indeterminate Forms](#)

[Linear Approximations](#)

[Differentials](#)

[Newton's Method](#)

[Business Applications](#)

Integrals

[Indefinite Integrals](#)

[Computing Indefinite Integrals](#)

[Substitution Rule for Indefinite Integrals](#)

[More Substitution Rule](#)

[Area Problem](#)

[Definition of the Definite Integral](#)

[Computing Definite Integrals](#)

[Substitution Rule for Definite Integrals](#)

[Applications of Integrals](#)

[Average Function Value](#)

[Area Between Two Curves](#)

[Volumes of Solids of Revolution / Method of Rings](#)

[Volumes of Solids of Revolution / Method of Cylinders](#)

[More Volume Problems](#)

[Work](#)

Review

Introduction

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[Review : Solving Trig Equations](#)

[Review : Solving Trig Equations with Calculators, Part I](#)

[Review : Solving Trig Equations with Calculators, Part II](#)

[Review : Exponential Functions](#)

[Review : Logarithm Functions](#)

[Review : Exponential and Logarithm Equations](#)

[Review : Common Graphs](#)

Review : Functions

For problems 1 – 4 the given functions perform the indicated function evaluations.

1. $f(x) = 3 - 5x - 2x^2$

(a) $f(4)$

(b) $f(0)$

(c) $f(-3)$

(d) $f(6-t)$

(e) $f(7-4x)$

(f) $f(x+h)$

2. $g(t) = \frac{t}{2t+6}$

(a) $g(0)$

(b) $g(-3)$

(c) $g(10)$

(d) $g(x^2)$

(e) $g(t+h)$

(f) $g(t^2-3t+1)$

3. $h(z) = \sqrt{1-z^2}$

(a) $h(0)$

(b) $h(-\frac{1}{2})$

(c) $h(\frac{1}{2})$

(d) $h(9z)$

(e) $h(z^2-2z)$

(f) $h(z+k)$

4. $R(x) = \sqrt{3+x} - \frac{4}{x+1}$

(a) $R(0)$

(b) $R(6)$

(c) $R(-9)$

(d) $R(x+1)$

(e) $R(x^4-3)$

(f) $R(\frac{1}{x}-1)$

The **difference quotient** of a function $f(x)$ is defined to be,

$$\frac{f(x+h) - f(x)}{h}$$

For problems 5 – 9 compute the difference quotient of the given function.

5. $f(x) = 4x - 9$

6. $g(x) = 6 - x^2$

7. $f(t) = 2t^2 - 3t + 9$

$$8. y(z) = \frac{1}{z+2}$$

$$9. A(t) = \frac{2t}{3-t}$$

For problems 10 – 17 determine all the roots of the given function.

$$10. f(x) = x^5 - 4x^4 - 32x^3$$

$$11. R(y) = 12y^2 + 11y - 5$$

$$12. h(t) = 18 - 3t - 2t^2$$

$$13. g(x) = x^3 + 7x^2 - x$$

$$14. W(x) = x^4 + 6x^2 - 27$$

$$15. f(t) = t^{\frac{5}{3}} - 7t^{\frac{4}{3}} - 8t$$

$$16. h(z) = \frac{z}{z-5} - \frac{4}{z-8}$$

$$17. g(w) = \frac{2w}{w+1} + \frac{w-4}{2w-3}$$

For problems 18 – 22 find the domain and range of the given function.

$$18. Y(t) = 3t^2 - 2t + 1$$

$$19. g(z) = -z^2 - 4z + 7$$

$$20. f(z) = 2 + \sqrt{z^2 + 1}$$

$$21. h(y) = -3\sqrt{14 + 3y}$$

$$22. M(x) = 5 - |x + 8|$$

For problems 23 – 31 find the domain of the given function.

$$23. f(w) = \frac{w^3 - 3w + 1}{12w - 7}$$

$$24. R(z) = \frac{5}{z^3 + 10z^2 + 9z}$$

$$25. g(t) = \frac{6t - t^3}{7 - t - 4t^2}$$

$$26. g(x) = \sqrt{25 - x^2}$$

$$27. h(x) = \sqrt{x^4 - x^3 - 20x^2}$$

$$28. P(t) = \frac{5t + 1}{\sqrt{t^3 - t^2 - 8t}}$$

$$29. f(z) = \sqrt{z - 1} + \sqrt{z + 6}$$

$$30. h(y) = \sqrt{2y + 9} - \frac{1}{\sqrt{2 - y}}$$

$$31. A(x) = \frac{4}{x - 9} - \sqrt{x^2 - 36}$$

$$32. Q(y) = \sqrt{y^2 + 1} - \sqrt[3]{1 - y}$$

For problems 33 – 36 compute $(f \circ g)(x)$ and $(g \circ f)(x)$ for each of the given pair of functions.

$$33. f(x) = 4x - 1, \quad g(x) = \sqrt{6 + 7x}$$

$$34. f(x) = 5x + 2, \quad g(x) = x^2 - 14x$$

$$35. f(x) = x^2 - 2x + 1, \quad g(x) = 8 - 3x^2$$

$$36. f(x) = x^2 + 3, \quad g(x) = \sqrt{5 + x^2}$$

Review : Inverse Functions

For each of the following functions find the inverse of the function. Verify your inverse by computing one or both of the composition as discussed in this section.

$$1. f(x) = 6x + 15$$

$$2. h(x) = 3 - 29x$$

$$3. R(x) = x^3 + 6$$

$$4. g(x) = 4(x - 3)^5 + 21$$

$$5. W(x) = \sqrt[5]{9 - 11x}$$

$$6. f(x) = \sqrt[3]{5x + 8}$$

$$7. h(x) = \frac{1 + 9x}{4 - x}$$

$$8. f(x) = \frac{6 - 10x}{8x + 7}$$

Review : Trig Functions

Determine the exact value of each of the following without using a calculator.

Note that the point of these problems is not really to learn how to find the value of trig functions but instead to get you comfortable with the unit circle since that is a very important skill that will be needed in solving trig equations.

$$1. \cos\left(\frac{5\pi}{6}\right)$$

2. $\sin\left(-\frac{4\pi}{3}\right)$

3. $\sin\left(\frac{7\pi}{4}\right)$

4. $\cos\left(-\frac{2\pi}{3}\right)$

5. $\tan\left(\frac{3\pi}{4}\right)$

6. $\sec\left(-\frac{11\pi}{6}\right)$

7. $\cos\left(\frac{8\pi}{3}\right)$

8. $\tan\left(-\frac{\pi}{3}\right)$

9. $\tan\left(\frac{15\pi}{4}\right)$

10. $\sin\left(-\frac{11\pi}{3}\right)$

11. $\sec\left(\frac{29\pi}{4}\right)$

Review : Solving Trig Equations

Without using a calculator find the solution(s) to the following equations. If an interval is given then find only those solutions that are in the interval. If no interval is given then find all solutions to the equation.

1. $4\sin(3t) = 2$

$$2. 4 \sin(3t) = 2 \text{ in } \left[0, \frac{4\pi}{3}\right]$$

$$3. 2 \cos\left(\frac{x}{3}\right) + \sqrt{2} = 0$$

$$4. 2 \cos\left(\frac{x}{3}\right) + \sqrt{2} = 0 \text{ in } [-7\pi, 7\pi]$$

$$5. 4 \cos(6z) = \sqrt{12} \text{ in } \left[0, \frac{\pi}{2}\right]$$

$$6. 2 \sin\left(\frac{3y}{2}\right) + \sqrt{3} = 0 \text{ in } \left[-\frac{7\pi}{3}, 0\right]$$

$$7. 8 \tan(2x) - 5 = 3 \text{ in } \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$$

$$8. 16 = -9 \sin(7x) - 4 \text{ in } \left[-2\pi, \frac{9\pi}{4}\right]$$

$$9. \sqrt{3} \tan\left(\frac{t}{4}\right) + 5 = 4 \text{ in } [0, 4\pi]$$

$$10. \sqrt{3} \csc(9z) - 7 = -5 \text{ in } \left[-\frac{\pi}{3}, \frac{4\pi}{9}\right]$$

$$11. 1 - 14 \cos\left(\frac{2x}{5}\right) = -6 \text{ in } \left[5\pi, \frac{40\pi}{3}\right]$$

$$12. 15 = 17 + 4 \cos\left(\frac{y}{7}\right) \text{ in } [10\pi, 15\pi]$$

Review : Solving Trig Equations with Calculators, Part I

Find the solution(s) to the following equations. If an interval is given then find only those solutions that are in the interval. If no interval is given then find all solutions to the equation. These will require the use of a calculator so use at least 4 decimal places in your work.

1. $7 \cos(4x) + 11 = 10$

2. $6 + 5 \cos\left(\frac{x}{3}\right) = 10$ in $[0, 38]$

3. $3 = 6 - 11 \sin\left(\frac{t}{8}\right)$

4. $4 \sin(6z) + \frac{13}{10} = -\frac{3}{10}$ in $[0, 2]$

5. $9 \cos\left(\frac{4z}{9}\right) + 21 \sin\left(\frac{4z}{9}\right) = 0$ in $[-10, 10]$

6. $3 \tan\left(\frac{w}{4}\right) - 1 = 11 - 2 \tan\left(\frac{w}{4}\right)$ in $[-50, 0]$

7. $17 - 3 \sec\left(\frac{z}{2}\right) = 2$ in $[20, 45]$

8. $12 \sin(7y) + 11 = 3 + 4 \sin(7y)$ in $\left[-2, -\frac{1}{2}\right]$

9. $5 - 14 \tan(8x) = 30$ in $[-1, 1]$

10. $0 = 18 + 2 \csc\left(\frac{t}{3}\right)$ in $[0, 5]$

11. $\frac{1}{2} \cos\left(\frac{x}{8}\right) + \frac{1}{4} = \frac{2}{3}$ in $[0, 100]$

12. $\frac{4}{3} = 1 + 3 \sec(2t)$ in $[-4, 6]$

Review : Solving Trig Equations with Calculators, Part II

Find all the solution(s) to the following equations. These will require the use of a calculator so use at least 4 decimal places in your work.

1. $3 - 14 \sin(12t + 7) = 13$

2. $3 \sec(4 - 9z) - 24 = 0$

3. $4 \sin(x + 2) - 15 \sin(x + 2) \tan(4x) = 0$

4. $3 \cos\left(\frac{3y}{7}\right) \sin\left(\frac{y}{2}\right) + 14 \cos\left(\frac{3y}{7}\right) = 0$

5. $7 \cos^2(3x) - \cos(3x) = 0$

6. $\tan^2\left(\frac{w}{4}\right) = \tan\left(\frac{w}{4}\right) + 12$

7. $4 \csc^2(1 - t) + 6 = 25 \csc(1 - t)$

8. $4y \sec(7y) = -21y$

9. $10x^2 \sin(3x + 2) = 7x \sin(3x + 2)$

10. $(2t - 3) \tan\left(\frac{6t}{11}\right) = 15 - 10t$

Review : Exponential Functions

Sketch the graphs of each of the following functions.

1. $f(x) = 3^{1+2x}$

2. $h(x) = 2^{3-\frac{x}{4}} - 7$

3. $h(t) = 8 + 3e^{2t-4}$

4. $g(z) = 10 - \frac{1}{4}e^{-2-3z}$

Review : Logarithm Functions

Without using a calculator determine the exact value of each of the following.

1. $\log_3 81$

2. $\log_5 125$

3. $\log_2 \frac{1}{8}$

4. $\log_{\frac{1}{4}} 16$

5. $\ln e^4$

6. $\log \frac{1}{100}$

Write each of the following in terms of simpler logarithms

7. $\log(3x^4y^{-7})$

8. $\ln\left(x\sqrt{y^2 + z^2}\right)$

9. $\log_4\left(\frac{x-4}{y^2\sqrt[5]{z}}\right)$

Combine each of the following into a single logarithm with a coefficient of one.

$$10. 2\log_4 x + 5\log_4 y - \frac{1}{2}\log_4 z$$

$$11. 3\ln(t+5) - 4\ln t - 2\ln(s-1)$$

$$12. \frac{1}{3}\log a - 6\log b + 2$$

Use the change of base formula and a calculator to find the value of each of the following.

$$13. \log_{12} 35$$

$$14. \log_{\frac{2}{3}} 53$$

Review : Exponential and Logarithm Equations

For problems 1 – 10 find all the solutions to the given equation. If there is no solution to the equation clearly explain why.

$$1. 12 - 4e^{7+3x} = 7$$

$$2. 1 = 10 - 3e^{z^2-2z}$$

$$3. 2t - te^{6t-1} = 0$$

$$4. 4x + 1 = (12x + 3)e^{x^2-2}$$

$$5. 2e^{3y+8} - 11e^{5-10y} = 0$$

$$6. 14e^{6-x} + e^{12x-7} = 0$$

$$7. 1 - 8\ln\left(\frac{2x-1}{7}\right) = 14$$

$$8. \ln(y-1) = 1 + \ln(3y+2)$$

9. $\log(w) + \log(w - 21) = 2$

10. $2\log(z) - \log(7z - 1) = 0$

11. $16 = 17^{t-2} + 11$

12. $2^{3-8w} - 7 = 11$

Compound Interest. If we put P dollars into an account that earns interest at a rate of r (written as a decimal as opposed to the standard percent) for t years then,

- a. if interest is compounded m times per year we will have,

$$A = P \left(1 + \frac{r}{m} \right)^{tm}$$

dollars after t years.

- b. if interest is compounded continuously we will have,

$$A = Pe^{rt}$$

dollars after t years.

13. We have \$10,000 to invest for 44 months. How much money will we have if we put the money into an account that has an annual interest rate of 5.5% and interest is compounded

- (a) quarterly (b) monthly (c) continuously

14. We are starting with \$5000 and we're going to put it into an account that earns an annual interest rate of 12%. How long should we leave the money in the account in order to double our money if interest is compounded

- (a) quarterly (b) monthly (c) continuously

Exponential Growth/Decay. Many quantities in the world can be modeled (at least for a short time) by the exponential growth/decay equation.

$$Q = Q_0 e^{kt}$$

If k is positive then we will get exponential growth and if k is negative we will get exponential decay.

15. A population of bacteria initially has 250 present and in 5 days there will be 1600 bacteria present.

- (a) Determine the exponential growth equation for this population.
(b) How long will it take for the population to grow from its initial population of 250 to

a population of 2000?

16. We initially have 100 grams of a radioactive element and in 1250 years there will be 80 grams left.

- (a) Determine the exponential decay equation for this element.
- (b) How long will it take for half of the element to decay?
- (c) How long will it take until there is only 1 gram of the element left?

Review : Common Graphs

Without using a graphing calculator sketch the graph of each of the following.

1. $y = \frac{4}{3}x - 2$

2. $f(x) = |x - 3|$

3. $g(x) = \sin(x) + 6$

4. $f(x) = \ln(x) - 5$

5. $h(x) = \cos\left(x + \frac{\pi}{2}\right)$

6. $h(x) = (x - 3)^2 + 4$

7. $W(x) = e^{x+2} - 3$

8. $f(y) = (y - 1)^2 + 2$

9. $R(x) = -\sqrt{x}$

10. $g(x) = \sqrt{-x}$

$$11. h(x) = 2x^2 - 3x + 4$$

$$12. f(y) = -4y^2 + 8y + 3$$

$$13. (x+1)^2 + (y-5)^2 = 9$$

$$14. x^2 - 4x + y^2 - 6y - 87 = 0$$

$$15. 25(x+2)^2 + \frac{y^2}{4} = 1$$

$$16. x^2 + \frac{(y-6)^2}{9} = 1$$

$$17. \frac{x^2}{36} - \frac{y^2}{49} = 1$$

$$18. (y+2)^2 - \frac{(x+4)^2}{16} = 1$$

Limits

Introduction

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Rates of Change and Tangent Lines

1. For the function $f(x) = 3(x+2)^2$ and the point P given by $x = -3$ answer each of the following questions.

(a) For the points Q given by the following values of x compute (accurate to at least 8 decimal places) the slope, m_{PQ} , of the secant line through points P and Q .

- | | | | | |
|-----------|------------|--------------|-------------|-------------|
| (i) -3.5 | (ii) -3.1 | (iii) -3.01 | (iv) -3.001 | (v) -3.0001 |
| (vi) -2.5 | (vii) -2.9 | (viii) -2.99 | (ix) -2.999 | (x) -2.9999 |

(b) Use the information from (a) to estimate the slope of the tangent line to $f(x)$ at $x = -3$ and write down the equation of the tangent line.

2. For the function $g(x) = \sqrt{4x+8}$ and the point P given by $x = 2$ answer each of the following questions.

(a) For the points Q given by the following values of x compute (accurate to at least 8 decimal places) the slope, m_{PQ} , of the secant line through points P and Q .

- | | | | | |
|----------|-----------|-------------|------------|------------|
| (i) 2.5 | (ii) 2.1 | (iii) 2.01 | (iv) 2.001 | (v) 2.0001 |
| (vi) 1.5 | (vii) 1.9 | (viii) 1.99 | (ix) 1.999 | (x) 1.9999 |

(b) Use the information from (a) to estimate the slope of the tangent line to $g(x)$ at $x = 2$ and write down the equation of the tangent line.

3. For the function $W(x) = \ln(1+x^4)$ and the point P given by $x = 1$ answer each of the following questions.

(a) For the points Q given by the following values of x compute (accurate to at least 8 decimal places) the slope, m_{PQ} , of the secant line through points P and Q .

- | | | | | |
|----------|-----------|-------------|------------|------------|
| (i) 1.5 | (ii) 1.1 | (iii) 1.01 | (iv) 1.001 | (v) 1.0001 |
| (vi) 0.5 | (vii) 0.9 | (viii) 0.99 | (ix) 0.999 | (x) 0.9999 |

(b) Use the information from (a) to estimate the slope of the tangent line to $W(x)$ at $x = 1$ and write down the equation of the tangent line.

4. The volume of air in a balloon is given by $V(t) = \frac{6}{4t+1}$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average rate of change of the volume of air in the balloon between $t = 0.25$ and the following values of t .

- | | | | | |
|--------|-----------|--------------|-------------|-------------|
| (i) 1 | (ii) 0.5 | (iii) 0.251 | (iv) 0.2501 | (v) 0.25001 |
| (vi) 0 | (vii) 0.1 | (viii) 0.249 | (ix) 0.2499 | (x) 0.24999 |

(b) Use the information from (a) to estimate the instantaneous rate of change of the volume of air in the balloon at $t = 0.25$.

5. The population (in hundreds) of fish in a pond is given by $P(t) = 2t + \sin(2t - 10)$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average rate of change of the population of fish between $t = 5$ and the following values of t . Make sure your calculator is set to radians for the computations.

- | | | | | |
|----------|-----------|-------------|------------|------------|
| (i) 5.5 | (ii) 5.1 | (iii) 5.01 | (iv) 5.001 | (v) 5.0001 |
| (vi) 4.5 | (vii) 4.9 | (viii) 4.99 | (ix) 4.999 | (x) 4.9999 |

(b) Use the information from (a) to estimate the instantaneous rate of change of the population of the fish at $t = 5$.

6. The position of an object is given by $s(t) = \cos^2\left(\frac{3t-6}{2}\right)$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average velocity of the object between $t = 2$ and the following values of t . Make sure your calculator is set to radians for the computations.

- | | | | | |
|----------|-----------|-------------|------------|------------|
| (i) 2.5 | (ii) 2.1 | (iii) 2.01 | (iv) 2.001 | (v) 2.0001 |
| (vi) 1.5 | (vii) 1.9 | (viii) 1.99 | (ix) 1.999 | (x) 1.9999 |

(b) Use the information from (a) to estimate the instantaneous velocity of the object at $t = 2$ and determine if the object is moving to the right (*i.e.* the instantaneous velocity is positive), moving to the left (*i.e.* the instantaneous velocity is negative), or not moving (*i.e.* the instantaneous velocity is zero).

7. The position of an object is given by $s(t) = (8-t)(t+6)^{\frac{3}{2}}$. Note that a negative position here simply means that the position is to the left of the “zero position” and is perfectly acceptable.

Answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average velocity of the object between $t = 10$ and the following values of t .

- | | | | | |
|----------|-----------|-------------|-------------|-------------|
| (i) 10.5 | (ii) 10.1 | (iii) 10.01 | (iv) 10.001 | (v) 10.0001 |
| (vi) 9.5 | (vii) 9.9 | (viii) 9.99 | (ix) 9.999 | (x) 9.9999 |

(b) Use the information from (a) to estimate the instantaneous velocity of the object at $t = 10$ and determine if the object is moving to the right (*i.e.* the instantaneous velocity is positive), moving to the left (*i.e.* the instantaneous velocity is negative), or not moving (*i.e.* the instantaneous velocity is zero).

The Limit

1. For the function $f(x) = \frac{8 - x^3}{x^2 - 4}$ answer each of the following questions.

(a) Evaluate the function the following values of x compute (accurate to at least 8 decimal places).

- | | | | | |
|----------|-----------|-------------|------------|------------|
| (i) 2.5 | (ii) 2.1 | (iii) 2.01 | (iv) 2.001 | (v) 2.0001 |
| (vi) 1.5 | (vii) 1.9 | (viii) 1.99 | (ix) 1.999 | (x) 1.9999 |

(b) Use the information from (a) to estimate the value of $\lim_{x \rightarrow 2} \frac{8 - x^3}{x^2 - 4}$.

2. For the function $R(t) = \frac{2 - \sqrt{t^2 + 3}}{t + 1}$ answer each of the following questions.

(a) Evaluate the function the following values of t compute (accurate to at least 8 decimal places).

- | | | | | |
|-----------|------------|--------------|-------------|-------------|
| (i) -0.5 | (ii) -0.9 | (iii) -0.99 | (iv) -0.999 | (v) -0.9999 |
| (vi) -1.5 | (vii) -1.1 | (viii) -1.01 | (ix) -1.001 | (x) -1.0001 |

(b) Use the information from (a) to estimate the value of $\lim_{t \rightarrow -1} \frac{2 - \sqrt{t^2 + 3}}{t + 1}$.

3. For the function $g(\theta) = \frac{\sin(7\theta)}{\theta}$ answer each of the following questions.

(a) Evaluate the function the following values of θ compute (accurate to at least 8 decimal places). Make sure your calculator is set to radians for the computations.

- | | | | | |
|-----------|------------|--------------|-------------|-------------|
| (i) 0.5 | (ii) 0.1 | (iii) 0.01 | (iv) 0.001 | (v) 0.0001 |
| (vi) -0.5 | (vii) -0.1 | (viii) -0.01 | (ix) -0.001 | (x) -0.0001 |

(b) Use the information from (a) to estimate the value of $\lim_{\theta \rightarrow 0} \frac{\sin(7\theta)}{\theta}$.

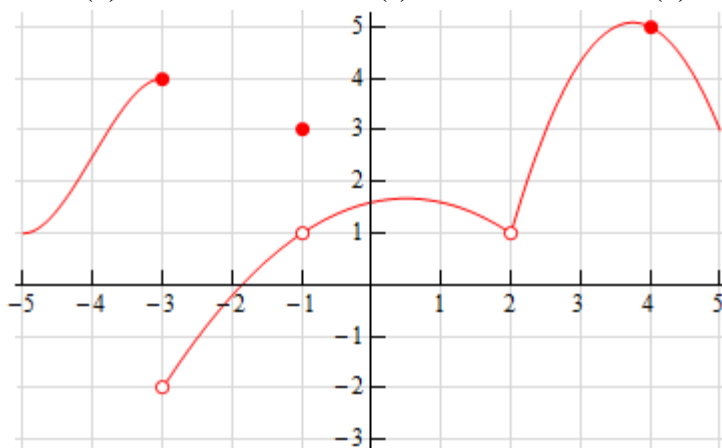
4. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim_{x \rightarrow a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -3$

(b) $a = -1$

(c) $a = 2$

(d) $a = 4$



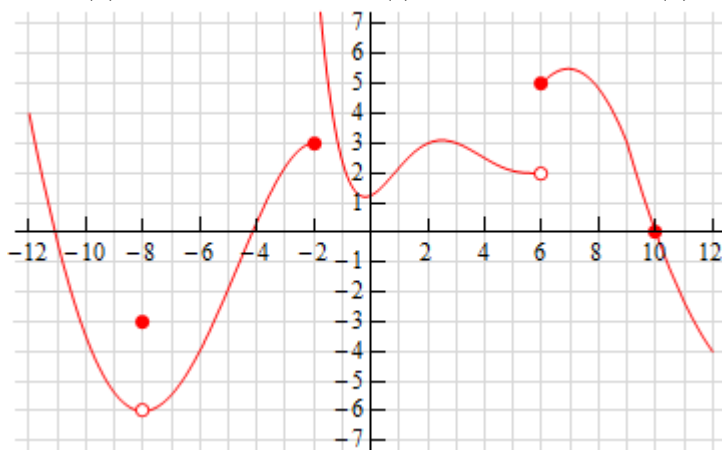
5. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim_{x \rightarrow a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -8$

(b) $a = -2$

(c) $a = 6$

(d) $a = 10$



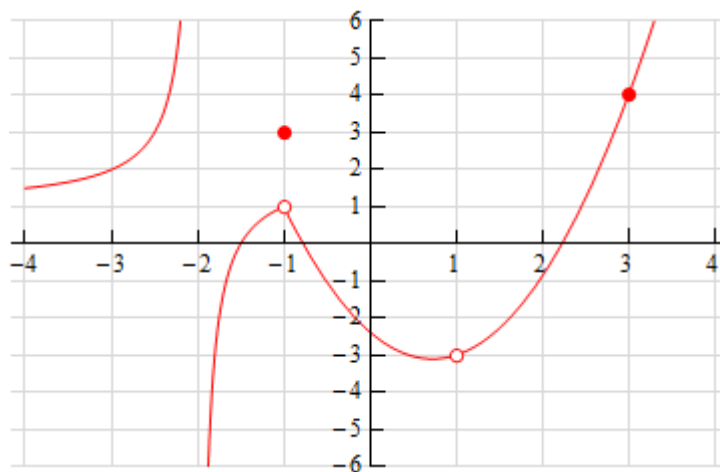
6. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim_{x \rightarrow a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -2$

(b) $a = -1$

(c) $a = 1$

(d) $a = 3$



One-Sided Limits

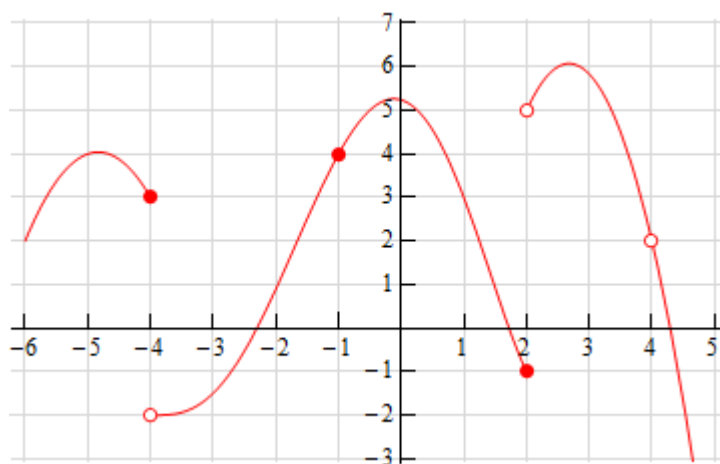
1. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$, $\lim_{x \rightarrow a^-} f(x)$, $\lim_{x \rightarrow a^+} f(x)$, and $\lim_{x \rightarrow a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -4$

(b) $a = -1$

(c) $a = 2$

(d) $a = 4$



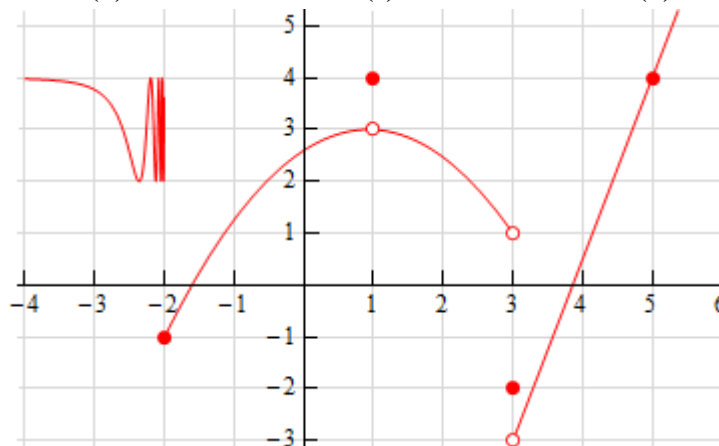
2. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$, $\lim_{x \rightarrow a^-} f(x)$, $\lim_{x \rightarrow a^+} f(x)$, and $\lim_{x \rightarrow a} f(x)$. If any of the quantities do not exist clearly explain why.

(a) $a = -2$

(b) $a = 1$

(c) $a = 3$

(d) $a = 5$



3. Sketch a graph of a function that satisfies each of the following conditions.

$$\lim_{x \rightarrow 2^-} f(x) = 1$$

$$\lim_{x \rightarrow 2^+} f(x) = -4$$

$$f(2) = 1$$

4. Sketch a graph of a function that satisfies each of the following conditions.

$$\lim_{x \rightarrow 3^-} f(x) = 0$$

$$\lim_{x \rightarrow 3^+} f(x) = 4$$

$$f(3) \text{ does not exist}$$

$$\lim_{x \rightarrow -1} f(x) = -3$$

$$f(-1) = 2$$

Limit Properties

1. Given $\lim_{x \rightarrow 8} f(x) = -9$, $\lim_{x \rightarrow 8} g(x) = 2$ and $\lim_{x \rightarrow 8} h(x) = 4$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) $\lim_{x \rightarrow 8} [2f(x) - 12h(x)]$

(b) $\lim_{x \rightarrow 8} [3h(x) - 6]$

(c) $\lim_{x \rightarrow 8} [g(x)h(x) - f(x)]$

(d) $\lim_{x \rightarrow 8} [f(x) - g(x) + h(x)]$

2. Given $\lim_{x \rightarrow -4} f(x) = 1$, $\lim_{x \rightarrow -4} g(x) = 10$ and $\lim_{x \rightarrow -4} h(x) = -7$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

$$(a) \lim_{x \rightarrow -4} \left[\frac{f(x)}{g(x)} - \frac{h(x)}{f(x)} \right]$$

$$(b) \lim_{x \rightarrow -4} [f(x)g(x)h(x)]$$

$$(c) \lim_{x \rightarrow -4} \left[\frac{1}{h(x)} + \frac{3 - f(x)}{g(x) + h(x)} \right]$$

$$(d) \lim_{x \rightarrow -4} \left[2h(x) - \frac{1}{h(x) + 7f(x)} \right]$$

3. Given $\lim_{x \rightarrow 0} f(x) = 6$, $\lim_{x \rightarrow 0} g(x) = -4$ and $\lim_{x \rightarrow 0} h(x) = -1$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

$$(a) \lim_{x \rightarrow 0} [f(x) + h(x)]^3$$

$$(b) \lim_{x \rightarrow 0} \sqrt{g(x)h(x)}$$

$$(c) \lim_{x \rightarrow 0} \sqrt[3]{11 + [g(x)]^2}$$

$$(d) \lim_{x \rightarrow 0} \sqrt{\frac{f(x)}{h(x) - g(x)}}$$

For each of the following limits use the limit properties given in this section to compute the limit. At each step clearly indicate the property being used. If it is not possible to compute any of the limits clearly explain why not.

$$4. \lim_{t \rightarrow -2} (14 - 6t + t^3)$$

$$5. \lim_{x \rightarrow 6} (3x^2 + 7x - 16)$$

$$6. \lim_{w \rightarrow 3} \frac{w^2 - 8w}{4 - 7w}$$

$$7. \lim_{x \rightarrow -5} \frac{x + 7}{x^2 + 3x - 10}$$

$$8. \lim_{z \rightarrow 0} \sqrt{z^2 + 6}$$

9. $\lim_{x \rightarrow 10} (4x + \sqrt[3]{x-2})$

Computing Limits

For problems 1 – 9 evaluate the limit, if it exists.

1. $\lim_{x \rightarrow 2} (8 - 3x + 12x^2)$

2. $\lim_{t \rightarrow -3} \frac{6 + 4t}{t^2 + 1}$

3. $\lim_{x \rightarrow -5} \frac{x^2 - 25}{x^2 + 2x - 15}$

4. $\lim_{z \rightarrow 8} \frac{2z^2 - 17z + 8}{8 - z}$

5. $\lim_{y \rightarrow 7} \frac{y^2 - 4y - 21}{3y^2 - 17y - 28}$

6. $\lim_{h \rightarrow 0} \frac{(6+h)^2 - 36}{h}$

7. $\lim_{z \rightarrow 4} \frac{\sqrt{z} - 2}{z - 4}$

8. $\lim_{x \rightarrow -3} \frac{\sqrt{2x+22} - 4}{x+3}$

9. $\lim_{x \rightarrow 0} \frac{x}{3 - \sqrt{x+9}}$

10. Given the function

$$f(x) = \begin{cases} 7 - 4x & x < 1 \\ x^2 + 2 & x \geq 1 \end{cases}$$

Evaluate the following limits, if they exist.

(a) $\lim_{x \rightarrow -6} f(x)$

(b) $\lim_{x \rightarrow 1} f(x)$

11. Given

$$h(z) = \begin{cases} 6z & z \leq -4 \\ 1 - 9z & z > -4 \end{cases}$$

Evaluate the following limits, if they exist.

(a) $\lim_{z \rightarrow 7} h(z)$

(b) $\lim_{z \rightarrow -4} h(z)$

For problems 12 & 13 evaluate the limit, if it exists.

12. $\lim_{x \rightarrow 5} (10 + |x - 5|)$

13. $\lim_{t \rightarrow -1} \frac{t+1}{|t+1|}$

14. Given that $7x \leq f(x) \leq 3x^2 + 2$ for all x determine the value of $\lim_{x \rightarrow 2} f(x)$.

15. Use the Squeeze Theorem to determine the value of $\lim_{x \rightarrow 0} x^4 \sin\left(\frac{\pi}{x}\right)$.

Infinite Limits

For problems 1 – 6 evaluate the indicated limits, if they exist.

1. For $f(x) = \frac{9}{(x-3)^5}$ evaluate,

(a) $\lim_{x \rightarrow 3^-} f(x)$

(b) $\lim_{x \rightarrow 3^+} f(x)$

(c) $\lim_{x \rightarrow 3} f(x)$

2. For $h(t) = \frac{2t}{6+t}$ evaluate,

(a) $\lim_{t \rightarrow -6^-} h(t)$

(b) $\lim_{t \rightarrow -6^+} h(t)$

(c) $\lim_{t \rightarrow -6} h(t)$

3. For $g(z) = \frac{z+3}{(z+1)^2}$ evaluate,

(a) $\lim_{z \rightarrow -1^-} g(z)$

(b) $\lim_{z \rightarrow -1^+} g(z)$

(c) $\lim_{z \rightarrow -1} g(z)$

4. For $g(x) = \frac{x+7}{x^2-4}$ evaluate,

(a) $\lim_{x \rightarrow 2^-} g(x)$

(b) $\lim_{x \rightarrow 2^+} g(x)$

(c) $\lim_{x \rightarrow 2} g(x)$

5. For $h(x) = \ln(-x)$ evaluate,

(a) $\lim_{x \rightarrow 0^-} h(x)$

(b) $\lim_{x \rightarrow 0^+} h(x)$

(c) $\lim_{x \rightarrow 0} h(x)$

6. For $R(y) = \tan(y)$ evaluate,

(a) $\lim_{y \rightarrow \frac{3\pi}{2}^-} R(y)$

(b) $\lim_{y \rightarrow \frac{3\pi}{2}^+} R(y)$

(c) $\lim_{y \rightarrow \frac{3\pi}{2}} R(y)$

For problems 7 & 8 find all the vertical asymptotes of the given function.

7. $f(x) = \frac{7x}{(10-3x)^4}$

8. $g(x) = \frac{-8}{(x+5)(x-9)}$

Limits At Infinity, Part I

1. For $f(x) = 4x^7 - 18x^3 + 9$ evaluate each of the following limits.

(a) $\lim_{x \rightarrow -\infty} f(x)$

(b) $\lim_{x \rightarrow \infty} f(x)$

2. For $h(t) = \sqrt[3]{t} + 12t - 2t^2$ evaluate each of the following limits.

(a) $\lim_{t \rightarrow -\infty} h(t)$

(b) $\lim_{t \rightarrow \infty} h(t)$

For problems 3 – 10 answer each of the following questions.

(a) Evaluate $\lim_{x \rightarrow -\infty} f(x)$.

(b) Evaluate $\lim_{x \rightarrow \infty} f(x)$.

(c) Write down the equation(s) of any horizontal asymptotes for the function.

$$3. f(x) = \frac{8 - 4x^2}{9x^2 + 5x}$$

$$4. f(x) = \frac{3x^7 - 4x^2 + 1}{5 - 10x^2}$$

$$5. f(x) = \frac{20x^4 - 7x^3}{2x + 9x^2 + 5x^4}$$

$$6. f(x) = \frac{x^3 - 2x + 11}{3 - 6x^5}$$

$$7. f(x) = \frac{x^6 - x^4 + x^2 - 1}{7x^6 + 4x^3 + 10}$$

$$8. f(x) = \frac{\sqrt{7 + 9x^2}}{1 - 2x}$$

$$9. f(x) = \frac{x + 8}{\sqrt{2x^2 + 3}}$$

$$10. f(x) = \frac{8 + x - 4x^2}{\sqrt{6 + x^2 + 7x^4}}$$

Limits At Infinity, Part II

For problems 1 – 6 evaluate (a) $\lim_{x \rightarrow -\infty} f(x)$ and (b) $\lim_{x \rightarrow \infty} f(x)$.

$$1. f(x) = e^{8+2x-x^3}$$

$$2. f(x) = e^{\frac{6x^2+x}{5+3x}}$$

$$3. f(x) = 2e^{6x} - e^{-7x} - 10e^{4x}$$

$$4. f(x) = 3e^{-x} - 8e^{-5x} - e^{10x}$$

$$5. f(x) = \frac{e^{-3x} - 2e^{8x}}{9e^{8x} - 7e^{-3x}}$$

$$6. f(x) = \frac{e^{-7x} - 2e^{3x} - e^x}{e^{-x} + 16e^{10x} + 2e^{-4x}}$$

For problems 7 – 12 evaluate the given limit.

$$7. \lim_{t \rightarrow -\infty} \ln(4 - 9t - t^3)$$

$$8. \lim_{z \rightarrow -\infty} \ln\left(\frac{3z^4 - 8}{2 + z^2}\right)$$

$$9. \lim_{x \rightarrow \infty} \ln\left(\frac{11 + 8x}{x^3 + 7x}\right)$$

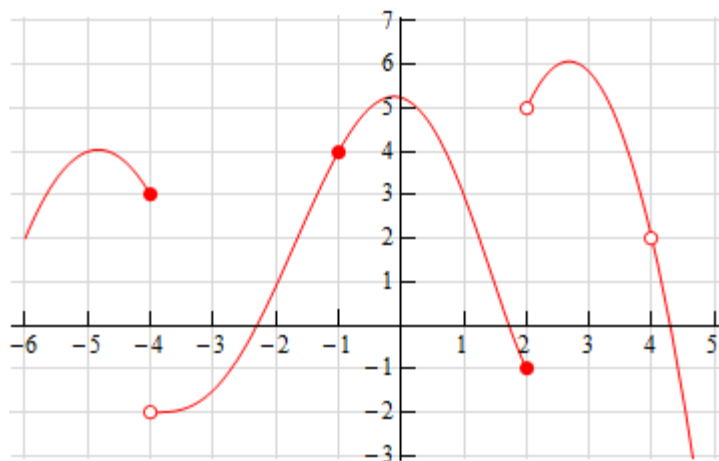
$$10. \lim_{x \rightarrow -\infty} \tan^{-1}(7 - x + 3x^5)$$

$$11. \lim_{t \rightarrow \infty} \tan^{-1}\left(\frac{4 + 7t}{2 - t}\right)$$

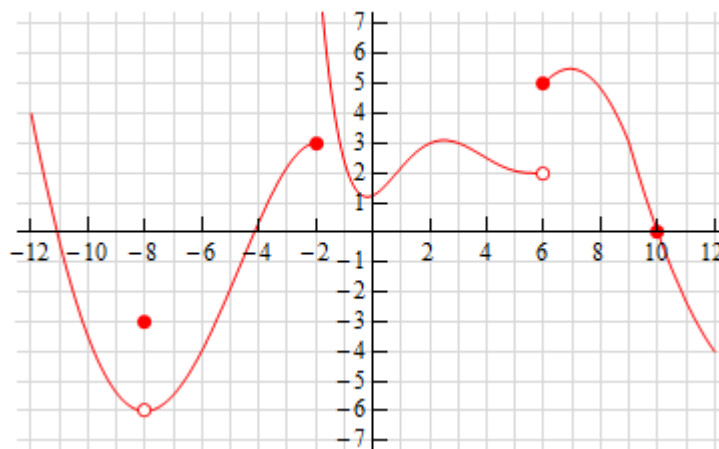
$$12. \lim_{w \rightarrow \infty} \tan^{-1}\left(\frac{3w^2 - 9w^4}{4w - w^3}\right)$$

Continuity

1. The graph of $f(x)$ is given below. Based on this graph determine where the function is discontinuous.



2. The graph of $f(x)$ is given below. Based on this graph determine where the function is discontinuous.



For problems 3 – 7 using only Properties 1 – 9 from the [Limit Properties](#) section, one-sided limit properties (if needed) and the definition of continuity determine if the given function is continuous or discontinuous at the indicated points.

3. $f(x) = \frac{4x+5}{9-3x}$

(a) $x = -1$, (b) $x = 0$, (c) $x = 3$?

4. $g(z) = \frac{6}{z^2 - 3z - 10}$

(a) $z = -2$, (b) $z = 0$, (c) $z = 5$?

5. $g(x) = \begin{cases} 2x & x < 6 \\ x-1 & x \geq 6 \end{cases}$

(a) $x = 4$, (b) $x = 6$?

$$6. h(t) = \begin{cases} t^2 & t < -2 \\ t+6 & t \geq -2 \end{cases}$$

(a) $t = -2$, (b) $t = 10$?

$$7. g(x) = \begin{cases} 1-3x & x < -6 \\ 7 & x = -6 \\ x^3 & -6 < x < 1 \\ 1 & x = 1 \\ 2-x & x > 1 \end{cases}$$

(a) $x = -6$, (b) $x = 1$?

For problems 8 – 12 determine where the given function is discontinuous.

$$8. f(x) = \frac{x^2 - 9}{3x^2 + 2x - 8}$$

$$9. R(t) = \frac{8t}{t^2 - 9t - 1}$$

$$10. h(z) = \frac{1}{2 - 4\cos(3z)}$$

$$11. y(x) = \frac{x}{7 - e^{2x+3}}$$

$$12. g(x) = \tan(2x)$$

For problems 13 – 15 use the Intermediate Value Theorem to show that the given equation has at least one solution in the indicated interval. Note that you are NOT asked to find the solution only show that at least one must exist in the indicated interval.

$$13. 25 - 8x^2 - x^3 = 0 \text{ on } [-2, 4]$$

$$14. w^2 - 4\ln(5w + 2) = 0 \text{ on } [0, 4]$$

$$15. 4t + 10e^t - e^{2t} = 0 \text{ on } [1, 3]$$

The Definition of the Limit

Use the definition of the limit to prove the following limits.

1. $\lim_{x \rightarrow 3} x = 3$

2. $\lim_{x \rightarrow -1} (x + 7) = 6$

3. $\lim_{x \rightarrow 2} x^2 = 4$

4. $\lim_{x \rightarrow -3} (x^2 + 4x + 1) = -2$

5. $\lim_{x \rightarrow 1} \frac{1}{(x-1)^2} = \infty$

6. $\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$

7. $\lim_{x \rightarrow \infty} \frac{1}{x^2} = 0$

Derivatives***Introduction***

Here are a set of practice problems for the Derivatives chapter of my Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a number of places on the site.

10. If you'd like a pdf document containing the solutions go to the note page for the section you'd like solutions for and select the download solutions link from there. Or,

11. Go to the download page for the site <http://tutorial.math.lamar.edu/download.aspx> and select the section you'd like solutions for and a link will be provided there.
12. If you'd like to view the solutions on the web or solutions to an individual problem you can go to the problem set web page, select the problem you want the solution for. At this point I do not provide pdf versions of individual solutions, but for a particular problem you can select "Printable View" from the "Solution Pane Options" to get a printable version.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have practice problems written for them.

[The Definition of the Derivative](#)

[Interpretation of the Derivative](#)

[Differentiation Formulas](#)

[Product and Quotient Rule](#)

[Derivatives of Trig Functions](#)

[Derivatives of Exponential and Logarithm Functions](#)

[Derivatives of Inverse Trig Functions](#)

[Derivatives of Hyperbolic Functions](#)

[Chain Rule](#)

[Implicit Differentiation](#)

[Related Rates](#)

[Higher Order Derivatives](#)

[Logarithmic Differentiation](#)

The Definition of the Derivative

Use the definition of the derivative to find the derivative of the following functions.

1. $f(x) = 6$

2. $V(t) = 3 - 14t$

3. $g(x) = x^2$

4. $Q(t) = 10 + 5t - t^2$

5. $W(z) = 4z^2 - 9z$

6. $f(x) = 2x^3 - 1$

7. $g(x) = x^3 - 2x^2 + x - 1$

8. $R(z) = \frac{5}{z}$

9. $V(t) = \frac{t+1}{t+4}$

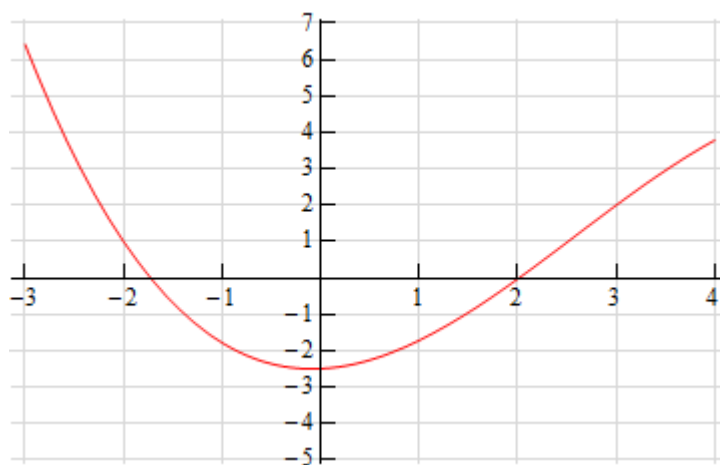
10. $Z(t) = \sqrt{3t-4}$

11. $f(x) = \sqrt{1-9x}$

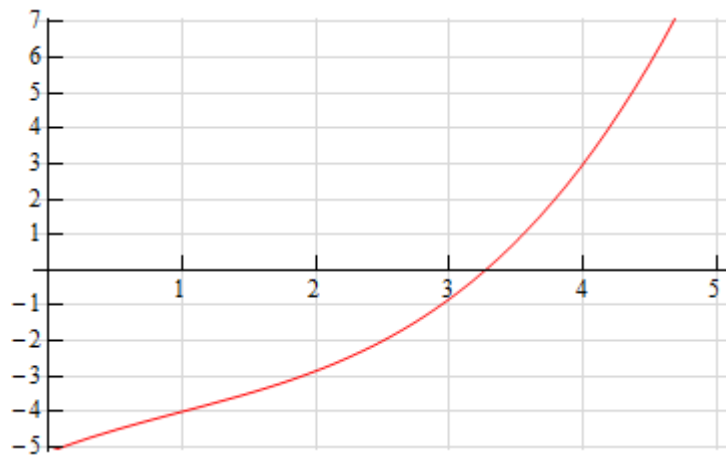
Interpretations of the Derivative

For problems 1 and 2 use the graph of the function, $f(x)$, estimate the value of $f'(a)$ for the given values of a .

1. (a)
- $a = -2$
- (b)
- $a = 3$



2. (a)
- $a = 1$
- (b)
- $a = 4$



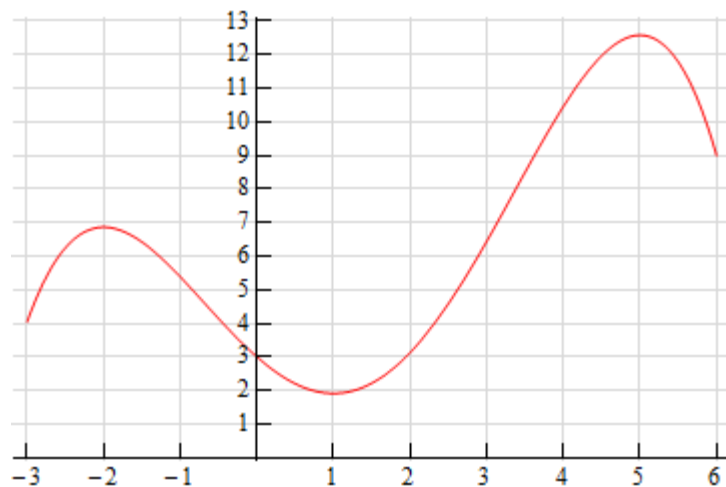
For problems 3 and 4 sketch the graph of a function that satisfies the given conditions.

3. $f(1) = 3$, $f'(1) = 1$, $f(4) = 5$, $f'(4) = -2$

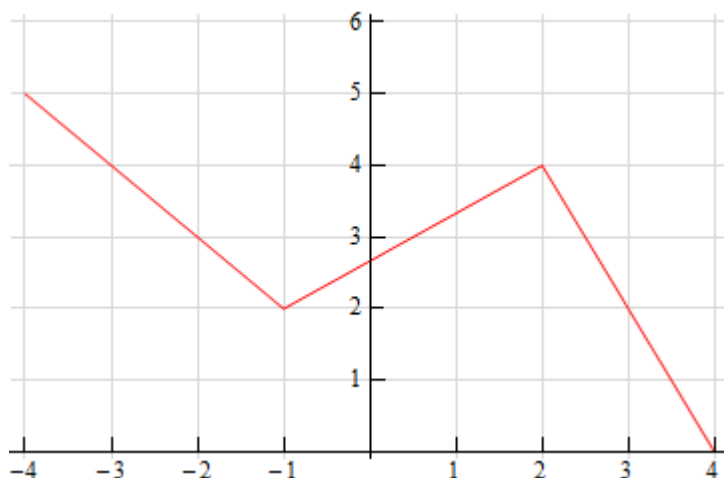
4. $f(-3) = 5$, $f'(-3) = -2$, $f(1) = 2$, $f'(1) = 0$, $f(4) = -2$, $f'(4) = -3$

For problems 5 and 6 the graph of a function, $f(x)$, is given. Use this to sketch the graph of the derivative, $f'(x)$.

5.



6.



7. Answer the following questions about the function $W(z) = 4z^2 - 9z$.
- (a) Is the function increasing or decreasing at $z = -1$?
 - (b) Is the function increasing or decreasing at $z = 2$?
 - (c) Does the function ever stop changing? If yes, at what value(s) of z does the function stop changing?
8. What is the equation of the tangent line to $f(x) = 3 - 14x$ at $x = 8$.
9. The position of an object at any time t is given by $s(t) = \frac{t+1}{t+4}$.
- (a) Determine the velocity of the object at any time t .
 - (b) Does the object ever stop moving? If yes, at what time(s) does the object stop moving?
10. What is the equation of the tangent line to $f(x) = \frac{5}{x}$ at $x = \frac{1}{2}$?
11. Determine where, if anywhere, the function $g(x) = x^3 - 2x^2 + x - 1$ stops changing.
12. Determine if the function $Z(t) = \sqrt{3t - 4}$ is increasing or decreasing at the given points.
- (a) $t = 5$
 - (b) $t = 10$
 - (c) $t = 300$
13. Suppose that the volume of water in a tank for $0 \leq t \leq 6$ is given by $Q(t) = 10 + 5t - t^2$.
- (a) Is the volume of water increasing or decreasing at $t = 0$?

- (b) Is the volume of water increasing or decreasing at $t = 6$?
- (c) Does the volume of water ever stop changing? If yes, at what times(s) does the volume stop changing?

Differentiation Formulas

For problems 1 – 12 find the derivative of the given function.

1. $f(x) = 6x^3 - 9x + 4$

2. $y = 2t^4 - 10t^2 + 13t$

3. $g(z) = 4z^7 - 3z^{-7} + 9z$

4. $h(y) = y^{-4} - 9y^{-3} + 8y^{-2} + 12$

5. $y = \sqrt{x} + 8\sqrt[3]{x} - 2\sqrt[4]{x}$

6. $f(x) = 10\sqrt[5]{x^3} - \sqrt{x^7} + 6\sqrt[3]{x^8} - 3$

7. $f(t) = \frac{4}{t} - \frac{1}{6t^3} + \frac{8}{t^5}$

8. $R(z) = \frac{6}{\sqrt{z^3}} + \frac{1}{8z^4} - \frac{1}{3z^{10}}$

9. $z = x(3x^2 - 9)$

10. $g(y) = (y - 4)(2y + y^2)$

11. $h(x) = \frac{4x^3 - 7x + 8}{x}$

12. $f(y) = \frac{y^5 - 5y^3 + 2y}{y^3}$

13. Determine where, if anywhere, the function $f(x) = x^3 + 9x^2 - 48x + 2$ is not changing.
14. Determine where, if anywhere, the function $y = 2z^4 - z^3 - 3z^2$ is not changing.
15. Find the tangent line to $g(x) = \frac{16}{x} - 4\sqrt{x}$ at $x = 4$.
16. Find the tangent line to $f(x) = 7x^4 + 8x^{-6} + 2x$ at $x = -1$.
17. The position of an object at any time t is given by $s(t) = 3t^4 - 40t^3 + 126t^2 - 9$.
- (a) Determine the velocity of the object at any time t .
 - (b) Does the object ever stop changing?
 - (c) When is the object moving to the right and when is the object moving to the left?
18. Determine where the function $h(z) = 6 + 40z^3 - 5z^4 - 4z^5$ is increasing and decreasing.
19. Determine where the function $R(x) = (x+1)(x-2)^2$ is increasing and decreasing.
20. Determine where, if anywhere, the tangent line to $f(x) = x^3 - 5x^2 + x$ is parallel to the line $y = 4x + 23$.

Product and Quotient Rule

For problems 1 – 6 use the Product Rule or the Quotient Rule to find the derivative of the given function.

1. $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$
2. $y = (1 + \sqrt{x^3})(x^{-3} - 2\sqrt[3]{x})$
3. $h(z) = (1 + 2z + 3z^2)(5z + 8z^2 - z^3)$
4. $g(x) = \frac{6x^2}{2-x}$

5. $R(w) = \frac{3w + w^4}{2w^2 + 1}$

6. $f(x) = \frac{\sqrt{x} + 2x}{7x - 4x^2}$

7. If $f(2) = -8$, $f'(2) = 3$, $g(2) = 17$ and $g'(2) = -4$ determine the value of $(fg)'(2)$.

8. If $f(x) = x^3 g(x)$, $g(-7) = 2$, $g'(-7) = -9$ determine the value of $f'(-7)$.

9. Find the equation of the tangent line to $f(x) = (1 + 12\sqrt{x})(4 - x^2)$ at $x = 9$.

10. Determine where $f(x) = \frac{x - x^2}{1 + 8x^2}$ is increasing and decreasing.

11. Determine where $V(t) = (4 - t^2)(1 + 5t^2)$ is increasing and decreasing.

Derivatives of Trig Functions

For problems 1 – 3 evaluate the given limit.

1. $\lim_{z \rightarrow 0} \frac{\sin(10z)}{z}$

2. $\lim_{\alpha \rightarrow 0} \frac{\sin(12\alpha)}{\sin(5\alpha)}$

3. $\lim_{x \rightarrow 0} \frac{\cos(4x) - 1}{x}$

For problems 4 – 10 differentiate the given function.

4. $f(x) = 2\cos(x) - 6\sec(x) + 3$

5. $g(z) = 10\tan(z) - 2\cot(z)$

6. $f(w) = \tan(w) \sec(w)$

7. $h(t) = t^3 - t^2 \sin(t)$

8. $y = 6 + 4\sqrt{x} \csc(x)$

9. $R(t) = \frac{1}{2 \sin(t) - 4 \cos(t)}$

10. $Z(v) = \frac{v + \tan(v)}{1 + \csc(v)}$

11. Find the tangent line to $f(x) = \tan(x) + 9 \cos(x)$ at $x = \pi$.

12. The position of an object is given by $s(t) = 2 + 7 \cos(t)$ determine all the points where the object is not moving.

13. Where in the range $[-2, 7]$ is the function $f(x) = 4 \cos(x) - x$ is increasing and decreasing.

Derivatives of Exponential and Logarithm Functions

For problems 1 – 6 differentiate the given function.

1. $f(x) = 2e^x - 8^x$

2. $g(t) = 4 \log_3(t) - \ln(t)$

3. $R(w) = 3^w \log(w)$

4. $y = z^5 - e^z \ln(z)$

5. $h(y) = \frac{y}{1 - e^y}$

6. $f(t) = \frac{1+5t}{\ln(t)}$

7. Find the tangent line to $f(x) = 7^x + 4e^x$ at $x = 0$.

8. Find the tangent line to $f(x) = \ln(x) \log_2(x)$ at $x = 2$.

9. Determine if $V(t) = \frac{t}{e^t}$ is increasing or decreasing at the following points.

(a) $t = -4$

(b) $t = 0$

(c) $t = 10$

10. Determine if $G(z) = (z-6)\ln(z)$ is increasing or decreasing at the following points.

(a) $z = 1$

(b) $z = 5$

(c) $z = 20$

Derivatives of Inverse Trig Functions

For each of the following problems differentiate the given function.

1. $T(z) = 2\cos(z) + 6\cos^{-1}(z)$

2. $g(t) = \csc^{-1}(t) - 4\cot^{-1}(t)$

3. $y = 5x^6 - \sec^{-1}(x)$

4. $f(w) = \sin(w) + w^2 \tan^{-1}(w)$

5. $h(x) = \frac{\sin^{-1}(x)}{1+x}$

Derivatives of Hyperbolic Functions

For each of the following problems differentiate the given function.

1. $f(x) = \sinh(x) + 2\cosh(x) - \operatorname{sech}(x)$

$$2. R(t) = \tan(t) + t^2 \operatorname{csch}(t)$$

$$3. g(z) = \frac{z+1}{\tanh(z)}$$

Chain Rule

For problems 1 – 26 differentiate the given function.

$$1. f(x) = (6x^2 + 7x)^4$$

$$2. g(t) = (4t^2 - 3t + 2)^{-2}$$

$$3. y = \sqrt[3]{1-8z}$$

$$4. R(w) = \csc(7w)$$

$$5. G(x) = 2 \sin(3x + \tan(x))$$

$$6. h(u) = \tan(4 + 10u)$$

$$7. f(t) = 5 + e^{4t+t^7}$$

$$8. g(x) = e^{1-\cos(x)}$$

$$9. H(z) = 2^{1-6z}$$

$$10. u(t) = \tan^{-1}(3t-1)$$

$$11. F(y) = \ln(1-5y^2+y^3)$$

$$12. V(x) = \ln(\sin(x) - \cot(x))$$

$$13. h(z) = \sin(z^6) + \sin^6(z)$$

14. $S(w) = \sqrt{7w} + e^{-w}$

15. $g(z) = 3z^7 - \sin(z^2 + 6)$

16. $f(x) = \ln(\sin(x)) - (x^4 - 3x)^{10}$

17. $h(t) = t^6 \sqrt{5t^2 - t}$

18. $q(t) = t^2 \ln(t^5)$

19. $g(w) = \cos(3w) \sec(1 - w)$

20. $y = \frac{\sin(3t)}{1 + t^2}$

21. $K(x) = \frac{1 + e^{-2x}}{x + \tan(12x)}$

22. $f(x) = \cos(x^2 e^x)$

23. $z = \sqrt{5x + \tan(4x)}$

24. $f(t) = (e^{-6t} + \sin(2 - t))^3$

25. $g(x) = (\ln(x^2 + 1) - \tan^{-1}(6x))^{10}$

26. $h(z) = \tan^4(z^2 + 1)$

27. $f(x) = (\sqrt[3]{12x} + \sin^2(3x))^{-1}$

28. Find the tangent line to $f(x) = 4\sqrt{2x} - 6e^{2-x}$ at $x = 2$.

29. Determine where $V(z) = z^4(2z - 8)^3$ is increasing and decreasing.

30. The position of an object is given by $s(t) = \sin(3t) - 2t + 4$. Determine where in the interval $[0, 3]$ the object is moving to the right and moving to the left.

31. Determine where $A(t) = t^2 e^{5-t}$ is increasing and decreasing.

32. Determine where in the interval $[-1, 20]$ the function $f(x) = \ln(x^4 + 20x^3 + 100)$ is increasing and decreasing.

Implicit Differentiation

For problems 1 – 3 do each of the following.

- (a) Find y' by solving the equation for y and differentiating directly.
- (b) Find y' by implicit differentiation.
- (c) Check that the derivatives in (a) and (b) are the same.

1. $\frac{x}{y^3} = 1$

2. $x^2 + y^3 = 4$

3. $x^2 + y^2 = 2$

For problems 4 – 9 find y' by implicit differentiation.

4. $2y^3 + 4x^2 - y = x^6$

5. $7y^2 + \sin(3x) = 12 - y^4$

6. $e^x - \sin(y) = x$

7. $4x^2 y^7 - 2x = x^5 + 4y^3$

8. $\cos(x^2 + 2y) + x e^{y^2} = 1$

9. $\tan(x^2 y^4) = 3x + y^2$

For problems 10 & 11 find the equation of the tangent line at the given point.

10. $x^4 + y^2 = 3$ at $(1, -\sqrt{2})$.

11. $y^2 e^{2x} = 3y + x^2$ at $(0, 3)$.

For problems 12 & 13 assume that $x = x(t)$, $y = y(t)$ and $z = z(t)$ and differentiate the given equation with respect to t .

12. $x^2 - y^3 + z^4 = 1$

13. $x^2 \cos(y) = \sin(y^3 + 4z)$

Related Rates

1. In the following assume that x and y are both functions of t . Given $x = -2$, $y = 1$ and $x' = -4$ determine y' for the following equation.

$$6y^2 + x^2 = 2 - x^3 e^{4-4y}$$

2. In the following assume that x , y and z are all functions of t . Given $x = 4$, $y = -2$, $z = 1$, $x' = 9$ and $y' = -3$ determine z' for the following equation.

$$x(1 - y) + 5z^3 = y^2 z^2 + x^2 - 3$$

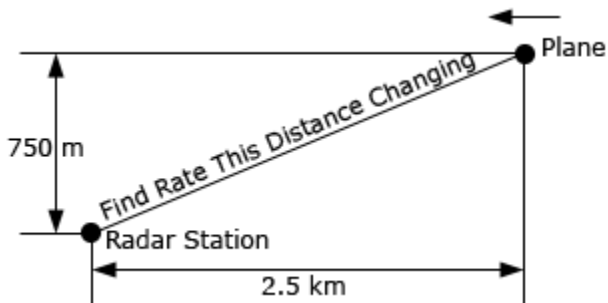
3. For a certain rectangle the length of one side is always three times the length of the other side.

- (a) If the shorter side is decreasing at a rate of 2 inches/minute at what rate is the longer side decreasing?
- (b) At what rate is the enclosed area decreasing when the shorter side is 6 inches long and is decreasing at a rate of 2 inches/minute?

4. A thin sheet of ice is in the form of a circle. If the ice is melting in such a way that the area of the sheet is decreasing at a rate of $0.5 \text{ m}^2/\text{sec}$ at what rate is the radius decreasing when the area of the sheet is 12 m^2 ?

5. A person is standing 350 feet away from a model rocket that is fired straight up into the air at a rate of 15 ft/sec. At what rate is the distance between the person and the rocket increasing (a) 20 seconds after liftoff? (b) 1 minute after liftoff?

6. A plane is 750 meters in the air flying parallel to the ground at a speed of 100 m/s and is initially 2.5 kilometers away from a radar station. At what rate is the distance between the plane and the radar station changing (a) initially and (b) 30 seconds after it passes over the radar station? See the (probably bad) sketch below to help visualize the problem.



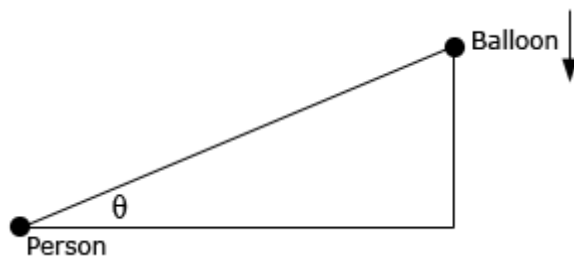
7. Two people are at an elevator. At the same time one person starts to walk away from the elevator at a rate of 2 ft/sec and the other person starts going up in the elevator at a rate of 7 ft/sec. What rate is the distance between the two people changing 15 seconds later?

8. Two people on bikes are at the same place. One of the bikers starts riding directly north at a rate of 8 m/sec. Five seconds after the first biker started riding north the second starts to ride directly east at a rate of 5 m/sec. At what rate is the distance between the two riders increasing 20 seconds after the second person started riding?

9. A light is mounted on a wall 5 meters above the ground. A 2 meter tall person is initially 10 meters from the wall and is moving towards the wall at a rate of 0.5 m/sec. After 4 seconds of moving is the tip of the shadow moving (a) towards or away from the person and (b) towards or away from the wall?

10. A tank of water in the shape of a cone is being filled with water at a rate of $12 \text{ m}^3/\text{sec}$. The base radius of the tank is 26 meters and the height of the tank is 8 meters. At what rate is the depth of the water in the tank changing with the radius of the top of the water is 10 meters?

11. The angle of elevation is the angle formed by a horizontal line and a line joining the observer's eye to an object above the horizontal line. A person is 500 feet away from the launch point of a hot air balloon. The hot air balloon is starting to come back down at a rate of 15 ft/sec. At what rate is the angle of elevation, θ , changing when the hot air balloon is 200 feet above the ground. See the (probably bad) sketch below to help visualize the angle of elevation if you are having trouble seeing it.



Higher Order Derivatives

For problems 1 – 5 determine the fourth derivative of the given function.

1. $h(t) = 3t^7 - 6t^4 + 8t^3 - 12t + 18$

2. $V(x) = x^3 - x^2 + x - 1$

3. $f(x) = 4\sqrt[5]{x^3} - \frac{1}{8x^2} - \sqrt{x}$

4. $f(w) = 7\sin\left(\frac{w}{3}\right) + \cos(1 - 2w)$

5. $y = e^{-5z} + 8\ln(2z^4)$

For problems 6 – 9 determine the second derivative of the given function.

6. $g(x) = \sin(2x^3 - 9x)$

7. $z = \ln(7 - x^3)$

8. $Q(v) = \frac{2}{(6 + 2v - v^2)^4}$

9. $H(t) = \cos^2(7t)$

For problems 10 & 11 determine the second derivative of the given function.

10. $2x^3 + y^2 = 1 - 4y$

11. $6y - xy^2 = 1$

Logarithmic Differentiation

For problems 1 – 3 use logarithmic differentiation to find the first derivative of the given function.

1. $f(x) = (5 - 3x^2)^7 \sqrt{6x^2 + 8x - 12}$

2. $y = \frac{\sin(3z + z^2)}{(6 - z^4)^3}$

3. $h(t) = \frac{\sqrt{5t+8} \sqrt[3]{1-9\cos(4t)}}{\sqrt[4]{t^2+10t}}$

For problems 4 & 5 find the first derivative of the given function.

4. $g(w) = (3w - 7)^{4w}$

5. $f(x) = (2x - e^{8x})^{\sin(2x)}$

Applications of Derivatives

Introduction

Here are a set of practice problems for the Applications of Derivatives chapter of my Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a number of places on the site.

13. If you'd like a pdf document containing the solutions go to the note page for the section you'd like solutions for and select the download solutions link from there. Or,

14. Go to the download page for the site <http://tutorial.math.lamar.edu/download.aspx> and select the section you'd like solutions for and a link will be provided there.
15. If you'd like to view the solutions on the web or solutions to an individual problem you can go to the problem set web page, select the problem you want the solution for. At this point I do not provide pdf versions of individual solutions, but for a particular problem you can select "Printable View" from the "Solution Pane Options" to get a printable version.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have practice problems written for them.

[Rates of Change](#)

[Critical Points](#)

[Minimum and Maximum Values](#)

[Finding Absolute Extrema](#)

[The Shape of a Graph, Part I](#)

[The Shape of a Graph, Part II](#)

[The Mean Value Theorem](#)

[Optimization Problems](#)

[More Optimization Problems](#)

[L'Hospital's Rule and Indeterminate Forms](#)

[Linear Approximations](#)

[Differentials](#)

[Newton's Method](#)

[Business Applications](#)

Rates of Change

As noted in the text for this section the purpose of this section is only to remind you of certain types of applications that were discussed in the previous chapter. As such there aren't any problems written for this section. Instead here is a list of links (note that these will only be active links in the web version and not the pdf version) to problems from the relevant sections from the previous chapter.

Each of the following sections has a selection of increasing/decreasing problems towards the bottom of the problem set.

[Differentiation Formulas](#)

[Product & Quotient Rules](#)

[Derivatives of Trig Functions](#)

Derivatives of Exponential and Logarithm Functions
Chain Rule

Related Rates problems are in the **Related Rates** section.

Critical Points

Determine the critical points of each of the following functions.

1. $f(x) = 8x^3 + 81x^2 - 42x - 8$

2. $R(t) = 1 + 80t^3 + 5t^4 - 2t^5$

3. $g(w) = 2w^3 - 7w^2 - 3w - 2$

4. $g(x) = x^6 - 2x^5 + 8x^4$

5. $h(z) = 4z^3 - 3z^2 + 9z + 12$

6. $Q(x) = (2 - 8x)^4 (x^2 - 9)^3$

7. $f(z) = \frac{z + 4}{2z^2 + z + 8}$

8. $R(x) = \frac{1 - x}{x^2 + 2x - 15}$

9. $r(y) = \sqrt[5]{y^2 - 6y}$

10. $h(t) = 15 - (3 - t) \left[t^2 - 8t + 7 \right]^{\frac{1}{3}}$

11. $s(z) = 4 \cos(z) - z$

12. $f(y) = \sin\left(\frac{y}{3}\right) + \frac{2y}{9}$

13. $V(t) = \sin^2(3t) + 1$

14. $f(x) = 5xe^{9-2x}$

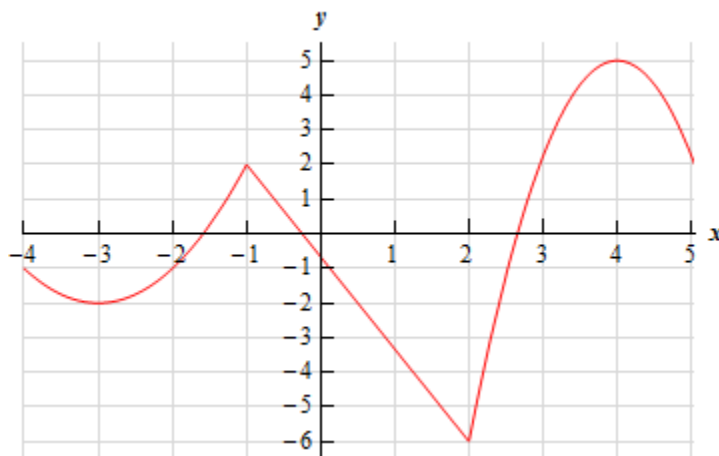
15. $g(w) = e^{w^3 - 2w^2 - 7w}$

16. $R(x) = \ln(x^2 + 4x + 14)$

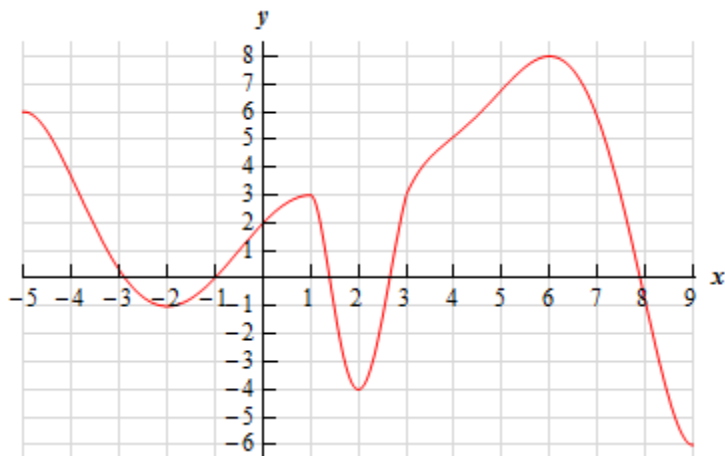
17. $A(t) = 3t - 7\ln(8t + 2)$

Minimum and Maximum Values

1. Below is the graph of some function, $f(x)$. Identify all of the relative extrema and absolute extrema of the function.



2. Below is the graph of some function, $f(x)$. Identify all of the relative extrema and absolute extrema of the function.



3. Sketch the graph of $g(x) = x^2 - 4x$ and identify all the relative extrema and absolute extrema of the function on each of the following intervals.

- (a) $(-\infty, \infty)$
- (b) $[-1, 4]$
- (c) $[1, 3]$
- (d) $[3, 5]$
- (e) $(-1, 5]$

4. Sketch the graph of $h(x) = -(x+4)^3$ and identify all the relative extrema and absolute extrema of the function on each of the following intervals.

- (a) $(-\infty, \infty)$
- (b) $[-5.5, -2]$
- (c) $[-4, -3)$
- (d) $[-4, -3]$

5. Sketch the graph of some function on the interval $[1, 6]$ that has an absolute maximum at $x = 6$ and an absolute minimum at $x = 3$.

6. Sketch the graph of some function on the interval $[-4, 3]$ that has an absolute maximum at $x = -3$ and an absolute minimum at $x = 2$.

7. Sketch the graph of some function that meets the following conditions :

- (a) The function is continuous.
- (b) Has two relative minimums.

- (b) One of relative minimums is also an absolute minimum and the other relative minimum is not an absolute minimum.
- (c) Has one relative maximum.
- (d) Has no absolute maximum.

Finding Absolute Extrema

For each of the following problems determine the absolute extrema of the given function on the specified interval.

1. $f(x) = 8x^3 + 81x^2 - 42x - 8$ on $[-8, 2]$

2. $f(x) = 8x^3 + 81x^2 - 42x - 8$ on $[-4, 2]$

3. $R(t) = 1 + 80t^3 + 5t^4 - 2t^5$ on $[-4.5, 4]$

4. $R(t) = 1 + 80t^3 + 5t^4 - 2t^5$ on $[0, 7]$

5. $h(z) = 4z^3 - 3z^2 + 9z + 12$ on $[-2, 1]$

6. $g(x) = 3x^4 - 26x^3 + 60x^2 - 11$ on $[1, 5]$

7. $Q(x) = (2 - 8x)^4 (x^2 - 9)^3$ on $[-3, 3]$

8. $h(w) = 2w^3 (w + 2)^5$ on $[-\frac{5}{2}, \frac{1}{2}]$

9. $f(z) = \frac{z + 4}{2z^2 + z + 8}$ on $[-10, 0]$

10. $A(t) = t^2 (10 - t)^{\frac{2}{3}}$ on $[2, 10.5]$

11. $f(y) = \sin\left(\frac{y}{3}\right) + \frac{2y}{9}$ on $[-10, 15]$

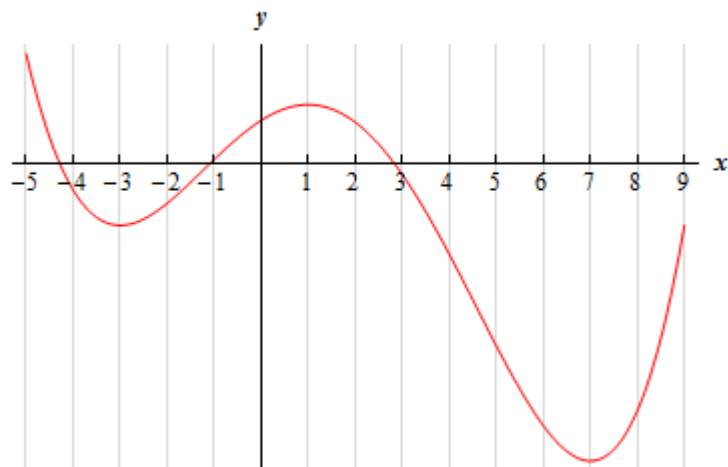
12. $g(w) = e^{w^3 - 2w^2 - 7w}$ on $[-\frac{1}{2}, \frac{5}{2}]$

13. $R(x) = \ln(x^2 + 4x + 14)$ on $[-4, 2]$

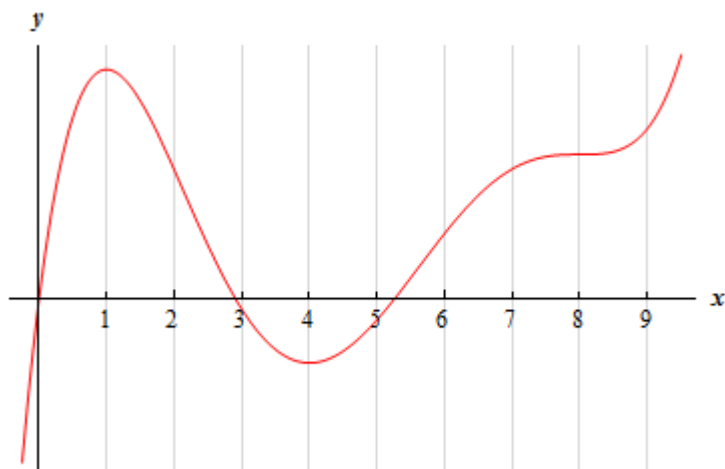
The Shape of a Graph, Part I

For problems 1 & 2 the graph of a function is given. Determine the open intervals on which the function increases and decreases.

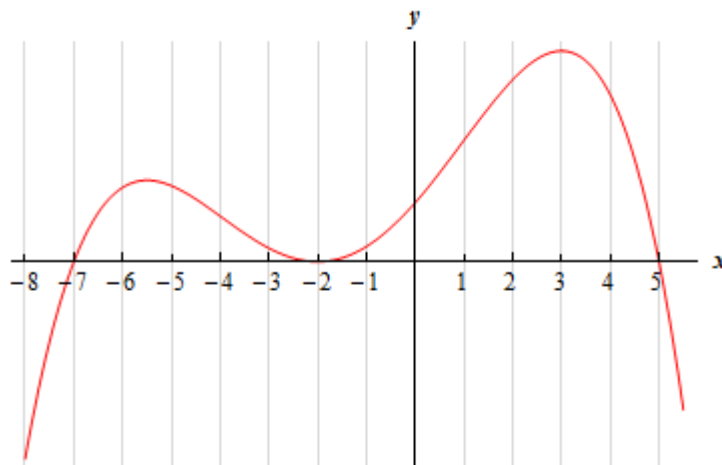
1.



2.



3. Below is the graph of the **derivative** of a function. From this graph determine the open intervals in which the **function** increases and decreases.



4. This problem is about some function. All we know about the function is that it exists everywhere and we also know the information given below about the derivative of the function. Answer each of the following questions about this function.

- (a) Identify the critical points of the function.
- (b) Determine the open intervals on which the function increases and decreases.
- (c) Classify the critical points as relative maximums, relative minimums or neither.

$$f'(-5) = 0 \quad f'(-2) = 0 \quad f'(4) = 0 \quad f'(8) = 0$$

$$f'(x) < 0 \quad \text{on} \quad (-5, -2), (-2, 4), (8, \infty) \quad f'(x) > 0 \quad \text{on} \quad (-\infty, -5), (4, 8)$$

For problems 5 – 12 answer each of the following.

- (a) Identify the critical points of the function.
- (b) Determine the open intervals on which the function increases and decreases.
- (c) Classify the critical points as relative maximums, relative minimums or neither.

5. $f(x) = 2x^3 - 9x^2 - 60x$

6. $h(t) = 50 + 40t^3 - 5t^4 - 4t^5$

7. $y = 2x^3 - 10x^2 + 12x - 12$

8. $p(x) = \cos(3x) + 2x$ on $[-\frac{3}{2}, 2]$

9. $R(z) = 2 - 5z - 14\sin(\frac{z}{2})$ on $[-10, 7]$

10. $h(t) = t^2 \sqrt[3]{t-7}$

11. $f(w) = we^{2 - \frac{1}{2}w^2}$

12. $g(x) = x - 2\ln(1 + x^2)$

13. For some function, $f(x)$, it is known that there is a relative maximum at $x = 4$. Answer each of the following questions about this function.

(a) What is the simplest form for the derivative of this function? Note : There really are many possible forms of the derivative so to make the rest of this problem as simple as possible you will want to use the simplest form of the derivative that you can come up with.

(b) Using your answer from (a) determine the most general form of the function.

(c) Given that $f(4) = 1$ find a function that will have a relative maximum at $x = 4$. Note :

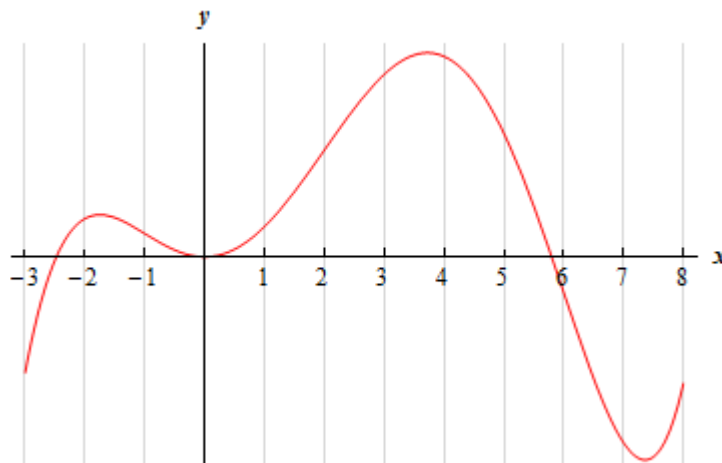
You should be able to use your answer from (b) to determine an answer to this part.

14. Given that $f(x)$ and $g(x)$ are increasing functions. If we define $h(x) = f(x) + g(x)$ show that $h(x)$ is an increasing function.

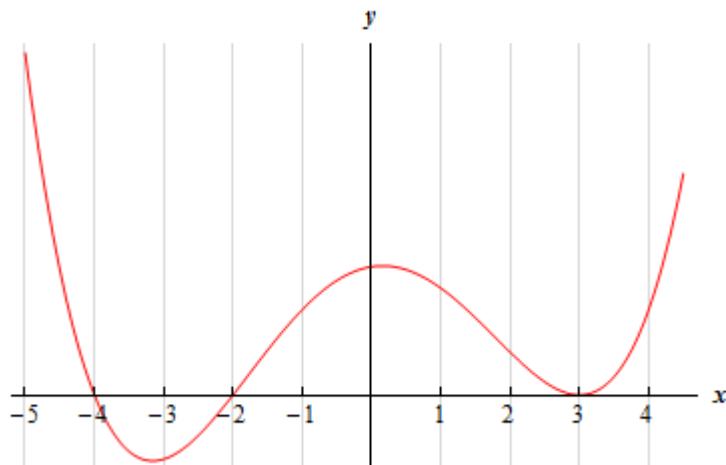
15. Given that $f(x)$ is an increasing function and define $h(x) = [f(x)]^2$. Will $h(x)$ be an increasing function? If yes, prove that $h(x)$ is an increasing function. If not, can you determine any other conditions needed on the function $f(x)$ that will guarantee that $h(x)$ will also increase?

The Shape of a Graph, Part II

1. The graph of a function is given below. Determine the open intervals on which the function is concave up and concave down.



2. Below is the graph the **2nd derivative** of a function. From this graph determine the open intervals in which the **function** is concave up and concave down.



For problems 3 – 8 answer each of the following.

- (a) Determine a list of possible inflection points for the function.
- (b) Determine the open intervals on which the function is concave up and concave down.
- (c) Determine the inflection points of the function.

3. $f(x) = 12 + 6x^2 - x^3$

4. $g(z) = z^4 - 12z^3 + 84z + 4$

5. $h(t) = t^4 + 12t^3 + 6t^2 - 36t + 2$

6. $h(w) = 8 - 5w + 2w^2 - \cos(3w)$ on $[-1, 2]$

7. $R(z) = z(z + 4)^{\frac{2}{3}}$

8. $h(x) = e^{4-x^2}$

For problems 9 – 14 answer each of the following.

- (a) Identify the critical points of the function.
- (b) Determine the open intervals on which the function increases and decreases.
- (c) Classify the critical points as relative maximums, relative minimums or neither.
- (d) Determine the open intervals on which the function is concave up and concave down.

(e) Determine the inflection points of the function.

(f) Use the information from steps (a) – (e) to sketch the graph of the function.

9. $g(t) = t^5 - 5t^4 + 8$

10. $f(x) = 5 - 8x^3 - x^4$

11. $h(z) = z^4 - 2z^3 - 12z^2$

12. $Q(t) = 3t - 8\sin\left(\frac{t}{2}\right)$ on $[-7, 4]$

13. $f(x) = x^{\frac{4}{3}}(x - 2)$

14. $P(w) = we^{4w}$ on $\left[-2, \frac{1}{4}\right]$

15. Determine the minimum degree of a polynomial that has exactly one inflection point.

16. Suppose that we know that $f(x)$ is a polynomial with critical points $x = -1$, $x = 2$ and $x = 6$. If we also know that the 2nd derivative is $f''(x) = -3x^2 + 14x - 4$. If possible, classify each of the critical points as relative minimums, relative maximums. If it is not possible to classify the critical points clearly explain why they cannot be classified.

The Mean Value Theorem

For problems 1 & 2 determine all the number(s) c which satisfy the conclusion of Rolle's Theorem for the given function and interval.

1. $f(x) = x^2 - 2x - 8$ on $[-1, 3]$

2. $g(t) = 2t - t^2 - t^3$ on $[-2, 1]$

For problems 3 & 4 determine all the number(s) c which satisfy the conclusion of the Mean Value Theorem for the given function and interval.

3. $h(z) = 4z^3 - 8z^2 + 7z - 2$ on $[2, 5]$

4. $A(t) = 8t + e^{-3t}$ on $[-2, 3]$
5. Suppose we know that $f(x)$ is continuous and differentiable on the interval $[-7, 0]$, that $f(-7) = -3$ and that $f'(x) \leq 2$. What is the largest possible value for $f(0)$?
6. Show that $f(x) = x^3 - 7x^2 + 25x + 8$ has exactly one real root.

Optimization

1. Find two positive numbers whose sum is 300 and whose product is a maximum.
2. Find two positive numbers whose product is 750 and for which the sum of one and 10 times the other is a minimum.
3. Let x and y be two positive numbers such that $x + 2y = 50$ and $(x+1)(y+2)$ is a maximum.
4. We are going to fence in a rectangular field. If we look at the field from above the cost of the vertical sides are \$10/ft, the cost of the bottom is \$2/ft and the cost of the top is \$7/ft. If we have \$700 determine the dimensions of the field that will maximize the enclosed area.
5. We have 45 m² of material to build a box with a square base and no top. Determine the dimensions of the box that will maximize the enclosed volume.
6. We want to build a box whose base length is 6 times the base width and the box will enclose 20 in³. The cost of the material of the sides is \$3/in² and the cost of the top and bottom is \$15/in². Determine the dimensions of the box that will minimize the cost.
7. We want to construct a cylindrical can with a bottom but no top that will have a volume of 30 cm³. Determine the dimensions of the can that will minimize the amount of material needed to construct the can.
8. We have a piece of cardboard that is 50 cm by 20 cm and we are going to cut out the corners and fold up the sides to form a box. Determine the height of the box that will give a maximum volume.

More Optimization Problems

1. We want to construct a window whose middle is a rectangle and the top and bottom of the window are semi-circles. If we have 50 meters of framing material what are the dimensions of the window that will let in the most light?
2. Determine the area of the largest rectangle that can be inscribed in a circle of radius 1.
3. Find the point(s) on $x = 3 - 2y^2$ that are closest to $(-4, 0)$.
4. An 80 cm piece of wire is cut into two pieces. One piece is bent into an equilateral triangle and the other will be bent into a rectangle with one side 4 times the length of the other side. Determine where, if anywhere, the wire should be cut to maximize the area enclosed by the two figures.
5. A line through the point $(2, 5)$ forms a right triangle with the x -axis and y -axis in the 1st quadrant. Determine the equation of the line that will minimize the area of this triangle.
6. A piece of pipe is being carried down a hallway that is 18 feet wide. At the end of the hallway there is a right-angled turn and the hallway narrows down to 12 feet wide. What is the longest pipe (always keeping it horizontal) that can be carried around the turn in the hallway?
7. Two 10 meter tall poles are 30 meters apart. A length of wire is attached to the top of each pole and it is staked to the ground somewhere between the two poles. Where should the wire be staked so that the minimum amount of wire is used?

Indeterminate Forms and L'Hospital's Rule

Use L'Hospital's Rule to evaluate each of the following limits.

1. $\lim_{x \rightarrow 2} \frac{x^3 - 7x^2 + 10x}{x^2 + x - 6}$
2. $\lim_{w \rightarrow -4} \frac{\sin(\pi w)}{w^2 - 16}$
3. $\lim_{t \rightarrow \infty} \frac{\ln(3t)}{t^2}$
4. $\lim_{z \rightarrow 0} \frac{\sin(2z) + 7z^2 - 2z}{z^2(z+1)^2}$

$$5. \lim_{x \rightarrow -\infty} \frac{x^2}{e^{1-x}}$$

$$6. \lim_{z \rightarrow \infty} \frac{z^2 + e^{4z}}{2z - e^z}$$

$$7. \lim_{t \rightarrow \infty} \left[t \ln \left(1 + \frac{3}{t} \right) \right]$$

$$8. \lim_{w \rightarrow 0^+} \left[w^2 \ln(4w^2) \right]$$

$$9. \lim_{x \rightarrow 1^+} \left[(x-1) \tan\left(\frac{\pi}{2}x\right) \right]$$

$$10. \lim_{y \rightarrow 0^+} \left[\cos(2y) \right]^{\frac{1}{y^2}}$$

$$11. \lim_{x \rightarrow \infty} \left[e^x + x \right]^{\frac{1}{x}}$$

Linear Approximations

For problems 1 & 2 find a linear approximation to the function at the given point.

$$1. f(x) = 3xe^{2x-10} \text{ at } x = 5$$

$$2. h(t) = t^4 - 6t^3 + 3t - 7 \text{ at } t = -3$$

3. Find the linear approximation to $g(z) = \sqrt[4]{z}$ at $z = 2$. Use the linear approximation to approximate the value of $\sqrt[4]{3}$ and $\sqrt[4]{10}$. Compare the approximated values to the exact values.

4. Find the linear approximation to $f(t) = \cos(2t)$ at $t = \frac{1}{2}$. Use the linear approximation to approximate the value of $\cos(1)$ and $\cos(9)$. Compare the approximated values to the exact values.

5. Without using any kind of computational aid use a linear approximation to estimate the value of $e^{0.1}$.

Differentials

For problems 1 – 3 compute the differential of the given function.

1. $f(x) = x^2 - \sec(x)$

2. $w = e^{x^4 - x^2 + 4x}$

3. $h(z) = \ln(2z) \sin(2z)$

4. Compute dy and Δy for $y = e^{x^2}$ as x changes from 3 to 3.01.

5. Compute dy and Δy for $y = x^5 - 2x^3 + 7x$ as x changes from 6 to 5.9.

6. The sides of a cube are found to be 6 feet in length with a possible error of no more than 1.5 inches. What is the maximum possible error in the volume of the cube if we use this value of the length of the side to compute the volume?

Newton's Method

For problems 1 & 2 use Newton's Method to determine x_2 for the given function and given value of x_0 .

1. $f(x) = x^3 - 7x^2 + 8x - 3$, $x_0 = 5$

2. $f(x) = x \cos(x) - x^2$, $x_0 = 1$

For problems 3 & 4 use Newton's Method to find the root of the given equation, accurate to six decimal places, that lies in the given interval.

3. $x^4 - 5x^3 + 9x + 3 = 0$ in $[4, 6]$

4. $2x^2 + 5 = e^x$ in $[3, 4]$

For problems 5 & 6 use Newton's Method to find all the roots of the given equation accurate to six decimal places.

5. $x^3 - x^2 - 15x + 1 = 0$

6. $2 - x^2 = \sin(x)$

Business Applications

1. A company can produce a maximum of 1500 widgets in a year. If they sell x widgets during the year then their profit, in dollars, is given by,

$$P(x) = 30,000,000 - 360,000x + 750x^2 - \frac{1}{3}x^3$$

How many widgets should they try to sell in order to maximize their profit?

2. A management company is going to build a new apartment complex. They know that if the complex contains x apartments the maintenance costs for the building, landscaping etc. will be,

$$C(x) = 4000 + 14x - 0.04x^2$$

The land they have purchased can hold a complex of at most 500 apartments. How many apartments should the complex have in order to minimize the maintenance costs?

3. The production costs, in dollars, per day of producing x widgets is given by,

$$C(x) = 1750 + 6x - 0.04x^2 + 0.0003x^3$$

What is the marginal cost when $x = 175$ and $x = 300$? What do your answers tell you about the production costs?

4. The production costs, in dollars, per month of producing x widgets is given by,

$$C(x) = 200 + 0.5x + \frac{10000}{x}$$

What is the marginal cost when $x = 200$ and $x = 500$? What do your answers tell you about the production costs?

5. The production costs, in dollars, per week of producing x widgets is given by,

$$C(x) = 4000 - 32x + 0.08x^2 + 0.00006x^3$$

and the demand function for the widgets is given by,

$$p(x) = 250 + 0.02x - 0.001x^2$$

What is the marginal cost, marginal revenue and marginal profit when $x = 200$ and $x = 400$?
What do these numbers tell you about the cost, revenue and profit?

Integrals

Introduction

Here are a set of practice problems for the Integrals chapter of my Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a number of places on the site.

16. If you'd like a pdf document containing the solutions go to the note page for the section you'd like solutions for and select the download solutions link from there. Or,
17. Go to the download page for the site <http://tutorial.math.lamar.edu/download.aspx> and select the section you'd like solutions for and a link will be provided there.
18. If you'd like to view the solutions on the web or solutions to an individual problem you can go to the problem set web page, select the problem you want the solution for. At this point I do not provide pdf versions of individual solutions, but for a particular problem you can select "Printable View" from the "Solution Pane Options" to get a printable version.

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[Indefinite Integrals](#)

[Computing Indefinite Integrals](#)

[Substitution Rule for Indefinite Integrals](#)

[More Substitution Rule](#)

[Area Problem](#)

[Definition of the Definite Integral](#)

[Computing Definite Integrals](#)

[Substitution Rule for Definite Integrals](#)

Indefinite Integrals

1. Evaluate each of the following indefinite integrals.

(a) $\int 6x^5 - 18x^2 + 7 \, dx$

(b) $\int 6x^5 \, dx - 18x^2 + 7$

2. Evaluate each of the following indefinite integrals.

(a) $\int 40x^3 + 12x^2 - 9x + 14 \, dx$

(b) $\int 40x^3 + 12x^2 - 9x \, dx + 14$

(c) $\int 40x^3 + 12x^2 \, dx - 9x + 14$

For problems 3 – 5 evaluate the indefinite integral.

3. $\int 12t^7 - t^2 - t + 3 \, dt$

4. $\int 10w^4 + 9w^3 + 7w \, dw$

5. $\int z^6 + 4z^4 - z^2 \, dz$

6. Determine $f(x)$ given that $f'(x) = 6x^8 - 20x^4 + x^2 + 9$.

7. Determine $h(t)$ given that $h'(t) = t^4 - t^3 + t^2 + t - 1$.

Computing Indefinite Integrals

For problems 1 – 21 evaluate the given integral.

1. $\int 4x^6 - 2x^3 + 7x - 4 \, dx$

$$2. \int z^7 - 48z^{11} - 5z^{16} dz$$

$$3. \int 10t^{-3} + 12t^{-9} + 4t^3 dt$$

$$4. \int w^{-2} + 10w^{-5} - 8 dw$$

$$5. \int 12 dy$$

$$6. \int \sqrt[3]{w} + 10 \sqrt[5]{w^3} dw$$

$$7. \int \sqrt{x^7} - 7 \sqrt[6]{x^5} + 17 \sqrt[3]{x^{10}} dx$$

$$8. \int \frac{4}{x^2} + 2 - \frac{1}{8x^3} dx$$

$$9. \int \frac{7}{3y^6} + \frac{1}{y^{10}} - \frac{2}{\sqrt[3]{y^4}} dy$$

$$10. \int (t^2 - 1)(4 + 3t) dt$$

$$11. \int \sqrt{z} \left(z^2 - \frac{1}{4z} \right) dz$$

$$12. \int \frac{z^8 - 6z^5 + 4z^3 - 2}{z^4} dz$$

$$13. \int \frac{x^4 - \sqrt[3]{x}}{6\sqrt{x}} dx$$

$$14. \int \sin(x) + 10 \csc^2(x) dx$$

$$15. \int 2 \cos(w) - \sec(w) \tan(w) dw$$

$$16. \int 12 + \csc(\theta) [\sin(\theta) + \csc(\theta)] d\theta$$

$$17. \int 4e^z + 15 - \frac{1}{6z} dz$$

$$18. \int t^3 - \frac{e^{-t} - 4}{e^{-t}} dt$$

$$19. \int \frac{6}{w^3} - \frac{2}{w} dw$$

$$20. \int \frac{1}{1+x^2} + \frac{12}{\sqrt{1-x^2}} dx$$

$$21. \int 6\cos(z) + \frac{4}{\sqrt{1-z^2}} dz$$

$$22. \text{Determine } f(x) \text{ given that } f'(x) = 12x^2 - 4x \text{ and } f(-3) = 17.$$

$$23. \text{Determine } g(z) \text{ given that } g'(z) = 3z^3 + \frac{7}{2\sqrt{z}} - e^z \text{ and } g(1) = 15 - e.$$

$$24. \text{Determine } h(t) \text{ given that } h''(t) = 24t^2 - 48t + 2, h(1) = -9 \text{ and } h(-2) = -4.$$

Substitution Rule for Indefinite Integrals

For problems 1 – 16 evaluate the given integral.

$$1. \int (8x - 12)(4x^2 - 12x)^4 dx$$

$$2. \int 3t^{-4} (2 + 4t^{-3})^{-7} dt$$

$$3. \int (3 - 4w)(4w^2 - 6w + 7)^{10} dw$$

$$4. \int 5(z - 4) \sqrt[3]{z^2 - 8z} dz$$

$$5. \int 90x^2 \sin(2 + 6x^3) dx$$

6. $\int \sec(1-z) \tan(1-z) dz$

7. $\int (15t^{-2} - 5t) \cos(6t^{-1} + t^2) dt$

8. $\int (7y - 2y^3) e^{y^4 - 7y^2} dy$

9. $\int \frac{4w+3}{4w^2+6w-1} dw$

10. $\int (\cos(3t) - t^2)(\sin(3t) - t^3)^5 dt$

11. $\int 4\left(\frac{1}{z} - e^{-z}\right) \cos(e^{-z} + \ln z) dz$

12. $\int \sec^2(v) e^{1+\tan(v)} dv$

13. $\int 10 \sin(2x) \cos(2x) \sqrt{\cos^2(2x) - 5} dx$

14. $\int \frac{\csc(x) \cot(x)}{2 - \csc(x)} dx$

15. $\int \frac{6}{7+y^2} dy$

16. $\int \frac{1}{\sqrt{4-9w^2}} dw$

17. Evaluate each of the following integrals.

(a) $\int \frac{3x}{1+9x^2} dx$

(b) $\int \frac{3x}{(1+9x^2)^4} dx$

(c) $\int \frac{3}{1+9x^2} dx$

More Substitution Rule

Evaluate each of the following integrals.

1. $\int 4\sqrt{5+9t} + 12(5+9t)^7 dt$

2. $\int 7x^3 \cos(2+x^4) - 8x^3 e^{2+x^4} dx$

3. $\int \frac{6e^{7w}}{(1-8e^{7w})^3} + \frac{14e^{7w}}{1-8e^{7w}} dw$

4. $\int x^4 - 7x^5 \cos(2x^6 + 3) dx$

5. $\int e^z + \frac{4 \sin(8z)}{1+9 \cos(8z)} dz$

6. $\int 20e^{2-8w} \sqrt{1+e^{2-8w}} + 7w^3 - 6 \sqrt[3]{w} dw$

7. $\int (4+7t)^3 - 9t \sqrt[4]{5t^2+3} dt$

8. $\int \frac{6x-x^2}{x^3-9x^2+8} - \csc^2\left(\frac{3x}{2}\right) dx$

9. $\int 7(3y+2)(4y+3y^2)^3 + \sin(3+8y) dy$

10. $\int \sec^2(2t) [9 + 7 \tan(2t) - \tan^2(2t)] dt$

11. $\int \frac{8-w}{4w^2+9} dw$

12. $\int \frac{7x+2}{\sqrt{1-25x^2}} dx$

13. $\int z^7 (8 + 3z^4)^8 dz$

Area Problem

For problems 1 – 3 estimate the area of the region between the function and the x -axis on the given interval using $n = 6$ and using,

- (a) the right end points of the subintervals for the height of the rectangles,
- (b) the left end points of the subintervals for the height of the rectangles and,
- (c) the midpoints of the subintervals for the height of the rectangles.

1. $f(x) = x^3 - 2x^2 + 4$ on $[1, 4]$

2. $g(x) = 4 - \sqrt{x^2 + 2}$ on $[-1, 3]$

3. $h(x) = -x \cos\left(\frac{x}{3}\right)$ on $[0, 3]$

4. Estimate the net area between $f(x) = 8x^2 - x^5 - 12$ and the x -axis on $[-2, 2]$ using $n = 8$ and the midpoints of the subintervals for the height of the rectangles. Without looking at a graph of the function on the interval does it appear that more of the area is above or below the x -axis?

The Definition of the Definite Integral

For problems 1 & 2 use the definition of the definite integral to evaluate the integral. Use the right end point of each interval for x_i^* .

1. $\int_1^4 2x + 3 dx$

2. $\int_0^1 6x(x-1) dx$

3. Evaluate : $\int_4^4 \frac{\cos(e^{3x} + x^2)}{x^4 + 1} dx$

For problems 4 & 5 determine the value of the given integral given that $\int_6^{11} f(x) dx = -7$ and

$$\int_6^{11} g(x) dx = 24.$$

4. $\int_{11}^6 9f(x) dx$

5. $\int_6^{11} 6g(x) - 10f(x) dx$

6. Determine the value of $\int_2^9 f(x) dx$ given that $\int_5^2 f(x) dx = 3$ and $\int_5^9 f(x) dx = 8$.

7. Determine the value of $\int_{-4}^{20} f(x) dx$ given that $\int_{-4}^0 f(x) dx = -2$, $\int_{31}^0 f(x) dx = 19$ and $\int_{20}^{31} f(x) dx = -21$.

For problems 8 & 9 sketch the graph of the integrand and use the area interpretation of the definite integral to determine the value of the integral.

8. $\int_1^4 3x - 2 dx$

9. $\int_0^5 -4x dx$

For problems 10 – 12 differentiate each of the following integrals with respect to x .

10. $\int_4^x 9 \cos^2(t^2 - 6t + 1) dt$

11. $\int_7^{\sin(6x)} \sqrt{t^2 + 4} dt$

12. $\int_{3x^2}^{-1} \frac{e^t - 1}{t} dt$

Computing Definite Integrals

1. Evaluate each of the following integrals.

$$\mathbf{a.} \int \cos(x) - \frac{3}{x^5} dx$$

$$\mathbf{b.} \int_{-3}^4 \cos(x) - \frac{3}{x^5} dx$$

$$\mathbf{c.} \int_1^4 \cos(x) - \frac{3}{x^5} dx$$

Evaluate each of the following integrals, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$2. \int_1^6 12x^3 - 9x^2 + 2 dx$$

$$3. \int_{-2}^1 5z^2 - 7z + 3 dz$$

$$4. \int_3^0 15w^4 - 13w^2 + w dw$$

$$5. \int_1^4 \frac{8}{\sqrt{t}} - 12\sqrt{t^3} dt$$

$$6. \int_1^2 \frac{1}{7z} + \frac{\sqrt[3]{z^2}}{4} - \frac{1}{2z^3} dz$$

$$7. \int_{-2}^4 x^6 - x^4 + \frac{1}{x^2} dx$$

$$8. \int_{-4}^{-1} x^2 (3 - 4x) dx$$

$$9. \int_2^1 \frac{2y^3 - 6y^2}{y^2} dy$$

$$10. \int_0^{\frac{\pi}{2}} 7 \sin(t) - 2 \cos(t) dt$$

$$11. \int_0^{\pi} \sec(z) \tan(z) - 1 dz$$

$$12. \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2 \sec^2(w) - 8 \csc(w) \cot(w) dw$$

$$13. \int_0^2 e^x + \frac{1}{x^2 + 1} dx$$

$$14. \int_{-5}^{-2} 7e^y + \frac{2}{y} dy$$

$$15. \int_0^4 f(t) dt \text{ where } f(t) = \begin{cases} 2t & t > 1 \\ 1 - 3t^2 & t \leq 1 \end{cases}$$

$$16. \int_{-6}^1 g(z) dz \text{ where } g(z) = \begin{cases} 2 - z & z > -2 \\ 4e^z & z \leq -2 \end{cases}$$

$$17. \int_3^6 |2x - 10| dx$$

$$18. \int_{-1}^0 |4w + 3| dw$$

Substitution Rule for Definite Integrals

Evaluate each of the following integrals, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$1. \int_0^1 3(4x + x^4)(10x^2 + x^5 - 2)^6 dx$$

$$2. \int_0^{\frac{\pi}{4}} \frac{8 \cos(2t)}{\sqrt{9 - 5 \sin(2t)}} dt$$

$$3. \int_{\pi}^0 \sin(z) \cos^3(z) dz$$

$$4. \int_1^4 \sqrt{w} e^{1 - \sqrt{w^3}} dw$$

$$5. \int_{-4}^{-1} \sqrt[3]{5-2y} + \frac{7}{5-2y} dy$$

$$6. \int_{-1}^2 x^3 + e^{\frac{1}{4}x} dx$$

$$7. \int_{\pi}^{\frac{3\pi}{2}} 6\sin(2w) - 7\cos(w) dw$$

$$8. \int_1^5 \frac{2x^3 + x}{x^4 + x^2 + 1} - \frac{x}{x^2 - 4} dx$$

$$9. \int_{-2}^0 t\sqrt{3+t^2} + \frac{3}{(6t-1)^2} dt$$

$$10. \int_{-2}^1 (2-z)^3 + \sin(\pi z) [3 + 2\cos(\pi z)]^3 dz$$

Applications of Integrals

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19. If you'd like a pdf document containing the solutions go to the note page for the section you'd like solutions for and select the download solutions link from there. Or,
20. Go to the download page for the site <http://tutorial.math.lamar.edu/download.aspx> and select the section you'd like solutions for and a link will be provided there.
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[Average Function Value](#)

[Area Between Two Curves](#)

[Volumes of Solids of Revolution / Method of Rings](#)

[Volumes of Solids of Revolution / Method of Cylinders](#)

[More Volume Problems](#)

[Work](#)

Average Function Value

For problems 1 & 2 determine f_{avg} for the function on the given interval.

1. $f(x) = 8x - 3 + 5e^{2-x}$ on $[0, 2]$

2. $f(x) = \cos(2x) - \sin\left(\frac{x}{2}\right)$ on $\left[-\frac{\pi}{2}, \pi\right]$

3. Find f_{avg} for $f(x) = 4x^2 - x + 5$ on $[-2, 3]$ and determine the value(s) of c in $[-2, 3]$ for which $f(c) = f_{\text{avg}}$.

Area Between Curves

1. Determine the area below $f(x) = 3 + 2x - x^2$ and above the x -axis.

2. Determine the area to the left of $g(y) = 3 - y^2$ and to the right of $x = -1$.

For problems 3 – 11 determine the area of the region bounded by the given set of curves.

3. $y = x^2 + 2$, $y = \sin(x)$, $x = -1$ and $x = 2$

4. $y = \frac{8}{x}$, $y = 2x$ and $x = 4$

5. $x = 3 + y^2$, $x = 2 - y^2$, $y = 1$ and $y = -2$
6. $x = y^2 - y - 6$ and $x = 2y + 4$
7. $y = x\sqrt{x^2 + 1}$, $y = e^{-\frac{1}{2}x}$, $x = -3$ and the y -axis
8. $y = 4x + 3$, $y = 6 - x - 2x^2$, $x = -4$ and $x = 2$
9. $y = \frac{1}{x+2}$, $y = (x+2)^2$, $x = -\frac{3}{2}$, $x = 1$
10. $x = y^2 + 1$, $x = 5$, $y = -3$ and $y = 3$
11. $x = e^{1+2y}$, $x = e^{1-y}$, $y = -2$ and $y = 1$

Volumes of Solids of Revolution / Method of Rings

For problems 1 – 8 use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by the given curves about the given axis.

1. Rotate the region bounded by $y = \sqrt{x}$, $y = 3$ and the y -axis about the y -axis.
2. Rotate the region bounded by $y = 7 - x^2$, $x = -2$, $x = 2$ and the x -axis about the x -axis.
3. Rotate the region bounded by $x = y^2 - 6y + 10$ and $x = 5$ about the y -axis.
4. Rotate the region bounded by $y = 2x^2$ and $y = x^3$ about the x -axis.
5. Rotate the region bounded by $y = 6e^{-2x}$ and $y = 6 + 4x - 2x^2$ between $x = 0$ and $x = 1$ about the line $y = -2$.
6. Rotate the region bounded by $y = 10 - 6x + x^2$, $y = -10 + 6x - x^2$, $x = 1$ and $x = 5$ about the line $y = 8$.
7. Rotate the region bounded by $x = y^2 - 4$ and $x = 6 - 3y$ about the line $x = 24$.
8. Rotate the region bounded by $y = 2x + 1$, $x = 4$ and $y = 3$ about the line $x = -4$.

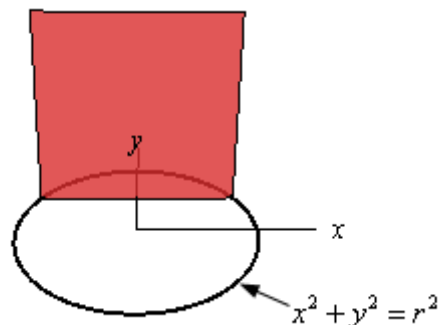
Volumes of Solids of Revolution / Method of Cylinders

For problems 1 – 8 use the method of cylinders to determine the volume of the solid obtained by rotating the region bounded by the given curves about the given axis.

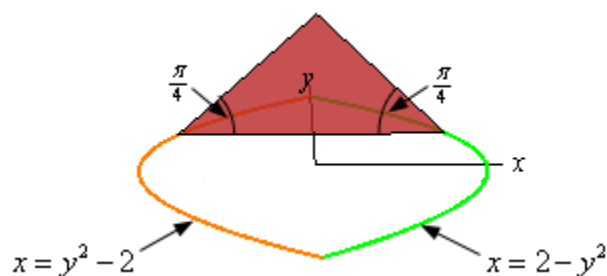
1. Rotate the region bounded by $x = (y - 2)^2$, the x -axis and the y -axis about the x -axis.
2. Rotate the region bounded by $y = \frac{1}{x}$, $x = \frac{1}{2}$, $x = 4$ and the x -axis about the y -axis.
3. Rotate the region bounded by $y = 4x$ and $y = x^3$ about the y -axis. For this problem assume that $x \geq 0$.
4. Rotate the region bounded by $y = 4x$ and $y = x^3$ about the x -axis. For this problem assume that $x \geq 0$.
5. Rotate the region bounded by $y = 2x + 1$, $y = 3$ and $x = 4$ about the line $y = 10$.
6. Rotate the region bounded by $x = y^2 - 4$ and $x = 6 - 3y$ about the line $y = -8$.
7. Rotate the region bounded by $y = x^2 - 6x + 9$ and $y = -x^2 + 6x - 1$ about the line $x = 8$.
8. Rotate the region bounded by $y = \frac{e^{\frac{1}{2}x}}{x+2}$, $y = 5 - \frac{1}{4}x$, $x = -1$ and $x = 6$ about the line $x = -2$.

More Volume Problems

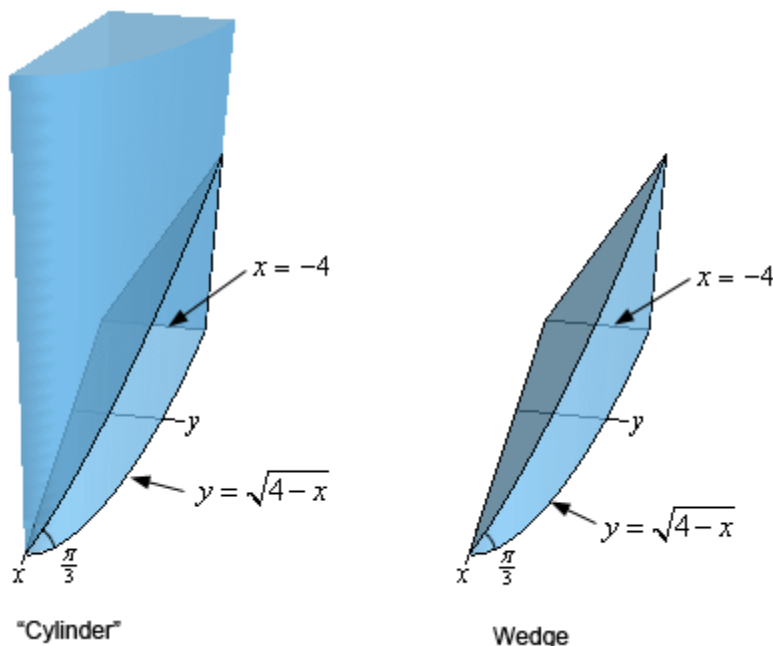
1. Find the volume of a pyramid of height h whose base is an equilateral triangle of length L .
2. Find the volume of the solid whose base is a disk of radius r and whose cross-sections are squares. See figure below to see a sketch of the cross-sections.



3. Find the volume of the solid whose base is the region bounded by $x = 2 - y^2$ and $x = y^2 - 2$ and whose cross-sections are isosceles triangles with the base perpendicular to the y -axis and the angle between the base and the two sides of equal length is $\frac{\pi}{4}$. See figure below to see a sketch of the cross-sections.



4. Find the volume of a wedge cut out of a “cylinder” whose base is the region bounded by $y = \sqrt{4 - x}$, $x = -4$ and the x -axis. The angle between the top and bottom of the wedge is $\frac{\pi}{3}$. See the figure below for a sketch of the “cylinder” and the wedge (the positive x -axis and positive y -axis are shown in the sketch – they are just in a different orientation).



Work

1. A force of $F(x) = x^2 - \cos(3x) + 2$, x is in meters, acts on an object. What is the work required to move the object from $x = 3$ to $x = 7$?
2. A spring has a natural length of 18 inches and a force of 20 lbs is required to stretch and hold the spring to a length of 24 inches. What is the work required to stretch the spring from a length of 21 inches to a length of 26 inches?
3. A cable that weighs $\frac{1}{2}$ kg/meter is lifting a load of 150 kg that is initially at the bottom of a 50 meter shaft. How much work is required to lift the load $\frac{1}{4}$ of the way up the shaft?
4. A tank of water is 15 feet long and has a cross section in the shape of an equilateral triangle with sides 2 feet long (point of the triangle points directly down). The tank is filled with water to a depth of 9 inches. Determine the amount of work needed to pump all of the water to the top of the tank. Assume that the density of water is 62 lb/ft^3 .