TVC Guide

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1 Introduction

TVC (Thrust Vector Control) is when you vector the thrust of a vehicle to keep it stable. In this case, it is referring to model rockets using low to mid power solid rocket engines (A-G class motors). This paper goes through the mathematics behind TVC.

2 Flight Mathematics

2.1 Quaternions

A gimbal is a pivoted support that permits rotation of an object about an axis. In order to stop gimbal lock, we need to use quaternions. Gimbal lock occurs when you rotate 2 or 3 of the successive gimbals and they align onto a plane. This results in you losing one direction as the angles now rotate the same direction. Quaternions solve this by representing orientations/rotations using four parameters instead of the three Euler angles ψ, θ, ϕ. a and b are two quaternions. This multiplication is not commutative meaning $a \otimes b \neq b \otimes a$.

$$a \otimes b = [a_1, a_2, a_3, a_4] \otimes [b_1, b_2, b_3, b_4]$$

For TVC, imu's will most likely put out an angular rate of x, y, z. So collectively, we will refer to these as

$$w_t = [0, x, y, z]$$

Next, we introduce q this is our base reference to the earth, so on the launch pad we assume the rocket is pointing upright so at launch:

$$q_{t-1} = [1, 0, 0, 0]$$

Now, we need to calculate the rate of change relative to the earth, to do this we use

$$\dot{q}_{t,w} = \frac{1}{2} q_{t-1} \otimes w_t$$

So now we have the rate of change relative to the earth, we can calculate our orientation q with

$$q = \sum \dot{q}_{t,w} \Delta t$$

Finally the quaternion has to be normalised. To normalise the quaternion we must first calculate the norm, n.

$$n = \sqrt{q_1^2 + q_2^2 + q_3^2 + q_4^2}$$

Then, we divide q by n

$$q = \frac{q}{n}$$

Great, we have our orientation relative to the earth, now we need to convert q to Euler angles.

$$\psi = atan2(2(q_2q_3 + q_1q_4), q_1^2 + q_2^2 - q_3^2 - q_4^2)$$

$$\theta = \sin^{-1}(2(q_2q_4 - q_1q_3))$$

$$\phi = atan2(2(q_1q_2 + q_3q_4), q_1^2 - q_2^2 - q_3^2 + q_4^2)$$

2.2 PID Control

PID control uses the formula:

$$u_t = k_p e_t + k_i \int_0^t e_t dt + k_d \frac{de_t}{dt}$$

In the formula: e is the error,k are the pid gains for p i and d, t is time and u is the PID output.

Due to the thrust curve of a solid rocket motor we have to calculate the amount of torque needed for the motor at any given time. To do this we take the pid output and convert it to torque with the following equation, however you can simply tune your rocket to output the torque value and this may not be necessary for your rocket.

$$T = f \times r \times \sin\theta$$

T is torque, f is the force the PID loop was tuned with, r is the distance of the tvc mount from pivot, θ is the pid output (u) from the PID equation. Then, we calculate the angle needed to create this torque with the current thrust of the motor:

$$\theta = \sin^{-1}\frac{T}{r \times f}$$

Now, θ is the angle the solid rocket motor should be at and f is the current thrust.

2.3 Kalman Filter

In TVC, Kalman filters are used to filter out barometer data and imu data as sensors can have noise. abc are all system properties that you define. q and r are the gains which affect the filter. The formula to give the next position is:

$$\dot{x} = ax + bu$$

With $\dot{x}$ updated the next step is to find the state error covariance, p.

$$p = apa + q$$

Now to find the state-feedback gain, k

$$k = pc\frac{1}{cpc + r}$$

Using all of these variables, the formula to update x is complete. x is the filtered data to use.

$$x = \sum k(y - c\dot{x})$$

Finally, for the next loop we must update p.

$$p = p(1 - kc)$$