linear_sir_vs_seir

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[1]:
```
from sympy import *
from sympy.vector import *
from sympy.matrices import Matrix

init_printing()
from IPython.display import display, Math
```

0.1 SIR model

$$\frac{dS}{dt} = -\frac{\beta IS}{N}, \tag{1}$$
$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I, \tag{2}$$
$$\frac{dR}{dt} = \gamma I, \tag{3}$$

Linearized SIR model

$$\frac{dS}{dt} = -\beta I, \tag{4}$$
$$\frac{dI}{dt} = (\beta - \gamma)I, \tag{5}$$
$$\frac{dR}{dt} = \gamma I, \tag{6}$$

Remove equations not relevant for I

$$\frac{dI}{dt} = (\beta - \gamma)I \tag{7}$$

Solution

$$I = I_0 e^{(\beta-\gamma)t} \tag{8}$$

Growth rate for $I(t)$

$$\beta - \gamma = \gamma(R_0 - 1) \tag{9}$$

0.2 SEIR model

$$\frac{dS}{dt} = -\frac{\beta IS}{N}, \tag{10}$$
$$\frac{dE}{dt} = \frac{\beta IS}{N} - \alpha E, \tag{11}$$
$$\frac{dI}{dt} = \alpha E - \gamma I, \tag{12}$$
$$\frac{dR}{dt} = \gamma I, \tag{13}$$

Linearized SEIR model

$$\frac{dS}{dt} = -\beta I, \tag{14}$$
$$\frac{dE}{dt} = \beta I - \alpha E, \tag{15}$$
$$\frac{dI}{dt} = \alpha E - \gamma I, \tag{16}$$
$$\frac{dR}{dt} = \gamma I, \tag{17}$$

Remove equations not relevant for I

$$\frac{dE}{dt} = \beta I - \alpha E, \tag{18}$$
$$\frac{dI}{dt} = \alpha E - \gamma I, \tag{19}$$
$$\tag{20}$$

Matrix form

$$\frac{d}{dt}\begin{pmatrix} E \\ I \end{pmatrix} = \begin{pmatrix} -\alpha & \beta \\ \alpha & -\gamma \end{pmatrix}\begin{pmatrix} E \\ I \end{pmatrix} \tag{21}$$

[2]:
```
alpha = Symbol('alpha', positive = True, real = True)
R0 = Symbol('R_0', positive = True, real = True)
gamma = Symbol('gamma', positive = True, real = True)
#beta = Symbol('beta')
beta = R0 * gamma

M = Matrix([[-alpha, beta],[alpha, -gamma]])
ll = list(M.eigenvals().keys())
[simplify(l) for l in ll]
```

[2]:

$$\left[-\frac{\alpha}{2} - \frac{\gamma}{2} - \frac{\sqrt{4R_0\alpha\gamma + \alpha^2 - 2\alpha\gamma + \gamma^2}}{2}, \ -\frac{\alpha}{2} - \frac{\gamma}{2} + \frac{\sqrt{4R_0\alpha\gamma + \alpha^2 - 2\alpha\gamma + \gamma^2}}{2} \right]$$

[3]:
```
[simplify(l.subs(alpha, gamma)) for l in ll]
```

[3]: $\left[-\gamma \left(\sqrt{R_0} + 1\right), \ \gamma \left(\sqrt{R_0} - 1\right)\right]$

Eigenvalues

$$\left[-\frac{\alpha}{2} - \frac{\gamma}{2} - \frac{\sqrt{4R_0\alpha\gamma + \alpha^2 - 2\alpha\gamma + \gamma^2}}{2}, \ -\frac{\alpha}{2} - \frac{\gamma}{2} + \frac{\sqrt{4R_0\alpha\gamma + \alpha^2 - 2\alpha\gamma + \gamma^2}}{2}\right] \tag{22}$$

Eigenvalues in case $\alpha = \gamma$

$$\left[-\gamma \left(\sqrt{R_0} + 1\right), \ \gamma \left(\sqrt{R_0} - 1\right)\right] \tag{23}$$

[4]:
```
growth = l1[1]
R0sir = Symbol('R_0^\mathrm{sir}', positive = True, real = True)
s = solve(growth -(gamma * (R0sir) - 1) , R0)[0]
s = s.subs([[gamma, 1/5.2], [alpha, 1/7]])
s = factor(s).n(4)
display(Math('R_0^\mathrm{seir}='), s)
```

$R_0^{\mathrm{seir}} =$

$1.346 \left(R_0^{\mathrm{sir}}\right)^2 - 11.65 R_0^{\mathrm{sir}} + 25.2$