

Given the total number of persons **n** and a number **k** which indicates that **k-1** persons are skipped and **k<sup>th</sup>** person is killed in circle in a fixed direction.

The task is to choose the **safe place in the circle** so that when you perform these operations starting from **1<sup>st</sup> place** in the circle, you are the last one remaining and survive.

Basically the last remaining person survives.  
∴ every recursive call reduces the no. of people ( $N-1$ )  
and at  $n == 1 \rightarrow$  return **1**.

If  $n = 7$  and  $k = 3$ , then the safe position is 4. The persons at positions 3, 6, 2, 7, 5, 1 are killed in order, and person at position 4 survives.

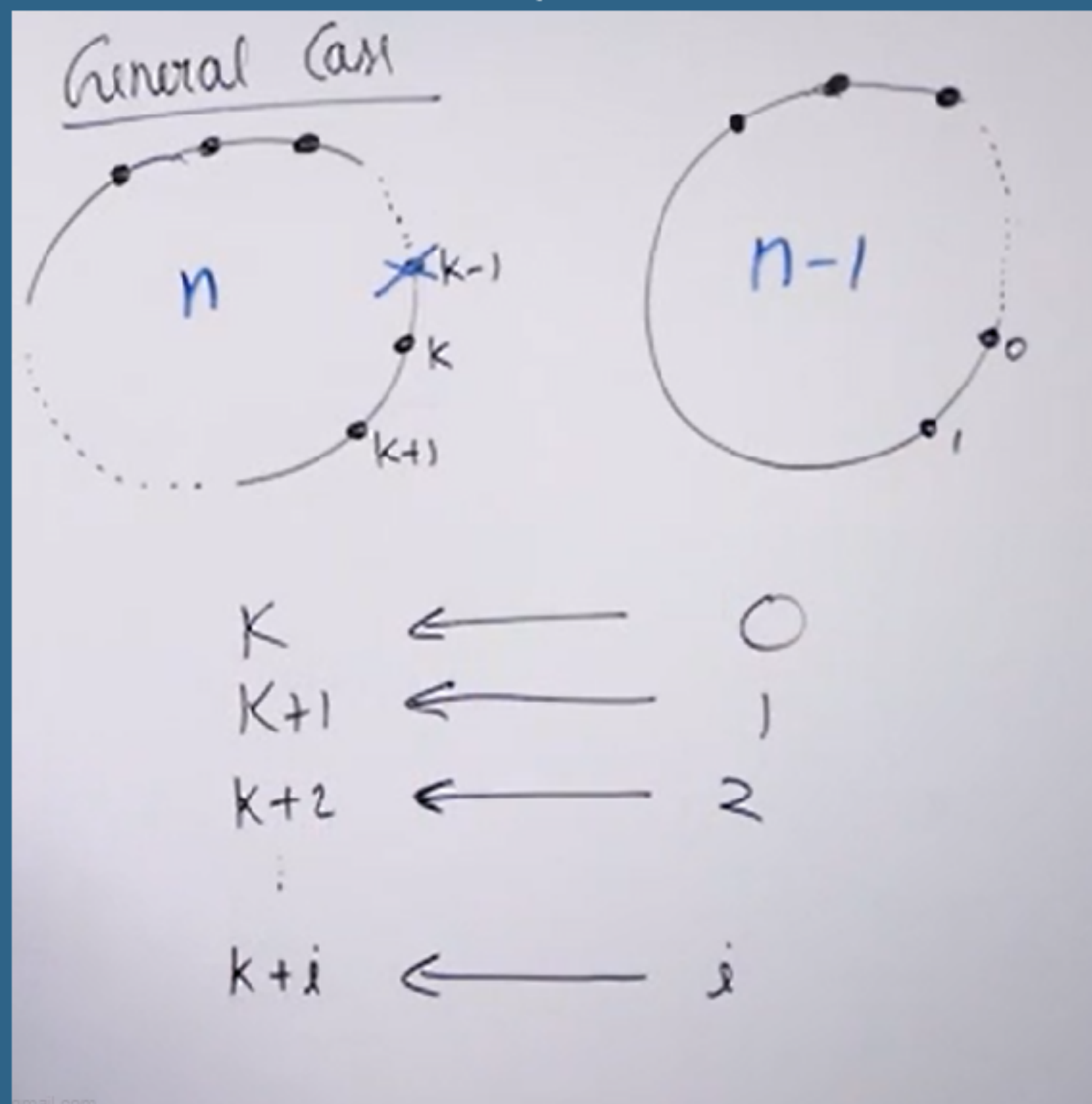
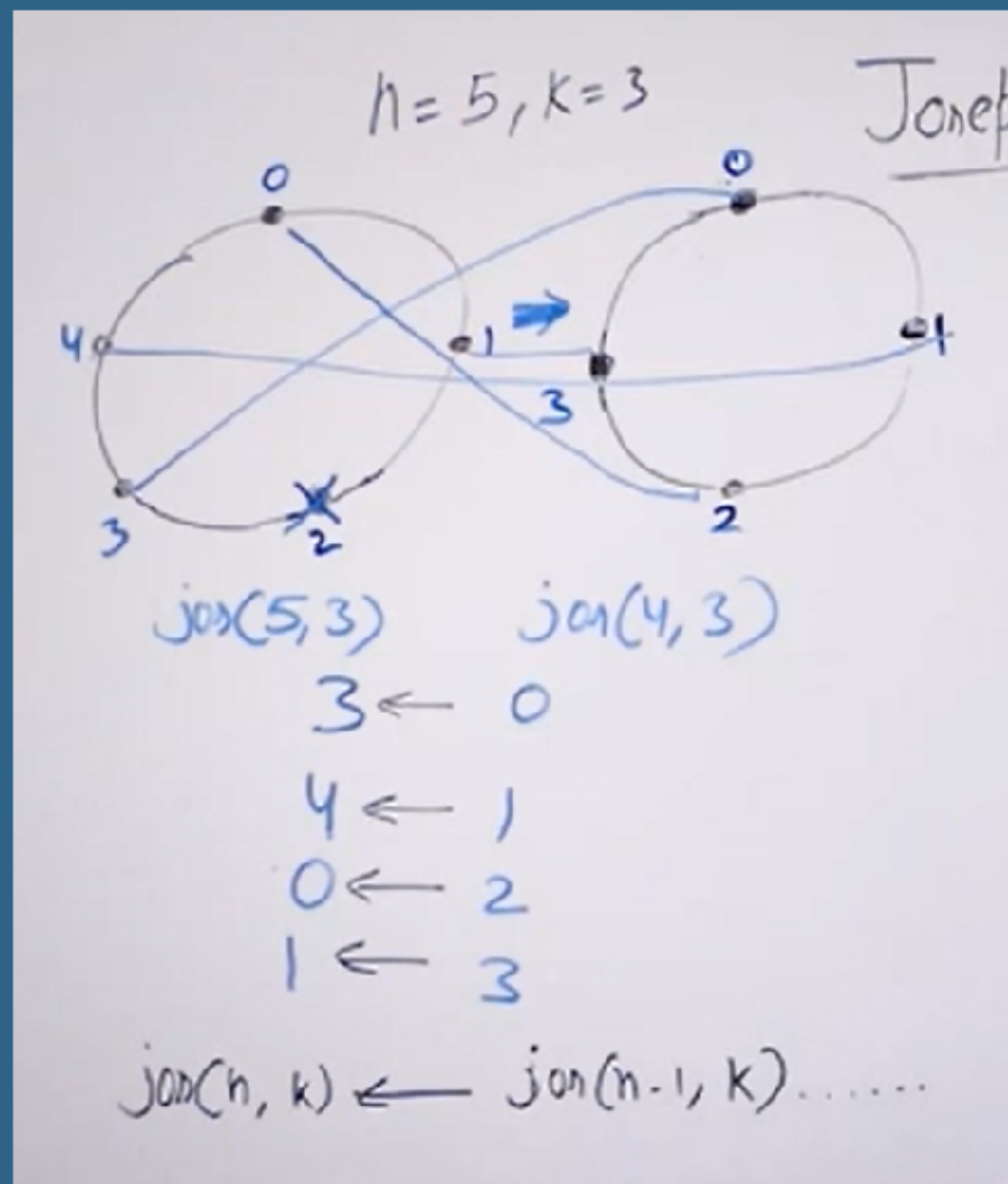
```
josephus(n, k) = (josephus(n - 1, k) + k - 1) % n + 1  
josephus(1, k) = 1
```

Now this  
is to be  
adjusted to implement  
the recursive function.

will return 1  
after every call.  
adjustment



Discerning the relationship b/w returned  
fn value and the original value:



→ Here we're  
solving our  
problem on a  
0-based index



Connecting the solution of  $(n-1)^{th}$  call and  $n^{th}$ :

① For  $i$ ' based index:

→ ① when  $k^{th}$  person is killed →  $k+1$  becomes the new  $i$ '.

$k+2 \rightarrow 2$

```
int josephus(int n, int k)
{
    if (n == 1) //base case
        return 1;
    else
        /* The position returned by josephus(n - 1, k) is adjusted because the
        recursive call josephus(n - 1, k) considers the original position
        k%n + 1 as position 1 */
        return (josephus(n - 1, k) + k - 1) % n + 1; //recursion
}
```

→ we are working on  $i$ ' based scale -  
 $O/P(n-1)$

→ Copy the starting position  
in  $i-1$  and we need to  
return it.

```

int josephus(int n, int k)
{
    if (n == 1) //base case
        return 1;
    else
        /* The position returned by josephus(n - 1, k) is adjusted because the
           recursive call josephus(n - 1, k) considers the original position
           k%n + 1 as position 1 */
        return (josephus(n - 1, k) + k - 1) % n + 1; //recursion
}

```

→ Base case, for  $N=1$ , it's the safest position.

→ Recursion steps:

① Assume we got the answer for  $(N-1)$  circle problem

② Now we need to build upon it till  $N$ th step, i.e.,  $(N-1+1)$

③ Since we need to return, the next surviving position, i.e., the person with whom the pistol starts

④ Include modulo so as we can handle the circular fashion of problem procedure.