

Exponential Population Growth:
Mathematical Modeling and Analysis
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Abstract

Exponential growth is an excellent example to demonstrate the difference between a numerical method/approximation and an exact solution. Through the derivative, Euler's method can be implemented to predict solutions through each iteration of time (t). Then by applying the exact solutions, two sets of comparable results are produced that can be then analyzed and marginalized through a relative error computation. To implement this algorithm, Excel, and C++ will be used. This solution is important as it can be applied to any derivative and its exact function. Utilizing this solution can help predict population growth in certain species in a particular area.

Table of Contents

Introduction

Methodology

Procedure

Assumptions

Results

Results and Explanation of Results

Errors and Error Analysis

Discussion

Future Improvements

Conclusion

Appendices

Pseudo-code

Data

References

Exponential Population Growth:
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Introduction

Exponential growth is applicable across many different facets of world problems, including population growth. One such derivative exists that defines this exponential growth written as $\frac{dp}{dt} = rp$ describing the rate of change in the population, with respect to time, is equivalent to the rate times the population. In order to approximate the solution to this derivative, a numerical method would have to be used. Equivocally, this exponential population growth can be described as $P = P_n * \exp(r*t)$ to represent an exact solution to a potential problem. This report examines one such exponential population growth problem and aims to solve the problem utilizing C++ code and Excel to produce applicable solutions.

Euler's method is one such numerical method that exists for approximating the solutions to first-order derivative functions. Excel boasts the ability to implement functions and solutions to these functions seamlessly within any worksheet. Utilizing this feature, a table can be created to represent: the number of times time (t) is increased or the iterator (i or n), the value of time (t) as we iterate, the value of the Population (P) as the solutions are iterated through, and for Euler's method a column for the function solution at the specified input values of t and P. C++ also offers an applicable solution to the problem through its function libraries and extensive programming capabilities. Through C++, a function can be devised that can handle all three necessary computations on its own. These three computations aim to solve the problem and examine these solutions These three computations are: estimation of the population growth

through Euler's method, exact or true solution through our derived function definition, and finally, the relative error between the previous two computations.

Methodology

Procedure.

In order to solve the solution through Excel, the three necessary computations were done on their own respective individual pages within the workbook. Each page was divided into two down the middle vertically, with the step size (or delta t) of 0.05 on the left side and the step size of 1.0 on the right. Both sides are labeled with the function/derivative being tested and the given variables of $P_0 = 1000$ for the initial population value, $r = 0.015$ indicating the growth rate, and delta t indicating the step size. Underneath this information, there is a table with each row representing another iteration/solution to the problem. The rows are labeled with the corresponding column headings to indicate what the data pertains to. They are labeled as i, t, P and in the Euler's method worksheet, $f(P, t)$, which is necessary to compile the values of P per the Euler method formula. In order to compute the values of P, the formula was applied at each step based on the variables necessary to make the computation. This process is repeated for the exact value with the exception of $f(P, t)$. In order to compute the relative error, the P values from each worksheet were brought into a table with three columns. One column is for the approximate value, the middle is for the exact value and the last column is the relative error between the first two columns computed by the absolute value of $|(approximate - exact)/exact|$.

In order to device a solution programmatically, C++ was selected as the most viable solution for the problem. A function can be used to create a solution as if any parameters in the problem were to change, (like the step size or delta t for example) the user would simply call the same function again but with the different parameters. The parameters used in the populationCalculator function consist of: the number of attempts that the user wants to make at the model (numAttempts), delta t to indicate the desired step size of the user, r to indicate the growth rate of the exponential function, and pZero to indicate the initial population size to be calculated with. Specifically for this problem, an array was chosen to hold the computation data through each solution iteration. The size of the array is determined by the (number of attempts/delta t). This calculation will also determine the run time or max number of iterations that are to be performed to solve the problem. These iterations are controlled utilizing for-loops and in the first for-loop, the estimate values are computed utilizing Euler's method and printed to the screen. The next for-loop performs the calculations necessary for the true value of the problem by utilizing the exact value function derived from the given derivative and prints those values to the screen as the loop iterates. The last for-loop loops through the estimate array and exact array, calculating the relative error and appending the result to the error array, and finally printing the value of the error as the loop iterates.

Assumptions.

One particular assumption of the model was within C++, arrays cannot be dynamically allocated, meaning an array size must be declared when creating the array. To solve this problem, the max array size necessary was computed by hand by dividing 20 (number of

attempts or desired end t value) by 0.05. As a result, the assumption is that this size will not need to increase to a larger one in order to account for a larger number of desired computations.

Results

Results and Explanation of Results

The results of the data show successful computations were made for all three necessary computations in both Excel and the executable C++ program. The primary difference between the two programs is in Excel the resulting computations account for up to six decimal places whereas in the C++ program only two are accounted for. Besides the significant figure difference, the whole numbers appear to match exactly between the two programs. The results of the population between the two different step sizes, 0.05 and 1.0, are extremely significant. This shows how much of an impact the step size can have on the final results. The smaller the step size, the more accurate the prediction becomes. The difference between the derivation formula and actual function can be seen as the iteration count increases. Essentially as each new solution is found the margin of error between the two methods becomes more and more significant. This margin of error shows the primary difference between numerical methods and exact solutions to a problem. While numerical methods are very good at getting extremely close to the actual value, there will always be that margin of error due to the fact that these methods are approximations and not exact solutions.

Errors and Error Analysis

In the C++ program, being limited to two decimal places will end up affecting the error calculation in a negative way. This loss of accuracy creates a slightly higher margin of error within the C++ program. Due to this, the most efficient program is the Excel program as it has a lower margin of relative error. Another error that arises are the margin of error between the approximate solution and the exact solution. This margin of error only furthers the solidification of the principles behind numerical methods: they are simply approximations of the solution and not the exact solution. The accuracy of the approximation directly correlates to the margin of error between the two methods; the more accurate a user is in the approximation, the smaller the margin of error.

Discussion

Future Improvements

In order to improve the solution in the future, the C++ arrays must be converted to vectors as vectors dynamically allocate memory as it is needed. This would allow the function to be implemented across any exponential growth problem, thus increasing the versatility of the solution. Another improvement would be to increase the number of significant figures computed in the C++ program to further reduce the margin of error.

Conclusion

In conclusion, the model did an excellent job of highlighting the primary differences between a numerical method and an exact method. While the margin of error can be minimal, it still, unfortunately, exists only proving that some of the solutions are truly approximations rather than

exact solutions. The created model/solution can be implemented on any exponential population growth problem to help find a solution. One such example would be to determine an overpopulation of a species in a certain area. This solution would help officials determine if a population cut-back would be necessary.

Appendices

Pseudo-code.

Pseudo-code for the C++ program:

Step 1:

Initialize any necessary variables for the computation

Step 2:

Run a controlled iteration loop that runs, starting at 1 and increasing by 1, until it reaches the set run time. The run time is determined by the desired value of t divided by the step size, or Δt . Through each iteration, a computation is made.

Step 3:

Set up a loop to determine all of the solutions to Eulers method/approximation

Step 4:

Set up a loop to determine all of the exact solutions to the problem

Step 5:

Set up a loop to determine the error between the two previously calculated computations

Data.

C++ Data:

Mathematical model displaying an exponential population growth rate. Our initial population size is 1000 and our growth rate is 0.015%

This is the computation with the change in time being 0.05, until time reaches 20

The estimated value at iteration 1 is 1000.75

The estimated value at iteration 2 is 1002.25

The estimated value at iteration 3 is 1004.51

The estimated value at iteration 4 is 1007.52

The estimated value at iteration 5 is 1011.3

The estimated value at iteration 6 is 1015.85

The estimated value at iteration 7 is 1021.18

The estimated value at iteration 8 is 1027.31

The estimated value at iteration 9 is 1034.24

The estimated value at iteration 10 is 1042

The estimated value at iteration 11 is 1050.6

The estimated value at iteration 12 is 1060.05

The estimated value at iteration 13 is 1070.39

The estimated value at iteration 14 is 1081.63

The estimated value at iteration 15 is 1093.79

The estimated value at iteration 16 is 1106.92

The estimated value at iteration 17 is 1121.03

The estimated value at iteration 18 is 1136.17

The estimated value at iteration 19 is 1152.36

The estimated value at iteration 20 is 1169.64

The estimated value at iteration 21 is 1188.07

The estimated value at iteration 22 is 1207.67

The estimated value at iteration 23 is 1228.5

The estimated value at iteration 24 is 1250.61

The estimated value at iteration 25 is 1274.06

The estimated value at iteration 26 is 1298.91

The estimated value at iteration 27 is 1325.21

The estimated value at iteration 28 is 1353.04

The estimated value at iteration 29 is 1382.47

The estimated value at iteration 30 is 1413.57

The estimated value at iteration 31 is 1446.44

The estimated value at iteration 32 is 1481.15

The estimated value at iteration 33 is 1517.81

The estimated value at iteration 34 is 1556.52

The estimated value at iteration 35 is 1597.37

The estimated value at iteration 36 is 1640.5

The estimated value at iteration 37 is 1686.03

The estimated value at iteration 38 is 1734.08

The estimated value at iteration 39 is 1784.8

The estimated value at iteration 40 is 1838.35

The estimated value at iteration 41 is 1894.87

The estimated value at iteration 42 is 1954.56

The estimated value at iteration 43 is 2017.6
The estimated value at iteration 44 is 2084.18
The estimated value at iteration 45 is 2154.52
The estimated value at iteration 46 is 2228.85
The estimated value at iteration 47 is 2307.42
The estimated value at iteration 48 is 2390.48
The estimated value at iteration 49 is 2478.33
The estimated value at iteration 50 is 2571.27
The estimated value at iteration 51 is 2669.62
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The estimated value at iteration 53 is 2884
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The estimated value at iteration 58 is 3542.68
The estimated value at iteration 59 is 3699.44
The estimated value at iteration 60 is 3865.92
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The estimated value at iteration 63 is 4430.68
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The estimated value at iteration 65 is 4869.71
The estimated value at iteration 66 is 5110.77
The estimated value at iteration 67 is 5367.58
The estimated value at iteration 68 is 5641.33
The estimated value at iteration 69 is 5933.27
The estimated value at iteration 70 is 6244.76
The estimated value at iteration 71 is 6577.3
The estimated value at iteration 72 is 6932.47
The estimated value at iteration 73 is 7312.02
The estimated value at iteration 74 is 7717.84
The estimated value at iteration 75 is 8151.97
The estimated value at iteration 76 is 8616.63
The estimated value at iteration 77 is 9114.24
The estimated value at iteration 78 is 9647.43
The estimated value at iteration 79 is 10219
The estimated value at iteration 80 is 10832.2
The estimated value at iteration 81 is 11490.2
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The estimated value at iteration 83 is 12956.1
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The estimated value at iteration 400 is 6.93399e+26
The actual value at iteration 1 is 1000.75
The actual value at iteration 2 is 1002.25
The actual value at iteration 3 is 1004.51
The actual value at iteration 4 is 1007.53
The actual value at iteration 5 is 1011.31
The actual value at iteration 6 is 1015.87
The actual value at iteration 7 is 1021.22
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The relative error at iteration 153 is: 0.274437
The relative error at iteration 154 is: 0.278919
The relative error at iteration 155 is: 0.28343
The relative error at iteration 156 is: 0.287968
The relative error at iteration 157 is: 0.292533
The relative error at iteration 158 is: 0.297125
The relative error at iteration 159 is: 0.301743
The relative error at iteration 160 is: 0.306386
The relative error at iteration 161 is: 0.311053

The relative error at iteration 162 is: 0.315744
The relative error at iteration 163 is: 0.320459
The relative error at iteration 164 is: 0.325197
The relative error at iteration 165 is: 0.329957
The relative error at iteration 166 is: 0.334738
The relative error at iteration 167 is: 0.339541
The relative error at iteration 168 is: 0.344363
The relative error at iteration 169 is: 0.349205
The relative error at iteration 170 is: 0.354066
The relative error at iteration 171 is: 0.358945
The relative error at iteration 172 is: 0.363842
The relative error at iteration 173 is: 0.368755
The relative error at iteration 174 is: 0.373685
The relative error at iteration 175 is: 0.37863
The relative error at iteration 176 is: 0.38359
The relative error at iteration 177 is: 0.388563
The relative error at iteration 178 is: 0.39355
The relative error at iteration 179 is: 0.39855
The relative error at iteration 180 is: 0.403562
The relative error at iteration 181 is: 0.408584
The relative error at iteration 182 is: 0.413617
The relative error at iteration 183 is: 0.41866
The relative error at iteration 184 is: 0.423712
The relative error at iteration 185 is: 0.428772
The relative error at iteration 186 is: 0.433839
The relative error at iteration 187 is: 0.438913
The relative error at iteration 188 is: 0.443993
The relative error at iteration 189 is: 0.449078
The relative error at iteration 190 is: 0.454167
The relative error at iteration 191 is: 0.459261
The relative error at iteration 192 is: 0.464357
The relative error at iteration 193 is: 0.469455
The relative error at iteration 194 is: 0.474555
The relative error at iteration 195 is: 0.479655
The relative error at iteration 196 is: 0.484756
The relative error at iteration 197 is: 0.489855
The relative error at iteration 198 is: 0.494953
The relative error at iteration 199 is: 0.500048
The relative error at iteration 200 is: 0.505141
The relative error at iteration 201 is: 0.510229
The relative error at iteration 202 is: 0.515313
The relative error at iteration 203 is: 0.520392
The relative error at iteration 204 is: 0.525465
The relative error at iteration 205 is: 0.53053
The relative error at iteration 206 is: 0.535588
The relative error at iteration 207 is: 0.540638

The relative error at iteration 208 is: 0.545679
The relative error at iteration 209 is: 0.55071
The relative error at iteration 210 is: 0.555731
The relative error at iteration 211 is: 0.56074
The relative error at iteration 212 is: 0.565738
The relative error at iteration 213 is: 0.570723
The relative error at iteration 214 is: 0.575695
The relative error at iteration 215 is: 0.580652
The relative error at iteration 216 is: 0.585595
The relative error at iteration 217 is: 0.590523
The relative error at iteration 218 is: 0.595434
The relative error at iteration 219 is: 0.600329
The relative error at iteration 220 is: 0.605207
The relative error at iteration 221 is: 0.610066
The relative error at iteration 222 is: 0.614907
The relative error at iteration 223 is: 0.619729
The relative error at iteration 224 is: 0.62453
The relative error at iteration 225 is: 0.629311
The relative error at iteration 226 is: 0.634071
The relative error at iteration 227 is: 0.638809
The relative error at iteration 228 is: 0.643525
The relative error at iteration 229 is: 0.648218
The relative error at iteration 230 is: 0.652887
The relative error at iteration 231 is: 0.657532
The relative error at iteration 232 is: 0.662152
The relative error at iteration 233 is: 0.666747
The relative error at iteration 234 is: 0.671317
The relative error at iteration 235 is: 0.67586
The relative error at iteration 236 is: 0.680376
The relative error at iteration 237 is: 0.684865
The relative error at iteration 238 is: 0.689326
The relative error at iteration 239 is: 0.693759
The relative error at iteration 240 is: 0.698163
The relative error at iteration 241 is: 0.702538
The relative error at iteration 242 is: 0.706883
The relative error at iteration 243 is: 0.711198
The relative error at iteration 244 is: 0.715483
The relative error at iteration 245 is: 0.719736
The relative error at iteration 246 is: 0.723958
The relative error at iteration 247 is: 0.728149
The relative error at iteration 248 is: 0.732307
The relative error at iteration 249 is: 0.736432
The relative error at iteration 250 is: 0.740525
The relative error at iteration 251 is: 0.744584
The relative error at iteration 252 is: 0.74861
The relative error at iteration 253 is: 0.752602

The relative error at iteration 254 is: 0.75656
The relative error at iteration 255 is: 0.760483
The relative error at iteration 256 is: 0.764371
The relative error at iteration 257 is: 0.768225
The relative error at iteration 258 is: 0.772043
The relative error at iteration 259 is: 0.775825
The relative error at iteration 260 is: 0.779571
The relative error at iteration 261 is: 0.783282
The relative error at iteration 262 is: 0.786956
The relative error at iteration 263 is: 0.790594
The relative error at iteration 264 is: 0.794195
The relative error at iteration 265 is: 0.797759
The relative error at iteration 266 is: 0.801287
The relative error at iteration 267 is: 0.804777
The relative error at iteration 268 is: 0.80823
The relative error at iteration 269 is: 0.811645
The relative error at iteration 270 is: 0.815024
The relative error at iteration 271 is: 0.818364
The relative error at iteration 272 is: 0.821667
The relative error at iteration 273 is: 0.824932
The relative error at iteration 274 is: 0.828159
The relative error at iteration 275 is: 0.831348
The relative error at iteration 276 is: 0.834499
The relative error at iteration 277 is: 0.837613
The relative error at iteration 278 is: 0.840688
The relative error at iteration 279 is: 0.843725
The relative error at iteration 280 is: 0.846725
The relative error at iteration 281 is: 0.849686
The relative error at iteration 282 is: 0.85261
The relative error at iteration 283 is: 0.855495
The relative error at iteration 284 is: 0.858343
The relative error at iteration 285 is: 0.861153
The relative error at iteration 286 is: 0.863925
The relative error at iteration 287 is: 0.86666
The relative error at iteration 288 is: 0.869356
The relative error at iteration 289 is: 0.872016
The relative error at iteration 290 is: 0.874638
The relative error at iteration 291 is: 0.877223
The relative error at iteration 292 is: 0.879771
The relative error at iteration 293 is: 0.882281
The relative error at iteration 294 is: 0.884755
The relative error at iteration 295 is: 0.887192
The relative error at iteration 296 is: 0.889593
The relative error at iteration 297 is: 0.891958
The relative error at iteration 298 is: 0.894286
The relative error at iteration 299 is: 0.896578

The relative error at iteration 300 is: 0.898834
The relative error at iteration 301 is: 0.901055
The relative error at iteration 302 is: 0.903241
The relative error at iteration 303 is: 0.905391
The relative error at iteration 304 is: 0.907506
The relative error at iteration 305 is: 0.909587
The relative error at iteration 306 is: 0.911633
The relative error at iteration 307 is: 0.913645
The relative error at iteration 308 is: 0.915624
The relative error at iteration 309 is: 0.917568
The relative error at iteration 310 is: 0.919479
The relative error at iteration 311 is: 0.921357
The relative error at iteration 312 is: 0.923202
The relative error at iteration 313 is: 0.925014
The relative error at iteration 314 is: 0.926794
The relative error at iteration 315 is: 0.928542
The relative error at iteration 316 is: 0.930258
The relative error at iteration 317 is: 0.931943
The relative error at iteration 318 is: 0.933597
The relative error at iteration 319 is: 0.93522
The relative error at iteration 320 is: 0.936812
The relative error at iteration 321 is: 0.938374
The relative error at iteration 322 is: 0.939907
The relative error at iteration 323 is: 0.941409
The relative error at iteration 324 is: 0.942883
The relative error at iteration 325 is: 0.944328
The relative error at iteration 326 is: 0.945744
The relative error at iteration 327 is: 0.947132
The relative error at iteration 328 is: 0.948492
The relative error at iteration 329 is: 0.949824
The relative error at iteration 330 is: 0.95113
The relative error at iteration 331 is: 0.952408
The relative error at iteration 332 is: 0.95366
The relative error at iteration 333 is: 0.954886
The relative error at iteration 334 is: 0.956085
The relative error at iteration 335 is: 0.95726
The relative error at iteration 336 is: 0.958409
The relative error at iteration 337 is: 0.959534
The relative error at iteration 338 is: 0.960634
The relative error at iteration 339 is: 0.96171
The relative error at iteration 340 is: 0.962762
The relative error at iteration 341 is: 0.963791
The relative error at iteration 342 is: 0.964796
The relative error at iteration 343 is: 0.965779
The relative error at iteration 344 is: 0.96674
The relative error at iteration 345 is: 0.967679

The relative error at iteration 346 is: 0.968596
The relative error at iteration 347 is: 0.969492
The relative error at iteration 348 is: 0.970367
The relative error at iteration 349 is: 0.971221
The relative error at iteration 350 is: 0.972055
The relative error at iteration 351 is: 0.972869
The relative error at iteration 352 is: 0.973663
The relative error at iteration 353 is: 0.974439
The relative error at iteration 354 is: 0.975195
The relative error at iteration 355 is: 0.975933
The relative error at iteration 356 is: 0.976652
The relative error at iteration 357 is: 0.977353
The relative error at iteration 358 is: 0.978037
The relative error at iteration 359 is: 0.978704
The relative error at iteration 360 is: 0.979354
The relative error at iteration 361 is: 0.979987
The relative error at iteration 362 is: 0.980604
The relative error at iteration 363 is: 0.981204
The relative error at iteration 364 is: 0.981789
The relative error at iteration 365 is: 0.982359
The relative error at iteration 366 is: 0.982914
The relative error at iteration 367 is: 0.983454
The relative error at iteration 368 is: 0.983979
The relative error at iteration 369 is: 0.98449
The relative error at iteration 370 is: 0.984988
The relative error at iteration 371 is: 0.985472
The relative error at iteration 372 is: 0.985942
The relative error at iteration 373 is: 0.9864
The relative error at iteration 374 is: 0.986844
The relative error at iteration 375 is: 0.987277
The relative error at iteration 376 is: 0.987697
The relative error at iteration 377 is: 0.988105
The relative error at iteration 378 is: 0.988502
The relative error at iteration 379 is: 0.988887
The relative error at iteration 380 is: 0.989261
The relative error at iteration 381 is: 0.989624
The relative error at iteration 382 is: 0.989977
The relative error at iteration 383 is: 0.990319
The relative error at iteration 384 is: 0.990651
The relative error at iteration 385 is: 0.990973
The relative error at iteration 386 is: 0.991286
The relative error at iteration 387 is: 0.991589
The relative error at iteration 388 is: 0.991883
The relative error at iteration 389 is: 0.992168
The relative error at iteration 390 is: 0.992445
The relative error at iteration 391 is: 0.992712

The relative error at iteration 392 is: 0.992972
The relative error at iteration 393 is: 0.993223
The relative error at iteration 394 is: 0.993467
The relative error at iteration 395 is: 0.993703
The relative error at iteration 396 is: 0.993931
The relative error at iteration 397 is: 0.994152
The relative error at iteration 398 is: 0.994366
The relative error at iteration 399 is: 0.994574
The relative error at iteration 400 is: 0.994774
This is the computation with the change in time being 1.0, until time reaches 20
The estimated value at iteration 1 is 1015
The estimated value at iteration 2 is 1030.99
The estimated value at iteration 3 is 1048
The estimated value at iteration 4 is 1066.08
The estimated value at iteration 5 is 1085.26
The estimated value at iteration 6 is 1105.61
The estimated value at iteration 7 is 1127.17
The estimated value at iteration 8 is 1150
The estimated value at iteration 9 is 1174.15
The estimated value at iteration 10 is 1199.69
The estimated value at iteration 11 is 1226.68
The estimated value at iteration 12 is 1255.2
The estimated value at iteration 13 is 1285.32
The estimated value at iteration 14 is 1317.14
The estimated value at iteration 15 is 1350.72
The estimated value at iteration 16 is 1386.18
The estimated value at iteration 17 is 1423.61
The estimated value at iteration 18 is 1463.11
The estimated value at iteration 19 is 1504.81
The estimated value at iteration 20 is 1548.83
The actual value at iteration 1 is 1015.11
The actual value at iteration 2 is 1031.23
The actual value at iteration 3 is 1048.38
The actual value at iteration 4 is 1066.63
The actual value at iteration 5 is 1086
The actual value at iteration 6 is 1106.55
The actual value at iteration 7 is 1128.34
The actual value at iteration 8 is 1151.42
The actual value at iteration 9 is 1175.86
The actual value at iteration 10 is 1201.72
The actual value at iteration 11 is 1229.06
The actual value at iteration 12 is 1257.97
The actual value at iteration 13 is 1288.53
The actual value at iteration 14 is 1320.82
The actual value at iteration 15 is 1354.93
The actual value at iteration 16 is 1390.97

The actual value at iteration 17 is 1429.04
 The actual value at iteration 18 is 1469.25
 The actual value at iteration 19 is 1511.72
 The actual value at iteration 20 is 1556.59
 The relative error at iteration 0 is: 0
 The relative error at iteration 1 is: 0.000111381
 The relative error at iteration 2 is: 0.000234104
 The relative error at iteration 3 is: 0.00036871
 The relative error at iteration 4 is: 0.000515737
 The relative error at iteration 5 is: 0.000675723
 The relative error at iteration 6 is: 0.000849205
 The relative error at iteration 7 is: 0.00103672
 The relative error at iteration 8 is: 0.00123879
 The relative error at iteration 9 is: 0.00145596
 The relative error at iteration 10 is: 0.00168875
 The relative error at iteration 11 is: 0.00193769
 The relative error at iteration 12 is: 0.0022033
 The relative error at iteration 13 is: 0.00248611
 The relative error at iteration 14 is: 0.00278663
 The relative error at iteration 15 is: 0.00310539
 The relative error at iteration 16 is: 0.0034429
 The relative error at iteration 17 is: 0.00379968
 The relative error at iteration 18 is: 0.00417622
 The relative error at iteration 19 is: 0.00457305
 The relative error at iteration 20 is: 0.00499066

Excel Data:

In the case of delta t being 0.05			Ap
Approximate	Actual	Error	
1000.75	1000.750281	2.81109E-07	
1002.251125	1002.252533	1.40498E-06	
1004.50619	1004.51014	3.93244E-06	10
1007.519709	1007.528195	8.42343E-06	11
1011.297908	1011.313519	1.54371E-05	12
1015.848748	1015.874685	2.55316E-05	13
1021.181954	1021.222052	3.92643E-05	14
1027.309046	1027.367803	5.71918E-05	16
1034.243382	1034.325993	7.98695E-05	19
1042.000207	1042.112601	0.000107852	21

1050.596709	1050.745592	0.000141693
1060.052079	1060.244986	0.000181945
1070.387587	1070.632933	0.00022916
1081.626657	1081.933805	0.000283888
1093.794957	1094.174284	0.000346679
1106.920496	1107.383472	0.000418081
1121.033732	1121.593004	0.000498641
1136.167688	1136.837176	0.000588905
1152.358077	1153.153081	0.000689417
1169.643448	1170.580758	0.000800722
1188.065333	1189.163359	0.00092336
1207.668411	1208.947323	0.001057873
1228.500691	1229.982572	0.001204798
1250.613703	1252.322716	0.001364675
1274.06271	1276.025284	0.001538037
1298.906933	1301.151966	0.00172542
1325.209798	1327.76888	0.001927355
1353.039204	1355.94686	0.002144373
1382.467807	1385.761766	0.002377002
1413.573333	1417.294822	0.00262577
1446.438913	1450.632981	0.002891199
1481.153446	1485.869318	0.003173813
1517.811994	1523.103455	0.003474131
1556.5162	1562.442028	0.003792671
1597.37475	1603.999183	0.004129948
1640.503869	1647.897116	0.004486474
1686.027851	1694.266663	0.00486276
1734.079645	1743.247931	0.005259313
1784.801474	1794.990986	0.005676636
1838.345519	1849.6566	0.006115233
1894.874643	1907.417056	0.0065756
1954.563195	1968.457025	0.007058234
2017.597858	2032.974517	0.007563626
2084.178587	2101.181909	0.008092265
2154.519614	2173.307064	0.008644637
2228.850541	2249.59455	0.009221221
2307.417522	2330.306957	0.009822498
2390.484553	2415.726331	0.010448939
2478.334861	2506.155737	0.011101016
2571.272418	2601.920952	0.011779195
2669.623588	2703.372317	0.012483937
2773.738908	2810.886742	0.013215699
2883.995029	2924.869897	0.013974935

25
30
36
43
53
66
83
10
13
17

3000.796828	3045.7586	0.014762093
3124.579697	3174.023418	0.015577617
3255.812045	3310.171498	0.016421945
3394.998009	3454.749669	0.017295511
3542.680423	3608.34782	0.018198744
3699.444032	3771.602585	0.019132067
3865.919013	3945.20138	0.020095899
4042.784808	4129.886811	0.021090651
4230.774301	4326.46149	0.022116732
4430.678387	4535.793315	0.023174541
4643.35095	4758.821245	0.024264474
4869.714308	4996.561621	0.02538692
5110.765167	5250.115099	0.026542262
5367.581116	5520.674252	0.027730877
5641.327753	5809.531902	0.028953133
5933.266465	6118.090267	0.030209395
6244.762954	6447.871008	0.031500018
6577.296581	6800.526258	0.032825353
6932.470597	7177.850751	0.034185742
7312.023362	7581.795159	0.03558152
7717.840658	8014.480757	0.037013015
8151.969195	8478.215573	0.038480548
8616.63144	8975.512179	0.03998443
9114.241905	9509.107276	0.041524968
9647.425057	10081.98331	0.043102457
10219.03499	10697.39228	0.044717187
10832.17709	11358.88208	0.046369439
11490.23185	12070.32547	0.048059485
12196.88111	12835.95221	0.049787588
12956.13696	13660.38448	0.051554004
13772.37358	14548.6761	0.05335898
14650.3624	15506.35593	0.055202753
15595.31078	16539.47586	0.057085551
16612.9048	17654.66404	0.059007593
17709.35652	18859.18381	0.060969091
18891.45607	20160.99909	0.062970243
20166.62935	21568.84689	0.065011243
21543.00181	23092.31781	0.067092269
23029.46893	24741.94546	0.069213495
24635.77439	26529.30577	0.071375082
26372.59648	28467.12749	0.073577181
28251.64398	30569.41502	0.075819934
30285.76235	32851.58522	0.078103472

32489.05156	35330.61965	0.080427915
34876.99685	38025.23431	0.082793374
37466.61387	40956.06882	0.085199948
40276.60991	44145.89746	0.087647727
43327.56311	47619.86471	0.090136787
46642.12169	51405.74826	0.092667196
50245.22559	55534.25291	0.095239011
54164.35318	60039.33916	0.097852276
58429.79599	64958.59065	0.100507024
63074.96478	70333.62558	0.103203279
68136.7307	76210.55727	0.105941052
73655.80589	82640.51032	0.108720341
79677.16802	89680.1992	0.111541135
86250.53438	97392.57737	0.11440341
93430.89136	105847.5657	0.117307132
101279.0862	115122.8708	0.120252253
109862.4888	125304.9041	0.123238714
119255.7316	136489.8155	0.126266446
129541.5384	148784.6562	0.129335364
140811.6523	162308.6874	0.132445376
153167.8748	177194.8545	0.135596374
166723.2317	193591.4479	0.13878824
181603.2801	211663.9764	0.142020842
197947.5753	231597.2797	0.14529404
215911.3178	253597.9148	0.148607677
235667.2034	277896.8495	0.151961587
257407.5029	304752.5086	0.15535559
281346.4006	334454.2169	0.158789495
307722.6257	367326.0966	0.162263099
336802.4138	403731.4786	0.165776186
368882.8438	444077.9011	0.169328528
404295.5968	488822.7748	0.172919885
443411.1957	538479.8085	0.176550005
486643.7873	593626.3019	0.180218623
534456.5394	654911.4266	0.183925463
587367.7368	723065.6354	0.187670236
645957.6686	798911.3592	0.191452642
710876.4143	883375.1775	0.195272369
782852.6512	977501.6697	0.199129091
862703.6216	1082469.193	0.203022472
951346.4188	1199607.861	0.206952164
1049810.773	1330420.052	0.210917806
1159253.546	1476603.8	0.214919028

1280975.169	1640079.508	0.218955445
1416438.293	1823020.464	0.223026663
1567288.971	2027887.723	0.227132275
1735380.713	2257470.009	0.231271864
1922801.83	2514929.373	0.235444999
2131906.529	2803853.5	0.239651241
2365350.294	3128315.626	0.243890139
2626130.164	3492943.257	0.24816123
2917630.612	3902997.006	0.25246404
3243675.833	4364461.094	0.256798087
3608589.364	4884147.327	0.261162876
4017262.109	5469814.595	0.265557902
4475229.99	6130306.33	0.269982649
4988762.631	6875708.694	0.274436592
5564964.715	7717532.75	0.278919197
6211891.863	8668924.382	0.283429917
6938683.211	9744906.332	0.287968198
7755713.159	10962657.46	0.292533477
8674765.169	12341835.13	0.297125178
9709230.915	13904947.62	0.301742719
10874338.63	15677784.67	0.306385509
12187415.01	17689915.32	0.311052948
13668185.94	19975264.26	0.315744425
15339121.67	22572779.22	0.320459323
17225833.63	25527204.36	0.325197018
19357530.55	28889977.08	0.329956874
21767543.1	32720268.61	0.334738252
24493927.87	37086192.15	0.339540501
27580162.78	42066206.44	0.344362967
31075948.42	47750747.49	0.349204985
35038131.84	54244126.59	0.354065886
39531772.25	61666739.72	0.358944993
44631370.87	70157640.77	0.363841623
50422291.24	79877540.73	0.368755087
57002400.25	91012305.35	0.37368469
64483965.28	103777036.8	0.378629731
72995848.7	118420840.1	0.383589505
82686047.61	135232392	0.388563299
93724634.97	154546453.5	0.3935504
106307167.2	176751487.6	0.398550085
120658634.8	202298579.2	0.40356163
137038044.5	231711884.1	0.408584307
155743737.5	265600877.4	0.413617383

177119565.5	304674721.6	0.418660122
201562065.5	349759131.2	0.423711785
229528802.1	401816180.3	0.428771629
261548070	461967582.6	0.43383891
298230186.9	531522070.1	0.438912881
340280643.2	612007612.4	0.443992793
388515424.4	705209358.4	0.449077895
443878872.3	813214344.4	0.454167434
507464520.8	938464211	0.459260657
580539411.8	1083817404	0.46435681
664572491.7	1252622611	0.469455137
761267789.2	1448805526	0.474554883
872603203.4	1676971423	0.479655293
1000875874	1942526494	0.484755612
1148755285	2251821495	0.489855085
1319345444	2612321898	0.494952959
1516257752	3032809574	0.500048481
1743696415	3523622015	0.505140901
2006558649	4096936265	0.510229469
2310552285	4767106142	0.515313438
2662333870	5551063006	0.520392064
3069670952	6468792381	0.525464604
3541632861	7543901149	0.53053032
4088815138	8804292988	0.535588474
4723603688	10282973221	0.540638336
5460485864	12019008546	0.545679176
6316417023	14058672175	0.55071027
7311252704	16456811131	0.555730898
8468258445	19278479887	0.560740344
9814711537	22600893526	0.565737898
11382611705	26515764477	0.570722854
13209520884	31132100025	0.575694512
15339556127	36579553693	0.580652179
17824564219	43012442833	0.585595166
20725512046	50614568103	0.590522792
24114133265	59604998740	0.59543438
28074879654	70245021827	0.600329263
32707234797	82846495367	0.605206778
38128458965	97781895472	0.610066273
44476847382	1.15496E+11	0.614907099
51915600107	1.36523E+11	0.619728618
60637420925	1.61497E+11	0.624530198
70869985706	1.91185E+11	0.629311218

82882448283	2.26499E+11	0.634071062
96993185104	2.68537E+11	0.638809126
1.13579E+11	3.18617E+11	0.643524811
1.33086E+11	3.7832E+11	0.648217531
1.56044E+11	4.49547E+11	0.652886707
1.83078E+11	5.34584E+11	0.657531769
2.14934E+11	6.36185E+11	0.66215216
2.52493E+11	7.57664E+11	0.666747328
2.96806E+11	9.03015E+11	0.671316735
3.49118E+11	1.07706E+12	0.675859852
4.10912E+11	1.28561E+12	0.68037616
4.83952E+11	1.5357E+12	0.68486515
5.70337E+11	1.83581E+12	0.689326324
6.7257E+11	2.19621E+12	0.693759197
7.93632E+11	2.62934E+12	0.698163292
9.37081E+11	3.15026E+12	0.702538145
1.10716E+12	3.7772E+12	0.706883302
1.30894E+12	4.53232E+12	0.711198321
1.54848E+12	5.44248E+12	0.715482773
1.83301E+12	6.54031E+12	0.719736237
2.1712E+12	7.86548E+12	0.723958307
2.57342E+12	9.46626E+12	0.728148588
3.05207E+12	1.14014E+13	0.732306696
3.62205E+12	1.37424E+13	0.736432259
4.30118E+12	1.65765E+13	0.740524919
5.11088E+12	2.001E+13	0.744584327
6.07683E+12	2.41729E+13	0.748610149
7.22991E+12	2.92238E+13	0.752602061
8.60721E+12	3.53566E+13	0.756559754
1.02533E+13	4.28084E+13	0.760482929
1.2222E+13	5.18697E+13	0.7643713
1.45778E+13	6.28961E+13	0.768224594
1.73986E+13	7.63238E+13	0.772042549
2.07782E+13	9.26876E+13	0.775824917
2.483E+13	1.12644E+14	0.779571462
2.96905E+13	1.37E+14	0.78328196
3.55246E+13	1.66748E+14	0.786956199
4.25319E+13	2.03107E+14	0.790593981
5.09532E+13	2.4758E+14	0.794195119
6.10801E+13	3.02017E+14	0.797759438
7.32656E+13	3.687E+14	0.801286777
8.79371E+13	4.50444E+14	0.804776987
1.05612E+14	5.50724E+14	0.80822993

1.2692E+14	6.73834E+14	0.81164548
1.52621E+14	8.25083E+14	0.815023525
1.83641E+14	1.01104E+15	0.818363964
2.21104E+14	1.23984E+15	0.821666707
2.66375E+14	1.52155E+15	0.824931678
3.21115E+14	1.86867E+15	0.828158812
3.87345E+14	2.29671E+15	0.831348054
4.67526E+14	2.82492E+15	0.834499363
5.64654E+14	3.47721E+15	0.837612709
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8.25173E+14	5.28028E+15	0.843725446
9.9846E+14	6.51416E+15	0.846724833
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1.46456E+15	9.93664E+15	0.852609719
1.77542E+15	1.22862E+16	0.855495279
2.15358E+15	1.52028E+16	0.858342976
2.61391E+15	1.88258E+16	0.861152869
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3.85792E+15	2.89329E+16	0.866659523
4.69124E+15	3.59087E+16	0.86935645
5.70806E+15	4.45998E+16	0.872015906
6.94957E+15	5.5436E+16	0.874637998
8.46631E+15	6.89567E+16	0.877222844
1.03204E+16	8.58395E+16	0.879770572
1.25883E+16	1.06936E+17	0.882281316
1.53641E+16	1.33317E+17	0.884755224
1.87634E+16	1.66331E+17	0.887192449
2.29288E+16	2.07676E+17	0.889593155
2.80362E+16	2.59493E+17	0.891957512
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4.19946E+16	4.06051E+17	0.89657791
5.14434E+16	5.08507E+17	0.898834335
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7.73392E+16	7.99294E+17	0.903240656
9.49145E+16	1.00323E+18	0.905390982
1.16555E+17	1.26014E+18	0.907506384
1.43217E+17	1.58403E+18	0.909587094
1.76085E+17	1.99267E+18	0.911633353
2.16629E+17	2.5086E+18	0.913645406
2.6667E+17	3.16048E+18	0.915623507
3.28471E+17	3.98475E+18	0.917567913
4.0484E+17	5.02776E+18	0.919478889
4.99269E+17	6.34853E+18	0.921356706

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9.39879E+17	1.28388E+19	0.926793988
1.16193E+18	1.62603E+19	0.928541979
1.4373E+18	2.06089E+19	0.930258242
1.77902E+18	2.61402E+19	0.931943077
2.20332E+18	3.31809E+19	0.933596788
2.73046E+18	4.21495E+19	0.935219684
3.38577E+18	5.35826E+19	0.936812078
4.20089E+18	6.81679E+19	0.938374287
5.21541E+18	8.67885E+19	0.939906629
6.47884E+18	1.10578E+20	0.941409429
8.0532E+18	1.40995E+20	0.942883011
1.00162E+19	1.79913E+20	0.944327706
1.24651E+19	2.29746E+20	0.945743845
1.55222E+19	2.93602E+20	0.947131761
1.93407E+19	3.75487E+20	0.948491792
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3.75485E+19	7.88966E+20	0.952407963
4.68981E+19	1.01204E+21	0.953659853
5.86109E+19	1.29916E+21	0.954885566
7.32929E+19	1.66899E+21	0.95608545
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1.14818E+20	2.76066E+21	0.958409115
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1.80301E+20	4.58009E+21	0.960633634
2.26143E+20	5.906E+21	0.961709586
2.8381E+20	7.62146E+21	0.962761799
3.56394E+20	9.84258E+21	0.963790622
4.47809E+20	1.27205E+22	0.964796404
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2.25153E+21	7.82351E+22	0.97122097
2.84256E+21	1.01719E+23	0.972054928
3.59086E+21	1.32352E+23	0.972868955
4.53885E+21	1.7234E+23	0.973663391
5.74051E+21	2.24577E+23	0.974438574
7.26461E+21	2.92867E+23	0.97519484

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1.47755E+22	6.52439E+23	0.97735346
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1.92288E+24	1.61655E+26	0.988105011
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2.43621E+26	4.01434E+28	0.993931236
3.16159E+26	5.40661E+28	0.994152368

4.10532E+26	7.28722E+28	0.994366412
5.33384E+26	9.82935E+28	0.994573557
6.93399E+26	1.32682E+29	0.99477399

References

Dawkins, Paul. *Differential Equations - Euler's Method*. 2019. 20 September 2019.