Atlas-PS 6

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Problem 1

a)

To show that the Metropolis-Hastings ratio will always be equal to 1 when $g(\cdot|x^{(t)}) = f(\cdot|x^{(t)})$, we first define the ratio

$$R(x^{(t)}, X^*) = \frac{f(X^*)g(x^{(t)}|X^*)}{f(x^{(t)})g(X^*|x^{(t)})}.$$

Note that $g(\cdot|x^{(t)}) = f(\cdot|x^{(t)}) \forall x^{(t)} \implies f(X^*|x^{(t)}) = g(X^*|x^{(t)})$ and \$ f(X^{(t)} | X^) = g(X^{(t)} / X^)\$. In the Metropolis-Hastings Ratio, f is not conditional on previous values of the simulation, as it is the target distribution that we want to converge on. Therefore, $f(X^*) = g(X^*|x^{(t)})$ and $f(X^{(t)}) = g(X^{(t)}|X^*) \forall x^{(t)}$.

Now, it is trivial to show that

$$R(x^{(t)}, X^*) = \frac{g(x^{(t)}|X^*)f(X^*)}{f(x^{(t)})g(X^*|x^{(t)})} = \frac{f(x^{(t)}|X^*)f(X^*)}{f(x^{(t)})f(X^*|x^{(t)})} = \frac{f(x^{(t)})f(X^*)}{f(x^{(t)})f(X^*)} = 1.$$

This is intuitive too, as if the proposal distribution is equal to the target distribution, any draw from the proposal distribution should be included in the chain, and this will lead to convergence on the target distribution.