625.664 Computational Statistics

Problem Set 2

Associated Reading: Chapter 1: Section 1.1 - 1.4
Chapter 2: Introduction - 2.1

Complete the problems either by hand or using the computer and upload your final document to the Blackboard course site. All final submittals are to be in PDF form. Please document any code used to solve the problems and include it with your submission.

1. The following data are an i.i.d. sample from a Normal$(\theta, 1)$ distribution: 28, 33, 22, 35, 31. We wish to estimate θ by minimizing residuals. We will use the L_2 norm squared as our metric.
   (a) What is the function $s_p(\theta)$ that we wish to minimize?
   (b) Graph $s_p(\theta)$.
   (c) Find the Minimum Residual Estimator for θ using the Bisection Method correct to 2 decimal places.
   (d) If we were to use Newton's Method to solve this optimization problem, what would the refinement increment $h(t)$ be?

2. Maximize the function $f(x) = -\frac{x^4}{4} + \frac{x^2}{2} - x + 2$ by using Newton's Method and the starting values below. For each value state the number of iterations Newton's Method takes converge if we want our solution to be correct to 2 decimal places.
   (a) $x_0 = -1$
   (b) $x_0 = 2$

3. Problem 2.1. For part (e) you only need to discuss your results, you do not need to reapply the methods to a random data set.

4. In each of the following, assume a random sample of size n, $x_1, x_2, \ldots, x_n$ and find the Maximum Likelihood Estimator for:
   (a) λ for the Poisson distribution.
   (b) θ for the Exponential distribution.
   Note: These are examples of distributions for which the MLE can be found analytically in terms of the data $x_1, \ldots, x_n$ and so no advanced computational methods are required.

5. Consider a sequence of n independent Bernoulli trials in which the probability of success is θ and the probability of failure is $1-\theta$. If A represents the observed number of success and B represents the observed number of failures, (with $A+B=n$), then find $I(\theta)$, the Fisher information matrix. (Hint: Recall that the sum of n Bernoulli trials is a Binomial random variable. Also assume that n, A and B are fixed and so the only unknown parameter is θ, in the case $I(\theta)$ will be a scalar.)