# JIOSI-GILLESPIE CTMDP ROLLOUT CONSTRUCTION & JIOSI-GILLESPIE SSA TRAJECTORY GENERATION FOR CONTINUOUS-TIME REINFORCEMENT LEARNING & INFERENCE

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## Abstract

This paper formalizes a continuous-time reinforcement learning and inverse reinforcement learning stack that uses stochastic simulation algorithm dynamics to generate event-timed trajectories. The Jiosi-Gillespie CTMDP Rollout Construction treats Gillespie sampling as an environment stepper mapping $(s,a)$ to $(s^{+},r,\tau)$ with an explicit event label, while Jiosi-Gillespie SSA Trajectory Generation produces full jump sequences suitable for likelihood-based inference. The framework supports event-driven or piecewise-constant control, preserves standard policy evaluation and improvement logic, and admits likelihoods over timestamped demonstrations. A scheduling toy problem is reformulated from discrete to continuous time, and a concrete SSA step is shown numerically. Discounting can be applied through exponential factors or reward folding, making the method compatible with standard RL objectives while retaining exact continuous-time semantics.

## 1. Introduction

Discrete-state systems often evolve with irregular event timing, yet most reinforcement learning (RL) rollouts assume a fixed time step. That assumption blurs the impact of dwell times on rewards and on inverse reinforcement learning (IRL) likelihoods. This paper presents the Jiosi-Gillespie CTMDP Rollout Construction and Jiosi-Gillespie SSA Trajectory Generation, two contracts that embed Gillespie's stochastic simulation algorithm (SSA) into a continuous-time Markov decision process (CTMDP) compatible with modern RL and IRL. The resulting trajectories retain event times, enabling value estimation, discounting, and likelihoods without time discretization. The contributions are: (i) a precise CTMDP stepper built on SSA rates; (ii) RL and IRL interfaces that operate on jump trajectories; and (iii) a continuous-time reformulation of a scheduling toy example inspired by [1].

## 2. Continuous-Time Decision Processes and Trajectories

Consider a CTMDP with state space $\mathscr{S}$, action set $\mathscr{A}$, event types $\Sigma$, and policy $\pi_\theta(a|s)$. For each event type $\sigma \in \Sigma$ and state-action pair $(s,a)$, the instantaneous rate is $\lambda_\sigma(s,a)$. The total rate is

$$\Lambda(s,a) \doteq \sum_{\sigma\in\Sigma} \lambda_\sigma(s,a), \tag{1}$$

and the event probability mass function is $p(\sigma \mid s,a) \doteq \lambda_\sigma(s,a)/\Lambda(s,a)$. Holding times satisfy $\tau_t \sim \mathrm{Exp}(\Lambda(s_t,a_t))$. A trajectory collects jump tuples

$$\tau = \left\{(s_t, a_t, \tau_t, \sigma_t, s_{t+1})\right\}_{t=0}^{T-1}, \tag{2}$$

where $s_{t+1}$ is drawn from the transition kernel conditioned on $(s_t, a_t, \sigma_t)$. A discounted return can be written as

$$F(\tau) = \sum_{t=0}^{T-1} \exp\left(-\beta \sum_{j=0}^{t} \tau_j\right) r(s_t, a_t, \sigma_t), \tag{3}$$

with rate-based discount $\beta \geq 0$. The objective is $\mathbb{E}_{\tau|\pi_\theta}[F(\tau)]$; value functions similarly use state-action marginals such as $\mathbb{E}_{s,a|\pi_\theta}[f(s,a)]$ without time discretization. This notation aligns with classical CTMDP treatments such as [2, 3].

## 3. Jiosi-Gillespie CTMDP Rollout Construction

Each SSA step instantiates an environment transition. Given $(s_t, a_t)$:

$$\Lambda(s_t, a_t) = \sum_{\sigma \in \Sigma} \lambda_\sigma(s_t, a_t), \tag{4}$$

$$\tau_t \sim \mathrm{Exp}(\Lambda(s_t, a_t)), \tag{5}$$

$$\sigma_t \sim p(\sigma \mid s_t, a_t), \tag{6}$$

$$s_{t+1} \sim P(\cdot \mid s_t, a_t, \sigma_t). \tag{7}$$

The stepper contract is

$$(s_{t+1}, r_t, \tau_t, \sigma_t) = \mathrm{Step}(s_t, a_t), \tag{8}$$

where $r_t = r(s_t, a_t, \sigma_t)$ and Step is simulated via SSA sampling. This construction retains $(\tau_t, \sigma_t)$ alongside $(s_t, a_t)$, enabling downstream RL to remain agnostic to the timing model while still obtaining time-aware returns. Piecewise-constant control holds $a_t$ fixed over the sampled holding time; event-driven control updates $a_t$ immediately after each jump.

**Algorithm 1** Jiosi-Gillespie SSA Trajectory Generation

Input: policy $\pi_\theta$, horizon $T$, discount $\beta$, initial state $s_0$
Initialize $G \leftarrow 0$, cumulative time $c \leftarrow 0$
**for** $t = 0$ to $T-1$ **do**
  Sample $a_t \sim \pi_\theta(a|s_t)$
  Compute $\Lambda(s_t, a_t)$ and $p(\sigma \mid s_t, a_t)$
  Draw $u_1, u_2 \sim \mathrm{Unif}(0,1)$
  Set $\tau_t = -\ln(u_1)/\Lambda(s_t, a_t)$ and update $c \leftarrow c + \tau_t$
  Sample $\sigma_t$ using $u_2$ and the cumulative mass of $p(\sigma \mid s_t, a_t)$
  Evaluate $r_t = r(s_t, a_t, \sigma_t)$
  Accumulate $G \leftarrow G + \exp(-\beta c)\, r_t$
  Sample $s_{t+1} \sim P(\cdot \mid s_t, a_t, \sigma_t)$
**end for**
Return trajectory $\{(s_t, a_t, \tau_t, \sigma_t, s_{t+1})\}_{t=0}^{T-1}$ and return $G$

## 4. Jiosi-Gillespie SSA Trajectory Generation

Gillespie sampling [4, 5] draws holding times and event types from two uniform random variables $u_1, u_2 \sim \mathrm{Unif}(0,1)$:

$$\tau_t = -\frac{1}{\Lambda(s_t, a_t)} \ln(u_1), \tag{9}$$

$$\sigma_t = \min\Big\{\sigma \in \Sigma : \sum_{\sigma' \preceq \sigma} p(\sigma' \mid s_t, a_t) \geq u_2\Big\}. \tag{10}$$

Algorithm Algorithm 1 generates a rollout episode. Discounting can be applied multiplicatively through $\exp(-\beta\tau_t)$ or folded into the reward as $\tilde{r}_t = \exp(-\beta\tau_t) r(s_t, a_t, \sigma_t)$; both choices keep policy evaluation unchanged while preserving continuous-time semantics.

## 5. Reinforcement Learning Interface

The CTMDP value of policy $\pi_\theta$ is $J(\theta) = \mathbb{E}_{\tau|\pi_\theta}[F(\tau)]$. Event-driven control selects $a_t$ after each jump; piecewise-constant control commits to $a_t$ over $(t, t+\tau_t)$. Both styles share the same return definition, so policy evaluation and improvement operators remain structurally identical to discrete-time RL [6]. For policy gradients,

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau|\pi_\theta}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t)\hat{A}_t\right], \tag{11}$$

where $\hat{A}_t$ may incorporate holding-time-aware baselines such as $\hat{A}_t = \exp(-\beta\tau_t)\hat{Q}(s_t,a_t) - b(s_t)$. Standard critics and actor updates can therefore consume SSA rollouts without altering optimizer structure.

## 6. Inverse Reinforcement Learning Interface

Expert demonstrations are timestamped jump trajectories $\mathscr{D} = \{\tau^{(n)}\}_{n=1}^{N}$ with $\tau^{(n)} = \{(\tau_t^{(n)}, \sigma_t^{(n)}, s_t^{(n)}, a_t^{(n)})\}$. The joint density for one step under parameters $\theta$ is $\lambda_{\sigma_t}(s_t,a_t)$ $\exp(-\Lambda(s_t,a_t)\tau_t)\pi_\theta(a_t|s_t)$. The trajectory log-likelihood is

$$\begin{aligned} \log p(\tau|\theta) = \sum_{t=0}^{T-1}\Big[&\log\lambda_{\sigma_t}(s_t,a_t) - \Lambda(s_t,a_t)\tau_t \\ &+ \log\pi_\theta(a_t|s_t)\Big], \end{aligned} \tag{12}$$

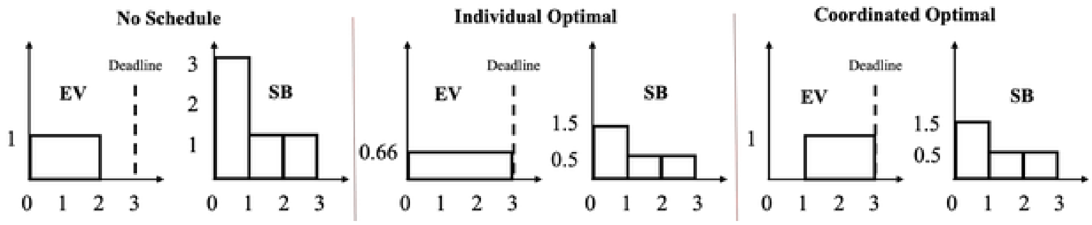
and IRL adapts either reward parameters (affecting $\lambda_\sigma$ or $r$) or policy parameters so that $\mathbb{E}_{\tau|\pi_\theta}[F(\tau)]$ aligns with expert likelihood [7]. Reward recovery constrains $\lambda_\sigma$ through optimality conditions; policy recovery directly maximizes the likelihood above, keeping the SSA dynamics fixed.

## 7. Continuous-Time Reformulation of a Scheduling Toy Example

The discrete scheduling chart in Figure Figure 1 allocates two jobs: an electric vehicle (EV) charge and a standby (SB) task, each with unit work across a three-slot horizon and a deadline at $t = 3$. In continuous time, the state records remaining work $(w^{\mathrm{EV}}, w^{\mathrm{SB}})$, the elapsed time $c$, and the deadline indicator. Event types are $\Sigma = \{\text{serve-EV}, \text{serve-SB}\}$ with rates $\lambda_{\mathrm{EV}}(s,a)$ and $\lambda_{\mathrm{SB}}(s,a)$ chosen by action $a \in \{\text{EV-first}, \text{SB-first}, \text{split}\}$. For action

**Figure 1**

*Discrete scheduling illustration adapted from [1]. SSA rollouts reuse the same prioritization logic but sample holding times and event order continuously.*




EV-first, set $\lambda_{\text{EV}} = 1.5$, $\lambda_{\text{SB}} = 0.5$, giving $\Lambda = 2.0$ and $p(\text{EV} \mid s,a) = 0.75$. The holding time is $\tau_t \sim \text{Exp}(2.0)$.

A worked SSA step: suppose $w^{\text{EV}} = 1$, $w^{\text{SB}} = 1$, deadline at $c = 0$, and $a_t = \text{EV-first}$. Draw $u_1 = 0.25$, $u_2 = 0.60$. Then $\tau_t = -\ln(0.25)/2.0 \approx 0.69$, $\sigma_t = \text{serve-EV}$ because $u_2 < 0.75$, and the state updates to $w^{\text{EV}} \leftarrow 0$. The reward can encode on-time completion, e.g., $r_t = 1$ if $c + \tau_t < 3$ and $w^{\text{EV}}$ hits zero, else $-1$. Subsequent steps continue with updated rates; the SSA keeps exact timing while preserving the discrete action logic from the original schedule.

## 8. Discussion and Limitations

SSA-based rollouts assume memoryless holding times and well-specified rate functions; systems with strong duration dependence may violate these assumptions. Identifiability in IRL is challenging when both rates and rewards are unknown: multiple $(\lambda_\sigma, r)$ pairs can induce the same likelihood over $\{(\tau_k, \sigma_k, s_k)\}$. Regularization, structural constraints on $\lambda_\sigma$, or anchoring to known physics can mitigate ambiguity. Finally, long holding times can create high-variance returns; control variates based on $\mathbb{E}_{s,a|\pi_\theta}[\Lambda(s,a)]$ help stabilize estimates.

## 9. Conclusion

The Jiosi-Gillespie CTMDP Rollout Construction and Jiosi-Gillespie SSA Trajectory Generation supply event-timed trajectories that keep RL and IRL objectives intact while avoiding time discretization. By treating SSA as the environment stepper, policies remain compatible with event-driven and piecewise-constant control, and likelihoods over timestamped demonstrations become straightforward. The continuous-time reformulation of the scheduling toy example illustrates how discrete action logic transfers directly to SSA rollouts, enabling precise reward attribution and inference.

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