

TikZ 几何作图

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第 1 章 基本作图命令

这里的命令都是通过 `/tikz/insert path[1]` 在当前路径上插入新的路径.

1.1 仿射组合 Affine Combination

调用方式

```
affine={A,B,k}
```

参数说明

A, B 两点坐标

k 系数

返回点 A, B 的仿射组合: $A + k \cdot (B - A)$.

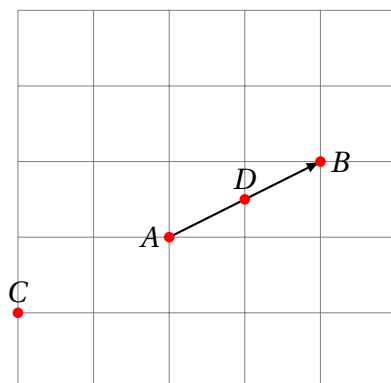
示例

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (5,5);
  \coordinate (A) at (2,2);
  \coordinate (B) at (4,3);
  \coordinate [affine={A,B,-1}] (C);
  \coordinate [affine={A,B,.5}] (D);
  \draw[thick, -latex] (A) -- (B);
  \foreach \p/\placement in {A/left,B/right,
```

```

C/above,D/above}{
  \fill[red] (\p) circle (2pt);
  \draw (\p) node[\placement] {$\p$};
}
\end{tikzpicture}

```



1.2 中点 Midpoint

调用方式

```
midpoint={A,B}
```

参数说明

A, B 两点坐标

返回点 A, B 的中点坐标.

示例

```

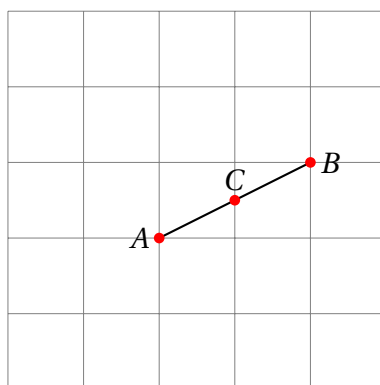
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (5,5);
  \coordinate (A) at (2,2);

```

```

\coordinate (B) at (4,3);
\coordinate [midpoint={A,B}] (C);
\draw[thick] (A) -- (B);
\foreach \p/\placement in {A/left,B/right,
C/above}{
  \fill[red] (\p) circle (2pt);
  \draw (\p) node[\placement] {$\p$};
}
\end{tikzpicture}

```



1.3 平移 Translate

调用方式

```
translate={A,B,C}
```

参数说明

A,B,C 三点坐标

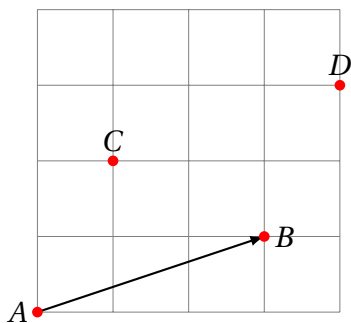
返回 C 按向量 AB 移动所得的坐标: $C + (B - A)$.

示例

```

\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,0);
  \coordinate (B) at (3,1);
  \coordinate (C) at (1,2);
  \coordinate [translate={A,B,C}] (D);
  \draw[thick, -latex] (A) -- (B);
  \foreach \p/\placement in
    {A/left,B/right,C/above,D/above}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}

```



1.4 对称点 Reflect

调用方式

```
reflect={A,B,C}
```

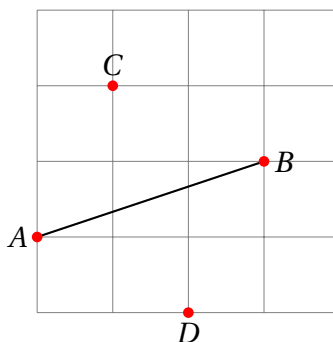
参数说明

A,B,C 三点坐标

返回 C 关于直线 AB 的对称点的坐标 (设 D 为 C 在 AB 的投影): $C + 2(D - C)$.

示例

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,1);
  \coordinate (B) at (3,2);
  \coordinate (C) at (1,3);
  \coordinate [reflect={A,B,C}] (D);
  \draw[thick] (A) -- (B);
  \foreach \p/\placement in
    {A/left,B/right,C/above,D/below}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.5 投影 Project

调用方式

```
project={A,B,C}
```

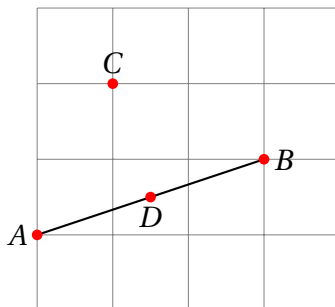
参数说明

A,B,C 三点坐标

返回 C 在直线 AB 的投影的坐标.

示例

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,1);
  \coordinate (B) at (3,2);
  \coordinate (C) at (1,3);
  \coordinate [project={A,B,C}] (D);
  \draw[thick] (A) -- (B);
  \foreach \p/\placement in
    {A/left,B/right,C/above,D/below}{
      \fill[red] (\p) circle (2pt);
      \draw (\p) node[\placement] {$\p$};
    }
\end{tikzpicture}
```



1.6 反演 Inverse

调用方式

```
inverse={O,A,P}
```

参数说明

O 圆心

A 圆上一点

P 平面上任一点

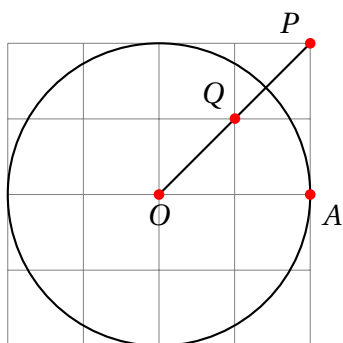
返回 P 关于圆 (O, A) 的反演点.

示例

```
\begin{tikzpicture}
  \draw[help lines] (-2,-2) grid (2,2);
  \coordinate (O) at (0,0);
  \coordinate (A) at +(0:2); % 圆上一点, 相对坐标
  \coordinate (P) at (2,2);
  \coordinate[inverse={O,A,P}] (Q);

  \draw[thick,circle={O,A}];
  \draw[thick] (O) -- (P);

  \foreach \p/\placement in {O/below,A/below right,
    P/above left,Q/above left}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.7 旋转 Revolve

调用方式

```
revolve={A,B}
```

参数说明

A,B 两点坐标

注 为了避免覆盖 tikz 的 rotate, 这里将旋转命令为 revolve.

返回 B 绕 A 旋转的点.

还需要指定 revolve/angle (default: 0) 和 revolve/angle scale(default: 1) 两个选项, 可以通过下面的方式来指定 /revolve/angle:

1. 直接指定角度: revolve/angle=60
2. 位置向量与 x 轴夹角: revolve/angle={P1}
3. 两位置向量的夹角: revolve/angle={P1,P2}
4. 由三点定义的角 (P_1 为顶点, P_2 为起点, P_3 为终点): revolve/angle={P1,P2,P3}
5. 两向量的夹角 (逆时针方向): revolve/angle={P1,P2,P3,P4}

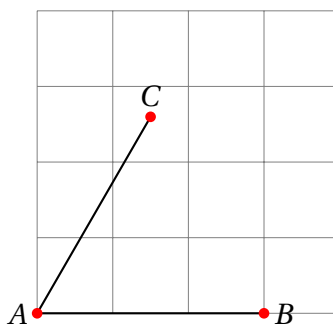
示例

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
```

```

\coordinate (A) at (0,0);
\coordinate (B) at (3,0);
\coordinate [revolve/angle=60, revolve={A,B}] (C);
\draw[thick] (A) -- (B) (A) -- (C);
\foreach \p/\placement in {A/left,B/right,C/above}{
  \fill[red] (\p) circle (2pt);
  \draw (\p) node[\placement] {$\p$};
}
\end{tikzpicture}

```



```

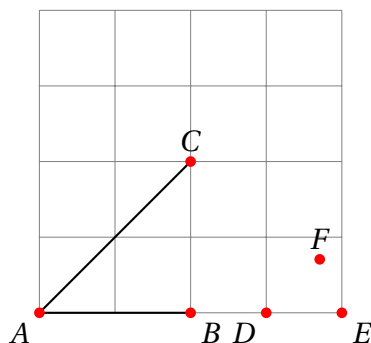
\begin{tikzpicture}
\draw[help lines] (0,0) grid (4,4);
\coordinate (A) at (0,0);
\coordinate (B) at (2,0);
\coordinate (C) at (2,2);
\coordinate (D) at (3,0);
\coordinate (E) at (4,0);
\coordinate [revolve/angle={A,B,C},revolve={D,E}] (F);
\draw[thick] (A) -- (B) (A) -- (C);
\foreach \p/\placement in {
  A/below left,B/below right,C/above,
  D/below left,E/below right,F/above}{

```

```

\fill[red] (\p) circle (2pt);
\draw (\p) node[\placement] {$\p$};
}
\end{tikzpicture}

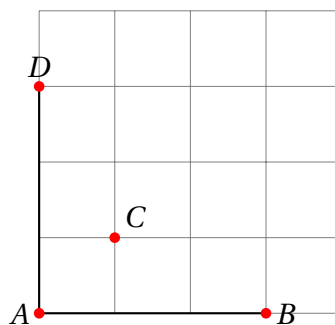
```



```

\begin{tikzpicture}
\draw[help lines] (0,0) grid (4,4);
\coordinate (A) at (0,0);
\coordinate (B) at (3,0);
\coordinate (C) at (1,1);
\coordinate [revolve/angle={C},
revolve/scale=2,
revolve={A,B}] (D);
\draw[thick] (A) -- (B) (A) -- (D);
\foreach \p/\placement in {A/left,B/right,
C/above right,D/above}{
\fill[red] (\p) circle (2pt);
\draw (\p) node[\placement] {$\p$};
}
\end{tikzpicture}

```

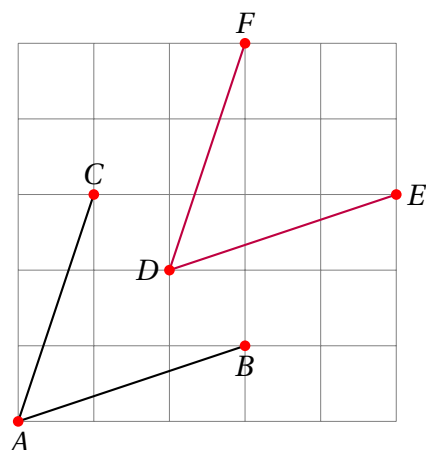


1.8 构造角 Angle

可以由 `resovle` 来构造一个角.

示例

```
\begin{tikzpicture}[scale=1]
  \draw[help lines] (0,0) grid (5,5);
  \coordinate (A) at (0,0);
  \coordinate (B) at (3,1);
  \coordinate (C) at (1,3);
  \coordinate (D) at (2,2);
  \coordinate (E) at (5,3);
  \coordinate [revolve/angle={A,B,C},
    revolve/scale=1,
    revolve={D,E}] (F);
  \draw[thick] (A) -- (B) (A) -- (C);
  \draw[thick, purple] (D) -- (E) (D) -- (F);
  \foreach \p/\placement in {A/below,B/below,C/above,
    D/left,E/right,F/above}{
    \fill[red] (\p) circle (2pt);
    \node[\placement] at (\p) {$\p$};
  }
\end{tikzpicture}
```



1.9 角平分线 Angle Bisector

调用方式

```
angle bisector={A,B,C}
```

参数说明

A,B,C 三点坐标, *A* 为顶点 (apex), *B* 为起点, *C* 为终点
 返回 $\angle BAC$ 角平分线上的一点. 实际上, 该操作等价于:

```
revolve/angle={A,B,C}, revolve/scale=.5, revolve={A,B}
```

示例

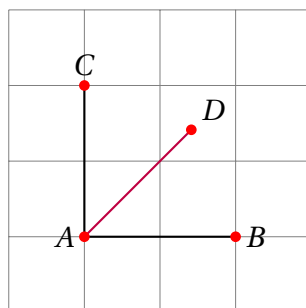
```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (1,1);
  \coordinate (B) at (3,1);
  \coordinate (C) at (1,3);
```



```

\coordinate [angle bisector={A,B,C}] (D);
\draw[thick] (A) -- (B) (A) -- (C);
\draw[thick, purple] (A) -- (D);
\foreach \p/\placement in {A/left,B/right,C/above,
D/above right}{
  \fill[red] (\p) circle (2pt);
  \draw (\p) node[\placement] {$\p$};
}
\end{tikzpicture}

```



1.10 等边三角形 Equilateral Triangle

调用方式

```
equilateral={A,B}
```

参数说明

A,B 两点坐标

返回以 AB 为边长的等边三角形的第 3 点 (位于向量 AB 的左侧). 实际上, 该操作等价于:

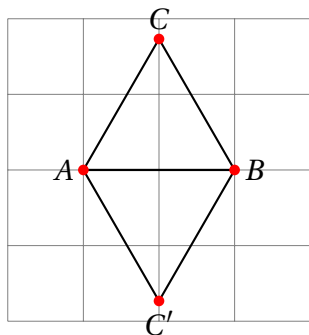
```
revolve/angle=60, revolve={A,B}
```

示例

```

\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (1,2);
  \coordinate (B) at (3,2);
  \coordinate [equilateral={A,B}] (C);
  \coordinate [equilateral={B,A}] (C');
  \draw[thick] (A) -- (B) -- (C) -- cycle
               (A) -- (B) -- (C') -- cycle;
  \foreach \p/\placement in
    {A/left,B/right,C/above,C'/below}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}

```



1.11 旋转 90° Erect

调用方式

```
erect={A,B}
```

参数说明

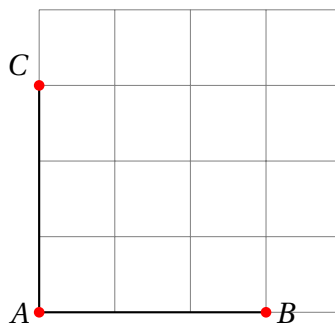
A,B 两点坐标

返回 B 绕 A 旋转 90° 的坐标. 实际上, 该操作等价于:

```
revolve/angle=90, revolve={A,B}
```

示例

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,0);
  \coordinate (B) at (3,0);
  \coordinate [erect={A,B}] (C);
  \draw[thick] (A) -- (B) (A) -- (C);
  \foreach \p/\placement in {A/left,B/right,C/above}
  {
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.12 截取 Intercept

调用方式

```
intercept={A,B}
```

参数说明

A,B 两点坐标

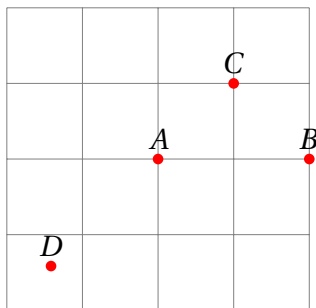
在直线 AB 截取指定长度线段, A 为新线段的起点, AB 是方向.

需要指定 `intercept/length` (default: 1cm) 和 `intercept/scale` (default: 1) 两个选项. 其中 `intercept/length` 有两种形式:

1. 直接指定长度: `intercept/length=2cm`
2. 指定线段长度: `intercept/length={P1,P2}`

示例

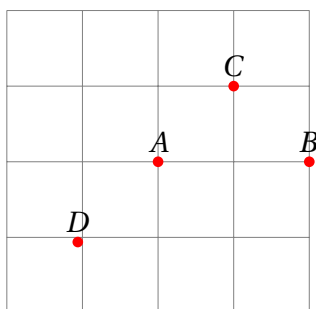
```
\begin{tikzpicture}
  \draw[help lines] (-2,-2) grid (2,2);
  \coordinate (A) at (0,0);
  \coordinate (B) at (2,0);
  \coordinate (C) at (1,1);
  \coordinate[intercept/length={A,B},
    intercept/scale=-1, intercept={A,C}] (D);
  \foreach \p/\placement in
    {A/above,B/above,C/above,D/above}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



```

\begin{tikzpicture}
  \draw[help lines] (-2,-2) grid (2,2);
  \coordinate (A) at (0,0);
  \coordinate (B) at (2,0);
  \coordinate (C) at (1,1);
  \coordinate[intercept/length=1.5cm,
    intercept/scale=-1,
    intercept={A,C}] (D);
  \foreach \p/\placement in
    {A/above,B/above,C/above,D/above}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}

```



1.13 直线与直线的交点 Line-Line Intersection

调用方式

```
intersect={A,B,C,D}
```

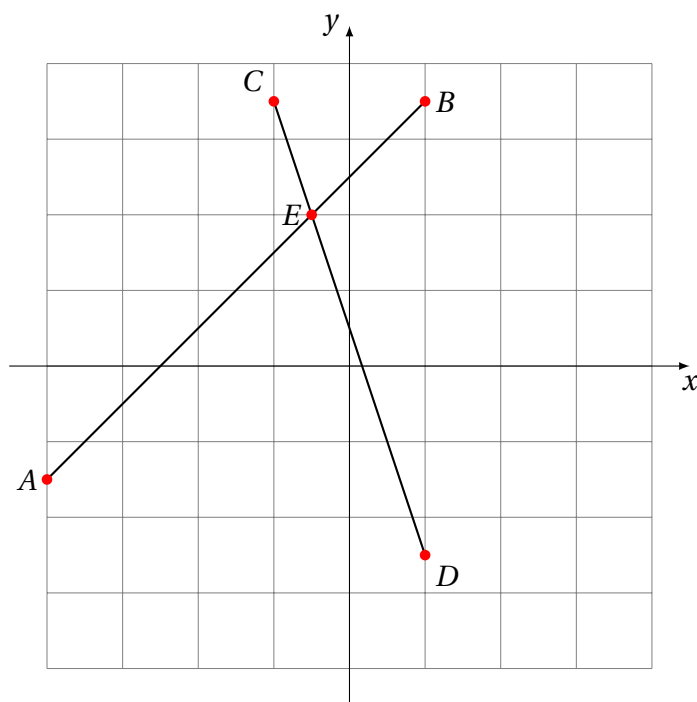
参数说明

A,B,C,D 四点坐标

返回 AB 与 CD 的交点 (可以是延长线相交点).

示例

```
\begin{tikzpicture}
  \draw[help lines] (-4,-4) grid[step=1] (4,4);
  \draw[-latex] (-4.5,0) -- (4.5,0) node[below] {$x$};
  \draw[-latex] (0,-4.5) -- (0,4.5) node[left] {$y$};
  \coordinate (A) at (-4,-1.5);
  \coordinate (B) at (1,3.5);
  \coordinate (C) at (-1,3.5);
  \coordinate (D) at (1,-2.5);
  \coordinate [intersect={A,B,C,D}] (E);
  \draw[thick] (A) -- (B) (C) -- (D);
  \foreach \p/\placement in {A/left,B/right,
    C/above left,D/below right,E/left}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.14 垂直平分线/中垂线 Perpendicular Bisector

调用方式

```
perpendicular bisector={A,B}
```

参数说明

A,B 两点坐标

构造 AB 的中垂线, 默认起点为 $.5(A+B) + (B-A) \cdot \mathbf{i}$, 终点为 $.5(A+B) - (B-A) \cdot \mathbf{i}$.
可以对起始点进行调整, 见1.23.

示例

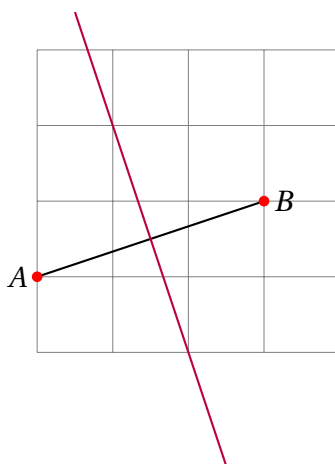
使用默认参数:

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
```

```

\coordinate (A) at (0,1);
\coordinate (B) at (3,2);
\draw[thick] (A) -- (B);
\draw[thick,purple,perpendicular bisector={A,B}];
\foreach \p/\placement in {A/left,B/right}{
  \fill[red] (\p) circle (2pt);
  \draw (\p) node[\placement] {$\p$};
}
\end{tikzpicture}

```



指定两端的长度:

```

\begin{tikzpicture}
\draw[help lines] (0,0) grid (4,4);
\coordinate (A) at (0,1);
\coordinate (B) at (3,2);
\draw[thick] (A) -- (B);
\draw[thick, purple,
start modifier=.5cm, end modifier=2.5cm,
perpendicular bisector={A,B}];

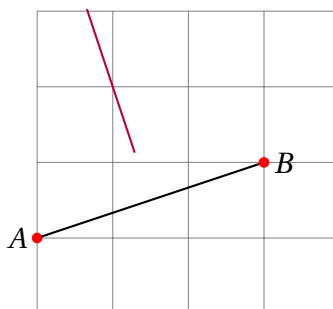
```



```

\foreach \p/\placement in {A/left,B/right}{
  \fill[red] (\p) circle (2pt);
  \draw (\p) node[\placement] {$\p$};
}
\end{tikzpicture}

```

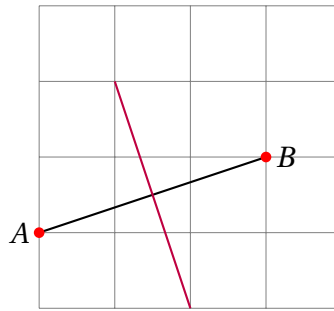


指定系数:

```

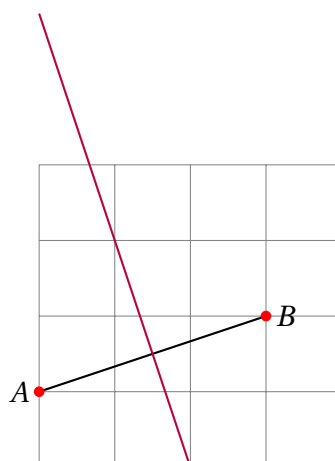
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,1);
  \coordinate (B) at (3,2);
  \draw[thick] (A) -- (B);
  \draw[thick,purple,
    start modifier=.25,end modifier=.75,
    perpendicular bisector={A,B}];
  \foreach \p/\placement in {A/left,B/right}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}

```



可以是负数, 这样就在相反方向:

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,1);
  \coordinate (B) at (3,2);
  \draw[thick] (A) -- (B);
  \draw[thick,purple,
    start modifier=-.25,end modifier=0.75,
    perpendicular bisector={A,B}];
  \foreach \p/\placement in {A/left,B/right}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.15 垂线 Perpendicular Line

调用方式

```
perpendicular={A,B,C}
```

参数说明

A,B,C 三点坐标

构造过 C 垂直于 AB 的直线 (设垂足为 D), 默认起点为 $D + (B - A) \cdot \mathbf{i}$, 终点为 $D - (B - A) \cdot \mathbf{i}$. 可以对起始点进行调整, 见1.23.

示例

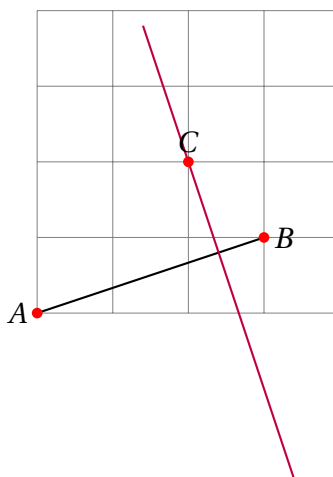
过直线外一点的垂线:

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,0);
  \coordinate (B) at (3,1);
  \coordinate (C) at (2,2);
  \draw[thick] (A) -- (B);
```

```

\path[draw, thick, purple, perpendicular={A,B,C}];
\foreach \p/\placement in {A/left,B/right,C/above}{
  \fill[red] (\p) circle (2pt);
  \draw (\p) node[\placement] {$\p$};
}
\end{tikzpicture}

```



过直线上一点的垂线:

```

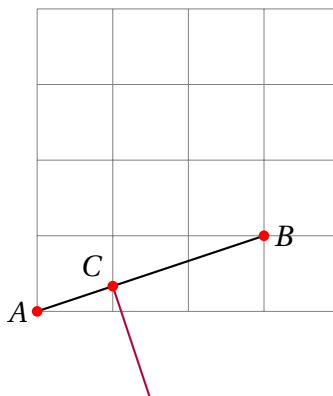
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,0);
  \coordinate (B) at (3,1);
  \coordinate (C) at ($(A)!1/3!(B)$);
  \draw[thick] (A) -- (B);
  \path[draw, thick, purple,
    start modifier=.5, end modifier=.75,
    perpendicular={A,B,C}];
  \foreach \p/\placement in {A/left,B/right,C/above
    ~ left}{

```

```

\fill[red] (\p) circle (2pt);
\draw (\p) node[\placement] {$\p$};
}
\end{tikzpicture}

```



1.16 平行线 Parallel Line

调用方式

```
parallel={A,B,C}
```

参数说明

过一点 C 作直线 AB 平行线, (如果 C 在 AB 上, 则重合).

首先将点 C 按向量 AB 平移至 D . 可以对起始点进行调整, 见1.23.

示例

指定起始点距离 C 的位置, 方向是 CD , 负值代表相反方向:

```

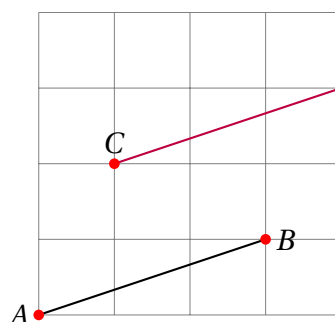
\begin{tikzpicture}
\draw[help lines] (0,0) grid (4,4);
\coordinate (A) at (0,0);

```

```

\coordinate (B) at (3,1);
\coordinate (C) at (1,2);
\draw[thick] (A) -- (B);
\path[draw, thick, purple, parallel={A,B,C}];
\foreach \p/\placement in {A/left,B/right,C/above}{
  \fill[red] (\p) circle (2pt);
  \draw (\p) node[\placement] {$\p$};
}
\end{tikzpicture}

```



指定系数:

```

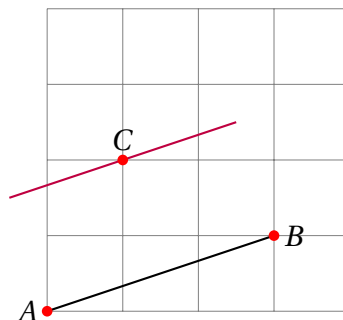
\begin{tikzpicture}
\draw[help lines] (0,0) grid (4,4);
\coordinate (A) at (0,0);
\coordinate (B) at (3,1);
\coordinate (C) at (1,2);
\draw[thick] (A) -- (B);
\path[draw, thick, purple,
start modifier=-.5, end modifier=.5,
parallel={A,B,C}];
\foreach \p/\placement in {A/left,B/right,C/above}{
  \fill[red] (\p) circle (2pt);

```

```

\draw (\p) node[\placement] {$\p$};
}
\end{tikzpicture}

```



1.17 延长线 *Extend*

调用方式

```
extend={A,B}
```

参数说明

作线段 AB 延长线, 可以对起始点进行调整, 见1.23. 实际上, 该操作等价于:

```
parallel={A,B,A}
```

示例

```

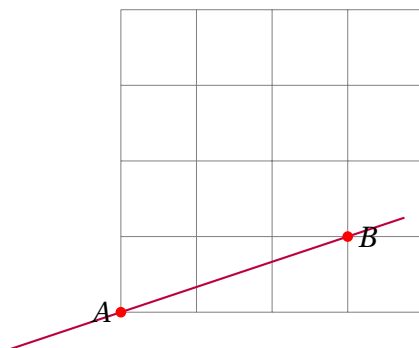
\begin{tikzpicture}
\draw[help lines] (0,0) grid (4,4);
\coordinate (A) at (0,0);
\coordinate (B) at (3,1);

```

```

\path[draw, thick, purple,
      start modifier=-.5, end modifier=1.25,
      extend={A,B}];
\foreach \p/\placement in {A/left,B/right}{
  \fill[red] (\p) circle (2pt);
  \draw (\p) node[\placement] {$\p$};
}
\end{tikzpicture}

```



1.18 圆 Circle

调用方式

```
circle={O,A}
```

参数说明

O 圆心

A 圆上一点

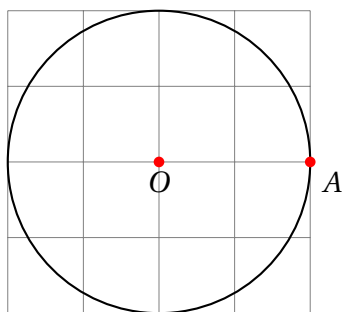
构造圆心为 O , 经过 A 的圆.

示例


```

\begin{tikzpicture}
  \draw[help lines] (-2,-2) grid (2,2);
  \coordinate (O) at (0,0);
  \coordinate (A) at +(0:2); % 圆上一点, 相对坐标
  \draw[thick,circle={O,A}];
  \foreach \p in {O,A}
    \fill[red] (\p) circle (2pt);
  \draw (O) node[below] {$O$};
  \draw (A) node[below right] {$A$};
\end{tikzpicture}

```



1.19 直线与圆的切点 Tangent Point

调用方式

```
tangent point={O,A,P}
```

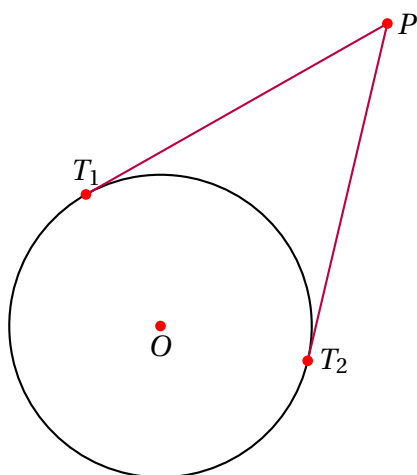
参数说明

- O** : 圆心坐标
- A** : 为圆上任意一点
- P** : 圆外一点坐标

过圆 (O 为圆心, A 为圆上任意一点) 外一点 P 作切线, 求得一个切点 (在向量 OP 的左边), 另外一点可以通过对称 (`reflect={O,P,T1}`) 求得.

示例

```
\begin{tikzpicture}
  \coordinate (O) at (0,0);
  \coordinate (A) at +(0:2); % 圆上一点, 相对坐标
  \coordinate (P) at (3,4);
  \coordinate[tangent point={O,A,P}] (T1);
  \coordinate[reflect={O,P,T1}] (T2);
  \draw[thick, circle={O,A}];
  \draw[thick, purple] (P) -- (T1) (P) -- (T2);
  \foreach \p in {O,P,T1,T2}
    \fill[red] (\p) circle (2pt);
  \draw (O) node[below] {$O$};
  \draw (P) node[right] {$P$};
  \draw (T1) node[above] {$T_1$};
  \draw (T2) node[right] {$T_2$};
\end{tikzpicture}
```



1.20 外位似中心 External Homothetic Center

调用方式

```
external center={O1,A1,O2,A2}
```

参数说明

求圆 1 (O_1 为圆心, A_1 为圆上任意一点) 和圆 2 (O_2 为圆心, A_2 为圆上任意一点) 的外位似中心 (external homothetic center)[2].

示例

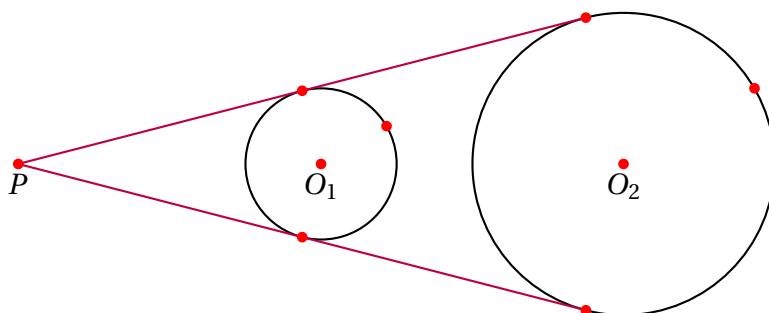
作外公切线: 先求位似中心, 可以求得两圆的外公切线.

```
\begin{tikzpicture}
  \tikzmath {
    \a = 30;
    \b = \a;
    \r1 = 1;
    \r2 = 2;
  }
  \coordinate (O1) at (0,0);
  \coordinate (A1) at ($(O1)+(\a:\r1)$);
  \coordinate (O2) at (4,0);
  \coordinate (A2) at ($(O2)+(\b:\r2)$);
  \coordinate[external center={O1,A1,O2,A2}] (P);
  \coordinate[tangent point={O1,A1,P}] (B);
  \coordinate[tangent point={O2,A2,P}] (C);
  \coordinate[reflect={O1,O2,B}] (D);
  \coordinate[reflect={O1,O2,C}] (E);
  \draw[thick,circle={O1,A1}];
  \draw[thick,circle={O2,A2}];
  \draw[thick,purple] (P) -- (C) (P) -- (E);
```

```

\foreach \p in {A1,A2,B,C,D,E,O1,O2,P}
  \fill[red] (\p) circle (2pt);
\draw (O1) node[below] {$O_1$};
\draw (O2) node[below] {$O_2$};
\draw (P) node[below] {$P$};
\end{tikzpicture}

```



1.21 内位似中心 Internal Homothetic Center

调用方式

```
internal center={O1,A1,O2,A2}
```

参数说明

求圆 1 (O_1 为圆心, A_1 为圆上任意一点) 和圆 2 (O_2 为圆心, A_2 为圆上任意一点) 的内位似中心 (internal homothetic center)[2].

示例

作内公切线: 先求位似中心, 可以求得两圆的内公切线.

```

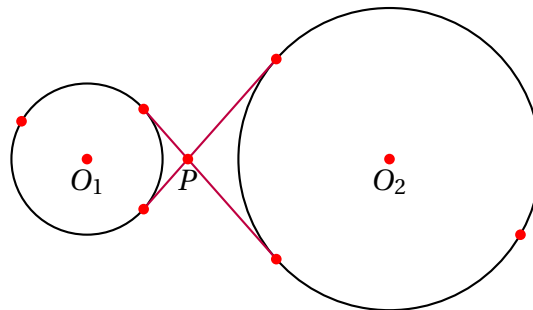
\begin{tikzpicture}
  \tikzmath {
    \a = 150;

```

```

    \b = \a - 180;
    \r1 = 1;
    \r2 = 2;
}
\coordinate (O1) at (0,0);
\coordinate (A1) at ($(O1)+(\a:\r1)$);
\coordinate (O2) at (4,0);
\coordinate (A2) at ($(O2)+(\b:\r2)$);
\coordinate[internal center={O1,A1,O2,A2}] (P);
\coordinate[tangent point={O1,A1,P}] (B);
\coordinate[tangent point={O2,A2,P}] (C);
\coordinate[reflect={O1,O2,B}] (D);
\coordinate[reflect={O1,O2,C}] (E);
\draw[thick, circle={O1,A1}];
\draw[thick, circle={O2,A2}];
\draw[thick, purple] (P) -- (B) (P) -- (C) (P) -- (D)
  _ (P) -- (E);
\foreach \p in {A1,A2,B,C,D,E,O1,O2,P}
  \fill[red] (\p) circle (2pt);
\draw (O1) node[below] {$O_1$};
\draw (O2) node[below] {$O_2$};
\draw (P) node[below] {$P$};
\end{tikzpicture}

```



1.22 根轴 Radical Axis

调用方式

```
radical axis={O1,A1,O2,A2}
```

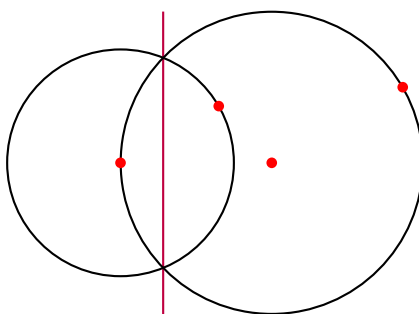
参数说明

构造两圆的根轴, 设与 O_1O_2 的交点为 P , 则默认起点为 $P + (O_2 - O_1) \cdot \mathbf{i}$, 终点为 $P - (O_2 - O_1) \cdot \mathbf{i}$. 可以对起始点进行调整, 见1.23.

示例

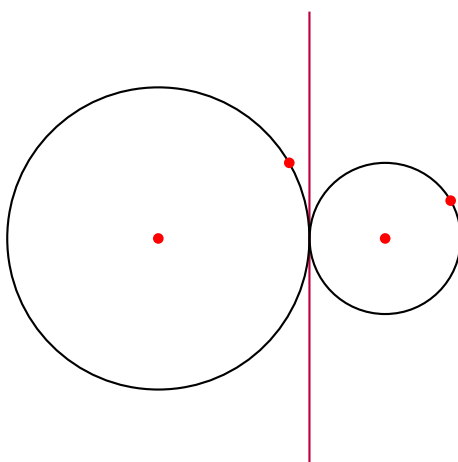
两圆相交:

```
\begin{tikzpicture}
  \tikzmath {
    \a = 30;
    \b = \a;
    \r1 = 1.5;
    \r2 = 2;
  }
  \coordinate (O1) at (0,0);
  \coordinate (A1) at ($(O1)+(\a:\r1)$);
  \coordinate (O2) at (2,0);
  \coordinate (A2) at ($(O2)+(\b:\r2)$);
  \draw[thick,purple,radical axis={O1,A1,O2,A2}];
  \draw[thick,circle={O1,A1}];
  \draw[thick,circle={O2,A2}];
  \foreach \p in {A1,A2,O1,O2}
    \fill[red] (\p) circle (2pt);
\end{tikzpicture}
```



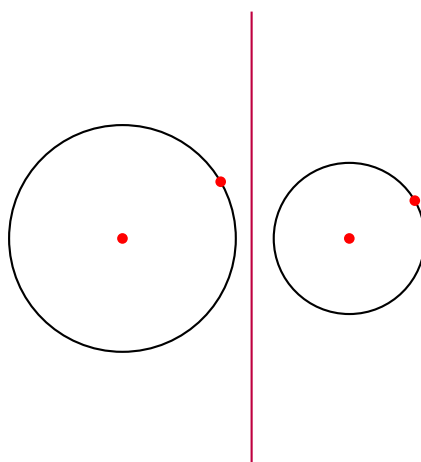
两圆外切:

```
\begin{tikzpicture}
  \tikzmath {
    \a = 30;
    \b = \a;
    \r1 = 2;
    \r2 = 1;
  }
  \coordinate (O1) at (0,0);
  \coordinate (A1) at ($(O1)+(\a:\r1)$);
  \coordinate (O2) at (3,0);
  \coordinate (A2) at ($(O2)+(\b:\r2)$);
  \draw[thick,purple,radical axis={O1,A1,O2,A2}];
  \draw[thick,circle={O1,A1}];
  \draw[thick,circle={O2,A2}];
  \foreach \p in {A1,A2,O1,O2}
    \fill[red] (\p) circle (2pt);
\end{tikzpicture}
```



两圆外离:

```
\begin{tikzpicture}
  \tikzmath {
    \a = 30;
    \b = \a;
    \r1 = 1.5;
    \r2 = 1;
  }
  \coordinate (O1) at (0,0);
  \coordinate (A1) at ($(O1)+(\a:\r1)$);
  \coordinate (O2) at (3,0);
  \coordinate (A2) at ($(O2)+(\b:\r2)$);
  \coordinate[radical axis={O1,A1,O2,A2}] (P);
  \draw[thick,purple,radical axis={O1,A1,O2,A2}];
  \draw[thick,circle={O1,A1}];
  \draw[thick,circle={O2,A2}];
  \foreach \p in {A1,A2,O1,O2}
    \fill[red] (\p) circle (2pt);
\end{tikzpicture}
```

1.23 Partway Modifiers and Distance Modifiers

perpendicular bisector, perpendicular, parallel, radical axis 等线段图形可以对起始点进行调整, 调整参数如下 [3]:

start modifier (default: 0), 长度或系数, 如: 1cm 或.75

end modifier (default: 1), 长度或系数, 如: 1cm 或.75

第 2 章 三角形的中心

2.1 重心 Centroid

调用方式

```
centroid={A,B,C}
```

参数说明

A, B, C 三角形的顶点

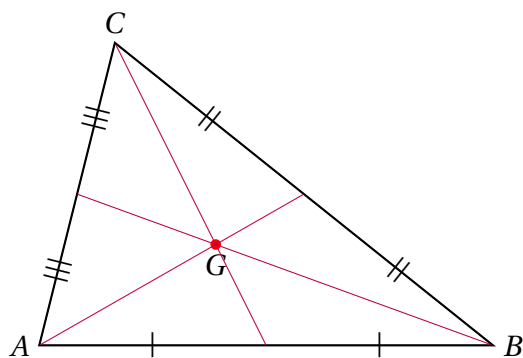
示例

```
\begin{tikzpicture}
  \coordinate (A) at (-2,0);
  \coordinate (B) at (4,0);
  \coordinate (C) at (-1,4);
  \coordinate (D) at ($(B)!0.5!(C)$);
  \coordinate (E) at ($(C)!0.5!(A)$);
  \coordinate (F) at ($(A)!0.5!(B)$);
  \path[centroid={A,B,C}] coordinate (G);
  \fill (G) [red] circle (2pt);
  \draw (G) node[below] {$G$};
  \draw[thick] (A) -- (B) -- (C) -- cycle;
  \draw[purple] (A) -- (D) (B) -- (E) (C) -- (F);
\end{tikzpicture}
```

```

\draw (A) node[left] {$A$};
\draw (B) node[right] {$B$};
\draw (C) node[above] {$C$};
\draw (A) -- (B) node[near start,sloped] {$|$}$
  \node[near end,sloped] {$|$}$;
\draw (B) -- (C) node[near start,sloped] {$||$}$
  \node[near end,sloped] {$||$}$;
\draw (C) -- (A) node[near start,sloped] {$|||$}$
  \node[near end,sloped] {$|||$}$;
\end{tikzpicture}

```



2.2 垂心 Orthocenter

调用方式

```
orthocenter={A,B,C}
```

参数说明

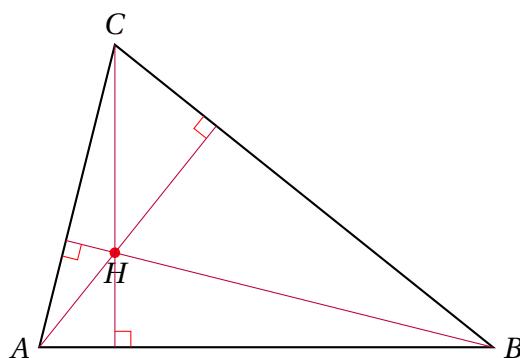
A, B, C 三角形的顶点

示例

```

\begin{tikzpicture}
  \coordinate (A) at (-2,0);
  \coordinate (B) at (4,0);
  \coordinate (C) at (-1,4);
  \path[orthocenter={A,B,C}] coordinate (H);
  \fill (H) [red] circle (2pt);
  \draw (H) node[below] {$H$};
  \draw[thick] (A) -- (B) -- (C) -- cycle;
  \coordinate (D) at ($(B)!(A)!(C)$);
  \coordinate (E) at ($(A)!(B)!(C)$);
  \coordinate (F) at ($(B)!(C)!(A)$);
  \draw[purple] (A) -- (D) (B) -- (E) (C) -- (F);
  \draw (A) node[left] {$A$};
  \draw (B) node[right] {$B$};
  \draw (C) node[above] {$C$};
  \pic [draw,red,angle radius=6pt] {right angle=H--D--C};
  \pic [draw,red,angle radius=6pt] {right angle=H--E--A};
  \pic [draw,red,angle radius=6pt] {right angle=H--F--B};
\end{tikzpicture}

```



2.3 外心 Circumcenter

调用方式

```
circumcenter={A,B,C}
```

参数说明

A, B, C 三角形的顶点

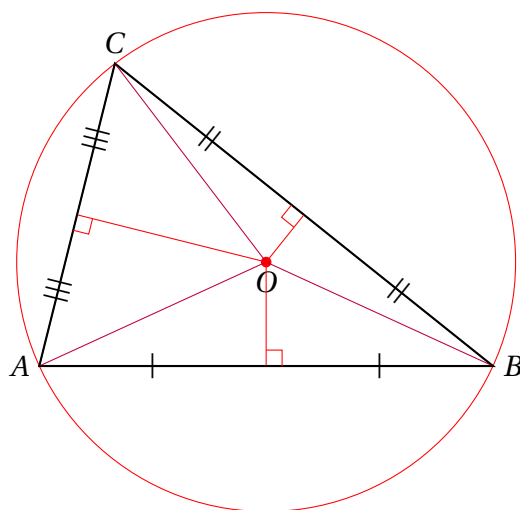
示例

```
\begin{tikzpicture}
  \coordinate (A) at (-2,0);
  \coordinate (B) at (4,0);
  \coordinate (C) at (-1,4);
  \path[circumcenter={A,B,C}] coordinate (O);
  \fill (O) [red] circle (2pt);
  \draw (O) node[below] {O$};
  \node[draw,red] at (O) [circle through=(A)]{};
  \draw[thick] (A) -- (B) -- (C) -- cycle;
  \draw[purple] (A) -- (O) (B) -- (O) (C) -- (O);
  \draw (A) node[left] {A$};
  \draw (B) node[right] {B$};
  \draw (C) node[above] {C$};
  \coordinate (D) at ({(B)!(O)!(C)});
  \coordinate (E) at ({(C)!(O)!(A)});
  \coordinate (F) at ({(A)!(O)!(B)});
  \draw[red] (O) -- (D) (O) -- (E) (O) -- (F);
  \draw (A) -- (B) node[near start,sloped] {|$|}
    \node[near end,sloped] {|$|};
```

```

\draw (B) -- (C) node[near start,sloped] {$||$}
      node[near end,sloped] {$||$};
\draw (C) -- (A) node[near start,sloped] {$|||}$
      node[near end,sloped] {$|||}$;
\pic [draw,red,angle radius=6pt] {right angle=O--D--C};
\pic [draw,red,angle radius=6pt] {right angle=O--E--A};
\pic [draw,red,angle radius=6pt] {right angle=O--F--B};
\end{tikzpicture}

```



2.4 内心 Incenter

调用方式

```
incenter={A,B,C}
```

参数说明

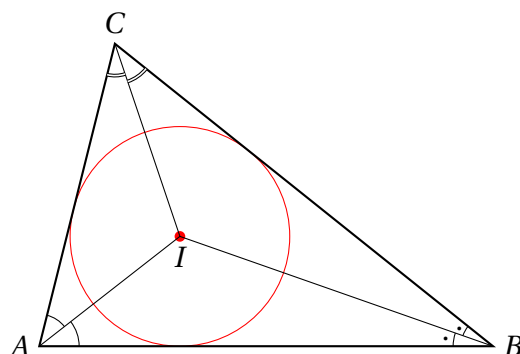
A, B, C 三角形的顶点

示例

```

\begin{tikzpicture}
  \coordinate (A) at (-2,0);
  \coordinate (B) at (4,0);
  \coordinate (C) at (-1,4);
  \path[incenter={A,B,C}] coordinate (I);
  \fill (I) [red] circle (2pt);
  \draw (I) node[below]  $\{I\}$ ;
  \node[draw,red] at (I) [circle
    through=($B!(I)!(C)$)] {};
  \draw[thick] (A) -- (B) -- (C) -- cycle;
  \draw (A) node[left]  $\{A\}$ ;
  \draw (B) node[right]  $\{B\}$ ;
  \draw (C) node[above]  $\{C\}$ ;
  \draw (A) -- (I) (B) -- (I) (C) -- (I);
  \pic [draw,angle radius=12pt] {angle=I--A--C};
  \pic [draw,angle radius=15pt] {angle=B--A--I};
  \pic [draw,double,angle radius=12pt] {angle=A--C--I};
  \pic [draw,double,angle radius=15pt] {angle=I--C--B};
  \pic [draw,pic text=.,angle radius=12pt,
    angle eccentricity=1.2] {angle=C--B--I};
  \pic [draw,pic text=.,angle radius=15pt,
    angle eccentricity=1.2] {angle=I--B--A};
\end{tikzpicture}

```

2.5 旁心 Excenter

调用方式

```
excenter={A,B,C}
```

参数说明

A, B, C 三角形的顶点, 返回与 A 相对的旁心, 调换顶点顺序就可以得到 3 个旁心.

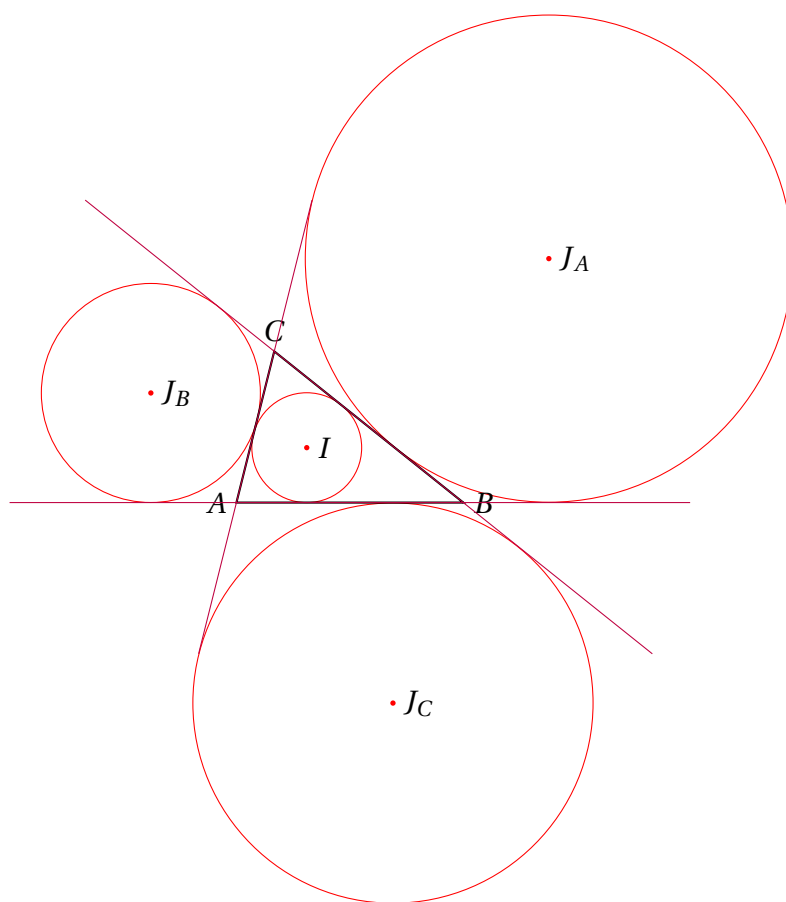
示例

```
\begin{tikzpicture}[scale=.5]
  \coordinate (A) at (-2,0);
  \coordinate (B) at (4,0);
  \coordinate (C) at (-1,4);
  \path[incenter={A,B,C}] coordinate (I);
  \path[excenter={A,B,C}] coordinate (JA);
  \path[excenter={B,A,C}] coordinate (JB);
  \path[excenter={C,A,B}] coordinate (JC);
  \foreach \point in {I,JA,JB,JC}
    \fill (\point) [red] circle (2pt);
\end{tikzpicture}
```

```

\draw[thick] (A) -- (B) -- (C) -- cycle;
\draw (A) node[left] {$A$};
\draw (B) node[right] {$B$};
\draw (C) node[above] {$C$};
\draw[purple] ($A$!-1!(B)$) -- ($A$!2!(B)$);
\draw[purple] ($B$!-1!(C)$) -- ($B$!2!(C)$);
\draw[purple] ($C$!-1!(A)$) -- ($C$!2!(A)$);
\draw (I) node[right] {$I$};
\draw (JA) node[right] {$J\_A$};
\draw (JB) node[right] {$J\_B$};
\draw (JC) node[right] {$J\_C$};
\end{tikzpicture}

```



2.6 九点圆圆心 Nine-Point Center

调用方式

```
nine-point center={A,B,C}
```

参数说明

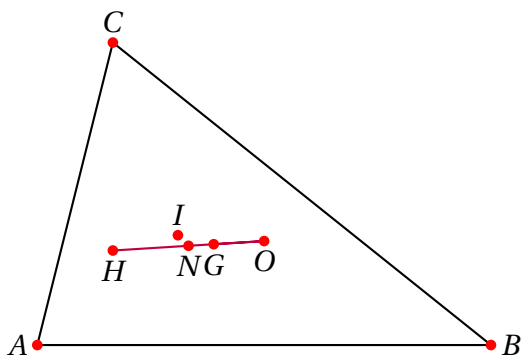
A, B, C 三角形的顶点

示例

```

\begin{tikzpicture}
  \coordinate (A) at (-2,0);
  \coordinate (B) at (4,0);
  \coordinate (C) at (-1,4);
  \path[orthocenter={A,B,C}] coordinate (H);
  \path[circumcenter={A,B,C}] coordinate (O);
  \path[centroid={A,B,C}] coordinate (G);
  \path[incenter={A,B,C}] coordinate (I);
  \path[nine-point center={A,B,C}] coordinate (N);
  \draw[thick] (A) -- (B) -- (C) -- cycle;
  \draw[thick,purple] (H) -- (O) -- (G);
  \foreach \p/\placement in {A/left,B/right,C/above,
    H/below,O/below,G/below,I/above,N/below}{
    \fill (\p) [red] circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}

```



附录 A 两直线的交点

求解两直线交点的方程 [4]:

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{vmatrix} = 0$$

注意, 两个方程的系数都是行列式, 解得:

$$x = \frac{\begin{vmatrix} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} & \begin{vmatrix} x_1 & 1 \\ x_2 & 1 \end{vmatrix} \\ \begin{vmatrix} x_3 & y_3 \\ x_4 & y_4 \end{vmatrix} & \begin{vmatrix} x_3 & 1 \\ x_4 & 1 \end{vmatrix} \end{vmatrix}}{\begin{vmatrix} \begin{vmatrix} x_1 & 1 \\ x_2 & 1 \end{vmatrix} & \begin{vmatrix} y_1 & 1 \\ y_2 & 1 \end{vmatrix} \\ \begin{vmatrix} x_3 & 1 \\ x_4 & 1 \end{vmatrix} & \begin{vmatrix} y_3 & 1 \\ y_4 & 1 \end{vmatrix} \end{vmatrix}} = \frac{\begin{vmatrix} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} & x_1 - x_2 \\ \begin{vmatrix} x_3 & y_3 \\ x_4 & y_4 \end{vmatrix} & x_3 - x_4 \end{vmatrix}}{\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_3 - x_4 & y_3 - y_4 \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \\ x_4 & 1 \end{vmatrix}} = \frac{\begin{vmatrix} x_1 & y_1 & y_1 - y_2 \\ x_2 & y_2 & y_1 - y_2 \\ x_3 & y_3 & y_3 - y_4 \\ x_4 & y_4 & y_3 - y_4 \end{vmatrix}}{\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_3 - x_4 & y_3 - y_4 \end{vmatrix}}$$

进一步化简得到¹:

$$x = \frac{(x_1 y_2 - y_1 x_2)(x_3 - x_4) - (x_1 - x_2)(x_3 y_4 - y_3 x_4)}{(x_1 - x_2)(y_3 - y_4) - (y_1 - y_2)(x_3 - x_4)}$$

$$y = \frac{(x_1 y_2 - y_1 x_2)(y_3 - y_4) - (y_1 - y_2)(x_3 y_4 - y_3 x_4)}{(x_1 - x_2)(y_3 - y_4) - (y_1 - y_2)(x_3 - x_4)}$$

上述方法给出的交点坐标公式在 TikZ 环境中的计算稳定性不够好, 经常出现 Dimension too large 错误, 究其原因是分母可能有时会比较小. 下面给出一个计算更稳定的公式.

我们可以给出两条直线的参数方程:

直线 L_1 的方程:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + s \begin{bmatrix} x_2 - x_1 \\ y_2 - y_1 \end{bmatrix}$$

直线 L_2 的方程:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_3 \\ y_3 \end{bmatrix} + t \begin{bmatrix} x_4 - x_3 \\ y_4 - y_3 \end{bmatrix}$$

¹https://en.wikipedia.org/wiki/Line_line_intersection

为了保证数值计算的稳定性, 可以对下面的方程进行列主元消元法求解:

$$x_1 + s(x_2 - x_1) = x_3 + t(x_4 - x_3)$$

$$y_1 + s(y_2 - y_1) = y_3 + t(y_4 - y_3)$$

附录 B 源代码

```
\ProvidesFile{tikzlibraryeuclidea.code.tex}[2023/12/21
  ~ v1.2.1 A tikz library for plane geometry]

\usetikzlibrary{math,calc}

% https://tex.stackexchange.com/questions/455991/
~ pgfmath-function-for-strings-and-numbers
% Solving the error:
% Package PGF Math: Could not parse input 'A' as a
~ floating
% point number, sorry. The unreadable part was near
'A'..
\pgfkeys{
  /pgf/fpu/handlers/invalid number/.code = {%
    \pgfmathfloatparsenumber{3Y0.0e0}}%
  }
}

\makeatletter

% 注意：计算过程是保留坐标单位（pt）的，所以存在乘除法单位
~ 的问题，首先数值始终携带单位，
```

```

% 在 calc 运算时有的需要转换为标量; 将坐标转换为 pt 值, 数
% 值可能超出限值, 出现
% Dimension too large 错误, 在计算长度时及时进行缩小
% https://tex.stackexchange.com/questions/475556/tikz-
% why-is-dimension-too-large
% 具体方法是修改默认的 1cm, 如:
% [scale=1.0,x=0.5cm,y=0.5cm]
% 注意此处的变量不要和 tikzpicture 环境重名, 否则被替换掉
% triangle centers:
% https://mathworld.wolfram.com/BarycentricCoordinates.
% html
\tikzmath{
  % 采用列主元消元法求直线 P1P2 与直线 P3P4 的交点 P 位置
  % 参数 s:  $s = P1P/P1P2$ 
  function intersectll(\x1,\y1,\x2,\y2,\x3,\y3,\x4,\y4)
  {
    \a1 = \x2-\x1; \b1 = \x3-\x4; \c1 = \x3-\x1;
    \a2 = \y2-\y1; \b2 = \y3-\y4; \c2 = \y3-\y1;
    \dmax = max(max(abs(\a1),abs(\a2)),
      max(abs(\b1),abs(\b2)));
    \a1 = \a1/\dmax; \b1 = \b1/\dmax; \c1=\c1/\dmax;
    \a2 = \a2/\dmax; \b2 = \b2/\dmax; \c2=\c2/\dmax;
    if abs(\a1) < abs(\a2) then {
      \temp = \a1; \a1 = \a2; \a2 = \temp;
      \temp = \b1; \b1 = \b2; \b2 = \temp;
      \temp = \c1; \c1 = \c2; \c2 = \temp;
    };
    \b1 = \b1/\a1; \c1 = \c1/\a1; \a1 = 1.0;
    \b2 = \b2-\a2*\b1; \c2 = \c2-\a2*\c1; \a2 = 0.0;
    \n2 = \c2/\b2; \n1 = \c1-\b1*\n2;
  }
}

```

```

    return \n1;
};
}

\tikzset{
  % specifying start and end with modifiers(see tikz
  ↳ manual 13.5)
  % commands supporting partway modifiers:
  % radical axis, perpendicular bisector, perpendicular,
  ↳ parallel
  start modifier/.initial = 0,
  start modifier/.default = 0,
  end modifier/.initial = 1,
  end modifier/.default = 1,
  % ===== Coordinates Transformations =====
  % affine={A,B,k}: returns affine combination of two
  ↳ points
  % with affine ratio, i.e.  $A + k * (B - A)$ 
  affine/.style args = {#1,#2,#3}{
    insert path = {
      ($(#1)!{#3}!(#2)$)
    }
  },
  % midpoint={A,B}: returns midpoint of AB.
  midpoint/.style args = {#1,#2}{
    insert path = {
      ($(#1)!.5!(#2)$)
    }
  },
  % translate={A,B,C}: returns translation of C by

```

```

% the vector AB, i.e. C + ( B - A )
translate/.style args = {#1,#2,#3}{
  insert path = {
    ($(#3)+(#2)-(#1)$)
  }
},
% reflect={A,B,C}: reflects point C across line AB.
reflect/.style args = {#1,#2,#3}{
  insert path = {
    let
      \p{ft} = ($(#1)!(#3)!(#2)$),% perpendicular foot
      in ($(#3)!2!(\p{ft})$)
    }
  },
% project={A,B,C}: projects point C onto line AB.
project/.style args = {#1,#2,#3}{
  insert path = {
    ($(#1)!(#3)!(#2)$)
  }
},
% inverse={O,A,P}: returns inverse point P with respect
  to
% a reference circle(O,A).
inverse/.style args = {#1,#2,#3}{
  insert path = {
    let
      \p{OA} = ($(#2)-(#1)$),
      \p{OP} = ($(#3)-(#1)$),
      \n{r} = {veclen(\p{OA})},
      \n{d} = {veclen(\p{OP})},

```

```

        \n1 = {scalar((\n{r}/\n{d}))},
in ($(#1)!\n1*\n1!(#3)$)
}
},
revolve/scale/.initial = 1,% angle scale
revolve/@angle/.initial = 90,
revolve/@argn/.initial = 1,% arguments count
% set revolve/@angle with certain degrees or angle of a
↳ vector
revolve/@set angle 1/.code args = {#1}{
    \pgfmathanglebetweenpoints
        {\pgfpoint{0cm}{0cm}}
        {\pgfpointanchor{#1}{center}}
    \pgfkeysalso{/tikz/revolve/@angle = \pgfmathresult}
    \typeout{=====}
    \typeout{/tikz/revolve/@angle:\pgfkeysvalueof{/tikz/}
↳ revolve/@angle}}
    \typeout{=====}
},
% set revolve/@angle with angle between two position
↳ vectors
revolve/@set angle 2/.code args = {#1,#2}{
    \pgfmathanglebetweenpoints
        {\pgfpointanchor{#1}{center}}
        {\pgfpointanchor{#2}{center}}
    \pgfkeysalso{/tikz/revolve/@angle = \pgfmathresult}
    \typeout{=====}
    \typeout{/tikz/revolve/@angle:\pgfkeysvalueof{/tikz/}
↳ revolve/@angle}}
    \typeout{=====}

```

```

},
% set revolve/@angle with angle {A,B,C}, angle between
  ↳ two sides
% (A is apex, B is the start point, C is the end point)
  ↳
revolve/@set angle 3/.code args = {#1,#2,#3}{
  \pgfmanglebetweenlines
    {\pgfpointanchor{#1}{center}}
    {\pgfpointanchor{#2}{center}}
    {\pgfpointanchor{#1}{center}}
    {\pgfpointanchor{#3}{center}}
  \pgfkeysalso{/tikz/revolve/@angle = \pgfmathresult}
  \typeout{=====}
  \typeout{/tikz/revolve/@angle:\pgfkeysvalueof{/tikz/}
    ↳ revolve/@angle}}
  \typeout{=====}
},
% set revolve/@angle with angle between two
  ↳ vectors(ccw, AB and CD)
revolve/@set angle 4/.code args = {#1,#2,#3,#4}{
  \pgfmanglebetweenlines
    {\pgfpointanchor{#1}{center}}
    {\pgfpointanchor{#2}{center}}
    {\pgfpointanchor{#3}{center}}
    {\pgfpointanchor{#4}{center}}
  \pgfkeysalso{/tikz/revolve/@angle = \pgfmathresult}
  \typeout{=====}
  \typeout{/tikz/revolve/@angle:\pgfkeysvalueof{/tikz/}
    ↳ revolve/@angle}}
  \typeout{=====}

```

```

},
revolve/angle/.code = {%
  \pgfmathfloatparsenumber{#1}
  \pgfmathfloattomacro{\pgfmathresult}{\F}{\M}{\E}
  \ifnum \F < 3%number
    \pgfmathparse{#1}
  \else
    \euclidea@ParseArguments#1\euclidea@stop
    \euclidea@ComputeAngle#1\euclidea@stop
  \fi
  \pgfkeysalso{/tikz/revolve/@angle = \pgfmathresult}
},
% revolve={A,B}: rotates point B by the angle around
%   point A.
revolve/.style args = {#1,#2}{
  insert path = {
    let
      \n1 = {\pgfkeysvalueof{/tikz/revolve/@angle}},
      \n2 = {\pgfkeysvalueof{/tikz/revolve/scale}}
    in ($(#1)!1!\n1*\n2:(#2)$)
  }
},
% angle bisector={A,B,C}: alias for [revolve/angle={
%   A,B,C},revolve/scale=.5,revolve={A,B}]
angle bisector/.style args = {#1,#2,#3}{
  revolve/angle={#1,#2,#3},revolve/scale=.5,revolve={#
    1,#2}
},
% erect={A,B}: alias for
%   [revolve/angle=90,revolve={A,B}]

```

```

erect/.style args = {#1,#2}{
  revolve/angle=90,revolve={#1,#2}
},
% equilateral={A,B}: alias for
  ↳ [revolve/angle=60,revolve={A,B}]
equilateral/.style args = {#1,#2}{
  revolve/angle=60,revolve={#1,#2}
},
% cut a line segment of a certain length on a straight
  ↳ line
intercept/@length/.initial = 1cm,
intercept/scale/.initial = 1,% length scale
intercept/length/.code = {% set length by distance of
  ↳ segment
  \pgfutil@in@{,}{#1}
  \ifpgfutil@in@%compute segment length
    \euclide@ComputeLength#1\euclide@stop
    \pgfkeysalso{/tikz/intercept/@length =
      ↳ \pgfmathresult}
  \else
    \pgfkeysalso{/tikz/intercept/@length = #1}
  \fi
  \typeout{=====}
  \typeout{/tikz/intercept/@length:\pgfkeysvalueof{/tikz/intercept/@length}}
  ↳ tikz/intercept/@length}}
  \typeout{=====}
},
% intercept={A,B}: intercepts a line segment(starting
% from point A) of a certain length on line AB.
intercept/.style args = {#1,#2}{

```



```

\p1 = (#1), \p2 = (#2), \p3 = (#3),
\p{AB} = ($(#2)-(#1)$),
\p{BC} = ($(#3)-(#2)$),
\p{CA} = ($(#1)-(#3)$),
\n{a} = {vecLen(\x{BC}, \y{BC})},
\n{b} = {vecLen(\x{CA}, \y{CA})},
\n{c} = {vecLen(\x{AB}, \y{AB})},
\n{s} = {\n{a}+\n{b}+\n{c}},
\n1 = {\n{a}/\n{s}},
\n2 = {\n{b}/\n{s}},
\n3 = {\n{c}/\n{s}},
in ({\n1*\x1+\n2*\x2+\n3*\x3,\n1*\y1+\n2*\y2+\n3*\y3})
}
},
% excenter = {A,B,C}, returns excenter opposite to the
% vertex A
excenter/.style args = {#1,#2,#3}{
  insert path = {
    let
      \p1 = (#1), \p2 = (#2), \p3 = (#3),
      \p{AB} = ($(#2)-(#1)$),
      \p{BC} = ($(#3)-(#2)$),
      \p{CA} = ($(#1)-(#3)$),
      \n{a} = {vecLen(\x{BC}, \y{BC})},
      \n{b} = {vecLen(\x{CA}, \y{CA})},
      \n{c} = {vecLen(\x{AB}, \y{AB})},
      \n{s} = {-\n{a}+\n{b}+\n{c}},
      \n1 = {\n{a}/\n{s}},
      \n2 = {\n{b}/\n{s}},

```

```

\nc3 = {\nc}/{\ns}},
in ({-\n1*\x1+\n2*\x2+\n3*\x3,-\n1*\y1+\n2*\y2+\n3*\y3})
}
},
% circumcenter = {A,B,C}
circumcenter/.style args = {#1,#2,#3}{
insert path = {
let
\p1 = (#1), \p2 = (#2), \p3 = (#3),
\p{AB} = ($(#2)-(#1)$),
\p{BC} = ($(#3)-(#2)$),
\p{CA} = ($(#1)-(#3)$),
\n{a} = {vecclen(\x{BC}, \y{BC})},
\n{b} = {vecclen(\x{CA}, \y{CA})},
\n{c} = {vecclen(\x{AB}, \y{AB})},
\n{m} = {max(max(\n{a}, \n{b}), \n{c})},
\n{a} = {\n{a}/\n{m}},
\n{a} = {\n{a}*\n{a}},
\n{b} = {\n{b}/\n{m}},
\n{b} = {\n{b}*\n{b}},
\n{c} = {\n{c}/\n{m}},
\n{c} = {\n{c}*\n{c}},
\n1 = {\n{a}*(\n{b}+\n{c}-\n{a})},
\n2 = {\n{b}*(\n{c}+\n{a}-\n{b})},
\n3 = {\n{c}*(\n{a}+\n{b}-\n{c})},
\n{s} = {\n1+\n2+\n3},
\n1 = {\n1/\n{s}},
\n2 = {\n2/\n{s}},
\n3 = {\n3/\n{s}},

```

```

in ({\n1*\x1+\n2*\x2+\n3*\x3,\n1*\y1+\n2*\y2+\n3*\y3})
}
},
% orthocenter = {A,B,C}
orthocenter/.style args = {#1,#2,#3}{
insert path = {
let
\p1 = (#1), \p2 = (#2), \p3 = (#3),
\p{AB} = ($(#2)-(#1)$),
\p{BC} = ($(#3)-(#2)$),
\p{CA} = ($(#1)-(#3)$),
\n{a} = {vecclen(\x{BC}, \y{BC})},
\n{b} = {vecclen(\x{CA}, \y{CA})},
\n{c} = {vecclen(\x{AB}, \y{AB})},
\n{m} = {max(max(\n{a}, \n{b}), \n{c})},
\n{a} = {\n{a}/\n{m}},
\n{a} = {\n{a}*\n{a}},
\n{b} = {\n{b}/\n{m}},
\n{b} = {\n{b}*\n{b}},
\n{c} = {\n{c}/\n{m}},
\n{c} = {\n{c}*\n{c}},
\n{a2} = {\n{b}+\n{c}-\n{a}},
\n{b2} = {\n{c}+\n{a}-\n{b}},
\n{c2} = {\n{a}+\n{b}-\n{c}},
\n1 = {\n{c2}*\n{b2}},
\n2 = {\n{a2}*\n{c2}},
\n3 = {\n{b2}*\n{a2}},
\n{s} = {\n1+\n2+\n3},
\n1 = {\n1/\n{s}},

```

```

\p2 = {\n2/\n{s}},
\p3 = {\n3/\n{s}},
in ({\n1*\x1+\n2*\x2+\n3*\x3,\n1*\y1+\n2*\y2+\n3*\y3})
}
},
% centroid = {A,B,C}
centroid/.style args = {#1,#2,#3}{
insert path = {
let
\p1 = (#1), \p2 = (#2), \p3 = (#3),
in ({(\x1+\x2+\x3)/3},{(\y1+\y2+\y3)/3})
}
},
% nine-point center = {A,B,C}
nine-point center/.style args = {#1,#2,#3}{
insert path = {
let
\p1 = (#1), \p2 = (#2), \p3 = (#3),
\p{AB} = ($(#2)-(#1)$),
\p{BC} = ($(#3)-(#2)$),
\p{CA} = ($(#1)-(#3)$),
\n{a} = {vecLen(\x{BC}, \y{BC})},
\n{b} = {vecLen(\x{CA}, \y{CA})},
\n{c} = {vecLen(\x{AB}, \y{AB})},
\n{m} = {max(max(\n{a},\n{b}),\n{c})},
\n{a} = {\n{a}/\n{m}},
\n{a} = {\n{a}*\n{a}},
\n{b} = {\n{b}/\n{m}},
\n{b} = {\n{b}*\n{b}},

```

```

\nc = {\nc}/\nm},
\nc = {\nc}*\nc},
\nc1 = {\nc}*(\nb+\nc)-(\nb-\nc)*(\nb-
    \nc)},
\nc2 = {\nb}*(\nc+\na)-(\nc-\na)*(\nc-
    \na)},
\nc3 = {\nc}*(\na+\nb)-(\na-\nb)*(\na-
    \nb)},
\ncs = {\nc1+\nc2+\nc3},
\nc1 = {\nc1}/\ncs},
\nc2 = {\nc2}/\ncs},
\nc3 = {\nc3}/\ncs},
in ({\nc1*\x1+\nc2*\x2+\nc3*\x3,\nc1*\y1+\nc2*\y2+\nc3*\y3})
}
},
% ===== Circle Operations =====
% circle = {0,A}, creates circle with the center (0)
    through A
circle/.style args = {#1,#2}{
    insert path = {
        let
            \p{OA} = ($(#2)-(#1)$),
            in (#1) circle ({veclen(\p{OA})})
        }
    },
% tagent point = {0,A,P}
% 0,A: center of circle and an abitary point on the
    circle
% P: a point outside the circle

```

```

tangent point/.style args = {#1,#2,#3}{
  insert path = {
    let
      \p{OA} = ($(#2)-(#1)$), % 半径
      \p{OP} = ($(#3)-(#1)$),
      \n1 = {vecLen(\p{OA})},
      \n2 = {vecLen(\p{OP})},
      \n3 = {scalar(\n1/\n2)}
    in ($(#1)!\n3!{acos(\n1/\n2)}:(#3)$)
  }
},
% external homothetic center
% O1,A1: center of circle 1 and an arbitrary point on the
  \draw circle
% O2,A2: center of circle 2 and an arbitrary point on the
  \draw circle
external center/.style args = {#1,#2,#3,#4}{
  insert path = {
    let
      \p{O1A1} = ($(#2)-(#1)$),% 半径 O1A1
      \p{O2A2} = ($(#4)-(#3)$),% 半径 O2A2
      \n{r1} = {vecLen(\p{O1A1})},
      \n{r2} = {vecLen(\p{O2A2})},
      \n1 = {scalar(\n{r1}/(\n{r1}-\n{r2}))}
    in ($(#1)!\n1!(#3)$)
  }
},
% internal homothetic center
% O1,A1: center of circle 1 and an arbitrary point on the
  \draw circle

```

```

% O2,A2: center of circle 2 and an abitary point on the
↳ circle
internal center/.style args = {#1,#2,#3,#4}{
  insert path = {
    let
      \p{O1A1} = ($(#2)-(#1)$),% 半径 O1A1
      \p{O2A2} = ($(#4)-(#3)$),% 半径 O2A2
      \n{r1} = {veclen(\p{O1A1})},
      \n{r2} = {veclen(\p{O2A2})},
      \n1 = {scalar(\n{r1}/(\n{r1}+\n{r2}))}
    in ($(#1)!\n1!(#3)$)
  }
},
% creates the radical axis of two non-concentric
↳ circles
% O1,A1: center of circle 1 and an abitary point on the
↳ circle
% O2,A2: center of circle 2 and an abitary point on the
↳ circle
radical axis/.style args = {#1,#2,#3,#4}{
  insert path = {
    let
      \n{s} = {\pgfkeysvalueof{/tikz/start modifier}},
      \n{e} = {\pgfkeysvalueof{/tikz/end modifier}},
      \p{O1A1} = ($(#2)-(#1)$),% 半径 O1A1
      \p{O2A2} = ($(#4)-(#3)$),% 半径 O2A2
      \p{O1O2} = ($(#3)-(#1)$),
      \n{r1} = {veclen(\p{O1A1})},
      \n{r2} = {veclen(\p{O2A2})},
      \n{d} = {veclen(\p{O1O2})},

```



```

\p{n1} = {scalar(\p{n1}/\p{n2})},
\p{n2} = {scalar(\p{n2}/\p{n3})},
\p{n3} = {.5*(1+\p{n1}*\p{n1}-\p{n2}*\p{n2})},
\p{ft} = ({\p{n1}}!\p{n3}!\p{n2}),% perpendicular foot
\p{s0} = ({\p{ft}}+(-\y{0102},\x{0102})),
\p{e0} = ({\p{ft}}+(\y{0102},-\x{0102})),
\p{s} = ({\p{s0}}!\p{n1}!\p{e0}),% start
\p{e} = ({\p{s0}}!\p{n2}!\p{e0})% end
in (\p{s}) -- (\p{e})
}
},
% ===== Path Definitions =====
% perpendicular bisector of the line segment (#1 -- #2)
perpendicular bisector/.style args = {#1,#2}{
  insert path = {
    let
      \p{s} = {\pgfkeysvalueof{/tikz/start modifier}},
      \p{e} = {\pgfkeysvalueof{/tikz/end modifier}},
      \p{AB} = ({\p{#2}}-{\p{#1}}),
      \p{m} = ({\p{#1}}!0.5!\p{#2}),% midpoint
      \p{s0} = ({\p{m}}+(-\y{AB},\x{AB})),% rotate
      ~ ccw, default start
      \p{e0} = ({\p{m}}+(\y{AB},-\x{AB})),% rotate
      ~ cw, default end
      \p{s} = ({\p{s0}}!\p{s}!\p{e0}),% start
      \p{e} = ({\p{s0}}!\p{e}!\p{e0})% end
    in (\p{s}) -- (\p{e})
  }
},
% perpendicular line of the line (#1 -- #2) through #3

```

```

% specifying start and end with modifiers(see tikz
↳ manual 13.5)
perpendicular/.style args = {#1,#2,#3}{
  insert path = {
    let
      \n{s} = {\pgfkeysvalueof{/tikz/start modifier}},
      \n{e} = {\pgfkeysvalueof{/tikz/end modifier}},
      \p{AB} = ($(#2)-(#1)$),
      \p{ft} = ($(#1)!(#3)!(#2)$),% perpendicular foot
      \p{s0} = ($(\p{ft})+(-\y{AB},\x{AB})$),
      \p{e0} = ($(\p{ft})+(\y{AB},-\x{AB})$),
      \p{s} = ($(\p{s0})!\n{s}!(\p{e0})$),% start
      \p{e} = ($(\p{s0})!\n{e}!(\p{e0})$)% end
    in (\p{s}) -- (\p{e})
  }
},
% parallel line of the line (#1 -- #2) through #3
% specifying start and end with modifiers(see tikz
↳ manual 13.5)
parallel/.style args = {#1,#2,#3}{
  insert path = {
    let
      \n{s} = {\pgfkeysvalueof{/tikz/start modifier}},
      \n{e} = {\pgfkeysvalueof{/tikz/end modifier}},
      \p{s0} = (#3),
      \p{e0} = ($(#3)+(#2)-(#1)$),
      \p{s} = ($(\p{s0})!\n{s}!(\p{e0})$),% start
      \p{e} = ($(\p{s0})!\n{e}!(\p{e0})$)% end
    in (\p{s}) -- (\p{e})
  }
}

```

```

    },
    % alias for parallel={A,B,A}
    extend/.style args = {#1,#2} {
        parallel={#1,#2,#1}
    },
}

% Utilities for implementation of 'revolve'
% parse comma separated arguments recursively
% store arguments number in /tikz/revolve/@argn
\def\euclidea@ParseArguments#1\euclidea@stop{%
    \pgfutil@in@{,}{#1}
    \ifpgfutil@in@%comma separated arguments
        \euclidea@ParseSeparatedArguments#1\euclidea@stop
    \fi
}

\def\euclidea@ParseSeparatedArguments#1,#2\j
    \euclidea@stop{%
        \pgfmathparse{int(add(\pgfkeysvalueof{/tikz/revolve/}
        \j @argn},1))}
        \pgfkeysalso{/tikz/revolve/@argn = \pgfmathresult}

        \pgfutil@in@{,}{#2}
        \ifpgfutil@in@%comma separated arguments
            \euclidea@ParseSeparatedArguments#2\euclidea@stop
        \fi
    }

% compute angle

```

```

\def\euclidea@ComputeAngle#1\euclidea@stop{
  \ifnum \pgfkeysvalueof{/tikz/revolve/@argn} = 1
    \tikzset{revolve/@set angle 1 = {#1}}
  \else
    \ifnum \pgfkeysvalueof{/tikz/revolve/@argn} = 2
      \tikzset{revolve/@set angle 2 = {#1}}
    \else
      \ifnum \pgfkeysvalueof{/tikz/revolve/@argn} = 3
        \tikzset{revolve/@set angle 3 = {#1}}
      \else
        \ifnum \pgfkeysvalueof{/tikz/revolve/@argn} = 4
          \tikzset{revolve/@set angle 4 = {#1}}
        \else
          \pgferror{"Incorrect number of arguments!"}
        \fi
      \fi
    \fi
  \fi
}

% Utilities for implementation of 'intercept'
\def\euclidea@ComputeLength#1,#2\euclidea@stop{
  \newdimen\euclidea@ax
  \newdimen\euclidea@ay
  \newdimen\euclidea@bx
  \newdimen\euclidea@by
  \pgfextractx{\euclidea@ax}{\pgfpointanchor{#1}{_center}}
  \pgfextracty{\euclidea@ay}{\pgfpointanchor{#1}{_center}}
  \pgfextractx{\euclidea@bx}{\pgfpointanchor{#2}{_center}}
  \pgfextracty{\euclidea@by}{\pgfpointanchor{#2}{_center}}
}

```

```

\pgfextractx{\euclidea@bx}{\pgfpointanchor{#2}{_
center}}
\pgfextracty{\euclidea@by}{\pgfpointanchor{#2}{_
center}}
% 以下 showthe 指令 overleaf.com 编译通过，而在
↳ macOS+texlive 2021 报错
% \showthe\euclidea@ax
% \showthe\euclidea@ay
% \showthe\euclidea@bx
% \showthe\euclidea@by
\pgfmathveclen{\euclidea@ax-\euclidea@bx}{\_
↳ euclidea@ay-\euclidea@by}
}

\makeatother

```


参考文献

- [1] Syntax for path specifications. <https://tikz.dev/tikz-paths>.
- [2] Homothetic center. https://en.wikipedia.org/wiki/Homothetic_center.
- [3] Coordinate calculations. <https://tikz.dev/tikz-coordinates#sec-13.5>.
- [4] Line-line intersection. <https://mathworld.wolfram.com/Line-LineIntersection.html>.