TikZ 几何作图

2024 年 1 月 11 日

目录

第 1 章　基本作图命令

这里的命令都是通过 /tikz/insert path[1] 在当前路径上插入新的路径, 或者是通过 def 组合一些 tikz 命令.

1.1　坐标轴 Axes

调用方式

```
\axes (xmin:xmain,ymin:ymax);
```

参数说明

xmin,xmax　x 范围

ymin,ymax　y 范围

示例

见1.2.

1.2　坐标变换 Coordinates Transformations

调用方式

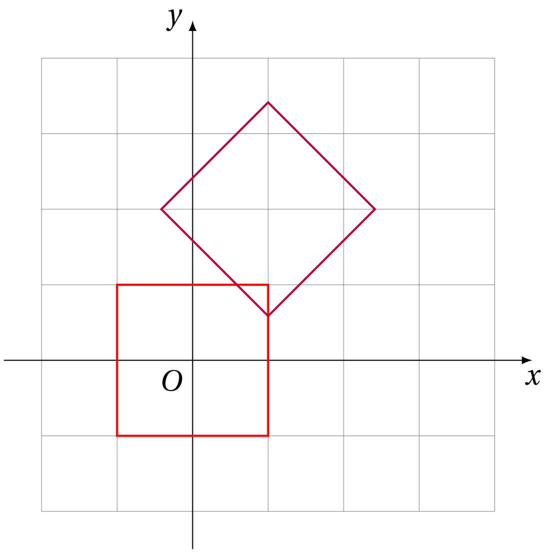
```
transform={angle:(xshift,yshift)}
```

参数说明

绕原点旋转 angle, 然后水平方向和竖直方向分别平移 xshift 和 yshift

示例

```
\begin{tikzpicture}
  % \draw[help lines] (-5,-5) grid (5,5);
  % \draw[-latex] (-5.5,0) -- (5.5,0) node[below] {$x$};
  % \draw[-latex] (0,-5.5) -- (0,5.5) node[left] {$y$};
  % \draw (0,0) node[below left] {$O$};
  \axes (-2:4, -2:4);
  \draw[thick,red] (-1,-1) rectangle (1,1);
  \draw[thick,purple,transform={45:(1,2)}] (-1,-1)
   ↪ rectangle (1,1);
\end{tikzpicture}
```



1.3 仿射组合 Affine Combination

调用方式

```
affine={A,B,k}
```

参数说明

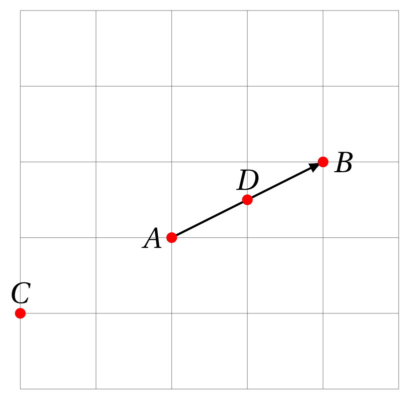
A, B 两点坐标

k 系数

返回点 A, B 的仿射组合: $A + k \cdot (B - A)$.

示例

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (5,5);
  \coordinate (A) at (2,2);
  \coordinate (B) at (4,3);
  \coordinate [affine={A,B,-1}] (C);
  \coordinate [affine={A,B,.5}] (D);
  \draw[thick, -latex] (A) -- (B);
  \foreach \p/\placement in {A/left,B/right,
  C/above,D/above}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.4 中点 Midpoint

调用方式

```
midpoint={A,B}
```

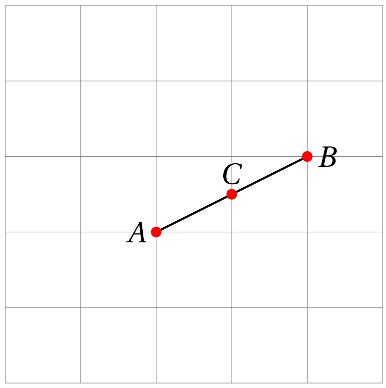
参数说明

A, B 两点坐标

返回点 A, B 的中点坐标.

示例

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (5,5);
  \coordinate (A) at (2,2);
  \coordinate (B) at (4,3);
  \coordinate [midpoint={A,B}] (C);
  \draw[thick] (A) -- (B);
  \foreach \p/\placement in {A/left,B/right,
  C/above}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.5 平移 Translate

调用方式

```
translate={A,B,C}
```

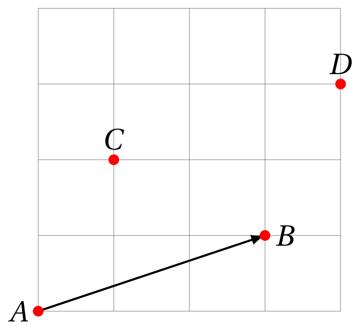
参数说明

A,B,C 三点坐标

返回 C 按向量 AB 移动所得的坐标: $C+(B-A)$.

示例

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,0);
  \coordinate (B) at (3,1);
  \coordinate (C) at (1,2);
  \coordinate [translate={A,B,C}] (D);
  \draw[thick, -latex] (A) -- (B);
  \foreach \p/\placement in
   ↪ {A/left,B/right,C/above,D/above}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.6 对称点 Reflect

调用方式

```
reflect={A,B,C}
```

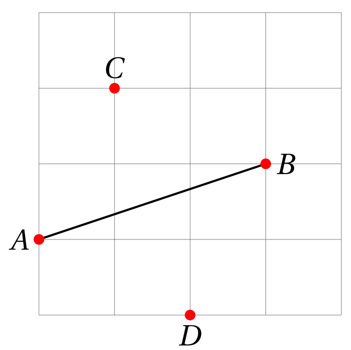
参数说明

A,B,C 三点坐标

返回 C 关于直线 AB 的对称点的坐标 (设 D 为 C 在 AB 的投影): $C+2(D-C)$.

示例

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,1);
  \coordinate (B) at (3,2);
  \coordinate (C) at (1,3);
  \coordinate [reflect={A,B,C}] (D);
  \draw[thick] (A) -- (B);
  \foreach \p/\placement in
   ↪ {A/left,B/right,C/above,D/below}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.7 投影 Project

调用方式

```
project={A,B,C}
```

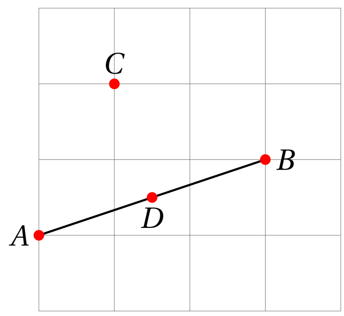
参数说明

A,B,C 三点坐标

返回 C 在直线 AB 的投影的坐标.

示例

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,1);
  \coordinate (B) at (3,2);
  \coordinate (C) at (1,3);
  \coordinate [project={A,B,C}] (D);
  \draw[thick] (A) -- (B);
  \foreach \p/\placement in
   ↪ {A/left,B/right,C/above,D/below}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.8 反演 Inverse

调用方式

```
inverse={O,A,P}
```

参数说明

O 圆心

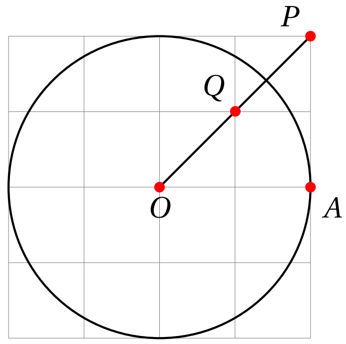
A 圆上一点

P 平面上任一点

返回 P 关于圆 (O, A) 的反演点.

示例

```
\begin{tikzpicture}
  \draw[help lines] (-2,-2) grid (2,2);
  \coordinate (O) at (0,0);
  \coordinate (A) at +(0:2);  % 圆上一点，相对坐标
  \coordinate (P) at (2,2);
  \coordinate[inverse={O,A,P}] (Q);

  \draw[thick,circle={O,A}];
  \draw[thick] (O) -- (P);

  \foreach \p/\placement in {O/below,A/below right,
  P/above left,Q/above left}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.9 旋转 Revolve

调用方式

```
revolve={A,B}
```

参数说明

A,B 两点坐标

注 为了避免覆盖 tikz 的 rotate, 这里将旋转命令为 revolve.

返回 B 绕 A 旋转的点.

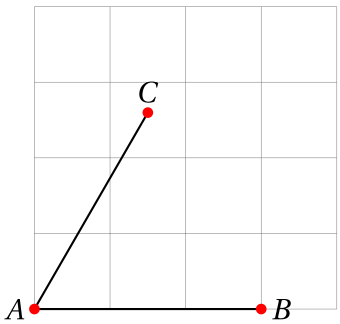
还需要指定 revolve/angle (default: 0) 和 revolve/angle scale(default: 1) 两个选项, 可以通过下面的方式来指定 /revolve/angle:

1. 直接指定角度: `revolve/angle=60`
2. 位置向量与 x 轴夹角: `revolve/angle={P1}`
3. 两位置向量的夹角: `revolve/angle={P1,P2}`
4. 由三点定义的角 (P_1 为顶点, P_2 为起点, P_3 为终点): `revolve/angle={P1,P2,P3}`
5. 两向量的夹角 (逆时针方向): `revolve/angle={P1,P2,P3,P4}`

示例

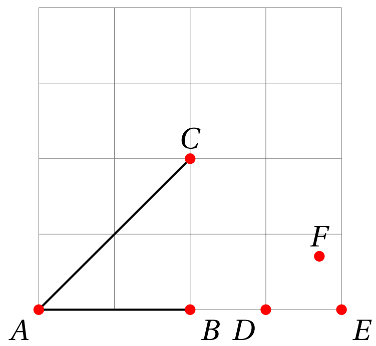
```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
```

```
  \coordinate (A) at (0,0);
  \coordinate (B) at (3,0);
  \coordinate [revolve/angle=60, revolve={A,B}] (C);
  \draw[thick] (A) -- (B) (A) -- (C);
  \foreach \p/\placement in {A/left,B/right,C/above}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```

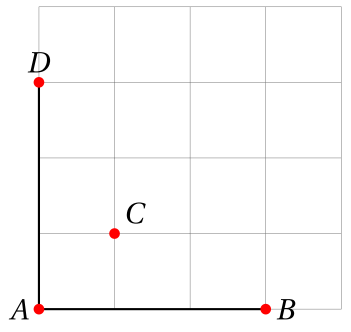



```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,0);
  \coordinate (B) at (2,0);
  \coordinate (C) at (2,2);
  \coordinate (D) at (3,0);
  \coordinate (E) at (4,0);
  \coordinate [revolve/angle={A,B,C},revolve={D,E}] (F);
  \draw[thick] (A) -- (B) (A) -- (C);
  \foreach \p/\placement in {
  A/below left,B/below right,C/above,
  D/below left,E/below right,F/above}{
```

```
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```




```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,0);
  \coordinate (B) at (3,0);
  \coordinate (C) at (1,1);
  \coordinate [revolve/angle={C},
    revolve/scale=2,
    revolve={A,B}] (D);
  \draw[thick] (A) -- (B) (A) -- (D);
  \foreach \p/\placement in {A/left,B/right,
  C/above right,D/above}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```

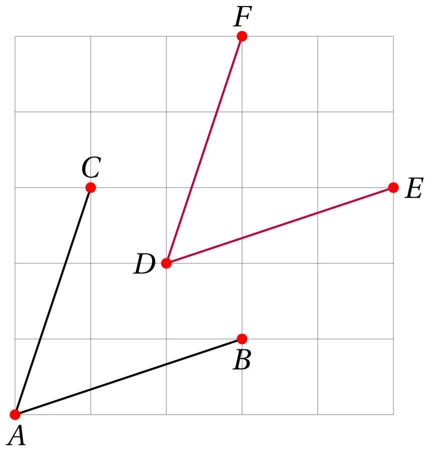


1.10 构造角 Angle

可以由 resovle 来构造一个角.

示例

```
\begin{tikzpicture}[scale=1]
  \draw[help lines] (0,0) grid (5,5);
  \coordinate (A) at (0,0);
  \coordinate (B) at (3,1);
  \coordinate (C) at (1,3);
  \coordinate (D) at (2,2);
  \coordinate (E) at (5,3);
  \coordinate [revolve/angle={A,B,C},
    revolve/scale=1,
    revolve={D,E}] (F);
  \draw[thick] (A) -- (B) (A) -- (C);
  \draw[thick, purple] (D) -- (E) (D) -- (F);
  \foreach \p/\placement in {A/below,B/below,C/above,
  D/left,E/right,F/above}{
    \fill[red] (\p) circle (2pt);
    \node[\placement] at (\p) {$\p$};
  }
\end{tikzpicture}
```



1.11 角平分线 Angle Bisector

调用方式

```
angle bisector={A,B,C}
```

参数说明

A,B,C 三点坐标, A 为顶点 (apex), B 为起点, C 为终点

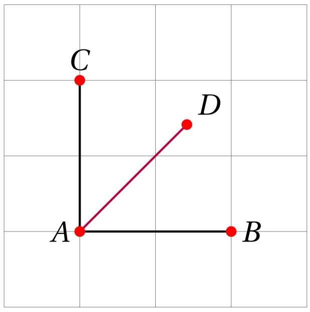
返回 $\angle BAC$ 角平分线上的一点. 实际上, 该操作等价于:

```
revolve/angle={A,B,C}, revolve/scale=.5, revolve={A,B}
```

示例

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (1,1);
  \coordinate (B) at (3,1);
  \coordinate (C) at (1,3);
```

```
  \coordinate [angle bisector={A,B,C}] (D);
  \draw[thick] (A) -- (B) (A) -- (C);
  \draw[thick, purple] (A) -- (D);
  \foreach \p/\placement in {A/left,B/right,C/above,
  D/above right}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.12 等边三角形 Equilateral Triangle

调用方式

```
equilateral={A,B}
```

参数说明

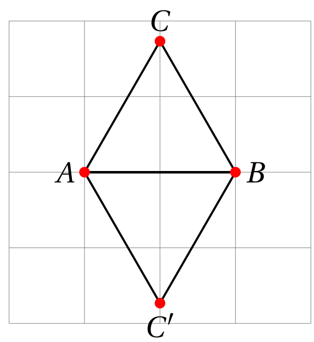
A,B 两点坐标

返回以 AB 为边长的等边三角形的第 3 点 (位于向量 AB 的左侧). 实际上, 该操作等价于:

```
revolve/angle=60, revolve={A,B}
```

示例

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (1,2);
  \coordinate (B) at (3,2);
  \coordinate [equilateral={A,B}] (C);
  \coordinate [equilateral={B,A}] (C');
  \draw[thick] (A) -- (B) -- (C) -- cycle
               (A) -- (B) -- (C') -- cycle;
  \foreach \p/\placement in
   ↪  {A/left,B/right,C/above,C'/below}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.13 旋转 90° Erect

调用方式

```
erect={A,B}
```

参数说明

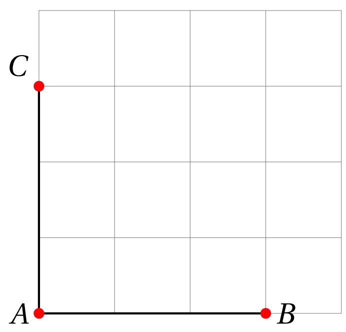
A,B 两点坐标

返回 B 绕 A 旋转 90° 的坐标. 实际上, 该操作等价于:

```
revolve/angle=90, revolve={A,B}
```

示例

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,0);
  \coordinate (B) at (3,0);
  \coordinate [erect={A,B}] (C);
  \draw[thick] (A) -- (B) (A) -- (C);
  \foreach \p/\placement in {A/left,B/right,C/above
  ↪  left}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.14 截取 Intercept

调用方式

```
intercept={A,B}
```

参数说明

A,B 两点坐标

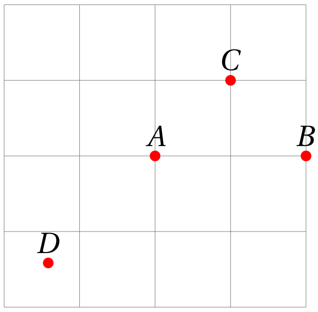
在直线 AB 截取指定长度线段, A 为新线段的起点, AB 是方向.

需要指定 intercept/length (default: 1cm) 和 intercept/scale(default: 1) 两个选项. 其中 intercept/length 有两种形式:

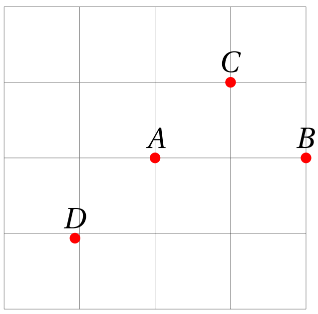
1. 直接指定长度: `intercept/length=2cm`
2. 指定线段长度: `intercept/length={P1,P2}`

示例

```
\begin{tikzpicture}
  \draw[help lines] (-2,-2) grid (2,2);
  \coordinate (A) at (0,0);
  \coordinate (B) at (2,0);
  \coordinate (C) at (1,1);
  \coordinate[intercept/length={A,B},
    intercept/scale=-1, intercept={A,C}] (D);
  \foreach \p/\placement in
   ↪  {A/above,B/above,C/above,D/above}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```




```
\begin{tikzpicture}
  \draw[help lines] (-2,-2) grid (2,2);
  \coordinate (A) at (0,0);
  \coordinate (B) at (2,0);
  \coordinate (C) at (1,1);
  \coordinate[intercept/length=1.5cm,
    intercept/scale=-1,
    intercept={A,C}] (D);
  \foreach \p/\placement in
   ↪ {A/above,B/above,C/above,D/above}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.15 直线与直线的交点 Line-Line Intersection

调用方式

```
intersect={A,B,C,D}
```

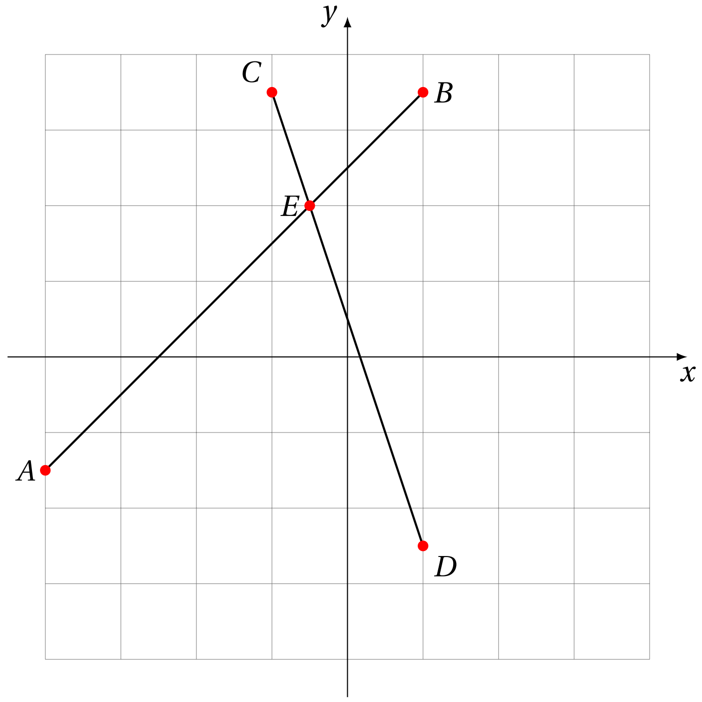
参数说明

A,B,C,D 四点坐标

返回 AB 与 CD 的交点 (可以是延长线相交点).

示例

```
\begin{tikzpicture}
  \draw[help lines] (-4,-4) grid[step=1] (4,4);
  \draw[-latex] (-4.5,0) -- (4.5,0) node[below] {$x$};
  \draw[-latex] (0,-4.5) -- (0,4.5) node[left] {$y$};
  \coordinate (A) at (-4,-1.5);
  \coordinate (B) at (1,3.5);
  \coordinate (C) at (-1,3.5);
  \coordinate (D) at (1,-2.5);
  \coordinate [intersect={A,B,C,D}] (E);
  \draw[thick] (A) -- (B) (C) -- (D);
  \foreach \p/\placement in {A/left,B/right,
  C/above left,D/below right,E/left}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.16 垂直平分线/中垂线 Perpendicular Bisector

调用方式

```
perpendicular bisector={A,B}
```

参数说明

A,B 两点坐标

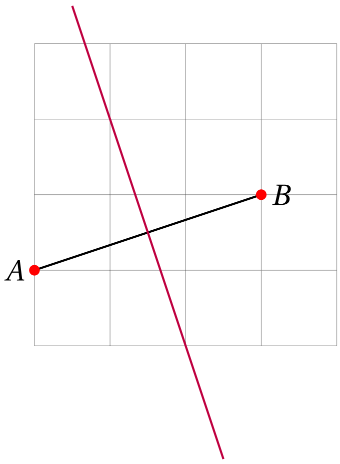
构造 AB 的中垂线, 默认起点为 $.5(A+B)+(B-A)\cdot\mathbf{i}$, 终点为 $.5(A+B)-(B-A)\cdot\mathbf{i}$. 可以对起始点进行调整, 见**??**.

示例

使用默认参数:

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
```

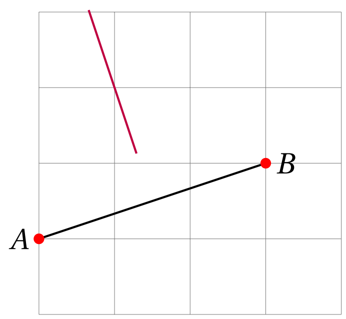
```
  \coordinate (A) at (0,1);
  \coordinate (B) at (3,2);
  \draw[thick] (A) -- (B);
  \draw[thick,purple,perpendicular bisector={A,B}];
  \foreach \p/\placement in {A/left,B/right}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



指定两端的长度:

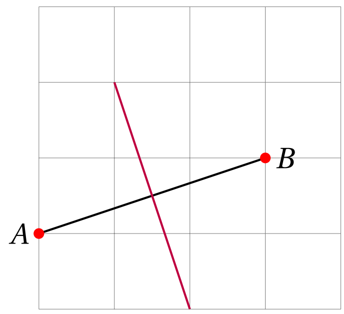
```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,1);
  \coordinate (B) at (3,2);
  \draw[thick] (A) -- (B);
  \draw[thick, purple,
    start modifier=.5cm, end modifier=2.5cm,
    perpendicular bisector={A,B}];
```

```
  \foreach \p/\placement in {A/left,B/right}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



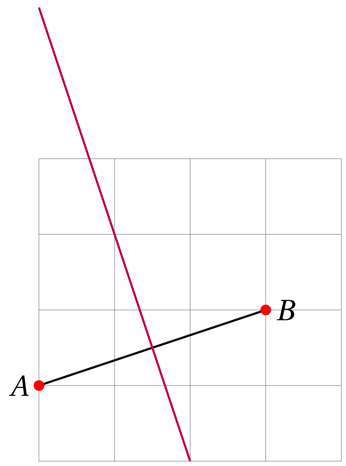
指定系数:

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,1);
  \coordinate (B) at (3,2);
  \draw[thick] (A) -- (B);
  \draw[thick,purple,
    start modifier=.25,end modifier=.75,
    perpendicular bisector={A,B}];
  \foreach \p/\placement in {A/left,B/right}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



可以是负数, 这样就在相反方向:

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,1);
  \coordinate (B) at (3,2);
  \draw[thick] (A) -- (B);
  \draw[thick,purple,
    start modifier=-.25,end modifier=0.75,
    perpendicular bisector={A,B}];
  \foreach \p/\placement in {A/left,B/right}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.17 垂线 Perpendicular Line

调用方式

```
perpendicular={A,B,C}
```

参数说明

A,B,C 三点坐标

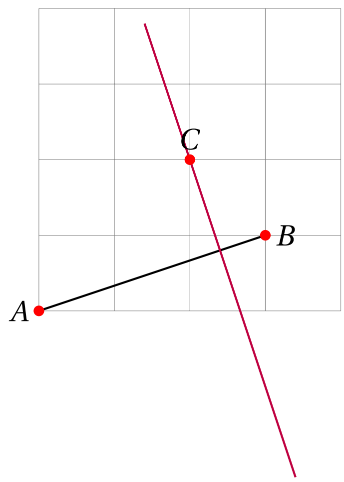
构造过 C 垂直于 AB 的直线 (设垂足为 D), 默认起点为 $D+(B-A)\cdot\mathbf{i}$, 终点为 $D-(B-A)\cdot\mathbf{i}$. 可以对起始点进行调整, 见**??**.

示例

过直线外一点的垂线:

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,0);
  \coordinate (B) at (3,1);
  \coordinate (C) at (2,2);
  \draw[thick] (A) -- (B);
```

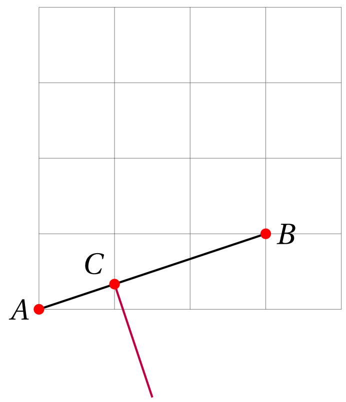
```
  \path[draw, thick, purple, perpendicular={A,B,C}];
  \foreach \p/\placement in {A/left,B/right,C/above}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



过直线上一点的垂线:

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,0);
  \coordinate (B) at (3,1);
  \coordinate (C) at ($(A)!1/3!(B)$);
  \draw[thick] (A) -- (B);
  \path[draw, thick, purple,
    start modifier=.5, end modifier=.75,
    perpendicular={A,B,C}];
  \foreach \p/\placement in {A/left,B/right,C/above
   ↪ left}{
```

```
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.18 平行线 Parallel Line

调用方式

```
parallel={A,B,C}
```

参数说明

过一点 C 作直线 AB 平行线, (如果 C 在 AB 上, 则重合).

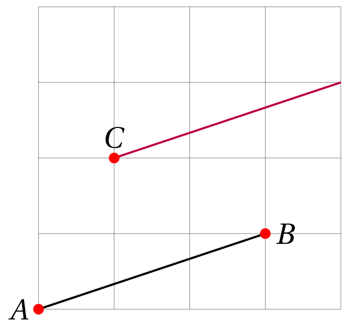
首先将点 C 按向量 AB 平移至 D. 可以对起始点进行调整, 见**??**.

示例

指定起始点距离 C 的位置, 方向是 CD, 负值代表相反方向:

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,0);
```

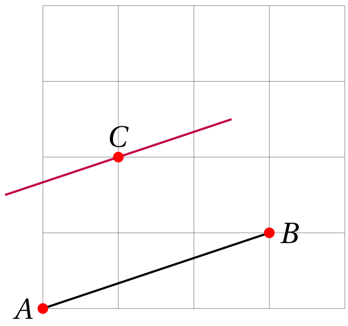
```
  \coordinate (B) at (3,1);
  \coordinate (C) at (1,2);
  \draw[thick] (A) -- (B);
  \path[draw, thick, purple, parallel={A,B,C}];
  \foreach \p/\placement in {A/left,B/right,C/above}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



指定系数:

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,0);
  \coordinate (B) at (3,1);
  \coordinate (C) at (1,2);
  \draw[thick] (A) -- (B);
  \path[draw, thick, purple,
    start modifier=-.5, end modifier=.5,
    parallel={A,B,C}];
  \foreach \p/\placement in {A/left,B/right,C/above}{
    \fill[red] (\p) circle (2pt);
```

```
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.19 延长线 Extend

调用方式

```
extend={A,B}
```

参数说明

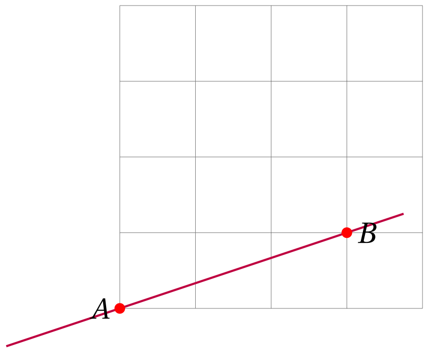
作线段 AB 延长线, 可以对起始点进行调整, 见**??**. 实际上, 该操作等价于:

```
parallel={A,B,A}
```

示例

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (4,4);
  \coordinate (A) at (0,0);
  \coordinate (B) at (3,1);
```

```
  \path[draw, thick, purple,
    start modifier=-.5, end modifier=1.25,
    extend={A,B}];
  \foreach \p/\placement in {A/left,B/right}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



1.20 圆 Circle

调用方式

```
circle={O,A}
```

参数说明

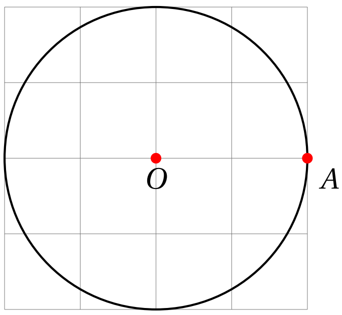
O 圆心

A 圆上一点

构造圆心为 O, 经过 A 的圆.

示例

```
\begin{tikzpicture}
  \draw[help lines] (-2,-2) grid (2,2);
  \coordinate (O) at (0,0);
  \coordinate (A) at +(0:2);  % 圆上一点，相对坐标
  \draw[thick,circle={O,A}];
  \foreach \p in {O,A}
    \fill[red] (\p) circle (2pt);
  \draw (O) node[below] {$O$};
  \draw (A) node[below right] {$A$};
\end{tikzpicture}
```



1.21 直线与圆的切点 Tangent Point

调用方式

```
tangent point={O,A,P}
```

参数说明

O ：圆心坐标

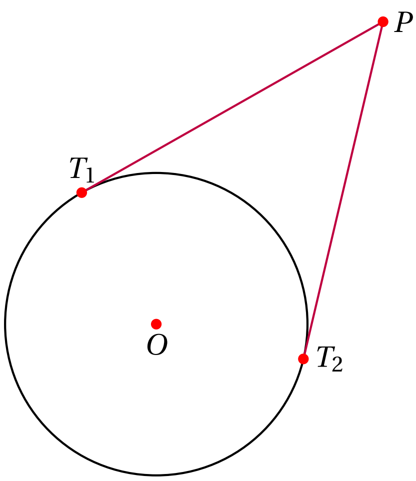
A ：为圆上任意一点

P ：圆外一点坐标

过圆 (O 为圆心, A 为圆上任意一点) 外一点 P 作切线, 求得一个切点 (在向量 OP 的左边), 另外一点可以通过对称 (`reflect={O,P,T}`) 求得.

示例

```
\begin{tikzpicture}
  \coordinate (O) at (0,0);
  \coordinate (A) at +(0:2);  % 圆上一点, 相对坐标
  \coordinate (P) at (3,4);
  \coordinate[tangent point={O,A,P}] (T1);
  \coordinate[reflect={O,P,T1}] (T2);
  \draw[thick,circle={O,A}];
  \draw[thick,purple] (P) -- (T1) (P) -- (T2);
  \foreach \p in {O,P,T1,T2}
    \fill[red] (\p) circle (2pt);
  \draw (O) node[below] {$O$};
  \draw (P) node[right] {$P$};
  \draw (T1) node[above] {$T_1$};
  \draw (T2) node[right] {$T_2$};
\end{tikzpicture}
```



1.22 外位似中心 External Homothetic Center

调用方式

```
external center={O1,A1,O2,A2}
```

参数说明

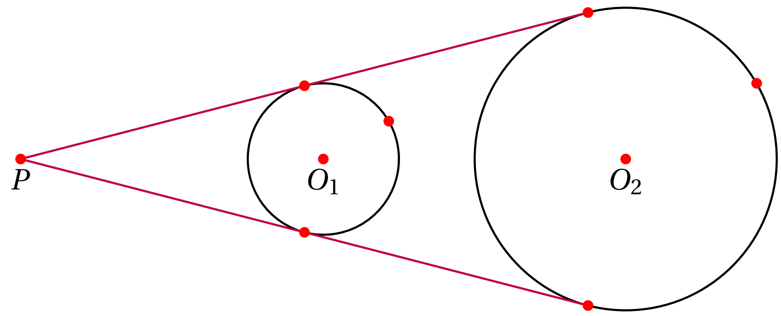
求圆 1 (O_1 为圆心, A_1 为圆上任意一点) 和圆 2 (O_2 为圆心, A_2 为圆上任意一点) 的外位似中心 (external homothetic center)[2].

示例

作外公切线: 先求位似中心, 可以求得两圆的外公切线.

```
\begin{tikzpicture}
  \tikzmath {
    \a = 30;
    \b = \a;
    \r1 = 1;
    \r2 = 2;
  }
  \coordinate (O1) at (0,0);
  \coordinate (A1) at ($(O1)+(\a:\r1)$);
  \coordinate (O2) at (4,0);
  \coordinate (A2) at ($(O2)+(\b:\r2)$);
  \coordinate[external center={O1,A1,O2,A2}] (P);
  \coordinate[tangent point={O1,A1,P}] (B);
  \coordinate[tangent point={O2,A2,P}] (C);
  \coordinate[reflect={O1,O2,B}] (D);
  \coordinate[reflect={O1,O2,C}] (E);
  \draw[thick,circle={O1,A1}];
  \draw[thick,circle={O2,A2}];
  \draw[thick,purple] (P) -- (C) (P) -- (E);
```

```
  \foreach \p in {A1,A2,B,C,D,E,O1,O2,P}
    \fill[red] (\p) circle (2pt);
  \draw (O1) node[below] {$O_1$};
  \draw (O2) node[below] {$O_2$};
  \draw (P) node[below] {$P$};
\end{tikzpicture}
```



1.23 内位似中心 Internal Homothetic Center

调用方式

```
internal center={O1,A1,O2,A2}
```

参数说明

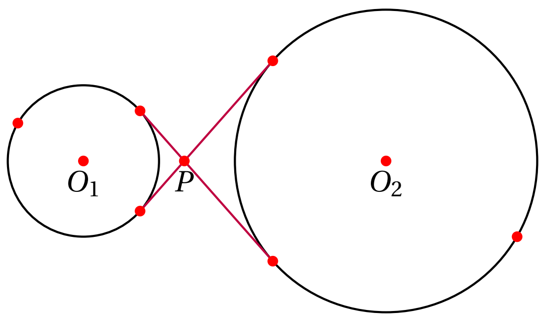
求圆 1 (O_1 为圆心, A_1 为圆上任意一点) 和圆 2 (O_2 为圆心, A_2 为圆上任意一点) 的内位似中心 (internal homothetic center)[2].

示例

作内公切线: 先求位似中心, 可以求得两圆的内公切线.

```
\begin{tikzpicture}
  \tikzmath {
    \a = 150;
```

```
    \b = \a - 180;
    \r1 = 1;
    \r2 = 2;
  }
  \coordinate (O1) at (0,0);
  \coordinate (A1) at ($(O1)+(\a:\r1)$);
  \coordinate (O2) at (4,0);
  \coordinate (A2) at ($(O2)+(\b:\r2)$);
  \coordinate[internal center={O1,A1,O2,A2}] (P);
  \coordinate[tangent point={O1,A1,P}] (B);
  \coordinate[tangent point={O2,A2,P}] (C);
  \coordinate[reflect={O1,O2,B}] (D);
  \coordinate[reflect={O1,O2,C}] (E);
  \draw[thick,circle={O1,A1}];
  \draw[thick,circle={O2,A2}];
  \draw[thick,purple] (P) -- (B) (P) -- (C) (P) -- (D)
  ↪  (P) -- (E);
  \foreach \p in {A1,A2,B,C,D,E,O1,O2,P}
    \fill[red] (\p) circle (2pt);
  \draw (O1) node[below] {$O_1$};
  \draw (O2) node[below] {$O_2$};
  \draw (P) node[below] {$P$};
\end{tikzpicture}
```



1.24 根轴 Radical Axis

调用方式

```
radical axis={O1,A1,O2,A2}
```

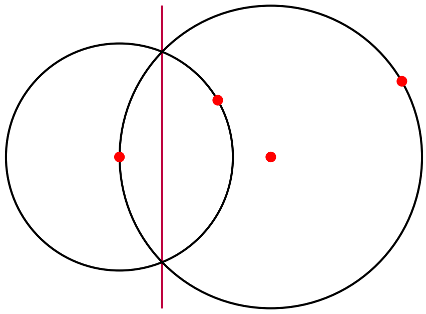
参数说明

构造两圆的根轴, 设与 O_1O_2 的交点为 P, 则默认起点为 $P+(O_2-O_1)\cdot\mathbf{i}$, 终点为 $P-(O_2-O_1)\cdot\mathbf{i}$. 可以对起始点进行调整, 见**??**.

示例

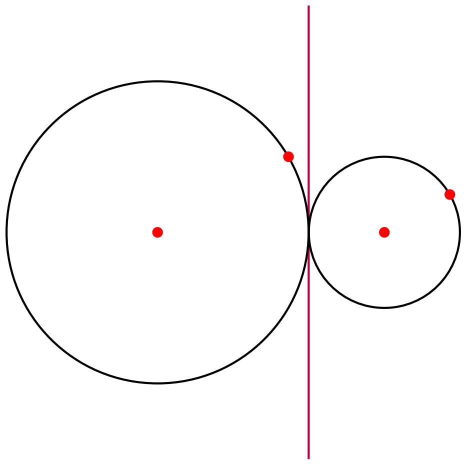
两圆相交:

```
\begin{tikzpicture}
  \tikzmath {
    \a = 30;
    \b = \a;
    \r1 = 1.5;
    \r2 = 2;
  }
  \coordinate (O1) at (0,0);
  \coordinate (A1) at ($(O1)+(\a:\r1)$);
  \coordinate (O2) at (2,0);
  \coordinate (A2) at ($(O2)+(\b:\r2)$);
  \draw[thick,purple,radical axis={O1,A1,O2,A2}];
  \draw[thick,circle={O1,A1}];
  \draw[thick,circle={O2,A2}];
  \foreach \p in {A1,A2,O1,O2}
    \fill[red] (\p) circle (2pt);
\end{tikzpicture}
```



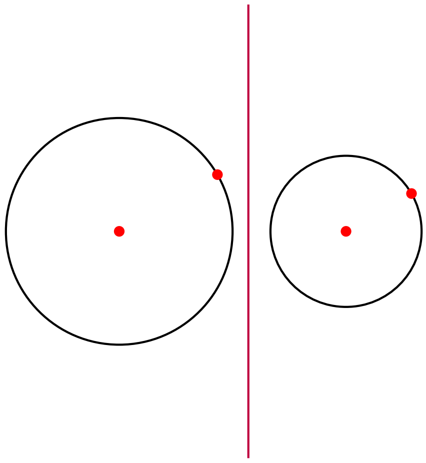
两圆外切:

```
\begin{tikzpicture}
  \tikzmath {
    \a = 30;
    \b = \a;
    \r1 = 2;
    \r2 = 1;
  }
  \coordinate (O1) at (0,0);
  \coordinate (A1) at ($(O1)+(\a:\r1)$);
  \coordinate (O2) at (3,0);
  \coordinate (A2) at ($(O2)+(\b:\r2)$);
  \draw[thick,purple,radical axis={O1,A1,O2,A2}];
  \draw[thick,circle={O1,A1}];
  \draw[thick,circle={O2,A2}];
  \foreach \p in {A1,A2,O1,O2}
    \fill[red] (\p) circle (2pt);
\end{tikzpicture}
```



两圆外离:

```
\begin{tikzpicture}
  \tikzmath {
    \a = 30;
    \b = \a;
    \r1 = 1.5;
    \r2 = 1;
  }
  \coordinate (O1) at (0,0);
  \coordinate (A1) at ($(O1)+(\a:\r1)$);
  \coordinate (O2) at (3,0);
  \coordinate (A2) at ($(O2)+(\b:\r2)$);
  \coordinate[radical axis={O1,A1,O2,A2}] (P);
  \draw[thick,purple,radical axis={O1,A1,O2,A2}];
  \draw[thick,circle={O1,A1}];
  \draw[thick,circle={O2,A2}];
  \foreach \p in {A1,A2,O1,O2}
    \fill[red] (\p) circle (2pt);
\end{tikzpicture}
```



1.25 Partway Modifiers and Distance Modifiers

`perpendicular bisector`,`perpendicular`,`parallel`,`radical axis` 等线段图形可以对起始点进行调整, 调整参数如下 [3]:

start modifier (default: 0), 长度或系数, 如: 1cm 或.75

end modifier (default: 1), 长度或系数, 如: 1cm 或.75

第 2 章　三角形的中心

2.1　重心 Centroid

调用方式

```
centroid={A,B,C}
```

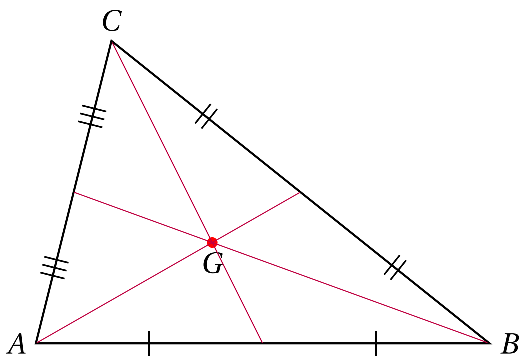
参数说明

A,B,C　三角形的顶点

示例

```
\begin{tikzpicture}
  \coordinate (A) at (-2,0);
  \coordinate (B) at (4,0);
  \coordinate (C) at (-1,4);
  \coordinate (D) at ($(B)!0.5!(C)$);
  \coordinate (E) at ($(C)!0.5!(A)$);
  \coordinate (F) at ($(A)!0.5!(B)$);
  \path[centroid={A,B,C}] coordinate (G);
  \fill (G) [red] circle (2pt);
  \draw (G) node[below] {$G$};
  \draw[thick] (A) -- (B) -- (C) -- cycle;
  \draw[purple] (A) -- (D) (B) -- (E) (C) -- (F);
```

```
  \draw (A) node[left] {$A$};
  \draw (B) node[right] {$B$};
  \draw (C) node[above] {$C$};
  \draw (A) -- (B) node[near start,sloped] {$|$}
   ↪ node[near end,sloped] {$|$};
  \draw (B) -- (C) node[near start,sloped] {$||$}
   ↪ node[near end,sloped] {$||$};
  \draw (C) -- (A) node[near start,sloped] {$|||$}
   ↪ node[near end,sloped] {$|||$};
\end{tikzpicture}
```



2.2 垂心 Orthocenter

调用方式

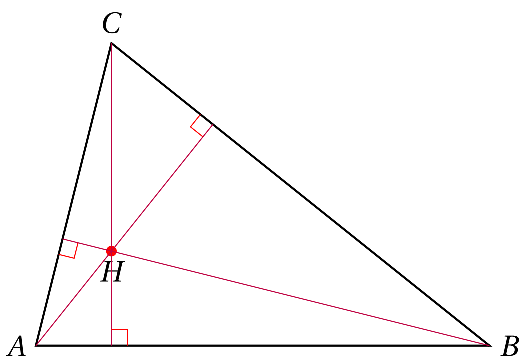
```
orthocenter={A,B,C}
```

参数说明

A, B, C 三角形的顶点

示例

```
\begin{tikzpicture}
  \coordinate (A) at (-2,0);
  \coordinate (B) at (4,0);
  \coordinate (C) at (-1,4);
  \path[orthocenter={A,B,C}] coordinate (H);
  \fill (H) [red] circle (2pt);
  \draw (H) node[below] {$H$};
  \draw[thick] (A) -- (B) -- (C) -- cycle;
  \coordinate (D) at ($(B)!(A)!(C)$);
  \coordinate (E) at ($(A)!(B)!(C)$);
  \coordinate (F) at ($(B)!(C)!(A)$);
  \draw[purple] (A) -- (D) (B) -- (E) (C) -- (F);
  \draw (A) node[left] {$A$};
  \draw (B) node[right] {$B$};
  \draw (C) node[above] {$C$};
  \pic [draw,red,angle radius=6pt] {right angle=H--D--C};
  \pic [draw,red,angle radius=6pt] {right angle=H--E--A};
  \pic [draw,red,angle radius=6pt] {right angle=H--F--B};
\end{tikzpicture}
```



2.3 外心 Circumcenter

调用方式

```
circumcenter={A,B,C}
```

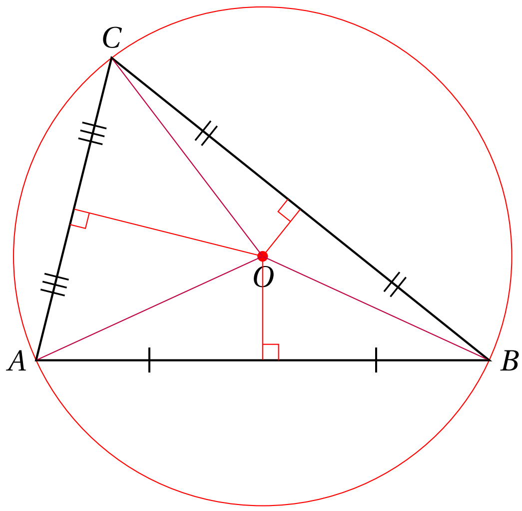
参数说明

A,B,C 三角形的顶点

示例

```
\begin{tikzpicture}
  \coordinate (A) at (-2,0);
  \coordinate (B) at (4,0);
  \coordinate (C) at (-1,4);
  \path[circumcenter={A,B,C}] coordinate (O);
  \fill (O) [red] circle (2pt);
  \draw (O) node[below] {$O$};
  \node[draw,red] at (O) [circle through=(A)]{};
  \draw[thick] (A) -- (B) -- (C) -- cycle;
  \draw[purple] (A) -- (O) (B) -- (O) (C) -- (O);
  \draw (A) node[left] {$A$};
  \draw (B) node[right] {$B$};
  \draw (C) node[above] {$C$};
  \coordinate (D) at ($(B)!(O)!(C)$);
  \coordinate (E) at ($(C)!(O)!(A)$);
  \coordinate (F) at ($(A)!(O)!(B)$);
  \draw[red] (O) -- (D) (O) -- (E) (O) -- (F);
  \draw (A) -- (B) node[near start,sloped] {$|$}
    node[near end,sloped] {$|$};
```

```
  \draw (B) -- (C) node[near start,sloped] {$||$}
  ↪  node[near end,sloped] {$||$};
  \draw (C) -- (A) node[near start,sloped] {$|||$}
  ↪  node[near end,sloped] {$|||$};
  \pic [draw,red,angle radius=6pt] {right angle=O--D--C};
  \pic [draw,red,angle radius=6pt] {right angle=O--E--A};
  \pic [draw,red,angle radius=6pt] {right angle=O--F--B};
\end{tikzpicture}
```



2.4 内心 Incenter

调用方式

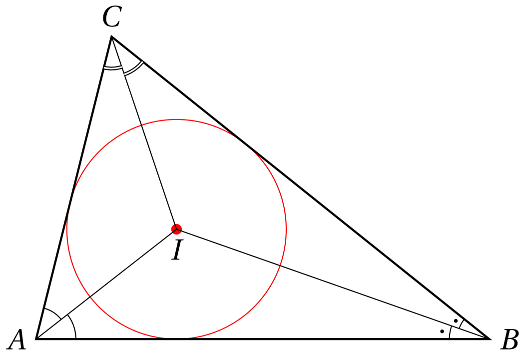
```
incenter={A,B,C}
```

参数说明

A, B, C 三角形的顶点

示例

```
\begin{tikzpicture}
  \coordinate (A) at (-2,0);
  \coordinate (B) at (4,0);
  \coordinate (C) at (-1,4);
  \path[incenter={A,B,C}] coordinate (I);
  \fill (I) [red] circle (2pt);
  \draw (I) node[below] {$I$};
  \node[draw,red] at (I) [circle
    ↪ through=($(B)!(I)!(C)$)]{};
  \draw[thick] (A) -- (B) -- (C) -- cycle;
  \draw (A) node[left] {$A$};
  \draw (B) node[right] {$B$};
  \draw (C) node[above] {$C$};
  \draw (A) -- (I) (B) -- (I) (C) -- (I);
  \pic [draw,angle radius=12pt] {angle=I--A--C};
  \pic [draw,angle radius=15pt] {angle=B--A--I};
  \pic [draw,double,angle radius=12pt] {angle=A--C--I};
  \pic [draw,double,angle radius=15pt] {angle=I--C--B};
  \pic [draw,pic text=.,angle radius=12pt,
    angle eccentricity=1.2] {angle=C--B--I};
  \pic [draw,pic text=.,angle radius=15pt,
    angle eccentricity=1.2] {angle=I--B--A};
\end{tikzpicture}
```



2.5 旁心 Excenter

调用方式

```
excenter={A,B,C}
```

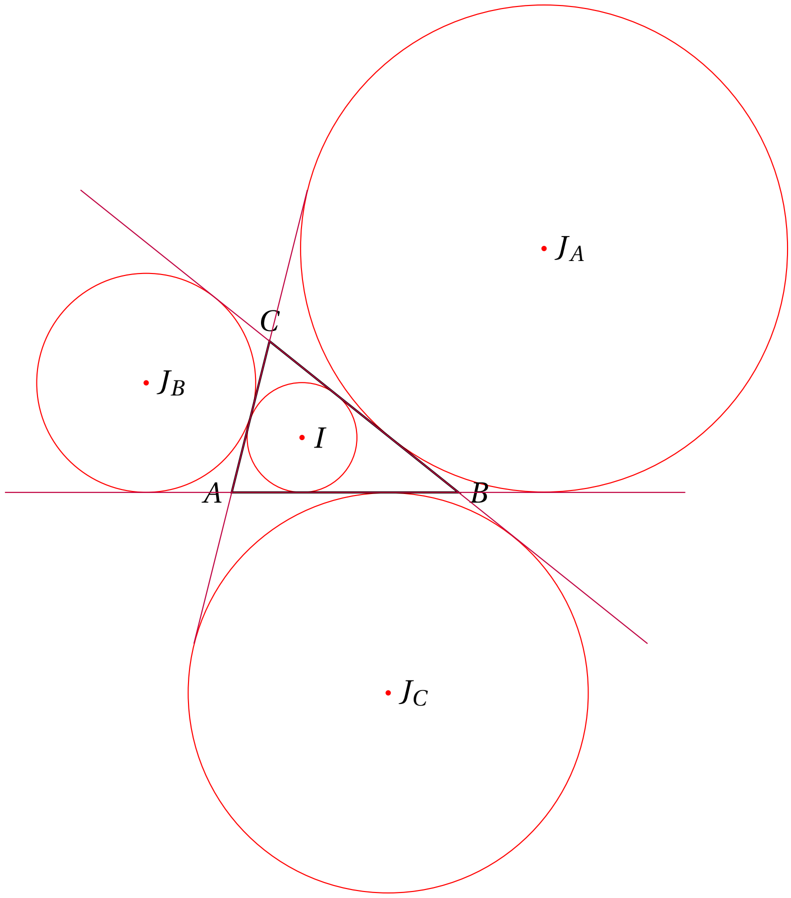
参数说明

A, B, C 三角形的顶点, 返回与 A 相对的旁心, 调换顶点顺序就可以得到 3 个旁心.

示例

```
\begin{tikzpicture}[scale=.5]
  \coordinate (A) at (-2,0);
  \coordinate (B) at (4,0);
  \coordinate (C) at (-1,4);
  \path[incenter={A,B,C}] coordinate (I);
  \path[excenter={A,B,C}] coordinate (JA);
  \path[excenter={B,A,C}] coordinate (JB);
  \path[excenter={C,A,B}] coordinate (JC);
  \foreach \point in {I,JA,JB,JC}
    \fill (\point) [red] circle (2pt);
```

```
  \node[draw,red] at (I) [circle
  ↪ through=($(B)!(I)!(C)$)]{};
  \node[draw,red] at (JA) [circle
  ↪ through=($(B)!(JA)!(C)$)]{};
  \node[draw,red] at (JB) [circle
  ↪ through=($(B)!(JB)!(C)$)]{};
  \node[draw,red] at (JC) [circle
  ↪ through=($(B)!(JC)!(C)$)]{};
  \draw[thick] (A) -- (B) -- (C) -- cycle;
  \draw (A) node[left] {$A$};
  \draw (B) node[right] {$B$};
  \draw (C) node[above] {$C$};
  \draw[purple] ($(A)!-1!(B)$) -- ($(A)!2!(B)$);
  \draw[purple] ($(B)!-1!(C)$) -- ($(B)!2!(C)$);
  \draw[purple] ($(C)!-1!(A)$) -- ($(C)!2!(A)$);
  \draw (I) node[right] {$I$};
  \draw (JA) node[right] {$J_A$};
  \draw (JB) node[right] {$J_B$};
  \draw (JC) node[right] {$J_C$};
\end{tikzpicture}
```



2.6 九点圆圆心 Nine-Point Center

调用方式

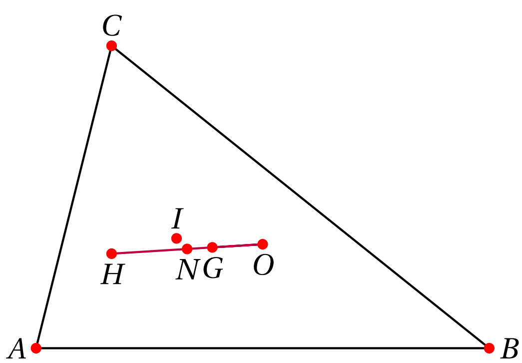
```
nine-point center={A,B,C}
```

参数说明

A, B, C 三角形的顶点

示例

```
\begin{tikzpicture}
  \coordinate (A) at (-2,0);
  \coordinate (B) at (4,0);
  \coordinate (C) at (-1,4);
  \path[orthocenter={A,B,C}] coordinate (H);
  \path[circumcenter={A,B,C}] coordinate (O);
  \path[centroid={A,B,C}] coordinate (G);
  \path[incenter={A,B,C}] coordinate (I);
  \path[nine-point center={A,B,C}] coordinate (N);
  \draw[thick] (A) -- (B) -- (C) -- cycle;
  \draw[thick,purple] (H) -- (O) -- (G);
  \foreach \p/\placement in {A/left,B/right,C/above,
  H/below,O/below,G/below,I/above,N/below}{
    \fill (\p) [red] circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



第 3 章　圆锥曲线 Conics

尽管 tikz 内置的 `ellipse` 和 `parabola` 绘制椭圆和抛物线, 这里定义了 `\ellipse` 和 `\parabola`.

3.1　椭圆 Ellipse

调用方式

```
\ellipse [options] (a,b)
```

参数说明

a, b 半长轴长 (semi-major axis) 和半短轴长 (semi-minor axis), 默认单位为 cm, 可指定单位, 如 `(4cm, 3cm)`

返回中心为原点的椭圆曲线: $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$.

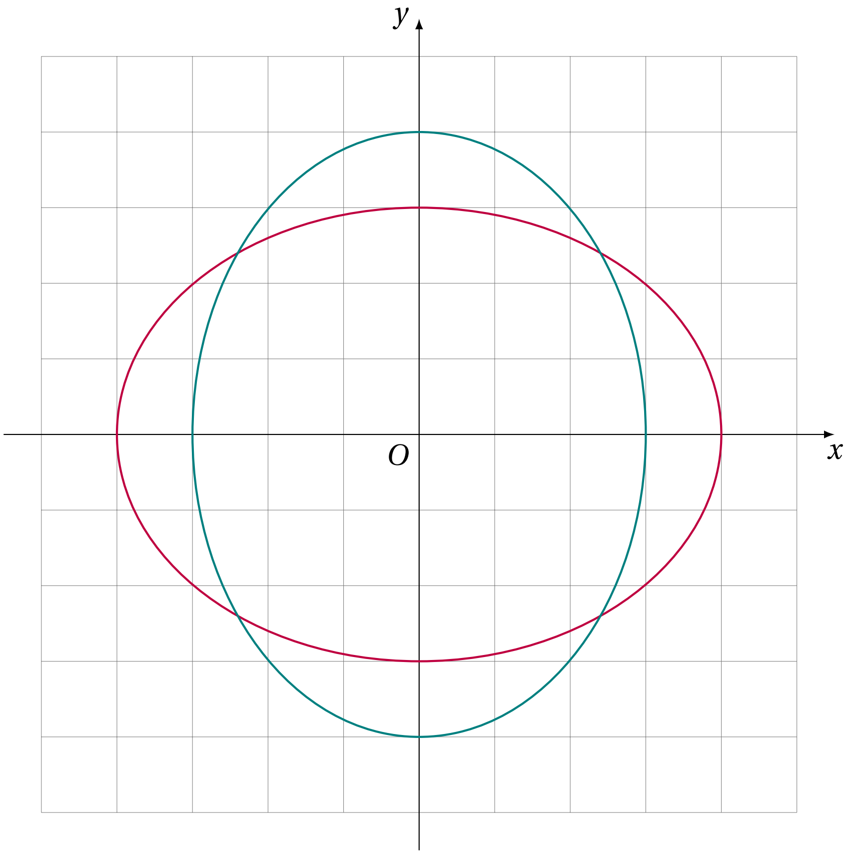
注 当指定椭圆曲线 (ellipse) 的 `domain` (default: `domain=-180:180`) 时, `domain` 是下列参数方程中 t 的取值范围:

$$\begin{cases} x = a\cos t, \\ y = b\sin t \end{cases}$$

示例

使用 tikz 内置曲线:

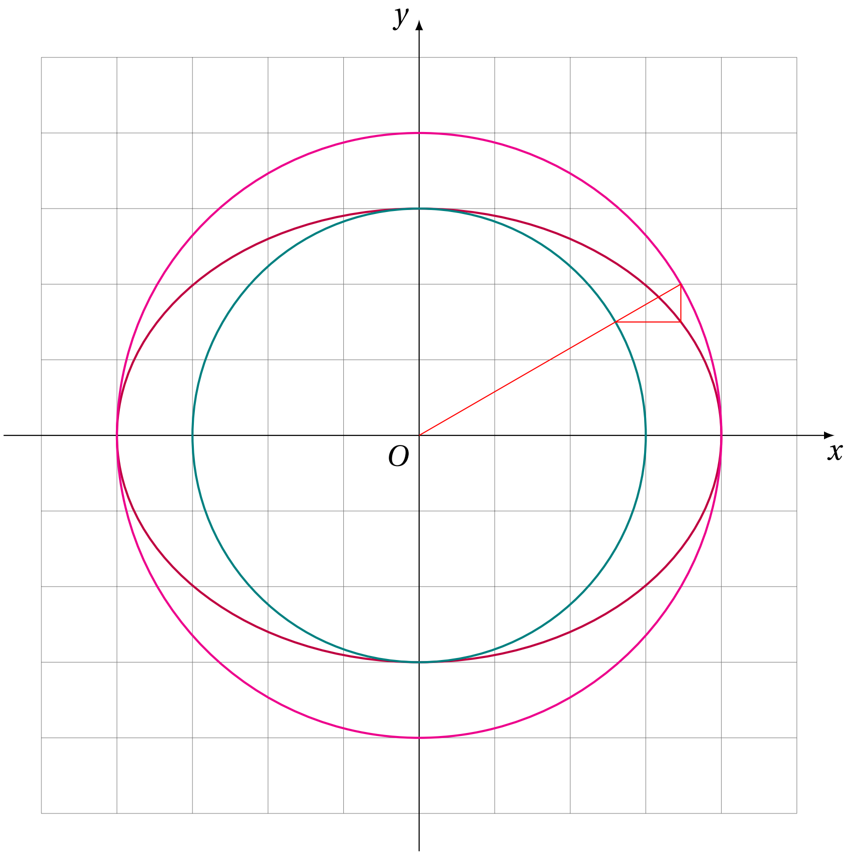
```
\begin{tikzpicture}
  \axes (-5:5,-5:5);
  \draw[thick,purple] (0,0) ellipse (4cm and 3cm);
  \draw[thick,teal] (0,0) ellipse [x radius=3cm,y
   ↪ radius=4cm];
\end{tikzpicture}
```



使用 `\ellipse` 命令:

```
\begin{tikzpicture}
  \axes (-5:5,-5:5);
  \ellipse[draw,thick,purple] (4,3);
```

```
  \ellipse[draw,thick,teal] (3,3);
  \ellipse[draw,thick,magenta] (4,4);

  \coordinate (O) at (0,0);
  \coordinate (A) at ({4*cos(30)},{4*sin(30)});
  \coordinate (B) at ({3*cos(30)},{3*sin(30)});
  \coordinate (C) at ({4*cos(30)},{3*sin(30)});
  \draw[red] (O) -- (A) -- (C) -- (B);
\end{tikzpicture}
```



3.2 抛物线 Parabola

调用方式

```
\parabola [options] (p)
```

参数说明

p 半正焦弦或半通径 (semi-latus rectum), 默认单位为 cm, 可指定单位, 如 `1cm`

返回中心为原点的抛物线: $x^2 = 2py$.

注 抛物线的焦点到顶点的距离为 $\frac{p}{2}$, 抛物线的准线到顶点的距离也是 $\frac{p}{2}$, 因此抛物线的焦准距也为 p(参考附录B).

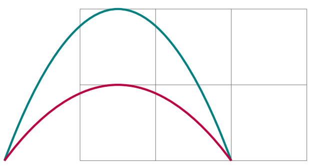
当指定绘制抛物线 (parabola) 的 `domain` (default: `domain=-2:2`) 时, `domain` 是下列参数方程中 t 的取值范围:

$$\begin{cases} x = 2pt, \\ y = 2pt^2 \end{cases}$$

示例

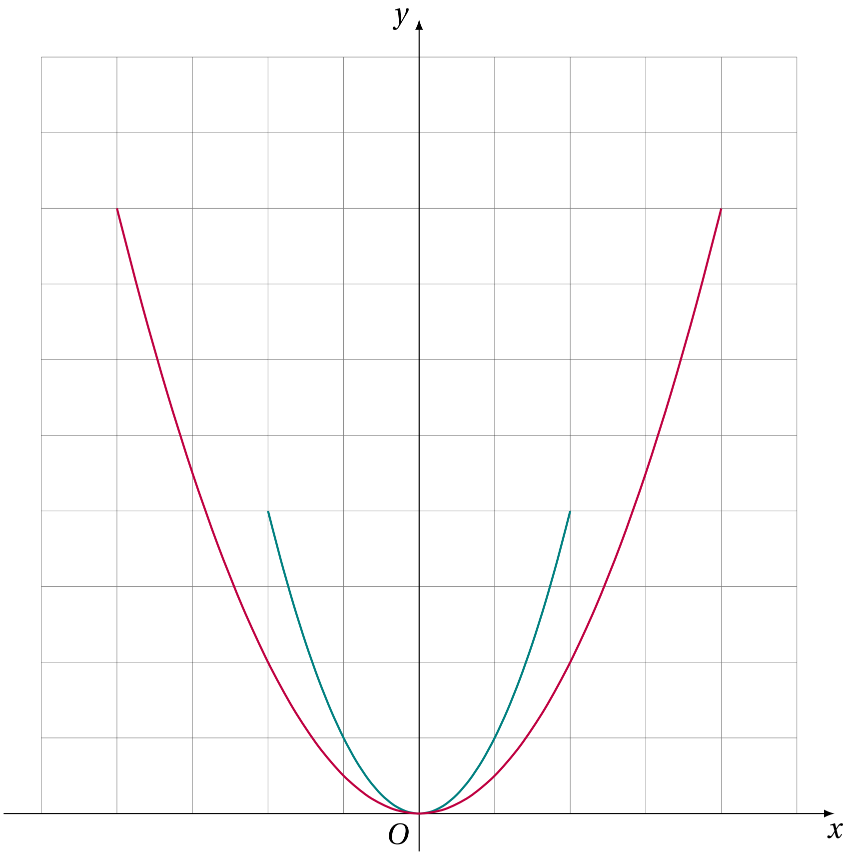
使用 tikz 内置曲线:

```
\begin{tikzpicture}
  \draw[help lines] (0,0) grid (3,2);
  \draw[thick,teal] (-1,0) parabola[bend pos=0.5] bend
   ↪ +(0,2) +(3,0);
  \draw[thick,purple] (-1,0) parabola[parabola
   ↪ height=1cm] +(3,0);
\end{tikzpicture}
```



使用 `\parabola` 命令:

```
\begin{tikzpicture}
  \axes (-5:5,0:10);
  \parabola[draw,thick,teal] (.5);
  \parabola[draw,thick,purple] (1);
\end{tikzpicture}
```



3.3 双曲线 Hyperbola 与渐近线 Asymptote

调用方式

```
\hyperbola [options] (a,b);
```

或

```
\asymptote [options] (a,b);
```

参数说明

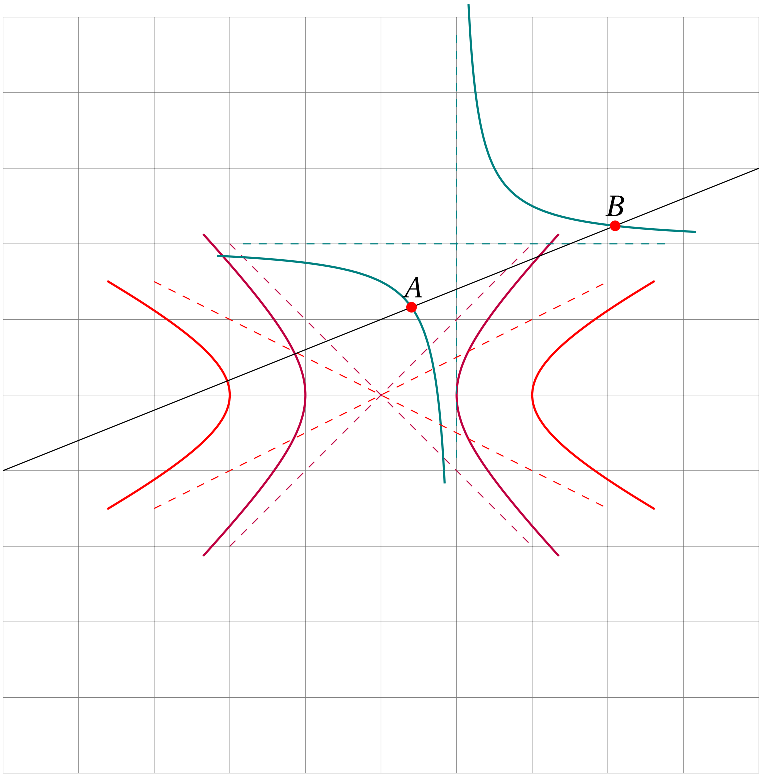
a, b 半实轴长 (semi-major axis) 和半虚轴长 (semi-minor axis), 默认单位为 cm, 可指定单位, 如 (4cm, 3cm)

分别返回中心在原点的双曲线: $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ 和渐近线: $y=\pm\dfrac{b}{a}x$.

示例

```
\begin{tikzpicture}
  \tikzmath{
    \a = 1;
    \b = 1;
  }
  \draw[help lines] (-5,-5) grid (5,5);

  \hyperbola [draw,thick,purple,name path=c1] (\a,\b);
  \asymptote [draw,dashed,purple] (\a,\b);

  \hyperbola [transform={45:(1,2)},draw,thick,teal,name
   ↪ path=c2] (1,1);
  \asymptote [transform={45:(1,2)},draw,dashed,teal]
   ↪ (1,1);

  \hyperbola [draw,thick,red,domain=-1.2:1.2,name
   ↪ path=c3] (2,1);
  \asymptote [draw,dashed,red,domain=-3:3] (2,1);

  \draw[name path=l] (-5,-1) -- (5,3);
```

```
  \path[name intersections={of=c2 and l, by={A,B}, sort
   ↪  by=l}];
  \foreach \p/\placement in {A/above, B/above}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```



注 当指定绘制双曲线 (hyperbola) 的 `domain` (default: `domain=-1.5:1.5`) 时, `domain` 是下列双曲线参数方程中 t 的取值范围:

$$\begin{cases} x = \cosh t, \\ y = \sinh t \end{cases}$$

t 的几何意义: 射线出原点交单位双曲线 $x^2 - y^2 = 1$ 于 $(\cosh t, y = \sinh t)$, t 是射

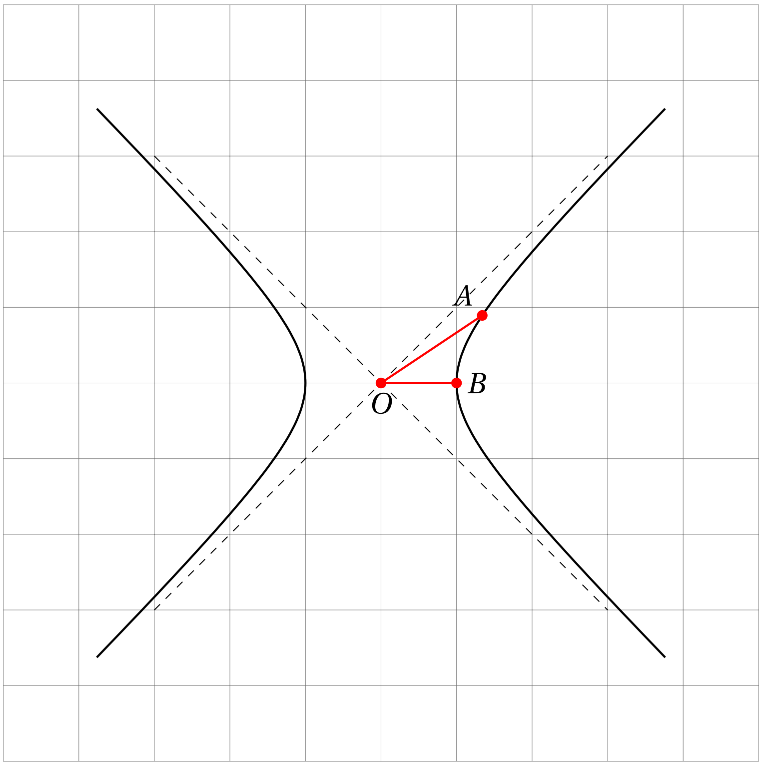
线, 双曲线和 x 轴围成的面积的二倍. 对于双曲线上位于 x 轴下方的点, 这个面积被认为是负值.

当指定绘制渐进线 (asymptote) 的 `domain` (default: `domain=-2:2`) 时, `domain` 是下列直线方程中 x 的取值范围:

$$y = \pm\frac{b}{a}x$$

示例

```
\begin{tikzpicture}
  \draw[help lines] (-5,-5) grid (5,5);

  \coordinate (O) at (0,0);
  \coordinate (A') at (3,2);
  \coordinate (B) at (1,0);

  \path[name path=ray] (O) -- (A');

  \hyperbola[draw,thick,domain=-2:2,name path=hyperbola]
   ↪ (1,1);
  \asymptote[draw,dashed,domain=-3:3] (1,1);

  \path[name intersections={of=ray and hyperbola, by=A}];

  \draw[thick,red] (A) -- (O) -- (B);

  \foreach \p/\placement in {O/below, A/above left,
   ↪ B/right}{
    \fill[red] (\p) circle (2pt);
    \draw (\p) node[\placement] {$\p$};
  }
\end{tikzpicture}
```


A
B
O

附录 A 两直线的交点

求解两直线交点的方程 [4]:

$$
\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0
$$

$$
\begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{vmatrix} = 0
$$

注意, 两个方程的系数都是行列式, 解得:

$$
x = \frac{\begin{vmatrix} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} & \begin{vmatrix} x_1 & 1 \\ x_2 & 1 \end{vmatrix} \\ \begin{vmatrix} x_3 & y_3 \\ x_4 & y_4 \end{vmatrix} & \begin{vmatrix} x_3 & 1 \\ x_4 & 1 \end{vmatrix} \end{vmatrix}}{\begin{vmatrix} \begin{vmatrix} x_1 & 1 \\ x_2 & 1 \end{vmatrix} & \begin{vmatrix} y_1 & 1 \\ y_2 & 1 \end{vmatrix} \\ \begin{vmatrix} x_3 & 1 \\ x_4 & 1 \end{vmatrix} & \begin{vmatrix} y_3 & 1 \\ y_4 & 1 \end{vmatrix} \end{vmatrix}} = \frac{\begin{vmatrix} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} & x_1 - x_2 \\ \begin{vmatrix} x_3 & y_3 \\ x_4 & y_4 \end{vmatrix} & x_3 - x_4 \end{vmatrix}}{\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_3 - x_4 & y_3 - y_4 \end{vmatrix}}
$$

$$y=\frac{\begin{vmatrix}\begin{vmatrix}x_1 & y_1\\ x_2 & y_2\end{vmatrix} & \begin{vmatrix}y_1 & 1\\ y_2 & 1\end{vmatrix}\\ \begin{vmatrix}x_3 & y_3\\ x_4 & y_4\end{vmatrix} & \begin{vmatrix}y_3 & 1\\ y_4 & 1\end{vmatrix}\end{vmatrix}}{\begin{vmatrix}\begin{vmatrix}x_1 & 1\\ x_2 & 1\end{vmatrix} & \begin{vmatrix}y_1 & 1\\ y_2 & 1\end{vmatrix}\\ \begin{vmatrix}x_3 & 1\\ x_4 & 1\end{vmatrix} & \begin{vmatrix}y_3 & 1\\ y_4 & 1\end{vmatrix}\end{vmatrix}}=\frac{\begin{vmatrix}\begin{vmatrix}x_1 & y_1\\ x_2 & y_2\end{vmatrix} & y_1-y_2\\ \begin{vmatrix}x_3 & y_3\\ x_4 & y_4\end{vmatrix} & y_3-y_4\end{vmatrix}}{\begin{vmatrix}x_1-x_2 & y_1-y_2\\ x_3-x_4 & y_3-y_4\end{vmatrix}}$$

进一步化简得到[1]:

$$x=\frac{(x_1y_2-y_1x_2)(x_3-x_4)-(x_1-x_2)(x_3y_4-y_3x_4)}{(x_1-x_2)(y_3-y_4)-(y_1-y_2)(x_3-x_4)}$$
$$y=\frac{(x_1y_2-y_1x_2)(y_3-y_4)-(y_1-y_2)(x_3y_4-y_3x_4)}{(x_1-x_2)(y_3-y_4)-(y_1-y_2)(x_3-x_4)}$$

上述方法给出的交点坐标公式在 TikZ 环境中的计算稳定性不够好, 经常出现 `Dimension too large` 错误, 究其原因是分母可能有时会比较小. 下面给出一个计算更稳定的公式.

我们可以给出两条直线的参数方程:

直线 L_1 的方程:

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}x_1\\ y_1\end{bmatrix}+s\begin{bmatrix}x_2-x_1\\ y_2-y_1\end{bmatrix}$$

直线 L_2 的方程:

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}x_3\\ y_3\end{bmatrix}+t\begin{bmatrix}x_4-x_3\\ y_4-y_3\end{bmatrix}$$

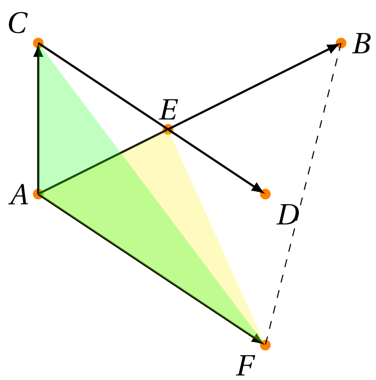
[1] https://en.wikipedia.org/wiki/Lineline_intersection

可以解出 s, t:

$$s = \frac{\begin{vmatrix} x_1 - x_3 & x_3 - x_4 \\ y_1 - y_3 & y_3 - y_4 \end{vmatrix}}{\begin{vmatrix} x_1 - x_2 & x_3 - x_4 \\ y_1 - y_2 & y_3 - y_4 \end{vmatrix}}$$

$$t = \frac{\begin{vmatrix} x_1 - x_3 & x_1 - x_2 \\ y_1 - y_3 & y_1 - y_2 \end{vmatrix}}{\begin{vmatrix} x_1 - x_2 & x_3 - x_4 \\ y_1 - y_2 & y_3 - y_4 \end{vmatrix}}$$

我们也可从几何的角度来分析:



$$\overrightarrow{AE} = s\overrightarrow{AB}$$

$$\begin{aligned} s &= \frac{S_{\triangle AEF}}{S_{\triangle ABF}} \\ &= \frac{S_{\triangle ACF}}{S_{\triangle ABF}} \\ &= \frac{\overrightarrow{AF} \times \overrightarrow{AC}}{\overrightarrow{AF} \times \overrightarrow{AB}} \\ &= \frac{\overrightarrow{CD} \times \overrightarrow{AC}}{\overrightarrow{CD} \times \overrightarrow{AB}} \end{aligned}$$

为了保证数值计算的稳定性, 可以对下面的方程进行列主元消元法求解:

$$x_1 + s(x_2 - x_1) = x3 + t(x_4 - x3)$$
$$y_1 + s(y_2 - y_1) = y3 + t(y_4 - y3)$$

附录 B 通径 Latus Rectum

通径 (latus rectum) 亦称" 正通径"," 首通径"," 直焦弦"," 主焦弦"," 正焦弦". 过圆锥曲线的焦点且与过焦点的轴垂直的弦称为通径, 清代明安图《割环密率捷法》中, 称圆的直径为通径.

B.1 椭圆的通径与焦准距

连接椭圆上任意两点的线段叫作这个**椭圆的弦**, 通过焦点的弦叫作这个椭圆的**焦点弦** (所以椭圆的长轴也是焦点弦), 和长轴垂直的焦点弦叫作这个椭圆的**通径 (正焦弦)**. 连接椭圆上任意一点与一个焦点的线段 (或这线段的长) 叫作椭圆在这点的**焦半径**, 椭圆上任意一点有两条焦半径.

设椭圆的方程为

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,(a>b>0)$$

其通径的长为 $\frac{2b^2}{a}$, 或 $2ep$, 其中: a 为**半长轴长**, b 为**半短轴长**, e 为椭圆的**离心率**, p 为椭圆的**焦准距**.

焦点与相应准线的距离称为椭圆的**焦准距**, 也叫**焦参数**.

$$p=\frac{a^2}{c}-c=\frac{a^2-c^2}{c}=\frac{b^2}{c}$$

B.2 抛物线的通径与焦准距

经过抛物线的焦点, 作一条垂直于它的对称轴的直线, 这直线与抛物线有两个交点, 这两个交点之间的线段叫做**抛物线的通径**.

抛物线的方程:

$$y^2 = 2px$$

抛物线的通径长为 $2p$, 其中: 抛物线的焦准距长为 p.

B.3 双曲线的通径与焦准距

过双曲线的焦点与双曲线的实轴垂直的直线被双曲线截得的线段的长, 称为双曲线的通径.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > 0, b > 0)$$

其通径长 $\frac{2b^2}{a}$, 或 $2ep$, 其中: a 为半实轴长, b 为半虚轴长, e 为双曲线的离心率, p 为双曲线的焦准距.

附录 C　源代码

```
\ProvidesFile{tikzlibraryeuclidea.code.tex}[2024/01/11
 ↪  v1.3.1 A tikz library for plane geometry]

\usetikzlibrary{math,calc,quotes}

% https://tex.stackexchange.com/questions/455991/␣
 ↪  pgfmath-function-for-strings-and-numbers
% Solving the error:
% Package PGF Math: Could not parse input 'A' as a
 ↪  floating
% point number, sorry. The unreadable part was near
'A'..
\pgfkeys{
  /pgf/fpu/handlers/invalid number/.code = {%
    \pgfmathfloatparsenumber{3Y0.0e0]}%
  }
}

\makeatletter

% 注意：计算过程是保留坐标单位（pt）的，所以存在乘除法单位
 ↪  的问题，首先数值始终携带单位，
```

```
% 在 calc 运算时有的需要转换为标量；将坐标转换为 pt 值，数
↪ 值可能超出限值，出现
% Dimension too large 错误，在计算长度时及时进行缩小
% https://tex.stackexchange.com/questions/475556/tikz-↵
↪ why-is-dimension-too-large
% 具体方法是修改默认的 1cm，如:
↪ [scale=1.0,x=0.5cm,y=0.5cm]
% 注意此处的变量不要和 tikzpicture 环境重名，否则被替换掉
% triangle centers:
% https://mathworld.wolfram.com/BarycentricCoordinates.↵
↪ html
\tikzmath{
  % 采用列主元消元法求直线 P1P2 与直线 P3P4 的交点 P 位置
  ↪ 参数 s: s = P1P/P1P2
  function intersectll(\x1,\y1,\x2,\y2,\x3,\y3,\x4,\y4)
{
    \a1 = \x2-\x1; \b1 = \x3-\x4; \c1 = \x3-\x1;
    \a2 = \y2-\y1; \b2 = \y3-\y4; \c2 = \y3-\y1;
    \dmax = max(max(abs(\a1),abs(\a2)),
    ↪ max(abs(\b1),abs(\b2)));
    \a1 = \a1/\dmax; \b1 = \b1/\dmax; \c1=\c1/\dmax;
    \a2 = \a2/\dmax; \b2 = \b2/\dmax; \c2=\c2/\dmax;
    if abs(\a1) < abs(\a2) then {
      \temp = \a1; \a1 = \a2; \a2 = \temp;
      \temp = \b1; \b1 = \b2; \b2 = \temp;
      \temp = \c1; \c1 = \c2; \c2 = \temp;
    };
    \b1 = \b1/\a1; \c1 = \c1/\a1; \a1 = 1.0;
    \b2 = \b2-\a2*\b1; \c2 = \c2-\a2*\c1; \a2 = 0.0;
    \n2 = \c2/\b2; \n1 = \c1-\b1*\n2;
```

```
    return \n1;
  };
}

\tikzset{
  % specifying start and end with modifiers(see tikz
   ↪ manual 13.5)
  % commands supporting partway modifiers:
  % radical axis, perpendicular bisector, perpendicular,
   ↪ parallel
  start modifier/.initial = 0,
  start modifier/.default = 0,
  end modifier/.initial = 1,
  end modifier/.default = 1,
  % ========== Coordinates Transformations ==========
  % affine={A,B,k}: returns affine combination of two
   ↪ points
  % with affine ratio, i.e. A + k * ( B - A )
  affine/.style args = {#1,#2,#3}{
    insert path = {
      ($(#1)!{#3}!(#2)$)
    }
  },
  % midpoint={A,B}: returns midpoint of AB.
  midpoint/.style args = {#1,#2}{
    insert path = {
      ($(#1)!.5!(#2)$)
    }
  },
  % translate={A,B,C}: returns translation of C by
```

```
% the vector AB, i.e. C + ( B - A )
translate/.style args = {#1,#2,#3}{
  insert path = {
    ($(#3)+(#2)-(#1)$)
  }
},
% reflect={A,B,C}: reflects point C across line AB.
reflect/.style args = {#1,#2,#3}{
  insert path = {
    let
      \p{ft} = ($(#1)!(#3)!(#2)$),% perpendicular foot
    in ($(#3)!2!(\p{ft})$)
  }
},
% project={A,B,C}: projects point C onto line AB.
project/.style args = {#1,#2,#3}{
  insert path = {
    ($(#1)!(#3)!(#2)$)
  }
},
% inverse={O,A,P}: returns inverse point P with respect
 ↪  to
% a reference circle(O,A).
inverse/.style args = {#1,#2,#3}{
  insert path = {
    let
      \p{OA} = ($(#2)-(#1)$),
      \p{OP} = ($(#3)-(#1)$),
      \n{r} = {veclen(\p{OA})},
      \n{d} = {veclen(\p{OP})},
```

```
      \n1 = {scalar((\n{r}/\n{d}))},
    in ($(#1)!\n1*\n1!(#3)$)
  }
},
revolve/scale/.initial = 1,% angle scale
revolve/@angle/.initial = 90,
revolve/@argn/.initial = 0,% arguments count
% set revolve/@angle with certain degrees or angle of a
 ↪  vector
revolve/@set angle 1/.code args = {#1}{
  \pgfmathanglebetweenpoints
    {\pgfpoint{0cm}{0cm}}
    {\pgfpointanchor{#1}{center}}
  \pgfkeysalso{/tikz/revolve/@angle = \pgfmathresult}
  \typeout{=========================}
  \typeout{/tikz/revolve/@angle:\pgfkeysvalueof{/tikz/⌋
   ↪  revolve/@angle}}
  \typeout{=========================}
},
% set revolve/@angle with angle between two position
 ↪  vectors
revolve/@set angle 2/.code args = {#1,#2}{
  \pgfmathanglebetweenpoints
    {\pgfpointanchor{#1}{center}}
    {\pgfpointanchor{#2}{center}}
  \pgfkeysalso{/tikz/revolve/@angle = \pgfmathresult}
  \typeout{=========================}
  \typeout{/tikz/revolve/@angle:\pgfkeysvalueof{/tikz/⌋
   ↪  revolve/@angle}}
  \typeout{=========================}
```

```
},
% set revolve/@angle with angle {A,B,C}, angle between
  ↪ two sides
% (A is apex, B is the start point, C is the end point)
  ↪
revolve/@set angle 3/.code args = {#1,#2,#3}{
  \pgfmathanglebetweenlines
    {\pgfpointanchor{#1}{center}}
    {\pgfpointanchor{#2}{center}}
    {\pgfpointanchor{#1}{center}}
    {\pgfpointanchor{#3}{center}}
  \pgfkeysalso{/tikz/revolve/@angle = \pgfmathresult}
  \typeout{=========================}
  \typeout{/tikz/revolve/@angle:\pgfkeysvalueof{/tikz/⌋
    ↪ revolve/@angle}}
  \typeout{=========================}
},
% set revolve/@angle with angle between two
  ↪ vectors(ccw, AB and CD)
revolve/@set angle 4/.code args = {#1,#2,#3,#4}{
  \pgfmathanglebetweenlines
    {\pgfpointanchor{#1}{center}}
    {\pgfpointanchor{#2}{center}}
    {\pgfpointanchor{#3}{center}}
    {\pgfpointanchor{#4}{center}}
  \pgfkeysalso{/tikz/revolve/@angle = \pgfmathresult}
  \typeout{=========================}
  \typeout{/tikz/revolve/@angle:\pgfkeysvalueof{/tikz/⌋
    ↪ revolve/@angle}}
  \typeout{=========================}
```

```
  },
  revolve/angle/.code = {%
    \pgfmathfloatparsenumber{#1}
    \pgfmathfloattomacro{\pgfmathresult}{\F}{\M}{\E}
    \ifnum \F < 3%number
      \pgfmathparse{#1}
    \else
      % Compute the Number of Arguments
      \pgfutil@for\arg:=#1\do{
        \pgfmathparse{int(add(\pgfkeysvalueof{/tikz/⌋
         ↪  revolve/@argn},1))}
        \pgfkeysalso{/tikz/revolve/@argn =
         ↪  \pgfmathresult}
      }
      \ifnum \pgfkeysvalueof{/tikz/revolve/@argn} = 1
        \tikzset{revolve/@set angle 1 = {#1}}
      \else\ifnum \pgfkeysvalueof{/tikz/revolve/@argn} =
       ↪  2
        \tikzset{revolve/@set angle 2 = {#1}}
      \else\ifnum \pgfkeysvalueof{/tikz/revolve/@argn} =
       ↪  3
        \tikzset{revolve/@set angle 3 = {#1}}
      \else\ifnum \pgfkeysvalueof{/tikz/revolve/@argn} =
       ↪  4
        \tikzset{revolve/@set angle 4 = {#1}}
      \else
        \pgferror{"Incorrect number of arguments!"}
      \fi\fi\fi
      \fi
    \fi
```

```
  \pgfkeysalso{/tikz/revolve/@angle = \pgfmathresult}
},
% revolve={A,B}: rotates point B by the angle around
 ↪  point A.
revolve/.style args = {#1,#2}{
  insert path = {
    let
      \n1 = {\pgfkeysvalueof{/tikz/revolve/@angle}},
      \n2 = {\pgfkeysvalueof{/tikz/revolve/scale}}
    in ($(#1)!1!\n1*\n2:(#2)$)
  }
},
% angle bisector={A,B,C}: alias for [revolve/angle={⌋
 ↪  A,B,C},revolve/scale=.5,revolve={A,B}]
angle bisector/.style args = {#1,#2,#3}{
  revolve/angle={#1,#2,#3},revolve/scale=.5,revolve={#⌋
   ↪  1,#2}
},
% erect={A,B}: alias for
 ↪  [revolve/angle=90,revolve={A,B}]
erect/.style args = {#1,#2}{
  revolve/angle=90,revolve={#1,#2}
},
% equilateral={A,B}: alias for
 ↪  [revolve/angle=60,revolve={A,B}]
equilateral/.style args = {#1,#2}{
  revolve/angle=60,revolve={#1,#2}
},
% cut a line segment of a certain length on a straight
 ↪  line
```

```
intercept/@length/.initial = 1cm,
intercept/scale/.initial = 1,% length scale
intercept/length/.code = {% set length by distance of
 ↪  segment
  \pgfutil@in@{,}{#1}
  \ifpgfutil@in@%compute segment length
    \euclidea@ComputeLength#1\euclidea@stop
    \pgfkeysalso{/tikz/intercept/@length =
     ↪  \pgfmathresult}
  \else
    \pgfkeysalso{/tikz/intercept/@length = #1}
  \fi
  \typeout{=========================}
  \typeout{/tikz/intercept/@length:\pgfkeysvalueof{/⌋
   ↪  tikz/intercept/@length}}
  \typeout{=========================}
},
% intercept={A,B}: intercepts a line segment(starting
% from point A) of a certain length on line AB.
intercept/.style args = {#1,#2}{
  insert path = {
    let
      \n1 = {\pgfkeysvalueof{/tikz/intercept/@length}},
      \n2 = {\pgfkeysvalueof{/tikz/intercept/scale}},
      \p{AB} = ($(#2)-(#1)$),
      \n{d} = {veclen(\p{AB})},
      \n3 = {scalar(\n1*\n2/\n{d})}
    in ($(#1)!\n3!(#2)$)
  }
},
```

```
% intersect={A,B,C,D}: returns the intersection
 ↪ coordinate
% of line AB and line CD.
% https://en.wikipedia.org/wiki/Line%E2%80%
 ↪ 93line_intersection
intersect/.style args = {#1,#2,#3,#4}{
  insert path = {
    let
      \p1 = (#1), \p2 = (#2), \p3 = (#3), \p4 = (#4),
      \n1 = {intersectll(\x1,\y1,\x2,\y2,\x3,\y3,\x4,\
       ↪ y4)},
    in ($(\p1)!\n1!(\p2)$)
  }
},
% ========== Triangle Centers ==========
% calculated from barycentric coordinates
% incenter = {A,B,C}
incenter/.style args = {#1,#2,#3}{
  insert path = {
    let
      \p1 = (#1), \p2 = (#2), \p3 = (#3),
      \p{AB} = ($(#2)-(#1)$),
      \p{BC} = ($(#3)-(#2)$),
      \p{CA} = ($(#1)-(#3)$),
      \n{a} = {veclen(\x{BC}, \y{BC})},
      \n{b} = {veclen(\x{CA}, \y{CA})},
      \n{c} = {veclen(\x{AB}, \y{AB})},
      \n{s} = {\n{a}+\n{b}+\n{c}},
      \n1 = {\n{a}/\n{s}},
      \n2 = {\n{b}/\n{s}},
```

```
      \n3 = {\n{c}/\n{s}},
    in ({\n1*\x1+\n2*\x2+\n3*\x3,\n1*\y1+\n2*\y2+\n3*\
    ↪ y3})
  }
},
% excenter = {A,B,C}, returns excenter opposite to the
↪ vertex A
excenter/.style args = {#1,#2,#3}{
  insert path = {
    let
      \p1 = (#1), \p2 = (#2), \p3 = (#3),
      \p{AB} = ($(#2)-(#1)$),
      \p{BC} = ($(#3)-(#2)$),
      \p{CA} = ($(#1)-(#3)$),
      \n{a} = {veclen(\x{BC}, \y{BC})},
      \n{b} = {veclen(\x{CA}, \y{CA})},
      \n{c} = {veclen(\x{AB}, \y{AB})},
      \n{s} = {-\n{a}+\n{b}+\n{c}},
      \n1 = {\n{a}/\n{s}},
      \n2 = {\n{b}/\n{s}},
      \n3 = {\n{c}/\n{s}},
    in ({-\n1*\x1+\n2*\x2+\n3*\x3,-\n1*\y1+\n2*\y2+\
    ↪ n3*\y3})
  }
},
% circumcenter = {A,B,C}
circumcenter/.style args = {#1,#2,#3}{
  insert path = {
    let
      \p1 = (#1), \p2 = (#2), \p3 = (#3),
```

```
      \p{AB} = ($(#2)-(#1)$),
      \p{BC} = ($(#3)-(#2)$),
      \p{CA} = ($(#1)-(#3)$),
      \n{a} = {veclen(\x{BC}, \y{BC})},
      \n{b} = {veclen(\x{CA}, \y{CA})},
      \n{c} = {veclen(\x{AB}, \y{AB})},
      \n{m} = {max(max(\n{a},\n{b}),\n{c})},
      \n{a} = {\n{a}/\n{m}},
      \n{a} = {\n{a}*\n{a}},
      \n{b} = {\n{b}/\n{m}},
      \n{b} = {\n{b}*\n{b}},
      \n{c} = {\n{c}/\n{m}},
      \n{c} = {\n{c}*\n{c}},
      \n1 = {\n{a}*(\n{b}+\n{c}-\n{a})},
      \n2 = {\n{b}*(\n{c}+\n{a}-\n{b})},
      \n3 = {\n{c}*(\n{a}+\n{b}-\n{c})},
      \n{s} = {\n1+\n2+\n3},
      \n1 = {\n1/\n{s}},
      \n2 = {\n2/\n{s}},
      \n3 = {\n3/\n{s}},
    in ({\n1*\x1+\n2*\x2+\n3*\x3,\n1*\y1+\n2*\y2+\n3*\
     ↪  y3})
  }
},
% orthocenter = {A,B,C}
orthocenter/.style args = {#1,#2,#3}{
  insert path = {
    let
      \p1 = (#1), \p2 = (#2), \p3 = (#3),
      \p{AB} = ($(#2)-(#1)$),
```

```
      \p{BC} = ($(#3)-(#2)$),
      \p{CA} = ($(#1)-(#3)$),
      \n{a} = {veclen(\x{BC}, \y{BC})},
      \n{b} = {veclen(\x{CA}, \y{CA})},
      \n{c} = {veclen(\x{AB}, \y{AB})},
      \n{m} = {max(max(\n{a},\n{b}),\n{c})},
      \n{a} = {\n{a}/\n{m}},
      \n{a} = {\n{a}*\n{a}},
      \n{b} = {\n{b}/\n{m}},
      \n{b} = {\n{b}*\n{b}},
      \n{c} = {\n{c}/\n{m}},
      \n{c} = {\n{c}*\n{c}},
      \n{a2} = {\n{b}+\n{c}-\n{a}},
      \n{b2} = {\n{c}+\n{a}-\n{b}},
      \n{c2} = {\n{a}+\n{b}-\n{c}},
      \n1 = {\n{c2}*\n{b2}},
      \n2 = {\n{a2}*\n{c2}},
      \n3 = {\n{b2}*\n{a2}},
      \n{s} = {\n1+\n2+\n3},
      \n1 = {\n1/\n{s}},
      \n2 = {\n2/\n{s}},
      \n3 = {\n3/\n{s}},
    in ({\n1*\x1+\n2*\x2+\n3*\x3,\n1*\y1+\n2*\y2+\n3*\
     ↪  y3})
  }
},
% centroid = {A,B,C}
centroid/.style args = {#1,#2,#3}{
  insert path = {
    let
```

```
      \p1 = (#1), \p2 = (#2), \p3 = (#3),
    in ({(\x1+\x2+\x3)/3},{(\y1+\y2+\y3)/3})
  }
},
% nine-pint center = {A,B,C}
nine-point center/.style args = {#1,#2,#3}{
  insert path = {
    let
      \p1 = (#1), \p2 = (#2), \p3 = (#3),
      \p{AB} = ($(#2)-(#1)$),
      \p{BC} = ($(#3)-(#2)$),
      \p{CA} = ($(#1)-(#3)$),
      \n{a} = {veclen(\x{BC}, \y{BC})},
      \n{b} = {veclen(\x{CA}, \y{CA})},
      \n{c} = {veclen(\x{AB}, \y{AB})},
      \n{m} = {max(max(\n{a},\n{b}),\n{c})},
      \n{a} = {\n{a}/\n{m}},
      \n{a} = {\n{a}*\n{a}},
      \n{b} = {\n{b}/\n{m}},
      \n{b} = {\n{b}*\n{b}},
      \n{c} = {\n{c}/\n{m}},
      \n{c} = {\n{c}*\n{c}},
      \n1 = {\n{a}*(\n{b}+\n{c})-(\n{b}-\n{c})*(\n{b}-
       ↪  \n{c})},
      \n2 = {\n{b}*(\n{c}+\n{a})-(\n{c}-\n{a})*(\n{c}-
       ↪  \n{a})},
      \n3 = {\n{c}*(\n{a}+\n{b})-(\n{a}-\n{b})*(\n{a}-
       ↪  \n{b})},
      \n{s} = {\n1+\n2+\n3},
      \n1 = {\n1/\n{s}},
```

```
      \n2 = {\n2/\n{s}},
      \n3 = {\n3/\n{s}},
    in ({\n1*\x1+\n2*\x2+\n3*\x3,\n1*\y1+\n2*\y2+\n3*\
     ↪  y3})
  }
},
% ========== Circle Operations ==========
% circle = {O,A}, creates circle with the center (O)
 ↪  through A
circle/.style args = {#1,#2}{
  insert path = {
    let
      \p{OA} = ($(#2)-(#1)$),
    in (#1) circle ({veclen(\p{OA})})
  }
},
% tagent point = {O,A,P}
% O,A: center of circle and an abitary point on the
 ↪  circle
% P: a point outside the circle
tangent point/.style args = {#1,#2,#3}{
  insert path = {
    let
      \p{OA} = ($(#2)-(#1)$), % 半径
      \p{OP} = ($(#3)-(#1)$),
      \n1 = {veclen(\p{OA})},
      \n2 = {veclen(\p{OP})},
      \n3 = {scalar(\n1/\n2)}
    in ($(#1)!\n3!{acos(\n1/\n2)}:(#3)$)
  }
```

```
},
% external homothetic center
% O1,A1: center of circle 1 and an abitary point on the
↪ circle
% O2,A2: center of circle 2 and an abitary point on the
↪ circle
external center/.style args = {#1,#2,#3,#4}{
  insert path = {
    let
      \p{O1A1} = ($(#2)-(#1)$),% 半径 O1A1
      \p{O2A2} = ($(#4)-(#3)$),% 半径 O2A2
      \n{r1} = {veclen(\p{O1A1})},
      \n{r2} = {veclen(\p{O2A2})},
      \n1 = {scalar(\n{r1}/(\n{r1}-\n{r2}))}
    in ($(#1)!\n1!(#3)$)
  }
},
% internal homothetic center
% O1,A1: center of circle 1 and an abitary point on the
↪ circle
% O2,A2: center of circle 2 and an abitary point on the
↪ circle
internal center/.style args = {#1,#2,#3,#4}{
  insert path = {
    let
      \p{O1A1} = ($(#2)-(#1)$),% 半径 O1A1
      \p{O2A2} = ($(#4)-(#3)$),% 半径 O2A2
      \n{r1} = {veclen(\p{O1A1})},
      \n{r2} = {veclen(\p{O2A2})},
      \n1 = {scalar(\n{r1}/(\n{r1}+\n{r2}))}
```

```
    in ($(#1)!\n1!(#3)$)
  }
},
% creates the radical axis of two non-concentric
↪  circles
% O1,A1: center of circle 1 and an abitary point on the
↪  circle
% O2,A2: center of circle 2 and an abitary point on the
↪  circle
radical axis/.style args = {#1,#2,#3,#4}{
  insert path = {
    let
      \n{s} = {\pgfkeysvalueof{/tikz/start modifier}},
      \n{e} = {\pgfkeysvalueof{/tikz/end modifier}},
      \p{O1A1} = ($(#2)-(#1)$),% 半径 O1A1
      \p{O2A2} = ($(#4)-(#3)$),% 半径 O2A2
      \p{O1O2} = ($(#3)-(#1)$),
      \n{r1} = {veclen(\p{O1A1})},
      \n{r2} = {veclen(\p{O2A2})},
      \n{d} = {veclen(\p{O1O2})},
      \n1 = {scalar(\n{r1}/\n{d})},
      \n2 = {scalar(\n{r2}/\n{d})},
      \n3 = {.5*(1+\n1*\n1-\n2*\n2)},
      \p{ft} = ($(#1)!\n3!(#3)$),% perpendicular foot
      \p{s0} = ($(\p{ft})+(-\y{O1O2},\x{O1O2})$),
      \p{e0} = ($(\p{ft})+(\y{O1O2},-\x{O1O2})$),
      \p{s} = ($(\p{s0})!\n{s}!(\p{e0})$),% start
      \p{e} = ($(\p{s0})!\n{e}!(\p{e0})$)% end
    in (\p{s}) -- (\p{e})
  }
```

```
},
% ========== Path Definitions ==========
% perpendicular bisector of the line segment (#1 -- #2)
perpendicular bisector/.style args = {#1,#2}{
  insert path = {
    let
      \n{s} = {\pgfkeysvalueof{/tikz/start modifier}},
      \n{e} = {\pgfkeysvalueof{/tikz/end modifier}},
      \p{AB} = ($(#2)-(#1)$),
      \p{m} = ($(#1)!0.5!(#2)$),% midpoint
      \p{s0} = ($(\p{m})+(-\y{AB},\x{AB})$),% rotate
       ↪ ccw, default start
      \p{e0} = ($(\p{m})+(\y{AB},-\x{AB})$),% rotate
       ↪ cw, default end
      \p{s} = ($(\p{s0})!\n{s}!(\p{e0})$),% start
      \p{e} = ($(\p{s0})!\n{e}!(\p{e0})$)% end
    in (\p{s}) -- (\p{e})
  }
},
% perpendicular line of the line (#1 -- #2) through #3
% specifying start and end with modifiers(see tikz
 ↪ manual 13.5)
perpendicular/.style args = {#1,#2,#3}{
  insert path = {
    let
      \n{s} = {\pgfkeysvalueof{/tikz/start modifier}},
      \n{e} = {\pgfkeysvalueof{/tikz/end modifier}},
      \p{AB} = ($(#2)-(#1)$),
      \p{ft} = ($(#1)!(#3)!(#2)$),% perpedicular foot
      \p{s0} = ($(\p{ft})+(-\y{AB},\x{AB})$),
```

```
      \p{e0} = ($(\p{ft})+(\y{AB},-\x{AB})$),
      \p{s} = ($(\p{s0})!\n{s}!(\p{e0})$),% start
      \p{e} = ($(\p{s0})!\n{e}!(\p{e0})$)% end
    in (\p{s}) -- (\p{e})
  }
},
% parallel line of the line (#1 -- #2) through #3
% specifying start and end with modifiers(see tikz
 ↪  manual 13.5)
parallel/.style args = {#1,#2,#3}{
  insert path = {
    let
      \n{s} = {\pgfkeysvalueof{/tikz/start modifier}},
      \n{e} = {\pgfkeysvalueof{/tikz/end modifier}},
      \p{s0} = (#3),
      \p{e0} = ($(#3)+(#2)-(#1)$),
      \p{s} = ($(\p{s0})!\n{s}!(\p{e0})$),% start
      \p{e} = ($(\p{s0})!\n{e}!(\p{e0})$)% end
    in (\p{s}) -- (\p{e})
  }
},
% alias for parallel={A,B,A}
extend/.style args = {#1,#2} {
  parallel={#1,#2,#1}
},
% rotate around the origin by `angle` and then shift by
 ↪  (`xshift`,`yshift`)
% tansform = {angle:(xshift,yshift)}
transform/.style args = {#1:(#2,#3)} {
  cm={cos(#1), sin(#1), -sin(#1), cos(#1), (#2,#3)}
```

```
  },
}

% Utilities for implementation of 'intercept'
\def\euclidea@ComputeLength#1,#2\euclidea@stop{
  \newdimen\euclidea@ax
  \newdimen\euclidea@ay
  \newdimen\euclidea@bx
  \newdimen\euclidea@by
  \pgfextractx{\euclidea@ax}{\pgfpointanchor{#1}{␣
center}}
  \pgfextracty{\euclidea@ay}{\pgfpointanchor{#1}{␣
center}}
  \pgfextractx{\euclidea@bx}{\pgfpointanchor{#2}{␣
center}}
  \pgfextracty{\euclidea@by}{\pgfpointanchor{#2}{␣
center}}
  % 以下 showthe 指令 overleaf.com 编译通过，而在
    ↪ macOS+texlive 2021 报错
  % \showthe\euclidea@ax
  % \showthe\euclidea@ay
  % \showthe\euclidea@bx
  % \showthe\euclidea@by
  \pgfmathveclen{\euclidea@ax-\euclidea@bx}{\␣
    ↪ euclidea@ay-\euclidea@by}
}


% Syntax of parabola: x^2=2py
%   \parabola [options] (p);
```

```
% wherein, p is the semi-latus rectum.
% x = 2pt, y = 2pt^2
\newcommand\parabola{} % just for safety
\def\parabola[#1]#2(#3){
  \path[smooth,domain=-2:2,#1,variable=\euclidea@var]
    plot({2*(#3)*\euclidea@var},{2*(#3)*\euclidea@var*\
     ↪ euclidea@var})
}

% Syntax of ellipse:
%   \ellipse [options] (a,b);
% wherein, a,b are major/minor semi axis.
\newcommand\ellipse{} % just for safety
\def\ellipse[#1]#2(#3,#4){
  \path[#1] (0,0) ellipse ({#3} and {#4})
}

% Syntax of hyperbola:
%   \hyperbola [options] (a,b);
% wherein, a,b major/minor semi axis.
\newcommand\hyperbola{} % just for safety
\def\hyperbola[#1]#2(#3,#4){
  \path[smooth,domain=-1.5:1.5,#1,variable=\
euclidea@var]
    plot({-(#3)*cosh(\euclidea@var)},{(#4)*sinh(\
     ↪ euclidea@var)})  % right arm
    plot({ (#3)*cosh(\euclidea@var)},{(#4)*sinh(\
     ↪ euclidea@var)})  % left arm
}
```

```
% Syntax of asymptote:
%   \asymptote [options] (a,b);
% wherein, a,b are major/minor semi axis.
\newcommand\asymptote{} % just for safety
\def\asymptote[#1]#2(#3,#4){
  \path[domain=-2:2,#1,variable=\euclidea@var,]
    plot({\euclidea@var},{ (#4)/(#3)*(\euclidea@var)})
    plot({\euclidea@var},{-(#4)/(#3)*(\euclidea@var)})
}

% Syntax of axes:
%   \axes (xmin:xmax, ymin:ymax);
\newcommand\axes{}
\def\axes(#1:#2,#3:#4){
  \draw[help lines] (#1,#3) grid[step=1] (#2,#4);
  \draw[-latex] ({(#1)-0.5},0) -- ({(#2)+0.5},0)
  ↪ node[below] {$x$};
  \draw[-latex] (0,{(#3)-0.5}) -- (0,{(#4)+0.5})
  ↪ node[left] {$y$};
  \draw (0,0) node[below left] {$O$};
}

\makeatother
```

参考文献

[1] Syntax for path specifications. https://tikz.dev/tikz-paths.

[2] Homothetic center. https://en.wikipedia.org/wiki/Homothetic_center.

[3] Coordinate calculations. https://tikz.dev/tikz-coordinates#sec-13.5.

[4] Line-line intersection. https://mathworld.wolfram.com/Line-LineIntersection.html.