2D HEAT CONDUCTION

YANNICK BONDO

Student ID: 50235476

University at Buffalo

MAE 339

In Collaboration with: N/A

12/12/2021

CONTENTS

ABSTRACT

Heat transfer has numerous applications in various engineering fields. It is particularly relevant to engineers as it plays a key role in material selection, machinery efficiency, and chemical kinetics. In this experiment, we asymmetrically heated a 2D conduction plate to a near steady-state and measured the temperature distribution along the plate using an infrared (IR) camera. We then performed a series of heat transfer analyses to compute for the natural convective heat transfer coefficient of the plate and numerically solved for the temperature distribution of the plate using the finite difference method. Consequently, comparing our calculated temperature distribution to our measured values yielded a relative error of 0.1 percent, making our approximation remarkably reliable. Our accurate estimation stemmed, in part, from our boundary condiction as only 0.8 percent of heat was lost to the foam. Although our calculation yielded a relatively accurate temperature distribution, we can improve our approximation by improving insulation to decrease the heat lost to the foam. We can also improve our approximation by increasing the number of nodes on the plate.

INTRODUCTION

In this lab, we were tasked with predicting the 2D temperature distribution in a heated steel plate. We used a combination of thermometers, thermocouples, and infrared cameras (see figure 1) to collect important data, namely the quasi-steady temperature at finite locations, and compared these values to test their validity. Unfortunately, the LabView application wasn't working on experiment day, which limited our data collection tool to the thermometer and the infrared camera. Also, using heat transfer principles, we solved for the convective heat transfer coefficient $\bar{h}$.

Figure 1: FLIR E60 IR camera [2]

Theory

Measuring thermally energized objects directly can be difficult for both safety and practical reasons. Therefore, for this experiment, we opted on utilizing a non-invasive measurement technique, that is using an infrared (IR) camera to obtain temperature data. However, measuring temperature data with an IR camera is limited to the type of material under observation. The IR light leaves a material surface following the radiation heat flux equation

$$q'' = \epsilon\sigma T_s^4 - \alpha\sigma T_{sur}^4 \quad (1)$$

Approximating most surfaces to be a gray body, we assume that the absorptivity α is equal to the emissivity ϵ, yielding the following relationship:

$$q^{''} = \epsilon\sigma\left(T_s^4 - T_{sur}^4\right) \tag{2}$$

Where $q^{''}$ is the rate of radiation heat flux, σ is the Stefan-Boltzmann constant, T_s is the surface temperature, and T_{sur} is the surrounding temperature. Therefore, studying materials with an IR camera requires a lowly reflective (ρ) and opaque ($\tau = 0$) material which in turn would result in a high absorptivity and emissivity based on the conservation law

$$\rho + \alpha + \tau = 1 \tag{3}$$

For smaller objects with high conductivity, we approximate these objects as isothermal using the Lumped Capacitance Method as long as the Biot number is less than 0.1

$$B_i = \frac{hL_c}{k_s} < 0.1 \tag{4}$$

Where h is the convective heat transfer coefficient, k_s is the thermal conductivity of the solid, and L_c is the characteristic length which is based on the object's geometry

$$L_c = \frac{V}{A_s} \tag{5}$$

Where V is the volume and A_s is the surface area. In absence of a forced fluid motion, heat transfer would be conducted by natural convection which is determined from the Rayleigh number

$$Ra_{L_c} = \frac{g\beta\left(T_s - T_\infty\right)L_c^3}{\nu\alpha} \tag{6}$$

Where ν is the kinematic viscosity, α is the thermal diffusivity, and β is the thermal coefficient, which is the inverse of the film temperature, $\beta = \frac{1}{T_f}$. From the

Rayleigh number, the Nusselt number could be determined from the following relationship

Natural Convention for a Horizontal Plate

$$\overline{Nu_L} = 0.54Ra_L^{\frac{1}{4}} \tag{7}$$

Where

$$\overline{Nu_{L_c}} = \frac{\bar{h}L_c}{k_f} \tag{8}$$

k_f is thermal conductivity of the fluid.

For multidimensional conduction, we will be referring to the following finite difference energy balance relations:

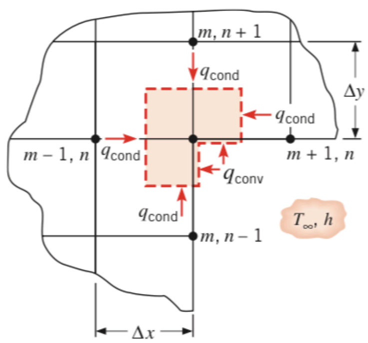


Figure 2: Finite Difference Method. [1]

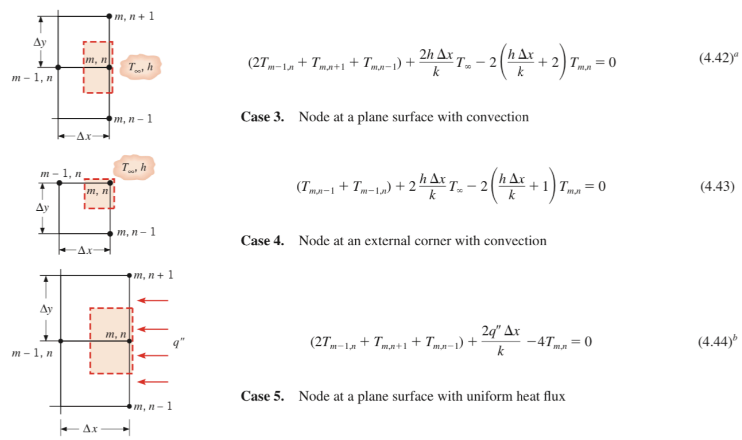


Figure 3: Solutions For $\Delta x = \Delta y$. [1]

METHOD

Experimental procedure

We turned on the IR camera and calibrated the camera by measuring the wall temperature. With a hanging thermometer, we measured the ambient temperature of the room. The measured temperature values turned to be approximately identical, therefore $T_{\infty} = T_{sur}$. Next, we mounted the FLIR E60 IR camera on a quadrapod facing down (see figure 4) and connected the camera to the computer via a USB. We opened the "ResearchIR(64-bit)" software and connected the computer to the camera's live feed. We had the IR camera facing a 2D conduction plate nested in a square insulating foam pit (see figure 4). The 2D conduction plate is coated with a high emissivity flat black spray to improve the accuracy of the temperature readings by the IR camera. Next, we drew a measurement box of 192 *pixels* x 192 *pixels* in the ResearchIR software around the edge of the plate, precisely enveloping the entire plate, to capture the temperature readings of the plate.

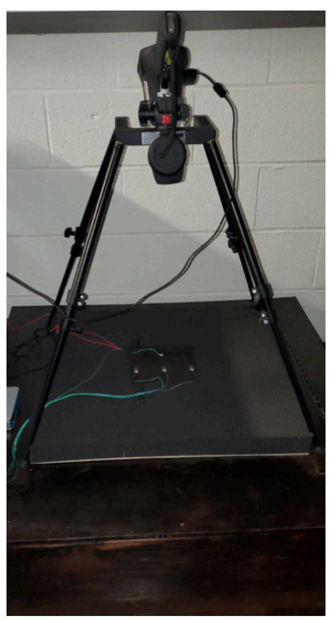


Figure 4: FLIR E60 IR camera setup [2]

Before heating the 2D conduction plate, we cold swapped our positive and ground leads, that is: disconnected the positive and ground leads from the DC power supply. Turned on the DC power supply setting the voltage to 20.1 V. Turned off the DC power supply, then reconnected the positive and ground leads whilst the DC power supply is still off. Then, we turned on the power supply, which in turn turned on the heater. The 2D conduction plate was heated by an Omega strip heater centered along the insulated edge of the plate, between the foam and the 2D conducting plate. The plate was heated for 2.5 hours to allow the plate to reach an approximate steady temperature. Finally, with the IR software, we captured an instantaneous snapshot of the temperature of every pixel within the measurement box (see figure 5) and exported our data for further analysis.

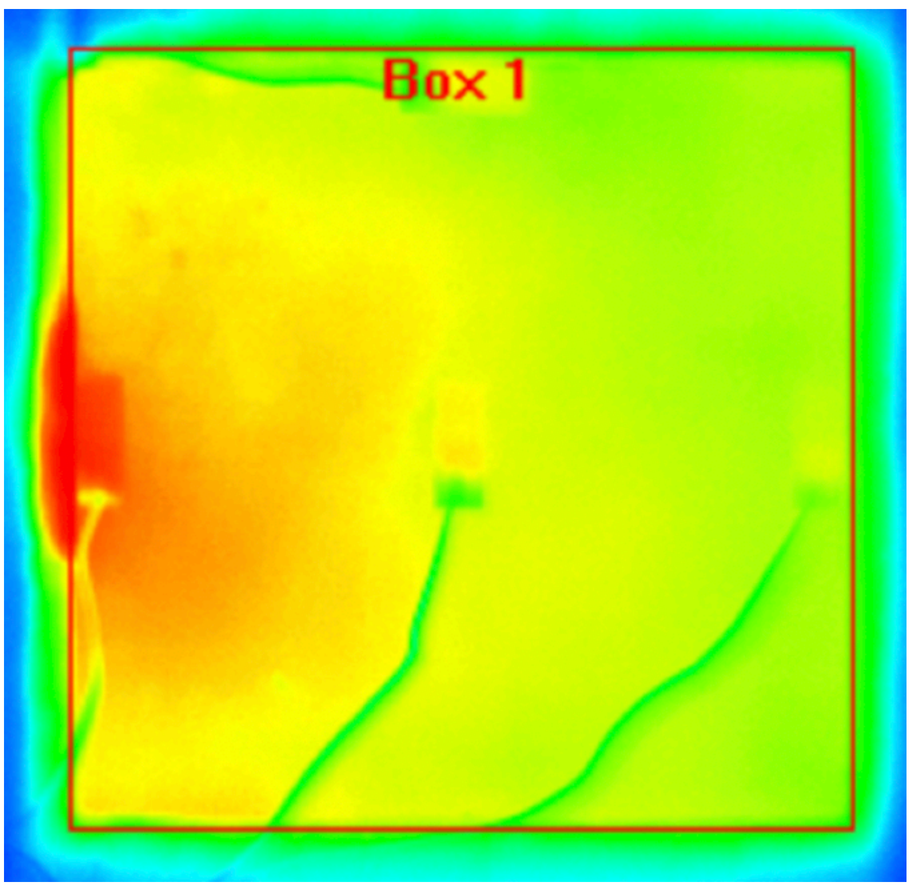


Figure 5: 2D Temperature Distribution Of The Plate

RESULTS

From our exported data, we measured the average temperature $\bar{T}$, for the entire plate to be, $\bar{T} = \mathbf{310.67\ K}$. Next, we defined the average temperature $\bar{T}$, as the temperature operating range for the plate and calculated for the average convective heat transfer coefficient $\bar{h}$ using equation 8, $\bar{h} = \mathbf{7.91\ W/m^2}$. To facilitate our data analysis, we opted for determining the scale factor $s_L = \frac{pix}{mm}$, from the physical plate dimensions and found the scale factor to be $S_L = \mathbf{1.2598}$ **pix/mm**. Next, we constructed a nodal network (see figure 5) such that $\Delta x = \Delta y = \mathbf{48\ pixels \equiv 38.1\ mm}$. We assumed a finite volume $\delta\forall = \Delta x \Delta y \Delta z$ to be isothermal. We did this by verifying if the Biot Number of the finite volume $\delta\forall$, is less than 0.1, using equation 4 and equation 5, we found the $Bi = \mathbf{0.0019 < 0.1}$.

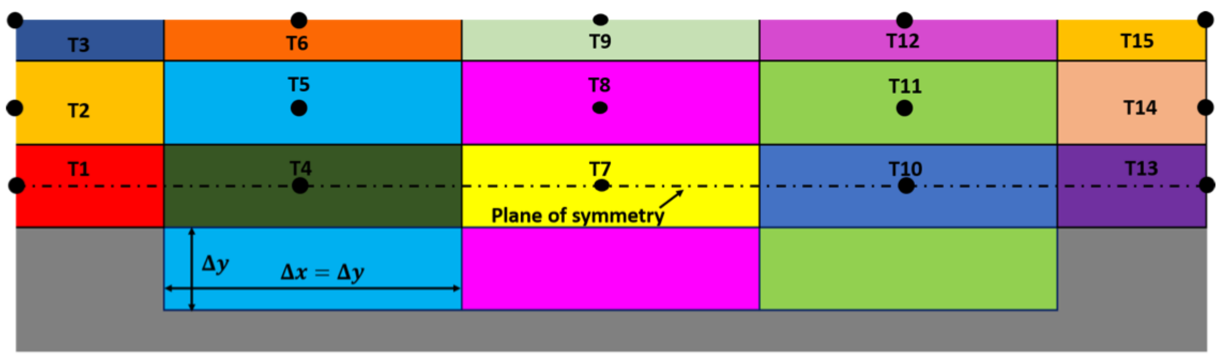


Figure 6: 2D Nodal Temperature Grid

For each colored finite-difference region shown in figure 5, we measured the temperature and displayed our results in a 3 x 5 matrix $T_{measured}$.

Table 1: The Data Nodal Temperature Locations In Pixels

Pixel location	Column 1	Column 25	Column 73	Column 121	Column 192
Row 1	T_3	T_6	T_9	T_{12}	T_{15}
Row 25	T_2	T_5	T_8	T_{11}	T_{14}
Row 73	T_1	T_4	T_7	T_{10}	T_{13}

Using Matlab, we arranged the measured temperature following the structure in table 1.

$$T_{measured}\,[K] = \begin{pmatrix} 310.4730 & 309.9537 & 309.7994 & 309.1867 & 309.4600 \\ 311.6590 & 311.3498 & 310.6990 & 309.8289 & 309.8020 \\ 313.8624 & 312.0541 & 310.8307 & 310.1899 & 309.7777 \end{pmatrix}$$

Next, we set out to solve the temperature for each colored finite-difference region using finite element analysis. To that end, we constructed an energy balance equation for each colored finite-difference region and rearranged the terms into a standard form.

Energy Balance Equation for node 1:

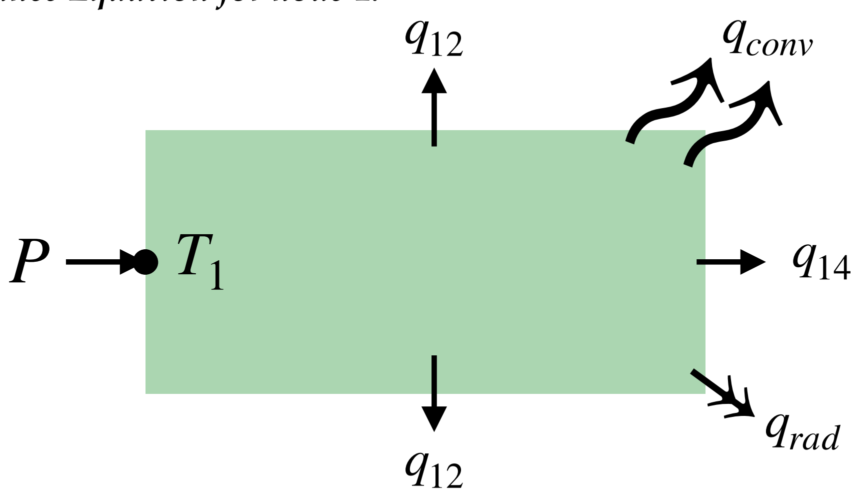


Figure 7: The Energy Balance Schematic For Node 1

The energy balance equation:

$$E_{in} - E_{out} = 0 \tag{9}$$

Where E the net energy of the system, with the subscript "*in*" signifying the net energy in and the subscript "*out*" signifying the net energy out. Completing equation 9 gives us the following relationship:

$$P - 2q_{12} - q_{14} - q_{conv} - q_{rad} = 0 \tag{10}$$

Where P is the power dissipated by the heater, $q_{12\&14}$ is the conductive heat transfer rate, q_{conv} is the convective heat transfer rate, and q_{rad} is the radiative heat transfer rate. For this experiment, we conveniently choose to express the net radiative heat transfer rate q_{rad} from equation 2 in the following form

$$q_{rad} = h_r A_s (T_s - T_\infty) \tag{11}$$

Where h_r is the radiation heat transfer coefficient, A_s is the surface area, T_∞ is the surrounding temperature, and h_r is:

$$h_r = \epsilon\sigma(T_s + T_\infty)(T_s^2 + T_\infty^2) \tag{12}$$

Similarly, the convective heat transfer rate q_{conv} is :

$$q_{conv} = \bar{h} A_s (T_s - T_\infty) \tag{13}$$

Where $\bar{h}$ is the convection heat transfer coefficient; finally the convective heat transfer rate q_{xy} is :

$$q_{xy} = k_s A_c \frac{T_x - T_y}{\Delta x} \tag{14}$$

Where A_c is the cross-sectional area; Therefore substituting equations 11 to 14 to equation 10, we get the following relationship:

$$P - 2k_s \Delta x \Delta z \frac{T_1 - T_2}{\Delta x} - k_s \Delta x \Delta z \frac{T_1 - T_4}{\Delta x} - \frac{\Delta x}{2} \Delta x \left(h_r + \bar{h}\right) \left(T_1 - T_\infty\right) = 0 \tag{15}$$

Rearranging this equation gives us our first nodal linear equation:

$$\left(2 + \frac{\left(\bar{h} + h_r\right) \Delta x^2}{2k_s \Delta z}\right) T_1 - T_2 - T_4 = \frac{P}{k_s \Delta z} + \frac{\left(\bar{h} + h_r\right) \Delta x^2}{2k_s \Delta z} T_\infty \tag{16}$$

Also for convenience purposes, we defined an expression Y to shorten our expressions.

$$Y = \frac{\left(\bar{h} + h_r\right) \Delta x^2}{k_s \Delta z} \tag{17}$$

Repeating the same procedure for nodes 2 to 15, we get the following relationships:

Node 2: $0.5T_1 - (2 + 0.5Y)T_2 + 0.5T_3 + T_5 = -0.5YT_\infty$ (18)

Node 3: $0.5T_2 - (1 + 0.25Y)T_3 + 0.5T_6 = -0.25YT_\infty$ (19)

Node 4: $T_1 - (4 + Y)T_4 + 2T_5 + T_7 = -YT_\infty$ (20)

Node 5: $T_2 + T_4 - (4 + Y)T_5 + T_6 + T_8 = -YT_\infty$ (21)

Node 6: $0.5T_3 + T_5 - (2 + 0.5Y)T_6 + 0.5T_9 = -0.5YT_\infty$ (22)

Node 7: $T_4 - (4 + Y)T_7 + 2T_8 + T_{10} = -YT_\infty$ (23)

Node 8: $T_5 + T_7 - (4 + Y)T_4 + T_9 + T_{11} = -YT_\infty$ (24)

Node 9: $0.5T_6 + T_8 - (2 + 0.5Y)T_9 + 0.5T_{12} = -0.5YT_\infty$ (25)

Node 10: $T_7 - (4 + Y)T_{10} + 2T_{11} + T_{13} = -YT_\infty$ (26)

Node 11: $T_8 + T_{10} - (4 + Y)T_{11} + T_{12} + T_{14} = -YT_\infty$ (27)

Node 12: $0.5T_9 + T_{11} - (2 + 0.5Y)T_{12} + 0.5T_{15} = -0.5YT_\infty$ (28)

Node 13: $T_{10} - (2 + 0.5Y)T_{13} + T_{14} = -0.5YT_\infty$ (29)

Node 14: $T_{11} + 0.5T_{13} - (2 + 0.5Y)T_{13} + 0.5T_{15} = -0.5YT_\infty$ (30)

Node 15: $0.5T_{12} + 0.5T_{14} - (1 + 0.25Y)T_{15} = -0.25YT_\infty$ (31)

Using Matlab, we arranged these equations in a 15x16 matrix and solved for the unknowns. We organized our results following the structure in table 1.

$$T_{calculated}[K] = \begin{pmatrix} 311.5420 & 311.0488 & 310.3749 & 309.8949 & 309.7268 \\ 312.2463 & 311.3429 & 310.4719 & 309.9259 & 309.7431 \\ 315.2001 & 312.0211 & 310.6344 & 309.9684 & 309.7633 \end{pmatrix}$$

DISCUSSION

I. Energy balance equation for the whole plate

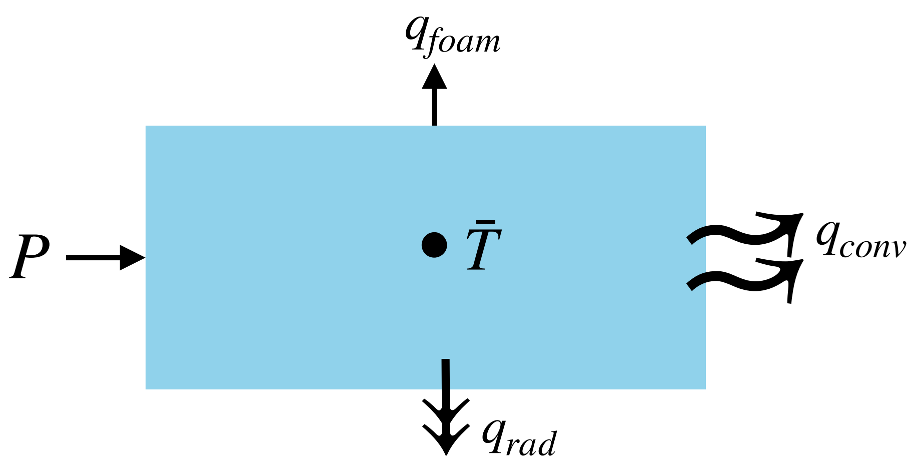


Figure 8: The Energy Balance Schematic For The Whole Plate

Using equation 9, we get the following relationship for the energy balance:

$$P - A_s\left(h_r + \bar{h}\right)\left(\bar{T} - T_\infty\right) - q_{foam} = 0 \tag{32}$$

Where q_{foam} is the conductive heat transfer rate into the foam; isolating q_{foam} we get the following equation:

$$q_{foam} = P - (L \times W)\left(h_r + \bar{h}\right)\left(\bar{T} - T_\infty\right) \tag{33}$$

Where L and W are the length and width of the 2D plate respectively. Next, we computed the heat transfer rate in the foam q_{foam}, and found that approximately **0.8 percent** of the total heat was lost to the foam.

II. Our approximation of the temperatures using the finite difference method turned out to be consistent with the measured temperature from ResearchIR, with a relative difference of approximately **0.1 percent**. Since the plate was insulated on every side but the top, these boundary conditions greatly simplified our equations, thereby allowing us to accurately analyze the heat transfer on top of the plate. Although our calculation yielded a relatively accurate temperature distribution, we can increase the accuracy of our approximation by improving the insulation, that is by making every side but the top of the plate nearly adiabatic. Also, we can improve our approximation by increasing the number of nodes of the plate; the higher the number of nodes the more accurate the approximation will be.

CONCLUSION

In this experiment, we performed a series of heat transfer analyses to compute for the natural convective heat transfer coefficient of a 2D conduction plate. We utilized a thermometer to measure the surrounding temperature and an infrared camera to measure the temperatures distribution across the plate. From our data, we computed for the heat transfer coefficient and determined the accuracy of some heat transfer analysis methods, namely the finite difference method. In conclusion, we can attest to the finite difference method being a reliable approximation method as it resulted in a relative error of approximately 0.1 percent. Although our analysis yielded an accurate estimation, we can additionally improve our results by making every side except the top of the plate adiabatic; which is possible by introducing better insulation, like aerogel, along the boundary of the 2D plate. Since from our current insulation, we lost approximately 0.8 percent of the heat to the foam. We can also improve the results by increasing the number of nodes on the plate, as a higher number of nodes would relatively improve our approximation.

REFERENCE

[1] Introduction to Heat Transfer (6e), Incropera, Lavine, Bergman, DeWitt.

[2] Burge. M. (Fall 2021). 2D Heat Conduction. Buffalo, NY: University at Buffalo.

APPENDIX

Relevant Matlab code:

```
close all; clear
Temp= importdata('lab3_heat_data.xls');
T= mean(Temp,'all')+273.15;
Tinf=24+273.15;
l=15.24/100;
w=15.24/100;
dz=1.28/100;
V=l*w*dz;
As=l*w;
rho_air=1.23;
rho_steel=7850;
rho_foam=2700;
k_air=0.0265;
k=54;
k_foam=0.0265;
cp_air=1007;
cp_steel=490;
nu=1.62e-5;
al=2.25e-5;
Pr=0.72;
g=9.81;
Lc=V/As;
Tf=(T+Tinf)/2;
Beta=1/Tf;
Ra=(g*Beta*(T-Tinf)*Lc^3)/(nu*al);
Nu=.54*Ra^(1/4);
h=Nu*k_air/Lc;
SL=152.4/192;
dx=48*SL*10^(-3);
dy=dx;

Temp3=0;
count=0;
for i=1:24
    for j=1:24
        Temp3=sum(Temp(i,j))+Temp3;
        count=count+1;
```

```
    end
end
T3= Temp3/count+273.15;

Temp2=0;
count=0;
for i=25:72
    for j=1:24
        Temp2=sum(Temp(i,j))+Temp2;
        count=count+1;
    end
end
T2= Temp2/count+273.15;

Temp1=0;
count=0;
for i=73:120
    for j=1:24
        Temp1=sum(Temp(i,j))+Temp1;
        count=count+1;
    end
end
T1= Temp1/count+273.15;

Temp6=0;
count=0;
for i=1:24
    for j=25:72
        Temp6=sum(Temp(i,j))+Temp6;
        count=count+1;
    end
end
T6= Temp6/count+273.15;

Temp5=0;
count=0;
for i=25:72
    for j=25:72
        Temp5=sum(Temp(i,j))+Temp5;
        count=count+1;
```

```
    end
end
T5= Temp5/count+273.15;

Temp4=0;
count=0;
for i=73:120
    for j=25:72
        Temp4=sum(Temp(i,j))+Temp4;
        count=count+1;
    end
end
T4= Temp4/count+273.15;

Temp9=0;
count=0;
for i=1:24
    for j=73:120
        Temp9=sum(Temp(i,j))+Temp9;
        count=count+1;
    end
end
T9= Temp9/count+273.15;

Temp8=0;
count=0;
for i=25:72
    for j=73:120
        Temp8=sum(Temp(i,j))+Temp8;
        count=count+1;
    end
end
T8= Temp8/count+273.15;

Temp7=0;
count=0;
for i=73:120
    for j=73:120
        Temp7=sum(Temp(i,j))+Temp7;
        count=count+1;
```

```
    end
end
T7= Temp7/count+273.15;

Temp12=0;
count=0;
for i=1:24
    for j=121:168
        Temp12=sum(Temp(i,j))+Temp12;
        count=count+1;
    end
end
T12= Temp12/count+273.15;

Temp11=0;
count=0;
for i=25:72
    for j=121:168
        Temp11=sum(Temp(i,j))+Temp11;
        count=count+1;
    end
end
T11= Temp11/count+273.15;

Temp10=0;
count=0;
for i=73:120
    for j=121:168
        Temp10=sum(Temp(i,j))+Temp10;
        count=count+1;
    end
end
T10= Temp10/count+273.15;

Temp15=0;
count=0;
for i=1:24
    for j=169:192
        Temp15=sum(Temp(i,j))+Temp15;
        count=count+1;
```

```
    end
end
T15= Temp15/count+273.15;

Temp14=0;
count=0;
for i=25:72
    for j=169:192
        Temp14=sum(Temp(i,j))+Temp14;
        count=count+1;
    end
end
T14= Temp14/count+273.15;

Temp13=0;
count=0;
for i=73:120
    for j=169:192
        Temp13=sum(Temp(i,j))+Temp13;
        count=count+1;
    end
end
T13= Temp13/count+273.15;

T_measured=[T3 T6 T9 T12 T15;T2 T5 T8 T11 T14;...
            T1 T4 T7 T10 T13];
eps=0.95;
sig=5.67e-8;
hr=eps*sig*(T+Tinf)*(T^2+Tinf^2);
P=20.1*.22;
f=((h+hr)*dx^2)/(dz);

M=zeros(15,16);
M(1,1)=-(2*k+.5*f);
M(1,2)=k;
M(1,4)=k;
M(1,16)=-((P/dz)+(.5*f*Tinf));
M(2,1)=k/2;
M(2,3)=k/2;
M(2,5)=k;
```

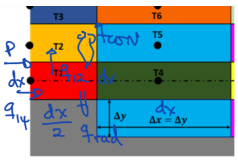
```
M(2,2)=-((2*k)+(.5*f));
M(2,16)=-(.5*f*Tinf);
M(3,2)=k/2;
M(3,6)=k/2;
M(3,3)=-(k+(.25*f));
M(3,16)=-(.25*f*Tinf);
M(4,1)=k;
M(4,5)=2*k;
M(4,7)=k;
M(4,16)=-(f*Tinf);
M(4,4)=-(4*k+(f));
M(5,2)=k;
M(5,4)=k;
M(5,6)=k;
M(5,8)=k;
M(5,5)=-(4*k+(f));
M(5,16)=-(f*Tinf);
M(6,3)=k/2;
M(6,5)=k;
M(6,9)=k/2;
M(6,6)=-(2*k+(.5*f));
M(6,16)=-(.5*f*Tinf);
M(7,4)=k;
M(7,8)=2*k;
M(7,10)=k;
M(7,7)=-(4*k+(f));
M(7,16)=-(f*Tinf);
M(8,5)=k;
M(8,7)=k;
M(8,9)=k;
M(8,11)=k;
M(8,8)=-(4*k+(f));
M(8,16)=-(f*Tinf);
M(9,6)=k/2;
M(9,8)=k;
M(9,12)=k/2;
M(9,9)=-(2*k+(.5*f));
M(9,16)=-(.5*f*Tinf);
M(10,7)=k;
M(10,11)=2*k;
```

```
M(10,13)=k;
M(10,10)=-(4*k+f);
M(10,16)=-(f*Tinf);
M(11,10)=k;
M(11,8)=k;
M(11,12)=k;
M(11,14)=k;
M(11,11)=-(4*k+f);
M(11,16)=-(f*Tinf);
M(12:14,16)=-(.5*f*Tinf);
M(12,11)=k;
M(12,9)=k/2;
M(12,15)=k/2;
M(12,12)=-(2*k+(.5*f));
M(13,10)=k;
M(13,14)=k;
M(13,13)=-(2*k+(.5*f));
M(14,11)=k;
M(14,13)=k/2;
M(14,15)=k/2;
M(14,14)=-((.5*f)+2*k);
M(15,12)=k/2;
M(15,14)=k/2;
M(15,15)=-((.25*f)+k);
M(15,16)=-(.25*f*Tinf);
M_solved=rref(M);
T_calculated=[M_solved(3,16),M_solved(6,16),M_solved(9,16),
M_solved(12,16)...
     ,M_solved(15,16);M_solved(2,16),M_solved(5,16),M_solved
(8,16)...
     ,M_solved(11,16),M_solved(14,16);M_solved(1,16),M_solve
d(4,16)...
    ,M_solved(7,16),M_solved(10,16),M_solved(13,16)];

Biot_num=(h*dz)/k; % my finite volume can be treated as
isothermal;

Relative_diff=((norm(T_calculated)-norm(T_measured))/
norm(T_measured))*100; % Relative difference in percent.
q_foam=P-As*(h+hr)*(T-Tinf);
```

Relevant Hand Written derivations:



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$P - q''_{conv} - q''_{rad} - q''_{14} - q''_{12} = 0$$

$$P - \bar{h}(T_1 - T_\infty) - h_r(T_1 - T_\infty) - \frac{k(T_1 - T_4)}{dx} - \frac{k(T_1 - T_2)}{dx} = 0$$

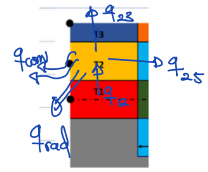
$$P - \bar{h}T_1 + \bar{h}T_\infty - h_rT_1 + h_rT_\infty - \frac{kT_1 + kT_4}{dx} - \frac{kT_1 + kT_2}{dx} = 0$$

$$\cancel{Pdx} - \cancel{dx\bar{h}T_1} + \cancel{dx\bar{h}T_\infty} - \cancel{dxh_rT_1} + \cancel{dxh_rT_\infty} - \cancel{kT_1} + kT_4 - \cancel{kT_1} + kT_2 = 0$$

$$Pdx + dx\bar{h}T_\infty + dx\,h_rT_\infty - T_1(dx\bar{h} + dx\,h_r + 2k) + k(T_4 + T_2) = 0$$

$$\boxed{k(T_2 + T_4) - \left(2k + 0.5\frac{(h_r + \bar{h})dx^2}{dz}\right) = -0.5T_\infty\frac{(h_r + \bar{h})dx^2}{dz} + \frac{P}{dz}}$$

$$Y = \frac{(h_r + \bar{h})dx^2}{dz}$$



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

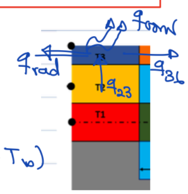
$$q'_{12} - q'_{23} - q'_{25} - q''_{rad} - q''_{conv} = 0$$

$$\frac{k(T_1 - T_2)}{dx} - \frac{k(T_2 - T_3)}{dx} - \frac{k(T_2 - T_5)}{dx} - h_r(T_2 - T_\infty) - \bar{h}(T_2 - T_\infty) = 0$$

$$\cancel{kT_1 - kT_2 - kT_2 + kT_3 - kT_2 + kT_5}(-\cancel{h_rT_2} + \cancel{h_rT_\infty} - \cancel{\bar{h}T_2} + \cancel{\bar{h}T_\infty})dx = 0$$

$$dxT_\infty(\bar{h} + h_r) + k(T_1 + T_3 + T_5) - T_2(3k + dx(h_r + \bar{h})) = 0$$

$$\boxed{0.5k(T_1 + T_3) - (2k + 0.5Y) + T_5 = -0.5YT_\infty}$$



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

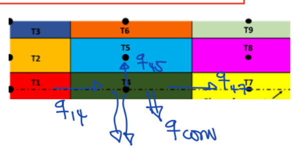
$$q'_{23} - q'_{36} - q''_{rad} - q'_{conv} = 0$$

$$\frac{k(T_2 - T_3)}{dx} - \frac{k(T_3 - T_6)}{dx} - h_r(T_3 - T_\infty) - \bar{h}(T_3 - T_\infty) = 0$$

$$\cancel{kT_2} - \cancel{kT_3} - \cancel{kT_3} + \cancel{kT_6} - \cancel{dxh_rT_3} + \cancel{dxh_rT_\infty} - \cancel{dx\bar{h}T_3} + \cancel{dx\bar{h}T_\infty} = 0$$

$$dxT_\infty(h_r + \bar{h}) + k(T_2 + T_6) - T_3(2k + dx(h_r + \bar{h})) = 0$$

$$\boxed{0.5k(T_2 + T_6) - (k + 0.25Y)T_3 = -0.25YT_\infty}$$



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

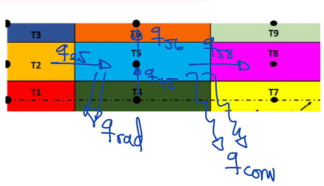
$$q'_{14} - q'_{45} - q'_{47} - q''_{conv} - q''_{rad} = 0$$

$$\frac{k(T_1 - T_4)}{dx} - \frac{k(T_4 - T_5)}{dx} - \frac{k(T_4 - T_7)}{dx} - \bar{h}(T_4 - T_\infty) - h_r(T_4 - T_\infty) = 0$$

$$kT_1 - \cancel{kT_4} - \cancel{kT_4} + kT_5 - \cancel{kT_4} + kT_7 - dx\bar{h}T_4 + \cancel{dx\bar{h}T_\infty} - dxh_rT_4 + \cancel{dxh_rT_\infty} = 0$$

$$dxT_\infty(\bar{h} + h_r) + k(T_1 + T_5 + T_7) - T_4(3k + dx(h_r + \bar{h})) = 0$$

$$\boxed{k(T_1 + T_7 + 2T_5) - (4k + Y)T_4 = -YT_\infty}$$



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

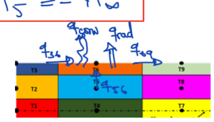
$$q''_{25} + q''_{45} - q''_{56} - q''_{58} - q''_{rad} - q''_{conv} = 0$$

$$\frac{k(T_2 - T_5)}{dx} + \frac{k(T_4 - T_5)}{dx} - \frac{k(T_5 - T_6)}{dx} - \frac{k(T_5 - T_8)}{dx} - h_r(T_5 - T_\infty) - \bar{h}(T_5 - T_\infty) = 0$$

$$\cancel{kT_2} - \cancel{kT_5} + \cancel{kT_4} - \cancel{kT_5} - \cancel{kT_5} + \cancel{kT_6} - \cancel{kT_5} + \cancel{kT_8} - dxh_rT_5 + \cancel{dxh_rT_\infty} - dx\bar{h}T_5 + \cancel{dx\bar{h}T_\infty} = 0$$

$$dxT_\infty(h_r + \bar{h}) + k(T_2 + T_4 + T_6 + T_8) - T_5(4k + dx(h_r + \bar{h})) = 0$$

$$\boxed{k(T_2 + T_4 + T_6 + T_8) - (4k + Y)T_5 = -YT_\infty}$$



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

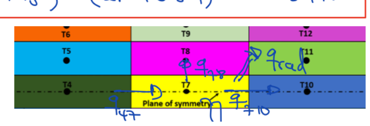
$$q'_{36} + q'_{56} - q''_{69} - q''_{conv} - q''_{rad} = 0$$

$$\frac{k(T_3 - T_6)}{dx} + \frac{k(T_5 - T_6)}{dx} - \frac{k(T_6 - T_9)}{dx} - \bar{h}(T_6 - T_\infty) - h_r(T_6 - T_\infty) = 0$$

$$\cancel{kT_3} - \cancel{kT_6} + \cancel{kT_5} - \cancel{kT_6} - \cancel{kT_6} + \cancel{kT_9} - dx\bar{h}T_6 + \cancel{dx\bar{h}T_\infty} - dxh_rT_6 + \cancel{dxh_rT_\infty} = 0$$

$$dxT_\infty(\bar{h} + h_r) + k(T_3 + T_5 + T_9) - T_6(3k + dx(\bar{h} + h_r)) = 0$$

$$\boxed{k(0.5(T_3 + T_9) + T_5) - (2k + 0.5Y) = -0.5YT_\infty}$$



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

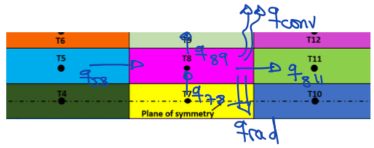
$$q''_{47} - q''_{78} - q''_{710} - q''_{rad} - q''_{conv} = 0$$

$$\frac{k(T_4 - T_7)}{dx} - \frac{k(T_7 - T_8)}{dx} - \frac{k(T_7 - T_{10})}{dx} - h_r(T_7 - T_\infty) - \bar{h}(T_7 - T_\infty) = 0$$

$$kT_4 - kT_7 - kT_7 + kT_8 - kT_7 + kT_{10} - dx\,h_r T_7 + dx\,h_r T_\infty - dx\,\bar{h} T_7 + dx\,\bar{h} T_\infty = 0$$

$$dx T_\infty(\bar{h} + h_r) + k(T_4 + T_8 + T_{10}) - T_7(3k + dx(h_r + \bar{h})) = 0$$

$$\boxed{k(T_4 + T_{10} + 2T_8) - (4k + Y)T_7 = -YT_\infty}$$



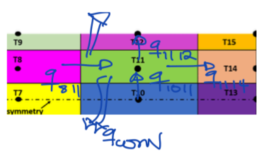
$$q''_{58} + q''_{78} - q''_{89} - q''_{811} - q''_{conv} - q''_{rad} = 0$$

$$\frac{k(T_5 - T_8)}{dx} + \frac{k(T_7 - T_8)}{dx} - \frac{k(T_8 - T_9)}{dx} - \frac{k(T_8 - T_{11})}{dx} - h_r(T_8 - T_\infty) - \bar{h}(T_8 - T_\infty) = 0$$

$$kT_5 - kT_8 + kT_7 - kT_8 - kT_8 + kT_9 - kT_8 + kT_{11} - dx\,h_r T_8 + dx\,h_r T_\infty - \bar{h}\,dx\,T_8 + \bar{h}\,dx\,T_\infty = 0$$

$$dx T_\infty(h_r + \bar{h}) + k(T_5 + T_7 + T_9 + T_{11}) - T_8(4k + dx(h_r + \bar{h})) = 0$$

$$\boxed{k(T_5 + T_7 + T_9 + T_{11}) - (4k + Y)T_8 = -YT_\infty}$$



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_{1011} + q''_{811} - q''_{1112} - q''_{1114} - q''_{rad} - q''_{conv} = 0$$

$$\frac{k(T_{10} - T_{11})}{dx} + \frac{k(T_8 - T_{11})}{dx} - \frac{k(T_{11} - T_{12})}{dx} - \frac{k(T_{11} - T_{14})}{dx} - h_r(T_{11} - T_\infty) - \bar{h}(T_{11} - T_\infty) = 0$$

$$kT_{10} - kT_{11} + kT_8 - kT_{11} - kT_{11} + kT_{12} - kT_{11} + kT_{14} - dx\,h_r T_{11} + dx\,h_r T_\infty - dx\,\bar{h} T_{11} + dx\,\bar{h} T_\infty = 0$$

$$dx T_\infty(\bar{h} + h_r) + k(T_{10} + T_8 + T_{12} + T_{14}) - T_{11}(4k + dx(h_r + \bar{h})) = 0$$

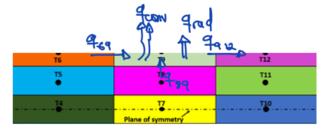
$$\boxed{k(T_8 + T_{10} + T_{12} + T_{14}) - (4k + Y)T_{11} = -YT_\infty}$$

$$q''_{912} + q''_{1112} - q''_{1215} - q''_{rad} - q''_{conv} = 0$$

$$\frac{k(T_9 - T_{12})}{dx} + \frac{k(T_{11} - T_{12})}{dx} - \frac{k(T_{12} - T_{15})}{dx} - h_r(T_{12} - T_\infty) - \bar{h}(T_{12} - T_\infty) = 0$$

$$kT_9 - kT_{12} + kT_{11} - kT_{12} - kT_{12} + kT_{15} - dx\,h_r T_{12} + dx\,h_r T_\infty - dx\,\bar{h} T_{12} + dx\,\bar{h} T_\infty = 0$$

$$\boxed{k(0.5(T_9 + T_{15}) + T_{11}) - (2k + 0.5Y) = -0.5YT_\infty}$$



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

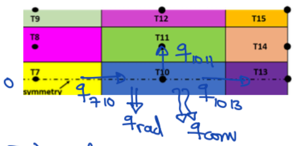
$$q''_{69} + q''_{89} - q''_{912} - q''_{conv} - q''_{rad} = 0$$

$$\frac{k(T_6 - T_9)}{dx} + \frac{k(T_8 - T_9)}{dx} - \frac{k(T_9 - T_{12})}{dx} - h_r(T_9 - T_\infty) - \bar{h}(T_9 - T_\infty) = 0$$

$$kT_6 - kT_9 + kT_8 - kT_9 - kT_9 + kT_{12} - dx\,h_r T_9 + dx\,h_r T_\infty - dx\,\bar{h} T_9 + dx\,\bar{h} T_\infty = 0$$

$$dx T_\infty(h_r + \bar{h}) + k(T_6 + T_8 + T_{12}) - T_9(3k + dx(h_r + \bar{h})) = 0$$

$$\boxed{k(0.5(T_6 + T_{12}) + T_8) - (2k + 0.5Y)T_9 = -0.5YT_\infty}$$



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

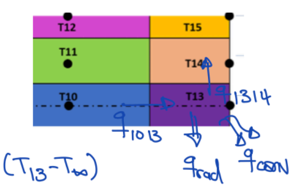
$$q''_{710} - q''_{1011} - q''_{1013} - q''_{rad} - q''_{conv} = 0$$

$$\frac{k(T_7 - T_{10})}{dx} - \frac{k(T_{10} - T_{11})}{dx} - \frac{k(T_{10} - T_{13})}{dx} - h_r(T_{10} - T_\infty) - \bar{h}(T_{10} - T_\infty) = 0$$

$$kT_7 - kT_{10} - kT_{10} + kT_{11} - kT_{10} + kT_{13} - dx\,h_r T_{10} + dx\,h_r T_\infty - dx\,\bar{h} T_{10} + dx\,\bar{h} T_\infty = 0$$

$$dx T_\infty(\bar{h} + h_r) + k(T_7 + T_{11} + T_{13}) - T_{10}(3k + dx(h_r + \bar{h})) = 0$$

$$\boxed{k(T_7 + 2T_{11} + T_{13}) - (4k + Y)T_{10} = -YT_\infty}$$



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

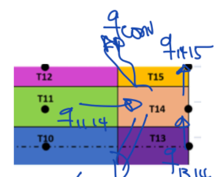
$$q''_{1013} - q''_{1314} - q''_{rad} - q''_{conv} = 0$$

$$\frac{k(T_{10} - T_{13})}{dx} - \frac{k(T_{13} - T_{14})}{dx} - h_r(T_{13} - T_\infty) - \bar{h}(T_{13} - T_\infty) = 0$$

$$kT_{10} - kT_{13} - kT_{13} + kT_{14} - dx\,h_r T_{13} + dx\,h_r T_\infty - dx\,\bar{h} T_{13} + dx\,\bar{h} T_\infty = 0$$

$$dx T_\infty(h_r + \bar{h}) + k(T_{10} + T_{14}) - T_{13}(2k + dx(h_r + \bar{h})) = 0$$

$$\boxed{k(T_{10} + T_{14}) - (2k + 0.5Y) = -0.5YT_\infty}$$



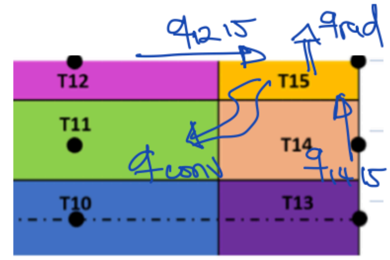
$$q''_{1114} + q''_{1314} - q''_{1415} - q''_{conv} - q''_{rad} = 0$$

$$\frac{k(T_{11} - T_{14})}{dx} + \frac{k(T_{13} - T_{14})}{dx} - \frac{k(T_{14} - T_{15})}{dx} - h_r(T_{14} - T_\infty) - \bar{h}(T_{14} - T_\infty) = 0$$

$$kT_{11} - kT_{14} + kT_{13} - kT_{14} - kT_{14} + kT_{15} - dx\,h_r T_{14} + dx\,h_r T_\infty - dx\,\bar{h} T_{14} + dx\,\bar{h} T_\infty = 0$$

$$dx T_\infty(h_r + \bar{h}) + k(T_{11} + T_{13} + T_{15}) - T_{14}(3k + dx(h_r + \bar{h})) = 0$$

$$\boxed{k(T_{11} + 0.5(T_{13} + T_{15})) - (2k + 0.5Y)T_{15} = -0.25YT_\infty}$$



$$E_{in} - E_{out} = 0$$

$$q''_{12\,15} + q''_{14\,15} - q''_{conv} - q''_{rad} = 0$$

$$\frac{k(T_{12}-T_{15})}{dx} + \frac{k(T_{14}-T_{15})}{dx}$$

$$- h_r(T_{15}-T_\infty) - \bar{h}(T_{15}-T_\infty) = 0$$

$$kT_{12} - kT_{15} + kT_{14} - kT_{15} - dx\,h_r T_{15} + dx\,h_r T_\infty$$

$$-dx\,\bar{h}T_{15} + dx\,\bar{h}T_\infty = 0$$

$$dx\,T_\infty(\bar{h} + h_r) + k(T_{12}+T_{14}) - T_{15}(2k + dx(h_r+\bar{h})) = 0$$

$$\boxed{0.5k(T_{12}+T_{14}) - (k+0.25)T_{15} = -0.25YT_\infty}$$