

#### Introduction

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# Sliding Window

#### A Sum Problem

#### Problem description

Write a program that, given an integer array of size N, finds the contiguous subarray of size K with the highest sum.

#### Input description

Input consist of two lines. The first line contains two space separated integers N, the size of the array, where  $1 \leq N \leq 10^6$ , and K, the size of the subarrays to consider, where  $1 \leq K \leq N$ . Then second line contains N space separated integers, the values of the array. Each value in the array is between  $-10^9$  and  $10^9$ .

#### Output description

Output one line, the sum of the highest valued contiguous subarray of size K.

# A Sum Problem

Sample input	Sample output
10 4	39
17 20 0 1 5 24 8 2 4 1	

```
n, k = map(int, input().split())
arr = list(map(int, input().split()))
highest = float('-inf')
for start in range(n-k+1):
   end = start + k
   total = 0
   for i in range(start, end):
        total += arr[i]
   highest = max((highest, total))
print(highest)
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- Too slow!

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- We subtract  $a_i$ .
- We add  $a_i + k$ .

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- A shift from the subarray starting at i to the subarray starting at i+1 takes  $\mathcal{O}(1)$  time then.

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- We subtract  $a_i$ .
- We add  $a_i + k$ .
- A shift from the subarray starting at i to the subarray starting at i+1 takes  $\mathcal{O}(1)$  time then.
- This is known as the sliding window technique.

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n, k = map(int, input().split())
arr = list(map(int, input().split()))
total = 0
for i in range(k):
    total += arr[i]
highest = total
for i in range(n - k):
    total -= arr[i]
    total += arr[i+k]
    highest = max((highest, total))
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- Then, N-K times, an element is removed and another added.

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- Subtracting and adding numbers is constant time so  $\mathcal{O}(N)$ .

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- ullet This solution constructs the first size K contiguous subarray.
- ullet Then, N-K times, an element is removed and another added.
- Subtracting and adding numbers is constant time so  $\mathcal{O}(N)$ .
- Fast enough!

# A Substring Problem

#### Problem description

Write a program that, give a string of size N, finds the longest substring with K distinct elements.

#### Input description

Input consist of two lines. The first line contains two space separated integers N, the size of the string, where  $1 \leq N \leq 10^6$ , and K, the size of the subarrays to consider, where  $1 \leq K \leq 26$ . Then second line contains a string of length N consisting of English lowercase characters.

#### Output description

Output one line, the longest substring with K distinct elements. If no such string exists, output "DOES NOT EXIST", without quotations.

# A Substring Problem

Sample input	Sample output
14 3	cdcbcbcb
bacdcbcbcbabdb	

#### General Framework

```
from string import ascii_lowercase
n, k = map(int, input().split())
s = input()

best_ind, best_len = distinct_k(n, k, s)

if best_len == -1:
    print("DOES NOT EXIST")
else:
    print(s[best_ind:best_ind + best_len])
```

```
def distinct_k(n, k, s):
 best_ind, best_len = -1, -1
 for start in range(n):
   for end in range(start, n+1):
      substring = s[start:end]
      distinct = 0
      for symbol in ascii_lowercase:
        if symbol in substring:
          distinct += 1
      cur_len = len(substring)
      if distinct == k and cur_len > best_len:
       best_ind = start
        best len = cur len
 return best_ind, best_len
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def distinct_k(n, k, s):
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- Checking each one takes us  $\mathcal{O}(N)$  time, so  $\mathcal{O}(N^3)$  in total.
- Way too slow!

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def distinct_k(n, k, s):
 best_ind, best_len = -1, -1
 for start in range(n):
   for end in range(start, n+1):
      substring = s[start:end]
      present = [False for _ in range(26)]
     for symbol in substring:
        present[ord(symbol) - ord('a')] = True
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This is a little faster, by a factor of 26 approximately.

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- Also note that once a character stops being distinct, expanding the substring will never make it distinct again

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- Also note that once a character stops being distinct, expanding the substring will never make it distinct again

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def distinct_k(n, k, s):
  best ind. best len = -1. -1
  start, end, distinct = 0, 0, 0
  count = [0 for _ in range(26)]
  while start < n:
    while end < n:
      c = ord(s[end]) - ord('a')
      if distinct == k and count[c] == 0:
        break
      count[c] += 1
      end += 1
      distinct = sum(x > 0 \text{ for } x \text{ in count})
    cur len = end - start
    if distinct == k and cur_len > best_len:
      best ind = start
      best_len = cur_len
    count[ord(s[start]) - ord('a')] -= 1
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# Sliding Window Improved

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def distinct_k(n, k, s):
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    c = ord(s[start]) - ord('a')
    count[c] -= 1
    if count[c] == 0:
      distinct -= 1
    start += 1
```

# Sliding Window Improved

```
def distinct_k(n, k, s):
  best_ind, best_len = -1, -1
  start, end, distinct = 0, 0, 0
  count = [0 for _ in range(26)]
  while start < n:
    while end < n:
      c = ord(s[end]) - ord('a')
      if distinct == k and count[c] == 0:
       break
      if count[c] == 0:
        distinct += 1
      count[c] += 1
      end += 1
    cur_len = end - start
    if distinct == k and cur len > best len:
      best_ind = start
      best len = cur len
    c = ord(s[start]) - ord('a')
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- Usually you want the maximal or the minimal window fulfilling a certain condition.

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- Time complexity is  $\mathcal{O}(N \cdot (X + Y))$  where X and Y are the cost of add and remove respectively.