Quantum Mechanics I: Homework 1

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1 Problem 1: Sakurai 1.1

We start with

$[AB, CD] = A[B, CD] + [A, CD]B$

$\rightarrow A([B, C]D + C[B, D]) + ([A, C]D + C[A, D])B$

Then, writing out each commutation relation

$\rightarrow ABCD - ACBD + ACBD - ACDB + ACDB - CADB + CADB - CDAB$

The second and fourth terms from the above line can be combined to give $-AC\{D, B\}$

$\rightarrow -AC\{D, B\} + ABCD + ACBD + ACDB - CADB + CADB - CDAB$

The second and third terms from the above line can be combined to give $A\{C, B\}D$

$\rightarrow -AC\{D, B\} + A\{C, B\}D + ACDB - CADB + CADB - CDAB$

The fourth and sixth terms from the above line can be combined to give $-C\{D, A\}B$

$\rightarrow -AC\{D, B\} + A\{C, B\}D - C\{D, A\}B + ACDB + CADB$

The last two terms from the above line are then combined to give $\{C, A\}DB$

$\rightarrow -AC\{D, B\} + A\{C, B\}D - C\{D, A\}B + \{C, A\}DB$

Thus we have proved that

$$\boxed{[AB, CD] = -AC\{D, B\} + A\{C, B\}D - C\{D, A\}B + \{C, A\}DB}$$

2 Problem 2