# Quantum Mechanics I Problem Set #1 (due Wed, Sept. 13 at 5pm)

**Reading:** Sakurai Chapter 1.1-1.3 (focusing on 1.2-1.3)

**Problem 1: [10 points]** Sakurai Problem #1.1.
**Problem 2: [20 points]** The orientation of a unit vector in 3D space is commonly described in terms of two variables, as $\hat{r} = \hat{z}cos(\theta) + sin(\theta)(\hat{x}\cos(\phi) + \hat{y}sin(\phi))$.

(a) A spin 1/2 state with arbitrary orientation can be written as $|\theta, \phi; +\rangle = a|S_z; +\rangle + b|S_z; -\rangle$, with appropriate complex coefficients $a$ and $b$, and the normalization constraint that $|a|^2 + |b|^2=1$. Identify two degrees of freedom that could take the place of $\theta$ and $\phi$ in the quantum state. (you do not need to establish an equivalence)

(b) Identify positive and negative eigenkets of the Pauli spin matrices in terms of $a$ and $b$. The Pauli matrices have eigenvalues of $\pm 1$. They act on the column vector $\begin{pmatrix} a \\ b \end{pmatrix}$, and are defined as:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

**Extra credit: [3 points]** Create a definition of $a$ and $b$ in terms of $\theta$ and $\phi$ that aligns the positive and negative eigenstates of the Pauli matrices with the corresponding positive and negative real space axes.