Improving Dark Matter Axion Searches with Active Resonators

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Axions are a well motivated candidate for dark matter. The most sensitive experiments searching for dark matter axions rely on the coupling of axions to the electromagnetic resonances of a microwave cavity immersed in a strong magnetic field. The sensitivity of the experiment is proportional to the Q of the resonance that is coupled to axions. To date, the resonators used in axion searches have all been passive, with Qs limited by power loss in the cavity walls. I propose the use of active feedback resonators to increase the Q of microwave cavity axion dark matter experiments by several orders of magnitude. This should allow experiments to significantly increase the rate at which they can test potential axion masses and couplings.

INTRODUCTION

The axion is a hypothetical particle that is both a candidate for dark matter and a result of the Peccei-Quinn solution to the strong CP problem [1–7]. Axions with masses below 10^{-3} eV are particularly interesting because they could be produced in sufficient quantities to account for dark matter [8]. For axions in this mass range the coupling between axions and photons , despite being exceptionally weak, provides the best chance of directly observing axion dark matter.

The most sensitive dark matter axion searches to date have been of the "microwave cavity" type. These experiments rely on the conversion of axions from the local dark matter halo into photons in a strong magnetic field. This conversion is resonantly enhanced when the resonant frequency of the microwave cavity is equal to the frequency of the photons produced from the axion conversion [9]. Operation of these experiments involves slowly tuning the resonant frequency of the microwave cavity to explore different potential axion masses and searching for an excess of power deposited from dark matter axion conversion. Microwave cavity experiments have been demonstrated to have the sensitivity required to detect optimistically coupled dark matter axions over a small mass range, but as of yet, axions have not been detected in a microwave cavity experiment [10].

The signal power in microwave cavity experiments is proportional to a number of factors, including the local density of dark matter, the strength of the magnetic field, the volume of the cavity, and the resonant quality factor Q of the resonance being used [**?**]:

$$P_{sig} = \eta g_{a\gamma\gamma}^2 B_{ext}^2 V C \frac{\rho_a}{m_a} \min(Q_{loaded}, Q_a) \tag{1}$$

where η is the coupling between the cavity and output, $g_{a\gamma\gamma}$ is the axion-photon coupling, B is the applied magnetic field, V is the volume of the cavity, C is the "form factor", or overlap between the cavity resonant mode and the axion field, ρ_a is the local density of dark matter axions, m_a is the axion mass, Q_{loaded} is the loaded Q of the cavity resonant mode, Q_a is the thermal width of the axion signal, taken to be 10^6 or higher. The primary background is the thermal noise from the physical temperature of the cavity and the electronic noise from the first stage amplifier. The figure of merit for a microwave cavity experiment is the instantaneous axion signal to noise ratio (SNR). The SNR is given by the ratio of the signal power to the fluctuations in the noise power. We assume that we measure the signal for time τ with a frequency resolution equal to the width of the axion signal, B_a:

$$SNR = \frac{P_{sig}}{\delta P_{noise}} = \frac{P_{sig}}{T_{noise}} \sqrt{B_a \tau} \tag{2}$$

Experiments with a larger SNR can be sensitive to more pessimistic axion photon couplings for a given frequency tuning speed or tune more quickly for a given axion photon coupling sensitivity.

Increasing the cavity Q is one way to increase SNR in a microwave cavity experiment; the speed at which the cavity frequency can be tuned while remaining sensitive to a given axion photon coupling is linearly proportional to Q [11]. The presence of a strong magnetic field precludes the use of high-Q superconducting cavities in these experiments, so the cavities are usually made of or coated with copper. Loaded Qs of 10^5 at 1 GHz have been achieved with copper cavities in axion experiments [11]. The Q of a cavity scales as $Q \propto f^{-2/3}$ due to skin depth effects [**?**], so that higher frequency cavities typically have smaller Qs. This has the effect of making higher mass axions more challenging to search for.

We present here a means of artificially increasing the Q of the cavity resonance using active feedback in order to improve sensitivity to dark matter axions and increase the speed at which different axion masses can be tested. In order to derive the effect of active feedback, we first study the classic axion haloscope in terms of the microwave S parameter formalism.

AXION HALOSCOPE AS A MICROWAVE DEVICE

Assume $\omega_a \simeq m_a$.

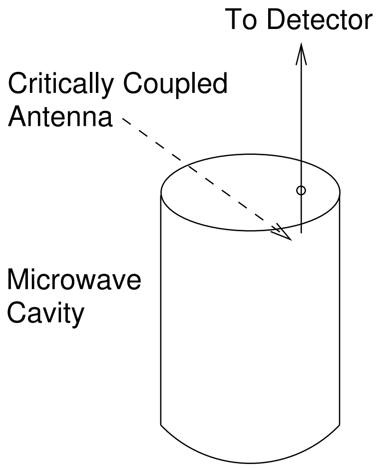

FIG. 1. Schematic of the classic axion haloscope experiment.

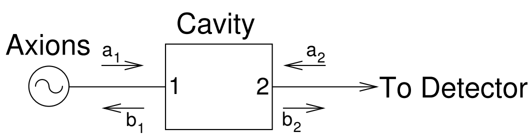

FIG. 2. Microwave diagram of the classic axion haloscope experiment as a two port device. Signals incoming to the cavity are denoted a_n, where n is the port number, and outgoing signals are denoted with b_n.

Consider the axion haloscope described in [?], and shown in Fig. 1. A microwave cavity is placed in a strong magnetic field. The unloaded Q of the resonance will be denoted Q_0 and the loaded Q will be denoted as Q_L. Axion dark matter passing through the field and cavity will excite the electromagnetic resonance as some axions are converted into photons. A critically coupled antenna extracts some of this power from the cavity and carries it to a detector. Inside the cavity, the presence of the axion field can be treated as a modification to Maxwell's equations, the relevant modification being: **Ana check this is right**

$$\nabla \times B - \frac{\partial E}{\partial t} = J + g_{a\gamma\gamma} B_0 m_a a, \tag{3}$$

where E and B are the electric and magnetic fields of the excitation, $g_{a\gamma\gamma}$ is the axion-photon coupling, B_0 is the applied magnetic field (assumed to be much larger than the excitation field), m_a is the axion mass, and a is the axion field. The energy flux of axion dark matter through the cavity is:

$$P_a = m_a^2 a^2 V \omega_a = \rho_{dm} V \omega_a, \tag{4}$$

where V is the volume of the cavity, ρ_{dm} is the density of dark matter axions, ω_a is the frequency of oscillation of the axion field. For cold dark matter, one can approximate $\hbar\omega_a \simeq m_a c^2$ and therefore m_a and ω_a can be used interchangeably.

From this it can be seen that the effect of an axion field in a strong magnetic field is equivalent to the effect of a current density. Therefore, all intuitions from antenna-resonator coupling are also relevant for axions, and the system can be treated as a cavity filter between the axion field and the output antenna, as shown in Fig. 2. Here the coupling between the output antenna and the cavity is $Q_2 = 2Q_L$ because of the assumption of critical coupling, and the coupling between the axion field and the cavity will be labeled Q_1. The value of Q_1 can be derived from the modified Maxwell's equation to be

$$\frac{1}{Q_1} = \frac{g_{a\gamma\gamma}^2 B_0^2 C^2}{m_a^2}, \tag{5}$$

where C is the 'form factor', or integral of $E \cdot B$ over the cavity volume as described in Ref. [?]. The loaded Q of the system is

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_1} + \frac{1}{Q_2}, \tag{6}$$

but Q_1 can be safely assumed to be so small as to not contribute to Q_L.

The S matrix for the cavity can be written:

$$S_{\text{haloscope}} = \begin{pmatrix} 1 - \frac{2Q_L}{Q_1} & \frac{2Q_L}{\sqrt{Q_1 Q_2}} \\ \frac{2Q_L}{\sqrt{Q_1 Q_2}} & 1 - \frac{2Q_L}{Q_1} \end{pmatrix}. \tag{7}$$

The signal towards the detector is therefore

$$b_2 = S_{21} a_1 + S_{11} b_2 \tag{8}$$

For a critically coupled antenna, $S_{11} = 0$, so the power at the detector:

$$P_{det} = |b_2^2| = S_{21}^2 P_a = \frac{1}{2} g_{a\gamma\gamma}^2 B_0^2 C^2 \frac{\rho_{dm}}{m_a}. \tag{9}$$

This number is a factor of 2 smaller than described in Ref. [?], because half of the axion power is dissipated by cavity losses due to critical coupling of the output antenna.

Thermal noise at the detector includes thermal noise from the cavity (we will use the term a_0 to represent this), as well as noise coupled in from the ports:

$$b_{2,noise} = S_{21} a_{1,noise} + S_{22} a_{2,noise} + \frac{Q_L}{\sqrt{Q_0 Q_2}} a_{0,noise} \tag{10}$$

In the simple haloscope case, only the cavity thermal noise is relevant, giving

$$P_{noise} = < b_{2,noise}^2 > = < a_{0,noise} >^2 = kTb, \tag{11}$$

where k is Boltzmann constant, T is the physical temperature of the cavity, and b is the bandwidth over which the noise is measured.

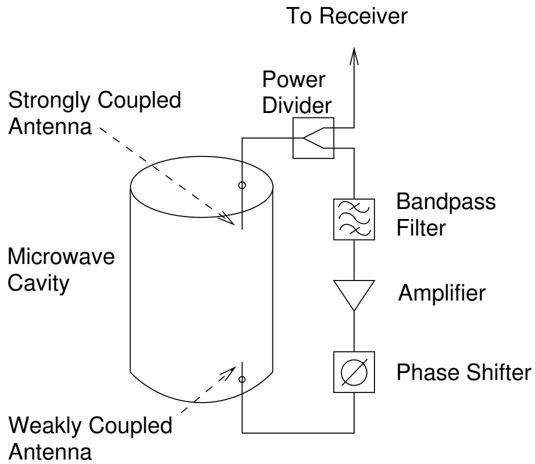

FIG. 3. Schematic of active feedback resonator for an axion experiment

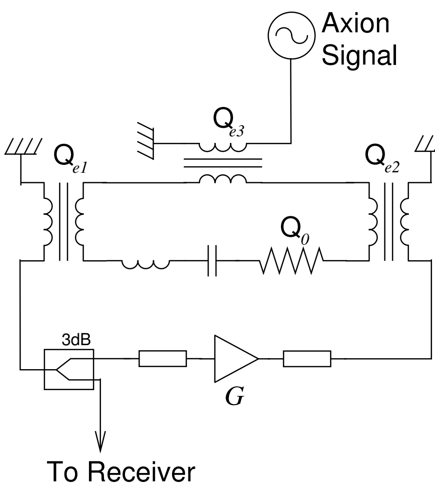

FIG. 4. Equivalent circuit to axion photon conversion in microwave cavity with active regeneration

THEORY OF METHOD

The use of active feedback resonators is a well established technique used in regenerative receivers [12]. It involves connecting positive feedback to a resonator at the appropriate phase and amplification such that the power lost each oscillation to the output and damping effects is nearly completely replaced. This has the effect of increasing the Q of the system. A schematic of a microwave cavity axion experiment with active feedback is shown in Fig. 3. Power from axion to photon conversion exits the cavity via a strongly coupled antenna, some of which is sent to a detector. The remainder of the power is amplified, phase shifted, and then fed back into the cavity via a weakly coupled antenna. A filter may also be needed to select the desired mode.

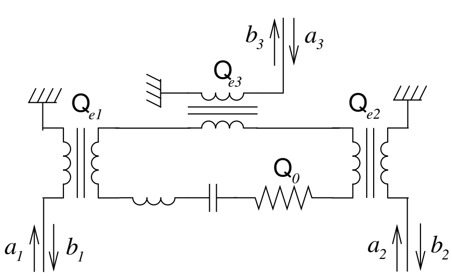

FIG. 5. Microwave cavity represented as three port device.

To calculate the effect of the active feedback, I will use the equivalent circuit to the cavity-amplifier system shown in Fig. 4. The cavity resonant mode is equivalent to an LRC circuit with an unloaded Q of Q_0 and the coupled antennas are equivalent to transformers with couplings Q_{e1} and Q_{e2} [13]. These external quality factors can be defined in terms of the unloaded Q and the coupling coefficients β_1 and β_2, which represent respectively the fraction of power lost out of port 1 or port 2 compared to the power lost to the walls:

$$\beta_1 = \frac{Q_{e1}}{Q_0}$$

and

$$\beta_2 = \frac{Q_{e2}}{Q_0}.$$

G is the combined voltage gain of the amplifier and power divider. There is also a phase shift between the two antennas around the loop, which will be denoted as δ. The coupling between the electromagnetic resonance in the cavity and dark matter axions is equivalent to a third antenna with coupling Q_{e3}. The passive loaded Q of the resonator is $Q_L = \left(Q_0^{-1} + Q_{e1}^{-1} + Q_{e2}^{-1} + Q_{e3}^{-1}\right)^{-1}$.

Let us look at the amplitude of the wave at the output of port 2, assuming that all components are impedance matched and that isolators are placed at the ports to prevent reflected waves. If port 2 is critically coupled, then the noise wave at its output is equal to $|c_2|^2 = kT_{cav}B$, where B is the bandwidth of the detector. We can add the noise of the cavity and the amplifier to get the system noise wave, $n(t) = c_2(t) + n_a(t)$.

The time averaged noise power of the output is then

$$P_n = \langle|n(t)+n(t-t_1)GS_{21}+(c_2(t-2t_1)+n_a(t-2t_1))G^2S_{21}^2+\ldots|^2\rangle \tag{12}$$

The amplifier noise n_a and the thermal noise from the cavity c_2 are independent, hence $< c_2 n_a >= 0$, and also in the steady state $< c_2(t) >^2 = \bar{c}_2$ is independent of time, so

$$P_n = (\bar{c}_2^2 + \bar{n}_a^2)\frac{1}{1 - G^2|S_{21}|^2} + \text{cross-terms} \tag{13}$$

where the cross-terms hold the information about the coherence of the noise between feedback rounds. Any signal entering the cavity undergoes exponential decay, so the expectation value of the cross terms goes as $< (c_2(t-nt_1)c_2(t-mt_1) >= \bar{c_2}^2 e^(- (|n-m|)t_1/\tau)$, where $\tau = Q_L/\omega_0$ is the coherence time of the cavity. One can see that when the roundtrip time around the feedback loop is much larger than the coherence time, $t_1 \gg \tau$, the cross terms are exponentially suppressed and the feedback loop and amplifier enhance the original noise temperature of the cavity by a factor:

$$T_n = T_{sys}\frac{1}{1-G^2|S_{21}|^2} \tag{14}$$

The analysis is slightly different for the signal power. We treat the axion-converted signal as a continuous source with amplitude e_a. Then the signal power observed at the output of port 2 (with the feedback loop is

$$P_s = \langle |e_a(t) + e_a(t-t_1)GS_{21} + e_a(t-2t_1)G^2S_{21}^2 + \dots|^2\rangle \tag{15}$$

and as long as ωt_1 is a multiple of 2π, $e_a(t-nt_1) = e_a(t)$.

Therefore the time averaged signal power is

$$P_s = \bar{e}_a^2\frac{1}{(1-GS_{21})^2} \tag{16}$$

This implies that the enhanced SNR from the feedback loop will be greater than the passive SNR, SNR_0, (assuming the same integration time and axion bandwidth) in the limit where the roundtrip time is much longer than the coherence time of the cavity, (need to add extra factor because axion signal will additionally be enhanced by Q).

$$\frac{SNR_e}{SNR_0} = \frac{P_{s,enhanced}}{P_{s,original}}\frac{T_o}{T_e} = \frac{Q_e}{Q_o}\frac{1-G^2|S_{21}|^2}{(1-GS_{21})^2} \tag{17}$$

$$= \frac{1}{1-GS_{21}}\left(1+\frac{2GS_{21}}{1-GS_{21}}\right) \tag{18}$$

In most cases we can achieve loaded Q's of only an order of magnitude below the axion linewidth, so we need a combined feedback of $GS_{21} = .8$ to improve the Q by an order of magnitude.

When $t_1 \approx \tau$ we can no longer ignore the cross terms and end up with a noise power of

$$P_n = (\bar{c}_2^2 + \bar{n}_a^2)\frac{1}{1-G^2|S_{21}|^2}\frac{1}{1-2e^{-t/\tau}GS_{21}} \tag{19}$$

One subtlety is that if there is only phase shifter for multiple frequencies within the feedback loop, one will not be able to hold the requirement that $\omega t = 2\pi$ when the roundtrip travel time t is long; that is to say, interference fringes will begin to be observed within the bandwidth of the cavity when tis of the order $t \approx \pi Q_e/\omega_0$, so in order to avoid this effect, one would want a delay which longer than the coherence time of the cavity, but not too long so as to avoid deconstructive interference among frequencies slightly off resonance. Another solution would be to multiplex the signal and give each frequency a slightly different phase shift so that all the frequencies in the cavity bandwidth receive a 2π phase shift around the loop.

assuming the resonant frequency of the cavity has been tuned to the axion frequency.

so that the enhanced cavity Q has a maximum value of $Q_e = \frac{Q_0}{1+\beta_1+\beta_2-2G\sqrt{\beta_1\beta_2}}$

Note that for a passive resonator $M = 1$ and that systems with $M < 0$ will oscillate regardless of the presence of an axion signal.

It is important to note that for the isothermal halo model of dark matter axions, the characteristic width of the axion signal is expected to correspond of a Q of 10^6 [16]. Experiments with a bandwidth smaller than the axion signal width sample only a subset of axions. Thus the conservative estimate for signal power is

$$P_{\text{signal}} = \frac{M}{2}\min\left(MQ_L, 10^6\right) g_{a\gamma\gamma}^2 V B_0^2 \rho_a C \frac{1}{m_a}, \tag{20}$$

though it could be larger in models where the axion dark matter is unusually cold.

THERMAL NOISE ANALYSIS

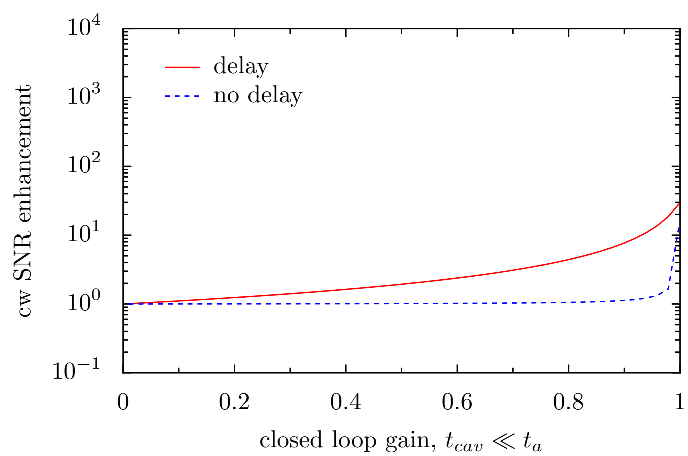

FIG. 6. Theoretical noise analysis

EXPERIMENTAL STUDY

Here we experimentally confirm the results of the analysis presented above. A microwave cavity with a feedback

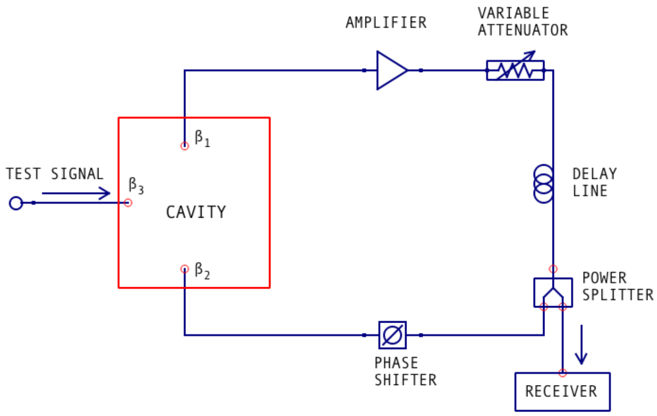

FIG. 7. Schematic of experimental setup

loop was built as in Fig. ??. The axion signal was simulated with a standard signal generator fed into a weakly coupled port in the cavity. The active Q of the system could be changed by modifying the phase and attenuation of the system. A network analyzer attached ******* was used to measure the active Q of the system. A specturm analyzer measured a fraction of the power in the feedback loop to measure signal and noise power. The relevant parameters of the setup are summarized in Table .

The signal to noise enhancement was measured as a function of active Q for three different delays in the feedback loop. First, the loop gain of the system was set by adjustion the variable attenuator. ***ANA MAKE SURE THIS IS THE RIGHT PROCEDURE*** Then the phase shifter was adjusted to maximize the Q measured in the network analyzer. This Q was recorded as the active Q. With the signal generator off, the power at the center of the Q was measured with the spectrum analyzer. This was recorded as the noise power. The signal generator was then turned on, the frequency tuned to correspond to the center of the resonance, and the power was measured and recorded as the signal power. The variable attenuation was then decreased slightly and the procedure was repeated for the new, higher, active Q until the loop gain was high enough that the system became unstable and began to oscillate.

Parameters	Values
f_0	$2.256{\times}10^9$ Hz
Q_L	1000
Q_0	?
β_1	?
β_2	?
T_{cav}	300 K
T_{amp}	300 K
G_a	? dB
Averaging Time	(what is 25 avgs in secs?)
Time Delay	2.4 μs
Bandwidth	? wrote it down on a napkin?

The procedure was followed for three time delays in the feedback loop. The noise power and signal power are shown in Fig. ??. The ratio of the two, SNR, is shown in Fig. ??. As expected, both the signal and noise scale quadratically for the shortest delay, while the noise scales only linearly for the delay greater than $\frac{Q}{f}$. For the intermediate delay, the transition in noise scaling from linear to quadratic is seen in the region where $t_{\text{delay}} \simeq \frac{Q}{f}$. Thus the SNR is seen to increase linearly only for delays greater than $\frac{Q_{\text{active}}}{f}$. In the longest delay, a Q enchancement of **** could be achieved, with a proportional gain in SNR.

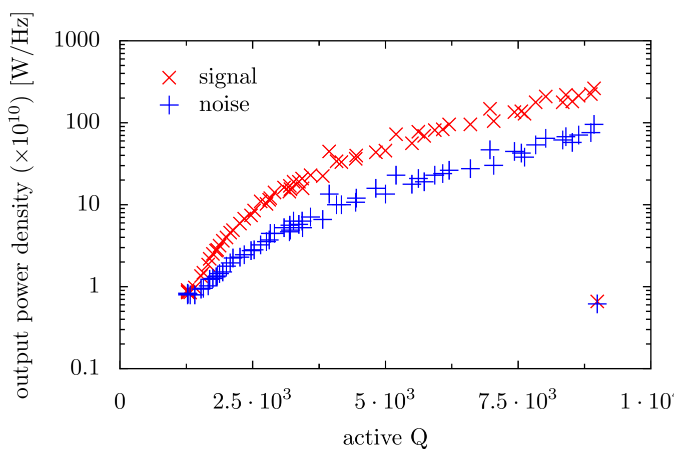

FIG. 8. No delay line

DISCUSSION

We see in this prototype that with a very low Q cavity one can enhance the Q by a factor of more than a thousand, and see a signal to noise improvement of a factor of 10. This is extremely encouraging as a proof of principle, and shows that one can get to Q's comparable with the axion linewidth with the simple technique of active feedback. The main sources of systematic noise would come from gain instability in the circuit; in this experi-

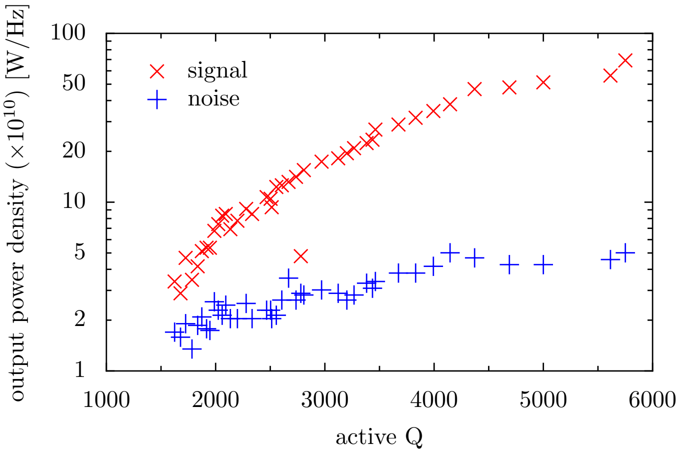

FIG. 9. Delay line

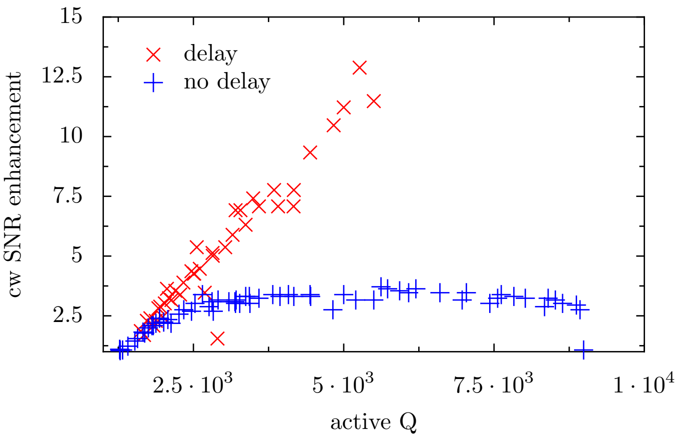

FIG. 10.

ment no noise additional to that expected was observed (or tested). This will increase the scan rate of microwave cavity experiments roughly by a factor of 3 and is simpler than research currently under way to improve the Q by making superconducting films on the walls of copper cavities. The active feedback technique becomes increasingly important for high-frequency axion experiments, as the passive limit for the Q scales as $f^{-1/3}$.

Delay lines with time delays longer than a few microseconds are difficult to make with analog technology; at 1 GHz and Q's of 100,000, we would need a delay of ten microseconds. Digital delay lines using FPGAs would be the natural way to implement the necessary delays.

The ADMX experiment is within an order of magnitude of the sensitivity necessary to be sensitive to pessimistically coupled axion dark matter. The addition of active feedback, along with other planned upgrades, should allow it to to be sensitive to pessimistically coupled axion dark matter even in models where axions constitute a very small fraction of the dark matter. The use of active feedback will also allow future experiments to operate at higher frequencies, where previous microwave cavity experiments have not had the sensitivity necessary to detect axion dark matter with a reasonable scan speed.

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