Experimental Study of the Frequency Stability of a Maser Oscillator Operated with an External Feedback Loop

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***Abstract*—In this paper, we report the results of an experimental study of the short- and mid-term frequency stability for both a hydrogen maser and a rubidium maser operated with an external feedback loop to modify the cavity quality factor. A revised version of the theoretical expression of the frequency stability for this type of maser is given and a numerical solution for various maser parameters is calculated. The predicted frequency stability exhibits an optimum when the cavity Q is varied. The experimental results presented in this study agree with this conclusion.**

I. Introduction

RECENTLY, much work has been devoted to hydrogen masers, in order to reduce their size, their weight, and to increase their long-term frequency stability. New technology and design improvements applied to conventional masers [1] has allowed the realization of atomic frequency standards exhibiting the best mid-term frequency stability [2] and a sufficiently light and rugged device to be space borne [3]. Small size masers with various types of microwave cavities were developed [4], [5]. Masers using a dielectric loaded cavity were operated as passive frequency standards [6], [7]. The latest realization is a small size cavity oscillating maser, equipped with an external feedback loop in order to enhance the cavity quality factor [8], [9].

A theoretical model has been developed to evaluate the amplitude noise and the phase noise in active and passive masers which allows the prediction of the short-term frequency stability of a maser equipped with a feedback loop [10]. It also makes it possible to compare the performances of a small size maser operated either actively or passively [11]. The present work gives an experimental check of that theory applied to a hydrogen maser and a rubidium maser of conventional design [12], [13], both equipped with a feedback loop.

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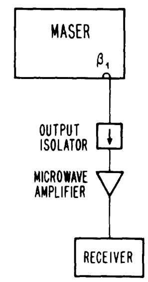

Fig. 1. Schematic diagram of a conventional oscillating maser frequency standard.

II. Theory

The short- and mid-term frequency stability of a maser is limited by the thermal noise within the microwave cavity and by the external noise added by the receiver used to detect the signal [14]. In the time domain, the frequency stability is commonly expressed by the two-sample variance of the mean relative frequency fluctuations over an averaging time τ, with no dead time (also known as the Allan variance) [15].

In this section, we give the theoretical expressions of the short- and mid-term frequency stability for a maser operated either conventionally or with an external feedback loop.

A. Conventionally Operated Maser

A schematic diagram of a conventional oscillating maser frequency standard is depicted in Fig. 1. The signal delivered at the output of the oscillating maser is applied to the microwave amplifier through an isolator, in order to match the input of the amplifier.

It can be shown [13], [16], [17] that the short- and mid-term frequency stability is given by

$$\sigma_y^2(\tau) = \frac{4k\theta_c}{P_0}\left\{\frac{3\pi f_c}{2\omega_0^2}\left[1+\frac{1}{4}\frac{Q_{\mathrm{ext}}}{Q_c}\frac{1}{\theta_c}\cdot\left(T_a+\frac{T_r}{G_a^2}\right)\right]\frac{1}{\tau^2}+\frac{1}{8Q_l^2}\frac{1}{\tau}\right\} \quad (1)$$

where k is Boltzmann's constant, θ_c, Q_c, and Q_{ext} are, respectively, the cavity temperature, and the loaded and the external cavity quality factors; T_a and G_a are, respectively, the noise temperature and the voltage gain of the microwave amplifier; T_r is the noise temperature of the receiver; ω_0 is the

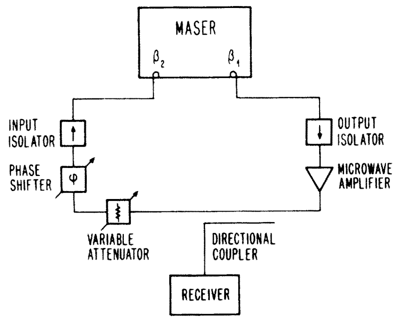

Fig. 2. Schematic diagram of a maser oscillator operated with an external feedback loop in order to modify the cavity Q.

maser angular frequency; Q_l is the atomic line Q; P_0 is the power delivered to the cavity by the atoms; and f_c denotes the cutoff frequency of the first-order low-pass filter introduced to describe the finite bandwidth of frequency fluctuations.

The first two terms of (1) are due to the cavity thermal noise and the receiver noise added to the maser signal; they lead to a white noise of phase. The third term is a white frequency noise resulting from the stimulated emission of radiation by the cavity thermal noise within the atomic linewidth.

The atomic power can be expressed in terms of the cavity Q by [18]

$$P_o = \frac{1}{2} I\hbar\omega_0 \frac{Q_c - Q_t}{Q_c} \tag{2}$$

where I is the source term, $\hbar$ is Planck's constant divided by 2π, and Q_t is the threshold cavity Q value, below which maser oscillation will not occur.

In comparison with previous work [10, eq. (9)] one can notice that the contribution of the white phase noise introduced by the external elements in (1) is reduced by a factor of 4. This comes from the fact that, for a given port of the receiver, the noise power and the signal power considered are the maximum ones available [17]. This assumption was not made in [10].

B. Maser Operated with an External Feedback Loop

The cavity quality factor can be artificially modified by use of an external feedback loop: part of the maser signal is taken out, amplified, and reinjected into the cavity. If the phase of the injected signal is properly adjusted, the maser signal will be increased, thus simulating a cavity with smaller losses.

A schematic diagram of the setup that we have used is shown in Fig. 2. The cavity operates in the transmission mode, with coupling loop coefficients β_1 and β_2 at the output and input ports, respectively. The loaded cavity Q becomes [19]

$$Q_c = \frac{Q_0}{1 + \beta_1 + \beta_2} \tag{3}$$

where Q_0 is the unloaded cavity Q.

The two external cavity quality factors are defined as follows:

$$Q_{\text{ext},1} = \frac{Q_0}{\beta_1} \quad \text{and} \quad Q_{\text{ext},2} = \frac{Q_0}{\beta_2}. \tag{4}$$

The enhanced cavity Q has a maximum value given by [20]

$$Q_e = \frac{Q_0}{1 + \beta_1 + \beta_2 - 2G\sqrt{\beta_1\beta_2}} \tag{5}$$

where G denotes the total voltage gain of the feedback loop. In this type of maser, the atomic power is given by

$$P_0 = \frac{1}{2} I\hbar\omega_0 \frac{Q_e - Q_t}{Q_e}. \tag{6}$$

The presence of the feedback loop increases the level of the thermal noise within the cavity: the additive internal noise of the loop amplifier generates noise inside the cavity and part of the cavity noise is circulated through the feedback loop. These supplementary contributions give the cavity an effective noise temperature which can be written as [17], [20]

$$T_c = \theta_c \frac{Q_e}{Q_c}\left[1 + \beta_2 \frac{T_a}{\theta_c} G^2 \frac{Q_c}{Q_0}\right]. \tag{7}$$

Using (3)-(7), it can easily be shown [17] that the short-term frequency stability of a maser operated with an external feedback loop can be expressed as

$$\sigma_y^2(\tau) = K_{-2}\tau^{-2} + K_{-1}\tau^{-1} \tag{8}$$

where the white phase noise and white frequency noise contributions are, respectively,

$$K_{-2} = \frac{12\pi k\theta_c f_c}{I\hbar\omega_0^3} \frac{Q_e}{Q_e - Q_t} \cdot \left\{\frac{Q_e}{Q_c} + \frac{1}{4\beta_1}\frac{Q_e Q_0}{Q_c^2}\frac{T_a}{\theta_c} + \frac{1}{4\beta_1 G_a^2}\frac{Q_0}{Q_e}\frac{T_r}{\theta_c}\right\} \tag{9}$$

and

$$K_{-1} = \frac{k\theta_c}{I\hbar\omega_0 Q_l^2} \frac{Q_e^2}{Q_e - Q_t} \frac{1}{Q_c} \cdot \left\{1 + \frac{1}{4\beta_1}\frac{Q_0}{Q_c}\left(1 - \frac{Q_c}{Q_e}\right)^2 \frac{T_a}{\theta_c}\right\}. \tag{10}$$

Computation of (8) has been performed with the parameters shown in Table I for both conventional hydrogen and rubidium masers and yields the results shown in Fig. 3(a) and (b), respectively. These figures point out that significant changes of the level of the white phase and white frequency noises can be observed by use of an external feedback loop.

III. Experimental Study

In order to check the validity of the theoretical analysis, both a hydrogen and a rubidium maser were operated with a feedback loop as illustrated in Fig. 2. Variations of the enhanced cavity Q were obtained by changing the attenuation inside the loop and measured with the usual technique of the RF pulse decay [18]. In the case of the hydrogen maser, for each attenuator setting, the phase shifter was adjusted so that the maser frequency remains the same when operated with or without the feedback loop. For the rubidium maser, the phase was adjusted to provide maximum power output.

The short-term frequency stability of each maser was measured by the usual techniques (see, for example, [13]). The

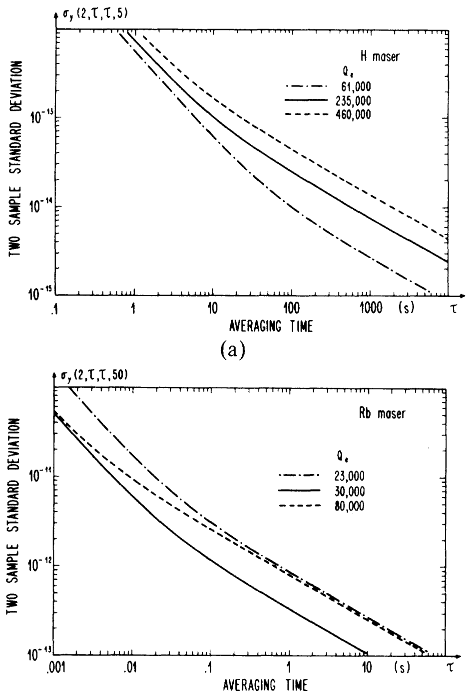

Fig. 3. Variations of the short-term frequency stability: theoretical evaluation. (a) H maser, and (b) Rb maser.

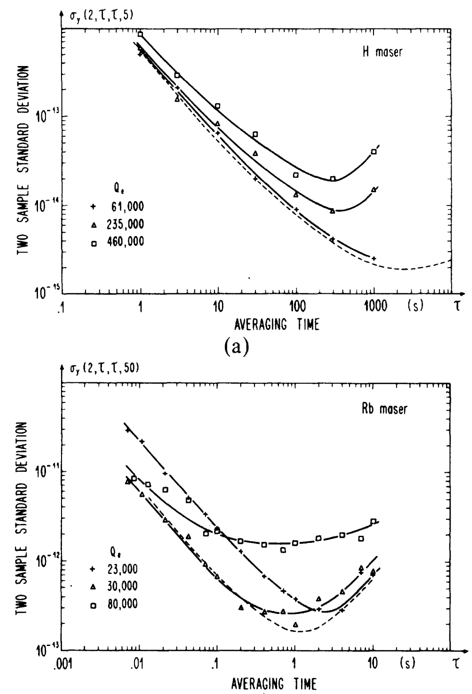

Fig. 4. Variations of the short-term frequency stability: experimental results. (a) H maser, and (b) Rb maser.

TABLE I
MASER PARAMETERS USED TO EVALUATE THE SHORT-TERM FREQUENCY STABILITY

Parameters	H maser	Rb maser
ω_0	$2\pi(1.42 \times 10^9)$ rad/sec	$2\pi(6.83 \times 10^9)$ rad/sec
Q_ℓ	2.2×10^9	5.5×10^7
I (normalization parameter)	3.1×10^{11} at./sec	1.8×10^{13} ph./sec
Q_t	40 000	22 500
Q_0	64 400	28 000
β_1	0.157	0.060
β_2	0.171	0.055
Θ_c	300 K	337 K
T_r	1000 K	1300 K
T_a	216 K	360 K
f_c	5 Hz	50 Hz
G_a	20 dB	28 dB

results obtained for several enhanced cavity Q's and for the value of the maser parameters given in Table I, are shown in Fig. 4(a) and (b). On these figures, the dotted lines represent the frequency stability of the masers when operated without a feedback loop. It is observed that the overall frequency stability decreases when the cavity Q increases. Furthermore, if these experimental data are compared to the theoretical results of Fig. 3(a) and (b), one can see that, for both systems, unpredicted sources of instabilities dominate for long averaging times. No attempt has been made in the frame of this work to find the origin of these instabilities, but the following discussion will be limited to averaging times sufficiently short so that their contribution can be neglected.

The solid lines are best fitted polynomials. From these polynomials, we have deduced the coefficients K_{-2} and K_{-1}, introduced in (8), which define the white phase noise and the white frequency noise contributions. The other terms of the polynomials are ascribed to long-term instabilities and are not considered in the present work. Variations of K_{-2} and K_{-1} with the enhanced cavity Q are depicted in Fig. 5(a) and (b). The solid lines are theoretical results deduced from (9) and (10), for different values of the enhanced cavity Q. They are normalized with the source term I, so that the $\sqrt{K_{-2}}$ theoretical curve has the same value as the experimental point indicated by an arrow. Fig. 5(a) shows that the white phase noise contribution is predominant in the case of the hydrogen maser while, as shown in Fig. 5(b), it is the white frequency noise contribution that dominates for the rubidium maser. These figures also show, especially for the rubidium maser, that each noise contribution can be minimized when varying the enhanced cavity Q and that the required Q value is not the same for each contribution. Consequently, if the overall frequency stability is to be optimized, the quality factor value has to be selected according to the averaging time considered.

A comparison between the experimental observation and the theoretical evaluation of the frequency stability for different fixed averaging times, as a function of the enhanced cavity Q, is given in Fig. 6(a) and (b). The averaging time τ was limited to values smaller than 300 s for the hydrogen maser and 0.07 s for the rubidium maser, in order to reduce the contribution due to long-term instabilities. The computed values (solid lines) show that the frequency stability exhibits an optimum for a value of the enhanced cavity Q which does not depend significantly on the counting time τ. Is also obvious that the agreement between the theoretical and the experimental results is very satisfactory for both masers. The experimental analysis pertaining to the rubidium maser confirms that an optimum of the frequency stability as a function of the cavity Q exists.

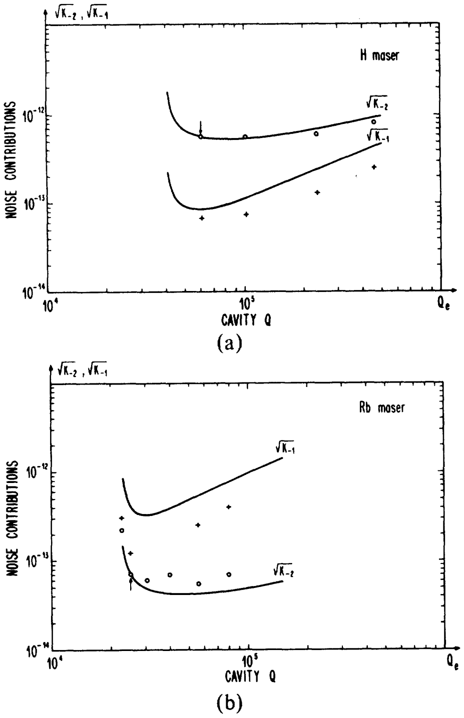

Fig. 5. Variations of the white phase and white frequency noise contributions as functions of the enhanced cavity Q. (a) H maser, and (b) Rb maser.

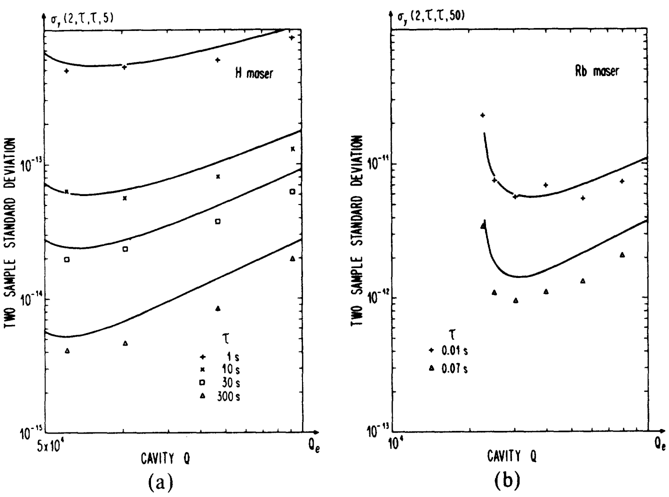

Fig. 6. Variations of the overall short-term frequency stability with the enhanced cavity Q. (a) H maser, and (b) Rb maser.

IV. Conclusion

The experimental study reported here has shown that the frequency stability of an oscillating maser operated with an external feedback loop to modify the cavity quality factor exhibits an optimum. The variations, with the enhanced cavity Q, of the white phase noise and white frequency noise contributions have been measured for both a hydrogen and a rubidium maser. The observed variations are in good agreement with theoretical predictions and show that the model developed to consider the effect of noise on actively operated masers equipped with an external feedback loop is satisfactory. Nevertheless, long-term and mid-term frequency instabilities are observed when the cavity Q is increased. A theoretical study will be established in order to understand their origin.

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