

Dimensionless units

$$\tilde{E} = E/E_0, \quad \tilde{x} = x/x_0$$

where

$$E_0 = \hbar^2 / (m x_0^2) \text{ unit of energy}$$

x_0 unit of length

it is convenient to take box sizes the unit of length $x_0 = L$

Iteration 0 (empty box)

1D) One dimensional box

$$\psi_n(x) = \sin k_n x, \quad n = 1, 2, 3, \dots$$

allowed momenta

$$k_n = n \cdot \frac{\pi}{L}$$

$$\text{Energy } E = \frac{\hbar^2 k_n^2}{2m} = \frac{\pi^2 \hbar^2 n^2}{2m L^2}$$

$$\tilde{E} = \frac{\pi^2 n^2}{2}$$

2D) Two dimensional box

$$\psi(x, y) = \psi_{n_x}(x) \cdot \psi_{n_y}(y)$$

Energy E

$$E = \frac{\pi^2 \hbar^2}{2m L^2} (n_x^2 + n_y^2)$$

$$\tilde{E} = \frac{\pi^2}{2} (n_x^2 + n_y^2)$$

$$\begin{array}{cc} n_x & n_y & \tilde{E} \\ 1 & 1 & \pi^2 \\ 1 & 2 & \left\{ \frac{5}{2} \pi^2 \right. \\ 2 & 1 & \end{array}$$