$$f''(x_i) = \frac{f_{i+1} - 2f_i + f_{i-1}}{\Delta x^2}$$

$$f''\left(\frac{L}{2}\right) = \varnothing \Rightarrow$$

$f(x_i) = f_i$, $i = \varnothing, 1, \ldots$ $\quad -1 \;\; \varnothing \;\; 1 \;\; 2 \;\; 3 \;\; \ldots$

$$x_i = -\frac{L}{2} + i\,\frac{L}{N-1}$$

p.b.c $f'_\varnothing = \varnothing$ $\qquad f'(x_i) = \dfrac{f_{i+1} - f_{i-1}}{2\Delta h}$

$$f'(x_0) = \frac{f_1 - f_{-1}}{2\Delta h} = \varnothing$$

$$\boxed{f_{-1} = f_1}$$

z.b.c $f_{-1} = \varnothing$

$$\Delta \rightarrow \begin{pmatrix} -2; & 1 \rightarrow 2 & \\ 1; & -2; & 1 \\ & 1; & -2; \; 1 \\ & & \ddots \end{pmatrix}$$

p.b.c $$f''_\varnothing = \frac{f_1 - 2f_0 + f_{-1}}{2\Delta x^2} = \frac{2f_1 - 2f_0}{2\Delta x^2}$$

$$\frac{\hbar^2}{2m}\left(-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right) f(x,y) = E f(x,y)$$

$$f(x,y) = f(x)\, f(y)$$

$$\frac{\hbar^2}{2m}\left(-\frac{\partial^2}{\partial x^2}\right) f(x) = E_x\, f(x)$$

$$\text{''}-\text{''} \qquad = E_y$$

$$f(x,y) = f(x)\, f(y)$$

$$\frac{\hbar^2}{2m}\left(-\frac{\partial^2}{\partial x^2}\right) f(x) = E_x\, f(x)$$

$$= E_y$$

$$\text{''} - \text{''}$$

$$= E_x\, f(x) f(y) + E_y\, f(x) f(y)$$

$$= (E_x + E_y)\, f(x,y)$$

$$E = E_x + E_y \underset{\text{g.s.}}{=} 2E_x$$

$$-\frac{\hbar^2}{2m} f''(x) = E_x\, f(x)$$

$$f(x) = A\sin(kx) + B\cos(kx)$$

$$f'(x) = Ak\cos(kx) - Bk\sin(kx)$$

$$f''(x) = -Ak^2\sin kx - Bk^2\cos kx$$

$$= -k^2 f(x)$$

$$\frac{\hbar^2 k^2}{2m} = E_x$$

Free parameters $A, B, k$

Boundary condition.

a) p.b.c $f'\left(\frac{L}{2}\right) = \emptyset$

$\Rightarrow k = \emptyset$ ; $\boxed{f(x) = B \cdot \cos \emptyset x = B = \text{Const}}$

$$\boxed{E = \frac{\hbar^2 k^2}{2m} = \emptyset}$$

b) z.b.c. $f\left(\frac{L}{2}\right) = \emptyset$

$$f\left(-\frac{L}{2}\right) = \emptyset$$

$$f\left(\frac{L}{2}\right) = A \sin \frac{kL}{2} + B \cos \frac{kL}{2} = \emptyset$$

$$f\left(-\frac{L}{2}\right) = -A \sin \frac{kL}{2} + B \cos \frac{kL}{2} = \emptyset$$

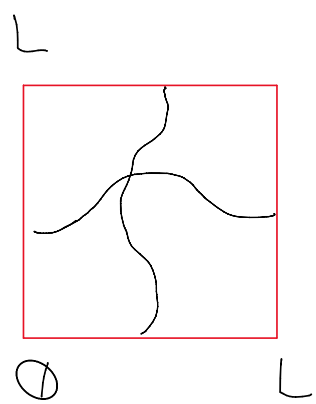
$$f\left(\frac{L}{2}\right) + f\left(-\frac{L}{2}\right) = 2B \cos \frac{kL}{2} = \emptyset$$

$$\frac{kL}{2} = \frac{\pi}{2}$$

$$k = \frac{\pi}{L}$$

$$E_x = \frac{\pi^2}{2m} \frac{\hbar^2}{L^2}$$

$$\boxed{E = \frac{\pi^2 \hbar^2}{m L^2}}$$




$$f(x,y) = \sin \frac{\pi}{L} x \cdot \sin \frac{\pi}{L} y$$

$$f(x=\emptyset, y) = \emptyset$$

$$f(x=L, y) = \sin \pi \cdot \ldots = \emptyset$$

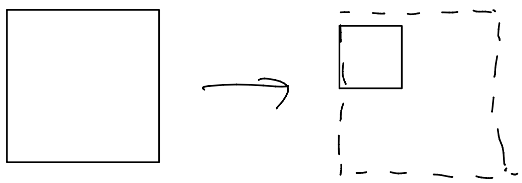
$$\boxed{f(x,y) = \sin \frac{\pi}{L}\left(x - \frac{L}{2}\right) \cdot \cos \frac{\pi}{2}\left(x - \frac{L}{2}\right)}$$

$$f\left(\frac{L}{2}, y\right) = \sin \emptyset \to \emptyset$$

$$f\left(-\frac{L}{2}, y\right) = \sin\frac{\pi}{L}(-L) \cdots = \sin(-\pi) = \emptyset$$

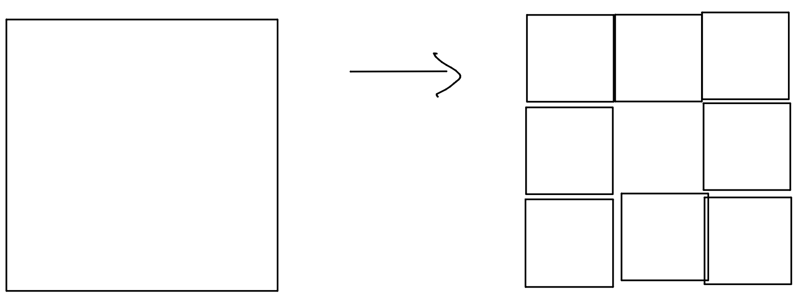
## Energy upper bound:

1) iteration $i$ : $\tilde{L} = \frac{L}{3^i}$



$$E = \pi^2 \frac{\hbar^2}{mL^2} \rightarrow \frac{\pi^2}{(1/3^i)^2}\frac{\hbar^2}{mL^2} = \pi^2 9^i \frac{\hbar^2}{mL^2}$$

2) Improved upper bound



iteration 1 $\quad N$ squares $\quad N = 8$

$$E = \pi^2 \frac{\hbar^2}{mL^2} = \pi^2 \frac{\hbar^2}{m} \frac{N_p}{L^2} = \pi^2 \frac{\hbar^2}{m} n$$

$n = \frac{N_p}{L^2}$ - density $\quad N_p \rightarrow$ number of particles

Iteration:

$N_{total} = 9$

$N_{occupied} = 8$

$\tilde{N}_p = \frac{N_p}{N_{occupied}}$

$$N_p = \sum_{occupied} \tilde{N}_p = N_{occupied} \cdot \tilde{N}_p$$

$$\tilde{N}_p = \frac{N_p}{N_{occupied}}$$

$$\tilde{n}_p = \frac{\tilde{N}_p}{\tilde{L}^2} = \frac{N_p \quad g^i}{N_{occupied} \; L^2}$$

$$E = N_{occupied} \cdot \pi^2 \cdot \frac{\hbar^2}{m} \frac{N_p \cdot g^i}{N_{occupied} \cdot L^2}$$

$$= \pi^2 g^i \frac{\hbar^2}{m L^2}$$