Chair of Scientific Computing
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Time stepping review of open-source solvers

Guided research

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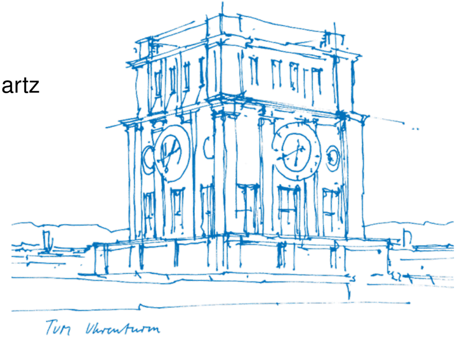

Outline

Motivation

- Simulations of physical phenomena are useful in many fields.

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- Can we reduce the error of solutions?
- Higher order of a numerical method $\Longrightarrow$ higher precision?
- How does this error behave when we couple simulations?

Open-source solvers

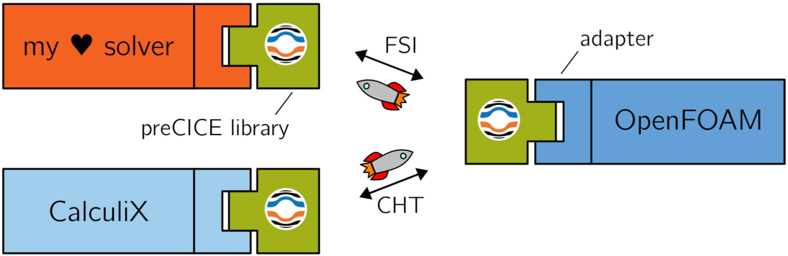

Figure 1 Diagram showing setup for coupling simulations with preCICE library.

Source: https://precice.org

Time stepping schemes

We focus on temporal discretization error $\varepsilon_{\Delta t}$.

Second-order time stepping schemes $\rightarrow \varepsilon_{\Delta t}$ decreases $\mathcal{O}(\Delta t^2)$.

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When solving a PDE of the form:

$$\frac{\partial u}{\partial t} = F(u, t) \tag{1}$$

we need to discretize the time-derivative. Easiest way is the Euler explicit method:

$$\frac{u^{n+1} - u^n}{\Delta t} = F(u^n, t^n) \tag{2}$$

Time stepping schemes

OpenFOAM

- Implicit Euler (1st order):

$$\frac{u^{n+1} - u^n}{\Delta t} = \frac{1}{2} \left[F(u^n, t^n) + F(u^{n+1}, t^{n+1}) \right] \tag{3}$$

- Crank-Nicolson (2nd order):

$$\frac{u^{n+1} - u^n}{\Delta t} = \frac{\theta}{2} F(u^n, t^n) + \left(1 - \frac{\theta}{2}\right) F(u^{n+1}, t^{n+1}) \tag{4}$$

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CalculiX

- Implicit α-method (2nd order):

$$\boldsymbol{v}^{n+1} = \boldsymbol{v}^n + \Delta t\left[(1-\gamma)\boldsymbol{a}^n + \gamma\boldsymbol{a}^{n+1}\right] \tag{5}$$

$$\boldsymbol{u}^{n+1} = \boldsymbol{u}^n + \Delta t\boldsymbol{v}^n + \frac{1}{2}(\Delta t)^2\left[(1-2\beta)\boldsymbol{a}^n + 2\beta\boldsymbol{a}^{n+1}\right] \tag{6}$$

Outline

OpenFOAM

The error has the form $\varepsilon_u = \varepsilon_{\Delta t} + \varepsilon_{\Delta x} + \varepsilon_{\text{num}}$.

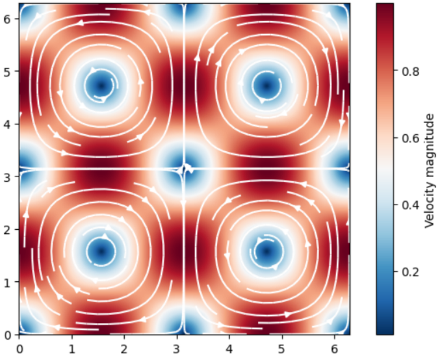
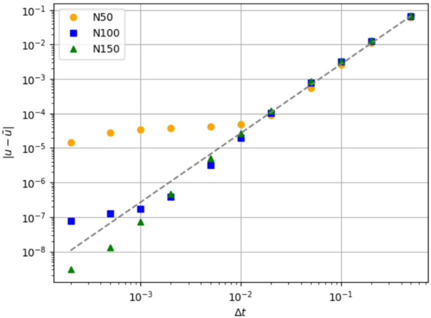


Figure 2 Visualization of the Taylor-Green vortex scenario.

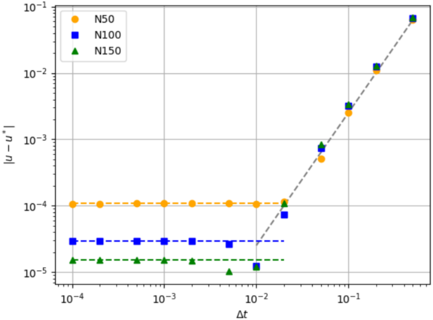
OpenFOAM - convergence study



(a) Error compared to reference ($\Delta t = 10^{-5}$).

$|u - \tilde{u}| = \varepsilon_{\Delta t} + \varepsilon_{\text{num}}$

plateau when $\varepsilon_{\Delta t} < \varepsilon_{\text{num}}$



(b) Error compared to analytical solution.

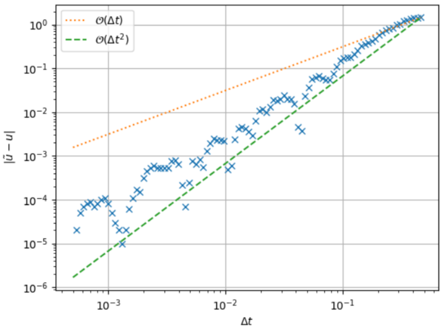
$|u - u^*| = \varepsilon_{\Delta t} + \varepsilon_{\Delta x} + \varepsilon_{\text{num}}$

plateau when $\varepsilon_{\Delta t} < \varepsilon_{\Delta x} + \varepsilon_{\text{num}}$

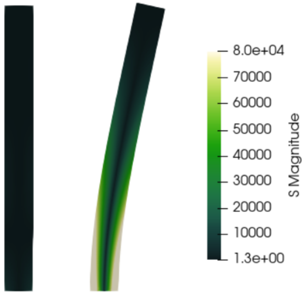
Outline

CalculiX

We use the α-method as time stepping scheme.



(a) Convergence study, showing higher-order convergence.



(b) Perpendicular elastic flap scenario.

Outline

FSI simulation

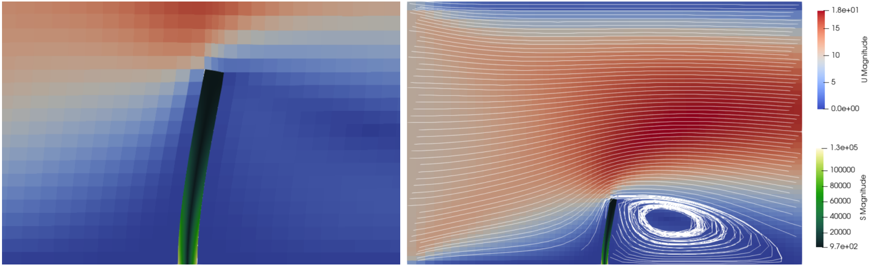


Figure 5 Example solution of the FSI simulation, coupled with preCICE.

FSI Simulation - convergence study

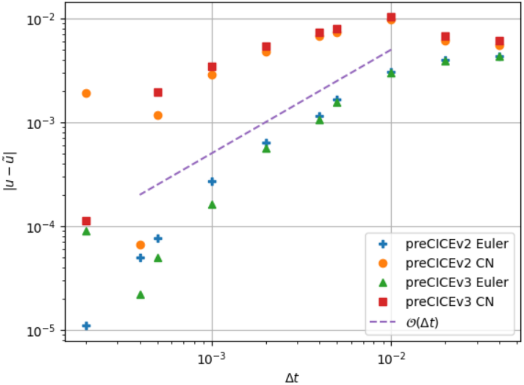


Figure 6 Convergence study of the coupled perpendicular flap scenario. Results with the Crank-Nicolson (CN) and the implicit Euler time stepping schemes, and using v2 and v3 of preCICE.

Outline

Verification of CalculiX adapter

We coupled the CalculiX adapter with a fake-fluid participant that applied a force $f^n = f_{max} \sin(t^n + \phi)$ on the tip.

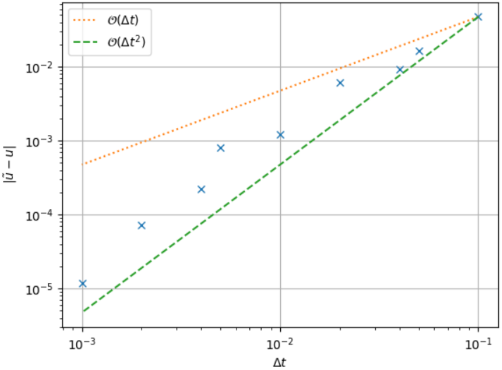
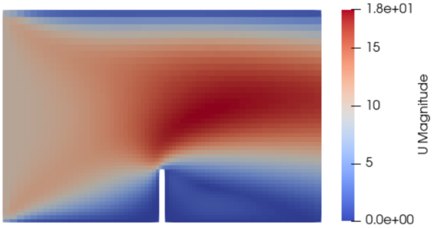


Figure 7 Convergence study of the CalculiX adapter coupled with a fake fluid participant. Reference solution is $\Delta t = 10^{-4}$.

Verification of fluid participant

- Single-solver simulations with fluid participant.
- Hard to convergence due to high CFL numbers.
- Precomputed initial conditions increased stability.
- Time variable inflow conditions.
- Sublinear error convergence.



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- Linear convergence error on FSI simulation achievable.
- CalculiX adapter maintains error convergence. OpenFOAM adapter?
- Try different setups and different numerical methods.
- This works offers some insights in the error behaviour.
- Useful for researchers surrounding preCICE.