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T81-558: Applications of Deep Neural Networks

Module 3: Introduction to TensorFlow

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- For more information visit the class website.

Module 3 Material

- Part 3.1: Deep Learning and Neural Network Introduction [Video] [Notebook]
- Part 3.2: Introduction to Tensorflow and Keras [Video] [Notebook]
- Part 3.3: Saving and Loading a Keras Neural Network [Video] [Notebook]
- **Part 3.4: Early Stopping in Keras to Prevent Overfitting** [Video] [Notebook]
- Part 3.5: Extracting Weights and Manual Calculation [Video] [Notebook]

Google CoLab Instructions

The following code ensures that Google CoLab is running the correct version of TensorFlow.

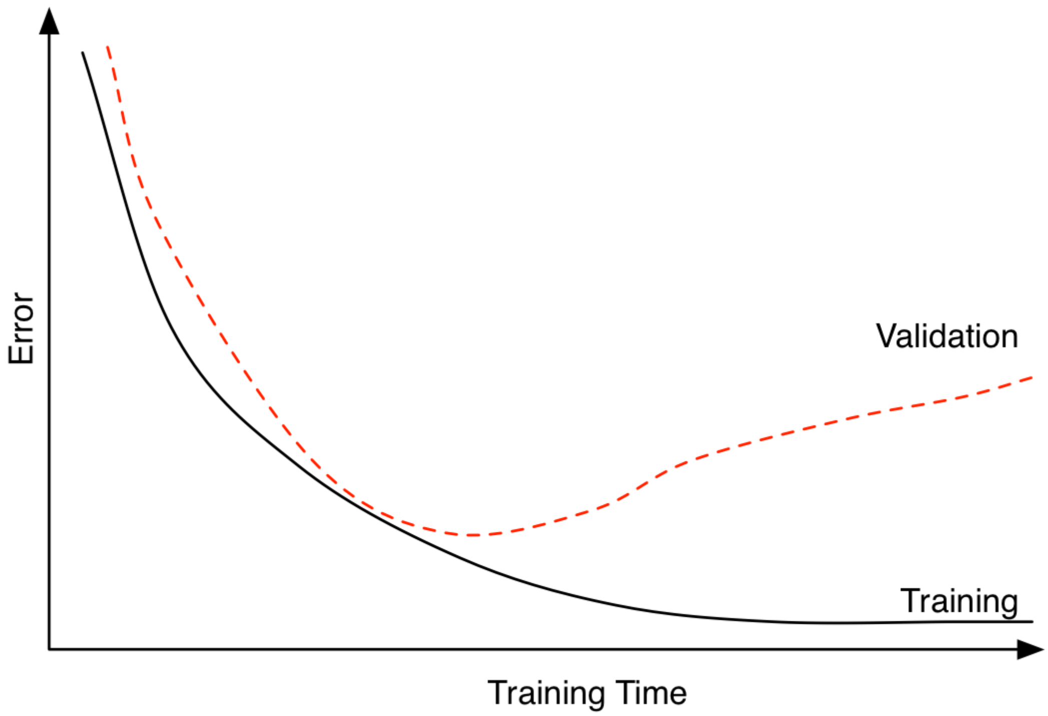
In [1]:
```
try:
    %tensorflow_version 2.x
    COLAB = True
    print("Note: using Google CoLab")
except:
    print("Note: not using Google CoLab")
    COLAB = False
```

```
Note: not using Google CoLab
```

Part 3.4: Early Stopping in Keras to Prevent Overfitting

It can be difficult to determine how many epochs to cycle through to train a neural network. Overfitting will occur if you train the neural network for too many epochs, and the neural network will not perform well on new data, despite attaining a good accuracy on the training set. Overfitting occurs when a neural network is trained to the point that it begins to memorize rather than generalize, as demonstrated in Figure 3.OVER.

Figure 3.OVER: Training vs. Validation Error for Overfitting



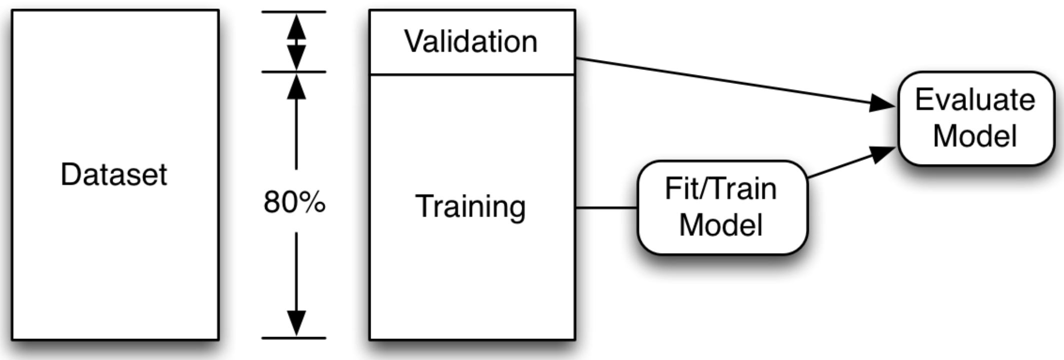
It is important to segment the original dataset into several datasets:

- **Training Set**
- **Validation Set**
- **Holdout Set**

You can construct these sets in several different ways. The following programs demonstrate some of these.

The first method is a training and validation set. We use the training data to train the neural network until the validation set no longer improves. This attempts to stop at a near-optimal training point. This method will only give accurate "out of sample" predictions for the validation set; this is usually 20% of the data. The predictions for the training data will be overly optimistic, as these were the data that we used to train the neural network. Figure 3.VAL demonstrates how we divide the dataset.

Figure 3.VAL: Training with a Validation Set



Early Stopping with Classification

We will now see an example of classification training with early stopping. We will train the neural network until the error no longer improves on the validation set.

In [2]:
```python
import pandas as pd
import io
import requests
import numpy as np
from sklearn import metrics
from sklearn.model_selection import train_test_split
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense, Activation
from tensorflow.keras.callbacks import EarlyStopping

df = pd.read_csv(
    "https://data.heatonresearch.com/data/t81-558/iris.csv",
    na_values=['NA', '?'])

# Convert to numpy - Classification
x = df[['sepal_l', 'sepal_w', 'petal_l', 'petal_w']].values
dummies = pd.get_dummies(df['species']) # Classification
species = dummies.columns
y = dummies.values

# Split into validation and training sets
x_train, x_test, y_train, y_test = train_test_split(
    x, y, test_size=0.25, random_state=42)

# Build neural network
model = Sequential()
model.add(Dense(50, input_dim=x.shape[1], activation='relu')) # Hidden 1
model.add(Dense(25, activation='relu')) # Hidden 2
model.add(Dense(y.shape[1],activation='softmax')) # Output
model.compile(loss='categorical_crossentropy', optimizer='adam')

monitor = EarlyStopping(monitor='val_loss', min_delta=1e-3, patience=5,
        verbose=1, mode='auto', restore_best_weights=True)
```

```
model.fit(x_train,y_train,validation_data=(x_test,y_test),
        callbacks=[monitor],verbose=2,epochs=1000)
```

```
Train on 112 samples, validate on 38 samples
Epoch 1/1000
112/112 - 0s - loss: 1.1940 - val_loss: 1.1126
Epoch 2/1000
112/112 - 0s - loss: 1.0545 - val_loss: 0.9984
Epoch 3/1000
112/112 - 0s - loss: 0.9533 - val_loss: 0.9130
Epoch 4/1000
112/112 - 0s - loss: 0.8823 - val_loss: 0.8365
Epoch 5/1000
112/112 - 0s - loss: 0.8243 - val_loss: 0.7619
Epoch 6/1000
112/112 - 0s - loss: 0.7592 - val_loss: 0.7059
Epoch 7/1000
112/112 - 0s - loss: 0.7142 - val_loss: 0.6644
Epoch 8/1000
112/112 - 0s - loss: 0.6788 - val_loss: 0.6302
Epoch 9/1000
112/112 - 0s - loss: 0.6481 - val_loss: 0.5979
Epoch 10/1000
112/112 - 0s - loss: 0.6198 - val_loss: 0.5698
Epoch 11/1000
112/112 - 0s - loss: 0.5957 - val_loss: 0.5434
Epoch 12/1000
112/112 - 0s - loss: 0.5738 - val_loss: 0.5189
Epoch 13/1000
112/112 - 0s - loss: 0.5539 - val_loss: 0.4964
Epoch 14/1000
112/112 - 0s - loss: 0.5344 - val_loss: 0.4771
Epoch 15/1000
112/112 - 0s - loss: 0.5177 - val_loss: 0.4601
Epoch 16/1000
112/112 - 0s - loss: 0.5022 - val_loss: 0.4455
Epoch 17/1000
112/112 - 0s - loss: 0.4869 - val_loss: 0.4334
Epoch 18/1000
112/112 - 0s - loss: 0.4786 - val_loss: 0.4236
Epoch 19/1000
112/112 - 0s - loss: 0.4634 - val_loss: 0.4096
Epoch 20/1000
112/112 - 0s - loss: 0.4521 - val_loss: 0.3980
Epoch 21/1000
112/112 - 0s - loss: 0.4409 - val_loss: 0.3872
Epoch 22/1000
112/112 - 0s - loss: 0.4296 - val_loss: 0.3776
Epoch 23/1000
112/112 - 0s - loss: 0.4204 - val_loss: 0.3688
Epoch 24/1000
112/112 - 0s - loss: 0.4113 - val_loss: 0.3598
Epoch 25/1000
112/112 - 0s - loss: 0.4025 - val_loss: 0.3519
Epoch 26/1000
112/112 - 0s - loss: 0.3970 - val_loss: 0.3478
Epoch 27/1000
112/112 - 0s - loss: 0.3860 - val_loss: 0.3382
Epoch 28/1000
```

```
112/112 - 0s - loss: 0.3763 - val_loss: 0.3297
Epoch 29/1000
112/112 - 0s - loss: 0.3678 - val_loss: 0.3213
Epoch 30/1000
112/112 - 0s - loss: 0.3600 - val_loss: 0.3137
Epoch 31/1000
112/112 - 0s - loss: 0.3535 - val_loss: 0.3062
Epoch 32/1000
112/112 - 0s - loss: 0.3451 - val_loss: 0.2995
Epoch 33/1000
112/112 - 0s - loss: 0.3380 - val_loss: 0.2940
Epoch 34/1000
112/112 - 0s - loss: 0.3301 - val_loss: 0.2860
Epoch 35/1000
112/112 - 0s - loss: 0.3228 - val_loss: 0.2791
Epoch 36/1000
112/112 - 0s - loss: 0.3152 - val_loss: 0.2726
Epoch 37/1000
112/112 - 0s - loss: 0.3084 - val_loss: 0.2668
Epoch 38/1000
112/112 - 0s - loss: 0.3009 - val_loss: 0.2608
Epoch 39/1000
112/112 - 0s - loss: 0.2945 - val_loss: 0.2558
Epoch 40/1000
112/112 - 0s - loss: 0.2874 - val_loss: 0.2516
Epoch 41/1000
112/112 - 0s - loss: 0.2818 - val_loss: 0.2437
Epoch 42/1000
112/112 - 0s - loss: 0.2744 - val_loss: 0.2364
Epoch 43/1000
112/112 - 0s - loss: 0.2689 - val_loss: 0.2313
Epoch 44/1000
112/112 - 0s - loss: 0.2612 - val_loss: 0.2268
Epoch 45/1000
112/112 - 0s - loss: 0.2556 - val_loss: 0.2219
Epoch 46/1000
112/112 - 0s - loss: 0.2498 - val_loss: 0.2179
Epoch 47/1000
112/112 - 0s - loss: 0.2443 - val_loss: 0.2111
Epoch 48/1000
112/112 - 0s - loss: 0.2381 - val_loss: 0.2053
Epoch 49/1000
112/112 - 0s - loss: 0.2331 - val_loss: 0.2008
Epoch 50/1000
112/112 - 0s - loss: 0.2273 - val_loss: 0.1956
Epoch 51/1000
112/112 - 0s - loss: 0.2249 - val_loss: 0.1906
Epoch 52/1000
112/112 - 0s - loss: 0.2172 - val_loss: 0.1909
Epoch 53/1000
112/112 - 0s - loss: 0.2170 - val_loss: 0.1943
Epoch 54/1000
112/112 - 0s - loss: 0.2099 - val_loss: 0.1791
Epoch 55/1000
112/112 - 0s - loss: 0.2073 - val_loss: 0.1758
Epoch 56/1000
```

```
112/112 - 0s - loss: 0.2031 - val_loss: 0.1712
Epoch 57/1000
112/112 - 0s - loss: 0.1970 - val_loss: 0.1717
Epoch 58/1000
112/112 - 0s - loss: 0.1907 - val_loss: 0.1648
Epoch 59/1000
112/112 - 0s - loss: 0.1862 - val_loss: 0.1606
Epoch 60/1000
112/112 - 0s - loss: 0.1831 - val_loss: 0.1572
Epoch 61/1000
112/112 - 0s - loss: 0.1840 - val_loss: 0.1590
Epoch 62/1000
112/112 - 0s - loss: 0.1753 - val_loss: 0.1518
Epoch 63/1000
112/112 - 0s - loss: 0.1721 - val_loss: 0.1470
Epoch 64/1000
112/112 - 0s - loss: 0.1706 - val_loss: 0.1443
Epoch 65/1000
112/112 - 0s - loss: 0.1660 - val_loss: 0.1488
Epoch 66/1000
112/112 - 0s - loss: 0.1643 - val_loss: 0.1441
Epoch 67/1000
112/112 - 0s - loss: 0.1598 - val_loss: 0.1390
Epoch 68/1000
112/112 - 0s - loss: 0.1566 - val_loss: 0.1334
Epoch 69/1000
112/112 - 0s - loss: 0.1554 - val_loss: 0.1316
Epoch 70/1000
112/112 - 0s - loss: 0.1519 - val_loss: 0.1315
Epoch 71/1000
112/112 - 0s - loss: 0.1483 - val_loss: 0.1396
Epoch 72/1000
112/112 - 0s - loss: 0.1502 - val_loss: 0.1327
Epoch 73/1000
112/112 - 0s - loss: 0.1441 - val_loss: 0.1229
Epoch 74/1000
112/112 - 0s - loss: 0.1417 - val_loss: 0.1198
Epoch 75/1000
112/112 - 0s - loss: 0.1411 - val_loss: 0.1189
Epoch 76/1000
112/112 - 0s - loss: 0.1365 - val_loss: 0.1207
Epoch 77/1000
112/112 - 0s - loss: 0.1350 - val_loss: 0.1229
Epoch 78/1000
112/112 - 0s - loss: 0.1355 - val_loss: 0.1182
Epoch 79/1000
112/112 - 0s - loss: 0.1320 - val_loss: 0.1152
Epoch 80/1000
112/112 - 0s - loss: 0.1300 - val_loss: 0.1092
Epoch 81/1000
112/112 - 0s - loss: 0.1285 - val_loss: 0.1091
Epoch 82/1000
112/112 - 0s - loss: 0.1258 - val_loss: 0.1140
Epoch 83/1000
112/112 - 0s - loss: 0.1308 - val_loss: 0.1144
Epoch 84/1000
```

```
112/112 - 0s - loss: 0.1259 - val_loss: 0.1027
Epoch 85/1000
112/112 - 0s - loss: 0.1237 - val_loss: 0.1022
Epoch 86/1000
112/112 - 0s - loss: 0.1202 - val_loss: 0.1022
Epoch 87/1000
112/112 - 0s - loss: 0.1180 - val_loss: 0.1049
Epoch 88/1000
112/112 - 0s - loss: 0.1174 - val_loss: 0.1028
Epoch 89/1000
112/112 - 0s - loss: 0.1153 - val_loss: 0.0974
Epoch 90/1000
112/112 - 0s - loss: 0.1167 - val_loss: 0.0946
Epoch 91/1000
112/112 - 0s - loss: 0.1149 - val_loss: 0.0966
Epoch 92/1000
112/112 - 0s - loss: 0.1157 - val_loss: 0.1050
Epoch 93/1000
112/112 - 0s - loss: 0.1122 - val_loss: 0.0930
Epoch 94/1000
112/112 - 0s - loss: 0.1136 - val_loss: 0.0905
Epoch 95/1000
112/112 - 0s - loss: 0.1086 - val_loss: 0.1000
Epoch 96/1000
112/112 - 0s - loss: 0.1118 - val_loss: 0.1087
Epoch 97/1000
112/112 - 0s - loss: 0.1095 - val_loss: 0.0923
Epoch 98/1000
112/112 - 0s - loss: 0.1096 - val_loss: 0.0864
Epoch 99/1000
112/112 - 0s - loss: 0.1138 - val_loss: 0.0856
Epoch 100/1000
112/112 - 0s - loss: 0.1096 - val_loss: 0.1144
Epoch 101/1000
112/112 - 0s - loss: 0.1197 - val_loss: 0.1026
Epoch 102/1000
112/112 - 0s - loss: 0.1064 - val_loss: 0.0827
Epoch 103/1000
112/112 - 0s - loss: 0.1069 - val_loss: 0.0823
Epoch 104/1000
112/112 - 0s - loss: 0.1022 - val_loss: 0.0863
Epoch 105/1000
112/112 - 0s - loss: 0.0992 - val_loss: 0.0933
Epoch 106/1000
112/112 - 0s - loss: 0.1017 - val_loss: 0.0926
Epoch 107/1000
Restoring model weights from the end of the best epoch.
112/112 - 0s - loss: 0.1001 - val_loss: 0.0869
Epoch 00107: early stopping
```

Out[2]:
```
<tensorflow.python.keras.callbacks.History at 0x22a9ad34708>
```

There are a number of parameters that are specified to the **EarlyStopping** object.

- **min_delta** This value should be kept small. It simply means the minimum change in error to be registered as an improvement. Setting it even smaller will not likely have a great deal of impact.
- **patience** How long should the training wait for the validation error to improve?
- **verbose** How much progress information do you want?
- **mode** In general, always set this to "auto". This allows you to specify if the error should be minimized or maximized. Consider accuracy, where higher numbers are desired vs log-loss/RMSE where lower numbers are desired.
- **restore_best_weights** This should always be set to true. This restores the weights to the values they were at when the validation set is the highest. Unless you are manually tracking the weights yourself (we do not use this technique in this course), you should have Keras perform this step for you.

As you can see from above, the entire number of requested epochs were not used. The neural network training stopped once the validation set no longer improved.

In [3]:
```
from sklearn.metrics import accuracy_score

pred = model.predict(x_test)
predict_classes = np.argmax(pred,axis=1)
expected_classes = np.argmax(y_test,axis=1)
correct = accuracy_score(expected_classes,predict_classes)
print(f"Accuracy: {correct}")
```

```
Accuracy: 1.0
```

Early Stopping with Regression

The following code demonstrates how we can apply early stopping to a regression problem. The technique is similar to the early stopping for classification code that we just saw.

In [4]:
```
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense, Activation
import pandas as pd
import io
import os
import requests
import numpy as np
from sklearn import metrics

df = pd.read_csv(
    "https://data.heatonresearch.com/data/t81-558/auto-mpg.csv",
    na_values=['NA', '?'])

cars = df['name']
```

```python
# Handle missing value
df['horsepower'] = df['horsepower'].fillna(df['horsepower'].median())

# Pandas to Numpy
x = df[['cylinders', 'displacement', 'horsepower', 'weight',
       'acceleration', 'year', 'origin']].values
y = df['mpg'].values # regression

# Split into validation and training sets
x_train, x_test, y_train, y_test = train_test_split(
    x, y, test_size=0.25, random_state=42)

# Build the neural network
model = Sequential()
model.add(Dense(25, input_dim=x.shape[1], activation='relu')) # Hidden 1
model.add(Dense(10, activation='relu')) # Hidden 2
model.add(Dense(1)) # Output
model.compile(loss='mean_squared_error', optimizer='adam')

monitor = EarlyStopping(monitor='val_loss', min_delta=1e-3,
        patience=5, verbose=1, mode='auto',
        restore_best_weights=True)
model.fit(x_train,y_train,validation_data=(x_test,y_test),
        callbacks=[monitor], verbose=2,epochs=1000)
```

```
Train on 298 samples, validate on 100 samples
Epoch 1/1000
298/298 - 0s - loss: 254618.1117 - val_loss: 104859.9187
Epoch 2/1000
298/298 - 0s - loss: 53735.2417 - val_loss: 10033.3467
Epoch 3/1000
298/298 - 0s - loss: 3456.0443 - val_loss: 2832.0205
Epoch 4/1000
298/298 - 0s - loss: 4912.1159 - val_loss: 5504.1926
Epoch 5/1000
298/298 - 0s - loss: 4154.7669 - val_loss: 2042.1780
Epoch 6/1000
298/298 - 0s - loss: 1411.5907 - val_loss: 1259.3724
Epoch 7/1000
298/298 - 0s - loss: 1189.8836 - val_loss: 1435.5145
Epoch 8/1000
298/298 - 0s - loss: 1207.4120 - val_loss: 1259.7002
Epoch 9/1000
298/298 - 0s - loss: 1069.7891 - val_loss: 1189.8975
Epoch 10/1000
298/298 - 0s - loss: 1068.2267 - val_loss: 1188.1633
Epoch 11/1000
298/298 - 0s - loss: 1068.9461 - val_loss: 1175.8650
Epoch 12/1000
298/298 - 0s - loss: 1044.6897 - val_loss: 1185.7492
Epoch 13/1000
298/298 - 0s - loss: 1056.0984 - val_loss: 1178.5605
Epoch 14/1000
298/298 - 0s - loss: 1041.7714 - val_loss: 1157.2365
Epoch 15/1000
298/298 - 0s - loss: 1031.7727 - val_loss: 1146.1638
Epoch 16/1000
298/298 - 0s - loss: 1026.6840 - val_loss: 1140.5295
Epoch 17/1000
298/298 - 0s - loss: 1019.7115 - val_loss: 1131.8495
Epoch 18/1000
298/298 - 0s - loss: 1010.8711 - val_loss: 1122.4224
Epoch 19/1000
298/298 - 0s - loss: 1013.6087 - val_loss: 1111.3609
Epoch 20/1000
298/298 - 0s - loss: 995.9503 - val_loss: 1105.5188
Epoch 21/1000
298/298 - 0s - loss: 987.5903 - val_loss: 1094.2863
Epoch 22/1000
298/298 - 0s - loss: 990.0723 - val_loss: 1089.7853
Epoch 23/1000
298/298 - 0s - loss: 968.7077 - val_loss: 1074.3502
Epoch 24/1000
298/298 - 0s - loss: 968.9280 - val_loss: 1065.4332
Epoch 25/1000
298/298 - 0s - loss: 955.3398 - val_loss: 1055.7287
Epoch 26/1000
298/298 - 0s - loss: 955.5287 - val_loss: 1052.8219
Epoch 27/1000
298/298 - 0s - loss: 935.7177 - val_loss: 1035.1746
Epoch 28/1000
```

```
298/298 - 0s - loss: 938.9435 - val_loss: 1026.7096
Epoch 29/1000
298/298 - 0s - loss: 921.2798 - val_loss: 1021.8623
Epoch 30/1000
298/298 - 0s - loss: 918.7541 - val_loss: 1021.8645
Epoch 31/1000
298/298 - 0s - loss: 903.5642 - val_loss: 994.2775
Epoch 32/1000
298/298 - 0s - loss: 896.2183 - val_loss: 984.4263
Epoch 33/1000
298/298 - 0s - loss: 886.1336 - val_loss: 978.4129
Epoch 34/1000
298/298 - 0s - loss: 877.7422 - val_loss: 964.1715
Epoch 35/1000
298/298 - 0s - loss: 871.3048 - val_loss: 956.3459
Epoch 36/1000
298/298 - 0s - loss: 861.6707 - val_loss: 948.6097
Epoch 37/1000
298/298 - 0s - loss: 850.0068 - val_loss: 932.7441
Epoch 38/1000
298/298 - 0s - loss: 846.9615 - val_loss: 921.6213
Epoch 39/1000
298/298 - 0s - loss: 830.3624 - val_loss: 913.5166
Epoch 40/1000
298/298 - 0s - loss: 831.6781 - val_loss: 907.8736
Epoch 41/1000
298/298 - 0s - loss: 814.4517 - val_loss: 889.8433
Epoch 42/1000
298/298 - 0s - loss: 804.2001 - val_loss: 879.6267
Epoch 43/1000
298/298 - 0s - loss: 793.5329 - val_loss: 869.0650
Epoch 44/1000
298/298 - 0s - loss: 786.6698 - val_loss: 857.7609
Epoch 45/1000
298/298 - 0s - loss: 775.7591 - val_loss: 847.1539
Epoch 46/1000
298/298 - 0s - loss: 767.7103 - val_loss: 836.7088
Epoch 47/1000
298/298 - 0s - loss: 756.9816 - val_loss: 825.8035
Epoch 48/1000
298/298 - 0s - loss: 747.9103 - val_loss: 819.3103
Epoch 49/1000
298/298 - 0s - loss: 739.1126 - val_loss: 805.0508
Epoch 50/1000
298/298 - 0s - loss: 734.6592 - val_loss: 795.2228
Epoch 51/1000
298/298 - 0s - loss: 724.3488 - val_loss: 783.2872
Epoch 52/1000
298/298 - 0s - loss: 710.7389 - val_loss: 779.2385
Epoch 53/1000
298/298 - 0s - loss: 702.9931 - val_loss: 762.7323
Epoch 54/1000
298/298 - 0s - loss: 694.2653 - val_loss: 751.5614
Epoch 55/1000
298/298 - 0s - loss: 682.2225 - val_loss: 744.4663
Epoch 56/1000
```

```
298/298 - 0s - loss: 683.8359 - val_loss: 738.8125
Epoch 57/1000
298/298 - 0s - loss: 673.9678 - val_loss: 723.7866
Epoch 58/1000
298/298 - 0s - loss: 655.8523 - val_loss: 715.6897
Epoch 59/1000
298/298 - 0s - loss: 649.6330 - val_loss: 704.0192
Epoch 60/1000
298/298 - 0s - loss: 643.9476 - val_loss: 691.1572
Epoch 61/1000
298/298 - 0s - loss: 627.9205 - val_loss: 685.6211
Epoch 62/1000
298/298 - 0s - loss: 630.9766 - val_loss: 675.3809
Epoch 63/1000
298/298 - 0s - loss: 620.4021 - val_loss: 664.9146
Epoch 64/1000
298/298 - 0s - loss: 601.4826 - val_loss: 655.5067
Epoch 65/1000
298/298 - 0s - loss: 602.5151 - val_loss: 644.4906
Epoch 66/1000
298/298 - 0s - loss: 584.9831 - val_loss: 631.7765
Epoch 67/1000
298/298 - 0s - loss: 582.3892 - val_loss: 620.7529
Epoch 68/1000
298/298 - 0s - loss: 580.3517 - val_loss: 617.2255
Epoch 69/1000
298/298 - 0s - loss: 575.3606 - val_loss: 603.6507
Epoch 70/1000
298/298 - 0s - loss: 551.4546 - val_loss: 598.6873
Epoch 71/1000
298/298 - 0s - loss: 552.0443 - val_loss: 583.6519
Epoch 72/1000
298/298 - 0s - loss: 536.8391 - val_loss: 576.5555
Epoch 73/1000
298/298 - 0s - loss: 529.9672 - val_loss: 564.6031
Epoch 74/1000
298/298 - 0s - loss: 522.4439 - val_loss: 556.2015
Epoch 75/1000
298/298 - 0s - loss: 513.4194 - val_loss: 548.1135
Epoch 76/1000
298/298 - 0s - loss: 505.7009 - val_loss: 537.8890
Epoch 77/1000
298/298 - 0s - loss: 496.8726 - val_loss: 530.3638
Epoch 78/1000
298/298 - 0s - loss: 488.8692 - val_loss: 520.3936
Epoch 79/1000
298/298 - 0s - loss: 481.2276 - val_loss: 512.5432
Epoch 80/1000
298/298 - 0s - loss: 477.8306 - val_loss: 503.1329
Epoch 81/1000
298/298 - 0s - loss: 473.3998 - val_loss: 494.9358
Epoch 82/1000
298/298 - 0s - loss: 465.8867 - val_loss: 490.6273
Epoch 83/1000
298/298 - 0s - loss: 453.1066 - val_loss: 479.8850
Epoch 84/1000
```

```
298/298 - 0s - loss: 445.6094 - val_loss: 471.7849
Epoch 85/1000
298/298 - 0s - loss: 444.9835 - val_loss: 462.8412
Epoch 86/1000
298/298 - 0s - loss: 443.5763 - val_loss: 456.5965
Epoch 87/1000
298/298 - 0s - loss: 436.6940 - val_loss: 453.5159
Epoch 88/1000
298/298 - 0s - loss: 414.3947 - val_loss: 447.0089
Epoch 89/1000
298/298 - 0s - loss: 416.7841 - val_loss: 433.9080
Epoch 90/1000
298/298 - 0s - loss: 403.5432 - val_loss: 423.4334
Epoch 91/1000
298/298 - 0s - loss: 403.1473 - val_loss: 415.1188
Epoch 92/1000
298/298 - 0s - loss: 390.5989 - val_loss: 408.5711
Epoch 93/1000
298/298 - 0s - loss: 385.0042 - val_loss: 400.7886
Epoch 94/1000
298/298 - 0s - loss: 380.2837 - val_loss: 394.4561
Epoch 95/1000
298/298 - 0s - loss: 382.1260 - val_loss: 388.9179
Epoch 96/1000
298/298 - 0s - loss: 371.1698 - val_loss: 380.5425
Epoch 97/1000
298/298 - 0s - loss: 359.9534 - val_loss: 373.7457
Epoch 98/1000
298/298 - 0s - loss: 358.0036 - val_loss: 366.5114
Epoch 99/1000
298/298 - 0s - loss: 348.6594 - val_loss: 359.0750
Epoch 100/1000
298/298 - 0s - loss: 344.6860 - val_loss: 352.5845
Epoch 101/1000
298/298 - 0s - loss: 338.0005 - val_loss: 345.9701
Epoch 102/1000
298/298 - 0s - loss: 331.2779 - val_loss: 340.2206
Epoch 103/1000
298/298 - 0s - loss: 325.3663 - val_loss: 334.1550
Epoch 104/1000
298/298 - 0s - loss: 319.3072 - val_loss: 327.7170
Epoch 105/1000
298/298 - 0s - loss: 313.7492 - val_loss: 322.3784
Epoch 106/1000
298/298 - 0s - loss: 313.7471 - val_loss: 315.4883
Epoch 107/1000
298/298 - 0s - loss: 304.8789 - val_loss: 309.6435
Epoch 108/1000
298/298 - 0s - loss: 301.6150 - val_loss: 304.9265
Epoch 109/1000
298/298 - 0s - loss: 300.2148 - val_loss: 299.9399
Epoch 110/1000
298/298 - 0s - loss: 289.3050 - val_loss: 292.6603
Epoch 111/1000
298/298 - 0s - loss: 282.8135 - val_loss: 286.8729
Epoch 112/1000
```

```
298/298 - 0s - loss: 283.6183 - val_loss: 281.0534
Epoch 113/1000
298/298 - 0s - loss: 274.6550 - val_loss: 275.6063
Epoch 114/1000
298/298 - 0s - loss: 269.9542 - val_loss: 271.9059
Epoch 115/1000
298/298 - 0s - loss: 265.6656 - val_loss: 265.1887
Epoch 116/1000
298/298 - 0s - loss: 262.1005 - val_loss: 260.1739
Epoch 117/1000
298/298 - 0s - loss: 256.3500 - val_loss: 255.2909
Epoch 118/1000
298/298 - 0s - loss: 251.3900 - val_loss: 252.0265
Epoch 119/1000
298/298 - 0s - loss: 247.2246 - val_loss: 245.5129
Epoch 120/1000
298/298 - 0s - loss: 241.7555 - val_loss: 240.8349
Epoch 121/1000
298/298 - 0s - loss: 237.9977 - val_loss: 236.3335
Epoch 122/1000
298/298 - 0s - loss: 233.5239 - val_loss: 231.7200
Epoch 123/1000
298/298 - 0s - loss: 229.3251 - val_loss: 227.2675
Epoch 124/1000
298/298 - 0s - loss: 225.5864 - val_loss: 222.6441
Epoch 125/1000
298/298 - 0s - loss: 221.2191 - val_loss: 218.1110
Epoch 126/1000
298/298 - 0s - loss: 217.8098 - val_loss: 213.9518
Epoch 127/1000
298/298 - 0s - loss: 214.3937 - val_loss: 210.5598
Epoch 128/1000
298/298 - 0s - loss: 210.2760 - val_loss: 205.6227
Epoch 129/1000
298/298 - 0s - loss: 206.5413 - val_loss: 202.4728
Epoch 130/1000
298/298 - 0s - loss: 202.3109 - val_loss: 197.9401
Epoch 131/1000
298/298 - 0s - loss: 199.8272 - val_loss: 196.1144
Epoch 132/1000
298/298 - 0s - loss: 197.1229 - val_loss: 190.0905
Epoch 133/1000
298/298 - 0s - loss: 192.5514 - val_loss: 186.7910
Epoch 134/1000
298/298 - 0s - loss: 189.2665 - val_loss: 184.1961
Epoch 135/1000
298/298 - 0s - loss: 185.1848 - val_loss: 179.9203
Epoch 136/1000
298/298 - 0s - loss: 186.1516 - val_loss: 176.2954
Epoch 137/1000
298/298 - 0s - loss: 182.4030 - val_loss: 173.4539
Epoch 138/1000
298/298 - 0s - loss: 177.4716 - val_loss: 169.6453
Epoch 139/1000
298/298 - 0s - loss: 173.9908 - val_loss: 166.0001
Epoch 140/1000
```

```
298/298 - 0s - loss: 173.2805 - val_loss: 162.8689
Epoch 141/1000
298/298 - 0s - loss: 172.0611 - val_loss: 159.8967
Epoch 142/1000
298/298 - 0s - loss: 165.5859 - val_loss: 157.3186
Epoch 143/1000
298/298 - 0s - loss: 162.3572 - val_loss: 153.6901
Epoch 144/1000
298/298 - 0s - loss: 161.0297 - val_loss: 151.6905
Epoch 145/1000
298/298 - 0s - loss: 164.1645 - val_loss: 148.8484
Epoch 146/1000
298/298 - 0s - loss: 156.3238 - val_loss: 145.0771
Epoch 147/1000
298/298 - 0s - loss: 152.3051 - val_loss: 142.4923
Epoch 148/1000
298/298 - 0s - loss: 149.8716 - val_loss: 140.4517
Epoch 149/1000
298/298 - 0s - loss: 147.7921 - val_loss: 137.0884
Epoch 150/1000
298/298 - 0s - loss: 144.5433 - val_loss: 134.4882
Epoch 151/1000
298/298 - 0s - loss: 144.0840 - val_loss: 134.6734
Epoch 152/1000
298/298 - 0s - loss: 142.7512 - val_loss: 129.2658
Epoch 153/1000
298/298 - 0s - loss: 138.6744 - val_loss: 126.8691
Epoch 154/1000
298/298 - 0s - loss: 136.3120 - val_loss: 125.7347
Epoch 155/1000
298/298 - 0s - loss: 134.8607 - val_loss: 122.1199
Epoch 156/1000
298/298 - 0s - loss: 132.3261 - val_loss: 120.8815
Epoch 157/1000
298/298 - 0s - loss: 130.2538 - val_loss: 117.9441
Epoch 158/1000
298/298 - 0s - loss: 127.5774 - val_loss: 116.9117
Epoch 159/1000
298/298 - 0s - loss: 128.5830 - val_loss: 114.8769
Epoch 160/1000
298/298 - 0s - loss: 123.8368 - val_loss: 112.2658
Epoch 161/1000
298/298 - 0s - loss: 121.8774 - val_loss: 110.3176
Epoch 162/1000
298/298 - 0s - loss: 121.1990 - val_loss: 108.8108
Epoch 163/1000
298/298 - 0s - loss: 119.1470 - val_loss: 106.4554
Epoch 164/1000
298/298 - 0s - loss: 117.1019 - val_loss: 104.7673
Epoch 165/1000
298/298 - 0s - loss: 114.4462 - val_loss: 102.9108
Epoch 166/1000
298/298 - 0s - loss: 113.8899 - val_loss: 100.6241
Epoch 167/1000
298/298 - 0s - loss: 113.7473 - val_loss: 99.1480
Epoch 168/1000
```

```
298/298 - 0s - loss: 109.9129 - val_loss: 98.5171
Epoch 169/1000
298/298 - 0s - loss: 111.6148 - val_loss: 95.8686
Epoch 170/1000
298/298 - 0s - loss: 109.5533 - val_loss: 97.8955
Epoch 171/1000
298/298 - 0s - loss: 110.5111 - val_loss: 92.7941
Epoch 172/1000
298/298 - 0s - loss: 110.6292 - val_loss: 96.9406
Epoch 173/1000
298/298 - 0s - loss: 108.2353 - val_loss: 90.7488
Epoch 174/1000
298/298 - 0s - loss: 103.7881 - val_loss: 88.2208
Epoch 175/1000
298/298 - 0s - loss: 100.4373 - val_loss: 89.0537
Epoch 176/1000
298/298 - 0s - loss: 100.0941 - val_loss: 85.4782
Epoch 177/1000
298/298 - 0s - loss: 97.8368 - val_loss: 85.8181
Epoch 178/1000
298/298 - 0s - loss: 95.8849 - val_loss: 83.0792
Epoch 179/1000
298/298 - 0s - loss: 94.7138 - val_loss: 84.0111
Epoch 180/1000
298/298 - 0s - loss: 93.9980 - val_loss: 80.8398
Epoch 181/1000
298/298 - 0s - loss: 92.4562 - val_loss: 81.9521
Epoch 182/1000
298/298 - 0s - loss: 91.9720 - val_loss: 81.2425
Epoch 183/1000
298/298 - 0s - loss: 93.9076 - val_loss: 77.1700
Epoch 184/1000
298/298 - 0s - loss: 92.0447 - val_loss: 76.0691
Epoch 185/1000
298/298 - 0s - loss: 92.4003 - val_loss: 77.9899
Epoch 186/1000
298/298 - 0s - loss: 87.6844 - val_loss: 73.9357
Epoch 187/1000
298/298 - 0s - loss: 86.4119 - val_loss: 74.5456
Epoch 188/1000
298/298 - 0s - loss: 85.1260 - val_loss: 73.0177
Epoch 189/1000
298/298 - 0s - loss: 85.2527 - val_loss: 71.2634
Epoch 190/1000
298/298 - 0s - loss: 84.7504 - val_loss: 73.4859
Epoch 191/1000
298/298 - 0s - loss: 83.9971 - val_loss: 70.3122
Epoch 192/1000
298/298 - 0s - loss: 82.2615 - val_loss: 68.4355
Epoch 193/1000
298/298 - 0s - loss: 86.2356 - val_loss: 76.1497
Epoch 194/1000
298/298 - 0s - loss: 82.0077 - val_loss: 70.1432
Epoch 195/1000
298/298 - 0s - loss: 84.0382 - val_loss: 74.0556
Epoch 196/1000
```

```
298/298 - 0s - loss: 79.0808 - val_loss: 65.3704
Epoch 197/1000
298/298 - 0s - loss: 77.5371 - val_loss: 65.6799
Epoch 198/1000
298/298 - 0s - loss: 76.7543 - val_loss: 63.9797
Epoch 199/1000
298/298 - 0s - loss: 75.4548 - val_loss: 65.2337
Epoch 200/1000
298/298 - 0s - loss: 75.2814 - val_loss: 62.7816
Epoch 201/1000
298/298 - 0s - loss: 78.7884 - val_loss: 66.5500
Epoch 202/1000
298/298 - 0s - loss: 74.7617 - val_loss: 62.7047
Epoch 203/1000
298/298 - 0s - loss: 73.3059 - val_loss: 63.5815
Epoch 204/1000
298/298 - 0s - loss: 73.3379 - val_loss: 60.1637
Epoch 205/1000
298/298 - 0s - loss: 72.8527 - val_loss: 59.5174
Epoch 206/1000
298/298 - 0s - loss: 71.3816 - val_loss: 58.9280
Epoch 207/1000
298/298 - 0s - loss: 71.0684 - val_loss: 58.5003
Epoch 208/1000
298/298 - 0s - loss: 69.5999 - val_loss: 58.6842
Epoch 209/1000
298/298 - 0s - loss: 69.6249 - val_loss: 63.3405
Epoch 210/1000
298/298 - 0s - loss: 70.2080 - val_loss: 56.3041
Epoch 211/1000
298/298 - 0s - loss: 68.7671 - val_loss: 56.1137
Epoch 212/1000
298/298 - 0s - loss: 67.4164 - val_loss: 56.0807
Epoch 213/1000
298/298 - 0s - loss: 67.4641 - val_loss: 60.8322
Epoch 214/1000
298/298 - 0s - loss: 70.2280 - val_loss: 55.4504
Epoch 215/1000
298/298 - 0s - loss: 70.7004 - val_loss: 56.4594
Epoch 216/1000
298/298 - 0s - loss: 69.3142 - val_loss: 66.7034
Epoch 217/1000
298/298 - 0s - loss: 70.9057 - val_loss: 52.7473
Epoch 218/1000
298/298 - 0s - loss: 63.8462 - val_loss: 53.7675
Epoch 219/1000
298/298 - 0s - loss: 65.2959 - val_loss: 56.9050
Epoch 220/1000
298/298 - 0s - loss: 63.8828 - val_loss: 52.9221
Epoch 221/1000
298/298 - 0s - loss: 66.2621 - val_loss: 61.4800
Epoch 222/1000
298/298 - 0s - loss: 66.0702 - val_loss: 51.9835
Epoch 223/1000
298/298 - 0s - loss: 62.1414 - val_loss: 50.4767
Epoch 224/1000
```

```
298/298 - 0s - loss: 60.9776 - val_loss: 51.0747
Epoch 225/1000
298/298 - 0s - loss: 61.1262 - val_loss: 49.3356
Epoch 226/1000
298/298 - 0s - loss: 59.9358 - val_loss: 56.2200
Epoch 227/1000
298/298 - 0s - loss: 61.8749 - val_loss: 48.5184
Epoch 228/1000
298/298 - 0s - loss: 59.3500 - val_loss: 49.2315
Epoch 229/1000
298/298 - 0s - loss: 58.7732 - val_loss: 49.6212
Epoch 230/1000
298/298 - 0s - loss: 59.0191 - val_loss: 47.4893
Epoch 231/1000
298/298 - 0s - loss: 58.5962 - val_loss: 52.0647
Epoch 232/1000
298/298 - 0s - loss: 57.5451 - val_loss: 47.0744
Epoch 233/1000
298/298 - 0s - loss: 57.4292 - val_loss: 48.5805
Epoch 234/1000
298/298 - 0s - loss: 57.2974 - val_loss: 46.4830
Epoch 235/1000
298/298 - 0s - loss: 59.5053 - val_loss: 48.0127
Epoch 236/1000
298/298 - 0s - loss: 57.6045 - val_loss: 48.8987
Epoch 237/1000
298/298 - 0s - loss: 55.6797 - val_loss: 45.2071
Epoch 238/1000
298/298 - 0s - loss: 54.9872 - val_loss: 46.9131
Epoch 239/1000
298/298 - 0s - loss: 55.2195 - val_loss: 44.9971
Epoch 240/1000
298/298 - 0s - loss: 54.4574 - val_loss: 47.3636
Epoch 241/1000
298/298 - 0s - loss: 55.7777 - val_loss: 43.9272
Epoch 242/1000
298/298 - 0s - loss: 56.6080 - val_loss: 43.6550
Epoch 243/1000
298/298 - 0s - loss: 53.3914 - val_loss: 44.0960
Epoch 244/1000
298/298 - 0s - loss: 53.3937 - val_loss: 46.4250
Epoch 245/1000
298/298 - 0s - loss: 52.5582 - val_loss: 43.4441
Epoch 246/1000
298/298 - 0s - loss: 52.2242 - val_loss: 42.5886
Epoch 247/1000
298/298 - 0s - loss: 53.1087 - val_loss: 45.4969
Epoch 248/1000
298/298 - 0s - loss: 51.2835 - val_loss: 42.2982
Epoch 249/1000
298/298 - 0s - loss: 51.9679 - val_loss: 42.0797
Epoch 250/1000
298/298 - 0s - loss: 50.6096 - val_loss: 41.9481
Epoch 251/1000
298/298 - 0s - loss: 52.3675 - val_loss: 42.1443
Epoch 252/1000
```

```
298/298 - 0s - loss: 52.0081 - val_loss: 41.5254
Epoch 253/1000
298/298 - 0s - loss: 52.4647 - val_loss: 46.1836
Epoch 254/1000
298/298 - 0s - loss: 49.0224 - val_loss: 40.2575
Epoch 255/1000
298/298 - 0s - loss: 50.8724 - val_loss: 40.5554
Epoch 256/1000
298/298 - 0s - loss: 48.6178 - val_loss: 40.2881
Epoch 257/1000
298/298 - 0s - loss: 48.1621 - val_loss: 40.1415
Epoch 258/1000
298/298 - 0s - loss: 47.9184 - val_loss: 39.6353
Epoch 259/1000
298/298 - 0s - loss: 47.7817 - val_loss: 44.1131
Epoch 260/1000
298/298 - 0s - loss: 48.0547 - val_loss: 38.6934
Epoch 261/1000
298/298 - 0s - loss: 49.1476 - val_loss: 38.5595
Epoch 262/1000
298/298 - 0s - loss: 48.3410 - val_loss: 38.4703
Epoch 263/1000
298/298 - 0s - loss: 47.1575 - val_loss: 43.8495
Epoch 264/1000
298/298 - 0s - loss: 47.5766 - val_loss: 37.7489
Epoch 265/1000
298/298 - 0s - loss: 45.9611 - val_loss: 37.8400
Epoch 266/1000
298/298 - 0s - loss: 45.3411 - val_loss: 37.4187
Epoch 267/1000
298/298 - 0s - loss: 44.8844 - val_loss: 40.0926
Epoch 268/1000
298/298 - 0s - loss: 45.0760 - val_loss: 36.9468
Epoch 269/1000
298/298 - 0s - loss: 45.1810 - val_loss: 40.3046
Epoch 270/1000
298/298 - 0s - loss: 44.8097 - val_loss: 37.5340
Epoch 271/1000
298/298 - 0s - loss: 44.2911 - val_loss: 38.6985
Epoch 272/1000
298/298 - 0s - loss: 43.8413 - val_loss: 37.3905
Epoch 273/1000
298/298 - 0s - loss: 43.3722 - val_loss: 36.7338
Epoch 274/1000
298/298 - 0s - loss: 43.0023 - val_loss: 35.9522
Epoch 275/1000
298/298 - 0s - loss: 43.3070 - val_loss: 42.2387
Epoch 276/1000
298/298 - 0s - loss: 43.3620 - val_loss: 35.6415
Epoch 277/1000
298/298 - 0s - loss: 44.2254 - val_loss: 35.0081
Epoch 278/1000
298/298 - 0s - loss: 43.6141 - val_loss: 35.4647
Epoch 279/1000
298/298 - 0s - loss: 42.5499 - val_loss: 37.3217
Epoch 280/1000
```

```
298/298 - 0s - loss: 42.4206 - val_loss: 36.6365
Epoch 281/1000
298/298 - 0s - loss: 41.8326 - val_loss: 33.9366
Epoch 282/1000
298/298 - 0s - loss: 40.7090 - val_loss: 35.2874
Epoch 283/1000
298/298 - 0s - loss: 41.1847 - val_loss: 35.7322
Epoch 284/1000
298/298 - 0s - loss: 40.2632 - val_loss: 33.2830
Epoch 285/1000
298/298 - 0s - loss: 40.4647 - val_loss: 33.4544
Epoch 286/1000
298/298 - 0s - loss: 41.8345 - val_loss: 33.3342
Epoch 287/1000
298/298 - 0s - loss: 40.1833 - val_loss: 33.5219
Epoch 288/1000
298/298 - 0s - loss: 42.5633 - val_loss: 45.5246
Epoch 289/1000
298/298 - 0s - loss: 43.4740 - val_loss: 32.2915
Epoch 290/1000
298/298 - 0s - loss: 40.7724 - val_loss: 35.0065
Epoch 291/1000
298/298 - 0s - loss: 40.1270 - val_loss: 41.1526
Epoch 292/1000
298/298 - 0s - loss: 41.5003 - val_loss: 35.0315
Epoch 293/1000
298/298 - 0s - loss: 39.4004 - val_loss: 33.8747
Epoch 294/1000
298/298 - 0s - loss: 41.5784 - val_loss: 31.3118
Epoch 295/1000
298/298 - 0s - loss: 38.1686 - val_loss: 31.1514
Epoch 296/1000
298/298 - 0s - loss: 38.6330 - val_loss: 37.5739
Epoch 297/1000
298/298 - 0s - loss: 38.8436 - val_loss: 30.6906
Epoch 298/1000
298/298 - 0s - loss: 37.6227 - val_loss: 32.6170
Epoch 299/1000
298/298 - 0s - loss: 36.6737 - val_loss: 30.3784
Epoch 300/1000
298/298 - 0s - loss: 36.7113 - val_loss: 30.9689
Epoch 301/1000
298/298 - 0s - loss: 36.3901 - val_loss: 31.8580
Epoch 302/1000
298/298 - 0s - loss: 36.4300 - val_loss: 29.8985
Epoch 303/1000
298/298 - 0s - loss: 36.6609 - val_loss: 31.8773
Epoch 304/1000
298/298 - 0s - loss: 39.6073 - val_loss: 29.4928
Epoch 305/1000
298/298 - 0s - loss: 37.2211 - val_loss: 29.9193
Epoch 306/1000
298/298 - 0s - loss: 38.2181 - val_loss: 42.4494
Epoch 307/1000
298/298 - 0s - loss: 41.9627 - val_loss: 36.3420
Epoch 308/1000
```

```
298/298 - 0s - loss: 35.2754 - val_loss: 29.0452
Epoch 309/1000
298/298 - 0s - loss: 34.5570 - val_loss: 28.5060
Epoch 310/1000
298/298 - 0s - loss: 35.0860 - val_loss: 28.4189
Epoch 311/1000
298/298 - 0s - loss: 34.4839 - val_loss: 29.8177
Epoch 312/1000
298/298 - 0s - loss: 37.1565 - val_loss: 27.9970
Epoch 313/1000
298/298 - 0s - loss: 38.1949 - val_loss: 34.7456
Epoch 314/1000
298/298 - 0s - loss: 35.7598 - val_loss: 29.5360
Epoch 315/1000
298/298 - 0s - loss: 34.7382 - val_loss: 30.6052
Epoch 316/1000
298/298 - 0s - loss: 34.0591 - val_loss: 29.3044
Epoch 317/1000
Restoring model weights from the end of the best epoch.
298/298 - 0s - loss: 32.9764 - val_loss: 29.1071
Epoch 00317: early stopping
```

Out[4]:
```
<tensorflow.python.keras.callbacks.History at 0x22a9acc8608>
```

Finally, we evaluate the error.

In [5]:
```
# Measure RMSE error.  RMSE is common for regression.
pred = model.predict(x_test)
score = np.sqrt(metrics.mean_squared_error(pred,y_test))
print(f"Final score (RMSE): {score}")
```

```
Final score (RMSE): 5.291219300799398
```

In [ ]:
```

```