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T81-558: Applications of Deep Neural Networks

Module 5: Regularization and Dropout

- Instructor: Jeff Heaton, McKelvey School of Engineering, Washington University in St. Louis
- For more information visit the class website.

Module 5 Material

- **Part 5.1: Part 5.1: Introduction to Regularization: Ridge and Lasso** [Video] [Notebook]
- Part 5.2: Using K-Fold Cross Validation with Keras [Video] [Notebook]
- Part 5.3: Using L1 and L2 Regularization with Keras to Decrease Overfitting [Video] [Notebook]
- Part 5.4: Drop Out for Keras to Decrease Overfitting [Video] [Notebook]
- Part 5.5: Benchmarking Keras Deep Learning Regularization Techniques [Video] [Notebook]

Google CoLab Instructions

The following code ensures that Google CoLab is running the correct version of TensorFlow.

In [1]:
```
try:
    %tensorflow_version 2.x
    COLAB = True
    print("Note: using Google CoLab")
except:
    print("Note: not using Google CoLab")
    COLAB = False
```

```
Note: not using Google CoLab
```

Part 5.1: Introduction to Regularization: Ridge and Lasso

Regularization is a technique that reduces overfitting, which occurs when neural networks attempt to memorize training data rather than learn from it. Humans are capable of overfitting as well. Before examining how a machine accidentally overfits, we will first explore how humans can suffer from it.

Human programmers often take certification exams to show their competence in a given programming language. To help prepare for these exams, the test makers often make practice exams available. Consider a programmer who enters a loop of taking the practice exam, studying more, and then retaking the practice exam. The programmer has memorized much of the practice exam at some point rather than learning the techniques necessary to figure out the individual questions. The programmer has now overfitted for the practice exam. When this programmer takes the real exam, his actual score will likely be lower than what he earned on the practice exam.

Although a neural network received a high score on its training data, this result does not mean that the same neural network will score high on data that was not inside the training set. A computer can overfit as well. Regularization is one of the techniques that can prevent overfitting. Several different regularization techniques exist. Most work by analyzing and potentially modifying the weights of a neural network as it trains.

L1 and L2 Regularization

L1 and L2 regularization are two standard regularization techniques that can reduce the effects of overfitting. These algorithms can either work with an objective function or as part of the backpropagation algorithm. The regularization algorithm is attached to the training algorithm by adding an objective in both cases.

These algorithms work by adding a weight penalty to the neural network training. This penalty encourages the neural network to keep the weights to small values. Both L1 and L2 calculate this penalty differently. You can add this penalty calculation to the calculated gradients for gradient-descent-based algorithms, such as backpropagation. The penalty is negatively combined with the objective score for objective-function-based training, such as simulated annealing.

We will look at linear regression to see how L1 and L2 regularization work. The following code sets up the auto-mpg data for this purpose.

In [2]:
```
from sklearn.linear_model import LassoCV
import pandas as pd
import os
import numpy as np
```

```
from sklearn import metrics
from scipy.stats import zscore
from sklearn.model_selection import train_test_split

df = pd.read_csv(
    "https://data.heatonresearch.com/data/t81-558/auto-mpg.csv",
    na_values=['NA', '?'])

# Handle missing value
df['horsepower'] = df['horsepower'].fillna(df['horsepower'].median())

# Pandas to Numpy
names = ['cylinders', 'displacement', 'horsepower', 'weight',
       'acceleration', 'year', 'origin']
x = df[names].values
y = df['mpg'].values # regression

# Split into train/test
x_train, x_test, y_train, y_test = train_test_split(
    x, y, test_size=0.25, random_state=45)
```

We will use the data just loaded for several examples. The first examples in this part use several forms of linear regression. For linear regression, it is helpful to examine the model's coefficients. The following function is utilized to display these coefficients.

In [3]:
```
# Simple function to evaluate the coefficients of a regression
%matplotlib inline
from IPython.display import display, HTML

def report_coef(names,coef,intercept):
    r = pd.DataFrame( { 'coef': coef, 'positive': coef>=0  }, index = names
    r = r.sort_values(by=['coef'])
    display(r)
    print(f"Intercept: {intercept}")
    r['coef'].plot(kind='barh', color=r['positive'].map(
        {True: 'b', False: 'r'}))
```

Linear Regression

Before jumping into L1/L2 regularization, we begin with linear regression. Researchers first introduced the L1/L2 form of regularization for linear regression. We can also make use of L1/L2 for neural networks. To fully understand L1/L2 we will begin with how we can use them with linear regression.

The following code uses linear regression to fit the auto-mpg data set. The RMSE reported will not be as good as a neural network.

In [4]:
```
import sklearn

# Create linear regression
```

```
regressor = sklearn.linear_model.LinearRegression()

# Fit/train linear regression
regressor.fit(x_train,y_train)
# Predict
pred = regressor.predict(x_test)

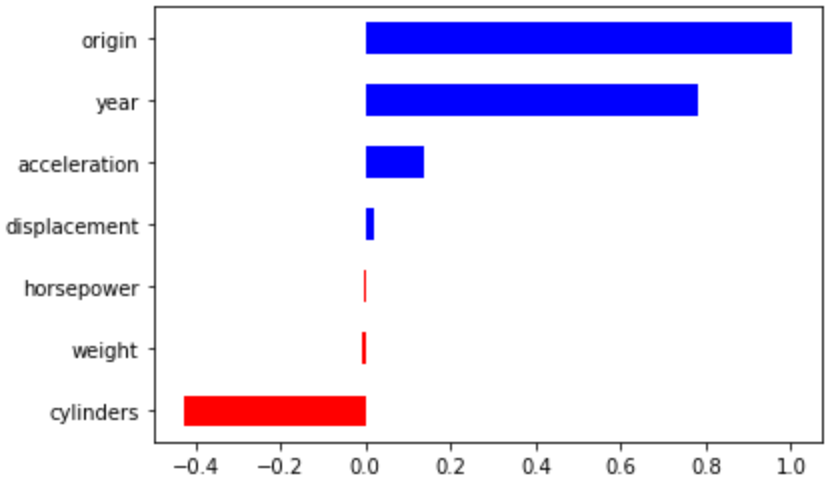
# Measure RMSE error.  RMSE is common for regression.
score = np.sqrt(metrics.mean_squared_error(pred,y_test))
print(f"Final score (RMSE): {score}")

report_coef(
  names,
  regressor.coef_,
  regressor.intercept_)
```

```
Final score (RMSE): 3.0019345985860784
```

	coef	positive
cylinders	-0.427721	False
weight	-0.007255	False
horsepower	-0.005491	False
displacement	0.020166	True
acceleration	0.138575	True
year	0.783047	True
origin	1.003762	True

```
Intercept: -19.101231042200112
```



L1 (Lasso) Regularization

L1 regularization, also called LASSO (Least Absolute Shrinkage and Selection Operator) should be used to create sparsity in the neural network. In other words, the L1 algorithm will push many weight connections to near 0. When the

weight is near 0, the program drops it from the network. Dropping weighted connections will create a sparse neural network.

Feature selection is a useful byproduct of sparse neural networks. Features are the values that the training set provides to the input neurons. Once all the weights of an input neuron reach 0, the neural network training determines that the feature is unnecessary. If your data set has many unnecessary input features, L1 regularization can help the neural network detect and ignore unnecessary features.

L1 is implemented by adding the following error to the objective to minimize:

$$E_1 = \alpha \sum_w |w|$$

You should use L1 regularization to create sparsity in the neural network. In other words, the L1 algorithm will push many weight connections to near 0. When the weight is near 0, the program drops it from the network. Dropping weighted connections will create a sparse neural network.

The following code demonstrates lasso regression. Notice the effect of the coefficients compared to the previous section that used linear regression.

In [5]:
```
import sklearn
from sklearn.linear_model import Lasso

# Create linear regression
regressor = Lasso(random_state=0,alpha=0.1)

# Fit/train LASSO
regressor.fit(x_train,y_train)
# Predict
pred = regressor.predict(x_test)

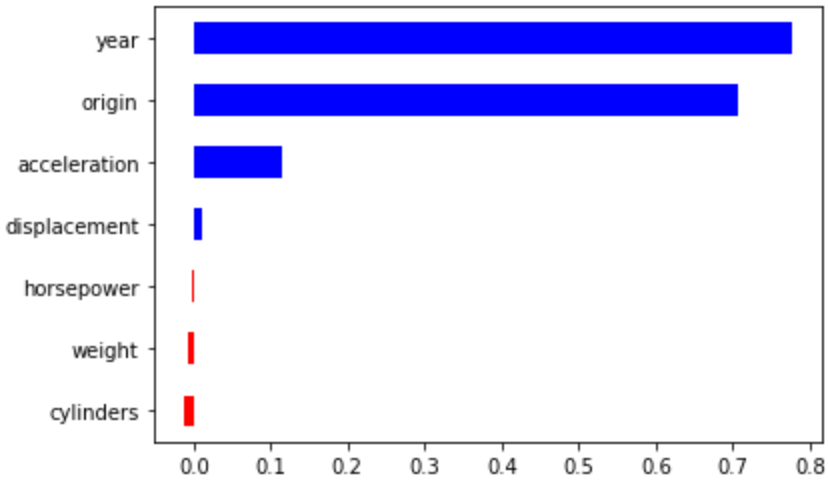
# Measure RMSE error.  RMSE is common for regression.
score = np.sqrt(metrics.mean_squared_error(pred,y_test))
print(f"Final score (RMSE): {score}")

report_coef(
  names,
  regressor.coef_,
  regressor.intercept_)
```

```
Final score (RMSE): 3.0604021904033303
```

	coef	positive
cylinders	-0.012995	False
weight	-0.007328	False
horsepower	-0.002715	False
displacement	0.011601	True
acceleration	0.114391	True
origin	0.708222	True
year	0.777480	True

```
Intercept: -18.506677982383252
```



L2 (Ridge) Regularization

You should use Tikhonov/Ridge/L2 regularization when you are less concerned about creating a space network and are more concerned about low weight values. The lower weight values will typically lead to less overfitting.

$$E_2 = \alpha \sum_w w^2$$

Like the L1 algorithm, the α value determines how important the L2 objective is compared to the neural network's error. Typical L2 values are below 0.1 (10%). The main calculation performed by L2 is the summing of the squares of all of the weights. The algorithm will not sum bias values.

You should use L2 regularization when you are less concerned about creating a space network and are more concerned about low weight values. The lower weight values will typically lead to less overfitting. Generally, L2 regularization will produce better overall performance than L1. However, L1 might be useful in situations with many inputs, and you can prune some of the weaker inputs.

The following code uses L2 with linear regression (Ridge regression):

In [7]:
```
import sklearn
from sklearn.linear_model import Ridge

# Create linear regression
regressor = Ridge(alpha=1)

# Fit/train Ridge
regressor.fit(x_train,y_train)
# Predict
pred = regressor.predict(x_test)

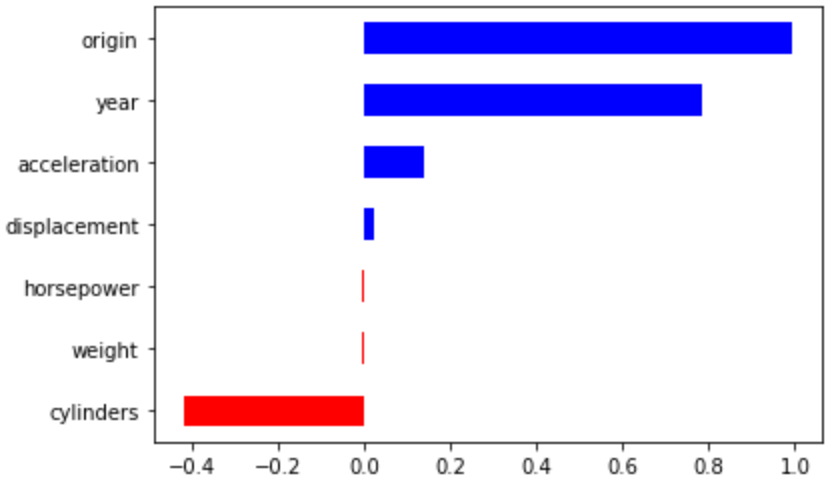
# Measure RMSE error.  RMSE is common for regression.
score = np.sqrt(metrics.mean_squared_error(pred,y_test))
print("Final score (RMSE): {score}")

report_coef(
  names,
  regressor.coef_,
  regressor.intercept_)
```

```
Final score (RMSE): {score}
```

	coef	positive
cylinders	-0.421393	False
weight	-0.007257	False
horsepower	-0.005385	False
displacement	0.020006	True
acceleration	0.138470	True
year	0.782889	True
origin	0.994621	True

```
Intercept: -19.07980074425469
```



ElasticNet Regularization

The ElasticNet regression combines both L1 and L2. Both penalties are applied. The amount of L1 and L2 are governed by the parameters alpha and beta.

$$a * \mathrm{L1} + b * \mathrm{L2}$$

In [8]:
```
import sklearn
from sklearn.linear_model import ElasticNet

# Create linear regression
regressor = ElasticNet(alpha=0.1, l1_ratio=0.1)

# Fit/train LASSO
regressor.fit(x_train,y_train)
# Predict
pred = regressor.predict(x_test)

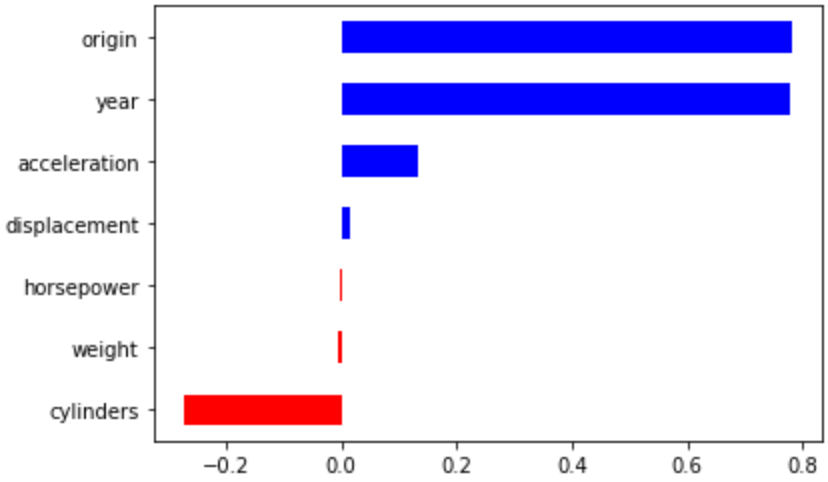
# Measure RMSE error.  RMSE is common for regression.
score = np.sqrt(metrics.mean_squared_error(pred,y_test))
print(f"Final score (RMSE): {score}")

report_coef(
  names,
  regressor.coef_,
  regressor.intercept_)
```

```
Final score (RMSE): 3.0450899960775013
```

	coef	positive
cylinders	-0.274010	False
weight	-0.007303	False
horsepower	-0.003231	False
displacement	0.016194	True
acceleration	0.132348	True
year	0.777482	True
origin	0.782781	True

```
Intercept: -18.389355690429767
```


origin
year
acceleration
displacement
horsepower
weight
cylinders
−0.2
0.0
0.2
0.4
0.6
0.8

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- Part 5.5: Benchmarking Keras Deep Learning Regularization Techniques [Video] [Notebook]

Google CoLab Instructions

The following code ensures that Google CoLab is running the correct version of TensorFlow.

In [1]:
```
import os
os.environ['TF_CPP_MIN_LOG_LEVEL'] = '3'

try:
    %tensorflow_version 2.x
    COLAB = True
    print("Note: using Google CoLab")
except:
    print("Note: not using Google CoLab")
    COLAB = False
```

```
Note: not using Google CoLab
```

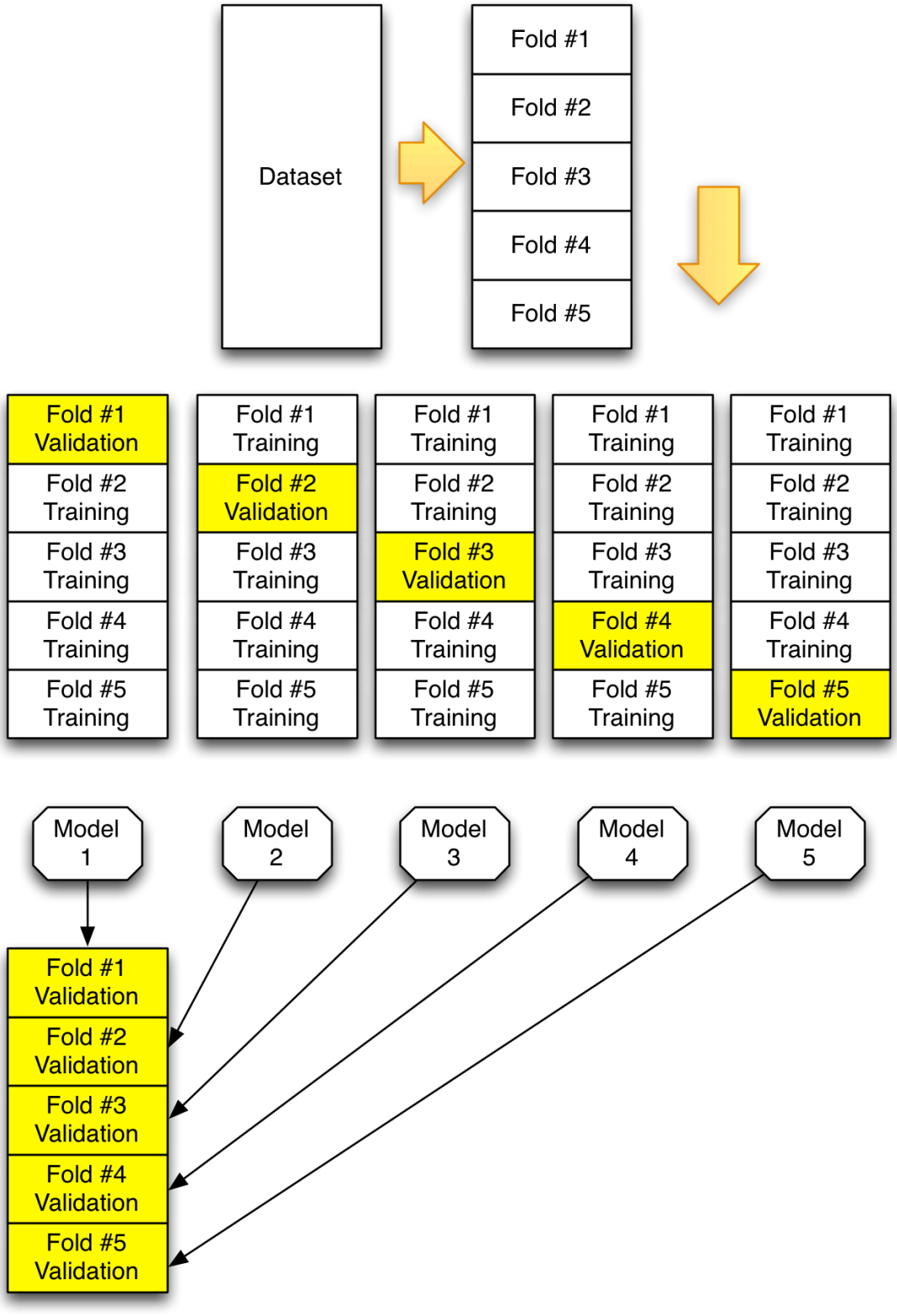
Part 5.2: Using K-Fold Cross-validation with Keras

You can use cross-validation for a variety of purposes in predictive modeling:

- Generating out-of-sample predictions from a neural network
- Estimate a good number of epochs to train a neural network for (early stopping)
- Evaluate the effectiveness of certain hyperparameters, such as activation functions, neuron counts, and layer counts

Cross-validation uses several folds and multiple models to provide each data segment a chance to serve as both the validation and training set. Figure 5.CROSS shows cross-validation.

Figure 5.CROSS: K-Fold Crossvalidation



It is important to note that each fold will have one model (neural network). To generate predictions for new data (not present in the training set), predictions from the fold models can be handled in several ways:

- Choose the model with the highest validation score as the final model.

- Preset new data to the five models (one for each fold) and average the result (this is an ensemble).
- Retrain a new model (using the same settings as the cross-validation) on the entire dataset. Train for as many epochs and with the same hidden layer structure.

Generally, I prefer the last approach and will retrain a model on the entire data set once I have selected hyper-parameters. Of course, I will always set aside a final holdout set for model validation that I do not use in any aspect of the training process.

Regression vs Classification K-Fold Cross-Validation

Regression and classification are handled somewhat differently concerning cross-validation. Regression is the simpler case where you can break up the data set into K folds with little regard for where each item lands. For regression, the data items should fall into the folds as randomly as possible. It is also important to remember that not every fold will necessarily have the same number of data items. It is not always possible for the data set to be evenly divided into K folds. For regression cross-validation, we will use the Scikit-Learn class **KFold**.

Cross-validation for classification could also use the **KFold** object; however, this technique would not ensure that the class balance remains the same in each fold as in the original. The balance of classes that a model was trained on must remain the same (or similar) to the training set. Drift in this distribution is one of the most important things to monitor after a trained model has been placed into actual use. Because of this, we want to make sure that the cross-validation itself does not introduce an unintended shift. This technique is called stratified sampling and is accomplished by using the Scikit-Learn object **StratifiedKFold** in place of **KFold** whenever you use classification. In summary, you should use the following two objects in Scikit-Learn:

- **KFold** When dealing with a regression problem.
- **StratifiedKFold** When dealing with a classification problem.

The following two sections demonstrate cross-validation with classification and regression.

Out-of-Sample Regression Predictions with K-Fold Cross-Validation

The following code trains the simple dataset using a 5-fold cross-validation. The expected performance of a neural network of the type trained here would be the score for the generated out-of-sample predictions. We begin by preparing a feature vector using the **jh-simple-dataset** to predict age. This model is set up as a regression problem.

In [2]:
```
import pandas as pd
from scipy.stats import zscore
from sklearn.model_selection import train_test_split

# Read the data set
df = pd.read_csv(
    "https://data.heatonresearch.com/data/t81-558/jh-simple-dataset.csv",
    na_values=['NA','?'])

# Generate dummies for job
df = pd.concat([df,pd.get_dummies(df['job'],prefix="job")],axis=1)
df.drop('job', axis=1, inplace=True)

# Generate dummies for area
df = pd.concat([df,pd.get_dummies(df['area'],prefix="area")],axis=1)
df.drop('area', axis=1, inplace=True)

# Generate dummies for product
df = pd.concat([df,pd.get_dummies(df['product'],prefix="product")],axis=1)
df.drop('product', axis=1, inplace=True)

# Missing values for income
med = df['income'].median()
df['income'] = df['income'].fillna(med)

# Standardize ranges
df['income'] = zscore(df['income'])
df['aspect'] = zscore(df['aspect'])
df['save_rate'] = zscore(df['save_rate'])
df['subscriptions'] = zscore(df['subscriptions'])

# Convert to numpy - Classification
x_columns = df.columns.drop('age').drop('id')
x = df[x_columns].values
y = df['age'].values
```

Now that the feature vector is created a 5-fold cross-validation can be performed to generate out-of-sample predictions. We will assume 500 epochs and not use early stopping. Later we will see how we can estimate a more optimal epoch count.

In [4]:
```
EPOCHS=500

import pandas as pd
import os
import numpy as np
```

```
from sklearn import metrics
from scipy.stats import zscore
from sklearn.model_selection import KFold
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense, Activation

# Cross-Validate
kf = KFold(5, shuffle=True, random_state=42) # Use for KFold classification
oos_y = []
oos_pred = []

fold = 0
for train, test in kf.split(x):
    fold+=1
    print(f"Fold #{fold}")

    x_train = x[train]
    y_train = y[train]
    x_test = x[test]
    y_test = y[test]

    model = Sequential()
    model.add(Dense(20, input_dim=x.shape[1], activation='relu'))
    model.add(Dense(10, activation='relu'))
    model.add(Dense(1))
    model.compile(loss='mean_squared_error', optimizer='adam')

    model.fit(x_train,y_train,validation_data=(x_test,y_test),verbose=0,
              epochs=EPOCHS)

    pred = model.predict(x_test)

    oos_y.append(y_test)
    oos_pred.append(pred)

    # Measure this fold's RMSE
    score = np.sqrt(metrics.mean_squared_error(pred,y_test))
    print(f"Fold score (RMSE): {score}")

# Build the oos prediction list and calculate the error.
oos_y = np.concatenate(oos_y)
oos_pred = np.concatenate(oos_pred)
score = np.sqrt(metrics.mean_squared_error(oos_pred,oos_y))
print(f"Final, out of sample score (RMSE): {score}")

# Write the cross-validated prediction
oos_y = pd.DataFrame(oos_y)
oos_pred = pd.DataFrame(oos_pred)
oosDF = pd.concat( [df, oos_y, oos_pred],axis=1 )
#oosDF.to_csv(filename_write,index=False)
```

```
Fold #1
Fold score (RMSE): 0.6814299426511208
Fold #2
Fold score (RMSE): 0.45486513719487165
Fold #3
Fold score (RMSE): 0.571615041876392
Fold #4
Fold score (RMSE): 0.46416356081116916
Fold #5
Fold score (RMSE): 1.0426518491685475
Final, out of sample score (RMSE): 0.678316077597408
```

As you can see, the above code also reports the average number of epochs needed. A common technique is to then train on the entire dataset for the average number of epochs required.

Classification with Stratified K-Fold Cross-Validation

The following code trains and fits the **jh**-simple-dataset dataset with cross-validation to generate out-of-sample. It also writes the out-of-sample (predictions on the test set) results.

It is good to perform stratified k-fold cross-validation with classification data. This technique ensures that the percentages of each class remain the same across all folds. Use the **StratifiedKFold** object instead of the **KFold** object used in the regression.

In [5]:
```
import pandas as pd
from scipy.stats import zscore

# Read the data set
df = pd.read_csv(
    "https://data.heatonresearch.com/data/t81-558/jh-simple-dataset.csv",
    na_values=['NA','?'])

# Generate dummies for job
df = pd.concat([df,pd.get_dummies(df['job'],prefix="job")],axis=1)
df.drop('job', axis=1, inplace=True)

# Generate dummies for area
df = pd.concat([df,pd.get_dummies(df['area'],prefix="area")],axis=1)
df.drop('area', axis=1, inplace=True)

# Missing values for income
med = df['income'].median()
df['income'] = df['income'].fillna(med)

# Standardize ranges
df['income'] = zscore(df['income'])
df['aspect'] = zscore(df['aspect'])
```

```
df['save_rate'] = zscore(df['save_rate'])
df['age'] = zscore(df['age'])
df['subscriptions'] = zscore(df['subscriptions'])

# Convert to numpy - Classification
x_columns = df.columns.drop('product').drop('id')
x = df[x_columns].values
dummies = pd.get_dummies(df['product']) # Classification
products = dummies.columns
y = dummies.values
```

We will assume 500 epochs and not use early stopping. Later we will see how we can estimate a more optimal epoch count.

In [6]:
```
import pandas as pd
import os
import numpy as np
from sklearn import metrics
from sklearn.model_selection import StratifiedKFold
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense, Activation

# np.argmax(pred,axis=1)
# Cross-validate
# Use for StratifiedKFold classification
kf = StratifiedKFold(5, shuffle=True, random_state=42)

oos_y = []
oos_pred = []
fold = 0

# Must specify y StratifiedKFold for
for train, test in kf.split(x,df['product']):
    fold+=1
    print(f"Fold #{fold}")

    x_train = x[train]
    y_train = y[train]
    x_test = x[test]
    y_test = y[test]

    model = Sequential()
    # Hidden 1
    model.add(Dense(50, input_dim=x.shape[1], activation='relu'))
    model.add(Dense(25, activation='relu')) # Hidden 2
    model.add(Dense(y.shape[1],activation='softmax')) # Output
    model.compile(loss='categorical_crossentropy', optimizer='adam')

    model.fit(x_train,y_train,validation_data=(x_test,y_test),
              verbose=0, epochs=EPOCHS)

    pred = model.predict(x_test)

    oos_y.append(y_test)
    # raw probabilities to chosen class (highest probability)
```

```
    pred = np.argmax(pred,axis=1)
    oos_pred.append(pred)

    # Measure this fold's accuracy
    y_compare = np.argmax(y_test,axis=1) # For accuracy calculation
    score = metrics.accuracy_score(y_compare, pred)
    print(f"Fold score (accuracy): {score}")

# Build the oos prediction list and calculate the error.
oos_y = np.concatenate(oos_y)
oos_pred = np.concatenate(oos_pred)
oos_y_compare = np.argmax(oos_y,axis=1) # For accuracy calculation

score = metrics.accuracy_score(oos_y_compare, oos_pred)
print(f"Final score (accuracy): {score}")

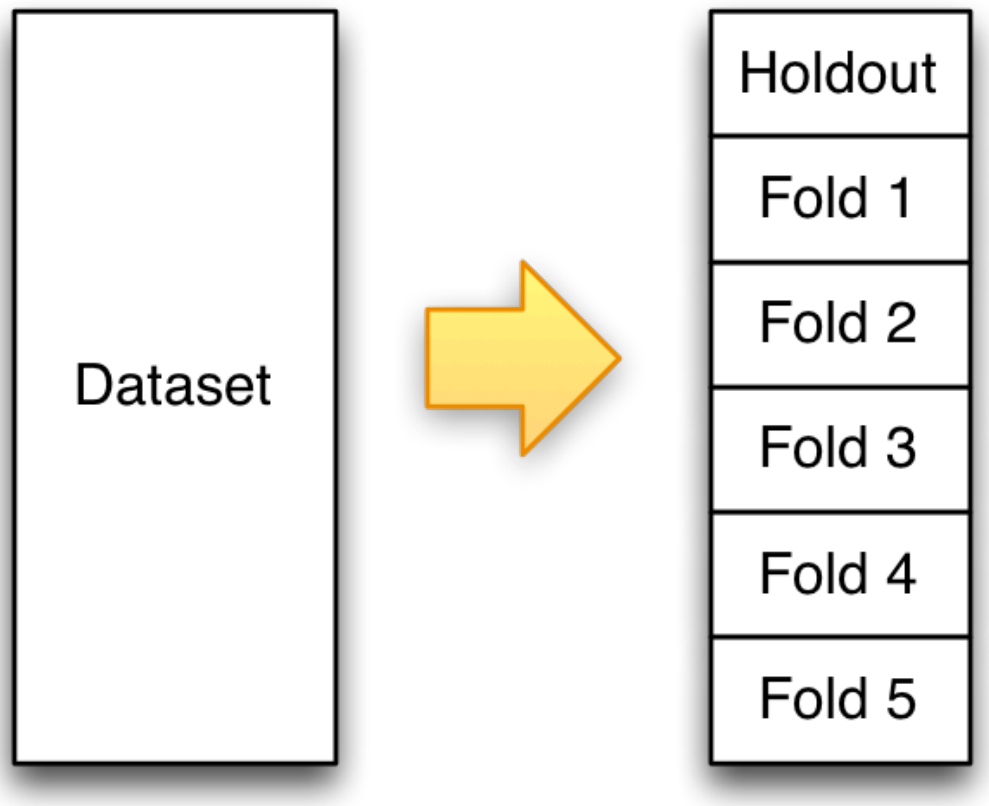
# Write the cross-validated prediction
oos_y = pd.DataFrame(oos_y)
oos_pred = pd.DataFrame(oos_pred)
oosDF = pd.concat( [df, oos_y, oos_pred],axis=1 )
#oosDF.to_csv(filename_write,index=False)
```

```
Fold #1
Fold score (accuracy): 0.6325
Fold #2
Fold score (accuracy): 0.6725
Fold #3
Fold score (accuracy): 0.6975
Fold #4
Fold score (accuracy): 0.6575
Fold #5
Fold score (accuracy): 0.675
Final score (accuracy): 0.667
```

Training with both a Cross-Validation and a Holdout Set

If you have a considerable amount of data, it is always valuable to set aside a holdout set before you cross-validate. This holdout set will be the final evaluation before using your model for its real-world use. Figure 5. HOLDOUT shows this division.

Figure 5. HOLDOUT: Cross-Validation and a Holdout Set



The following program uses a holdout set and then still cross-validates.

In [7]:
```
import pandas as pd
from scipy.stats import zscore
from sklearn.model_selection import train_test_split

# Read the data set
df = pd.read_csv(
    "https://data.heatonresearch.com/data/t81-558/jh-simple-dataset.csv",
    na_values=['NA','?'])

# Generate dummies for job
df = pd.concat([df,pd.get_dummies(df['job'],prefix="job")],axis=1)
df.drop('job', axis=1, inplace=True)

# Generate dummies for area
df = pd.concat([df,pd.get_dummies(df['area'],prefix="area")],axis=1)
df.drop('area', axis=1, inplace=True)

# Generate dummies for product
df = pd.concat([df,pd.get_dummies(df['product'],prefix="product")],axis=1)
df.drop('product', axis=1, inplace=True)

# Missing values for income
med = df['income'].median()
```

```
df['income'] = df['income'].fillna(med)

# Standardize ranges
df['income'] = zscore(df['income'])
df['aspect'] = zscore(df['aspect'])
df['save_rate'] = zscore(df['save_rate'])
df['subscriptions'] = zscore(df['subscriptions'])

# Convert to numpy - Classification
x_columns = df.columns.drop('age').drop('id')
x = df[x_columns].values
y = df['age'].values
```

Now that the data has been preprocessed, we are ready to build the neural network.

In [8]:
```
from sklearn.model_selection import train_test_split
import pandas as pd
import os
import numpy as np
from sklearn import metrics
from scipy.stats import zscore
from sklearn.model_selection import KFold

# Keep a 10% holdout
x_main, x_holdout, y_main, y_holdout = train_test_split(
    x, y, test_size=0.10)


# Cross-validate
kf = KFold(5)

oos_y = []
oos_pred = []
fold = 0
for train, test in kf.split(x_main):
    fold+=1
    print(f"Fold #{fold}")

    x_train = x_main[train]
    y_train = y_main[train]
    x_test = x_main[test]
    y_test = y_main[test]

    model = Sequential()
    model.add(Dense(20, input_dim=x.shape[1], activation='relu'))
    model.add(Dense(5, activation='relu'))
    model.add(Dense(1))
    model.compile(loss='mean_squared_error', optimizer='adam')

    model.fit(x_train,y_train,validation_data=(x_test,y_test),
              verbose=0,epochs=EPOCHS)

    pred = model.predict(x_test)
```

```
    oos_y.append(y_test)
    oos_pred.append(pred)

    # Measure accuracy
    score = np.sqrt(metrics.mean_squared_error(pred,y_test))
    print(f"Fold score (RMSE): {score}")


# Build the oos prediction list and calculate the error.
oos_y = np.concatenate(oos_y)
oos_pred = np.concatenate(oos_pred)
score = np.sqrt(metrics.mean_squared_error(oos_pred,oos_y))
print()
print(f"Cross-validated score (RMSE): {score}")

# Write the cross-validated prediction (from the last neural network)
holdout_pred = model.predict(x_holdout)

score = np.sqrt(metrics.mean_squared_error(holdout_pred,y_holdout))
print(f"Holdout score (RMSE): {score}")
```

```
Fold #1
Fold score (RMSE): 0.544195299216696
Fold #2
Fold score (RMSE): 0.48070599342910353
Fold #3
Fold score (RMSE): 0.7034584765928998
Fold #4
Fold score (RMSE): 0.5397141785190473
Fold #5
Fold score (RMSE): 24.126205213080077

Cross-validated score (RMSE): 10.801732731207947
Holdout score (RMSE): 24.097657947297677
```

In [ ]:

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T81-558: Applications of Deep Neural Networks

Module 5: Regularization and Dropout

- Instructor: Jeff Heaton, McKelvey School of Engineering, Washington University in St. Louis
- For more information visit the class website.

Module 5 Material

- Part 5.1: Part 5.1: Introduction to Regularization: Ridge and Lasso [Video] [Notebook]
- Part 5.2: Using K-Fold Cross Validation with Keras [Video] [Notebook]
- **Part 5.3: Using L1 and L2 Regularization with Keras to Decrease Overfitting** [Video] [Notebook]
- Part 5.4: Drop Out for Keras to Decrease Overfitting [Video] [Notebook]
- Part 5.5: Benchmarking Keras Deep Learning Regularization Techniques [Video] [Notebook]

Google CoLab Instructions

The following code ensures that Google CoLab is running the correct version of TensorFlow.

In [1]:
```
try:
    %tensorflow_version 2.x
    COLAB = True
    print("Note: using Google CoLab")
except:
    print("Note: not using Google CoLab")
    COLAB = False
```

```
Note: not using Google CoLab
```

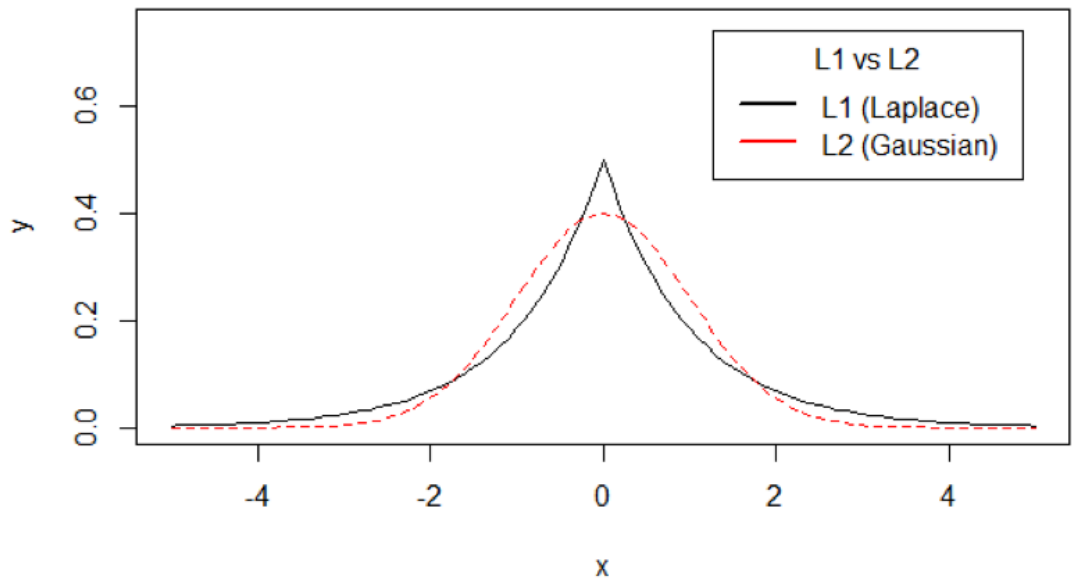
Part 5.3: L1 and L2 Regularization to Decrease Overfitting

L1 and L2 regularization are two common regularization techniques that can reduce the effects of overfitting [Cite:ng2004feature]. These algorithms can either work with an objective function or as a part of the backpropagation algorithm. In both cases, the regularization algorithm is attached to the training algorithm by adding an objective.

These algorithms work by adding a weight penalty to the neural network training. This penalty encourages the neural network to keep the weights to small values. Both L1 and L2 calculate this penalty differently. You can add this penalty calculation to the calculated gradients for gradient-descent-based algorithms, such as backpropagation. The penalty is negatively combined with the objective score for objective-function-based training, such as simulated annealing.

Both L1 and L2 work differently in that they penalize the size of the weight. L2 will force the weights into a pattern similar to a Gaussian distribution; the L1 will force the weights into a pattern similar to a Laplace distribution, as demonstrated in Figure 5.L1L2.

Figure 5.L1L2: L1 vs L2



As you can see, L1 algorithm is more tolerant of weights further from 0, whereas the L2 algorithm is less tolerant. We will highlight other important differences between L1 and L2 in the following sections. You also need to note that both L1 and L2 count their penalties based only on weights; they do not count penalties on bias values. Keras allows l1/l2 to be directly added to your network.

In [2]:
```
import pandas as pd
from scipy.stats import zscore
```

```
# Read the data set
df = pd.read_csv(
    "https://data.heatonresearch.com/data/t81-558/jh-simple-dataset.csv",
    na_values=['NA','?'])

# Generate dummies for job
df = pd.concat([df,pd.get_dummies(df['job'],prefix="job")],axis=1)
df.drop('job', axis=1, inplace=True)

# Generate dummies for area
df = pd.concat([df,pd.get_dummies(df['area'],prefix="area")],axis=1)
df.drop('area', axis=1, inplace=True)

# Missing values for income
med = df['income'].median()
df['income'] = df['income'].fillna(med)

# Standardize ranges
df['income'] = zscore(df['income'])
df['aspect'] = zscore(df['aspect'])
df['save_rate'] = zscore(df['save_rate'])
df['age'] = zscore(df['age'])
df['subscriptions'] = zscore(df['subscriptions'])

# Convert to numpy - Classification
x_columns = df.columns.drop('product').drop('id')
x = df[x_columns].values
dummies = pd.get_dummies(df['product']) # Classification
products = dummies.columns
y = dummies.values
```

We now create a Keras network with L1 regression.

In [3]:
```
import pandas as pd
import os
import numpy as np
from sklearn import metrics
from sklearn.model_selection import KFold
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense, Activation
from tensorflow.keras import regularizers

# Cross-validate
kf = KFold(5, shuffle=True, random_state=42)

oos_y = []
oos_pred = []
fold = 0

for train, test in kf.split(x):
    fold+=1
    print(f"Fold #{fold}")

    x_train = x[train]
```

```
    y_train = y[train]
    x_test = x[test]
    y_test = y[test]

    #kernel_regularizer=regularizers.l2(0.01),

    model = Sequential()
    # Hidden 1
    model.add(Dense(50, input_dim=x.shape[1],
            activation='relu',
             activity_regularizer=regularizers.l1(1e-4)))
    # Hidden 2
    model.add(Dense(25, activation='relu',
                    activity_regularizer=regularizers.l1(1e-4)))
     # Output
    model.add(Dense(y.shape[1],activation='softmax'))
    model.compile(loss='categorical_crossentropy', optimizer='adam')

    model.fit(x_train,y_train,validation_data=(x_test,y_test),
              verbose=0,epochs=500)

    pred = model.predict(x_test)

    oos_y.append(y_test)
    # raw probabilities to chosen class (highest probability)
    pred = np.argmax(pred,axis=1)
    oos_pred.append(pred)

    # Measure this fold's accuracy
    y_compare = np.argmax(y_test,axis=1) # For accuracy calculation
    score = metrics.accuracy_score(y_compare, pred)
    print(f"Fold score (accuracy): {score}")


# Build the oos prediction list and calculate the error.
oos_y = np.concatenate(oos_y)
oos_pred = np.concatenate(oos_pred)
oos_y_compare = np.argmax(oos_y,axis=1) # For accuracy calculation

score = metrics.accuracy_score(oos_y_compare, oos_pred)
print(f"Final score (accuracy): {score}")

# Write the cross-validated prediction
oos_y = pd.DataFrame(oos_y)
oos_pred = pd.DataFrame(oos_pred)
oosDF = pd.concat( [df, oos_y, oos_pred],axis=1 )
#oosDF.to_csv(filename_write,index=False)
```

```
Fold #1
Fold score (accuracy): 0.64
Fold #2
Fold score (accuracy): 0.6775
Fold #3
Fold score (accuracy): 0.6825
Fold #4
Fold score (accuracy): 0.6675
Fold #5
Fold score (accuracy): 0.645
Final score (accuracy): 0.6625
```

In [ ]:

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T81-558: Applications of Deep Neural Networks

Module 5: Regularization and Dropout

- Instructor: Jeff Heaton, McKelvey School of Engineering, Washington University in St. Louis
- For more information visit the class website.

Module 5 Material

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Google CoLab Instructions

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In [1]:
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try:
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    COLAB = True
    print("Note: using Google CoLab")
except:
    print("Note: not using Google CoLab")
    COLAB = False
```

```
Note: not using Google CoLab
```

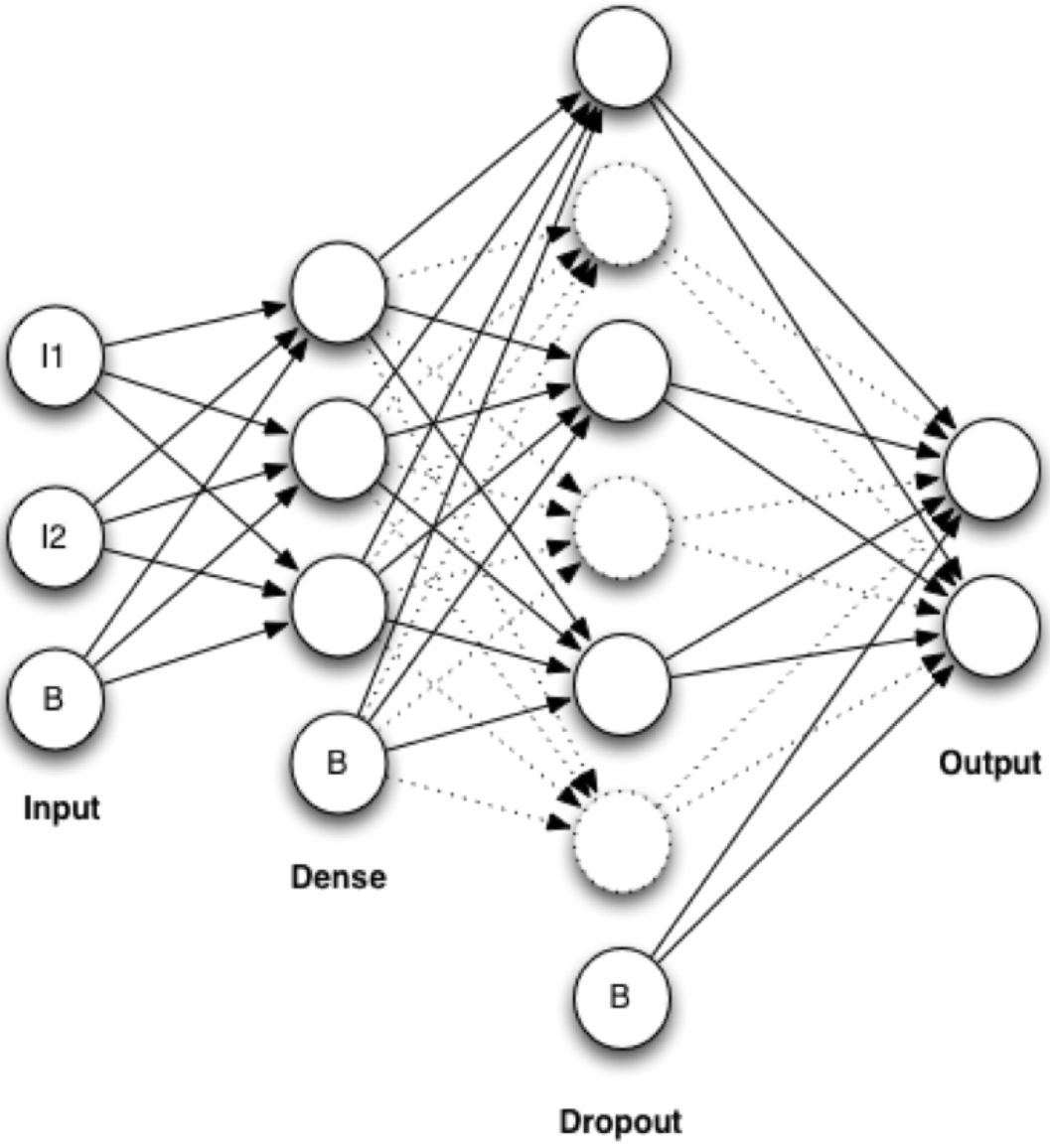
Part 5.4: Drop Out for Keras to Decrease Overfitting

Hinton, Srivastava, Krizhevsky, Sutskever, & Salakhutdinov (2012) introduced the dropout regularization algorithm. [Cite:srivastava2014dropout] Although dropout works differently than L1 and L2, it accomplishes the same goal—the prevention of overfitting. However, the algorithm does the task by actually removing neurons and connections—at least temporarily. Unlike L1 and L2, no weight penalty is added. Dropout does not directly seek to train small weights. Dropout works by causing hidden neurons of the neural network to be unavailable during part of the training. Dropping part of the neural network causes the remaining portion to be trained to still achieve a good score even without the dropped neurons. This technique decreases co-adaptation between neurons, which results in less overfitting.

Most neural network frameworks implement dropout as a separate layer. Dropout layers function like a regular, densely connected neural network layer. The only difference is that the dropout layers will periodically drop some of their neurons during training. You can use dropout layers on regular feedforward neural networks.

The program implements a dropout layer as a dense layer that can eliminate some of its neurons. Contrary to popular belief about the dropout layer, the program does not permanently remove these discarded neurons. A dropout layer does not lose any of its neurons during the training process, and it will still have the same number of neurons after training. In this way, the program only temporarily masks the neurons rather than dropping them. Figure 5.DROPOUT shows how a dropout layer might be situated with other layers.

Figure 5.DROPOUT: Dropout Regularization



The discarded neurons and their connections are shown as dashed lines. The input layer has two input neurons as well as a bias neuron. The second layer is a dense layer with three neurons and a bias neuron. The third layer is a dropout layer with six regular neurons even though the program has dropped 50% of them. While the program drops these neurons, it neither calculates nor trains them. However, the final neural network will use all of these neurons for the output. As previously mentioned, the program only temporarily discards the neurons.

The program chooses different sets of neurons from the dropout layer during subsequent training iterations. Although we chose a probability of 50% for dropout, the computer will not necessarily drop three neurons. It is as if we flipped a coin for each of the dropout candidate neurons to choose if that neuron was dropped out. You must know that the program should never drop the bias

neuron. Only the regular neurons on a dropout layer are candidates. The implementation of the training algorithm influences the process of discarding neurons. The dropout set frequently changes once per training iteration or batch. The program can also provide intervals where all neurons are present. Some neural network frameworks give additional hyper-parameters to allow you to specify exactly the rate of this interval.

Why dropout is capable of decreasing overfitting is a common question. The answer is that dropout can reduce the chance of codependency developing between two neurons. Two neurons that develop codependency will not be able to operate effectively when one is dropped out. As a result, the neural network can no longer rely on the presence of every neuron, and it trains accordingly. This characteristic decreases its ability to memorize the information presented, thereby forcing generalization.

Dropout also decreases overfitting by forcing a bootstrapping process upon the neural network. Bootstrapping is a prevalent ensemble technique. Ensembling is a technique of machine learning that combines multiple models to produce a better result than those achieved by individual models. The ensemble is a term that originates from the musical ensembles in which the final music product that the audience hears is the combination of many instruments.

Bootstrapping is one of the most simple ensemble techniques. The bootstrapping programmer simply trains several neural networks to perform precisely the same task. However, each neural network will perform differently because of some training techniques and the random numbers used in the neural network weight initialization. The difference in weights causes the performance variance. The output from this ensemble of neural networks becomes the average output of the members taken together. This process decreases overfitting through the consensus of differently trained neural networks.

Dropout works somewhat like bootstrapping. You might think of each neural network that results from a different set of neurons being dropped out as an individual member in an ensemble. As training progresses, the program creates more neural networks in this way. However, dropout does not require the same amount of processing as bootstrapping. The new neural networks created are temporary; they exist only for a training iteration. The final result is also a single neural network rather than an ensemble of neural networks to be averaged together.

The following animation shows how dropout works: animation link

In [2]:
```
import pandas as pd
from scipy.stats import zscore

```

```
# Read the data set
df = pd.read_csv(
    "https://data.heatonresearch.com/data/t81-558/jh-simple-dataset.csv",
    na_values=['NA','?'])

# Generate dummies for job
df = pd.concat([df,pd.get_dummies(df['job'],prefix="job")],axis=1)
df.drop('job', axis=1, inplace=True)

# Generate dummies for area
df = pd.concat([df,pd.get_dummies(df['area'],prefix="area")],axis=1)
df.drop('area', axis=1, inplace=True)

# Missing values for income
med = df['income'].median()
df['income'] = df['income'].fillna(med)

# Standardize ranges
df['income'] = zscore(df['income'])
df['aspect'] = zscore(df['aspect'])
df['save_rate'] = zscore(df['save_rate'])
df['age'] = zscore(df['age'])
df['subscriptions'] = zscore(df['subscriptions'])

# Convert to numpy - Classification
x_columns = df.columns.drop('product').drop('id')
x = df[x_columns].values
dummies = pd.get_dummies(df['product']) # Classification
products = dummies.columns
y = dummies.values
```

Now we will see how to apply dropout to classification.

In [3]:

```
########################################
# Keras with dropout for Classification
########################################

import pandas as pd
import os
import numpy as np
from sklearn import metrics
from sklearn.model_selection import KFold
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense, Activation, Dropout
from tensorflow.keras import regularizers

# Cross-validate
kf = KFold(5, shuffle=True, random_state=42)

oos_y = []
oos_pred = []
fold = 0

for train, test in kf.split(x):
    fold+=1
```

```
    print(f"Fold #{fold}")

    x_train = x[train]
    y_train = y[train]
    x_test = x[test]
    y_test = y[test]

    #kernel_regularizer=regularizers.l2(0.01),

    model = Sequential()
    model.add(Dense(50, input_dim=x.shape[1], activation='relu')) # Hidden 1
    model.add(Dropout(0.5))
    model.add(Dense(25, activation='relu', \
                activity_regularizer=regularizers.l1(1e-4))) # Hidden 2
    # Usually do not add dropout after final hidden layer
    #model.add(Dropout(0.5))
    model.add(Dense(y.shape[1],activation='softmax')) # Output
    model.compile(loss='categorical_crossentropy', optimizer='adam')

    model.fit(x_train,y_train,validation_data=(x_test,y_test),\
              verbose=0,epochs=500)

    pred = model.predict(x_test)

    oos_y.append(y_test)
    # raw probabilities to chosen class (highest probability)
    pred = np.argmax(pred,axis=1)
    oos_pred.append(pred)

    # Measure this fold's accuracy
    y_compare = np.argmax(y_test,axis=1) # For accuracy calculation
    score = metrics.accuracy_score(y_compare, pred)
    print(f"Fold score (accuracy): {score}")


# Build the oos prediction list and calculate the error.
oos_y = np.concatenate(oos_y)
oos_pred = np.concatenate(oos_pred)
oos_y_compare = np.argmax(oos_y,axis=1) # For accuracy calculation

score = metrics.accuracy_score(oos_y_compare, oos_pred)
print(f"Final score (accuracy): {score}")

# Write the cross-validated prediction
oos_y = pd.DataFrame(oos_y)
oos_pred = pd.DataFrame(oos_pred)
oosDF = pd.concat( [df, oos_y, oos_pred],axis=1 )
#oosDF.to_csv(filename_write,index=False)
```

```
Fold #1
Fold score (accuracy): 0.68
Fold #2
Fold score (accuracy): 0.695
Fold #3
Fold score (accuracy): 0.7425
Fold #4
Fold score (accuracy): 0.71
Fold #5
Fold score (accuracy): 0.6625
Final score (accuracy): 0.698
```

In [ ]:

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Google CoLab Instructions

The following code ensures that Google CoLab is running the correct version of TensorFlow.

In [5]:
```
try:
    %tensorflow_version 2.x
    COLAB = True
    print("Note: using Google CoLab")
except:
    print("Note: not using Google CoLab")
    COLAB = False
```

```
Note: using Google CoLab
```

Part 5.5: Benchmarking Regularization Techniques

Quite a few hyperparameters have been introduced so far. Tweaking each of these values can have an effect on the score obtained by your neural networks. Some of the hyperparameters seen so far include:

- Number of layers in the neural network
- How many neurons in each layer
- What activation functions to use on each layer
- Dropout percent on each layer
- L1 and L2 values on each layer

To try out each of these hyperparameters you will need to run train neural networks with multiple settings for each hyperparameter. However, you may have noticed that neural networks often produce somewhat different results when trained multiple times. This is because the neural networks start with random weights. Because of this it is necessary to fit and evaluate a neural network times to ensure that one set of hyperparameters are actually better than another. Bootstrapping can be an effective means of benchmarking (comparing) two sets of hyperparameters.

Bootstrapping is similar to cross-validation. Both go through a number of cycles/folds providing validation and training sets. However, bootstrapping can have an unlimited number of cycles. Bootstrapping chooses a new train and validation split each cycle, with replacement. The fact that each cycle is chosen with replacement means that, unlike cross validation, there will often be repeated rows selected between cycles. If you run the bootstrap for enough cycles, there will be duplicate cycles.

In this part we will use bootstrapping for hyperparameter benchmarking. We will train a neural network for a specified number of splits (denoted by the SPLITS constant). For these examples we use 100. We will compare the average score at the end of the 100. By the end of the cycles the mean score will have converged somewhat. This ending score will be a much better basis of comparison than a single cross-validation. Additionally, the average number of epochs will be tracked to give an idea of a possible optimal value. Because the early stopping validation set is also used to evaluate the the neural network as well, it might be slightly inflated. This is because we are both stopping and evaluating on the same sample. However, we are using the scores only as relative measures to determine the superiority of one set of hyperparameters to another, so this slight inflation should not present too much of a problem.

Because we are benchmarking, we will display the amount of time taken for each cycle. The following function can be used to nicely format a time span.

In [6]:
```
# Nicely formatted time string
def hms_string(sec_elapsed):
```

```
    h = int(sec_elapsed / (60 * 60))
    m = int((sec_elapsed % (60 * 60)) / 60)
    s = sec_elapsed % 60
    return "{}:{:>02}:{:>05.2f}".format(h, m, s)
```

Bootstrapping for Regression

Regression bootstrapping uses the **ShuffleSplit** object to perform the splits. This technique is similar to **KFold** for cross-validation; no balancing occurs. We will attempt to predict the age column for the **jh-simple-dataset**; the following code loads this data.

In [7]:
```
import pandas as pd
from scipy.stats import zscore
from sklearn.model_selection import train_test_split

# Read the data set
df = pd.read_csv(
    "https://data.heatonresearch.com/data/t81-558/jh-simple-dataset.csv",
    na_values=['NA','?'])

# Generate dummies for job
df = pd.concat([df,pd.get_dummies(df['job'],prefix="job")],axis=1)
df.drop('job', axis=1, inplace=True)

# Generate dummies for area
df = pd.concat([df,pd.get_dummies(df['area'],prefix="area")],axis=1)
df.drop('area', axis=1, inplace=True)

# Generate dummies for product
df = pd.concat([df,pd.get_dummies(df['product'],prefix="product")],axis=1)
df.drop('product', axis=1, inplace=True)

# Missing values for income
med = df['income'].median()
df['income'] = df['income'].fillna(med)

# Standardize ranges
df['income'] = zscore(df['income'])
df['aspect'] = zscore(df['aspect'])
df['save_rate'] = zscore(df['save_rate'])
df['subscriptions'] = zscore(df['subscriptions'])

# Convert to numpy - Classification
x_columns = df.columns.drop('age').drop('id')
x = df[x_columns].values
y = df['age'].values
```

The following code performs the bootstrap. The architecture of the neural network can be adjusted to compare many different configurations.

In [8]:

```
import pandas as pd
import os
import numpy as np
import time
import statistics
from sklearn import metrics
from sklearn.model_selection import StratifiedKFold
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense, Activation
from tensorflow.keras import regularizers
from tensorflow.keras.callbacks import EarlyStopping
from sklearn.model_selection import ShuffleSplit

SPLITS = 50

# Bootstrap
boot = ShuffleSplit(n_splits=SPLITS, test_size=0.1, random_state=42)

# Track progress
mean_benchmark = []
epochs_needed = []
num = 0

# Loop through samples
for train, test in boot.split(x):
    start_time = time.time()
    num+=1

    # Split train and test
    x_train = x[train]
    y_train = y[train]
    x_test = x[test]
    y_test = y[test]

    # Construct neural network
    model = Sequential()
    model.add(Dense(20, input_dim=x_train.shape[1], activation='relu'))
    model.add(Dense(10, activation='relu'))
    model.add(Dense(1))
    model.compile(loss='mean_squared_error', optimizer='adam')

    monitor = EarlyStopping(monitor='val_loss', min_delta=1e-3,
        patience=5, verbose=0, mode='auto', restore_best_weights=True)

    # Train on the bootstrap sample
    model.fit(x_train,y_train,validation_data=(x_test,y_test),
              callbacks=[monitor],verbose=0,epochs=1000)
    epochs = monitor.stopped_epoch
    epochs_needed.append(epochs)

    # Predict on the out of boot (validation)
    pred = model.predict(x_test)

    # Measure this bootstrap's log loss
    score = np.sqrt(metrics.mean_squared_error(pred,y_test))
```

```
    mean_benchmark.append(score)
    m1 = statistics.mean(mean_benchmark)
    m2 = statistics.mean(epochs_needed)
    mdev = statistics.pstdev(mean_benchmark)

    # Record this iteration
    time_took = time.time() - start_time
    print(f"#{num}: score={score:.6f}, mean score={m1:.6f},"
          f" stdev={mdev:.6f}",
          f" epochs={epochs}, mean epochs={int(m2)}",
          f" time={hms_string(time_took)}")
```

```
#1: score=0.630750, mean score=0.630750, stdev=0.000000  epochs=147, mean ep
ochs=147  time=0:00:12.56
#2: score=1.020895, mean score=0.825823, stdev=0.195072  epochs=101, mean ep
ochs=124  time=0:00:08.70
#3: score=0.803801, mean score=0.818482, stdev=0.159614  epochs=155, mean ep
ochs=134  time=0:00:20.85
#4: score=0.540871, mean score=0.749079, stdev=0.183188  epochs=122, mean ep
ochs=131  time=0:00:10.64
#5: score=0.802589, mean score=0.759781, stdev=0.165240  epochs=116, mean ep
ochs=128  time=0:00:10.84
#6: score=0.862807, mean score=0.776952, stdev=0.155653  epochs=108, mean ep
ochs=124  time=0:00:10.65
#7: score=0.550373, mean score=0.744584, stdev=0.164478  epochs=131, mean ep
ochs=125  time=0:00:10.85
#8: score=0.659148, mean score=0.733904, stdev=0.156428  epochs=118, mean ep
ochs=124  time=0:00:10.10
#9: score=0.606425, mean score=0.719740, stdev=0.152826  epochs=99, mean epo
chs=121  time=0:00:10.64
#10: score=1.169816, mean score=0.764748, stdev=0.198120  epochs=101, mean e
pochs=119  time=0:00:10.65
#11: score=0.985013, mean score=0.784772, stdev=0.199231  epochs=106, mean e
pochs=118  time=0:00:09.02
#12: score=0.857432, mean score=0.790827, stdev=0.191803  epochs=113, mean e
pochs=118  time=0:00:09.46
#13: score=0.495272, mean score=0.768092, stdev=0.200402  epochs=151, mean e
pochs=120  time=0:00:20.88
#14: score=1.079376, mean score=0.790326, stdev=0.209092  epochs=104, mean e
pochs=119  time=0:00:10.65
#15: score=0.616606, mean score=0.778745, stdev=0.206597  epochs=130, mean e
pochs=120  time=0:00:10.70
#16: score=0.781853, mean score=0.778939, stdev=0.200038  epochs=123, mean e
pochs=120  time=0:00:10.69
#17: score=0.781730, mean score=0.779103, stdev=0.194067  epochs=116, mean e
pochs=120  time=0:00:10.64
#18: score=0.845470, mean score=0.782790, stdev=0.189211  epochs=143, mean e
pochs=121  time=0:00:20.89
#19: score=0.643181, mean score=0.775442, stdev=0.186784  epochs=124, mean e
pochs=121  time=0:00:10.63
#20: score=1.026157, mean score=0.787978, stdev=0.190078  epochs=91, mean ep
ochs=119  time=0:00:10.63
#21: score=0.587819, mean score=0.778447, stdev=0.190332  epochs=106, mean e
pochs=119  time=0:00:10.62
#22: score=0.600830, mean score=0.770373, stdev=0.189600  epochs=117, mean e
pochs=119  time=0:00:10.32
#23: score=0.662913, mean score=0.765701, stdev=0.186723  epochs=126, mean e
pochs=119  time=0:00:20.89
#24: score=0.671352, mean score=0.761770, stdev=0.183762  epochs=130, mean e
pochs=119  time=0:00:20.87
#25: score=0.647940, mean score=0.757217, stdev=0.181425  epochs=143, mean e
pochs=120  time=0:00:12.29
#26: score=0.684534, mean score=0.754421, stdev=0.178450  epochs=94, mean ep
ochs=119  time=0:00:08.29
#27: score=0.534195, mean score=0.746265, stdev=0.179986  epochs=149, mean e
pochs=120  time=0:00:20.91
#28: score=0.901485, mean score=0.751808, stdev=0.179074  epochs=110, mean e
pochs=120  time=0:00:10.66
```

```
#29: score=0.696614, mean score=0.749905, stdev=0.176248  epochs=117, mean e
pochs=120  time=0:00:10.46
#30: score=0.656065, mean score=0.746777, stdev=0.174102  epochs=109, mean e
pochs=120  time=0:00:10.63
#31: score=0.749652, mean score=0.746870, stdev=0.171272  epochs=118, mean e
pochs=119  time=0:00:10.66
#32: score=0.508090, mean score=0.739408, stdev=0.173619  epochs=106, mean e
pochs=119  time=0:00:10.66
#33: score=0.732891, mean score=0.739210, stdev=0.170971  epochs=124, mean e
pochs=119  time=0:00:10.76
#34: score=1.089590, mean score=0.749516, stdev=0.178539  epochs=95, mean ep
ochs=118  time=0:00:08.24
#35: score=0.568665, mean score=0.744349, stdev=0.178530  epochs=115, mean e
pochs=118  time=0:00:10.64
#36: score=0.523255, mean score=0.738207, stdev=0.179744  epochs=108, mean e
pochs=118  time=0:00:09.23
#37: score=1.082163, mean score=0.747503, stdev=0.185865  epochs=87, mean ep
ochs=117  time=0:00:10.62
#38: score=0.752920, mean score=0.747646, stdev=0.183405  epochs=125, mean e
pochs=117  time=0:00:10.66
#39: score=0.587106, mean score=0.743529, stdev=0.182808  epochs=118, mean e
pochs=117  time=0:00:10.18
#40: score=0.781335, mean score=0.744474, stdev=0.180605  epochs=103, mean e
pochs=117  time=0:00:10.64
#41: score=1.209243, mean score=0.755810, stdev=0.192257  epochs=82, mean ep
ochs=116  time=0:00:07.39
#42: score=0.650733, mean score=0.753308, stdev=0.190628  epochs=141, mean e
pochs=117  time=0:00:21.19
#43: score=0.622103, mean score=0.750257, stdev=0.189434  epochs=116, mean e
pochs=117  time=0:00:09.85
#44: score=0.519172, mean score=0.745005, stdev=0.190409  epochs=135, mean e
pochs=117  time=0:00:11.94
#45: score=0.926205, mean score=0.749032, stdev=0.190167  epochs=87, mean ep
ochs=116  time=0:00:07.78
#46: score=0.604350, mean score=0.745887, stdev=0.189268  epochs=78, mean ep
ochs=116  time=0:00:10.64
#47: score=0.690874, mean score=0.744716, stdev=0.187412  epochs=136, mean e
pochs=116  time=0:00:20.86
#48: score=0.719645, mean score=0.744194, stdev=0.185484  epochs=112, mean e
pochs=116  time=0:00:09.33
#49: score=0.911419, mean score=0.747607, stdev=0.185098  epochs=124, mean e
pochs=116  time=0:00:10.66
#50: score=0.599252, mean score=0.744639, stdev=0.184411  epochs=132, mean e
pochs=116  time=0:00:20.91
```

The bootstrapping process for classification is similar, and I present it in the next section.

Bootstrapping for Classification

Regression bootstrapping uses the **StratifiedShuffleSplit** class to perform the splits. This class is similar to **StratifiedKFold** for cross-validation, as the classes are balanced so that the sampling does not affect proportions. To demonstrate

this technique, we will attempt to predict the product column for the **jh-simple-dataset**; the following code loads this data.

In [9]:
```
import pandas as pd
from scipy.stats import zscore

# Read the data set
df = pd.read_csv(
    "https://data.heatonresearch.com/data/t81-558/jh-simple-dataset.csv",
    na_values=['NA','?'])

# Generate dummies for job
df = pd.concat([df,pd.get_dummies(df['job'],prefix="job")],axis=1)
df.drop('job', axis=1, inplace=True)

# Generate dummies for area
df = pd.concat([df,pd.get_dummies(df['area'],prefix="area")],axis=1)
df.drop('area', axis=1, inplace=True)

# Missing values for income
med = df['income'].median()
df['income'] = df['income'].fillna(med)

# Standardize ranges
df['income'] = zscore(df['income'])
df['aspect'] = zscore(df['aspect'])
df['save_rate'] = zscore(df['save_rate'])
df['age'] = zscore(df['age'])
df['subscriptions'] = zscore(df['subscriptions'])

# Convert to numpy - Classification
x_columns = df.columns.drop('product').drop('id')
x = df[x_columns].values
dummies = pd.get_dummies(df['product']) # Classification
products = dummies.columns
y = dummies.values
```

We now run this data through a number of splits specified by the SPLITS variable. We track the average error through each of these splits.

In [10]:
```
import pandas as pd
import os
import numpy as np
import time
import statistics
from sklearn import metrics
from sklearn.model_selection import StratifiedKFold
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense, Activation
from tensorflow.keras import regularizers
from tensorflow.keras.callbacks import EarlyStopping
from sklearn.model_selection import StratifiedShuffleSplit

SPLITS = 50
```

```
# Bootstrap
boot = StratifiedShuffleSplit(n_splits=SPLITS, test_size=0.1,
                              random_state=42)

# Track progress
mean_benchmark = []
epochs_needed = []
num = 0

# Loop through samples
for train, test in boot.split(x,df['product']):
    start_time = time.time()
    num+=1

    # Split train and test
    x_train = x[train]
    y_train = y[train]
    x_test = x[test]
    y_test = y[test]

    # Construct neural network
    model = Sequential()
    model.add(Dense(50, input_dim=x.shape[1], activation='relu')) # Hidden 1
    model.add(Dense(25, activation='relu')) # Hidden 2
    model.add(Dense(y.shape[1],activation='softmax')) # Output
    model.compile(loss='categorical_crossentropy', optimizer='adam')
    monitor = EarlyStopping(monitor='val_loss', min_delta=1e-3,
        patience=25, verbose=0, mode='auto', restore_best_weights=True)

    # Train on the bootstrap sample
    model.fit(x_train,y_train,validation_data=(x_test,y_test),
              callbacks=[monitor],verbose=0,epochs=1000)
    epochs = monitor.stopped_epoch
    epochs_needed.append(epochs)

    # Predict on the out of boot (validation)
    pred = model.predict(x_test)

    # Measure this bootstrap's log loss
    y_compare = np.argmax(y_test,axis=1) # For log loss calculation
    score = metrics.log_loss(y_compare, pred)
    mean_benchmark.append(score)
    m1 = statistics.mean(mean_benchmark)
    m2 = statistics.mean(epochs_needed)
    mdev = statistics.pstdev(mean_benchmark)

    # Record this iteration
    time_took = time.time() - start_time
    print(f"#{num}: score={score:.6f}, mean score={m1:.6f}," +\
          f"stdev={mdev:.6f}, epochs={epochs}, mean epochs={int(m2)}," +\
          f" time={hms_string(time_took)}")
```

```
#1: score=0.666342, mean score=0.666342,stdev=0.000000, epochs=66, mean epoc
hs=66, time=0:00:06.31
#2: score=0.645598, mean score=0.655970,stdev=0.010372, epochs=59, mean epoc
hs=62, time=0:00:10.63
#3: score=0.676924, mean score=0.662955,stdev=0.013011, epochs=66, mean epoc
hs=63, time=0:00:10.64
#4: score=0.672602, mean score=0.665366,stdev=0.012017, epochs=84, mean epoc
hs=68, time=0:00:08.20
#5: score=0.667274, mean score=0.665748,stdev=0.010776, epochs=73, mean epoc
hs=69, time=0:00:10.65
#6: score=0.706372, mean score=0.672518,stdev=0.018055, epochs=50, mean epoc
hs=66, time=0:00:04.81
#7: score=0.687937, mean score=0.674721,stdev=0.017565, epochs=71, mean epoc
hs=67, time=0:00:06.89
#8: score=0.734794, mean score=0.682230,stdev=0.025781, epochs=43, mean epoc
hs=64, time=0:00:05.51
#9: score=0.623972, mean score=0.675757,stdev=0.030431, epochs=65, mean epoc
hs=64, time=0:00:10.66
#10: score=0.650303, mean score=0.673212,stdev=0.029862, epochs=109, mean ep
ochs=68, time=0:00:10.63
#11: score=0.679500, mean score=0.673783,stdev=0.028529, epochs=83, mean epo
chs=69, time=0:00:10.63
#12: score=0.736851, mean score=0.679039,stdev=0.032403, epochs=51, mean epo
chs=68, time=0:00:05.51
#13: score=0.703048, mean score=0.680886,stdev=0.031782, epochs=92, mean epo
chs=70, time=0:00:08.48
#14: score=0.733015, mean score=0.684609,stdev=0.033439, epochs=52, mean epo
chs=68, time=0:00:05.13
#15: score=0.664863, mean score=0.683293,stdev=0.032679, epochs=77, mean epo
chs=69, time=0:00:10.62
#16: score=0.740248, mean score=0.686853,stdev=0.034514, epochs=79, mean epo
chs=70, time=0:00:10.94
#17: score=0.639677, mean score=0.684078,stdev=0.035276, epochs=82, mean epo
chs=70, time=0:00:10.64
#18: score=0.648893, mean score=0.682123,stdev=0.035216, epochs=64, mean epo
chs=70, time=0:00:06.14
#19: score=0.603215, mean score=0.677970,stdev=0.038541, epochs=60, mean epo
chs=69, time=0:00:10.72
#20: score=0.691074, mean score=0.678625,stdev=0.037673, epochs=49, mean epo
chs=68, time=0:00:05.07
#21: score=0.649008, mean score=0.677215,stdev=0.037302, epochs=54, mean epo
chs=68, time=0:00:05.51
#22: score=0.745487, mean score=0.680318,stdev=0.039121, epochs=39, mean epo
chs=66, time=0:00:05.54
#23: score=0.588884, mean score=0.676343,stdev=0.042563, epochs=74, mean epo
chs=67, time=0:00:07.11
#24: score=0.697504, mean score=0.677224,stdev=0.041881, epochs=61, mean epo
chs=66, time=0:00:05.86
#25: score=0.569334, mean score=0.672909,stdev=0.046161, epochs=64, mean epo
chs=66, time=0:00:10.67
#26: score=0.632199, mean score=0.671343,stdev=0.045936, epochs=65, mean epo
chs=66, time=0:00:06.16
#27: score=0.707666, mean score=0.672688,stdev=0.045597, epochs=74, mean epo
chs=66, time=0:00:07.34
#28: score=0.747781, mean score=0.675370,stdev=0.046894, epochs=48, mean epo
chs=66, time=0:00:05.56
```

```
#29: score=0.648160, mean score=0.674432,stdev=0.046345, epochs=61, mean epo
chs=66, time=0:00:05.80
#30: score=0.695912, mean score=0.675148,stdev=0.045729, epochs=70, mean epo
chs=66, time=0:00:06.72
#31: score=0.692880, mean score=0.675720,stdev=0.045094, epochs=61, mean epo
chs=66, time=0:00:10.65
#32: score=0.675613, mean score=0.675717,stdev=0.044384, epochs=73, mean epo
chs=66, time=0:00:10.66
#33: score=0.625625, mean score=0.674199,stdev=0.044542, epochs=57, mean epo
chs=65, time=0:00:05.34
#34: score=0.571148, mean score=0.671168,stdev=0.047210, epochs=130, mean ep
ochs=67, time=0:00:20.88
#35: score=0.542365, mean score=0.667488,stdev=0.051240, epochs=75, mean epo
chs=68, time=0:00:10.67
#36: score=0.645099, mean score=0.666866,stdev=0.050657, epochs=59, mean epo
chs=67, time=0:00:05.59
#37: score=0.639249, mean score=0.666119,stdev=0.050168, epochs=78, mean epo
chs=68, time=0:00:10.68
#38: score=0.684326, mean score=0.666598,stdev=0.049589, epochs=75, mean epo
chs=68, time=0:00:10.63
#39: score=0.728835, mean score=0.668194,stdev=0.049928, epochs=79, mean epo
chs=68, time=0:00:07.78
#40: score=0.706089, mean score=0.669142,stdev=0.049654, epochs=46, mean epo
chs=67, time=0:00:04.59
#41: score=0.727177, mean score=0.670557,stdev=0.049855, epochs=68, mean epo
chs=67, time=0:00:10.69
#42: score=0.653240, mean score=0.670145,stdev=0.049329, epochs=53, mean epo
chs=67, time=0:00:05.17
#43: score=0.692113, mean score=0.670656,stdev=0.048864, epochs=51, mean epo
chs=67, time=0:00:05.56
#44: score=0.745355, mean score=0.672353,stdev=0.049572, epochs=66, mean epo
chs=67, time=0:00:10.66
#45: score=0.631125, mean score=0.671437,stdev=0.049393, epochs=75, mean epo
chs=67, time=0:00:07.16
#46: score=0.664004, mean score=0.671276,stdev=0.048865, epochs=57, mean epo
chs=67, time=0:00:05.69
#47: score=0.686937, mean score=0.671609,stdev=0.048395, epochs=52, mean epo
chs=66, time=0:00:05.07
#48: score=0.760827, mean score=0.673468,stdev=0.049555, epochs=40, mean epo
chs=66, time=0:00:04.14
#49: score=0.665493, mean score=0.673305,stdev=0.049060, epochs=60, mean epo
chs=66, time=0:00:10.65
#50: score=0.692625, mean score=0.673691,stdev=0.048642, epochs=55, mean epo
chs=65, time=0:00:05.22
```

Benchmarking

Now that we've seen how to bootstrap with both classification and regression, we can start to try to optimize the hyperparameters for the **jh-simple-dataset** data. For this example, we will encode for classification of the product column. Evaluation will be in log loss.

In [11]:
```
import pandas as pd
from scipy.stats import zscore

# Read the data set
df = pd.read_csv(
    "https://data.heatonresearch.com/data/t81-558/jh-simple-dataset.csv",
    na_values=['NA','?'])

# Generate dummies for job
df = pd.concat([df,pd.get_dummies(df['job'],prefix="job")],axis=1)
df.drop('job', axis=1, inplace=True)

# Generate dummies for area
df = pd.concat([df,pd.get_dummies(df['area'],prefix="area")],
               axis=1)
df.drop('area', axis=1, inplace=True)

# Missing values for income
med = df['income'].median()
df['income'] = df['income'].fillna(med)

# Standardize ranges
df['income'] = zscore(df['income'])
df['aspect'] = zscore(df['aspect'])
df['save_rate'] = zscore(df['save_rate'])
df['age'] = zscore(df['age'])
df['subscriptions'] = zscore(df['subscriptions'])

# Convert to numpy - Classification
x_columns = df.columns.drop('product').drop('id')
x = df[x_columns].values
dummies = pd.get_dummies(df['product']) # Classification
products = dummies.columns
y = dummies.values
```

I performed some optimization, and the code has the best settings that I could determine. Later in this book, we will see how we can use an automatic process to optimize the hyperparameters.

In [13]:
```
import pandas as pd
import os
import numpy as np
import time
import tensorflow.keras.initializers
import statistics
from sklearn import metrics
from sklearn.model_selection import StratifiedKFold
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense, Activation, Dropout
from tensorflow.keras import regularizers
from tensorflow.keras.callbacks import EarlyStopping
from sklearn.model_selection import StratifiedShuffleSplit
from tensorflow.keras.layers import LeakyReLU,PReLU
```

```
SPLITS = 100

# Bootstrap
boot = StratifiedShuffleSplit(n_splits=SPLITS, test_size=0.1)

# Track progress
mean_benchmark = []
epochs_needed = []
num = 0

# Loop through samples
for train, test in boot.split(x,df['product']):
    start_time = time.time()
    num+=1

    # Split train and test
    x_train = x[train]
    y_train = y[train]
    x_test = x[test]
    y_test = y[test]

    # Construct neural network
    model = Sequential()
    model.add(Dense(100, input_dim=x.shape[1], activation=PReLU(), \
        kernel_regularizer=regularizers.l2(1e-4))) # Hidden 1
    model.add(Dropout(0.5))
    model.add(Dense(100, activation=PReLU(), \
        activity_regularizer=regularizers.l2(1e-4))) # Hidden 2
    model.add(Dropout(0.5))
    model.add(Dense(100, activation=PReLU(), \
        activity_regularizer=regularizers.l2(1e-4)
    )) # Hidden 3
#    model.add(Dropout(0.5)) - Usually better performance
# without dropout on final layer
    model.add(Dense(y.shape[1],activation='softmax')) # Output
    model.compile(loss='categorical_crossentropy', optimizer='adam')
    monitor = EarlyStopping(monitor='val_loss', min_delta=1e-3,
        patience=100, verbose=0, mode='auto', restore_best_weights=True)

    # Train on the bootstrap sample
    model.fit(x_train,y_train,validation_data=(x_test,y_test), \
              callbacks=[monitor],verbose=0,epochs=1000)
    epochs = monitor.stopped_epoch
    epochs_needed.append(epochs)

    # Predict on the out of boot (validation)
    pred = model.predict(x_test)

    # Measure this bootstrap's log loss
    y_compare = np.argmax(y_test,axis=1) # For log loss calculation
    score = metrics.log_loss(y_compare, pred)
    mean_benchmark.append(score)
    m1 = statistics.mean(mean_benchmark)
    m2 = statistics.mean(epochs_needed)
    mdev = statistics.pstdev(mean_benchmark)
```

```
    # Record this iteration
    time_took = time.time() - start_time
    print(f"#{num}: score={score:.6f}, mean score={m1:.6f},"
          f"stdev={mdev:.6f}, epochs={epochs},"
          f"mean epochs={int(m2)}, time={hms_string(time_took)}")
```

#1: score=0.642887, mean score=0.642887,stdev=0.000000, epochs=325,mean epoc
hs=325, time=0:00:42.10
#2: score=0.555518, mean score=0.599202,stdev=0.043684, epochs=208,mean epoc
hs=266, time=0:00:41.74
#3: score=0.605537, mean score=0.601314,stdev=0.035793, epochs=187,mean epoc
hs=240, time=0:00:24.22
#4: score=0.609415, mean score=0.603339,stdev=0.031195, epochs=250,mean epoc
hs=242, time=0:00:41.72
#5: score=0.619657, mean score=0.606603,stdev=0.028655, epochs=201,mean epoc
hs=234, time=0:00:26.10
#6: score=0.638641, mean score=0.611943,stdev=0.028755, epochs=172,mean epoc
hs=223, time=0:00:41.73
#7: score=0.671137, mean score=0.620399,stdev=0.033731, epochs=203,mean epoc
hs=220, time=0:00:26.58
#8: score=0.635294, mean score=0.622261,stdev=0.031935, epochs=209,mean epoc
hs=219, time=0:00:41.74
#9: score=0.633694, mean score=0.623531,stdev=0.030322, epochs=162,mean epoc
hs=213, time=0:00:41.78
#10: score=0.596081, mean score=0.620786,stdev=0.029921, epochs=197,mean epo
chs=211, time=0:00:41.74
#11: score=0.583717, mean score=0.617416,stdev=0.030454, epochs=232,mean epo
chs=213, time=0:00:41.77
#12: score=0.686736, mean score=0.623193,stdev=0.034889, epochs=216,mean epo
chs=213, time=0:00:28.25
#13: score=0.684454, mean score=0.627905,stdev=0.037284, epochs=134,mean epo
chs=207, time=0:00:21.58
#14: score=0.573696, mean score=0.624033,stdev=0.038545, epochs=184,mean epo
chs=205, time=0:00:23.81
#15: score=0.723944, mean score=0.630694,stdev=0.044808, epochs=170,mean epo
chs=203, time=0:00:41.80
#16: score=0.659891, mean score=0.632519,stdev=0.043957, epochs=203,mean epo
chs=203, time=0:00:41.80
#17: score=0.569637, mean score=0.628820,stdev=0.045139, epochs=204,mean epo
chs=203, time=0:00:41.77
#18: score=0.608905, mean score=0.627713,stdev=0.044103, epochs=233,mean epo
chs=205, time=0:00:41.76
#19: score=0.734381, mean score=0.633328,stdev=0.049092, epochs=193,mean epo
chs=204, time=0:00:25.25
#20: score=0.587099, mean score=0.631016,stdev=0.048899, epochs=252,mean epo
chs=206, time=0:00:42.07
#21: score=0.661902, mean score=0.632487,stdev=0.048171, epochs=211,mean epo
chs=206, time=0:00:41.79
#22: score=0.656783, mean score=0.633591,stdev=0.047335, epochs=145,mean epo
chs=204, time=0:00:19.19
#23: score=0.611230, mean score=0.632619,stdev=0.046519, epochs=201,mean epo
chs=204, time=0:00:41.77
#24: score=0.638759, mean score=0.632875,stdev=0.045556, epochs=223,mean epo
chs=204, time=0:00:28.67
#25: score=0.635676, mean score=0.632987,stdev=0.044639, epochs=240,mean epo
chs=206, time=0:00:41.77
#26: score=0.599321, mean score=0.631692,stdev=0.044248, epochs=199,mean epo
chs=205, time=0:00:42.10
#27: score=0.696892, mean score=0.634107,stdev=0.045133, epochs=146,mean epo
chs=203, time=0:00:21.28
#28: score=0.637397, mean score=0.634224,stdev=0.044324, epochs=179,mean epo
chs=202, time=0:00:23.53

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#29: score=0.645323, mean score=0.634607,stdev=0.043600, epochs=256,mean epo
chs=204, time=0:00:32.44
#30: score=0.588104, mean score=0.633057,stdev=0.043672, epochs=199,mean epo
chs=204, time=0:00:25.96
#31: score=0.676097, mean score=0.634445,stdev=0.043630, epochs=229,mean epo
chs=205, time=0:00:41.74
#32: score=0.667709, mean score=0.635485,stdev=0.043331, epochs=155,mean epo
chs=203, time=0:00:20.61
#33: score=0.616544, mean score=0.634911,stdev=0.042793, epochs=283,mean epo
chs=206, time=0:00:36.55
#34: score=0.622340, mean score=0.634541,stdev=0.042212, epochs=174,mean epo
chs=205, time=0:00:22.82
#35: score=0.665123, mean score=0.635415,stdev=0.041916, epochs=205,mean epo
chs=205, time=0:00:27.05
#36: score=0.573597, mean score=0.633698,stdev=0.042560, epochs=205,mean epo
chs=205, time=0:00:41.81
#37: score=0.617111, mean score=0.633249,stdev=0.042067, epochs=253,mean epo
chs=206, time=0:00:31.92
#38: score=0.627494, mean score=0.633098,stdev=0.041520, epochs=205,mean epo
chs=206, time=0:00:41.76
#39: score=0.669212, mean score=0.634024,stdev=0.041380, epochs=193,mean epo
chs=206, time=0:00:42.14
#40: score=0.684894, mean score=0.635296,stdev=0.041624, epochs=171,mean epo
chs=205, time=0:00:21.86
#41: score=0.648313, mean score=0.635613,stdev=0.041162, epochs=205,mean epo
chs=205, time=0:00:41.74
#42: score=0.679919, mean score=0.636668,stdev=0.041226, epochs=251,mean epo
chs=206, time=0:00:41.74
#43: score=0.701787, mean score=0.638183,stdev=0.041909, epochs=146,mean epo
chs=204, time=0:00:21.29
#44: score=0.660646, mean score=0.638693,stdev=0.041566, epochs=168,mean epo
chs=204, time=0:00:41.80
#45: score=0.660335, mean score=0.639174,stdev=0.041225, epochs=136,mean epo
chs=202, time=0:00:21.67
#46: score=0.656875, mean score=0.639559,stdev=0.040856, epochs=154,mean epo
chs=201, time=0:00:21.33
#47: score=0.679169, mean score=0.640402,stdev=0.040821, epochs=286,mean epo
chs=203, time=0:00:41.78
#48: score=0.608082, mean score=0.639728,stdev=0.040656, epochs=173,mean epo
chs=202, time=0:00:22.20
#49: score=0.590421, mean score=0.638722,stdev=0.040839, epochs=185,mean epo
chs=202, time=0:00:24.23
#50: score=0.616646, mean score=0.638281,stdev=0.040546, epochs=273,mean epo
chs=203, time=0:00:41.76
#51: score=0.683312, mean score=0.639163,stdev=0.040629, epochs=163,mean epo
chs=202, time=0:00:41.76
#52: score=0.686289, mean score=0.640070,stdev=0.040754, epochs=166,mean epo
chs=202, time=0:00:22.03
#53: score=0.701892, mean score=0.641236,stdev=0.041235, epochs=185,mean epo
chs=201, time=0:00:24.02
#54: score=0.647809, mean score=0.641358,stdev=0.040861, epochs=171,mean epo
chs=201, time=0:00:22.38
#55: score=0.678673, mean score=0.642036,stdev=0.040793, epochs=160,mean epo
chs=200, time=0:00:41.77
#56: score=0.594752, mean score=0.641192,stdev=0.040910, epochs=185,mean epo
chs=200, time=0:00:25.42
```

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#57: score=0.719842, mean score=0.642572,stdev=0.041843, epochs=124,mean epo
chs=198, time=0:00:17.03
#58: score=0.689348, mean score=0.643378,stdev=0.041926, epochs=223,mean epo
chs=199, time=0:00:29.47
#59: score=0.657452, mean score=0.643617,stdev=0.041608, epochs=220,mean epo
chs=199, time=0:00:28.31
#60: score=0.611100, mean score=0.643075,stdev=0.041470, epochs=226,mean epo
chs=200, time=0:00:29.49
#61: score=0.660965, mean score=0.643368,stdev=0.041191, epochs=162,mean epo
chs=199, time=0:00:21.28
#62: score=0.669189, mean score=0.643785,stdev=0.040987, epochs=147,mean epo
chs=198, time=0:00:21.31
#63: score=0.652563, mean score=0.643924,stdev=0.040675, epochs=187,mean epo
chs=198, time=0:00:41.74
#64: score=0.590525, mean score=0.643090,stdev=0.040896, epochs=275,mean epo
chs=199, time=0:00:35.78
#65: score=0.699827, mean score=0.643963,stdev=0.041176, epochs=182,mean epo
chs=199, time=0:00:23.86
#66: score=0.665028, mean score=0.644282,stdev=0.040944, epochs=214,mean epo
chs=199, time=0:00:28.54
#67: score=0.729557, mean score=0.645554,stdev=0.041932, epochs=225,mean epo
chs=199, time=0:00:41.79
#68: score=0.586906, mean score=0.644692,stdev=0.042217, epochs=219,mean epo
chs=200, time=0:00:28.43
#69: score=0.717007, mean score=0.645740,stdev=0.042792, epochs=124,mean epo
chs=199, time=0:00:21.30
#70: score=0.670428, mean score=0.646093,stdev=0.042586, epochs=198,mean epo
chs=199, time=0:00:41.92
#71: score=0.717004, mean score=0.647091,stdev=0.043103, epochs=203,mean epo
chs=199, time=0:00:42.09
#72: score=0.582071, mean score=0.646188,stdev=0.043474, epochs=174,mean epo
chs=198, time=0:00:41.77
#73: score=0.723909, mean score=0.647253,stdev=0.044110, epochs=199,mean epo
chs=198, time=0:00:41.78
#74: score=0.685384, mean score=0.647768,stdev=0.044032, epochs=145,mean epo
chs=198, time=0:00:19.13
#75: score=0.584444, mean score=0.646924,stdev=0.044336, epochs=205,mean epo
chs=198, time=0:00:41.77
#76: score=0.681646, mean score=0.647381,stdev=0.044221, epochs=160,mean epo
chs=197, time=0:00:21.30
#77: score=0.585961, mean score=0.646583,stdev=0.044480, epochs=195,mean epo
chs=197, time=0:00:42.19
#78: score=0.626380, mean score=0.646324,stdev=0.044252, epochs=231,mean epo
chs=198, time=0:00:41.76
#79: score=0.700790, mean score=0.647014,stdev=0.044391, epochs=188,mean epo
chs=197, time=0:00:24.41
#80: score=0.664455, mean score=0.647232,stdev=0.044155, epochs=164,mean epo
chs=197, time=0:00:21.95
#81: score=0.601657, mean score=0.646669,stdev=0.044169, epochs=205,mean epo
chs=197, time=0:00:41.80
#82: score=0.661004, mean score=0.646844,stdev=0.043927, epochs=151,mean epo
chs=197, time=0:00:19.90
#83: score=0.693299, mean score=0.647404,stdev=0.043955, epochs=161,mean epo
chs=196, time=0:00:21.32
#84: score=0.732184, mean score=0.648413,stdev=0.044649, epochs=147,mean epo
chs=196, time=0:00:20.04
```

#85: score=0.628028, mean score=0.648173,stdev=0.044440, epochs=197,mean epochs=196, time=0:00:25.61
#86: score=0.626073, mean score=0.647916,stdev=0.044245, epochs=176,mean epochs=195, time=0:00:23.27
#87: score=0.632806, mean score=0.647742,stdev=0.044019, epochs=261,mean epochs=196, time=0:00:41.76
#88: score=0.694768, mean score=0.648277,stdev=0.044051, epochs=204,mean epochs=196, time=0:00:41.80
#89: score=0.699703, mean score=0.648855,stdev=0.044137, epochs=183,mean epochs=196, time=0:00:23.10
#90: score=0.611230, mean score=0.648437,stdev=0.044068, epochs=270,mean epochs=197, time=0:00:34.62
#91: score=0.637264, mean score=0.648314,stdev=0.043841, epochs=257,mean epochs=197, time=0:00:41.78
#92: score=0.678976, mean score=0.648647,stdev=0.043718, epochs=158,mean epochs=197, time=0:00:21.24
#93: score=0.627937, mean score=0.648424,stdev=0.043534, epochs=218,mean epochs=197, time=0:00:41.74
#94: score=0.644387, mean score=0.648381,stdev=0.043304, epochs=197,mean epochs=197, time=0:00:41.76
#95: score=0.660005, mean score=0.648504,stdev=0.043092, epochs=167,mean epochs=197, time=0:00:21.69
#96: score=0.674187, mean score=0.648771,stdev=0.042946, epochs=174,mean epochs=197, time=0:00:22.79
#97: score=0.654942, mean score=0.648835,stdev=0.042729, epochs=162,mean epochs=196, time=0:00:21.30
#98: score=0.644139, mean score=0.648787,stdev=0.042513, epochs=173,mean epochs=196, time=0:00:22.70
#99: score=0.697473, mean score=0.649279,stdev=0.042577, epochs=172,mean epochs=196, time=0:00:41.79
#100: score=0.678298, mean score=0.649569,stdev=0.042462, epochs=169,mean epochs=196, time=0:00:21.90