

Modelling of Atmospheric Clouds

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lecture+lab 3

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previously on... (transport problem)

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$$\partial_t G\psi + \nabla \cdot (G\vec{u}\psi) = GR$$

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$$\frac{\psi_i^{n+1} - \psi_i^n}{\Delta t} = - \begin{cases} u \frac{\psi_i - \psi_{i-1}}{\Delta x} & u > 0 \\ u \frac{\psi_{i+1} - \psi_i}{\Delta x} & u < 0 \end{cases}$$

previously on ... upwind/upstream/donor-cell discretisation

$$\psi_i^{n+t} = \psi_i^n - \underbrace{u \frac{\Delta t}{\Delta x}}_C \begin{cases} \psi_i - \psi_{i-1} & u > 0 \\ \psi_{i+1} - \psi_i & u < 0 \end{cases}$$

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$$\psi_i^{n+1} = \psi_i^n - \left[\text{flux}(\psi_i^n, \psi_{i+1}^n, C) - \text{flux}(\psi_{i-1}^n, \psi_i^n, C) \right]$$

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$$\text{flux}(\psi_l, \psi_r, C) = \frac{C+|C|}{2} \psi_l + \frac{C-|C|}{2} \psi_r$$

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```
In [2]: def flux(psi_l, psi_r, C):  
        return .5 * (C + abs(C)) * psi_l + \  
               .5 * (C - abs(C)) * psi_r
```

```
In [3]: def upwind(psi, i, C):  
        return psi[i] - flux(psi[i-1], psi[i], C[i]) + \  
               flux(psi[i], psi[i+1], C[i+1])
```

not previously on: modified equation analysis (1/3)

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$$\psi_i^{n+1} = \psi_i^n - \left[\frac{C+|C|}{2} (\psi_i^n - \psi_{i-1}^n) + \frac{C-|C|}{2} (\psi_{i+1}^n - \psi_i^n) \right]$$

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$$\psi_i^{n+1} = \psi_i^n + \partial_t \psi|_i^n \Delta t + \partial_t^2 \psi|_i^n (\Delta t)^2/2 + \dots$$

$$\psi_{i+1}^n = \psi_i^n + \partial_x \psi|_i^n \Delta x + \partial_x^2 \psi|_i^n (\Delta x)^2/2 + \dots$$

$$\psi_{i-1}^n = \psi_i^n + \partial_x \psi|_i^n (-\Delta x) + \partial_x^2 \psi|_i^n (-\Delta x)^2/2 + \dots$$

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$$\begin{aligned} & \partial_t \psi \Delta t + \partial_t^2 \psi \frac{(\Delta t)^2}{2} = \\ &= - \left[\frac{C+|C|}{2} \left(-\partial_x \psi(-\Delta x) - \partial_x^2 \psi \frac{(-\Delta x)^2}{2} \right) + \right. \\ & \quad \left. \frac{C-|C|}{2} \left(\partial_x \psi \Delta x + \partial_x^2 \psi \frac{(\Delta x)^2}{2} \right) \right] \end{aligned}$$

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$$\partial_t \psi + \partial_t^2 \psi \frac{\Delta t}{2} = -\frac{u + |u|}{2} \left(\partial_x \psi - \partial_x^2 \psi \frac{\Delta x}{2} \right) - \frac{u - |u|}{2} \left(\partial_x \psi + \partial_x^2 \psi \frac{\Delta x}{2} \right)$$

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$$\partial_t \psi = -u \partial_x \psi \quad \rightsquigarrow \quad \partial_t^2 \psi = -u \partial_x \partial_t \psi = u^2 \partial_x^2 \psi$$

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$$\partial_t \psi + u \partial_x \psi + \underbrace{\left(u^2 \frac{\Delta t}{2} - |u| \frac{\Delta x}{2} \right)}_{\text{numerical diff. coeff.}} \partial_x^2 \psi + \dots = 0$$

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$$\partial_t \psi + \underbrace{\partial_x (u \psi)}_{\text{flux}} + \underbrace{\partial_x \left(u^2 \frac{\Delta t}{2} - |u| \frac{\Delta x}{2} \right) \frac{\partial_x \psi}{\psi}}_{\text{diffusive velocity}} \psi + \dots = 0$$

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antidiffusive Courant number

$$C' = -\frac{\Delta t}{\Delta x} \left(u^2 \frac{\Delta t}{2} - |u| \frac{\Delta x}{2} \right) \frac{\partial_x \psi}{\psi}$$

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$$C' \approx - \left(C^2 \frac{\Delta x}{2} - |C| \frac{\Delta x}{2} \right) \frac{\psi_{i+1}^n - \psi_i^n}{\Delta x} \frac{2}{\psi_{i+1}^n + \psi_i^n}$$

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$$C' = (|C| - C^2) \frac{\psi_{i+i}^n - \psi_i^n}{\psi_{i+i}^n + \psi_i^n}$$

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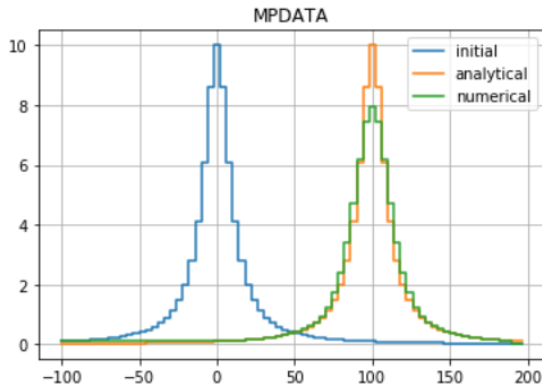
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```
In [9]: def C_corr(C, nx, psi):  
        j = slice(0, nx-1)  
        return (abs(C[j]) - C[j]**2) * (psi[j+one] - psi[j]) / (psi[j+one] + psi[j])
```

previously on...

```
In [10]: psi = psi_0(x)
         for _ in range(nt):
             psi[i] = upwind(psi, i, C_phys)
             psi[i] = upwind(psi, i, C_corr(C_phys, nx, psi))
         plot(x, psi, psi_0, 'MPDATA', v, nt)
```



attendance list!!!

job offer @ ECMWF

https://www.ecmwf.int/sites/default/files/vacancies/_VNVN20-06_en.pdf



EUROPEAN CENTRE FOR MEDIUM-RANGE WEATHER FORECASTS

Analyst - Full Stack Web Developer (Python)

Position information

Vacancy No.: VN20-06	Department: Forecasts
Grade: A2	Section: Development
Job Ref. No.: STF-C/20-06	Reports to: Team Leader
Publication Date: 28 February 2020	Closing Date: 6 April 2020

LAB: github.com/atmos-cloud-sim-uj/MoAC-2020

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
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MoAC-2020

Materials accompanying the "Modelling of Atmospheric Clouds" lecture series

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LAB: plan

- ▶ MPDATA convergence analysis
- ▶ lognormal spectrum, mass vs. number distribution, PM2.5
- ▶ change of coordinates, advection, conservation

Thank you for your attention!