# Modelling of Atmospheric Clouds

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lecture+lab 3
Mar. 9 2020

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$$\frac{\psi_i^{n+1} - \psi_i^n}{\Delta t} = -\begin{cases} u\frac{\psi_i - \psi_{i-1}}{\Delta x} & u > 0 \\ u\frac{\psi_{i+1} - \psi_i}{\Delta x} & u < 0 \end{cases}$$

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$$\psi_i^{n+t} = \psi_i^n - \underbrace{u\frac{\Delta t}{\Delta x}}_{C} \begin{cases} \psi_i - \psi_{i-1} & u > 0 \\ \psi_{i+1} - \psi_i & u < 0 \end{cases}$$

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$$\psi_i^{n+1} = \psi_i^n - \left[\mathrm{flux}\left(\psi_i^n, \psi_{i+1}^n, C\right) - \mathrm{flux}\left(\psi_{i-1}^n, \psi_i^n, C\right)\right]$$

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$$\mathrm{flux}(\psi_l, \psi_r, C) = \tfrac{C+|C|}{2}\psi_l + \tfrac{C-|C|}{2}\psi_r$$

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In [2]:
```
def flux(psi_l, psi_r, C):
    return .5 * (C + abs(C)) * psi_l + \
           .5 * (C - abs(C)) * psi_r
```

In [3]:
```
def upwind(psi, i, C):
    return psi[i] - flux(psi[i    ], psi[i+one], C[i]) + \
                    flux(psi[i-one], psi[i    ], C[i-one])
```

# not previously on: modified equation analysis (1/3)

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$$\psi_i^{n+1} = \psi_i^n - \left[\frac{C+|C|}{2}(\psi_i^n - \psi_{i-1}^n) + \frac{C-|C|}{2}(\psi_{i+1}^n - \psi_i^n)\right]$$

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$$\begin{aligned}
\psi_i^{n+1} &= \psi_i^n + \partial_t\psi|_i^n \,\Delta t + \partial_t^2\psi\Big|_i^n \,{}^{(\Delta t)^2}\!/_2 + \ldots \\
\psi_{i+1}^n &= \psi_i^n + \partial_x\psi|_i^n \,\Delta x + \partial_x^2\psi\Big|_i^n \,{}^{(\Delta x)^2}\!/_2 + \ldots \\
\psi_{i-1}^n &= \psi_i^n + \partial_x\psi|_i^n \,(-\Delta x) + \partial_x^2\psi\Big|_i^n \,{}^{(-\Delta x)^2}\!/_2 + \ldots
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\end{aligned}$$

$$\begin{aligned}
\partial_t\psi\Delta t + \partial_t^2\psi\frac{(\Delta t)^2}{2} &= \\
= -\Bigg[\frac{C+|C|}{2}\left(-\partial_x\psi(-\Delta x) - \partial_x^2\psi\frac{(-\Delta x)^2}{2}\right) &+ \\
\frac{C-|C|}{2}\left(\partial_x\psi\Delta x + \partial_x^2\psi\frac{(\Delta x)^2}{2}\right)\Bigg]
\end{aligned}$$

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$$\partial_t \psi + \partial_t^2 \psi \frac{\Delta t}{2} = -\frac{u + |u|}{2} \left( \partial_x \psi - \partial_x^2 \psi \frac{\Delta x}{2} \right) - \frac{u - |u|}{2} \left( \partial_x \psi + \partial_x^2 \psi \frac{\Delta x}{2} \right)$$

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$$\partial_t \psi = -u \partial_x \psi \quad \rightsquigarrow \quad \partial_t^2 \psi = -u \partial_x \partial_t \psi = u^2 \partial_x^2 \psi$$

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$$\partial_t \psi + u \partial_x \psi + \underbrace{\left( u^2 \frac{\Delta t}{2} - |u| \frac{\Delta x}{2} \right)}_{\text{numerical diff. coeff.}} \partial_x^2 \psi + \ldots = 0$$

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$$\partial_t\psi + \partial_x\underbrace{(u\psi)}_{\text{flux}} + \partial_x\underbrace{\left(u^2\frac{\Delta t}{2} - |u|\frac{\Delta x}{2}\right)\frac{\partial_x\psi}{\psi}}_{\text{diffusive velocity}}\psi + \ldots = 0$$

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antidiffusive Courant number

$$C' = -\frac{\Delta t}{\Delta x}\left(u^2\frac{\Delta t}{2} - |u|\frac{\Delta x}{2}\right)\frac{\partial_x \psi}{\psi}$$

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$$C' \approx -\left(C^2\frac{\Delta x}{2} - |C|\frac{\Delta x}{2}\right)\frac{\psi^n_{i+1} - \psi^n_i}{\Delta x}\frac{2}{\psi^n_{i+1} + \psi^n_i}$$

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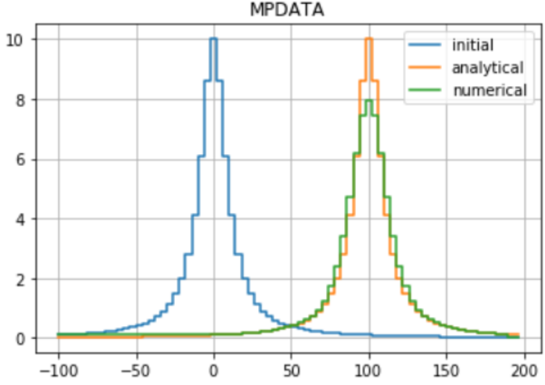
In [9]:
```
def C_corr(C, nx, psi):
    j = slice(0, nx-1)
    return (abs(C[j]) - C[j]**2) * (psi[j+one] - psi[j]) / (psi[j+one] + psi[j])
```

# previously on...

In [10]:
```
psi = psi_0(x)
for _ in range(nt):
    psi[i] = upwind(psi, i, C_phys)
    psi[i] = upwind(psi, i, C_corr(C_phys, nx, psi))
plot(x, psi, psi_0, 'MPDATA', v, nt)
```



attendance list!!!

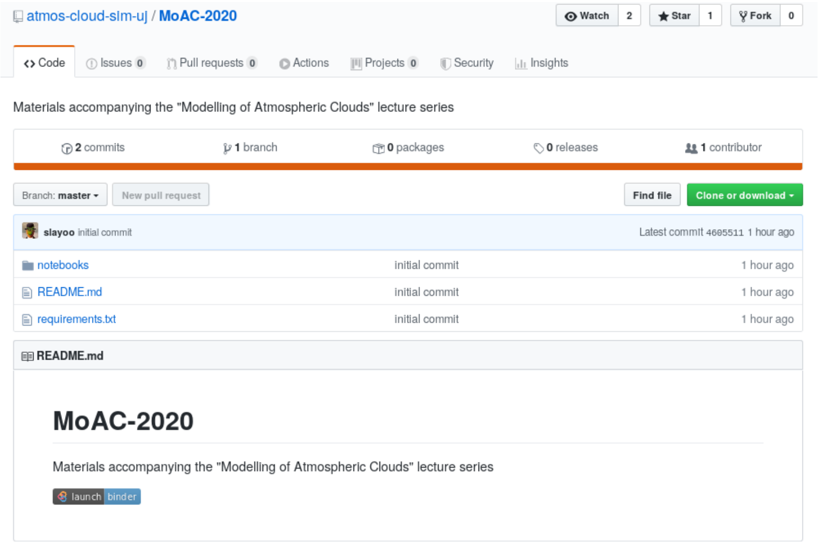
# job offer @ ECMWF

https://www.ecmwf.int/sites/default/files/vacancies/_VNVN20-06_en.pdf


## Analyst - Full Stack Web Developer (Python)
## Position information

| | |
|---|---|
| **Vacancy No.:** VN20-06 | **Department:** Forecasts |
| **Grade:** A2 | **Section:** Development |
| **Job Ref. No.:** STF-C/20-06 | **Reports to:** Team Leader |
| **Publication Date:** 28 February 2020 | **Closing Date:** 6 April 2020 |

# LAB: github.com/atmos-cloud-sim-uj/MoAC-2020




please clone or launch on binder and open notebooks/01_...

# LAB: plan

- MPDATA convergence analysis
- lognormal spectrum, mass vs. number distribution, PM2.5
- change of coordinates, advection, conservation

# seminarium 13-go marca

Równania Różniczkowe i Zagadnienia Pokrewne
13.03.2020 (piątek), godz. 12:15-13:45, sala 1016

Piotr Dziekan (UW)
Modelowanie zderzeń kropelek chmurowych: od opisu mikroskopowego do makroskopowego

Opis: Deszcz w atmosferze ziemskiej powstaje na skutek zderzeń kropelek chmurowych. Poprawne modelowanie zderzeń jest więc istotne dla numerycznych modeli chmur. W takich modelach stosuje się zazwyczaj uproszczony opis zderzeń, ze względu na ogromną liczbę kropelek w każdej chmurze. Przedstawione zostaną metody modelowania zderzeń o rożnym stopniu dokładności: od mikroskopowych, w których śledzone są trajektorie kropelek, poprzez mezoskopowe, oparte o opis statystyczny, po makroskopowe. Porównane zostaną wyniki uzyskane za pomocą tych metod oraz omówione zostaną niektóre nierozwiązane problemy dotyczące modelowania zderzeń kropelek chmurowych.

Referat na podstawie: Stochastic coalescence in Lagrangian cloud microphysics, Dziekan and Pawłowska (2017), Atmos. Chem. Phys., 17, https://doi.org/10.5194/acp-17-13509-2017.

Thank you for your attention!