Advanced radiation and remote sensing

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Exercise No. 6 – Inversion theory: Optimal Estimation Method (OEM)

In this exercise you will work with "realistic" data measured by a water vapor radiometer. The data is not real but has been simulated for a well-known atmospheric state using ARTS. Simulated measurements allow to compare retrieval results to the true atmospheric state. The radiometer (Fig. 1) measures thermal radiation in a frequency range around the 22 GHz water vapor absorption line. As the pressure broadening of absorption lines varies with height the measurement contains information about the vertical water vapor profile. This information can be retrieved using the "Optimal Estimation Method" (OEM). The radiometer is placed in 10 km height, which resembles an upward looking airborne measurement. The scarce concentration of water vapor in the stratosphere allows to perform a linear retrieval approach. Retrievals that cover the whole atmosphere, including the highly absorbent lower troposphere, need more advanced retrieval approaches like an iterative OEM.

1. Run the Jupyter Notebook `oem.ipynb` and plot the observed brightness temperature spectrum `y_measurement` as function of frequency `f_grid`.

2. Uncomment the function call `forward_model()` in the 6th cell from top and run the cell to simulate the brightness temperature spectrum `y` and the water vapor Jacobian `K` for the *a priori* state.

   (a) Add the simulated brightness temeperature spectrum to the plot of the observed brightness temperature spectrum.

   (b) Plot the Jacobians `K` in a suitable way. Explain the plot.

3. Plot the measurement covariance matrix `S_y` and the *a priori* covariance matrix `S_xa` in a suitable way. What do the covariance matrices mean?

4. Implement the function `retrieve()` according to the OEM solution:

$$\hat{\mathbf{x}} = \mathbf{x}_a + \left(\mathbf{K}^T\mathbf{S}_y^{-1}\mathbf{K} + \mathbf{S}_{xa}^{-1}\right)^{-1}\mathbf{K}^T\mathbf{S}_y^{-1}\left(\mathbf{y}_{measure} - \mathbf{y}_a\right) \tag{1}$$

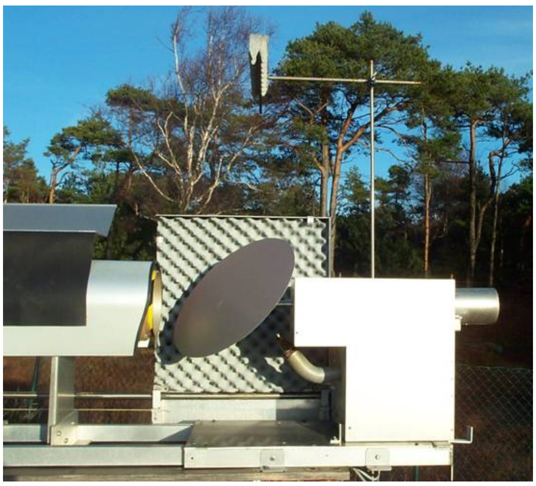

Figure 1: Onsala water vapor radiometer (photo: Peter Forkman)

with $\mathbf{x}_a$ the a priori profile, $\mathbf{K}$ the Jacobian, $\mathbf{S}_y$ the measurement covariance matrix, $\mathbf{S}_{xa}$ the *a priori* covariance matrix, $\mathbf{y}_{measure}$ the observed brightness temperature spectrum and $\mathbf{y}_a$ the simulated brightness temperature spectrum of profile $\mathbf{x}_a$.

In Python, a matrix `M` can be transposed using `M.T` and inversed using `inv(M)`[1]. Two matrices `M1` and `M2` can be multiplied using `M1 @ M2`.

5. Use the function `retrieve()` to retrieve the water vapor profile.
6. Plot the retrieved water vapor `x_oem` and the *a priori* water vapor profile as function of height `z`.
7. Load the true water vapor retrieval (`input/x_true.xml`) and add it to the previous plot. Dicuss the results.
8. Implement the function `averaging_kernel_matrix()` to calculate the same-named matrix:

$$\mathbf{A} = \left(\mathbf{K}^T\mathbf{S}_y^{-1}\mathbf{K} + \mathbf{S}_{xa}^{-1}\right)^{-1}\mathbf{K}^T\mathbf{S}_y^{-1}\mathbf{K} \tag{2}$$

9. Plot the kernels (columns) of $\mathbf{A}$ as function of height `z` and interpret the results.
10. The measurement response is defined as the sum over all averaging kernels in a given height (row). The measurement response indicates in which heights the measurement actually adds information to the retrieval result.
    (a) Calculate the measurement response and plot it together with the averaging kernels.
    (b) In which heights does the measurement provide useful information?
    (c) Is it possible to estimate the vertical resolution?

[1]We are using the inverse function `scipy.linalg.inv()` provided by the SciPy package.