Floquet Wannier-Stark brief

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STATEMENT OF PROMBLEM

Consider a time-independent Hamiltonian operator $H_0 = \varepsilon_a$, which forms the static part of a time-dependent Hamiltonian $\mathcal{H}(t) = H_0 + H(t)$. H_0 has only one single eigenstate $|a\rangle$ corresponding to the only eigenvalue ε_a, i.e. the Hilbert space of H_0 is one-dimensional. $H(t) = 2\gamma\cos(\omega t)$ depends periodically on time, i.e. $H(t) = H(t+T)$. Here $\omega T = 2\pi$.

Now we are to solve for the time-dependent Shrödinger equation

$$i\partial_t \psi(t) = H(t)\psi(t) \tag{1}$$

We tackle this issue using Floquet Approach. The Floquet Hamiltonian writes

$$\begin{pmatrix}
\ddots & \ddots & & & & & \\
\ddots & \varepsilon_a + 2\omega & \gamma & 0 & 0 & 0 & \\
 & \gamma & \varepsilon_a + \omega & \gamma & 0 & 0 & \\
 & 0 & \gamma & \varepsilon_a & \gamma & 0 & \\
 & 0 & 0 & \gamma & \varepsilon_a - \omega & \gamma & \\
 & 0 & 0 & 0 & \gamma & \varepsilon_a - 2\omega & \ddots \\
 & & & & & \ddots & \ddots
\end{pmatrix}$$

In principle, it's infinite dimensional. However, we expect a convergent series of truncated Floquet Hamiltonian matrices which would give good numerical results, to quite high precision, on the region of quasibands and corresponding wave functions we focus on.

This is based on the Wannier-Stark localization argument.

Of course, we expect the rate of convergence depends on dimensionless parameters ω/ε, γ/ε_a.

CONCLUSION

1. We find indeed convergence of the series of Floquet matrices with respect to the truncation of dimensions of matrices of n.

2. Rate of convergence depends on γ/ω.

3. To the region we interested in, namely the midest quasiband[1] $(\varepsilon_a - \omega/2, \varepsilon_a + \omega/2)$, Floquet matrices truncated to $n \gtrsim \mathcal{O}[10(\gamma/\omega)]$ (i.e., one order higher than the order of γ/ω) would give convergent results to quite high precision. In addition, the wave function corresponding to the midest quasienergy are seen to be localized within number of sites of roughly also this order.

RESULTS

We calculated several cases for different values of parameters ω and γ. Three things are considered: quasienergy differences, overlap of wave functions and also the behavior of wave function. As an specific example, we showed below the case that $\omega = 5.6$, $\gamma = 28.8$. ($\omega/\gamma = 5.14$)

1. We look at the middlemost 7 quasibands.

TABLE I: quasi energy spectrum (in unit of ε_a)

n	5	8	10	12	15	18	20	30	in principle
$\vdots$									
ε_3	40.509	26.9219	21.9183	19.1083	17.8599	17.8003	17.8	17.8	17.8
ε_2	28.2457	18.2618	14.6279	12.7847	12.2132	12.2	12.2	12.2	12.2
ε_1	15.1094	9.6966	7.7195	6.7990	6.6023	6.6	6.6	6.6	6.6
ε_0	1.	1.	1.	1.	1.	1.	1.	1.	1
ε_{-1}	-13.1094	-7.6966	-5.7195	-4.7990	-4.6023	-4.6	-4.6	-4.6	-4.6
ε_{-2}	-26.2457	-16.2618	-12.6279	-10.7847	-10.2132	-10.2	-10.2	-10.2	-10.2
ε_{-3}	-38.509	-24.9219	-19.9183	-17.1083	-15.8599	-15.8003	-15.8	-15.8	-15.8
$\vdots$									

In principle, the quasi energy spectrum should be rigorously periodic in energy.

2. We evaluate the overlap of wave function with a previous one when we enlarge the matrix truncation. Wave funciton considered here is the one corresponding to the middlemost quasiband, namely $\Psi_n^{(0)}$. Subscript denotes the truncation.

TABLE II: overlap of $\Psi_n^{(0)}$ with a previous one $\Psi_{n-1}^{(0)}$

n	5	8	10	12	15	18	20	30	in principle
$\langle\Psi_{n-1}^{(0)}\vert\Psi_n^{(0)}\rangle$	0.524	0.795	0.934	-0.993	-0.999	-1.	1.	1.	1

3. The series of truncated wave functions are seen to be converging to a localised one. Plotting of wave function as well as modulus square of wave function are shown below.

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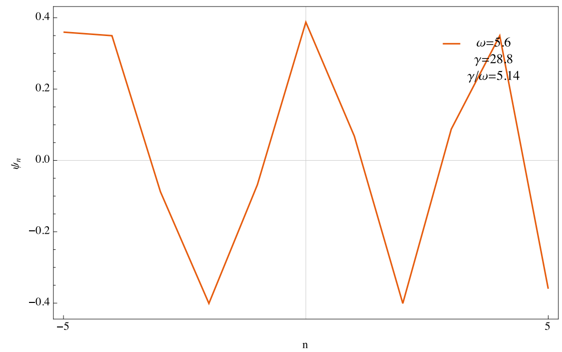
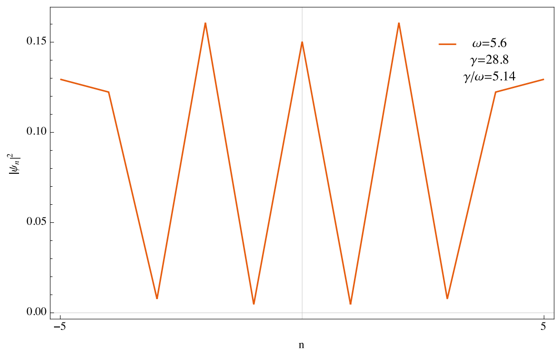
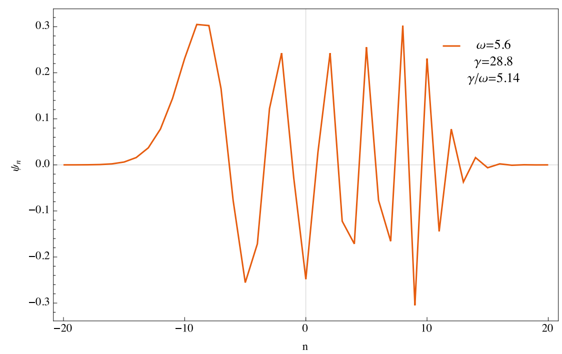
[1] Actually it is midest several quasibands more rigorously.



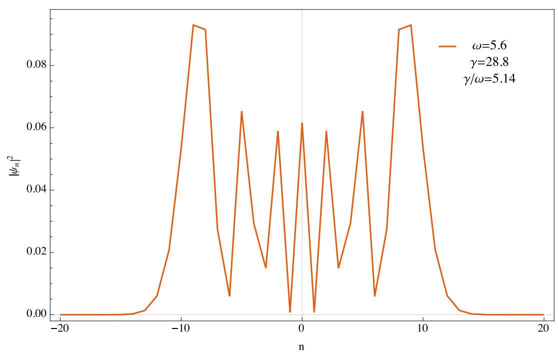
(a)Ψ_l, $l = (-5, 5)$



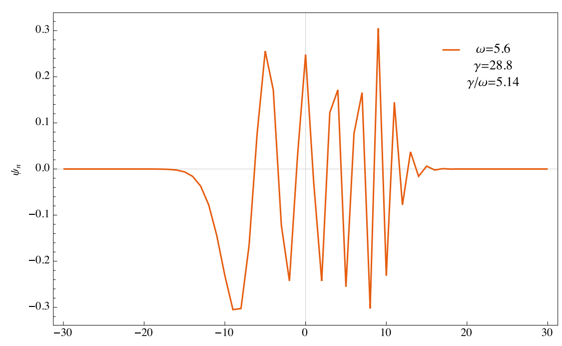
(b)$|\Psi_l|^2$, $l = (-5, 5)$



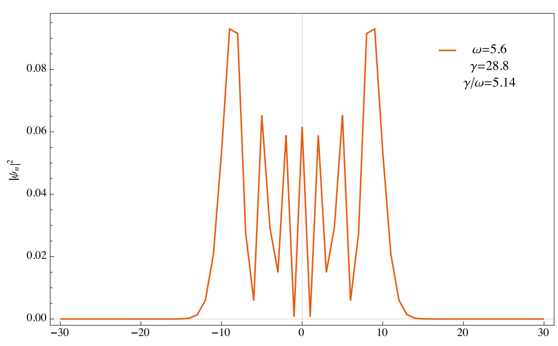
(c)Ψ_l, $l = (-20, 20)$



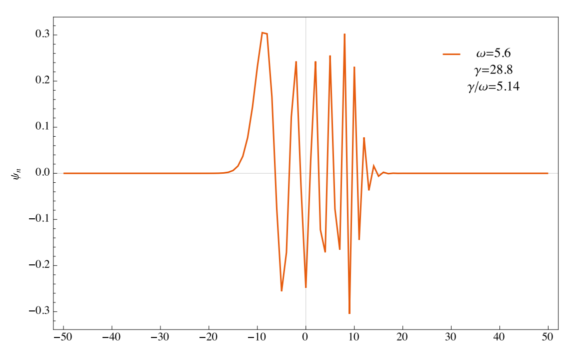
(d)$|\Psi_l|^2$, $l = (-20, 20)$



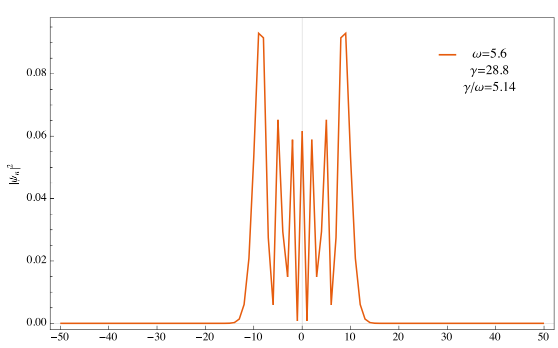
(e)Ψ_l, $l = (-30, 30)$



(f)$|\Psi_l|^2$, $l = (-30, 30)$



(g)Ψ_l, $l = (-50, 50)$



(h)$|\Psi_l|^2$, $l = (-50, 50)$

FIG. 1: $\omega = 5.6$, $\gamma = 28.8$, $\gamma/\omega = 5.14$, convergent $n \gtrsim 30$