Two Experiments

Aharonov-Bohm interferometry and Wilson lines

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Outline

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An Aharonov-Bohm interferometer for determining Bloch band topology

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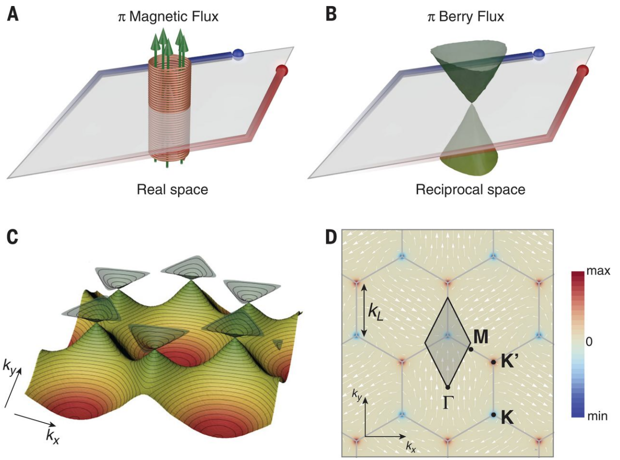

Berry connection and Berry curvature

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- Bloch state

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$$\mathbf{A}_n(\mathbf{k}) = \mathrm{i}\langle u^n_{\mathbf{k}} | \nabla_{\mathbf{k}} | u^n_{\mathbf{k}} \rangle$$

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- Berry phase

$$\phi_{\text{Berry}} = \oint_C \mathbf{A}_n(\mathbf{k}) d\mathbf{k} = \int_S \Omega_n(\mathbf{k}) d^2\mathbf{k}$$

Hexagonal lattice in real space

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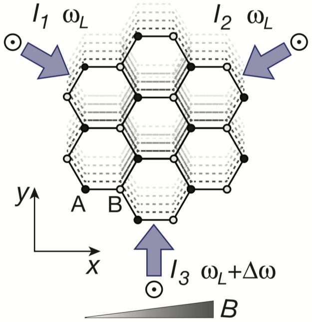
Magnetic field $B = B_0 + \mathbf{r} \cdot \nabla B$ combined with an orthogonal acceleration $\mathbf{a} \perp \nabla B$ of the lattice:



The Hamiltonian is

$$H(t) = \frac{\mathbf{p}^2}{2m} + V[\mathbf{r} - \mathbf{R}] - \mu \mathbf{r} \cdot \nabla B - \mu B_0$$

in co-moving frame

$$H(t) = \frac{\mathbf{p}^2}{2m} + V(\mathbf{r}) - \mathcal{F}_\mu \cdot \mathbf{r} + \varepsilon_\mu(t)$$

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Zeeman energy $\varepsilon_\mu(t) = -\mu[\mathbf{R}(t) \cdot \nabla B + B_0]$

$\mathcal{F}_\mu$: $\quad \mathbf{k} \to \mathbf{k} + \mathcal{F}_\mu t$

Evolution of a state in such a lattice

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Hamiltonian

$$H = H_0 - \mathcal{F}_\mu \cdot \mathbf{r}$$

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Bloch state

$$H_0 \psi_{\mathbf{k}}^n(\mathbf{r}) = E_n(\mathbf{k}) \psi_{\mathbf{k}}^n(\mathbf{r})$$
$$\psi_{\mathbf{k}}^n(\mathbf{r}) = e^{\mathbf{ik}\cdot\mathbf{r}} u_{\mathbf{k}}^n(\mathbf{r})$$

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Ansatz

$$\tilde{\Psi}(t) = e^{\mathrm{i}\eta(t)} \psi^n_{k_0 + \mathcal{F}_\mu t}$$
$$\eta = \phi_{\mathsf{dyn}} + \phi_{\mathsf{Berry}}$$

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Phase

$$\phi_{\text{dyn}} = \int_0^T [E_1(\mathbf{k} + \mathcal{F}_\mu t) + \varepsilon_\mu t] dt$$
$$\phi_{\text{Berry}} = \mathrm{i} \int_C \langle u^1_{\mathbf{k}} | \nabla_{\mathbf{k}} | u^1_{\mathbf{k}} \rangle d\mathbf{k}$$

Experiment procedure

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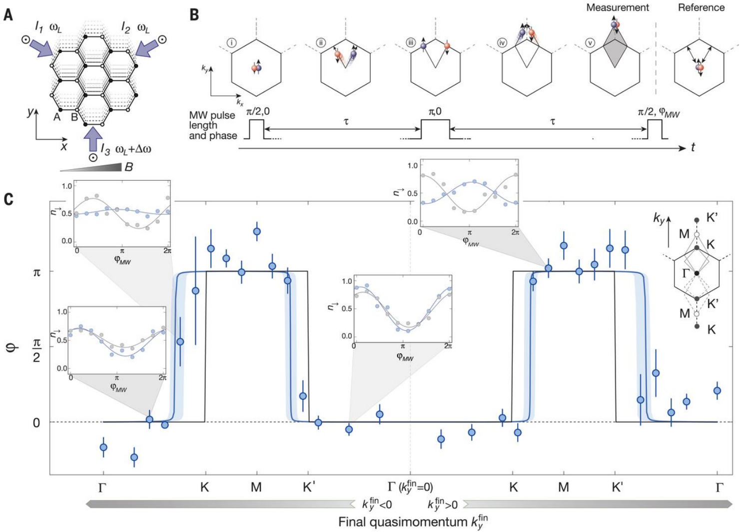
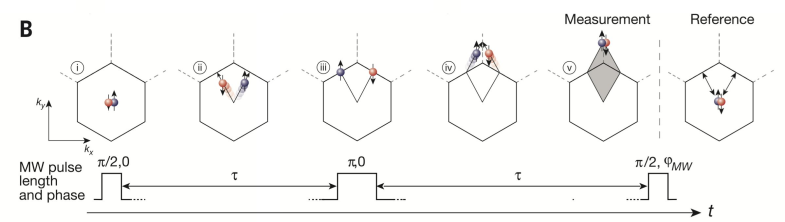
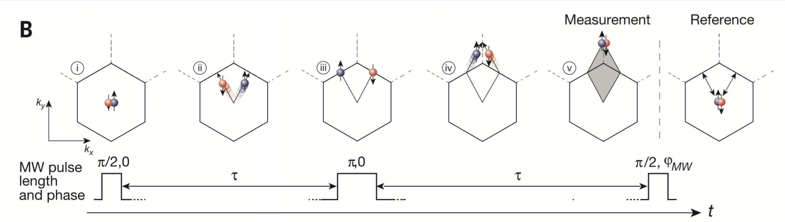



B
Measurement
Reference
i
ii
iii
iv
v
k_y
k_x
MW pulse length and phase
π/2,0
π,0
π/2, φ_{MW}
τ
τ
t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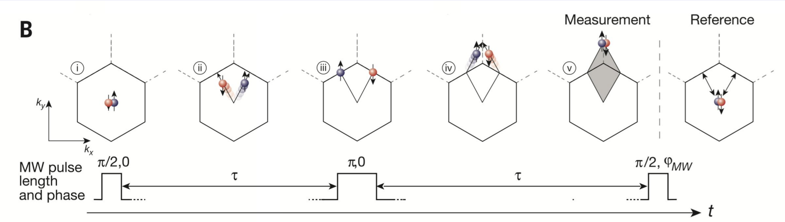
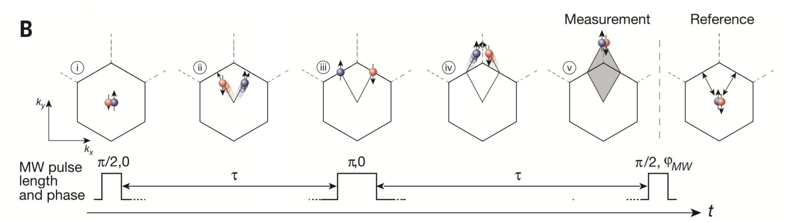


1. ^{87}Rb BEC initial state $|\uparrow\rangle = |F=2, m_F=1\rangle$; $\pi/2$-microwave pulse;



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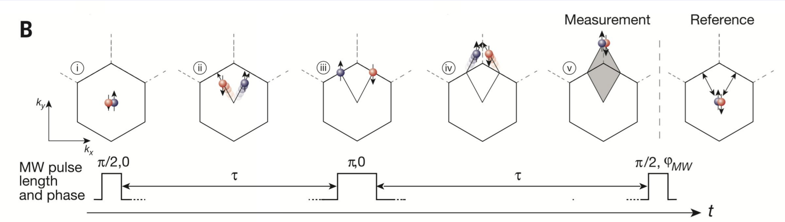

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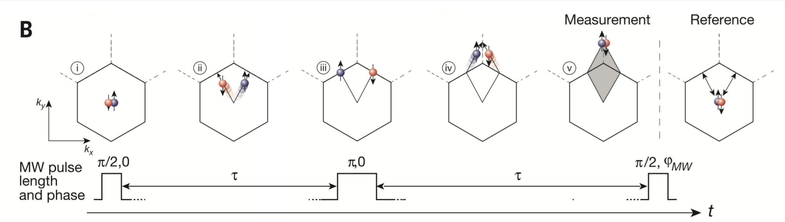

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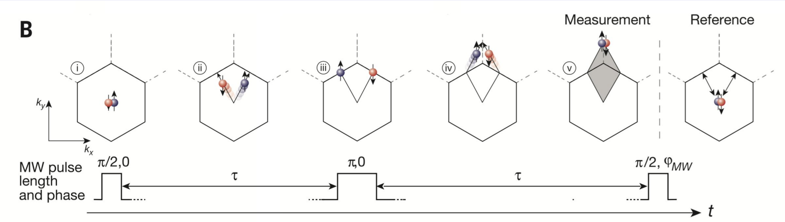

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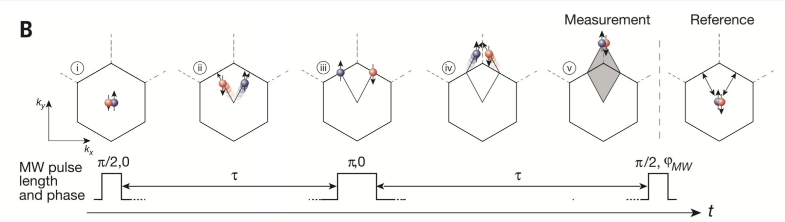

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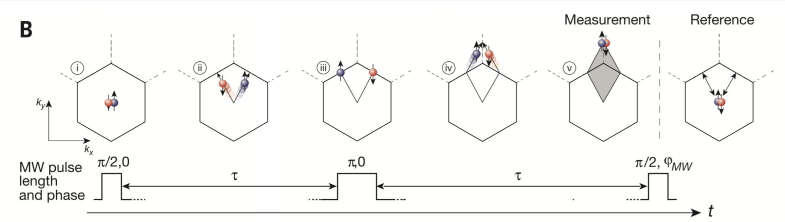
zero-area reference:

V-shape path.



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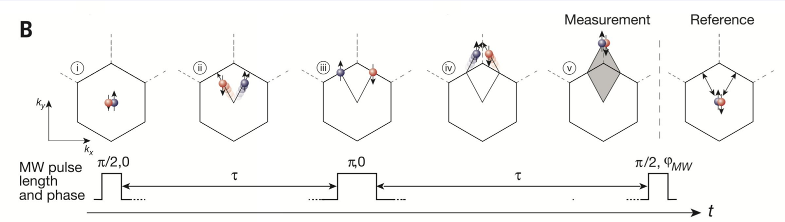
Reversing the lattice acceleration after π pulse.



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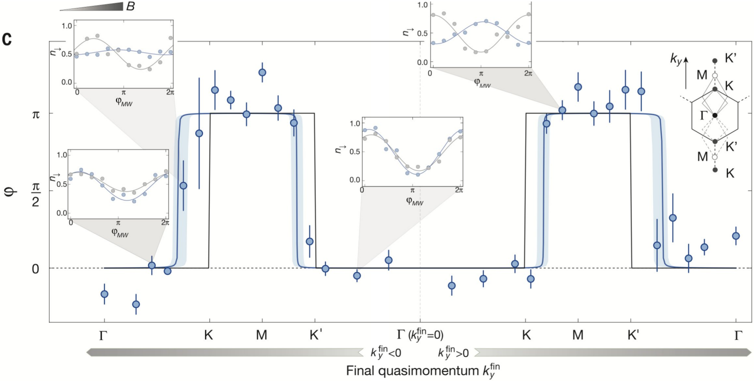
Reversing the lattice acceleration after π pulse.

$$n_{\uparrow,\downarrow} \propto 1 \pm \cos(\varphi + \varphi_{MW})$$

Main result I



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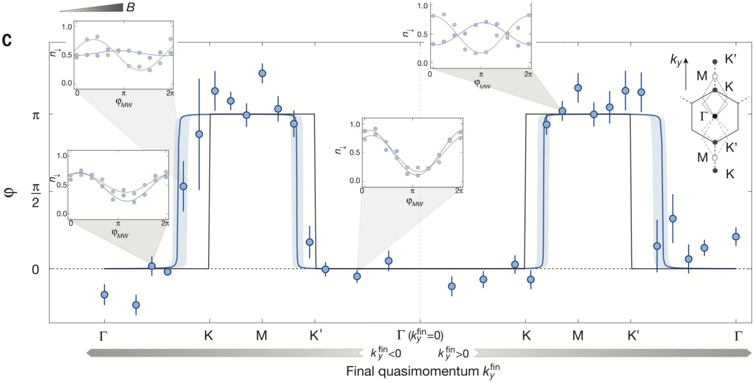

- Broadening of the edges — caused by momentum spread.

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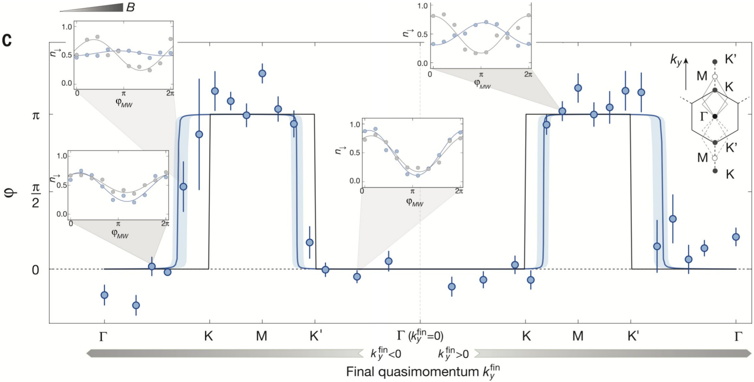

- Broadening of the edges — caused by momentum spread.
- Systematic errors.

Self-referenced interferometry at Dirac point

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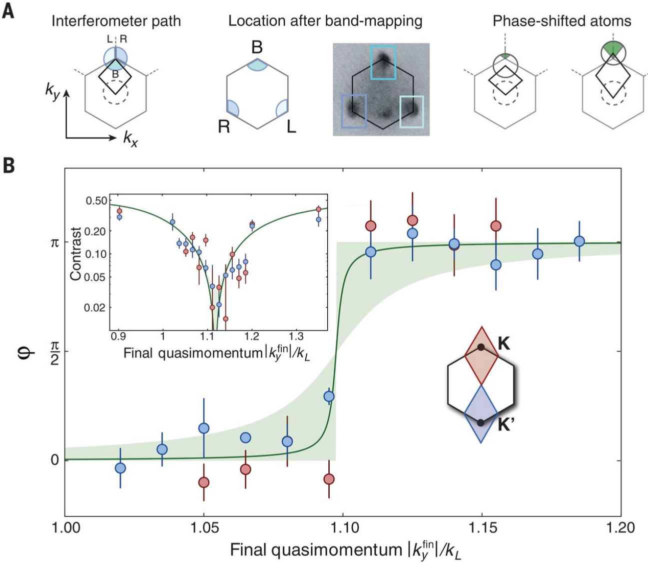
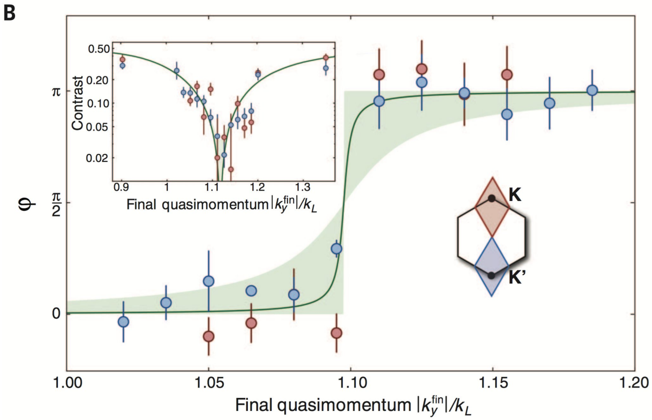


Main result II



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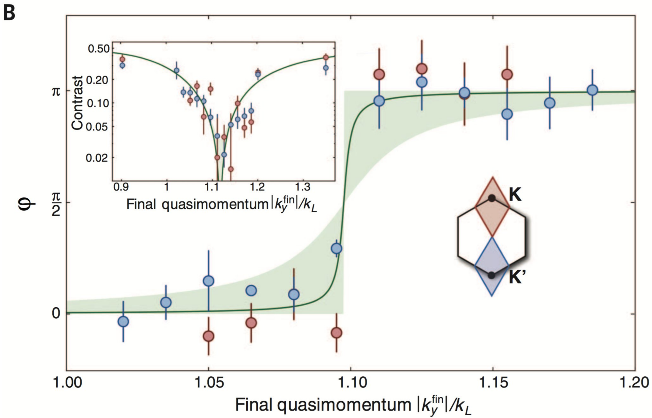

- Contrast: $(n_\downarrow^{\max} - n_\downarrow^{\min})/(n_\downarrow^{\max} + n_\downarrow^{\min})$

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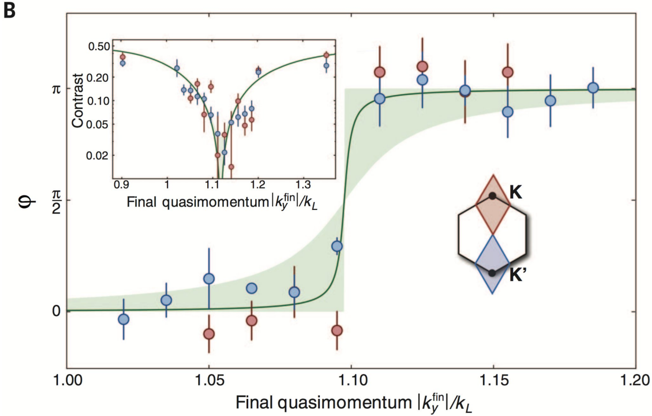
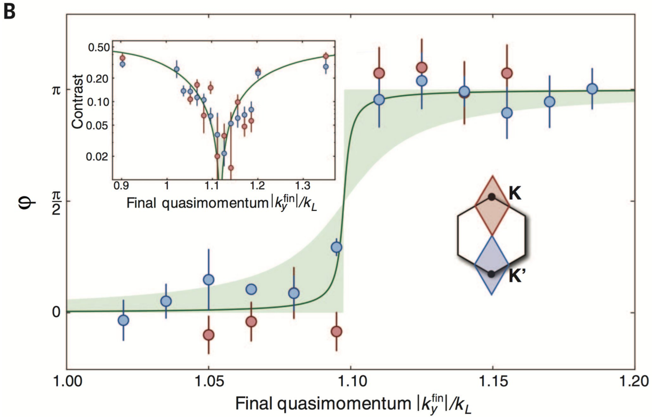


- Contrast: $(n_\downarrow^{\max} - n_\downarrow^{\min})/(n_\downarrow^{\max} + n_\downarrow^{\min})$
- $\varphi = (\varphi_L + \varphi_R)/2 - \varphi_B$

Main result II



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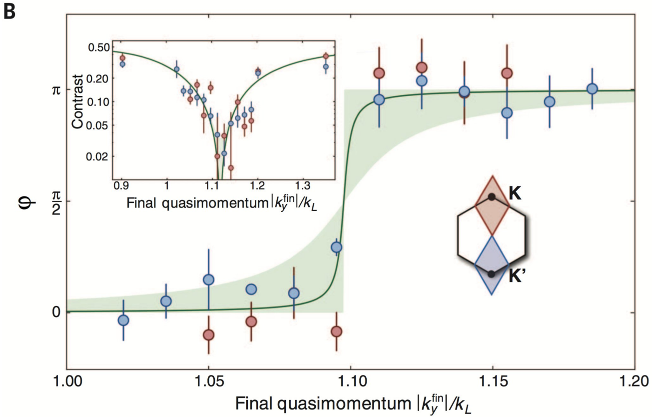

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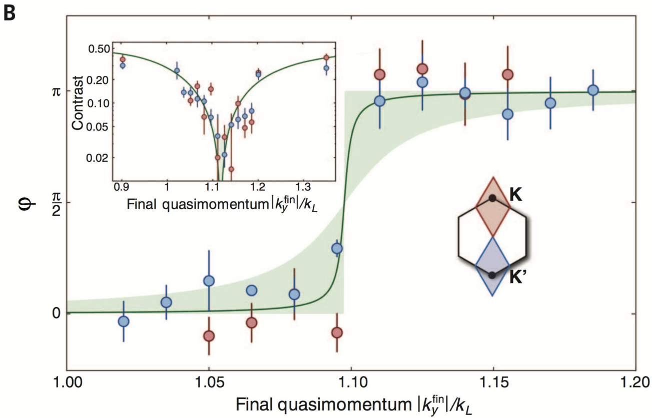

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- $\varphi = (\varphi_L + \varphi_R)/2 - \varphi_B = 0.95(10)\pi$
- Berry curvature localization $\delta k_w \simeq 10^{-4} k_L (\Delta \simeq h \times 3\text{Hz})$

Imbalance lattice mapping

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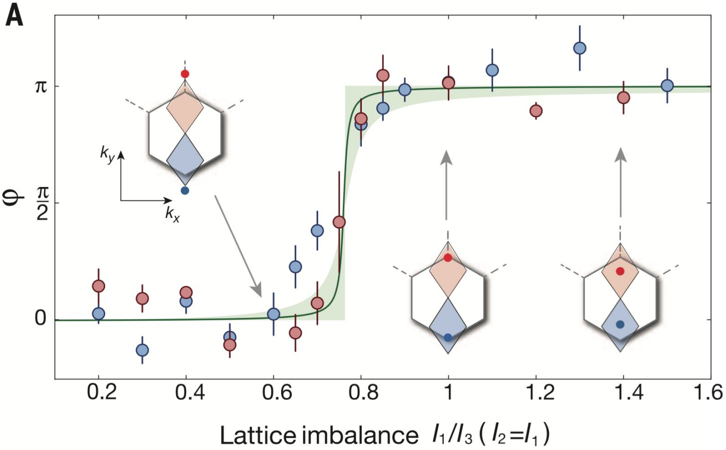

Imbalance lattice mapping

Self-referenced phase

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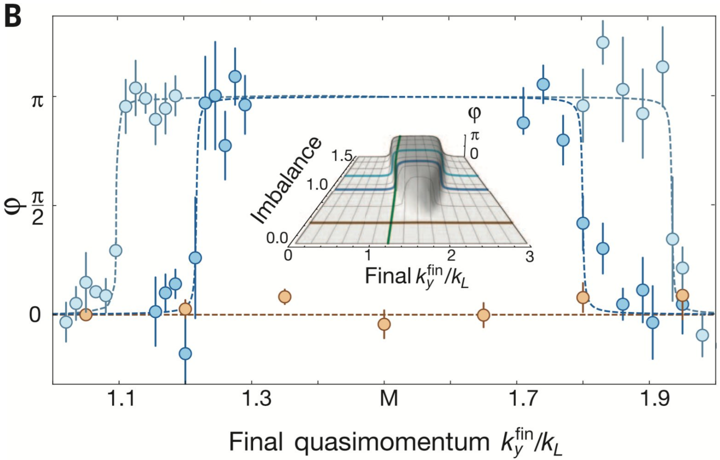
Self-referenced phase



ab initio calculation of imbalanced lattice

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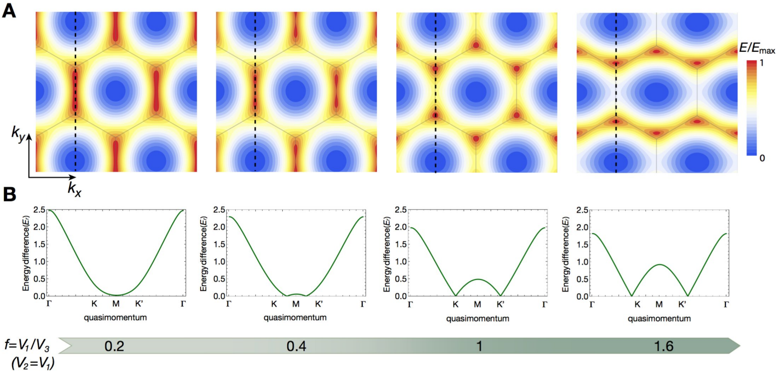

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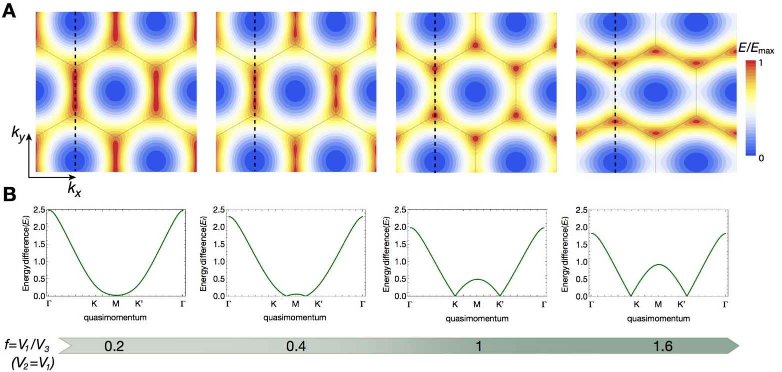
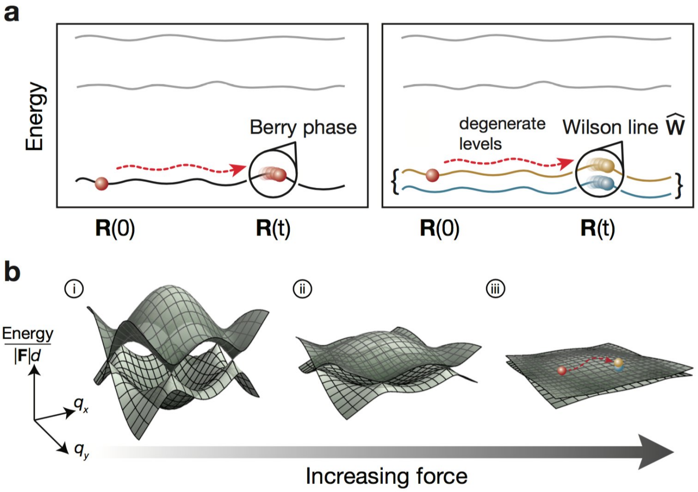


Seeing Dirac points annihilating clear.

Outline

Experimental reconstruction of Wilson lines in Bloch bands



Wilson line regime in honeycomb lattice

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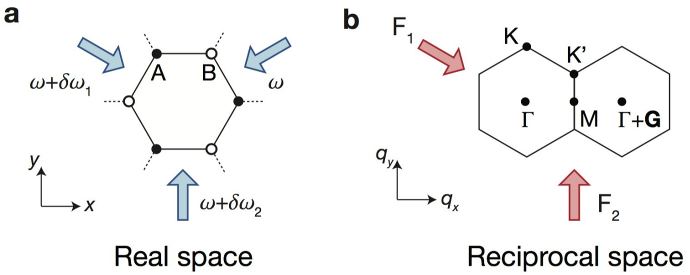

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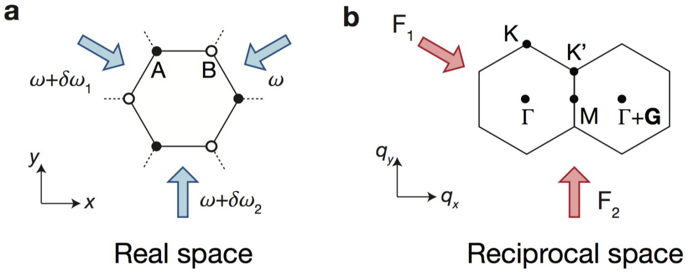
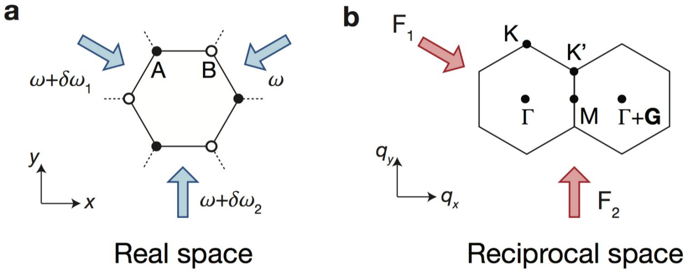


- force $\mathbf{F}$: $\quad \mathbf{q}(t) = \mathbf{q}(0) + \mathbf{F}t$

Wilson line regime in honeycomb lattice



- force $\mathbf{F}$: $\mathbf{q}(t) = \mathbf{q}(0) + \mathbf{F}t$
- unitary time-evolution operator (Wilson line matrix)

$$W_{\mathbf{q}(0)\to\mathbf{q}(t)} = \mathcal{P}\exp(\mathrm{i}\int_C \mathrm{A}_{\mathbf{q}} d\mathbf{q})$$

$\mathrm{A}_{\mathbf{q}}$: Wilczek-Zee connection
$\mathcal{P}$: Path-ordering (non-Abelian)

Wilson line

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- $n = n'$: Berry connections of individual Bloch bands

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  yields Berry phase along a closed path

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Wilson lines and Wilczek-Zee connections

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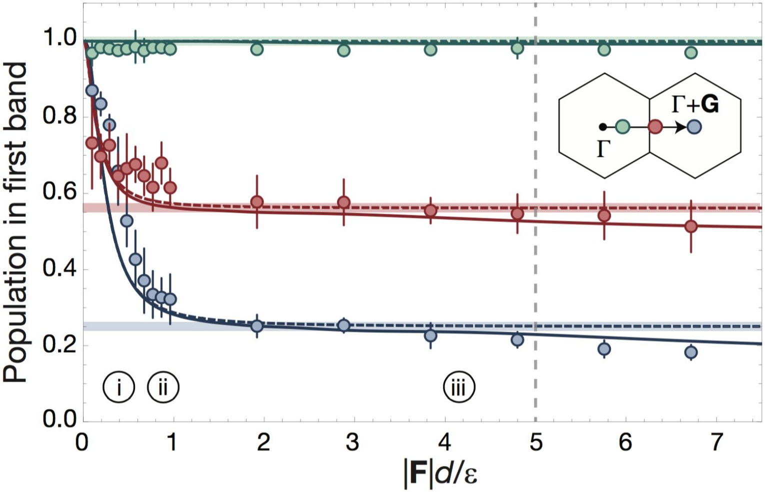
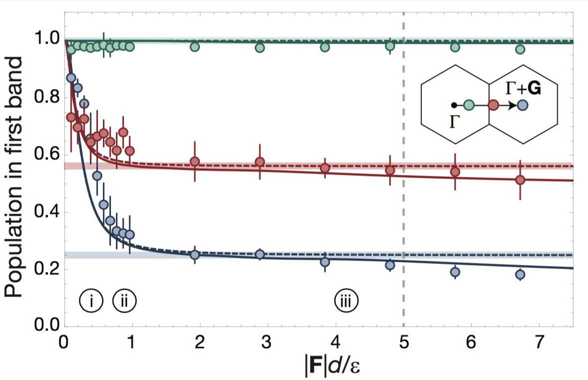
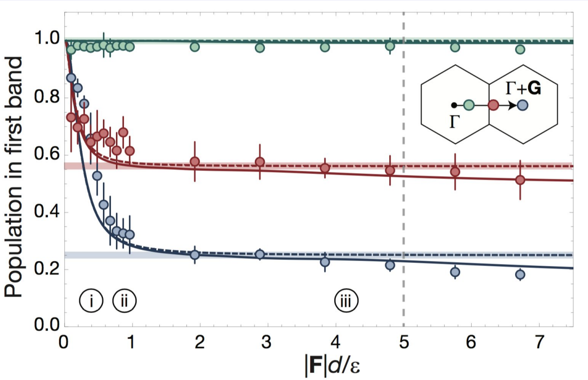
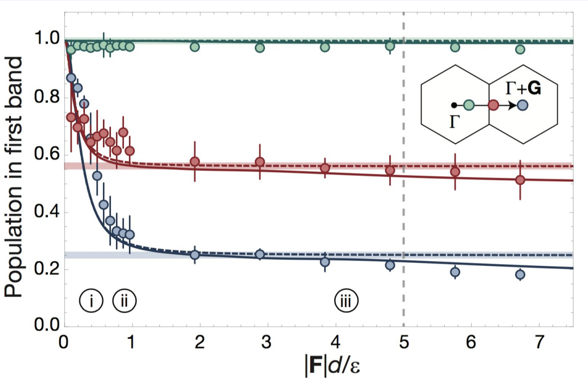
Measuring Wilson lines




Population in first band
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
7
|F|d/ε
Γ+G
Γ
i
ii
iii



- Matrix elements of Wilson line operator



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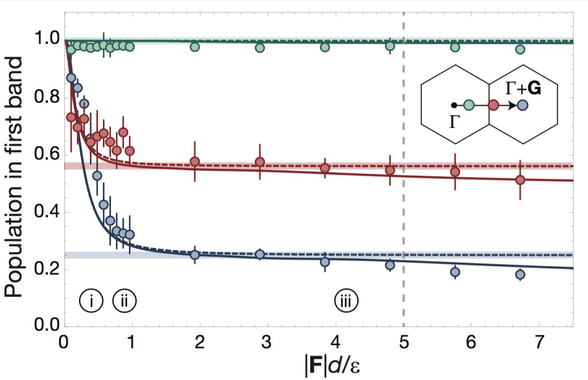

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- Throughout this work basis states are chosen at reference point $\mathbf{Q} = \Gamma$

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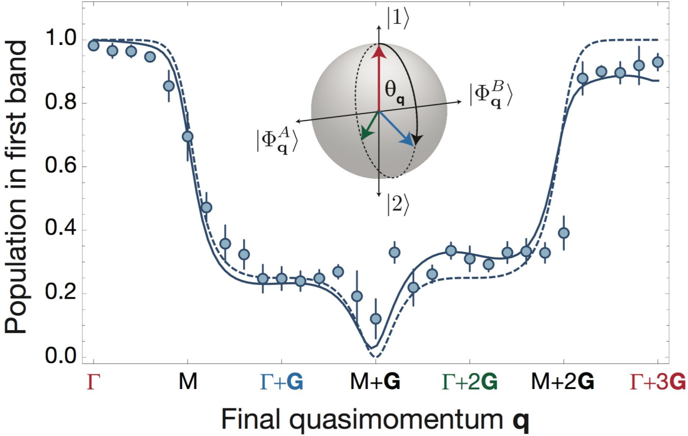

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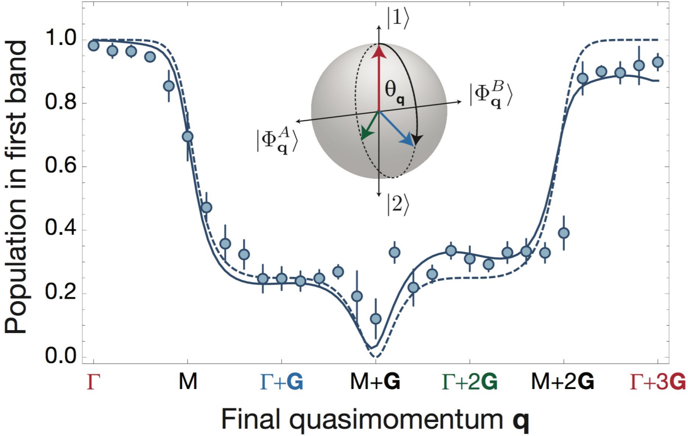

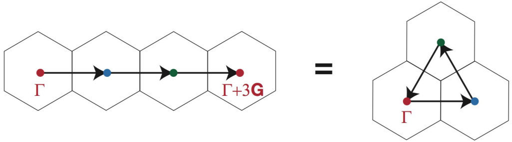

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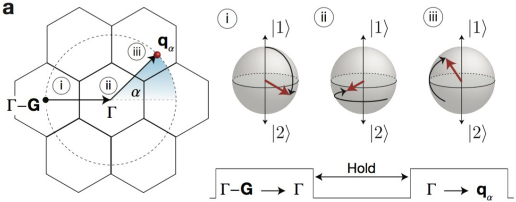

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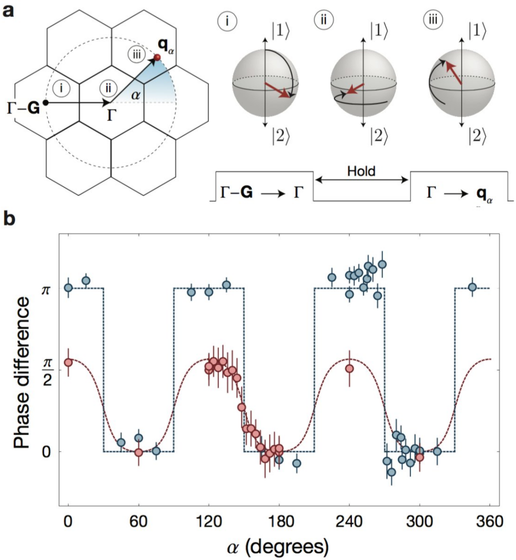
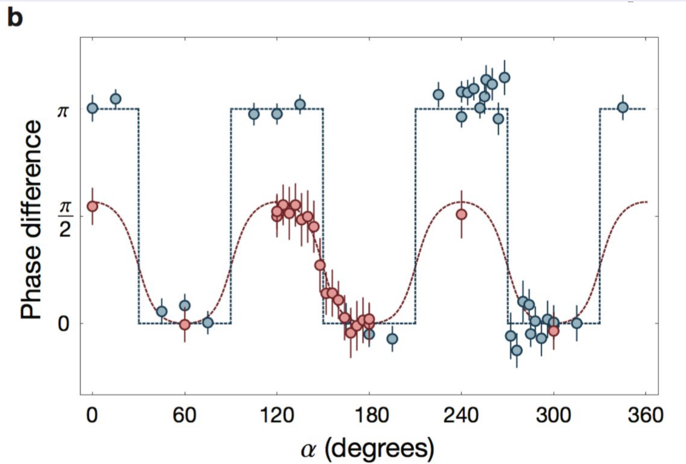
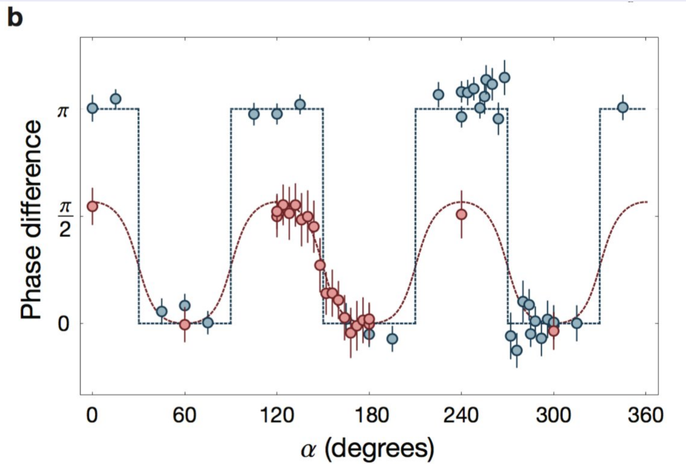



b
Phase difference
π
π/2
0
0
60
120
180
240
300
360
α (degrees)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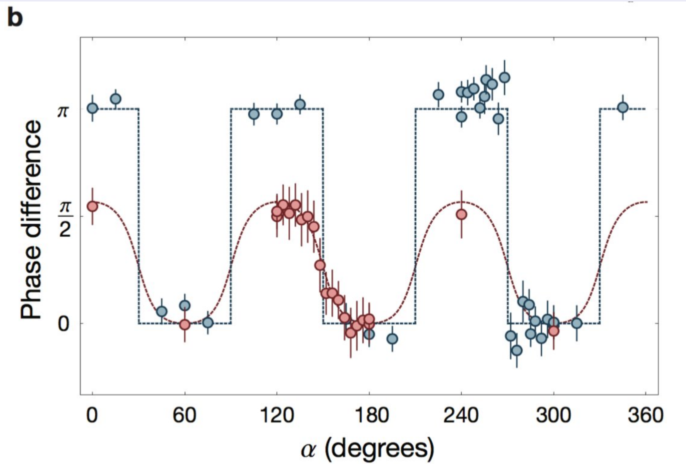
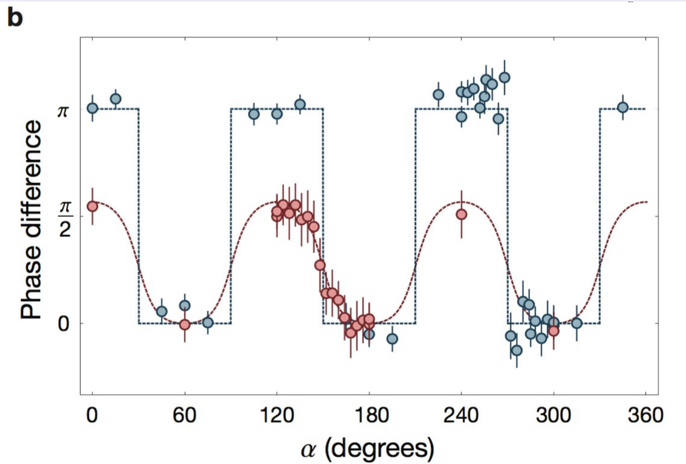


- AB-site degeneracy (blue) — π jump



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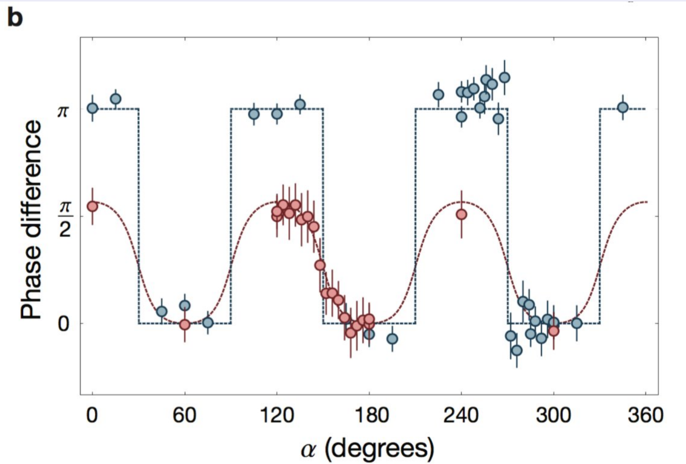
- AB-site offset (red) by elliptically-polarized lattice beam — continuous varying phase



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- 3-fold symmetry of system (no matter whether offset)

Determining Wilson line eigenvaluse

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Wilson-Zak loop

$$W_{\mathbf{q}\to\mathbf{q}+\mathbf{G}}$$

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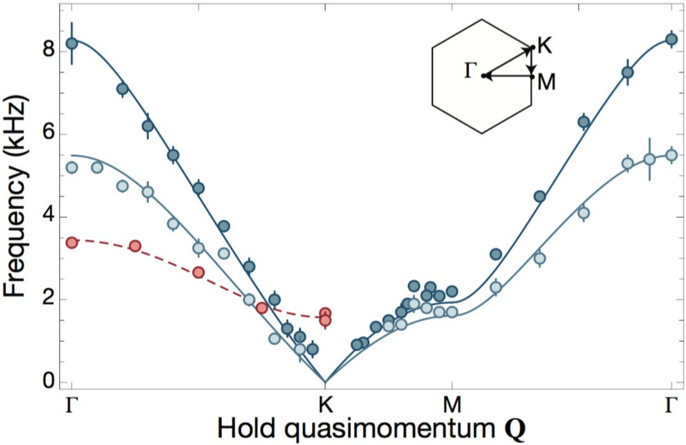
- experimental data analysis gives: $\xi = 1.03(2)\pi/3$
- theoretical expected value: $\xi = \pi/3$

Mapping dispersion relation



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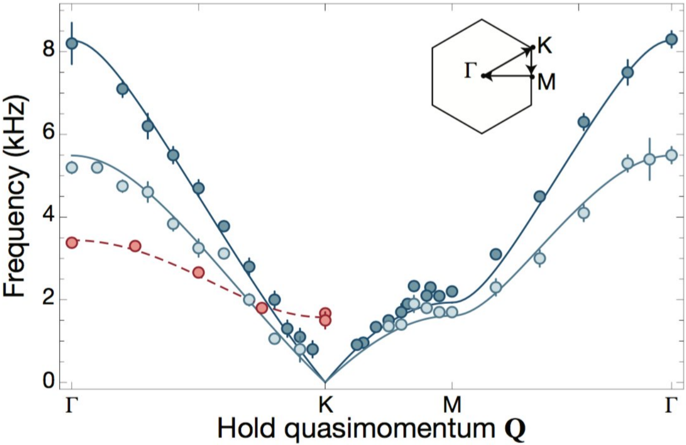

by varying the reference quasimomentum $\mathbf{Q}$

Reference

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- Immanuel Bloch *et al.*, *Experimental reconstruction of Wilson lines in Bloch bands*, arXiv:1509.02185v2 [cond-mat.quant-gas].
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Thank you !