# Floquet Wannier-Stark brief

Ning Sun

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## STATEMENT OF PROMBLEM

Consider a time-independent Hamiltonian operator $H_0 = \varepsilon_a$, which forms the static part of a time-dependent Hamiltonian $\mathcal{H}(t) = H_0 + H(t)$. $H_0$ has only one single eigenstate $|a\rangle$ corresponding to the only eigenvalue $\varepsilon_a$, i.e. the Hilbert space of $H_0$ is one-dimensional. $H(t) = 2\gamma\cos(\omega t)$ depends periodically on time, i.e. $H(t) = H(t+T)$. Here $\omega T = 2\pi$.

Now we are to solve for the time-dependent Shrödinger equation

$$i\partial_t\psi(t) = H(t)\psi(t) \tag{1}$$

We tackle this issue using Floquet Approach. The Floquet Hamiltonian writes

$$\begin{pmatrix}
\ddots & \ddots & & & & & \\
\ddots & \varepsilon_a + 2\omega & \gamma & 0 & 0 & 0 & \\
 & \gamma & \varepsilon_a + \omega & \gamma & 0 & 0 & \\
 & 0 & \gamma & \varepsilon_a & \gamma & 0 & \\
 & 0 & 0 & \gamma & \varepsilon_a - \omega & \gamma & \\
 & 0 & 0 & 0 & \gamma & \varepsilon_a - 2\omega & \ddots \\
 & & & & & \ddots & \ddots
\end{pmatrix}$$

In principle, it's infinite dimensional. However, we expect a convergent series of truncated Floquet Hamiltonian matrices which would give good numerical results, to quite high precision, on the region of quasibands and corresponding wave functions we focus on.

This is based on the Wannier-Stark localization argument.

Of course, we expect the rate of convergence depends on dimensionless parameters $\omega/\varepsilon$, $\gamma/\varepsilon_a$.

## CONCLUSION

1. We find indeed convergence of the series of Floquet matrices with respect to the truncation of dimensions of matrices of $n$.
2. Rate of convergence depends on $\gamma/\omega$.

3. To the region we interested in, namely the midest quasiband[1] $(\varepsilon_a - \omega/2, \varepsilon_a + \omega/2)$, Floquet matrices truncated to $n \gtrsim \mathcal{O}[10(\gamma/\omega)]$ (i.e., one order higher than the order of $\gamma/\omega$) would give convergent results to quite high precision. In addition, the wave function corresponding to the midest quasienergy are seen to be localized within number of sites of roughly also this order.

## RESULTS

We calculated several cases with different parameters of $\omega$ and $\gamma$. Results are shown below.

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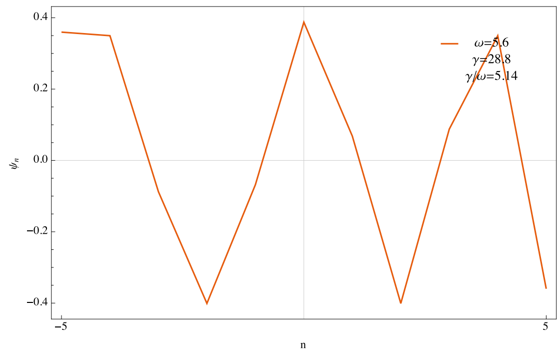
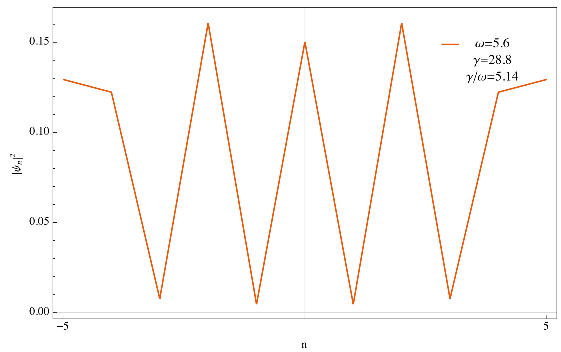
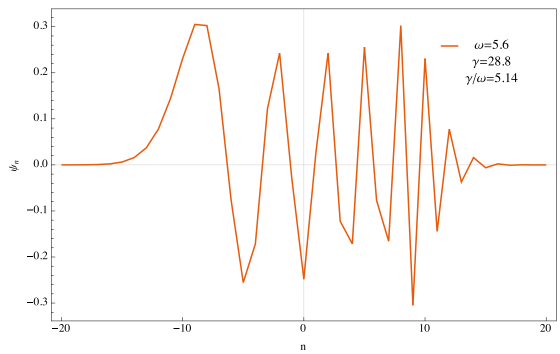
[1] Actually it is midest several quasibands more rigorously.



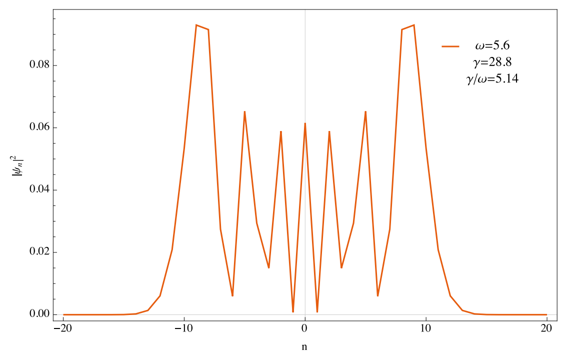

(a)$\Psi_l$, $\quad l = (-5, 5)$



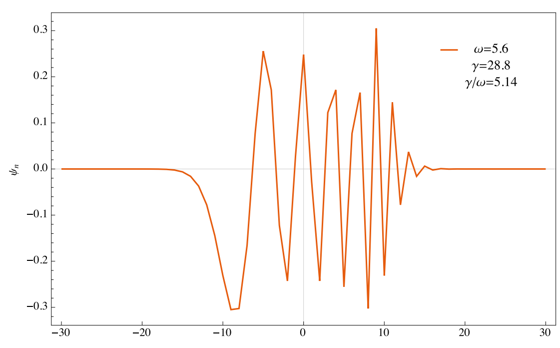

(b)$|\Psi_l|^2$, $\quad l = (-5, 5)$



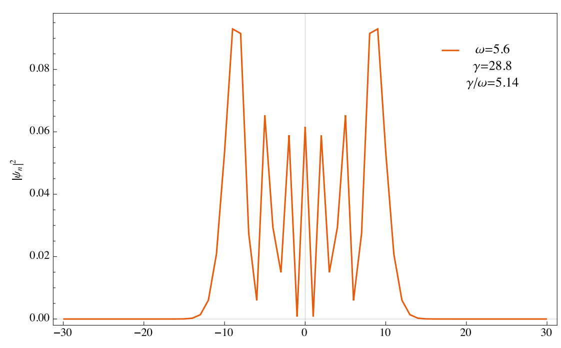

(c)$\Psi_l$, $\quad l = (-20, 20)$



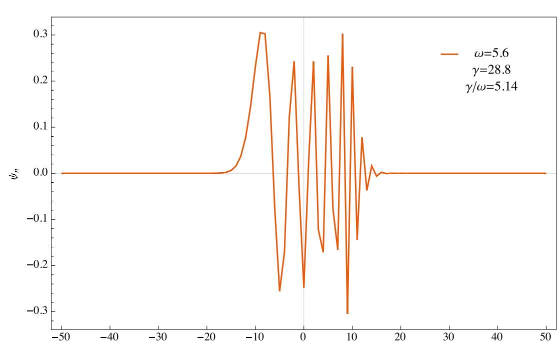

(d)$|\Psi_l|^2$, $\quad l = (-20, 20)$



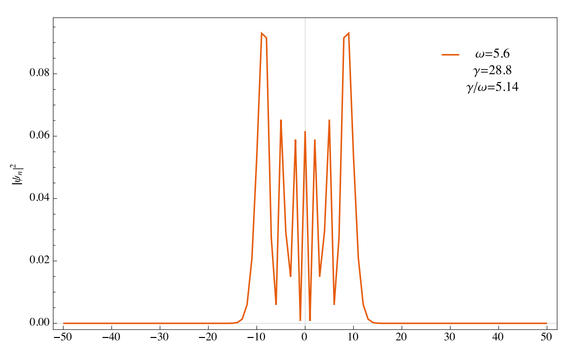

(e)$\Psi_l$, $\quad l = (-30, 30)$




(f)$|\Psi_l|^2$, $\quad l = (-30, 30)$

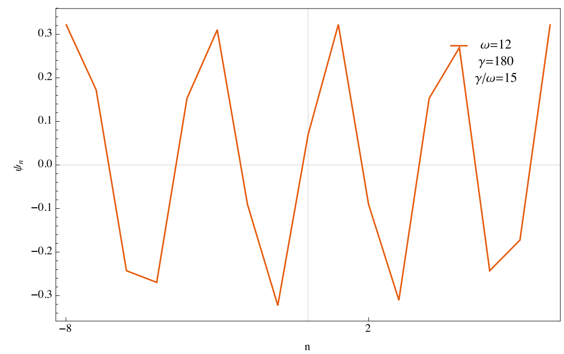



(g)$\Psi_l$, $\quad l = (-50, 50)$

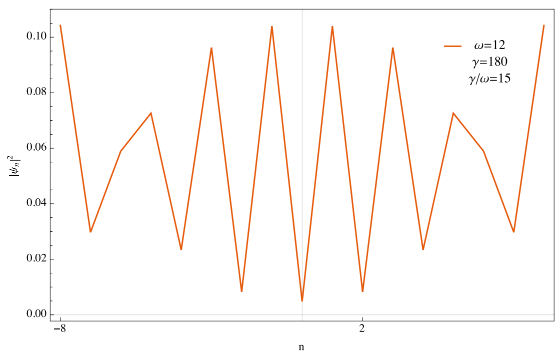



(h)$|\Psi_l|^2$, $\quad l = (-50, 50)$

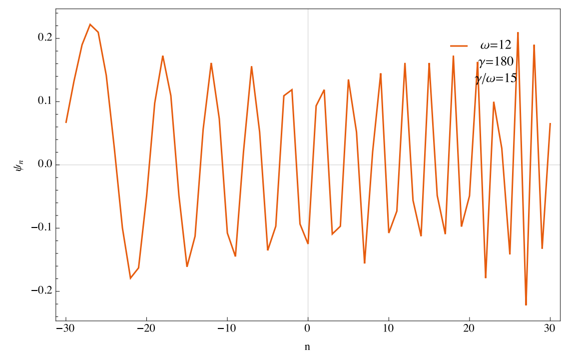
FIG. 1: $\omega = 5.6$, $\gamma = 28.8$, $\gamma/\omega = 5.14$, convergent $n \gtrsim 30$



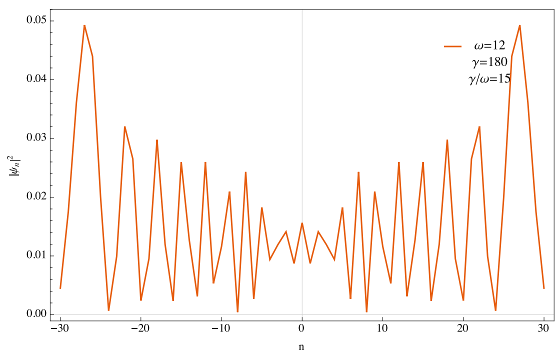

(a)$\Psi_l$, $\quad l = (-8, 8)$



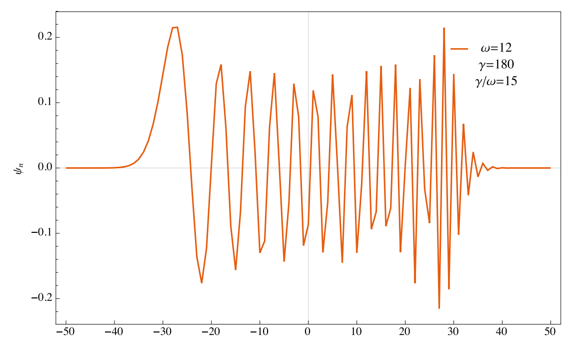

(b)$|\Psi_l|^2$, $\quad l = (-8, 8)$



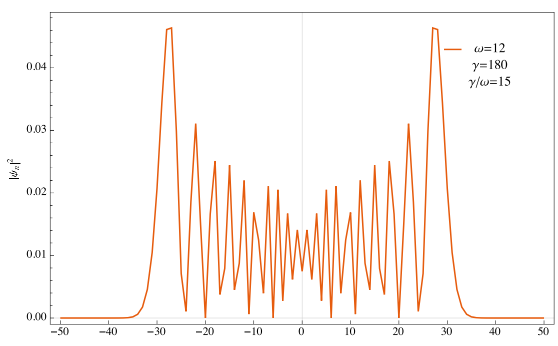

(c)$\Psi_l$, $\quad l = (-30, 30)$



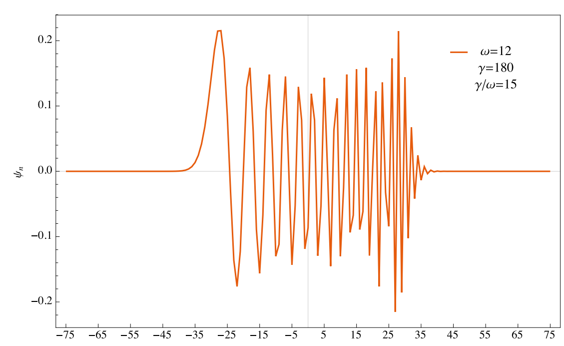

(d)$|\Psi_l|^2$, $\quad l = (-30, 30)$



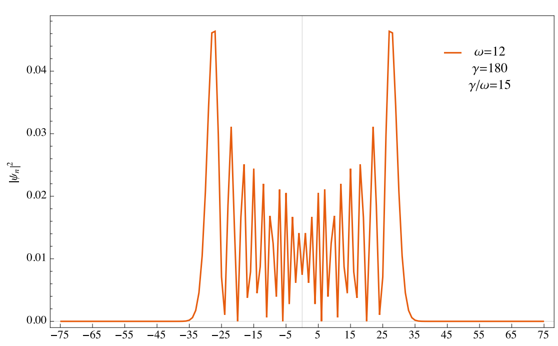

(e)$\Psi_l$, $\quad l = (-50, 50)$




(f)$|\Psi_l|^2$, $\quad l = (-50, 50)$

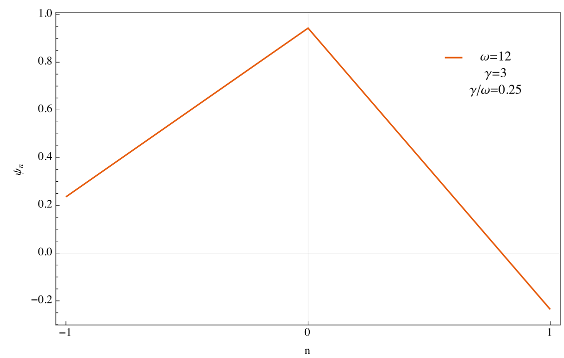



(g)$\Psi_l$, $\quad l = (-75, 75)$

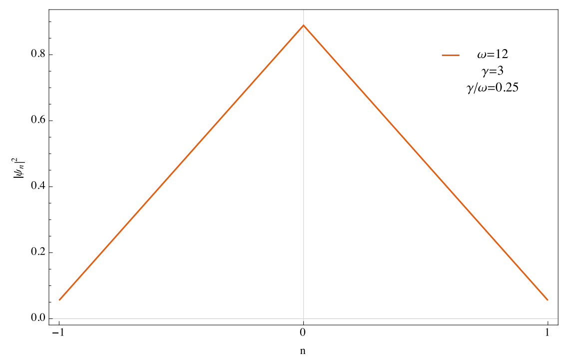



(h)$|\Psi_l|^2$, $\quad l = (-75, 75)$

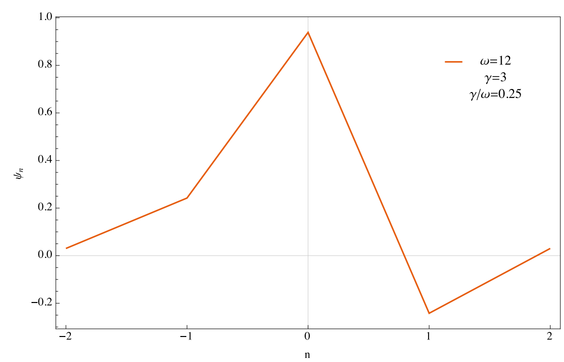
FIG. 2: $\omega = 12$, $\gamma = 180$, $\gamma/\omega = 15$, convergent $n \gtrsim 50$



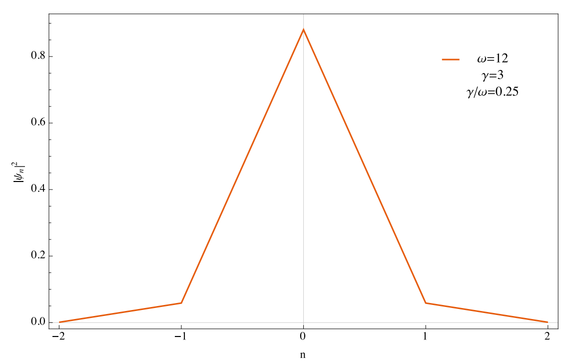

(a)$\Psi_l$, $\quad l = (-1, 1)$



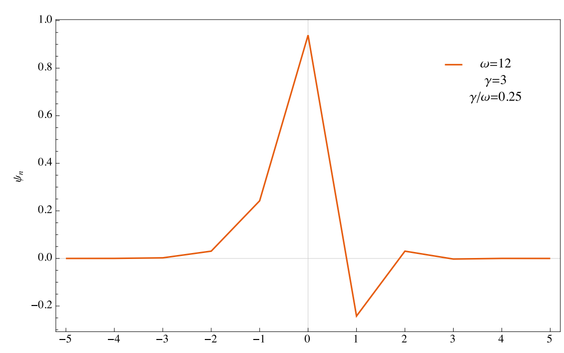

(b)$|\Psi_l|^2$, $\quad l = (-1, 1)$



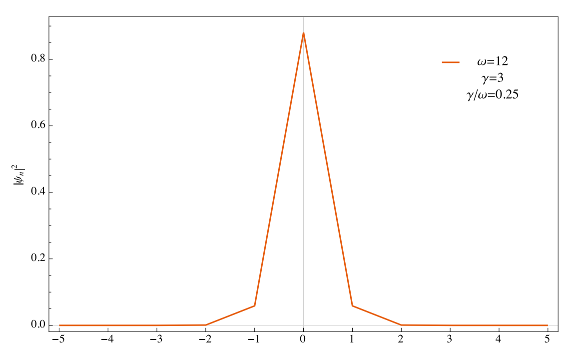

(c)$\Psi_l$, $\quad l = (-2, 2)$



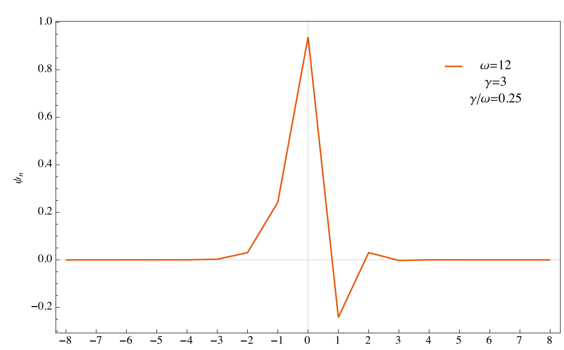

(d)$|\Psi_l|^2$, $\quad l = (-2, 2)$



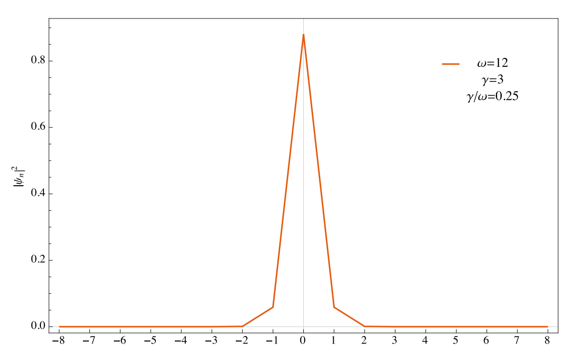

(e)$\Psi_l$, $\quad l = (-5, 5)$




(f)$|\Psi_l|^2$, $\quad l = (-5, 5)$

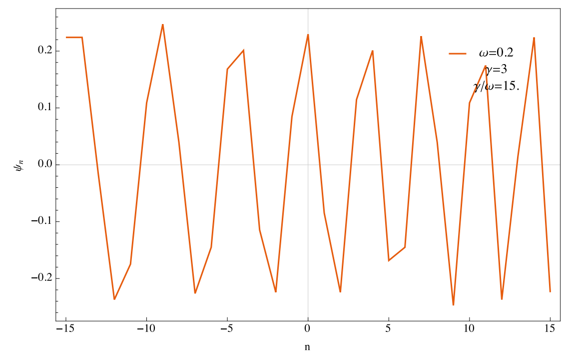



(g)$\Psi_l$, $\quad l = (-8, 8)$

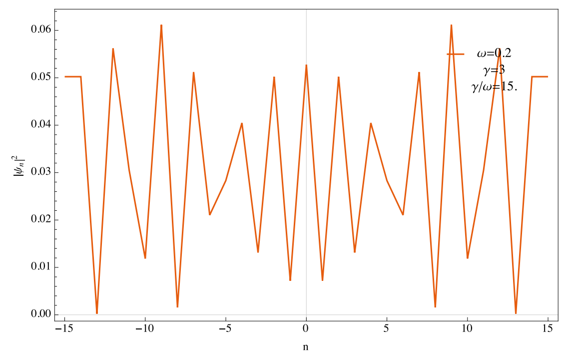



(h)$|\Psi_l|^2$, $\quad l = (-8, 8)$

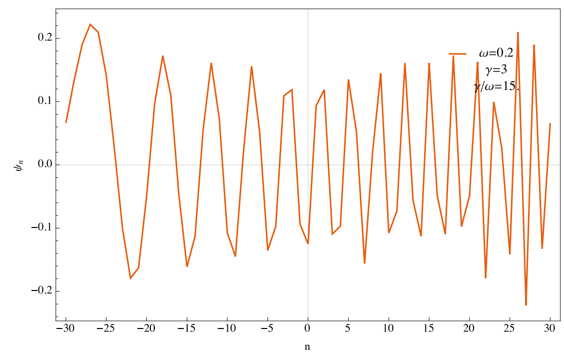
FIG. 3: $\omega = 12$, $\gamma = 3$, $\gamma/\omega = 0.25$, convergent $n \gtrsim 5$



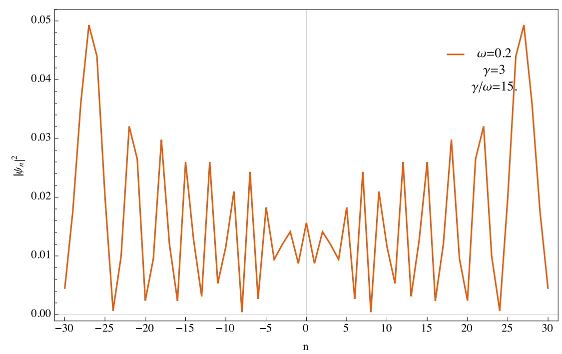

(a)$\Psi_l$, $\quad l = (-15, 15)$



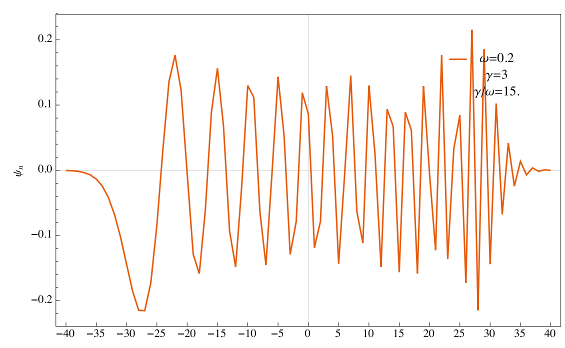

(b)$|\Psi_l|^2$, $\quad l = (-15, 15)$



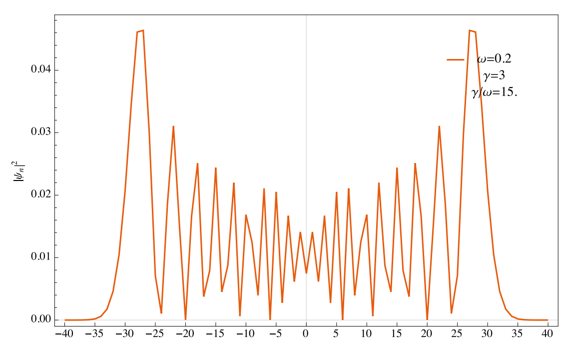

(c)$\Psi_l$, $\quad l = (-30, 30)$



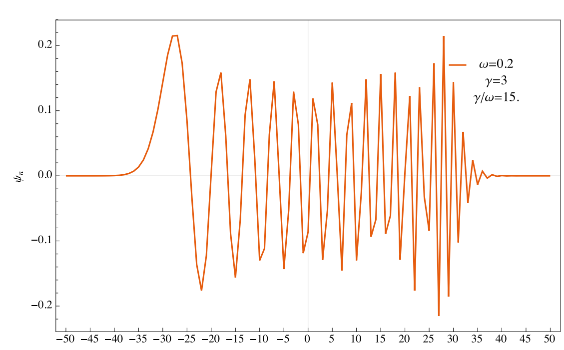

(d)$|\Psi_l|^2$, $\quad l = (-30, 30)$



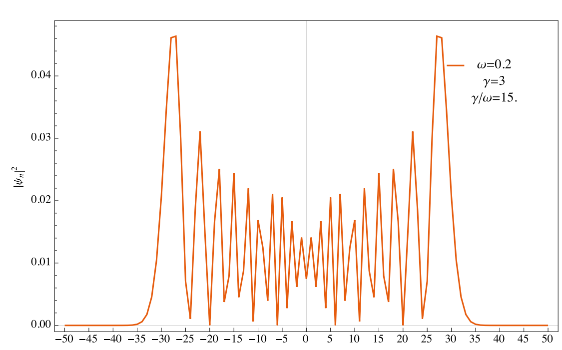

(e)$\Psi_l$, $\quad l = (-40, 40)$




(f)$|\Psi_l|^2$, $\quad l = (-40, 40)$

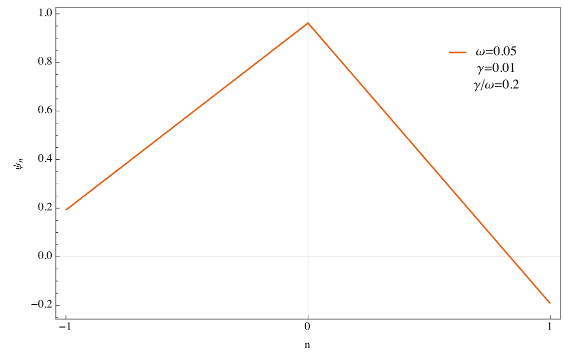



(g)$\Psi_l$, $\quad l = (-50, 50)$



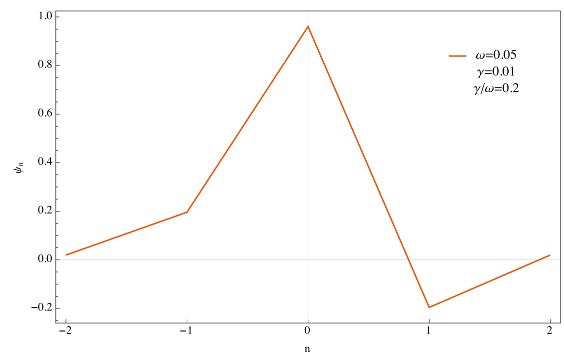

(h)$|\Psi_l|^2$, $\quad l = (-50, 50)$

FIG. 4: $\omega = 0.2$, $\gamma = 3$, $\gamma/\omega = 15$, convergent $n \gtrsim 50$

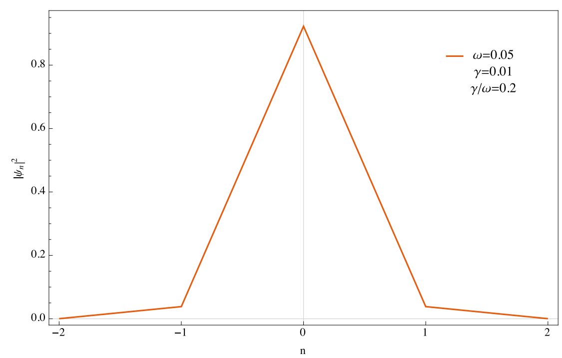



(a)$\Psi_l$, $\quad l = (-1, 1)$

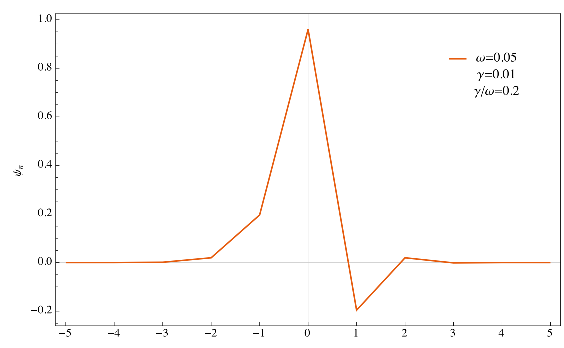

(b)$|\Psi_l|^2$, $\quad l = (-1, 1)$



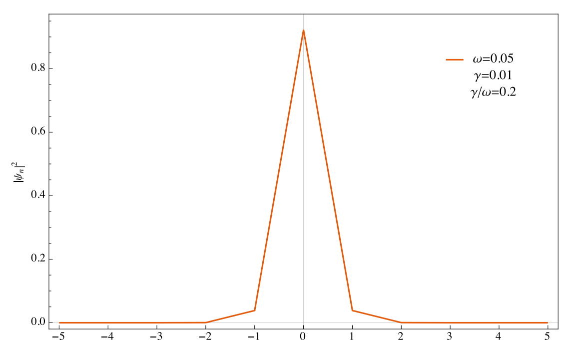

(c)$\Psi_l$, $\quad l = (-2, 2)$



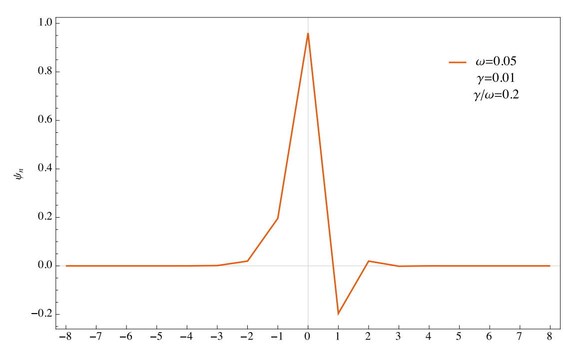

(d)$|\Psi_l|^2$, $\quad l = (-2, 2)$



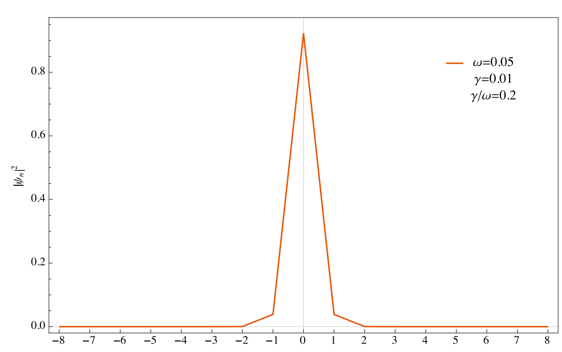

(e)$\Psi_l$, $\quad l = (-5, 5)$




(f)$|\Psi_l|^2$, $\quad l = (-5, 5)$




(g)$\Psi_l$, $\quad l = (-8, 8)$




(h)$|\Psi_l|^2$, $\quad l = (-8, 8)$

FIG. 5: $\omega = 0.05$, $\gamma = 0.01$, $\gamma/\omega = 0.2$, convergent $n \gtrsim 5$