Topological Floquet States in Two-Dimensional Triangle Shaking Optical Lattice

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May 30, 2015

Outline

- Floquet Theoty and Effective Hamiltonian
- Model
- Solving and Results
- Discussion for Further Study

Time-dependent system

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Shrödinger equation reads

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$$\mathbf{i}\hbar\partial_t\psi = H\psi$$

For a periodically driven system,

$$H(t+T) = H(t)$$

$$T = \frac{2\pi}{\omega}$$

Time-dependent system

we have

$$\psi_\lambda = e^{-\mathrm{i}(\lambda t/\hbar)} u_\lambda(\vec{r}, \omega t)$$

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and

$$u_\lambda(\vec{r}, \omega(t+T)) = u_\lambda(\vec{r}, \omega t)$$

Floquet Theory

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$$
\begin{aligned}
\hat{F} &= \hat{U}(T_i + T, T_i) \\
&= \hat{\mathcal{T}} \exp\left(-\mathrm{i} \int_{T_i}^{T_i+T} \hat{H}(t)\mathrm{d}t \right)
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$$

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$$
\hat{F}|\varphi_n\rangle = e^{-\mathrm{i}\varepsilon_n T}|\varphi_n\rangle
$$

Floquet Theory

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$$\hat{F}|\varphi_n\rangle = e^{-\mathrm{i}\varepsilon_n T}|\varphi_n\rangle$$

$\{\varepsilon_n\}$: the quasi-energy bands

Effective Hamiltonian

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$$\hat{H}_{\mathrm{eff}} = \hat{H}_0 + \sum_{n=1}^{\infty}\left(\frac{[\hat{H}_n, \hat{H}_{-n}]}{n\omega} - \frac{[\hat{H}_n, \hat{H}_0]}{e^{-2\pi n i\alpha} n\omega} + \frac{[\hat{H}_{-n}, \hat{H}_0]}{e^{2\pi n i\alpha} n\omega}\right)$$

Hamiltonian

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$$\mathcal{H}(t) \quad = T + \mathcal{V}(\vec{r}, t)$$

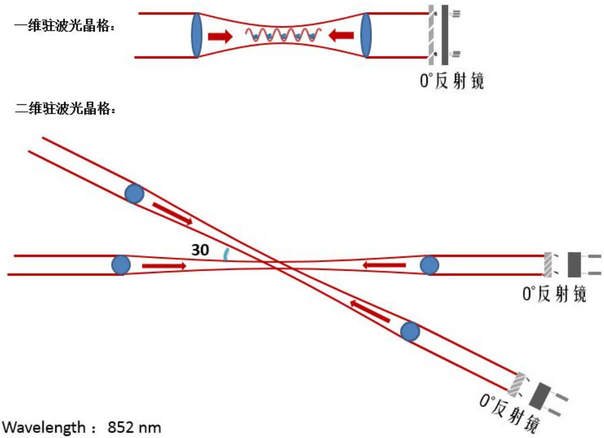
Hamiltonian

$$\mathcal{H}(t) \;= T + \mathcal{V}(\vec{r}, t)$$

$$\begin{aligned}\mathcal{V} = &-V_1 \cos(2\vec{k_1} \cdot \vec{r_1} + f\cos(\omega t)) \\ &- V_2 \cos(2\vec{k_2} \cdot \vec{r_2} + f\cos(\omega t + \alpha))\end{aligned}$$

Setup

Setup



Wavelength： 852 nm
两束光都为线偏且偏振相互垂直
两束光的频率差为211M

Translation

Translation

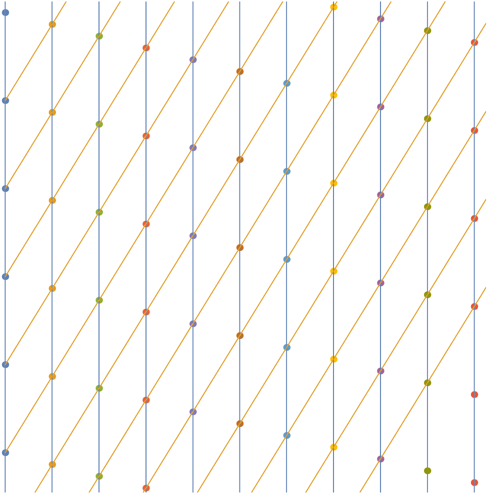
Translational transformation into the co-moving frame of reference,

$$\vec{r} \to \vec{r} + \vec{\xi}(t)$$

$$\begin{aligned} H(t) \;&=\; \frac{p^2}{2\mu} - V_1 \cos(2\vec{k_1} \cdot \vec{r_1}) - V_2 \cos(2\vec{k_2} \cdot \vec{r_2}) \\ &\quad + i\frac{\hbar\omega f}{2k_0}\Bigg[\Big(\sin(\omega t) + \sin(\omega t + \alpha)\cos(30^\circ)\Big)\partial_x \\ &\quad - \sin(\omega t + \alpha)\sin(30^\circ)\partial_y\Bigg] \end{aligned}$$

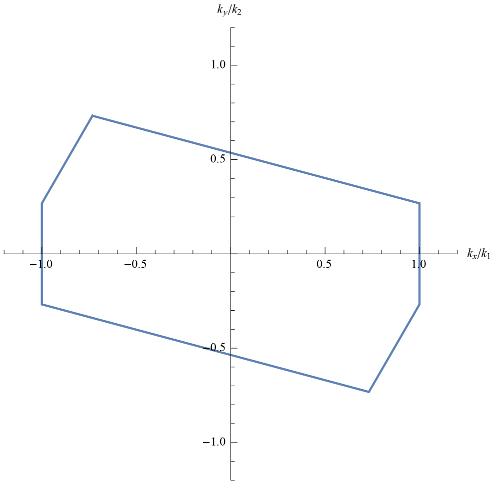
Lattice Structure

Lattice Structure



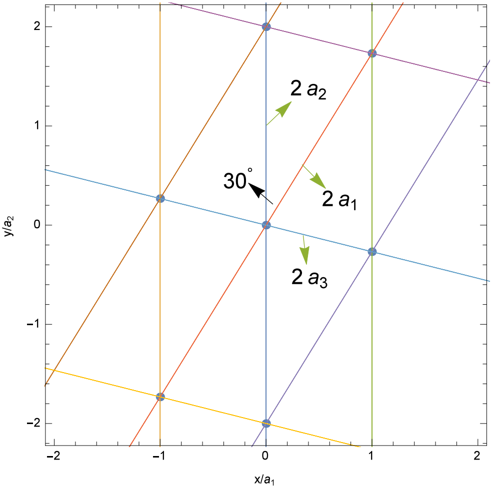
First Brillouin Zone in Reciprocal Lattice

First Brillouin Zone in Reciprocal Lattice



Hopping term

Hopping term



Eigen problem under Bloch bases

Eigen equation reads

$$H|\psi_q^{(n)}(\vec{r})\rangle = \varepsilon^{(n)}(\vec{k})|\psi_q^{(n)}(\vec{r})\rangle$$

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$$H = \frac{p^2}{2m} + V_1 \cos(2\vec{k_1} \cdot \vec{r_1}) + V_2 \cos(2\vec{k_2} \cdot \vec{r_2})$$

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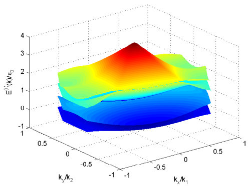
with

$$H = \frac{p^2}{2m} + V_1 \cos(2\vec{k_1} \cdot \vec{r_1}) + V_2 \cos(2\vec{k_2} \cdot \vec{r_2})$$

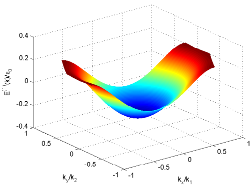
$$\psi_q(\vec{r}) = \sum_{l_1,l_2} C_{l_1,l_2}(\vec{q}) e^{i(2l_1\vec{k_1} + 2l_2\vec{k_2} + \vec{q})\cdot\vec{r}}$$

Energy bands with V=1.0

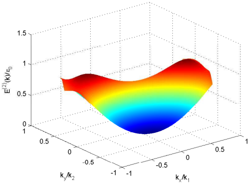
Energy bands with V=1.0



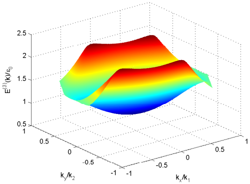
(a) The lowest four bands



(b) Band 1



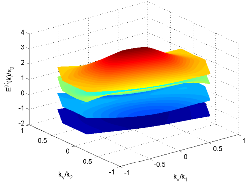
(c) Band 2



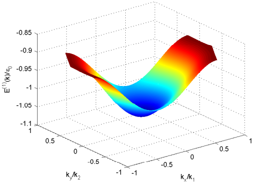
(d) Band 3

Energy bands with V=2.0

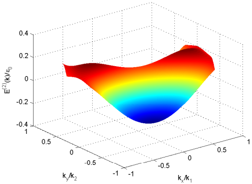
Energy bands with V=2.0



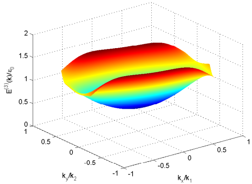
(a) The lowest four bands



(b) Band 1



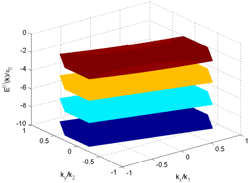
(c) Band 2



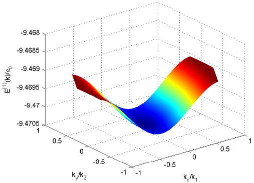
(d) Band 3

Energy bands with V=8.0

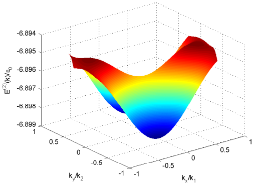
Energy bands with V=8.0



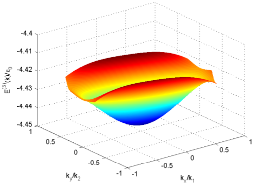
(a) The lowest four bands



(b) Band 1



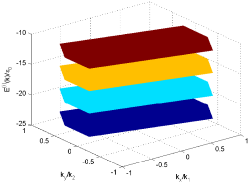
(c) Band 2



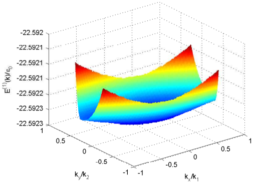
(d) Band 3

Energy bands with V=16.0

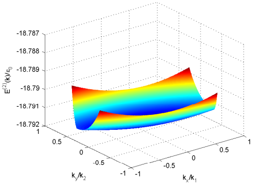
Energy bands with V=16.0



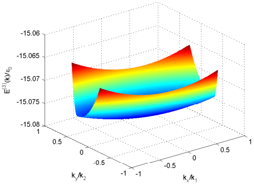
(a) The lowest four bands



(b) Band 1



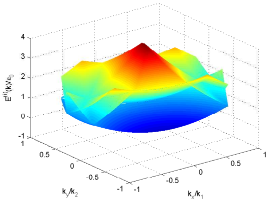
(c) Band 2



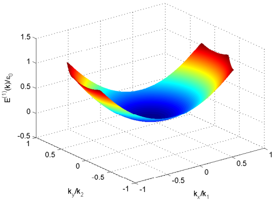
(d) Band 3

Energy bands with V=0.05(≈ 0)

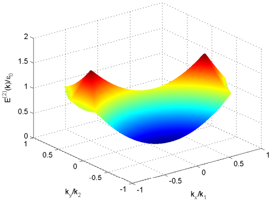
Energy bands with V=0.05(≈ 0)



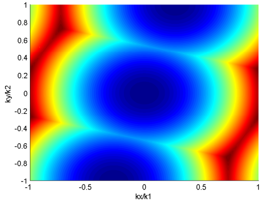
(a) The lowest four bands



(b) Band 1



(c) Band 2



(d) Top View for the

Wannier functions

Wannier functions

$$
\begin{aligned}
w_n(\vec{R}_m, \vec{r}) &= w_n(\vec{r} - \vec{R}_m) \\
&= \frac{1}{\sqrt{N}} \sum_{\vec{k} \in BZ} e^{-i\vec{k}\cdot\vec{R}_m} \psi_{n\vec{k}}(\vec{r})
\end{aligned}
$$

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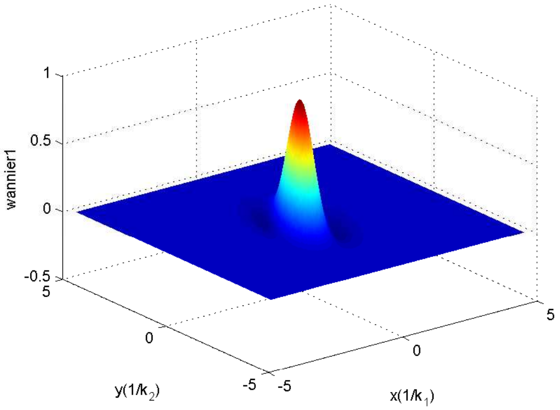
\end{aligned}
$$

$$
\begin{aligned}
w_n(\vec{r}) &= w_n(0, \vec{r}) \\
&= \frac{1}{\sqrt{N}} \sum_{\vec{q} \in BZ} \psi_{n\vec{q}}(\vec{r}) \\
&= \frac{1}{\sqrt{N}} \sum_{\vec{q} \in BZ} \sum_{l_1, l_2} C_{l_1,l_2}(\vec{q}) e^{i(2l_1\vec{k}_1 + 2l_2\vec{k}_2 + \vec{q})\cdot\vec{r}}
\end{aligned}
$$

Wannier 1

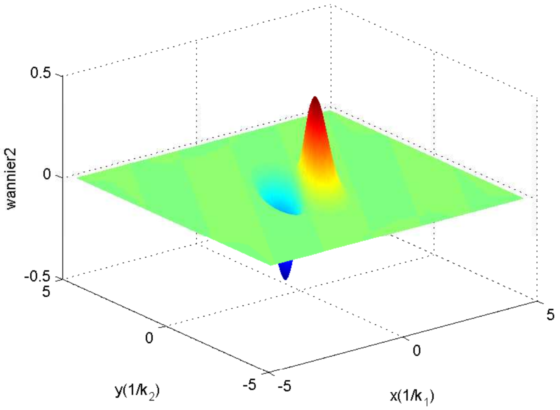
Wannier 1



(b) Wannier function for the 1^{st} band $w_1(x, y)$

Wannier 2

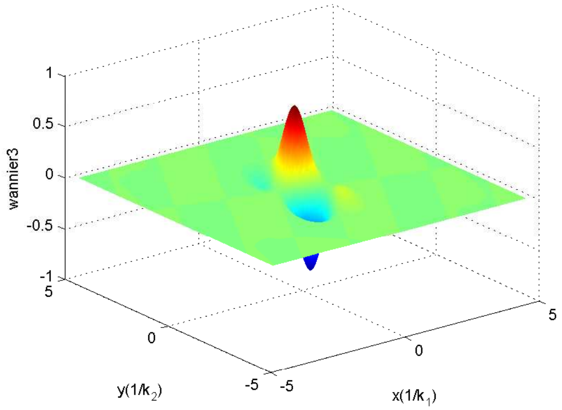
Wannier 2



(d) Wannier function for the 2^{nd} band $w_2(x,y)$

Wannier 3

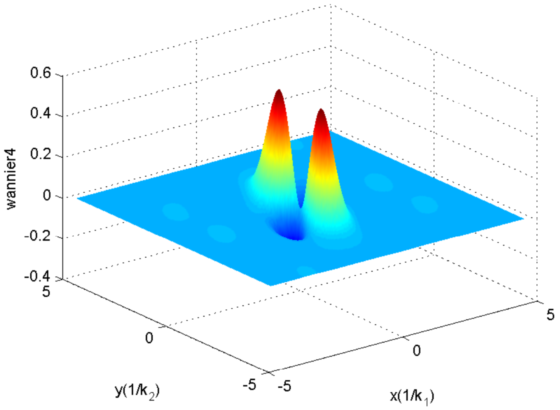
Wannier 3



(f) Wannier function for the 3^{th} band $w_3(x, y)$

Wannier 4

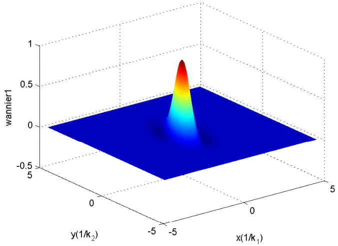
Wannier 4



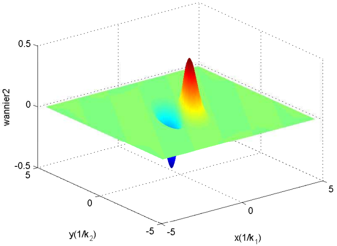
(h) Wannier function for the 4^{th} band $w_4(x, y)$

Wannier functions with V=8.0

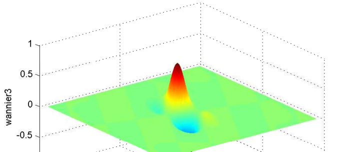
Wannier functions with V=8.0

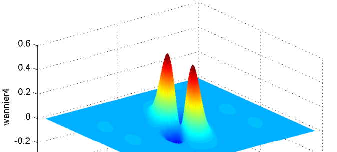


(a) Wannier function for the 1^{st} band $w_1(x, y)$



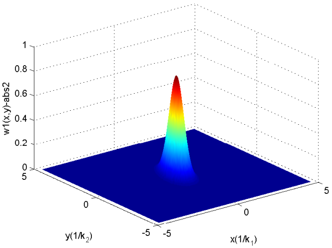
(b) Wannier function for the 2^{nd} band $w_2(x, y)$

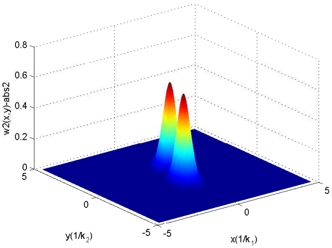




Wannier functions with V=8.0

Wannier functions with V=8.0





(a) Square Modulus of $w_1(x, y)$ (b) Square Modulus of $w_2(x, y)$

Figure: Square modulus of the Wannier functions in Figure 6

Tight-bounding model

Tight-bounding model

$$\hat{H}(t) = \sum_{\vec{k}} \left(\hat{a}^{\dagger}_{s\vec{k}} \ \ \hat{a}^{\dagger}_{p\vec{k}}\right) H(\vec{k}, t) \begin{pmatrix} \hat{a}_{s\vec{k}} \\ \hat{a}_{p\vec{k}} \end{pmatrix}$$

Tight-bounding model

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$$H(\vec{k}, t) = H_0(\vec{k}) + H_1(\vec{k}) e^{i\omega t} + H_{-1}(\vec{k}) e^{-i\omega t}$$

Tight-bounding model

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$$H(\vec{k}, t) = H_0(\vec{k}) + H_1(\vec{k})e^{i\omega t} + H_{-1}(\vec{k})e^{-i\omega t}$$

Tight-bounding model

$$H_0(\vec{k}) = \begin{pmatrix} E_s & iK \\ -iK & E_p \end{pmatrix}$$

$$H_1(\vec{k}) = \begin{pmatrix} \Lambda_s & \Omega \\ -\Omega & \Lambda_p \end{pmatrix}$$

$$H_{-1}(\vec{k}) = H_1(\vec{k})^\dagger = \begin{pmatrix} \Lambda_s^* & -\Omega^* \\ \Omega^* & \Lambda_p^* \end{pmatrix}$$

One-photon resonance

One-photon resonance

$$
\begin{aligned}
\widetilde{H}_{\text{eff}}(\vec{k}) &= \varepsilon_0 + \vec{B}(\vec{k}) \cdot \vec{\sigma} \\
&= \varepsilon_0 + B_x(\vec{k})\sigma_x + B_y(\vec{k})\sigma_y + B_z(\vec{k})\sigma_z
\end{aligned}
$$

One-photon resonance

$$
\begin{aligned}
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&= \varepsilon_0 + B_x(\vec{k})\sigma_x + B_y(\vec{k})\sigma_y + B_z(\vec{k})\sigma_z
\end{aligned}
$$

$$
\varepsilon_0 = \frac{1}{2}(E_s + \omega + E_p)
$$

$$
B_x(\vec{k}) = \Re(\Omega) + \frac{K}{\omega}\big(\Im(\Lambda_p) - \Im(\Lambda_s)\big)
$$

$$
B_y(\vec{k}) = -\Im(\Omega) + \frac{K}{\omega}\big(\Re(\Lambda_p) - \Re(\Lambda_s)\big)
$$

$$
B_z(\vec{k}) = \frac{E_s + \omega - E_p}{2} - \frac{K^2}{\omega} - \frac{|\Omega|^2}{2\omega}
$$

Two-photon resonance

Two-photon resonance

$$
\begin{aligned}
H_{\text{eff}} \;=\; & \frac{E_s + E_p}{2} \\
& + \Big[\frac{E_s - E_p + 2\omega}{2} - \frac{1}{\omega} \left(\frac{4}{3} |\Omega|^2 + \frac{K^2}{2} \right) \Big] \sigma_z \\
& + \frac{1}{\omega} \Re \Big(\Omega (\Lambda_s - \Lambda_p) \Big) \sigma_x \\
& - \frac{1}{\omega} \Im \Big(\Omega (\Lambda_s - \Lambda_p) \Big) \sigma_y
\end{aligned}
$$

which can be written as

$$\begin{aligned} H_{\text{eff}} &= \varepsilon_0 + \vec{B}(\vec{k}) \cdot \vec{\sigma} \\ &= \varepsilon_0 + \vec{B}_z(\vec{k}) \cdot \vec{\sigma_z} + \vec{B}_x(\vec{k}) \cdot \vec{\sigma_x} + \vec{B}_y(\vec{k}) \cdot \vec{\sigma_y} \end{aligned}$$

with

$$\begin{aligned} \varepsilon_0 &= \frac{E_s + E_p}{2} \\ \vec{B}_z(\vec{k}) &= \frac{E_s - E_p + 2\omega}{2} - \frac{1}{\omega}\left(\frac{4}{3}|\Omega|^2 + \frac{K^2}{2}\right) \\ \vec{B}_x(\vec{k}) &= \frac{1}{\omega}\Re\Big(\Omega(\Lambda_s - \Lambda_p)\Big) \\ \vec{B}_y(\vec{k}) &= -\frac{1}{\omega}\Im\Big(\Omega(\Lambda_s - \Lambda_p)\Big) \end{aligned}$$

Chern numbers

Chern numbers

$$H_{\text{eff}} = \vec{B}(\vec{k}) \cdot \vec{\sigma} = \varepsilon(\mathbf{k})\hat{\mathbf{n}}(\mathbf{k}) \cdot \tilde{\sigma}$$

Chern numbers

$$H_{\text{eff}} = \vec{B}(\vec{k}) \cdot \vec{\sigma} = \varepsilon(\mathbf{k})\hat{\mathbf{n}}(\mathbf{k}) \cdot \tilde{\sigma}$$

$$C_{\pm} = \frac{\pm 1}{4\pi} \int_{\text{FBZ}} \hat{\mathbf{n}} \cdot (\partial_{\mathbf{k_x}} \hat{\mathbf{n}} \times \partial_{\mathbf{k_y}} \hat{\mathbf{n}}) \mathbf{d^2 k}$$

Chern numbers

Chern numbers

One-photon resonance

None topological non-trivial states found.

Chern numbers

One-photon resonance

None topological non-trivial states found.

Tne-photon resonance

For small potential, $V < 1.5$, non-zero Chern number cases found. Shaking amplitudes f and frequencies ω allowed for topological non-trivial states differs for different potential depths. While the range of f is approximately several k_0, the range of ω allowed from tens to hundruds units comparing to the energy unit of the system.

Thank you !