Note on Rice Mele model

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I. RICE-MELE MODEL

One-dimensional Hamiltonian writes

$$H = \sum_j -(J+\delta)a_j^\dagger b_j - (J-\delta)a_{j+1}^\dagger b_j + h.c. + \Delta(a_j^\dagger a_j - b_j^\dagger b_j)$$

Fourier transformation (assuming periodical boundary conditions):

$$H = \sum_q \begin{pmatrix} a_q^\dagger & b_q^\dagger \end{pmatrix} \mathcal{H}(q) \begin{pmatrix} a_q \\ b_q \end{pmatrix}$$

where $\mathcal{H}(q) = \boldsymbol{h}(q)\cdot\boldsymbol{\sigma}$, and

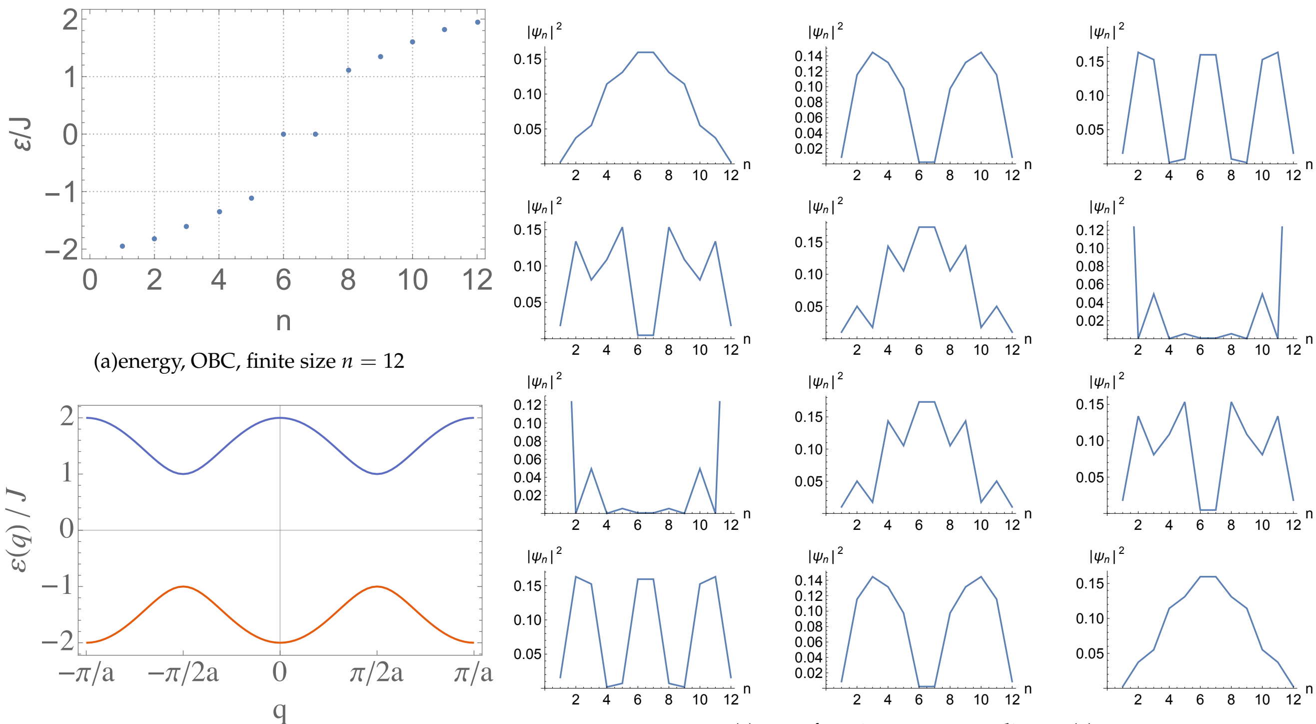
$$\boldsymbol{h}(q) = (-2J\cos(\frac{qa}{2}), -2\delta\sin(\frac{qa}{2}), \Delta)$$
$$\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$$

A typical example is shown in FIG 1.

II. TIME-DEPENDENT RM

Suppose time dependence

$$\delta(t) = \delta\sin(\omega t)$$
$$\Delta(t) = \Delta\cos(\omega t)$$



(a)energy, OBC, finite size $n = 12$

(b)energy, PBC

(c)wave functions corresponding to (a)

FIG. 1: A typical example with parameters $\delta/J = -0.5, \Delta = 0$, which underlines the celebrated SSH model.

then

$$H(t) = \sum_j -(J + \delta \sin(\omega t))a_j^\dagger b_j - (J - \delta \sin(\omega t))a_{j+1}^\dagger b_j + h.c. + \Delta \cos(\omega t)(a_j^\dagger a_j - b_j^\dagger b_j)$$

and $\mathcal{F}.\mathcal{T}$.

$$\boldsymbol{h}(q,t) = (-2J\cos(\frac{qa}{2}), -2\delta \sin(\omega t)\sin(\frac{qa}{2}), \Delta\cos(\omega t))$$

Considering t as a parameter, solve the energy band at given t. A typical case is shown in FIG 2

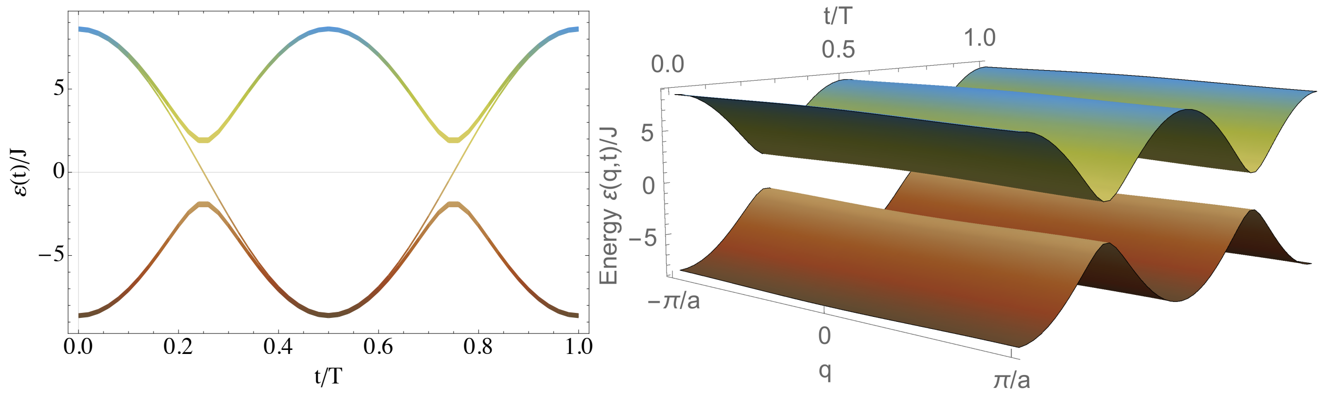
A. current operator

In the adiabatic limit $\omega \ll J$, current operator could be calculated as

$$\begin{aligned} j^{(n)}(q) &= \frac{\partial \varepsilon(q)}{\hbar \partial q} - \mathrm{i}\Big[\langle \partial_q u_n | \partial_t u_n \rangle - \langle \partial_t u_n | \partial_q u_n \rangle\Big] \\ &= \frac{\partial \varepsilon(q)}{\hbar \partial q} - \Omega_{qt} \end{aligned}$$

Then the particle transport in a period is given by

$$c = -\frac{1}{2\pi}\int_0^T dt \int_{BZ} dq \Omega_{qt}$$



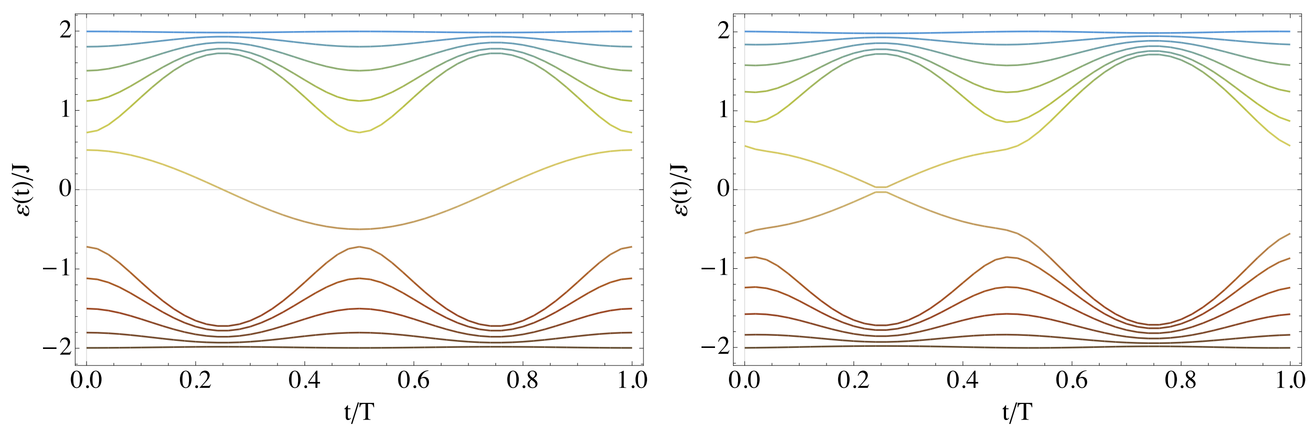
(a)obc, finite size $n = 21$

(b)pbc

FIG. 2: Time-dependent RM with $\delta/J = -0.85$, $\Delta/J = 8.5$

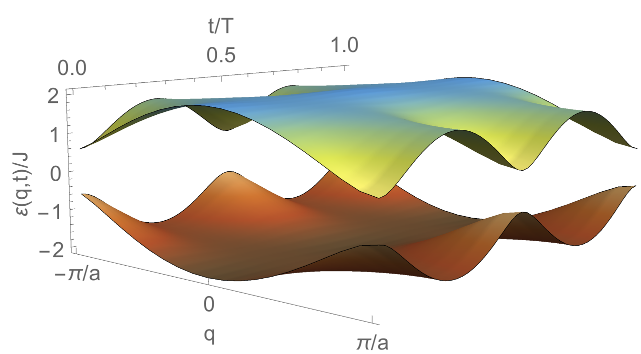
B. eg

Take a look at another case of parameters where $\delta/J = -0.85$, $\Delta/J = .5$. Still consider t as an explicit parameter. We solve the energy bands for finite size of either odd or even number of sites ($n = 11$ or $n = 12$). Also, we solve it using periodic boundary conditions (PBC)



(a)OBC, finite size $n = 11$

(b)OBC, finite size $n = 12$



(c)PBC, finite size $n = 11$

FIG. 3: Time-dependent RM with $\delta/J = -0.85$, $\Delta/J = 0.5$

III. FLOQUET RM

For a periodic time-dependent system with period T, the Hamiltonian could be expanded as $H(t) = \sum_n H_n e^{\mathrm{i}n\omega t}$ where $\omega = 2\pi/T$.

Here t serves no longer as some parameter, but eliminated totally in the frequency domian, by which we mean that it is the Floquet Hamiltonian H_F we are considering now, which is the Fourier transformation of $H(t)$ into frequency domain as $H_F(\omega)$, where ω serves as some given parameter assuming totally periodicity in time dimension, just like in the case of periodicity in space dimensions quasimomentum q as an explicit parameter of $H(q)$.

For a simple harmonic time-dependent case,

$$H(t) = H_0 + H_1 e^{\mathrm{i}\omega t} + H_{-1} e^{-\mathrm{i}\omega t}$$

Then block Floquet Hamiltonian is of form

$$(H_F)_{nn} = n\omega I + H_0$$
$$(H_F)_{n,n\pm 1} = H_{\pm 1}$$

$$\boldsymbol{H_F} = \left(\begin{array}{c:c:c:c:c} & & & & \\ \hdashline & H_0+\omega & H_1 & & \\ \hdashline & H_{-1} & H_0 & H_1 & \\ \hdashline & & H_{-1} & H_0-\omega & \\ \hdashline & & & & \end{array}\right)$$

A. Periodical boundary conditions

$\mathcal{F}.\mathcal{T}.$ of the Hamiltonian:

$$\begin{aligned} \mathcal{H}(q,t) &= -2J\cos(\frac{qa}{2})\sigma_x - 2\delta\sin(\omega t)\sin(\frac{qa}{2})\sigma_y + \Delta\cos(\omega t)\sigma_z \\ &= \mathcal{H}_0(q) + \mathcal{H}_1(q)e^{\mathrm{i}\omega t} + \mathcal{H}_{-1}(q) \end{aligned}$$

where

$$\begin{aligned} \mathcal{H}_0(q) &= -2J\cos(\frac{qa}{2})\sigma_x = \begin{pmatrix} 0 & -2J\cos(qa/2) \\ -2J\cos(qa/2) & 0 \end{pmatrix} \\ \mathcal{H}_1(q) &= \mathrm{i}\delta\sin(\frac{qa}{2})\sigma_y + \frac{\Delta}{2}\sigma_z = \begin{pmatrix} \Delta/2 & \delta\sin(qa/2) \\ -\delta\sin(qa/2) & -\Delta/2 \end{pmatrix} \\ \mathcal{H}_{-1}(q) &= H_1^\dagger(q) = \begin{pmatrix} \Delta/2 & -\delta\sin(qa/2) \\ \delta\sin(qa/2) & -\Delta/2 \end{pmatrix} \end{aligned}$$

B. Open boundary conditions

Assume n sites in all (sum ove j from 1 to n in following):

$$\begin{aligned} H(t) &= \sum_j -(J+\delta\sin(\omega t))a_j^\dagger b_j - (J-\delta\sin(\omega t))a_{j+1}^\dagger b_j + h.c. + \Delta\cos(\omega t)(a_j^\dagger a_j - b_j^\dagger b_j) \\ &= H_0 + H_1 e^{\mathrm{i}\omega t} + H_{-1}e^{-\mathrm{i}\omega t} \end{aligned}$$

with

$$\begin{aligned} H_0 &= \sum_j -Ja_j^\dagger b_j - Ja_{j+1}^\dagger b_j + h.c. \\ H_1 &= \sum_j \mathrm{i}\frac{\delta}{2}a_j^\dagger b_j - \mathrm{i}\frac{\delta}{2}a_{j+1}^\dagger b_j + h.c. + \frac{\Delta}{2}(a_j^\dagger a_j - b_j^\dagger b_j) \\ H_{-1} &= \sum_j -\mathrm{i}\frac{\delta}{2}a_j^\dagger b_j + \mathrm{i}\frac{\delta}{2}a_{j+1}^\dagger b_j + h.c. + \frac{\Delta}{2}(a_j^\dagger a_j - b_j^\dagger b_j) \end{aligned}$$

[1] Di Xiao, Ming-Che Chang, and Qian Niu, *Berry phase effects on electronic properties*, Rev. Mod. Phys. **82**, 1959 (2010).