

Molecule Dynamics with Quench

Ning Sun and Pengfei Zhang

January 28, 2018

Method

The Hamiltonian of spin-1/2 fermions interacting via a contact potential writes

$$\mathcal{H} = \psi_\sigma^\dagger \left(-\frac{\hbar^2 \nabla^2}{2m} - \mu \right) \psi_\sigma + g \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow \quad (1)$$

The BCS molecule creation operator is defined as

$$b_q = \int \frac{d^3 k}{(2\pi)^3} \varphi_f^*(\mathbf{k}) c_{q/2+\mathbf{k},\uparrow}^\dagger c_{q/2-\mathbf{k},\downarrow}^\dagger \quad (2)$$

Hence the molecule density is given by [1]

$$n_q = \langle b_q^\dagger b_q \rangle \quad (3)$$

$$= \int \frac{d^3 k}{(2\pi)^3} \frac{d^3 k'}{(2\pi)^3} \varphi^*(\mathbf{k}) \varphi(\mathbf{k}') \langle c_{q/2+\mathbf{k},\uparrow}^\dagger c_{q/2-\mathbf{k},\downarrow}^\dagger c_{q/2-\mathbf{k},\downarrow} c_{q/2+\mathbf{k},\uparrow} \rangle \quad (4)$$

$$= \left| \int \frac{d^3 k}{(2\pi)^3} \varphi(\mathbf{k}) \langle c_{k,\uparrow}^\dagger c_{-k,\downarrow} \rangle \right|^2 \delta(q) + \int \frac{d^3 k}{(2\pi)^3} |\varphi(\mathbf{k})|^2 \langle n_{q/2+\mathbf{k},\uparrow} \rangle \langle n_{q/2-\mathbf{k},\downarrow} \rangle \quad (5)$$

where we assume the expectation value is taken under the BCS mean field wave functions (initial state in the unitary regime).¹

In regarding we consider the dynamical behavior after quench, only the first term in above contributes. (the second term remains constant.)

¹That is to say, we use the BCS mean field wave function as the initial state before and after quench, while using the value of parameters μ and Δ in the unitary regime.

1 Quench to the BCS limit

Experimental process. To start from the unitary regime and then quench to the BCS limit and evolve a period of time afterwards. Then try to measure the BCS molecule by sweeping in a proper rate² to the BEC side to photograph the molecules. As a function of evolving time after quench, the molecule density shows revival under large quench parameters. Ref [2].

Back-of-envelope calculation. Since we focus on the molecule density after quench to the BCS limit, we choose $\varphi_f(k) = b_0 v_k / u_k$. Ref [3]. Meanwhile

$$\langle c_{k,\uparrow}^\dagger c_{-k,\downarrow} \rangle = u_k v_k = \frac{\Delta/2}{\sqrt{(\varepsilon_k - \mu)^2 + \Delta^2}}$$

Thus

$$N_1 = N_0 \left| \int_0^{k_F} 4\pi k^2 dk \frac{\Delta/2}{\sqrt{\Delta^2 + (\varepsilon_k - \mu)^2}} e^{i2H_{\text{BCS}}t} \right|^2 \quad (6)$$

where N_0 is some normalization constant (consistent with b_0). Here we use the value of parameters approximately in the unitary regime:

$$\Delta = 0.6E_F, \quad \mu = 0.37E_F.$$

And choose $\phi_f(k)$ to be a step function which truncates the integral at k_F .

The results are as shown in Figure 1.

²Not too fast to stay coherence and not too slow to avoid molecule's life decay.

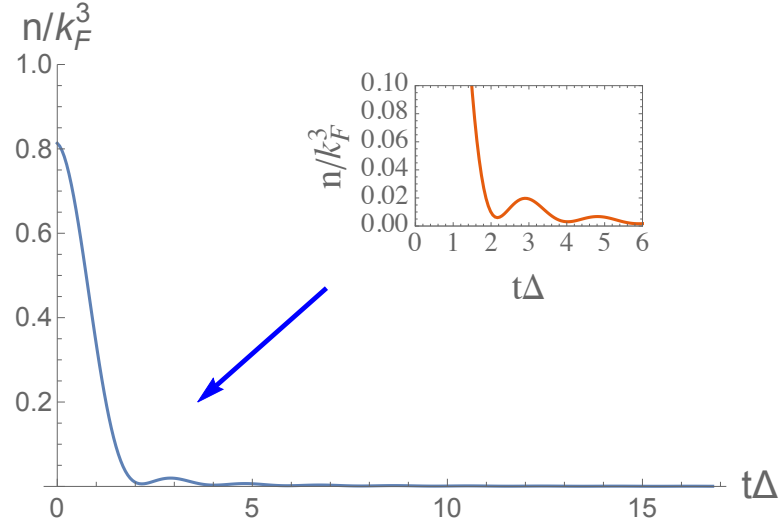


Figure 1: $\Delta = 0.6E_F$, $\mu = 0.37E_F$. We use the free Hamiltonian $H_{\text{free}} = (\epsilon_k - \mu)c_k^\dagger c_k$ to replace the weakly interacting H_{BCS} in the calculation.

2 Sweep to the BEC limit

Another way to consider is assign $\varphi_f(\mathbf{k})$ a BEC wave function, as is to first quench to the BCS limit and sweeping to the BEC side. Similar to Ref [1], we can assign some "hard core" wave function cutoff at some a_s^* . However, the a_s^* is determined by the sweeping rate which we are not clear. Make use of this, the basic formula is of the same form as Eq.(6), but the result might be different quantitatively.

References

- [1] Ehud Altman and Ashvin Vishwanath, *Dynamic Projection on Feshbach Molecules: A Probe of Pairing and Phase Fluctuations*, [Phys. Rev. Lett. **95** 110404 \(2005\)](#).
- [2] Kuiyi Gao's informal talk (2017).
- [3] Hui Zhai, Lecture Note on Cold Atom Physics (2017).