Two Experiments

Aharonov-Bohm interferometry and Wilson lines

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Outline

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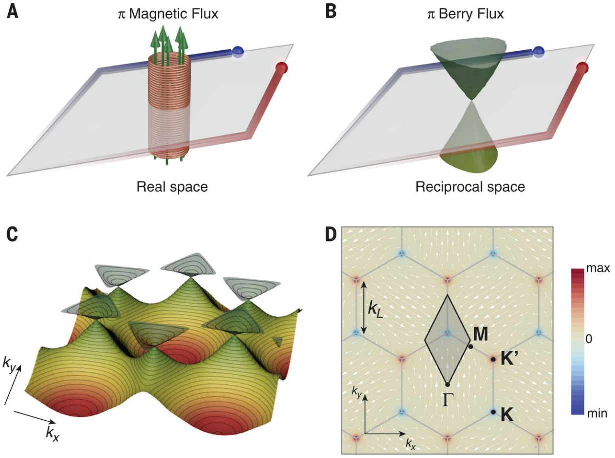
An Aharonov-Bohm interferometer for determining Bloch band topology



Berry connection and Berry curvature

- Bloch state

$$\psi_{\mathbf{k}}^{n}(\mathbf{r}) = e^{\mathrm{i}\mathbf{k}\mathbf{r}} u_{\mathbf{k}}^{n}(\mathbf{r})$$

- Berry connection

$$\mathbf{A}_n(\mathbf{k}) = \mathrm{i}\langle u_{\mathbf{k}}^{n} | \nabla_{\mathbf{k}} | u_{\mathbf{k}}^{n} \rangle$$

- Berry curvature

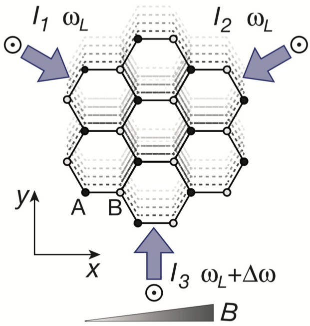
$$\Omega_n = \nabla_{\mathbf{k}} \times \mathbf{A}_n(\mathbf{k})$$

- Berry phase

$$\phi_{\mathsf{Berry}} = \oint_C \mathbf{A}_n(\mathbf{k}) d\mathbf{k} = \int_S \Omega_n(\mathbf{k}) d^2\mathbf{k}$$

Hexagonal lattice in real space

Magnetic field $B = B_0 + \mathbf{r} \cdot \nabla B$ combined with an orthogonal acceleration $\mathbf{a} \perp \nabla B$ of the lattice:



The Hamiltonian is

$$H(t) = \frac{\mathbf{p}^2}{2m} + V[\mathbf{r} - \mathbf{R}] - \mu \mathbf{r} \cdot \nabla B - \mu B_0$$

in co-moving frame

$$H(t) = \frac{\mathbf{p}^2}{2m} + V(\mathbf{r}) - \mathcal{F}_\mu \cdot \mathbf{r} + \varepsilon_\mu(t)$$

How arbitrary path in reciprocal space comes about

The Hamiltonian in co-moving frame is

$$H(t) = \frac{\mathbf{p}^2}{2m} + V(\mathbf{r}) - \mathcal{F}_\mu \cdot \mathbf{r} + \varepsilon_\mu(t) \quad (+\frac{1}{2}m|\mathbf{a}t|^2)$$

external force inducing moving in reciprocal space

$$\mathcal{F}_\mu = \mu \nabla B - m\mathbf{a}$$

Zeeman energy $\varepsilon_\mu(t) = -\mu[\mathbf{R}(t) \cdot \nabla B + B_0]$

$\mathcal{F}_\mu$: $\qquad \mathbf{k} \to \mathbf{k} + \mathcal{F}_\mu t$

Evolution of a state in such a lattice

Hamiltonian

$$H = H_0 - \mathcal{F}_\mu \cdot \mathbf{r}$$

Bloch state

$$H_0 \psi^n_{\mathbf{k}}(\mathbf{r}) = E_n(\mathbf{k}) \psi^n_{\mathbf{k}}(\mathbf{r})$$
$$\psi^n_{\mathbf{k}}(\mathbf{r}) = e^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} u^n_{\mathbf{k}}(\mathbf{r})$$

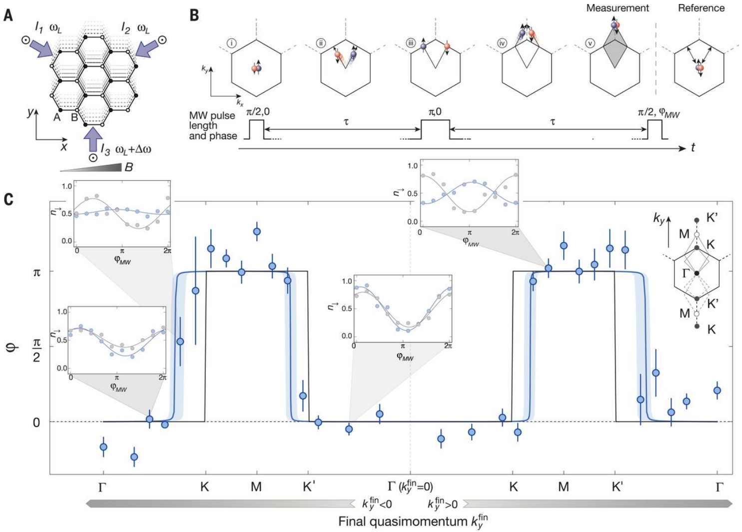
Ansatz

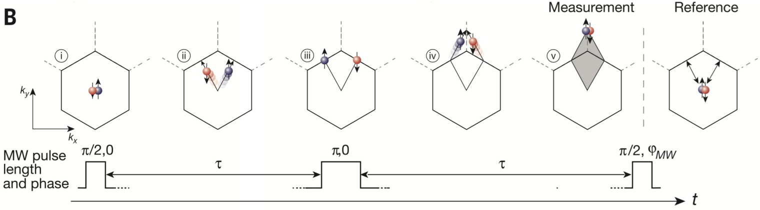
$$\tilde{\Psi}(t) = e^{\mathrm{i}\eta(t)} \psi^n_{k_0 + \mathcal{F}_\mu t}$$
$$\eta = \phi_{\mathsf{dyn}} + \phi_{\mathsf{Berry}}$$

Phase

$$\phi_{\mathsf{dyn}} = \int_0^T [E_1(\mathbf{k} + \mathcal{F}_\mu t) + \varepsilon_\mu t] dt$$
$$\phi_{\mathsf{Berry}} = \mathbf{i} \int_C \langle u^1_{\mathbf{k}} | \nabla_{\mathbf{k}} | u^1_{\mathbf{k}} \rangle d\mathbf{k}$$

Experiment procedure





1. ^{87}Rb BEC initial state $|\uparrow\rangle = |F=2, m_F=1\rangle$; $\pi/2$-microwave pulse;
2. τ evolution;
3. microwave π pulse;
4. τ evolution;
5. $\pi/2$ pulse;

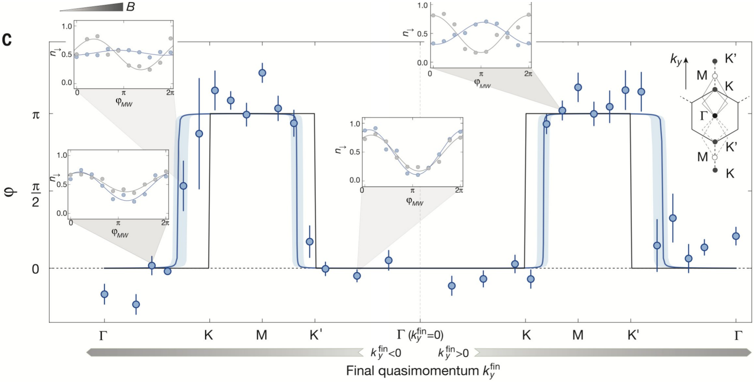
zero-area reference:

V-shape path.

Reversing the lattice acceleration after π pulse.

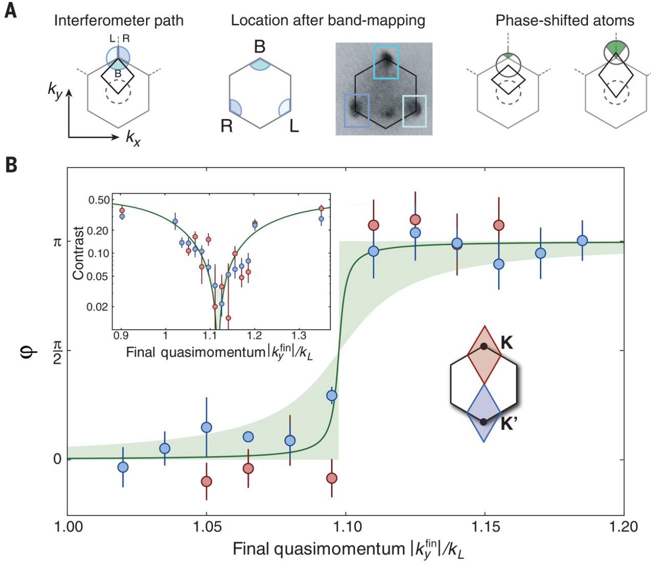
$$n_{\uparrow,\downarrow} \propto 1 \pm \cos(\varphi + \varphi_{MW})$$

Main result I

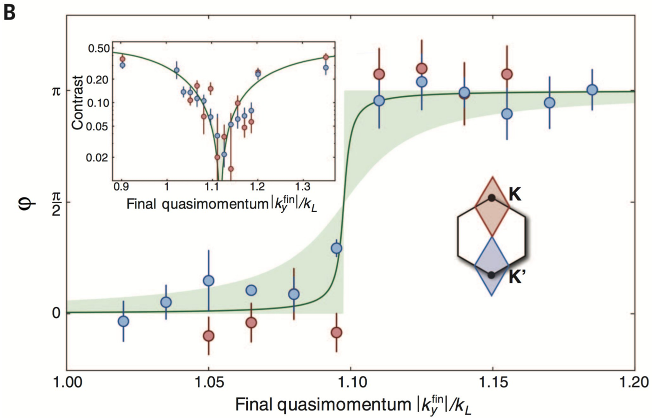


- Broadening of the edges — caused by momentum spread.
- Systematic errors.

Self-referenced interferometry at Dirac point

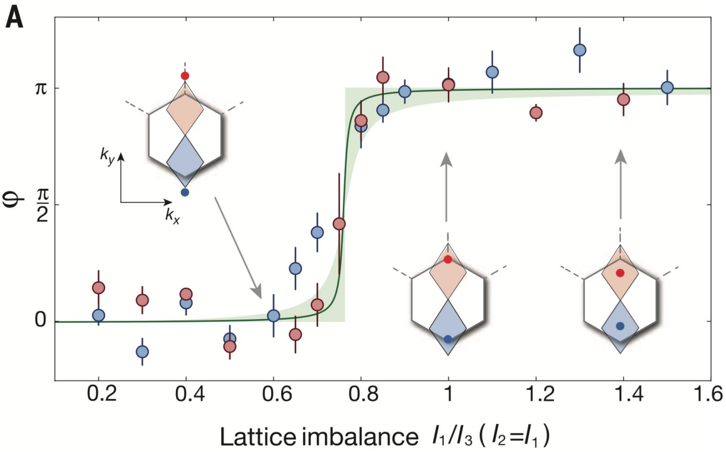


Main result II



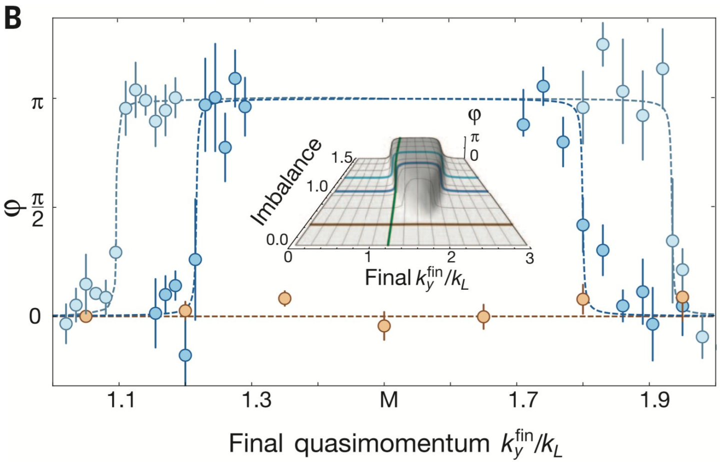
- Contrast: $(n_\downarrow^{\mathrm{max}} - n_\downarrow^{\mathrm{min}})/(n_\downarrow^{\mathrm{max}} + n_\downarrow^{\mathrm{min}})$
- $\varphi = (\varphi_L + \varphi_R)/2 - \varphi_B = 0.95(10)\pi$
- Berry curvature localization $\delta k_w \simeq 10^{-4} k_L (\Delta \simeq h \times 3\mathrm{Hz})$

Imbalance lattice mapping

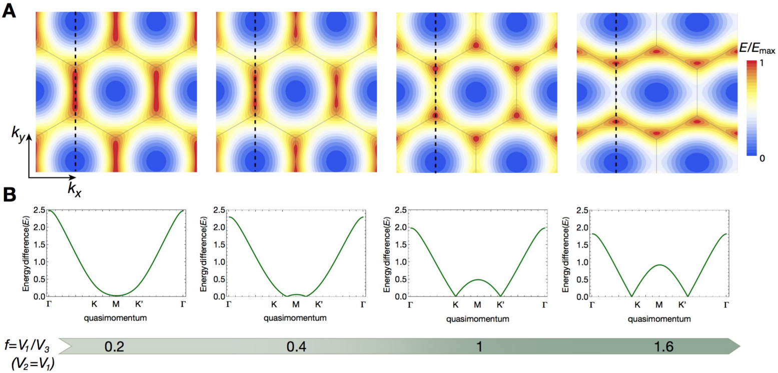


Imbalance lattice mapping

Self-referenced phase



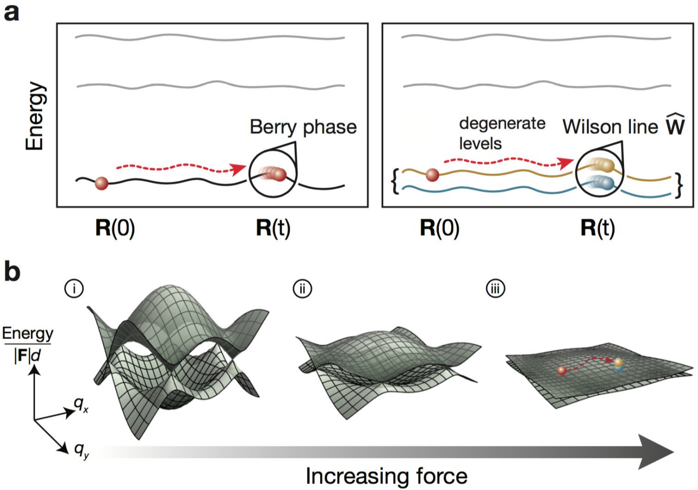
ab initio calculation of imbalanced lattice



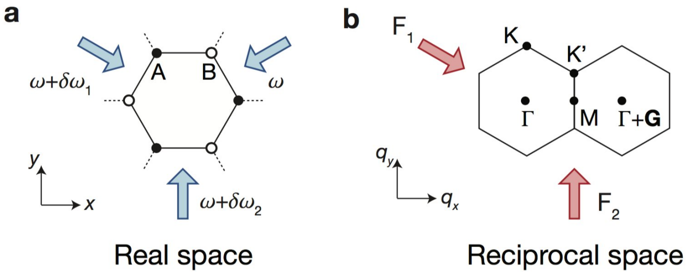
Seeing Dirac points annihilating clear.

Outline

Experimental reconstruction of Wilson lines in Bloch bands



Wilson line regime in honeycomb lattice



- force $\mathbf{F}$: $\mathbf{q}(t) = \mathbf{q}(0) + \mathbf{F}t$
- unitary time-evolution operator (Wilson line matrix)

$$\mathrm{W}_{\mathbf{q}(0)\to\mathbf{q}(t)} = \mathcal{P}\exp(\mathrm{i}\int_C \mathrm{A}_{\mathbf{q}} d\mathbf{q})$$

$\mathrm{A}_{\mathbf{q}}$: Wilczek-Zee connection
$\mathcal{P}$: Path-ordering (non-Abelian)

Wilson line

- $\mathrm{W}_{\mathbf{q}(0)\to\mathbf{q}(t)} = \mathcal{P}\exp\left(\mathrm{i}\int_C \mathrm{A}_{\mathbf{q}} d\mathbf{q}\right)$
- Bloch state (presence of lattice) $|\Phi^n_{\mathbf{q}}\rangle = e^{\mathbf{i}\mathbf{q}\cdot\mathbf{r}}|u^n_{\mathbf{q}}\rangle$
- Wilczek-Zee connection $\mathrm{A}^{n,n'}_{\mathbf{q}} = \mathrm{i}\langle u^n_{\mathbf{q}}|\nabla_{\mathbf{q}}|u^{n'}_{\mathbf{q}}\rangle$
- $n = n'$: Berry connections of individual Bloch bands yields Berry phase along a closed path
- $n \neq n'$: inter-band Berry connections induce inter-band transition

Dynamics in combined lattice with external force

- total Hamiltonian: $H = H_0 - \mathbf{F} \cdot \mathbf{r}$
- lattice
$$H_0 = \sum_{\mathbf{q},n} E^n_{\mathbf{q}} |\Phi^n_{\mathbf{q}}\rangle\langle\Phi^n_{\mathbf{q}}|$$
constant force $\mathbf{F}$
- initial state $|\psi(0)\rangle = \sum_n \alpha^n(0)|\Phi^n_{\mathbf{q}_0}\rangle$
$|\alpha^n(0)|^2$ gives the population in the n^{th} band at $t = 0$
- ansatz
$$|\psi(t)\rangle = \sum_n \alpha^n(t)|\Phi^n_{\mathbf{q}(t)}\rangle$$
$$\mathbf{q}(t) = \mathbf{q}_0 + \mathbf{F}t$$

Wilson lines and Wilczek-Zee connections

Example (two-band system)

$$\mathrm{i}\partial_t \begin{pmatrix} \alpha^1(t) \\ \alpha^2(t) \end{pmatrix} = \begin{pmatrix} E^1_{\mathbf{q}(t)} - \xi^{1,1}_{\mathbf{q}(t)} & -\xi^{1,2}_{\mathbf{q}(t)} \\ -\xi^{2,1}_{\mathbf{q}(t)} & E^1_{\mathbf{q}(t)} - \xi^{2,2}_{\mathbf{q}(t)} \end{pmatrix} \begin{pmatrix} \alpha^1(t) \\ \alpha^2(t) \end{pmatrix}$$

where

$$\xi^{n,n'}_{\mathbf{q}(t)} = \mathrm{A}^{n,n'}_{\mathbf{q}(t)} \cdot \mathbf{F} = \mathrm{i}\langle u^n_{\mathbf{q}(t)} | \partial_t | u^{n'}_{\mathbf{q}(t)} \rangle$$

and thus

$$\mathrm{A}^{n,n'}_{\mathbf{q}(t)} = \mathrm{i}\langle u^n_{\mathbf{q}} | \nabla_{\mathbf{q}} | u^{n'}_{\mathbf{q}} \rangle \Big|_{\mathbf{q}=\mathbf{q}(t)}$$

Wilson line regime of *the* two-band model

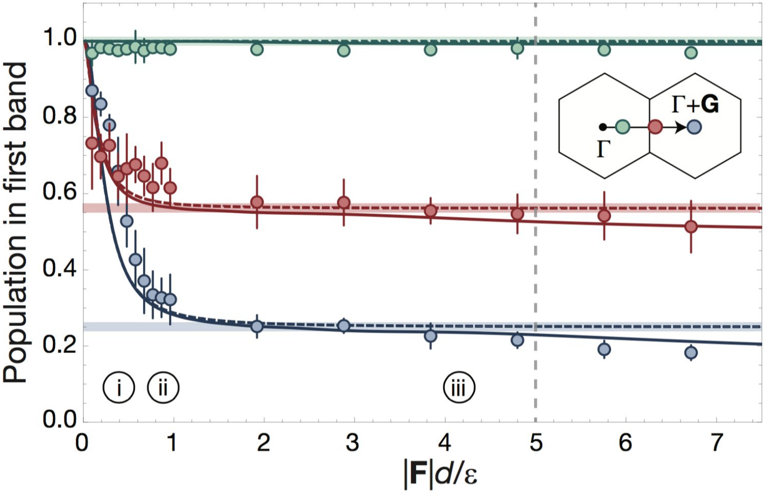
$$\mathrm{i}\partial_t \begin{pmatrix} \alpha^1(t) \\ \alpha^2(t) \end{pmatrix} = \begin{pmatrix} -\xi^{1,1}_{\mathbf{q}(t)} & -\xi^{1,2}_{\mathbf{q}(t)} \\ -\xi^{2,1}_{\mathbf{q}(t)} & -\xi^{2,2}_{\mathbf{q}(t)} \end{pmatrix} \begin{pmatrix} \alpha^1(t) \\ \alpha^2(t) \end{pmatrix}$$

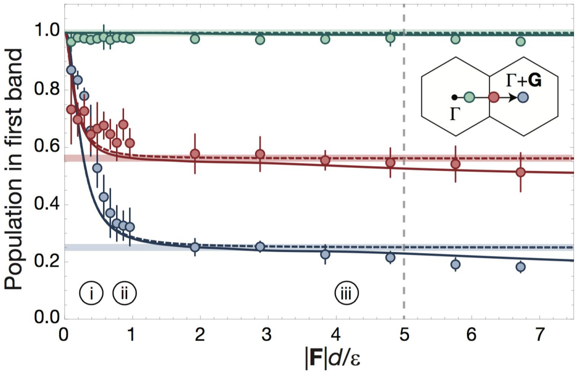
- Defining $\xi_{\mathbf{q}(t)}$ as the matrix with elements $\xi^{n,n'}_{\mathbf{q}(t)}$
- state evolution

$$|\psi(t)\rangle = \mathcal{T}\exp(\mathrm{i}\int \xi_{\mathbf{q}(t)} dt)|\psi(0)\rangle \equiv \mathrm{W}|\psi(0)\rangle$$

- thus $\mathrm{W}_{\mathbf{q}(0)\to\mathbf{q}(t)} = \mathcal{P}\exp\left(\mathrm{i}\int_C \mathrm{A}_{\mathbf{q}} d\mathbf{q}\right)$

Measuring Wilson lines





- Matrix elements of Wilson line operator

$$W^{m,n}_{\mathbf{Q}\to\mathbf{q}} = \langle \Phi^m_{\mathbf{q}} | e^{i(\mathbf{q}-\mathbf{Q})\cdot\hat{\mathbf{r}}} | \Phi^n_{\mathbf{Q}} \rangle = \langle u^m_{\mathbf{q}} | u^n_{\mathbf{Q}} \rangle$$

- saturation value $W^{11}_{\Gamma\to\mathbf{q}} = \langle u^1_{\mathbf{q}} | u^1_{\Gamma} \rangle$ of population after transport measures overlap between $|u\rangle$

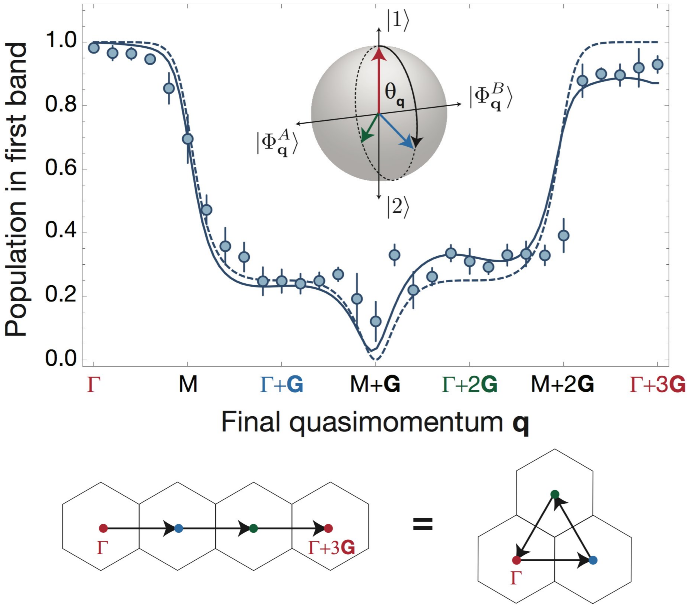
Reconstructing band eigenstates

- cell-periodic Bloch state as pseudo-spin
- cell-periodic Bloch functions at a fixed reference quasimomentum $\mathbf{Q}$

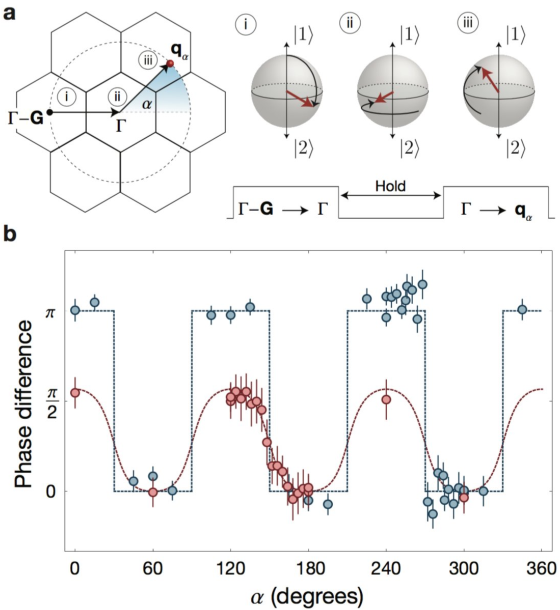
$$|1\rangle = |u^1_{\mathbf{Q}}\rangle \qquad |2\rangle = |u^2_{\mathbf{Q}}\rangle$$

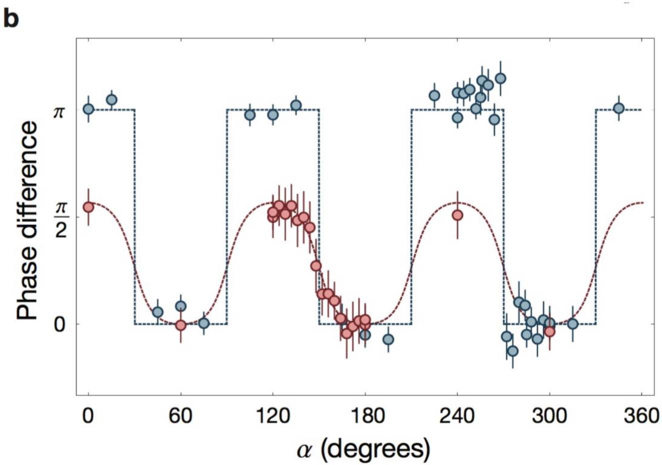
- such that $|u^1_{\mathbf{q}}\rangle = \cos(\frac{\theta_{\mathbf{q}}}{2})|1\rangle + \sin(\frac{\theta_{\mathbf{q}}}{2})e^{\mathrm{i}\phi_{\mathbf{q}}}|2\rangle$
- thus $(\theta_{\mathbf{q}}, \phi_{\mathbf{q}})$ characterize a state
- Throughout this work basis states are chosen at reference point $\mathbf{Q} = \Gamma$

Measuring mixing angle $\theta_{\mathbf{q}}$



Measuring relative phase $\phi_{\mathbf{q}}$





- AB-site degeneracy (blue) — π jump
- AB-site offset (red) by elliptically-polarized lattice beam — continuous varying phase
- 3-fold symmetry of system (no matter whether offset)

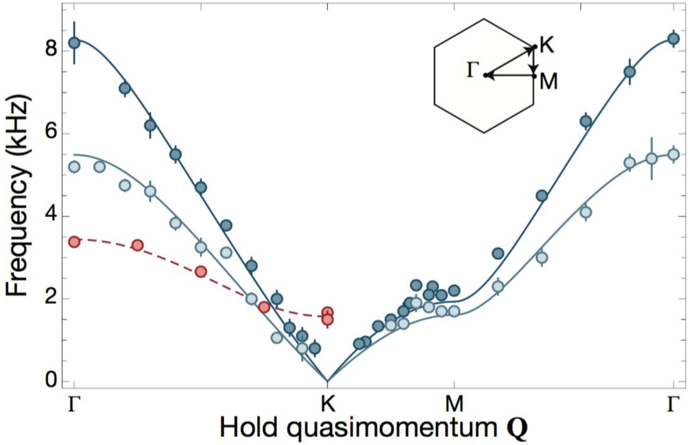
Determining Wilson line eigenvaluse

Wilson-Zak loop

$$\mathrm{W}_{\mathbf{q}\to\mathbf{q}+\mathbf{G}}$$

- $\hat{\mathrm{A}} \Longrightarrow \hat{\mathrm{A}}_{U(1)} + \hat{\mathrm{A}}_{SU(2)}$
- the $U(2)$ Wilson line $\Longrightarrow U(1)$ global phase multiplied by a $SU(2)$ matrix.
- $SU(2)$ matrix eigenvalues $e^{\pm \mathrm{i}\xi}$
- experimental data analysis gives: $\xi = 1.03(2)\pi/3$
- theoretical expected value: $\xi = \pi/3$

Mapping dispersion relation



by varying the reference quasimomentum $\mathbf{Q}$

Reference

Immanuel Bloch *et al.*, *An Aharonov-Bohm interferometer for determining Bloch band topology*, Science **347**, 288-292 (2015).

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Thank you !