Unit 2 Lesson 8

- Consider again the DE

$$m\ddot{x} + b\dot{x} + kx = B\cos(\omega t)$$

- The complex gain is ~~defined as~~

$$\frac{1}{P(i\omega)}$$

- The gain is

$$\frac{1}{|P(i\omega)|},$$

- ~~The~~ and phase lag is

$$\tan^{-1}\left(\frac{b\omega}{k - m\omega^2}\right)$$

- We have general particular solution to the above DE is

$$x_p = (\text{gain})\, B\cos(\omega t - (\text{Phase lag}))$$

- For systems with $mk - (b^2/2) > 0$, there exists "practical resonant frequency

$$\omega_r = \sqrt{\frac{k}{m} - \frac{b^2}{2m^2}}$$

Example Problem

1: Noting $P(iw) = -w^2 + \frac{iw}{4} + 2$, then the complex gain is

$$\frac{1}{-w^2 + \frac{iw}{4} + 2}.$$

2: ~~Solving~~

$$\frac{1}{|-w^2 + iw/4 + 2|} = \frac{1}{\sqrt{(2-w^2)^2 + (iw/4)^2}} = \left(w^4 - \frac{63}{16}w^2 + 4\right)^{-1/2}$$

3: When $w=1$, we have gain $4/\sqrt{17}$