Unit 4 Lesson 2

- To spite the presenter I'll try and prove

  $\det(AB) = \det(A)\det(B)$

- Let

$$A = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix},\quad B = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}; \text{then}$$

$$AB = \begin{bmatrix} a_1a_2 + b_1c_2 & a_1b_2 + b_1d_2 \\ c_1a_2 + d_1c_2 & c_1b_2 + d_1d_2 \end{bmatrix}$$

I suddenly understand that when they said "no-one can prove" they meant impossibly tedious.

- We can reformat a linear DE to the form

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

or in turn, solving

$$\lambda \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

for λ gets us the same roots

Example Problem

Solving

$$\left|\begin{bmatrix} -3-\lambda & 4 \\ -2 & 3-\lambda \end{bmatrix}\right| = 0 \text{ gets us } \lambda = \pm 1$$

Then, solving for a_1, a_2 for $\lambda = \pm 1$ in

$$\begin{bmatrix} -3-\lambda & 4 \\ -2 & 3-\lambda \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

gets us eigenvectors

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \text{ and solution}$$

$$x = C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{t} + C_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-t}.$$