18.03 Differential Equations: Week 10

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March 9th, 2020

Progress Update

Over the past week we have covered:

1. Operators
2. Resonance

Operators

We define operators as objects that act to modify functions; consider these example operators:

1. The multiplication operator: $F(f(x), g(x)) = f(x) \cdot g(x)$
2. The differentiation operator: $D(f(x)) = f'(x)$

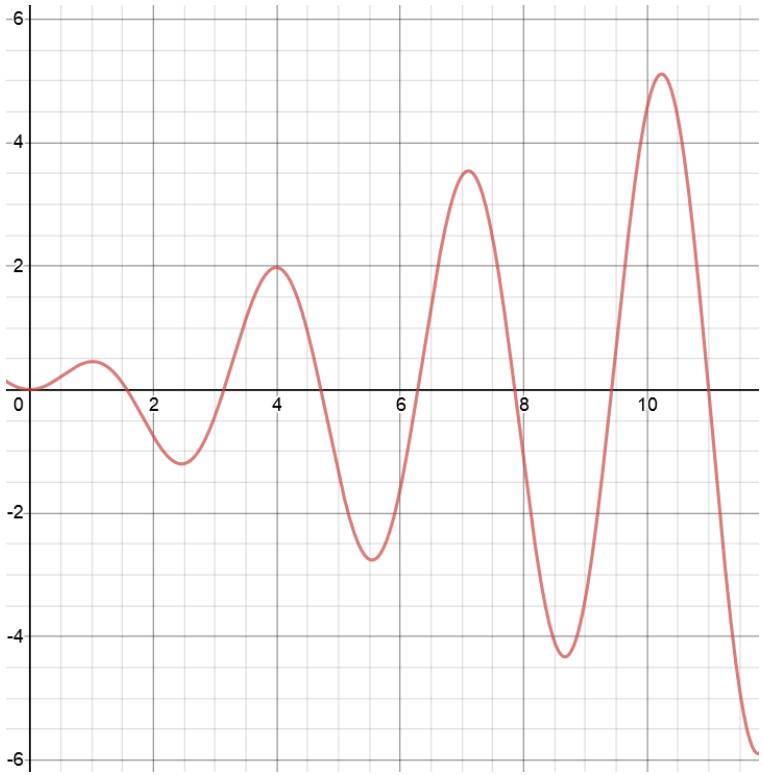
And basically any other ones you can come up with; this notation will be useful when we begin talking about non-linear ODEs.

Resonance

We define resonance as a correlation between an acting force on a system and some changing property in the system; consider the DE

$$\ddot{x} + 4x = 2\cos(2t) \tag{1}$$

with its solution



Example Problem

Consider the DE on the previous slide that I borrowed from the example problem:

$$\ddot{x} + 4x = 2\cos(2t) \tag{2}$$

What is the particular solution for x?

Example solution

We complex replace using Euler's formula, defining $z = x + iy$:

$$\ddot{z} + 4z = 2e^{2it} \tag{3}$$

with characteristic polynomial

$$p(a) = a^2 + 4. \tag{4}$$

Since $p(2i) = 0$, we have to use the g