18.03 Differential Equations: Week 11

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Progress Update

Over the past week we have covered:

1. Frequency Response
2. RLC circuits

Frequency response

For a differential equation of the form

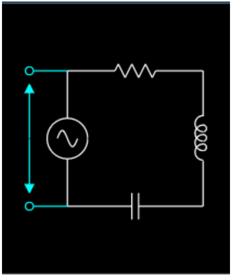
$$m\ddot{x} + b\dot{x} + kx = B\cos(\omega t) \tag{1}$$

there are a few variables that are "applied" to the input signal that get you your output signal:

1. Complex gain $= (p(i\omega))^{-1}$, the ratio between the input and output signals when you consider the complex case of each.
2. Gain $= |(\text{Complex Gain})|$, is the ratio between the real parts of the input and output.
3. Phase lag $= \tan^{-1}(\frac{b\omega}{k - m\omega^2})$

RLC Circuits

Consider this circuit of a resistor, coil, and capacitor in series:



Unfortunately my understanding of this scenario is very poor, but there is one significant piece of information I was able to extract; for certain values of resistance, inductance, and capacitance, you can generate voltage drops perfectly in phase, out of phase, etc by supplying a sinuisoidal input.

Example Problem

Consider this problem:

1. Write down the equation for the system we are working with. Compute its complex gain $H(\omega)$. (That is, make the complex replacement $F_{cx} = e^{i\omega t}$ for the input signal, and express the exponential system response z_p as a complex multiple of F_{cx}: $z_p = H(\omega)F_{cx}$.)

And the corresponding solution: We get the equation

$$\ddot{x} + \frac{1}{4}\dot{x} + 2x = \cos wt \tag{2}$$

and using the characteristic polynomial of the complex replacement

$$p(i\omega) = -\omega^2 + \frac{i\omega}{4} + 2 \tag{3}$$

we have the complex gain

$$(\text{Complex Gain}) = \frac{1}{-\omega^2 + \frac{i\omega}{4} + 2} \tag{4}$$