Exam 2

Problem 1

(a): Recall that this system is critically damped when there is one root with order n multiplicity; thus we need

$$B^2 = 4ac \quad \text{or}$$

$$1 = 4k$$

$$k = \frac{1}{4}$$

(b): Over damped

(c): $k = \pi^2 + 1/4$

Problem 2

(a): Suppose $x = ue^{2t}$; then

$\dot{x} = u2e^{2t} + \dot{u}e^{2t} = (\dot{u} + 2u)e^{2t}$

$\ddot{x} = \cancel{4ue^{2t} + 2\dot{u}e} = (\ddot{u} + 4\dot{u} + 5u)e^{2t}$

and thus.

$\ddot{x} + x = 5te^{2t}$ implies $\ddot{u} + 4\dot{u} + 5u = 5t$;

$u = t - 4/5$, $x_p = (t - 4/5)e^{2t}$

(b): $x = y + 2\cos t + 3\sin t$

Problem 3

(a) Consider the complex replacement

$\ddot{z} + b\dot{z} + kz = e^{i\omega t}$, then the amplitude of the real part is

$$\frac{1}{|p(i\omega)|} = \frac{1}{|(i\omega)^2 + bi\omega + k|} = \frac{1}{|k - \omega^2 + bi\omega|}$$

which is maximized for $k = \omega^2$.

(b): $p(s) = s(s-1)(s+1)$ implies

$$x = a + be^{-t} + ce^{t}$$

Problem 4

(a): Plugging in, we get

$$H(\omega) = \frac{6}{(6-\omega^2)+i\omega}$$

(b) $$|H(2)| = \frac{3}{|1+i|} = \frac{3}{\sqrt{(1+i)(1-i)}} = \frac{3}{1+1-1} = \frac{3}{\sqrt{2}}$$

(c) $\pi/4$

Problems

(1): A potential solution to this is

$$x = \frac{1}{2}\left(t - \frac{1}{2}\right)\sin\left(2\left(t - \frac{1}{2}\right)\right)$$

(2): $x = t\sin(2t)$

(3): $m = 2, b = 0, k = 8$