18.03 Differential Equations: Week 8

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Progress Update

Over the past week we have covered:

1. Damped Harmonic Oscillators
2. Solving Second Order Linear DEs with exponential/sinusoidal input

Damped harmonic oscillators

Consider some mass on a spring set to vibrate; with its vertical height above the axis represented by x and time represented by t; we have the "damped harmonic oscillator equation":

$$m\ddot{x} + b\dot{x} + kx = 0 \tag{1}$$

where m is the mass constant, b is the damping constant, and k is the spring constant. Now consider this equations characteristic polynomial:

$$mr^2 + br + k = 0 \tag{2}$$

Second order linear DEs with exponential/sinusoidal input

Solving these second order LDEs occurs almost just as it did in the first order case:

1. If exponential input, make an exponential ansatz with matching power and solve for leading A, then add homogeneous solution.
2. If sinusoidal input, complex replace (add an imaginary equation of similar form) and apply (1), then find the real part; add homogeneous solution.

A formula for the particular solution exists:

$$x_p = \frac{Bt^n e^{at}}{P^{(n)}(a)} \text{ for the first integer n such that } P^{(n)} \neq 0 \tag{3}$$

Example problem

Consider this problem from the practice problems:

Find the general solution to

$$x'' + 8x' + 7x = 2e^{-t}.$$