Unit 1 Lesson 5: Linear ODE's

Notes

- A first order linear differential equation of some $X(t)$ is of the form

$$A(t)\frac{dX}{dt} + B(t)\,X(t) = C(t)$$

where $A(t)$ and $B(t)$ are called coefficients, and dividing by $A(t)$ on each side yields the standard form.

- If $C(t) = 0$, then the equation is homogeneous

Unit 1 Lesson 5 Quiz:

A: 1: No
2: ~~No~~ Yes
3: Yes

B: Deposit

Unit 1 Lesson 5 Problems

For equations of standard form

$$y' + a(x)\, y = f(x),$$ we define $u(x) = \exp\left(\int a(x)\, dx\right)$,

the general solution is

$$y = \frac{\int u(x)\, f(x)\, dx + C}{u(x)}$$

1: Plugging in,

$$y = \frac{\int (e^{\int 1\, dx}) \cdot 2\, dx) + C}{e^x} = \frac{2e^x + C}{e^x}$$

$$y(0) = 2 + C = 0 \Rightarrow C = -2, \text{ thus}$$

$$y = \frac{2(e^x - 1)}{e^x}.$$

2: $$y = \frac{\int (\exp(\int -2\, dx)\, 3e^{2x})\, dx + C}{\exp(\int -2\, dx)}$$

$$= \frac{3x + C}{e^{-2x}}$$

$$y(0) = 0 = \frac{C}{1} \Rightarrow C = 0 \Rightarrow y = \frac{3x}{e^{-2x}}$$

3: $$y = \frac{\int (\exp(\int 3\, dx))\, 2x e^{-3x}\, dx + C}{e^{3x}}$$

$$= \frac{x^2 + C}{e^{3x}};$$