# Unit 1 Lesson 10 Notes

- The general solution to a DE is composed of the addition of the particular and homogeneous solutions.

- One can solve a linear constant coefficient DE by making an ansatz of the form

$$Ae^{rt}$$

- One can use Euler's form to solve in case of sinusoidal input, solving for $x = Re(z)$.

# Unit 1 Lesson 10 problems

1: First make the ansatz

$x_p = Ae^t$; $\Rightarrow Ae^t + 2Ae^t = e^t \Rightarrow A = 1/3$, thus

$x_p = \frac{1}{3}e^t$

The homogeneous solution would be

$x_h = e^{-kt}$, thus $x_h = Ce^{-2t}$ and thus $x = \frac{1}{3}e^t + Ce^{-2t}$

2: Making the ansatz $x_p = A e^{2it}$,

$$\dot{A} e^{2it} + 2iA e^{2it} + A e^{2it} = e^{2it}$$

$$\dot{A} + A(2i+1) = 1 \Rightarrow A = (2i+1)^{-1},$$

Thus

$$x_p = \frac{e^{2it}}{2i+1}, \quad x_h = Ce^{-2t}, \quad x = x_p + x_h$$

3: Recall Euler's formula:

$$e^{it} = \cos(t) + i\sin(t);$$ letting

$z = x + iy$, note that

$$\dot{x} + 2x = \cos(2t)$$

$$+$$

$$i(\dot{y} + 2y) = i\sin(2t)$$

$$\Downarrow$$

$$\dot{z} + 2z = e^{2it} \Rightarrow x = \frac{e^{2it}}{1+2i} + Ce^{-2t}$$