Unit 4 Lesson 6

- The idea of this lesson is that, ~~nea~~ sufficiently close to some point,

$$\dot{x} = f(x, y) \approx a_1 x + b_1 y + \dots$$
$$\dot{y} = g(x, y) \approx a_2 x + b_2 y + \dots$$

- This is called linearization, and the general method is to make a change of variables to eliminate higher-order terms.

Example Problem

1: We begin our analysis of this system by computing the ~~Jac~~ Jacobian; given

$$\dot{x} = x - y + xy$$
$$\dot{y} = 3x - 2y - xy$$

We have

$$J = \begin{bmatrix} \frac{\dot{x}}{\partial x} & \frac{\dot{x}}{\partial y} \\ \frac{\dot{y}}{\partial x} & \frac{\dot{y}}{\partial y} \end{bmatrix} = \begin{bmatrix} 1-y & -1+x \\ 3-y & -2-x \end{bmatrix}.$$

They state the critical point is at the origin, so

$$J(0,0) = \begin{bmatrix} 1 & -1 \\ 3 & -2 \end{bmatrix}$$

Find the eigenvalues:

$$\det\left(\begin{bmatrix} 1-\lambda & -1 \\ -3 & -2-\lambda \end{bmatrix}\right) = \lambda^2 + \lambda + 1 = 0 \Rightarrow \lambda = \frac{-1 \pm \sqrt{-3}}{2}$$

Since this has imaginary ~~to~~ eigenvalues with negative real part, this is a spiral sink portrait.