Unit 1 Exam: Problem 1

(a) $k = 1$

(b) $\dot{x} = \left(1 - \frac{x}{1000}\right) x$

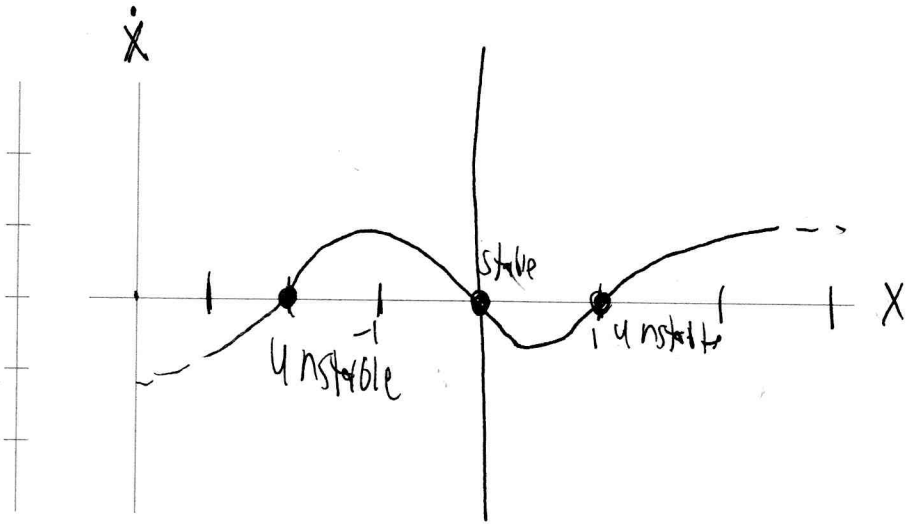
(c) 187.5

Problem 2

2. For the autonomous equation $\dot{x} = x(x-1)(x+2)$, please sketch:

(a) the phase line, identifying the critical points and whether they are stable, unstable, or neither. [4]

(b) at least one solution of each basic type (so that every solution is a time-translate of one you have drawn) [4]



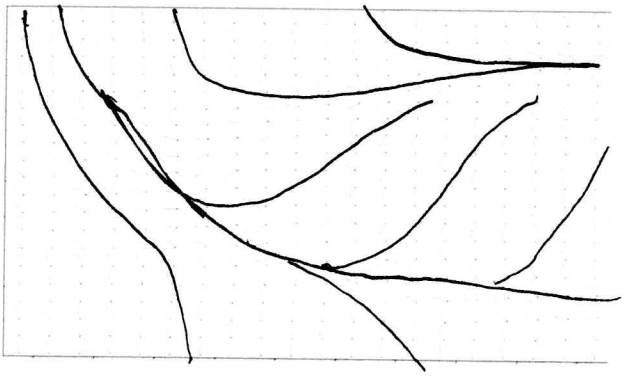
Below is a diagram of a direction field of the differential equation $y' = (1/4)(x - y^2)$. On it please plot and label:

(c) the nullcline [3]

(d) at least two quite different solutions [3]

(e) the separatrix (if there is one) [3]

(f) True or false: If $y(x)$ is a solution with a minimum, then for all large enough x, $y(x) < \sqrt{x}$. (No explanation needed: just circle one.) [3]



Problem 3

(a)

Step #	X	y	slope
0	0	1	1
1	0.5	1.25	2.25
2	1	3.125	4.125
3	1.5	5.4375	6.9375

(b) $x = \frac{\pi + \sin(t)}{7}$

Problem 4

(a): Multiply by 1:

$$\frac{3-2i}{(3+2i)(3-2i)} = \frac{3-2i}{9+4} = \frac{3}{13} - \frac{2}{13}i$$

(b): $1 - i = re^{i\theta} = r(\cos\theta - i\sin\theta) \Rightarrow r = \sqrt{2}, \theta = \frac{\pi}{4}$

(c): $a = 16, b = 0$

(d): $a = \frac{1}{2}, b = \frac{\sqrt{3}}{2}$

(e): $a = -2, b = 0$

(f): $a = 2\sqrt{2}, \phi = \frac{-\pi}{4}$

Problem 5

(a): Make exponential ansatz

$X = Ae^{2t}$, then

$$2Ae^{2t} + \dot{A}e^{2t} + 3Ae^{2t}$$

$$5Ae^{2t} = e^{2t} \Rightarrow A = 1/5, \text{ thus}$$

$X_p = \frac{1}{5}e^{2t}$; X_h is s.t. $\ddot{x} + 3x = 0 \Rightarrow x_h = ce^{-3t}$, so

$$x = \frac{1}{5}e^{2t} + Ce^{-3t}$$

(b): Plug in:

$$x(0) = 1 = \frac{1}{5} + C \Rightarrow C = \frac{4}{5}, \text{ thus}$$

$$x = \frac{1}{5}e^{2t} + \frac{4}{5}e^{-3t}$$

(c): For $z = (x + iy)$, where $i\dot{y} + 3iy = i\sin(2t)$,

$$\dot{z} + 3z = e^{2it}$$

(d): $x_p = \frac{3}{13}\cos(2t) + \frac{2}{13}\sin(2t)$