Unit 2 Lesson 3

- The principle of super position in DE's states that, if two DEs differ ~~by a~~ only by an input, then the solution to the addition of the two equations is the sum of the original solutions:

$$a_2 \ddot{x} + a_1 \dot{x} + a_0 x = b_1 \text{ has sol. } y_1$$

$$\text{〃} = b_2 \text{ has sol. } y_2$$

$$\text{〃} = b_1 + b_2 \text{ has sol. } \overset{c_2}{\underset{\vee}{}} y_1 + y_2$$

- To solve for exponential input of second order constant linear DE, make an exponential input with common power.

- To solve for sinusoidal input, complex replace and solve as in (2).

- Defining $P(r)$ = the characteristic polynomial of a given equation, we have general solution for the particular solution

$$x_p = \begin{cases} Be^{at}/P(a) & \text{if } P(a) \neq 0 \\ Bte^{at}/P'(a) & \text{if } P'(a) \neq 0 \\ Bt^2e^{at}/P''(a) & \text{if } P''(a) \neq 0 \\ \text{etc.} & \end{cases} \quad \text{for } p(D)x = Be^{at}$$

Practice Problem

Consider the differential equation

$$\ddot{x} + 8\dot{x} + 7x = 2e^{-t}$$

We have

$$P(r) = r^2 + 8r + 7 \Rightarrow P(-1) = 1 + -8 + 7 = 0;$$

$$P'(r) = 2r + 8 \Rightarrow P(-1) = 6, \text{Thus}$$

$$x_p = \frac{2te^{-t}}{6} = \frac{1}{3}te^{-t}$$

We also gethomogenous solution

$$x_h = C_1 e^{-t} + C_2 e^{-7t}$$

and thus

$$x = x_h + x_p$$