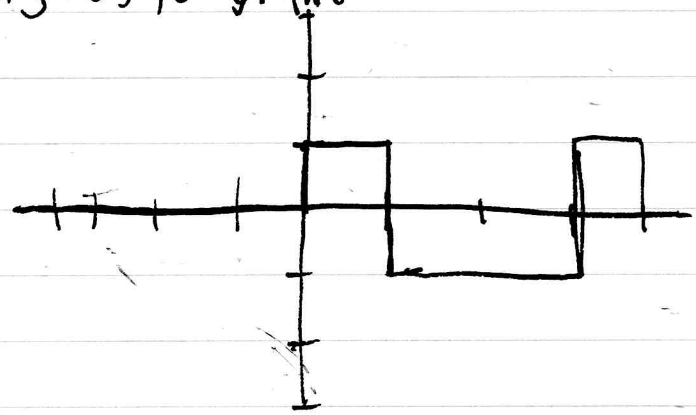
Final Exam

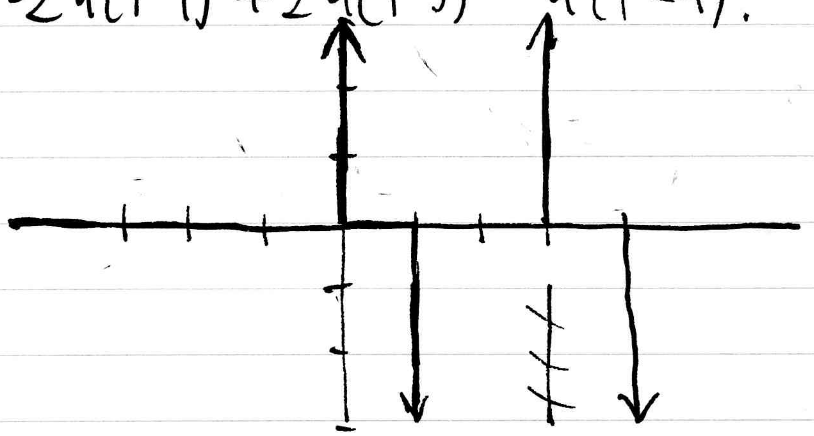
Problem 1

(a): By inspection, $v(t)$ is graphed



(b): $v(t) = u(t) - 2u(t-1) + 2u(t-3) - u(t-4)$.

(c):



(d): Since this infinitely fast impulses, we don't use $u(t)$;

$$\dot{v}(t) = \delta(t) - 2\delta(t-1) + 2\delta(t-3) - \delta(t-4)$$

(e): By Green's Formula,

$$~~\text{[scribbled]}~~\; a(t) = (t-1)\,u(t-1)$$

Problem2

(a): Just take the multiplicative inverse of $P(s)$:

$$W(s) = \frac{1}{2s^2+8s+16} = \frac{1}{2} \frac{1}{s^2+4s+8}$$

(b): By (A), taking inverse Laplace Transform gets us

$$w(t) = \frac{1}{4} e^{-2t} \sin(2t)$$

(c):

$$X(s) = \frac{1}{2s^2+8s+16}\left(\frac{1}{s^2+1} + 2s + 12\right)$$

Problem 3

(a): Lets solve

$$\begin{vmatrix} 2-\lambda & 12 \\ 3 & 2-\lambda \end{vmatrix} = 0 = (2-\lambda)^2 - 36$$

$$= 4 + \lambda^2 - 4\lambda - 36$$

$$= \lambda^2 - 4\lambda - 32$$

$$= (\lambda - 8)(\lambda + 4) \Rightarrow \lambda = 8, -4$$

(b): Now we must solve

$$(A - 8I)v = 0 \quad \text{and} \quad (A + 4I)v = 0$$

By inspection,

$$\begin{bmatrix} -6 & 12 \\ 3 & -6 \end{bmatrix} v_1 = 0$$

has

$$v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Similarly,

$$\begin{bmatrix} 6 & 12 \\ 3 & -6 \end{bmatrix} v_2 = 0$$

has

$$v_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

(c): use the given eigen-pairs to note that

$$F(t) = \begin{bmatrix} e^t & -e^{2t} \\ e^t & e^{2t} \end{bmatrix}$$

Recalling

$$e^{tB} = F(t)\,F(0)^{-1}, \text{ then}$$

Problem #3 continued

$$e^{tB} = \begin{bmatrix} e^{t} & -e^{2t} \\ e^{t} & e^{2t} \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} e^{t} + e^{2t} & e^{t} - e^{2t} \\ e^{t} - e^{2t} & e^{t} + e^{2t} \end{bmatrix}$$

(d): $u(t) = \frac{1}{2} \begin{bmatrix} 3e^{t} + e^{2t} \\ 3e^{t} - e^{2t} \end{bmatrix}$

Problem 4

(a): ~~dr~~ Since the equilibria are where each DE is zero,

$$x^2 + y^2 - 8 = 0$$
$$\underline{x^2 - y^2 = 0} \quad \text{adds and subtracts to}$$
$$2x^2 - 8 = 0, \text{ and}$$
$$\underline{2y^2 - 8 = 0}, \text{ thus}$$
$$x^2 = 4$$
$$y^2 = 4,$$

and

$$(x,y) = (2,2), (-2,2), (-2,-2), (2,-2).$$

(b): We have

$$J(x,y) = \begin{bmatrix} 2x & 2y \\ 2x & -2y \end{bmatrix} \text{ and thus } J((-2,2)) = \begin{bmatrix} -4 & 4 \\ -4 & -4 \end{bmatrix}$$