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Independent Study - Differential Equations
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Unit 2 Lesson 2

- For a differential equation of this form:
  - $m\ddot{x} + b\dot{x} + kx = 0$
- We define m as the mass constant, b as the damping constant, and k as the spring constant.
  - If $b = 0$, then $x = A\cos(\omega t - \phi);\ \omega = \sqrt{\frac{k}{m}}$
- Different values of b have different values based off of the solutions to r in the differential equation:
  - $b = 0$ is the *simple harmonic oscillator/undamped.* This differential equation and its graph is of the shape $A\cos(\omega t - \phi)$.
  - $b^2 \leq 4mk$ is *underdamped.*
  - $b^2 = 4mk$ is *critically damped.*
  - $b^2 \geq 4mk$ is *overdamped.*
- Practice problem:
  - Start with $\ddot{x} + \omega^2 x = 0$. What is the characteristic polynomial? What are its roots? What are the exponential solutions – the solutions of the form e^{at}? These may be complex exponentials. What are their real and imaginary parts? Check that these are also solutions to the original equation. What is the general real solution?
- Practice solution:
  - We get polynomial $r^2 + \omega^2 = 0$; solving yields $r = \pm i\omega$. We get exponential solutions $e^{\omega it}, e^{-\omega it}$; in turn we get real solutions $\pm - \cos(\omega t)$ and imaginary solutions $\pm \sin(\omega t)$. The general solution is of the form $x = A\cos(\omega t - \phi)$.