

# 18.03 Differential Equations: Week 9

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March 2nd, 2020

# Progress Update

Over the past week we have covered:

- ① Stability of ODEs
- ② Polynomial Input

# Stability of ODEs

We call an ODE stable if its homogeneous solution (now also known as the transient solution)  $x_h(t)$  goes to 0 as  $t \rightarrow \infty$ ; we can determine this trait using the transience theorem:

$$\text{For } x(t) \text{ satisfying } p(D)x = 0, x(t) \rightarrow 0 \text{ if } \text{Real}(r_k) < 0 \quad (1)$$

# Polynomial Input

Solving for polynomial input is incredibly easy; make a polynomial ansatz of the form:

$$a_k t^k + a_{k-1} t^{k-1} + \cdots + a_0. \quad (2)$$

In this form, we set  $k$  to the degree of the input, and let  $a_n$  be solved for later. We then match corresponding coefficients to get a system of linear equations, and solve.

# Example Problem

Consider the following example problem:

Find the polynomial solution of  $\ddot{x} - x = t^2 + t + 1$ .