18.03 Differential Equations: Week 16

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April 20th, 2020

Progress Update

Over the past week we have covered:

1. Solving linear systems of DE's

Solving linear systems of DE's

Consider what we define as a 2-by-2 linear system of differential equations:

$$\dot{x} = ax + by \tag{1}$$
$$\dot{y} = cx + dy \tag{2}$$

To solve any system of this form, we follow a few simple steps:

1. If solving for x, solve for x in (2) and substitute into (1) after manipulation, and vice versa.
2. Solve the acquired DE, get your solutions for that variable.

Example Problem

Consider the example problem where we want to solve for x in

$$\dot{x} = x + 3y \tag{3}$$
$$\dot{y} = x - y. \tag{4}$$

Begin by solving for y in the first line:

$$y = \frac{1}{3}\dot{x} - \frac{1}{3}x \tag{5}$$

Then substitute into the second line:

$$\frac{1}{3}\ddot{x} - \frac{1}{3}\dot{x} = x - \frac{1}{3}\dot{x} + \frac{1}{3}x \tag{6}$$

Example Problem (continued)

Multiply each side by 3 for clarity:

$$\ddot{x} - \dot{x} = 3x - \dot{x} + x \tag{7}$$

Solve the characteristic polynomial for

$$\ddot{x} - 4x = 0 \tag{8}$$

to get

$$x = c_1 e^{2t} + c_2 e^{-2t} \tag{9}$$