18.03 Differential Equations: Week 15

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Progress Update

Over the past week we have covered:

1. Inverse Laplace Transform
2. Solving DE's via Laplace transforms

Referencing the table of Laplace transforms, we can work backwards. Consider the example function we are asked to find the inverse of:

$$X(s) = \frac{2s+1}{s^2+9}. \tag{1}$$

Example Problem Solution

We begin by separating the numerator:

$$\frac{2s+1}{s^2+9} = \frac{2s}{s^2+9} + \frac{1}{s^2+9}. \tag{2}$$

Then, since we know

$$L(\cos(\omega t)) = \frac{s}{s^2+\omega}, \; L(\sin(\omega t)) = \frac{\omega}{s^2+\omega^2} \tag{3}$$

then factoring gets us

$$x = 2\cos(3t) + \frac{1}{3}\sin(3t). \tag{4}$$

Solving DE's via Laplace Transforms

There is a simple series of steps to solve DE's using Laplace transforms:

1. Take the Laplace transform of each side, with $x^{(n)}$ being replaced by $s^n X$.
2. Solve for X.
3. Take the inverse Laplace transform.

Example DE Solution

Consider the DE

$$x + 3x = e^{-t} \text{ with rest conditions.} \tag{5}$$

Start by taking the Laplace transform of each side:

$$sX + 3X = \frac{1}{s+1} \tag{6}$$

Move all non-X items to the right side:

$$X = \frac{1}{(s+1)(s+3)} \tag{7}$$

Heaviside coverup:

$$X = \frac{-1/2}{s+1} + \frac{1/2}{s+3} \tag{8}$$

and then take the inverse Laplace transform:

$$x = \frac{1}{2}e^{-3t} - \frac{1}{2}e^{-t} \tag{9}$$