Unit 1 Lesson 2 Notes: Basic DE's

- $\dot{y} = ay$ is solved by $y = Ce^{at}$ where C is a constant.

- For an object represented by y above earth's surface,
$$\frac{d^2y}{dt^2} = -g$$

Separation of Variables

- Consider the case
$$\frac{dy}{dx} = x(y-1)$$

Separate variables:

$$\int \frac{dy}{y-1} = \int dx\ x$$

$$\log(y-1) = \frac{x^2}{2} + C$$

$$|y-1| = e^{\frac{x^2}{2}} e^C$$

$$y = \left(\pm e^{\frac{x^2}{2}} e^C\right) + 1$$

Unit 1 Lesson 2 Quiz 1: Basic DE's

$$\frac{dy}{dx} = 2y + 1$$

$$\int \frac{dy}{2y} = \int 1\, dx$$

$$\log(2y) = x$$

$$|2y| = e^x$$

$$2y = \pm \frac{e^x + C}{2e^{1/2}}$$

I was supposed to move everything that wasn't x to the left side:

$$\int \frac{dy}{2y+1} = \int 1\, dx$$

$$\frac{1}{2}\log(2y+1) = x + C$$

$$\log(2y+1) = 2x + C$$

$$|2y+1| = e^{2x} C$$

$$2y + 1 = Ce^{2x} - \frac{1}{2}$$

Unit 1 Lesson 2 Quiz 2: Basic DE's

~~No~~. Yes) $y' + xy = x$ implies

$$y' = y(x-1)$$

$$\frac{dy}{dx} = y(x-1)$$

$$\int \frac{dx}{y} = \int x-1 \; dx$$

$$\log y = \frac{x^2}{2} - x + C$$

and so on

Unit 1 Lesson 2 Problems: Basic DE's

1: Is $y' = 3x^2$ satisfied by $y = x^3 + 7$?

$3x^2 = 3x^2$ ✓

13: For what r does $y = e^{rx}$ satisfy $3y' = 2y$?

$3re^{rx} = 2e^{rx} \Rightarrow r = \frac{2}{3}$ ✓

37: Find a potential solution to $y'' = 0$; $y = x \Rightarrow y' = 1 \Rightarrow y'' = 0$ ✓