Unit 2 Lesson 1: Characteristic Equation

- A linear differential equation is of the form

$$a_n x^{(n)} + a_{n-1} x^{(n-1)} + \dots + a_0 x = q(t),$$

where a_k are called coefficients; ~~the~~ n is the order.

- For constant a_k, a ~~pere~~ constant to the front

- For a spring system of the form

$$m\ddot{x} + b\dot{x} + kx = F_{ext} \text{ with}$$

m is mass
b is damping
k is spring constant,

- We call $F_{ext} = 0$ and $b = 0$ the undamped case/ simple harmonic oscillator; if $w = \sqrt{k/m}$ we have

$$\ddot{x} + w^2 x = 0$$

and $x_1 = \cos(wt)$, $x_2 = \sin(wt)$

- In the homogenous case of above linear DE, if we find some r s.t.

$$mr^2 + br + k = 0, \text{ then}$$

e^{rt} is a solution

Unit 2 Lesson 1 Quiz

Pi ; $\cos(2t)$, $\sin(2t)$ have period π.

Continued notes

- For any n-order ~~lin tt~~ homogeneous linear DE, we have characteristic polynomial

$$a_n r^n + a_{n-1} r^{n-1} + \dots + a_0 = 0$$

where ~~if~~ r is a solution, e^{rt} is a solution of the DE.

- A solution of the form ce^{rt} is called modal

Practice Problems

1: Plugging $x = \cos(wt)$ into

$$\ddot{x} + W^2 x = 0,$$ we get

$$-W^2\cos(wt) + W^2\cos(Wt) \overset{\checkmark}{=} 0,$$ likewise

$X = \sin(Wt)$ results in

$$-W^2\sin(Wt) + W^2\sin(Wt) \overset{\checkmark}{=} 0$$

2: Plugging in, we get

$$-AW^2\cos(Wt + -\phi) + AW^2\cos(Wt - \phi) \overset{\checkmark}{=} 0.$$

3: Those with $\phi = C(\frac{\pi}{2})$ where c is an integer. No, this is not first order.