18.03 Differential Equations: Week 1

Logan Pachulski

January 6th, 2019

Progress Update

Over the past week we have covered:

1. Basic Notation
2. Definition of differential equations

Basic Notation

We began by defining some notation that will be used throughout the course:

$$\dot{x} = \frac{dx}{dt} \tag{1}$$

$$x' = \frac{dx}{dy} = Dx \tag{2}$$

are various ways of describing derivatives. For equations where a set of variables and a constant is a valid result, such as c in

$$\int 2x \, dx = x^2 + c, \tag{3}$$

c is a *parameter*.

Differential equations

A differential equation is some equation composed of a function and/or said function's derivatives, such as

$$x' + x = 1 \tag{4}$$

where this is a order-1 differential equation.

Example Problem

Problem: For $\ddot{x} = 2t$, we have initial values $x(1) = 1$, $\dot{x}(1) = 2$. What is x?

Example Solution

Integrating each side, we get

$$\dot{x} = t^2 + c_1 \implies 2 = 1 + c_1 \implies c_1 = 1 \tag{5}$$

$$x = \frac{t^3}{3} + t + c_2 \implies 1 = \frac{1}{3} + 1 + c_2 \implies c_2 = \frac{-1}{3} \tag{6}$$

and thus

$$x = \frac{t^3}{3} + t + \frac{-1}{3} \tag{7}$$