Unit 3 Lesson 1

- We define a periodic function as one satisfying

$$f(t+P)=f(t)$$

for constant P and all t.

Obviously ~~all~~ sine and cosine satisfy this definition, as do the square wave, triangle wave, etc.

- The Fourier series for period 2π is defined and computed as

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nt) + b_n \sin(nt)$$

where

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\,dt$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(nt)\,dt$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(nt)\,dt$$

- Of for general period $P=2L$,

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(t)\,dt$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\left(n+\frac{\pi}{L}\right)dt$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin\left(n+\frac{\pi}{L}\right)dt$$

Problems

1: All with ~~even~~ integer angular frequency

2: 1

3: We have $a_0 = 0$, $a_n = 0$ since there is no symmetric behavior, and

$$b_n = \frac{1}{\pi}\left(\int_{-\pi}^{0} -\sin(nt)\,dt + \int_{0}^{\pi} \sin(nt)\,dt \right)$$

$$= \frac{1}{\pi}\left(\frac{\cos(nt)}{n}\Big|_{-\pi}^{0} + \frac{-\cos(nt)}{n}\Big|_{0}^{\pi} \right)$$

$$\cancel{= \frac{1}{\pi}\left(\frac{1-(-1)^n}{n} \right)}$$

$$= \frac{1}{\pi}\left(\frac{\cos(n0) - \cos(-\pi n)}{n} - \frac{\cos(\pi n) - \cos(0n)}{n} \right)$$

$$= \frac{1}{\pi}\left(\frac{1 - \cos(+\pi n)}{n} - \frac{\cos(\pi n) - 1}{n} \right)$$

$$= \frac{1}{\pi}\left(\frac{1 - (-1)^{+n} - ((-1)^n - 1)}{n} \right)$$

$$= \frac{1}{\pi}\left(\frac{2(1-(-1)^n)}{n} \right)$$

$$= \frac{2}{\pi n}\left(1-(-1)^n\right)$$

0 for n even

$4/(n\pi)$ for n odd