18.06 Linear Algebra: Week 7

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Progress Update

Over the past week we have covered:

1. Eigenvalues and eigenvectors.
2. Eigenvalue decomposition.

Eigenvalues and eigenvectors.

For a given square matrix A, we have eigenvalue(s) λ corresponding to a set of eigenvectors x, where

$$Ax = \lambda x. \tag{1}$$

One can find λ by solving

$$\det(A - \lambda I) = 0 \tag{2}$$

for λ, and then finding eigenvectors in the nullspace of $A - \lambda I$.

Eigenvalue decomposition.

Given a n by n square matrix A with n independent eigenvectors, we can rewrite

$$A = S\Lambda S^{-1} \tag{3}$$

where S is the matrix with the eigenvectors of A for columns, and Λ is the matrix with all λ on the diagonal.

Example Problem

Consider the following problem from the Lecture 22 Problems:

Problem 22.1: (6.2 #6. *Introduction to Linear Algebra:* Strang) Describe all matrices S that diagonalize this matrix A (find all eigenvectors):

$$A = \begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}.$$

Then describe all matrices that diagonalize A^{-1}.