18.06 Linear Algebra: Week 6

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Progress Update

Over the past week we have covered:

1. Determinants and properties of them.
2. Cofactor matrices.
3. A method of finding A^{-1}.

Determinants.

A determinant is some number associated with a square matrix A, denoted $\det(A)$. For a 2x2 matrix,

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc \tag{1}$$

and for higher dimensional matrices, one applies "expansion," for example

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}. \tag{2}$$

Other properties of determinant evaluation can be discussed in voice.

Cofactor matrix

The cofactor matrix of some matrix A is, for a given position, the determinant of what is left of the matrix when you eliminate the row and column associated with a position in the matrix, and overlay a grid of negatives. Consider

$$\begin{pmatrix} 1 & 3 \\ 7 & 4 \end{pmatrix} \text{ has } C = \begin{pmatrix} 4 & -7 \\ -3 & 1 \end{pmatrix} \tag{3}$$

The cofactor matrix (or more specifically, its transpose) is used on the next slide as a considerably more efficient way of finding a matrix's inverse.

Finding A^{-1}.

We are told that for a square matrix A which is invertible (and thus $\det(A) = 0$)

$$A^{-1} = \frac{1}{\det(A)} C^T, \tag{4}$$

Example Problem

Problem 20.1: (5.3 #8. *Introduction to Linear Algebra:* Strang) Suppose

$$A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}.$$

Find its cofactor matrix C and multiply AC^T to find $\det(A)$.

$$C = \begin{bmatrix} 6 & -3 & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \text{ and } AC^T = \underline{\qquad}.$$

If you change $a_{1,3} = 4$ to 100, why is $\det(A)$ unchanged?