

# Lecture 12 Problems: Problem 1

~~$$A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$~~

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

Solving  $Ax=0$ ,

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ x_3 - x_4 \\ x_4 - x_1 \end{bmatrix}$$

implies

$x_1 = x_2 = x_3 = x_4$ . and thus the nullspace is

$$N(A) = \text{span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

## Problem 2

Given  $C$ , note that

$$A^T C A = \begin{bmatrix} 2 & -1 & 0 & 1 \\ -1 & 3 & -2 & 0 \\ 0 & -2 & 4 & -2 \\ -1 & 0 & -2 & 3 \end{bmatrix}$$

And that reducing

$$\begin{bmatrix} -1 & 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ -0 & 1 & 1 & 0 & -1 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$

returns

$$x = \begin{bmatrix} 3/4 \\ 1/4 \\ 0 \\ 1/4 \end{bmatrix}$$

when assuming  $x_3 = 0$ ,

which sketch out shows

