Lecture 29 Problems i Problem

We see that

$A^TA = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ and that

$\det(A^TA - \lambda' I) = \lambda'^2 - 3\lambda' + 1 \overset{!}{=} 0 \Rightarrow \lambda' = \frac{3 \pm \sqrt{5}}{2}$,

comparing to given λ in Σ,

$\lambda_1^2 = \left(\frac{1+\sqrt{5}}{2}\right)^2 = \frac{6+2\sqrt{5}}{4} = \frac{3+\sqrt{5}}{2} = \lambda'_1$, and

$\lambda_2^2 = \left(\frac{-1+\sqrt{5}}{2}\right)^2 = \frac{6-2\sqrt{5}}{4} = \frac{3-\sqrt{5}}{2} = \lambda'_2$, thus Σ is corre

Problem 2

Since

$A^TA = V\Sigma^2V^T$, we see that $\Sigma^2 = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 & \ddots \end{bmatrix}$

and thus

$\Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 & \ddots \end{bmatrix}$;

we also have $V = I$ and that

$U = \begin{bmatrix} \frac{1}{\sigma_1}w_1 & \frac{1}{\sigma_2}w_2 & \cdots \end{bmatrix}$.