18.06 Linear Algebra: Week 4

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Progress Update

Over the past week we have covered:

1. Completing exam 1.
2. Orthogonality of vectors and subspaces.

Orthogonality of vectors and subspaces

We call two vectors a and b *orthogonal* if they meet at a 90 degree angle, or if

$$a^T \cdot b = 0. \tag{1}$$

We also call two subspaces orthogonal if all vectors in each are orthogonal to eachother. We also learned that for a matrix A,

$$R(A) \perp N(A) \tag{2}$$

$$C(A) \perp N(A^T) \tag{3}$$

Example problem

Problem 16.1: (4.1 #7. *Introduction to Linear Algebra:* Strang) For every system of m equations with no solution, there are numbers $y_1, ..., y_m$ that multiply the equations so they add up to $0 = 1$. This is called *Fredholm's Alternative*:

> Exactly one of these problems has a solution:
> $A\mathbf{x} = \mathbf{b}$ OR $A^T\mathbf{y} = \mathbf{0}$ with $\mathbf{y}^T\mathbf{b} = 1$.

If $\mathbf{b}$ is not in the column space of A it is not orthogonal to the nullspace of A^T. Multiply the equations $x_1 - x_2 = 1$, $x_2 - x_3 = 1$ and $x_1 - x_3 = 1$ by numbers y_1, y_2 and y_3 chosen so that the equations add up to $0 = 1$.