Lecture 9 Problems

Problem 1

Write out the vectors as a set of columns to start:

$$
\begin{array}{cccccc}
\multicolumn{6}{c}{\left[\begin{array}{cccccc} 1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & -1 & -1 \end{array}\right]} \\
V_1 & V_2 & V_3 & V_4 & V_5 & V_6
\end{array}
$$

We see that $V_5 = V_3 - V_1$, $V_4 = V_5 - V_6$, and $V_6 = V_3 - V_2$, thus we can rewrite the matrix as

$$
\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}
\xrightarrow{\text{reducing}}
\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}
\rightarrow
\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}
\rightarrow
\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}
$$

Three pivots for three vectors implies that these are independent, and thus 3 is the langest number of independent vectors.

Problem 2

Let

$A = \cancel{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}\ [1\ \ -2\ \ 3]$; then by inspection there are special solutions

$$A\cancel{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}(\underbrace{[2\ \ 1\ \ 0]^T}_{v_1}) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \ ; \ \overset{A}{\cancel{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}}(\underbrace{[3\ \ 0\ \ -1]^T}_{v_2}) = 0.$$

And these two form the basis for the plane. v_1 lies in the xy plane and thus provides a basis for that intersection, and by taking the cross product of v_1 and v_2 we get the perpendicular basis

$$\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix},$$