Lecture 31 Problem 1

Said vectors are

$$V_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, V_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}, V_3 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, V_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, V_5 = \begin{bmatrix} 1 \\ -1 \\ \\ \\ \\ \\ \\ \\ \end{bmatrix}, V_6 = \begin{bmatrix} \\ \\ 1 \\ -1 \\ \\ \\ \\ \\ \end{bmatrix}, V_7 = \begin{bmatrix} \\ \\ \\ \\ 1 \\ -1 \\ \\ \\ \end{bmatrix}, V_8 = \begin{bmatrix} \\ \\ \\ \\ \\ \\ 1 \\ -1 \end{bmatrix}$$

all

$V_1^T V_n$ where $n \neq 1$ equals zero due to V_{2-8} having equal 1s and -1s.

$V_2^T V_n$ for $n > 2 = 0$ by matching the top 1s and -1s without overlap

$V_3^T V_n$ for $n > 3 = 0$ for the same reason, ~~thu~~ and all equal zero

$V_i^T V_j = 0$ for $i \neq j$, thus orthogonal; to make orthonormal, divide by length:

$V_1' = 1/\sqrt{8}\ V_1$, $V_2' = V_2/\sqrt{8}$, $V_3' = V_3/\sqrt{4}$, $V_4' = V_4/\sqrt{4}$,

$V_5' = V_5/\sqrt{2}$, $V_6' = V_6/\sqrt{2}$, $V_7' = V_7/\sqrt{2}$, $V_8' = V8/\sqrt{2}$.

Problem 2:

Consider

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix};$$

The former is better for all purposes.