Lecture 20 problems: Problem 1

We have

$$C = \begin{bmatrix} \begin{vmatrix} 2 & 2 \\ 2 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 1 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} \\ -\begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 1 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 4 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 6 & -3 & 0 \\ 1 & 1 & -1 \\ -6 & 2 & 1 \end{bmatrix}$$

and thus

$$A C^T = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} 6 & 1 & -6 \\ -3 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \Rightarrow \det(A) = 3$$

Det(A) does not change when $A_{1,3} = 4 \to 100$ because $C_{1,3} = 0$.

Problem 2

$$A = \begin{bmatrix} \sin\phi\cos\theta & \rho\cos\phi\cos\theta & -\rho\sin\phi\sin\theta \\ \sin\phi\sin\theta & \rho\cos\phi\sin\theta & \rho\sin\phi\cos\theta \\ \cos\phi & -\rho\sin\phi & 0 \end{bmatrix}$$

Expanding,

$$\det(A) = \cos\phi \begin{vmatrix} \rho\cos\phi\cos\theta & -\rho\sin\phi\sin\theta \\ \rho\cos\phi\sin\theta & \rho\sin\phi\cos\theta \end{vmatrix} + \rho\sin\phi \begin{vmatrix} \sin\phi\cos\theta & -\rho\sin\phi\sin\theta \\ \sin\phi\sin\theta & \rho\sin\phi\cos\theta \end{vmatrix}$$

$$= \cos\phi \, \rho^2\cos\phi\sin\phi + \rho^2\sin^3(\phi)$$

$$= \rho^2\sin(\phi)\left(\cos^2\phi + \sin^2\phi\right)$$

$$= \rho^2\sin\phi$$