Lecture21 Problems
Problem 1

(a): 2

(b): det $(B) = 0$ since $\lambda = 0$ is present, thus

$$\det(B)\det(B^T) = \det(BB^T) = 0$$

(c): NEI

(d): Adding n multiples of the identity matrix increases all eigenvalues by n, and inverting a matrix inverts the eigenvalues, thus

$$Bx = \lambda x$$
$$Bx = \lambda x$$
$$B^2 x = \lambda^2 x$$
$$(B^2 + I)x = (\lambda^2 + 1)x,$$ and thus
$(B^2+I)^2$ has eigenvalues

$$\frac{1}{(\lambda^2+1)} = \frac{1}{1}, \frac{1}{2}, \frac{1}{5}.$$

Problem 2

A is triangular and thus the eigenvalues are on the diagonal:

$\lambda_1 = 1$

$\lambda_2 = 4$

$\lambda_3 = 6$

B:

$$\det B - \lambda I = \det \begin{pmatrix} -\lambda & 0 & 1 \\ 0 & 2-\lambda & 0 \\ 3 & 0 & -\lambda \end{pmatrix} \stackrel{!}{=} 0$$

$$= -\lambda\left(-\lambda(2-\lambda)\right) + 1 \begin{vmatrix} 0 & 2-\lambda \\ 3 & 0 \end{vmatrix}$$

$$= \lambda^2 (2-\lambda) \quad +3(2-\lambda)$$

$$0 \stackrel{!}{=} -\lambda^3 + 2\lambda^2 + 6 - 3\lambda$$

$\lambda = 2, \pm\sqrt{3}.$

C:

$$\det(C - \lambda I) = (2-\lambda)\left((2-\lambda^2)-4\right) - 2\left(2(2-\lambda)-4\right) + 2\left(4-2(2-\lambda)\right)$$

$$0 \stackrel{!}{=} \lambda^2(\lambda-6)$$

implies $\lambda = 0, 0, 6.$