18.06 Linear Algebra: Week 8

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Progress Update

Over the past week we have covered:

1. Markov Matrices
2. Exam 2
3. Symmetric Matrices

Markov Matrices

A Markov matrix is one where each column sums to 1; consider the 2 by 2 example

$$\begin{bmatrix} 0.1 & 0.01 \\ 0.9 & 0.99 \end{bmatrix}. \tag{1}$$

which has eigenvalues $\lambda = 1, 0.09$; All Markov matrices have $\lambda_1 = 1$ and $\lambda_n < 1$ when $n \neq 1$.

Symmetric Matrices

Now consider an example of a symmetric matrix:

$$\begin{bmatrix} 1 & 3 & 17 \\ 3 & 2 & 14 \\ 17 & 14 & 3 \end{bmatrix} \tag{2}$$

A symmetric matrix provably will always have real eigenvalues, and in this case we have $\lambda \approx 25.73, -18.39, -1.34$. A subset of symmetric matrices is positive definite matrices; those which have strictly positive eigenvalues.

Example Problem

Consider the following problem from the Lecture 24 Problems:

Problem 24.1: (6.4 #7. *Introduction to Linear Algebra:* Strang)

a) Find a symmetric matrix $\begin{bmatrix} 1 & b \\ b & 1 \end{bmatrix}$ that has a negative eigenvalue.

b) How do you know it must have a negative pivot?

c) How do you know it can't have two negative eigenvalues?