# Lecture 28 Problems: Problem 1

For a "generic matrix of the form

$$M=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix},$$

then

$$JM=\begin{bmatrix} a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \\ 0 & 0 & 0 & 0 \end{bmatrix} \overset{!}{=} \begin{bmatrix} 0 & m_{11} & m_{12} & 0 \\ 0 & m_{21} & m_{22} & 0 \\ 0 & m_{31} & m_{32} & 0 \\ 0 & m_{41} & m_{42} & 0 \end{bmatrix} = MK$$

implies $x_{ij}=0$ $\forall$ which implies $\det(M)=0$, thus $M$ is not invertible and $J$ is not similar to $K$.

# Problem 2

(a): Since $A$ is similar to $B$,

$$A = M^{-1} B M \Rightarrow A^2 = M^{-1} B M M^{-1} B M$$

$$= M^{-1} B^2 M$$

and thus $A^2$ is similar to $B^2$

(b): For

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, A^2 = B^2 \text{ but } A \text{ is not similar to } B,$$

(c): For

$$M = \begin{bmatrix} 1 & +1 \\ 0 & 1 \end{bmatrix}$$

we have that

$$\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$$

(d): For $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ there is a plane of eigenvectors but for $\begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$ there is a line.

(e): The row and column exchange matrix is

$$\begin{bmatrix} 0 & 1 & \cdots & 1 \\ 1 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

which is its own inverse and thus $A = M B M \Rightarrow A$ is similar to $B$.