Lecture 23 Problems: Problem 1

Using given equation,

$$\|u(t)\|^2 = u_1^2 + u_2^2 + u_3^2 \Rightarrow \frac{d\|u(t)\|^2}{dt} = 2u_1 u_1' + 2u_2 u_2' + 2u_3 u_3'$$

and by substituting given values of u',

$$\cdots = 2au_1 u_2 - 2b u_1 u_3 + 2a u_2 u_3 - 2c u_1 u_2 + 2b u_1 u_3 - 2a u_2 u_3$$

$$= 0$$

by zeroing out. Thus, $u(t)$ is constant and equal to $u(0)$

Problem 2

A is triangular and thus $\lambda = 1, 3$

Then, finding null space of

$\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$ is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and of $\begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}$ is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, then

$S = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$, $S^{-1} = \begin{bmatrix} 1 & -1/2 \\ 0 & 1/2 \end{bmatrix}$

and thus

$S e^{\Lambda t} S^{-1} = \begin{bmatrix} e^t & 0.5 e^{3t} - 0.5 e^{t} \\ 0 & e^{t} \end{bmatrix} = e^{At}$