

Lecture 18 Problems: Problem

By inspection, we get $AX=0$ when $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; since $x \in \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$, then A is not invertible and thus $\det(A) = 0$. ~~Moreover~~

~~Moreover~~
 $A-I$ also has a nonzero nullspace and thus $\det(A-I) = 0$, and
No, as in the case of $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Problem 2

$$\det \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ & 1 & b+a \\ & & (-a)(-b) \end{vmatrix} (b-a)$$

$$= \begin{vmatrix} 1 & a & a^2 \\ & 1 & b+a \\ & & 1 \end{vmatrix} (b-a)(-a)(-b)$$

$$= (b-a)(-a)(-b)$$