18.06 Linear Algebra: Week 1

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Progress Update

Over the past 2 weeks I have been introduced to:

1. Matrix multiplication
2. Solving linear systems of equations with matrices
3. Identifying when a matrix is not invertible

Matrix multiplication

We began the course my reviewing basic principles of matrices; first, multiplication of matrices:

$$\begin{array}{ccccccc} & & & & \begin{array}{cc}\vec{b_1} & \vec{b_2} \\ \downarrow & \downarrow\end{array} & & \\ \begin{array}{c}\vec{a_1}\rightarrow \\ \vec{a_2}\rightarrow\end{array} & & \begin{bmatrix}1 & 7\\ 2 & 4\end{bmatrix} & \cdot & \begin{bmatrix}3 & 3\\ 5 & 2\end{bmatrix} & = & \begin{bmatrix}\vec{a_1}\cdot\vec{b_1} & \vec{a_1}\cdot\vec{b_2}\\ \vec{a_2}\cdot\vec{b_1} & \vec{a_2}\cdot\vec{b_2}\end{bmatrix} \\ & & A & & B & & C\end{array}$$

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Which allows us to make our first foray into linear algebra, by rewriting a system of equations as matrix multiplication; consider the system

$$\begin{aligned} 3x + 4y &= 7 \\ x + y &= 2 \end{aligned}$$

Matrix multiplication and linear systems

We can encode the system of equations seen on the previous slide into the matrix multiplication

$$\begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}. \tag{1}$$

We say that the leftmost matrix, which we shall call M, is invertible because none of the columns are scalar multiples of each-other (likewise but not discussed yet, it has non-zero determinant).

Example Problem

Consider the following problem from lecture:

Problem 2.1: In the two-by-two system of linear equations below, what multiple of the first equation should be subtracted from the second equation when using the method of elimination? Convert this system of equations to matrix form, apply elimination (what are the pivots?), and use back substitution to find a solution. Try to check your work before looking up the answer.

$$\begin{aligned} 2x + 3y &= 5 \\ 6x + 15y &= 12 \end{aligned}$$