Exam 1

Problem 1

(a): We see that $r = n$, $m = 3$, and $r < m$.

(b): By (A), we know there exists only one solution to $Ax = 0$!

$x = [0]$ or $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ depending on r.

(c): $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

Problem 2

(a): The inverse is those row operations applied ~~in reverse order~~ to I

$$\begin{bmatrix}1&&\\&1&\\&&1\end{bmatrix} \to \begin{bmatrix}1&&\\-4&1&\\&&1\end{bmatrix} \to \begin{bmatrix}1&&\\-4&1&\\-3&&1\end{bmatrix} \to \begin{bmatrix}1&0&0\\-4&1&-1\\-3&0&1\end{bmatrix} = A^{-1}$$

(b): ~~The inverse~~ A is those operations applied to I in reverse order:

$$\begin{bmatrix}1&&\\&1&\\&&1\end{bmatrix} \to \begin{bmatrix}1&&\\&1&+1\\&&1\end{bmatrix} \to \begin{bmatrix}1&&\\&1&+1\\+3&&1\end{bmatrix} \to \begin{bmatrix}1&0&0\\+4&1&+1\\+3&0&1\end{bmatrix} = A.$$

(c):

$$L = \begin{bmatrix}1&&\\4&1&\\3&&1\end{bmatrix}$$

Problem 3

(a): for $c \neq 3$, we have that the first 3 columns are pivot columns, thus

$C(A)$ has basis $\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ c \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\}$

(a): for $c=3$, only columns 1 and 3 are pivot columns, thus

$C(A)$ has basis $\begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

(b): for $c \neq 3$,

$$N(A) = c_1 \begin{bmatrix} -2 \\ 0 \\ -1 \\ 1 \end{bmatrix},$$

for $c=3$,

$$N(A) = c_1 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -2 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

(c):

$$x_0 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix},$$

and the full solutions are that added to each case of (b).

(a): $N(a)$ has $2 \le \dim \le 5$ Problem 4

(b): Columns 1, 4, and 5 are basis for $C(A)$.

(c): All upper triangular matrices,

$$U = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$