Lecture 22 Problems

Problem 1

Begin by finding λ:

$$\det\begin{pmatrix} 4-\lambda & 0 \\ 1 & 2-\lambda \end{pmatrix} = (4-\lambda)(2-\lambda) = 0$$

implies $\lambda = 4, 2$ which is obvious because A is triangular.
Then, we have

$$A - 4I = \begin{bmatrix} 0 & 0 \\ 1 & -2 \end{bmatrix}, \quad A - 2I = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}$$

and thus we have ~~th in~~ the nullspace bases

$$x_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad x_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \text{ thus}$$

$$S = \begin{bmatrix} c_1(x_1 \text{ or } x_2) & c_2(\text{other}) \end{bmatrix}$$

The same matrices diagonalize A^{-1}

Problem 2

The matrix is Markov, thus $\lambda_1 = 1$; since

$$\lambda_1 + \lambda_2 = 0.6 + 0.1 = 1 + \lambda_2, \text{ then } \lambda_2 = -0.3.$$

We then want to find the nullspace of

$$A - I = \begin{bmatrix} -0.4 & 0.9 \\ 0.4 & -0.9 \end{bmatrix}$$

which is

$$x_1 = \begin{bmatrix} 9 \\ 4 \end{bmatrix},$$

and then find the nullspace of

$$A + 0.3I = \begin{bmatrix} 0.9 & 0.9 \\ 0.4 & 0.4 \end{bmatrix}$$

which is

$$x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

and thus

$$\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & -0.3 \end{bmatrix}, \quad S = \begin{bmatrix} 9 & 1 \\ 4 & -1 \end{bmatrix}$$

$$\lim_{k \to \infty} \Lambda^k = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

and in turn

$$S \lim_{k \to \infty} \Lambda^k S^{-1} = \frac{1}{13} \begin{bmatrix} 9 & 9 \\ 4 & 4 \end{bmatrix}$$