Problem 1

If such al ine existed, it would satisfy

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \\ 21 \end{bmatrix};$$

no such solution exists, but we get the best fit r/aleast samres solution

$$\hat{x} = \begin{bmatrix} 9 \\ 4 \end{bmatrix};$$

We get $P = (5, 13, 17)$ and thus $b - p = 2 - 6, 4$; since

$Pe = Pb - Pp = Pp = 0.$

Problem 3

If $b = e$, then $b \perp C(A)$.

Problem 4

$e = 0$ since $b \in C(A)$

Problem 5

$e \in N(A^T)$,
$p \in C(A)$
$\hat{x} \in C(A^T)$, and
$N(A) = \{0\}$

Problem 6

We get least squares system

$$\begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} c \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$, which has obvious visual solution $c = 1, b = -1$, and thus line $1 - t$.