



t-test za aritmetičku sredinu

Statistički metodi 2019/2020

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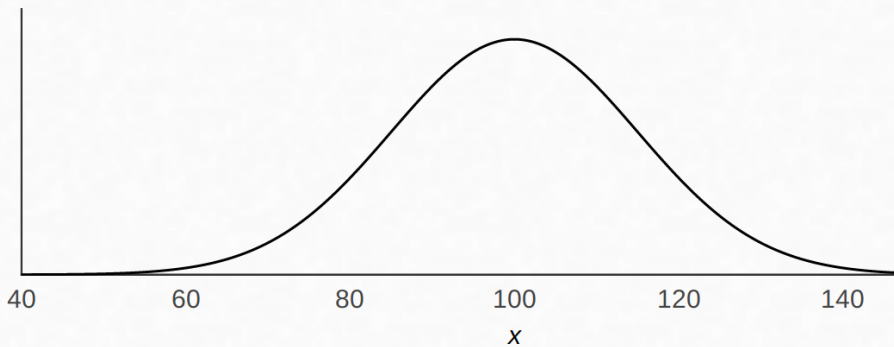
1. Primer sa prethodnog časa
2. z-tablica
3. t-test za aritmetičku sredinu
4. Interval poverenja

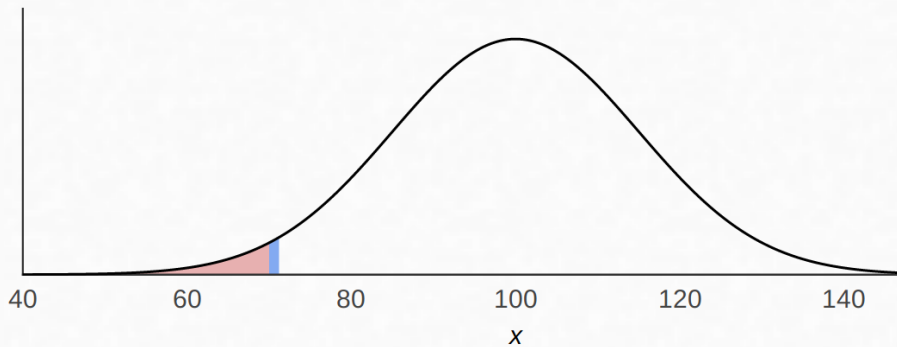
Primer sa prethodnog časa

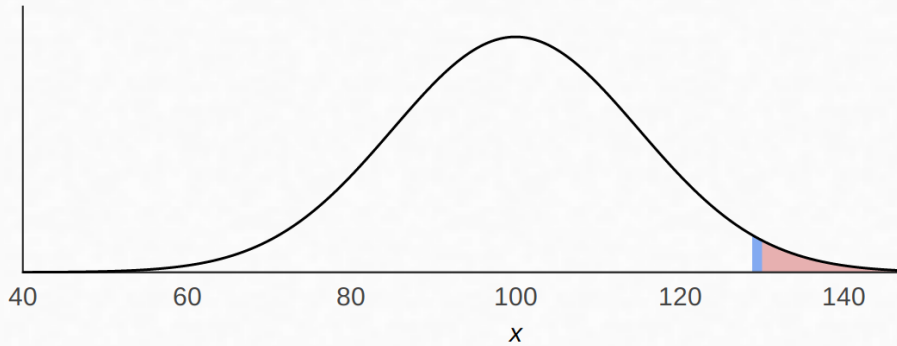
$$\bar{X} = 125$$

$$n = 100$$

$$\sigma = 15$$







$$Z = \frac{\bar{X} - \mu_0}{\sigma_{\bar{X}}}$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

$$Z > Z_{\frac{\alpha}{2}}$$

Kritične vrednosti

$$\alpha = 10\% \Rightarrow Z_{\frac{\alpha}{2}} = 1.645$$

$$\alpha = 5\% \Rightarrow Z_{\frac{\alpha}{2}} = 1.96$$

$$\alpha = 1\% \Rightarrow Z_{\frac{\alpha}{2}} = 2.58$$

z-tablica

z-tablica

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986

Pronaći $1 - \frac{\alpha}{2}$
unutar polja tablice



z-tablica

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
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0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6773	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
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1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
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2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
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2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986

- 1) Iz reda tablice pročitati prva dva prve cifre pre i posle decimalnog zareza
- 2) Iz kolone tablice pročitati drugu cifru nakon decimalnog zareza
- 3) Zajedno ove tri cifre zajedno sa zarezom predstavljaju $Z_{\frac{\alpha}{2}}$

$$\bar{X} - z_{\frac{\alpha}{2}} \sigma_{\bar{X}} \leq \mu \leq \bar{X} + z_{\frac{\alpha}{2}} \sigma_{\bar{X}}$$

Prvi interval poverenja

$$\alpha = 5\%$$

$$101,3 \leq \mu \leq 103,97$$

Pronaći intervale poverenja za druge rizike greške

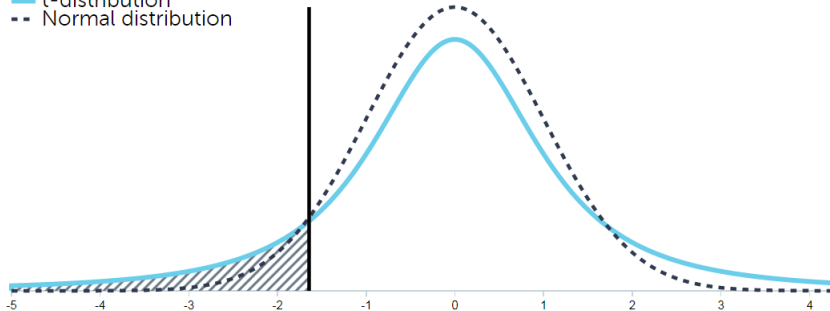
$$\alpha = 1\%$$

$$\alpha = 10\%$$

t-test za aritmetičku sredinu

t-distribucija

— t-distribution
-- Normal distribution



$$H_0 : \mu = \mu_0$$

$$H_1 : \mu \neq \mu_0$$

$$t = \frac{\bar{X} - \mu_0}{S_{\bar{X}}}$$

Ocena standardne greške AS

$$S_{\bar{X}} = \frac{s}{\sqrt{n}}$$

$$t > t_{n-1; \frac{\alpha}{2}}$$

t-tablica

df	α									
	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098
Z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%
										99.9%

- 1) U naslovu redova pronaći $n-1$
- 2) U naslovu kolona pronaći $\frac{\alpha}{2}$
- 3) Ukrštanjem pronađenog reda i pronađene kolone nalazi se vrednost

$$t_{n-1; \frac{\alpha}{2}}$$

Na uzorku od 144 stanovnika jedne opštine izračunata je prosečna plata i ona iznosi 406,5€. Standardna devijacija u uzorku je 52€. Da li se **prosečna plata svih stanovnika ove opštine** razlikuje od *republičke prosečne plate* koja je 400€? (Analizu izvršiti uz rizik greške od 5 %)

Interval poverenja

$$\bar{X} - t_{n-1; \frac{\alpha}{2}} S_{\bar{X}} \leq \mu \leq \bar{X} + t_{n-1; \frac{\alpha}{2}} S_{\bar{X}}$$