Basic Combinatorics

Sets

- Explicit : A={1,4,2}
- Implicit: A={i|i is an odd number}
- Intersection: $x \in A \cap B \;\; if \;\; x \in A \; and \; x \in B$
- Union: $x \in A \cup B \;\; if \;\; x \in A \; or \; x \in B$

Products of sets

- Taking all possible combinations.

Style

Color

Size

XL

L

M

S

XS

Product Set={ (, ,),(, ,) ,... }

Size of Product Set=

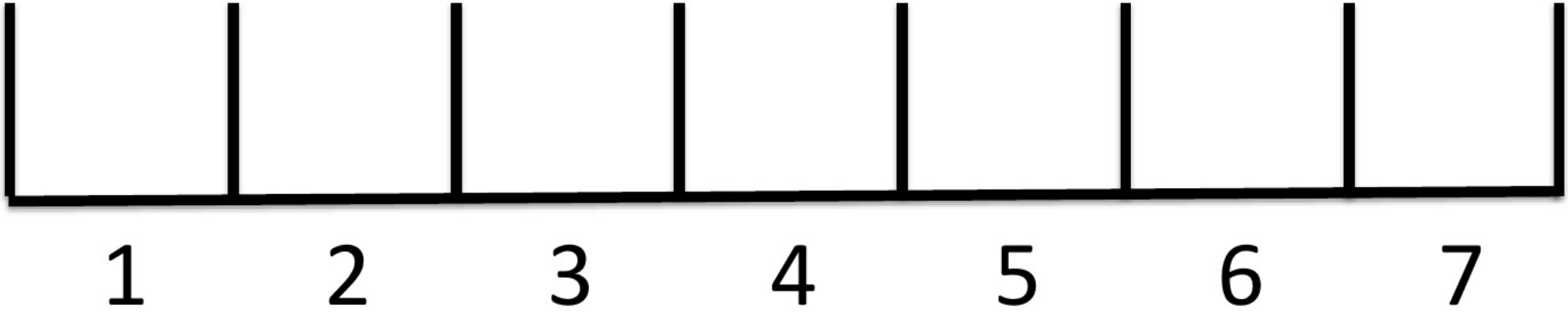
Raising a set to a power

- The set of all binary sequences of length 7:
  - 0000000, 0000001, 0000010,…
  - 1111101, 1111110, 1111111
- Using product notation:
  - $\{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\} = \{0,1\}^7$
- Size:
  - $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7 = 128$

The Factorial Function

How many ways are there to order n different objects?

How many ways are there to order 7 different objects?



The Factorial function

- The number of possible ways to put n different objects into n different slots is

$$n * (n-1) * (n-2) * \cdots * 2 * 1 \doteq n!$$

- We say “n factorial”

Permutations

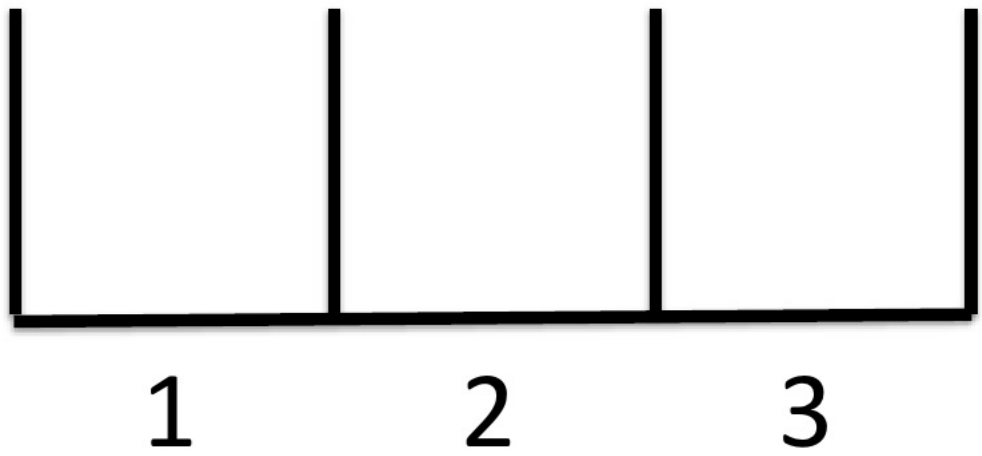
How many ways are there

To pick k out of n elements

When the order matters

How many ways are there to pick 3 out of 7 elements

When the order matters



The Permutation Function

- The number of possible ways to put k<n different objects into n different slots is
- For n=7, k=3:

$$7 * 6 * 5 = \frac{7 * 6 * 5 * 4 * 3 * 2 * 1}{4 * 3 * 2 * 1} = \frac{7!}{(7-3)!}$$

- In general:

$$P(n,k) \doteq \frac{n!}{(n-k)!}$$

combinations

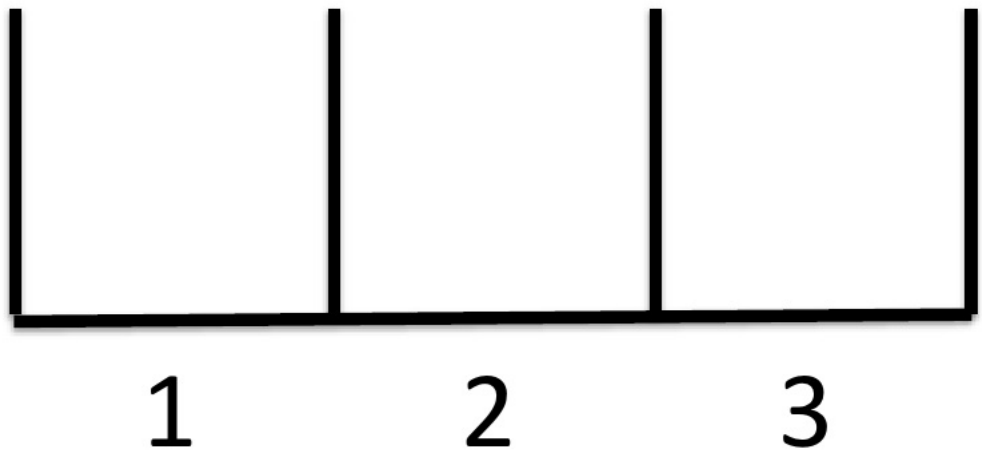
How many ways are there

To pick k out of n elements

When the order does not matter

How many ways are there to pick 3 out of 7 elements

When the order does not matter



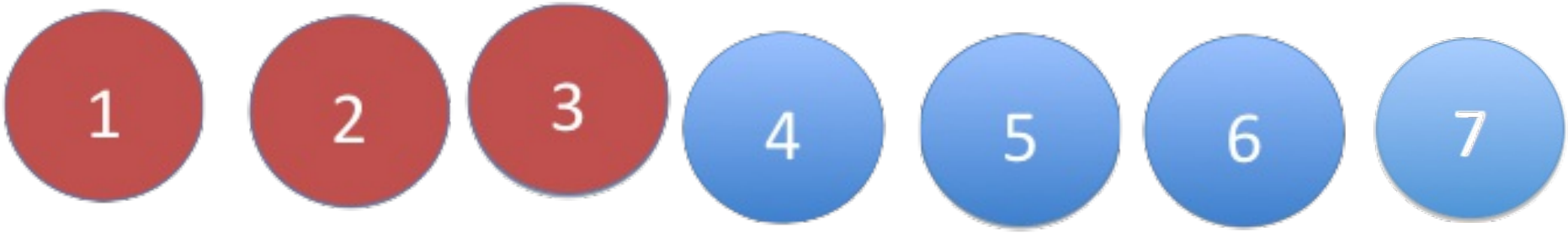
The Combinatorial function

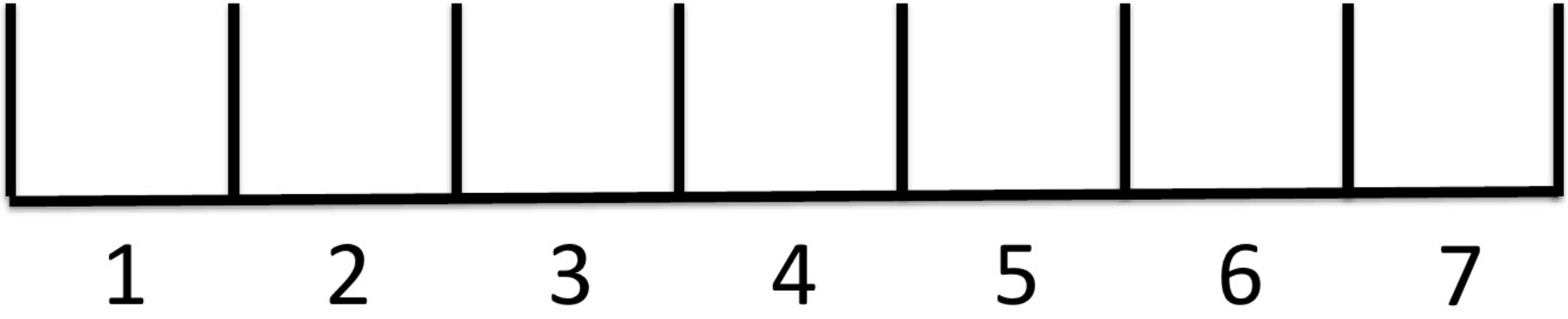
- The number of possible ways to place k identical objects into n different slots is

- $C(n,k) \doteq \binom{n}{k} \doteq \frac{P(n,k)}{k!} = \frac{n!}{k!(n-k)!}$

- We Say “n choose k”

How many ways are there to order 3 red and 4 blue balls?





How many different ways to arrange (all) the letters in

M I S S I S S I P P I

Consider all length 10 binary sequences.
How many contain exactly one 1?

How many contain five 1s?

For Friday

1. Read Chapter 3

2. Do Webwork problem 1-7 in week2