# CSE103: Introduction to Probability and Statistics

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# Why should you care about prob&stat?

- Learn to reason under uncertainty.
- Navigation software:
  - **Certainty**: Find the shortest route from UCSD to UCLA.
  - **Uncertainty**: Find the fastest route from UCSD to UCLA.
- Search Engine:
  - **Certainty**: Find all web pages that contain the words "Trump","Hillary" and "debate"
  - **Uncertainty**: Find the 10 most relevant pages for the query "Trump, Hillary debate"
- Insurance Company:
  - **Certainty**: If a person with life insurance dies, the insurance company has to pay $X
  - **Uncertainty**: What is the minimal life insurance premium such that the probability that the life insurance company will be bankrupt in 10 years is smaller than 1% ?

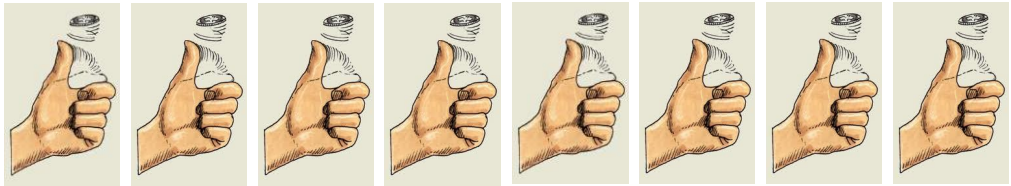
# What you will learn

- The navigation and search engine problems are advanced, in this class you will learn the conceptual foundation of P&S and solve much simpler problems.
- Examples:
  - If you flip a coin 100 times, what is the probability of getting at most 10 "heads" ?
  - What is the probability of getting a "4 of a kind" hand in poker.
  - If you want to hash 1,000,000 elements and cannot allow more than 5 indirections, how big does the table need to be?
  - Suppose that the expected time between failures for a router is one year. What is the probability that the router will fail during the first month?

# Flipping a coin

0+ 1+ 0 +0 +0+ 0 +0 +1=2 heads

T H T T T T T H

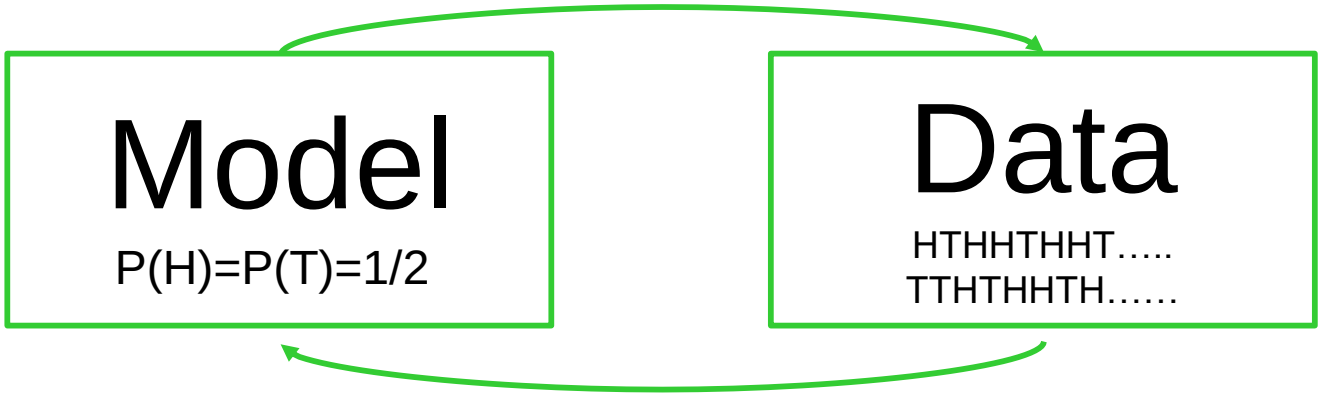


Suppose that we flip the coin 100 times, and count the number of heads

# Probability vs. statistics

**Probability**

What is the probability that 60 out of 100 flips of fair coin are heads?




**Statistics**

Given that 60 out of 100 coin flips are "heads", how sure can we be that the coin is not fair?

# Flipping two dice

- 1 dice-What is the probability that it will land on 6 ? Or on 5 ?
  - 1/6
- G,R dice. What is the probability of green=6 and red=5
  - 1/6 X 1/6 = 1/36
- R,R dice. What is the probability of red=5 and red=6?
  - 2 X 1/36 = 1/18
- R,R dice. What is the probability of red=5 and red=5 ?
  - 1/36
- Dice of the same color are

**indistinguishable or interchangeable**

# Indistinguishability / Exchangeability

- Two object are indistinguishable if exchanging them makes not difference.
- What makes two objects indistinguishable?
- Mathematical objects (points, lines) are indistinguishable
  - Physical objects are distinguishable (we can mark them)
- Poker cards are indistinguishable, unless marked (illegal)
- iPhones (of the same model) are indistinguishable
  - iPhone covers make the iPhones distinguishable.
- Dollars are indistinguishable – that is what makes the economy work (compare that to bartering).
- Are fruit indistinguishable? (same DNA)
- Are animals of a species distinguishable?
- Are people distinguishable?

# Probabilities regarding people

- Which of the following is more correct?
  1. Each of us is unique, we have our own free will.
  2. We belong to groups, our opinions are the opinions of the group.
- When the number of people is large, a very effective way to reason is to think of people as interchangeable:
  1. How many children in this district have special needs?
  2. How many voters in San Diego county will vote republican?
  3. Does using seat belts save lifes?
- Are we all the same or are we all different?

# We are individuals – The life of Brian

# Concepts, not formulas!

- The most important skill that you will learn in this class is the ability to analyze problems that involve uncertainty and express your understanding in a precise mathematical way.
- Computing the correct final answer is much less important!
- Different formulas are relevant for Machine Learning, Search engines, route planning, medical research, computer vision etc. etc.
- The fundamental concepts remain the same: Probability, expectation, central limit theorem …
- I supply a cheat sheet with all of the relevant formulas,
  - You don't need to memorize formulas.
  - You need to understand the concepts!

# What to expect?

- This is a math Class!
  - No Programming.
  - When solving problems, you need to find a short expression, the final numerical answer is much less important.
  - A computer can easily compute the value of an expression, the computer cannot translate the problem into an expression.
- You will learn important **concepts**:
  - Probability distribution
  - Random variables
  - Dependence
  - Expectation and Variance
  - …
- The examples I give in class and the examples in your homework will often be different, but they will use the same concepts.
- Concepts are harder than recipes: they require understanding and adopting.

# Tips on Learning concepts

- Words in math have precise meaning, be sure you understand that meaning.
- Don't let new words/concepts wash over you.
  To do the HW, you will need to understand those concepts.
- **Ask questions!**
  - Ask questions during Class.
  - Ask questions during Discussion.
  - Form study groups and discuss concepts.
  - Post questions on Piazza.
- Asking good questions will earn you points!

# The mechanics

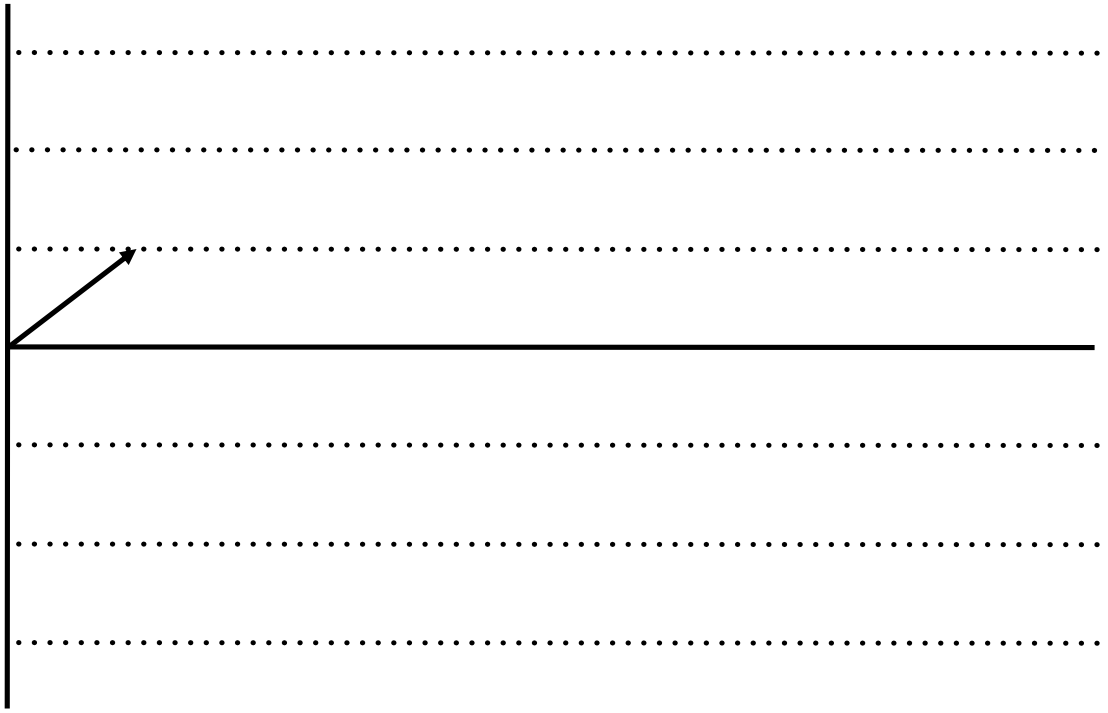
- Final Grade:
  - 50% HW assignments. Bottom 2/9 HW grades dropped.
  - 40% Final
  - 20% Classroom participation (asking good questions).
- HW:
  - Delivered using the open-edx platform.
  - Assignments will open and close at noon on Friday.
  - You can attempt each problem several times:
    - Initially: unlimited
    - Close to final: limited.
- Hint system:
  - Will sometimes send you a hint to help you towards the answer.
  - The most common hint: "Please write an expression, not just the final result".
  - Part of a research project: we ask for your consent to use results in publication.
  - Published results are anonymized.

# The mechanics (continued)

- Final:
  - similar questions to the HWs.
  - But you get only one attempt.
  - 3 hours, most finish in < 2hours.
- Classroom participation:
  - Each good question earns you 5 points (to a max of 20 points)
  - If you asked a good question in class/section, post the question and the answer to it on Piazza.

# PROBABILITY

# A Basic Problem: Is a Coin Fair?



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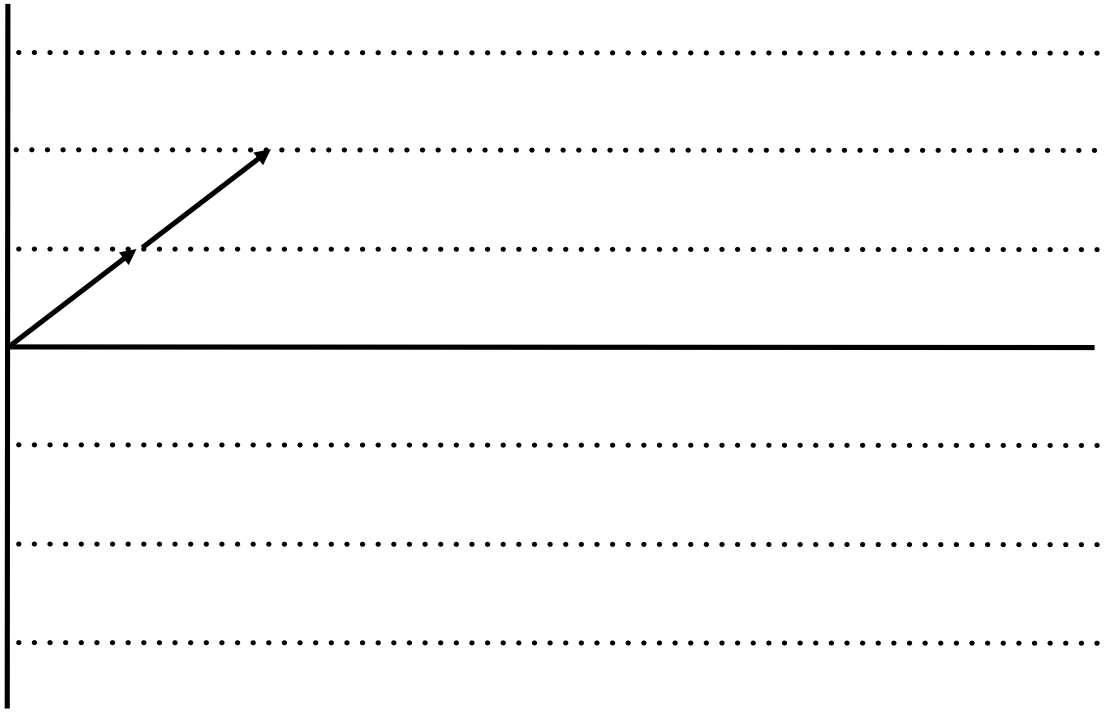

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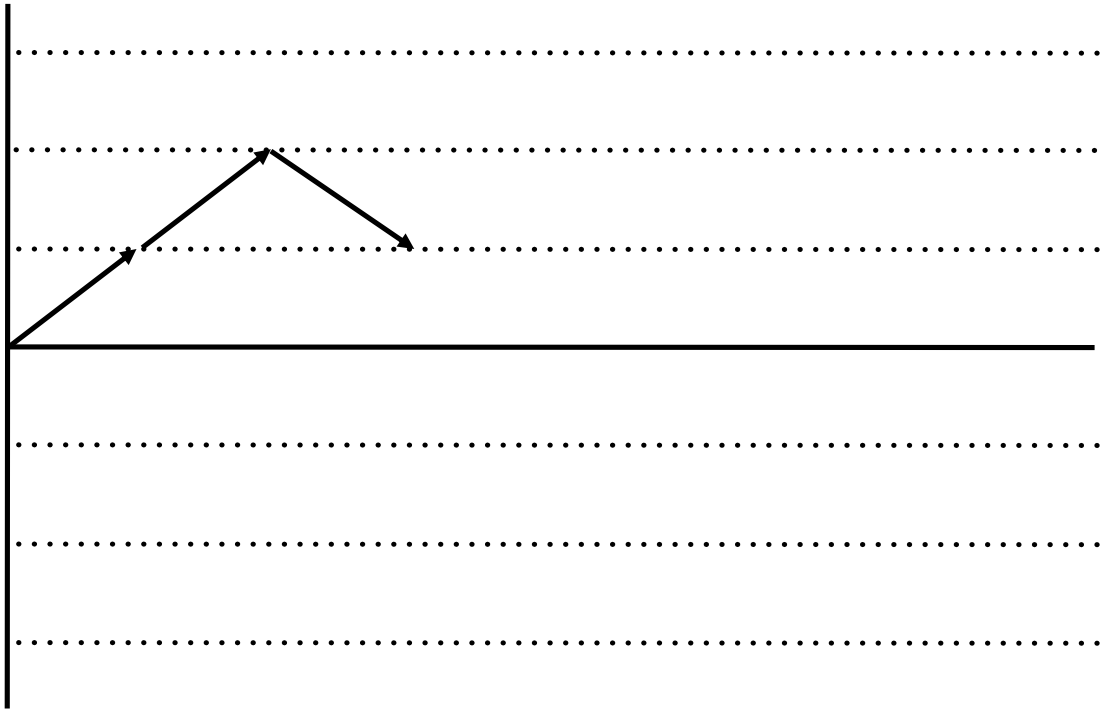


# A Basic Problem: Is a Coin F

Null hypothesis $H_0$: $h = 0.5$

Alternative hypothesis $H_1$: $h \neq 0.5$

# The law of large numbers

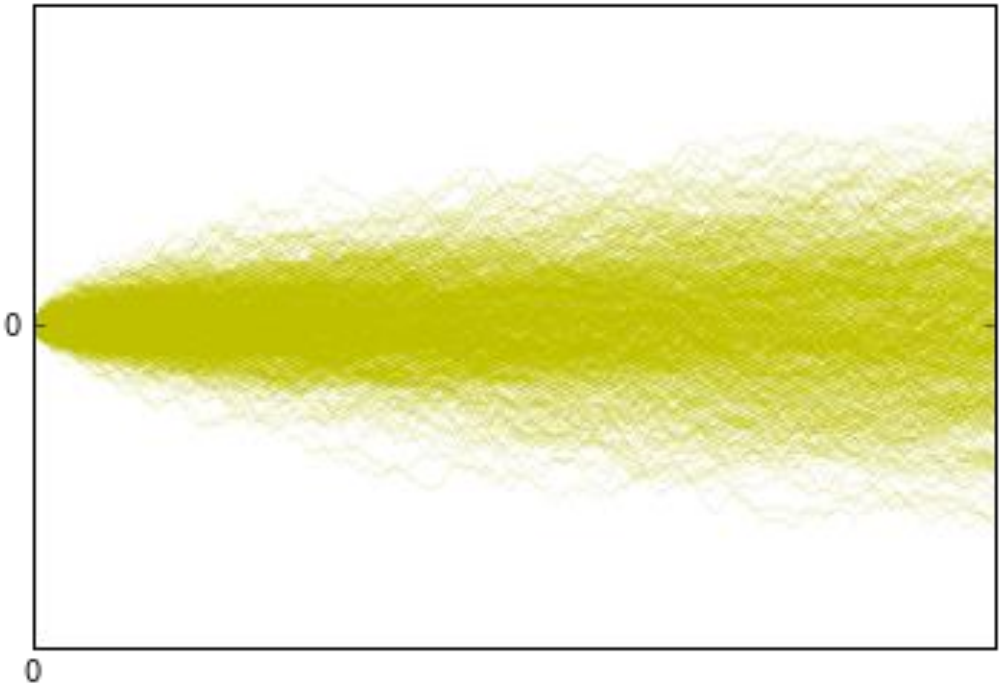
- If we flip forever, the fraction of times that we get "heads" converges to the true bias of the coin.
- This is called "the law of large numbers"
- If bias =1/2 then the coin is fair.
- But we cannot wait forever!
- How many times do we need to flip the coin in order to know whether the true bias is between 0.45 and 0.55 ?
- We will give increasingly more accurate answers to this questions throughout the quarter.
- For now, consider some examples.

# Statistical terminology and notation

- h = the true bias of the coin = probability of Heads.
- Null hypothesis
  - = our default assumption
  - = the coin is unbiased
  - = the bias of the coin is between 0.45 and 0.55
- Alternate hypothesis
  - = the coin is biased
  - = the bias of the coin is outside 0.45 and 0.55

# Fair Coin (Null Hypothesis)

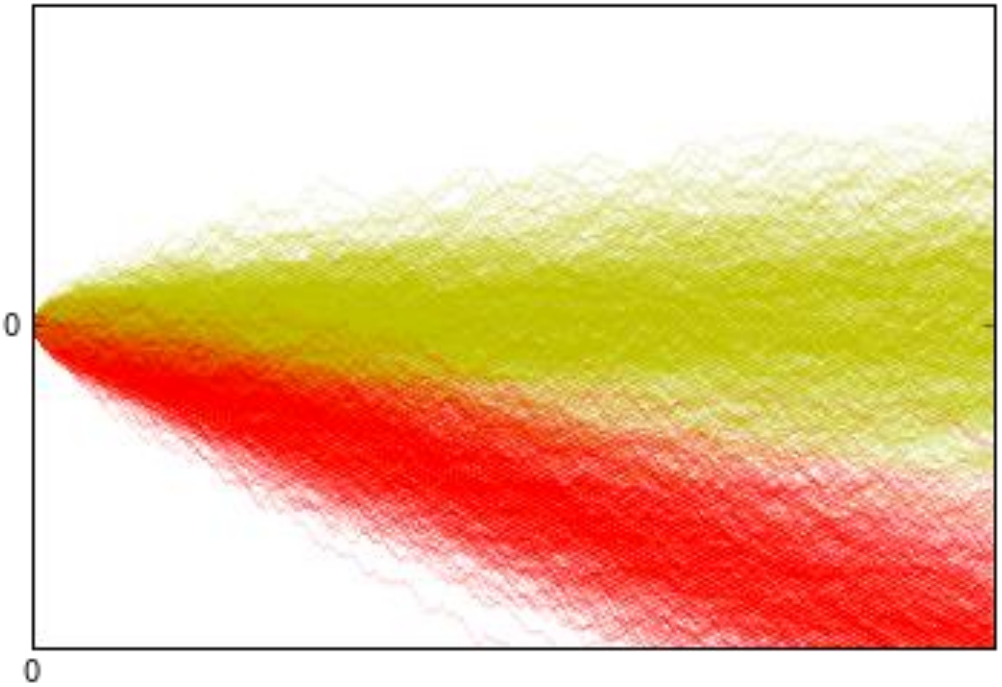
h = 0.5



# Biased Coin (Alternate Hypothesis)

h = 0.5

h = 0.4

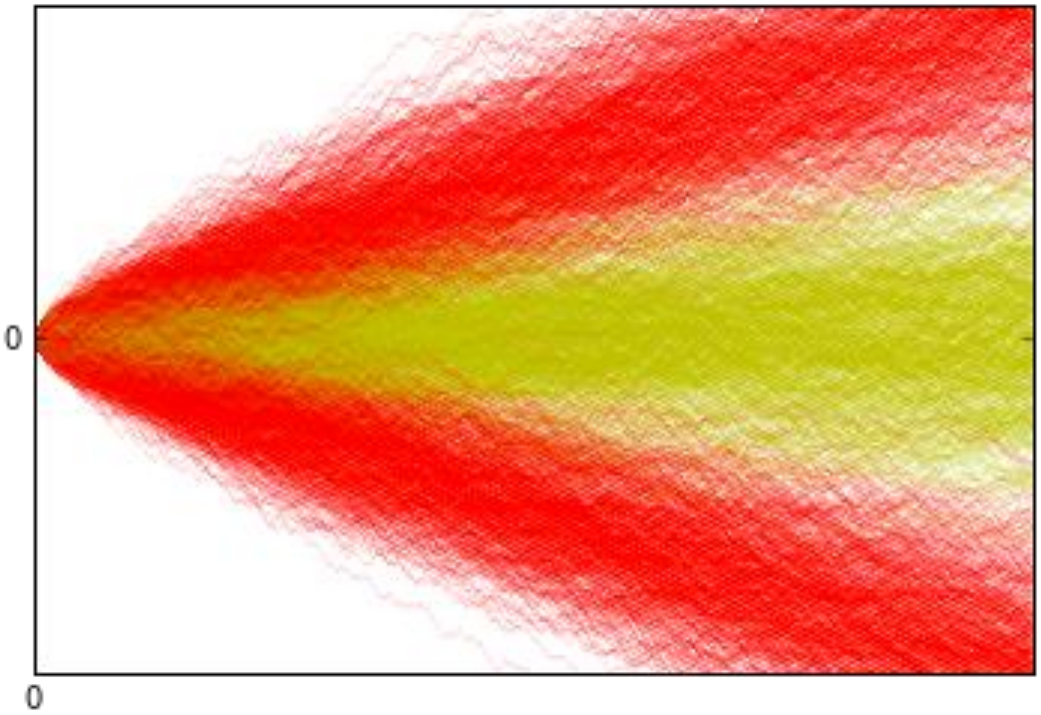


# Biased Coin (Alternate Hypothesis)

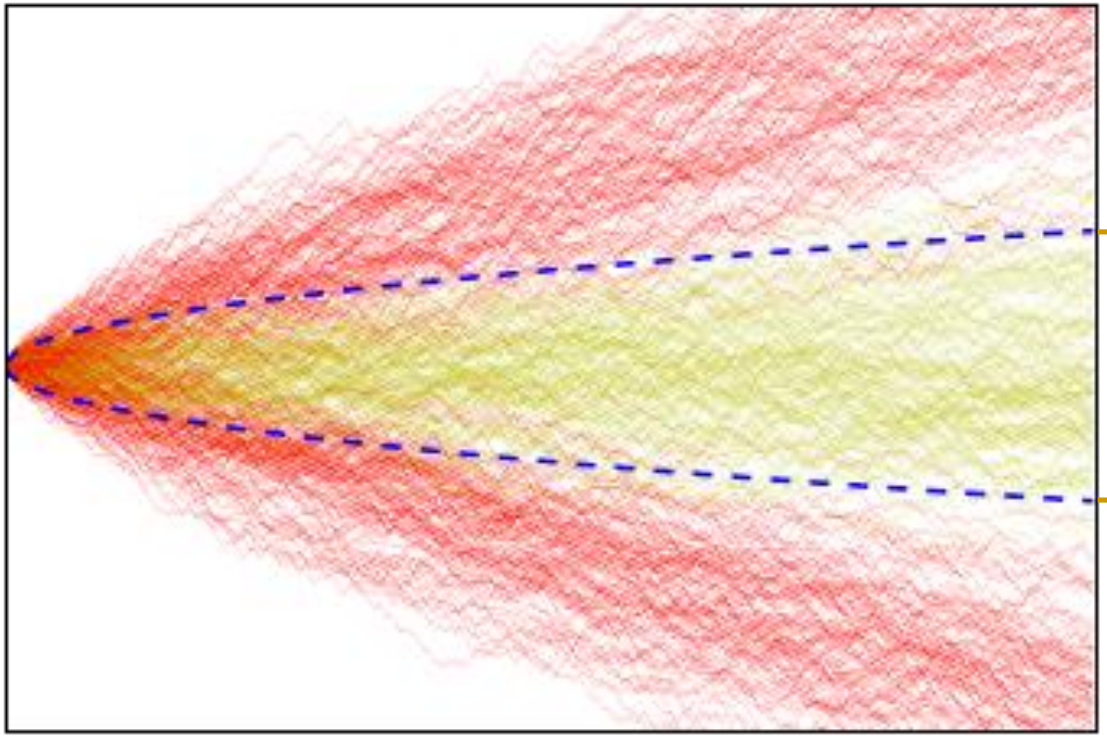
h = 0.6

h = 0.5

h = 0.4



# Testing a Coin



Reject null
(i.e. coin is unfair)

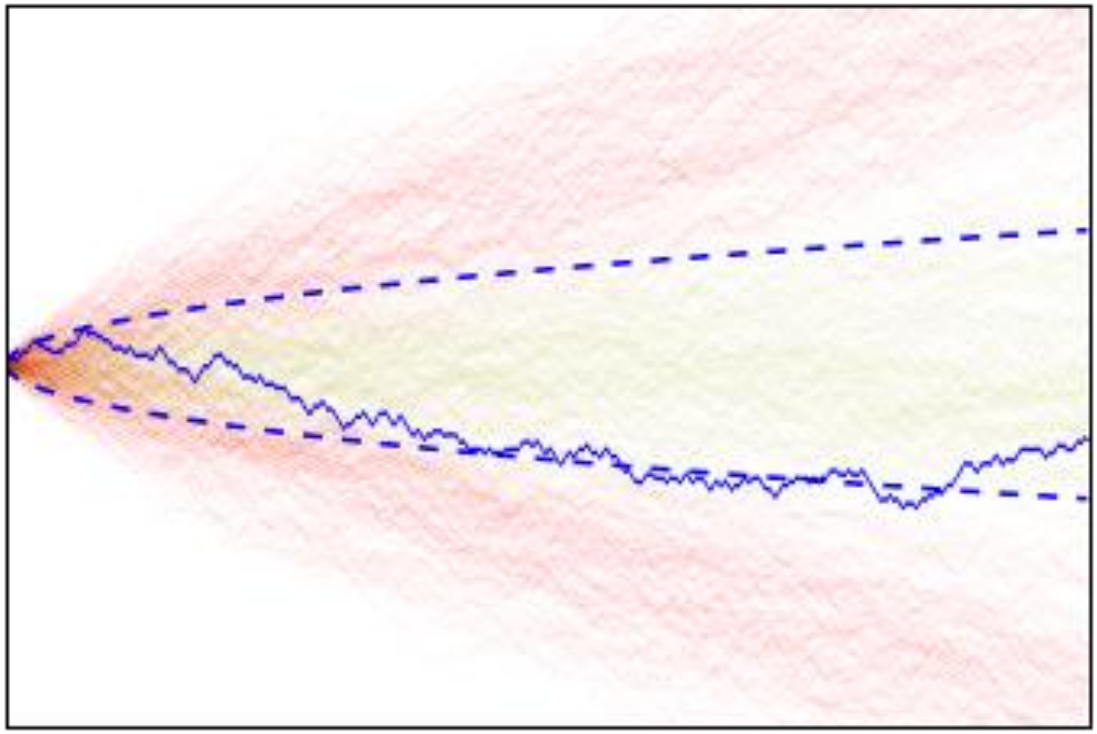
Can't reject null
Don't know whether
or not coin is fair

Reject null
(i.e. coin is
unfair)

N flips

# Testing a Coin

Remember that our test is **a single** random path



# STATISTICS

# Who cares about coin flips?

- Gamblers, sport referees, …
- Coin flips, especially biased coin flips, can be used to represent many types of random events:
  1. Car insurance: T=day without accident, H=day with accident.
  2. Polls: T=polled leans republican, H= polled leans democratic
  3. Poker: T=your hand is the highest, H=your hand is not the highest
  4. Internet: H=the sent packet will arrive within 1 second, T= it will not arrive within 1 second.
- Each of these examples can be seen as a flip of a biased coin.
  - 1,2,4 are case in which you want to estimate the bias from data (Statistics)
  - 3: Poker is a case in which you want to calculate the bias from first principles (Probability)

# Stents for preventing heart attacks

## Wingspan Stents

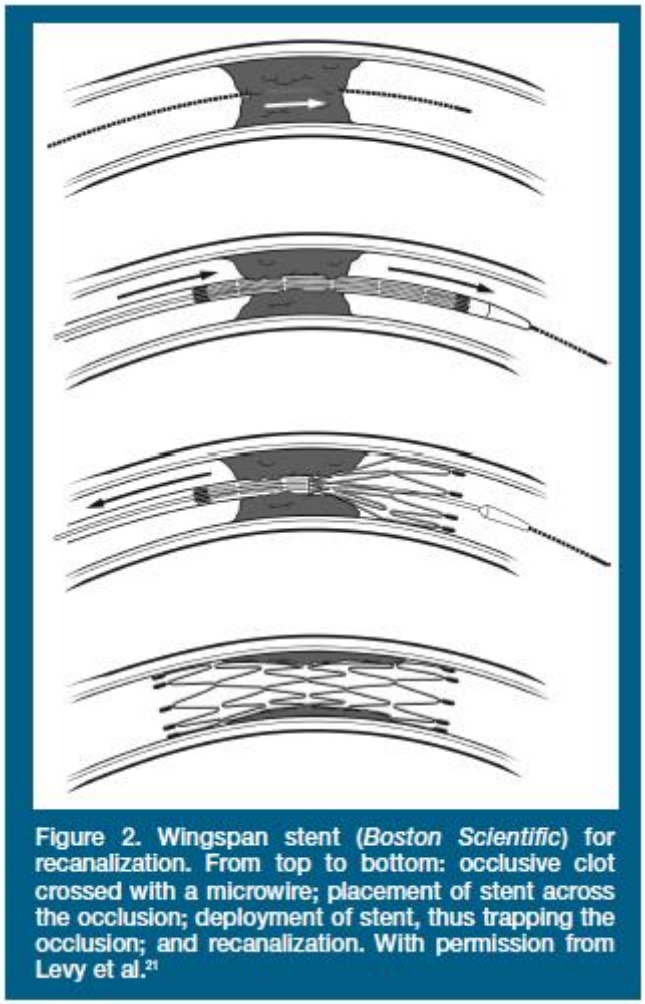


Figure 2. Wingspan stent (*Boston Scientific*) for recanalization. From top to bottom: occlusive clot crossed with a microwire; placement of stent across the occlusion; deployment of stent, thus trapping the occlusion; and recanalization. With permission from Levy et al.[21]

- A stent is a medical device that is inserted into a blood vessel with a clot and expands to remove the clot.
- Stents have been shown to help patients after a heart attack by reducing the chance of additional heart attacks.
- Doctors hoped that stents can be used to prevent heart attacks (before an attack has occurred).
- A study was conducted, involving 451 patients at high risk for heart attack.

# The study

- Chimowitz MI, Lynn MJ, Derdeyn CP, et al. 2011. Stenting versus Aggressive Medical Therapy for Intracranial Arterial Stenosis. New England Journal of Medicine 365:993- 1003.
  - More details in the book "Open Intro to Statistics"
- 451 patients with high risk of heart attack.
  - Divided randomly to "Treatment" and "Control"
  - Treatment received standard care and a Stent
  - Control received standard care but no Stent

| Patient | group | 0-30 days | 0-365 days |
|---|---|---|---|
| 1 | treatment | no event | no event |
| 2 | treatment | stroke | stroke |
| 3 | treatment | no event | no event |
| ⋮ | ⋮ | ⋮ | |
| 450 | control | no event | no event |
| 451 | control | no event | no event |

Table 1.1: Results for five patients from the stent study.

# The results of the study

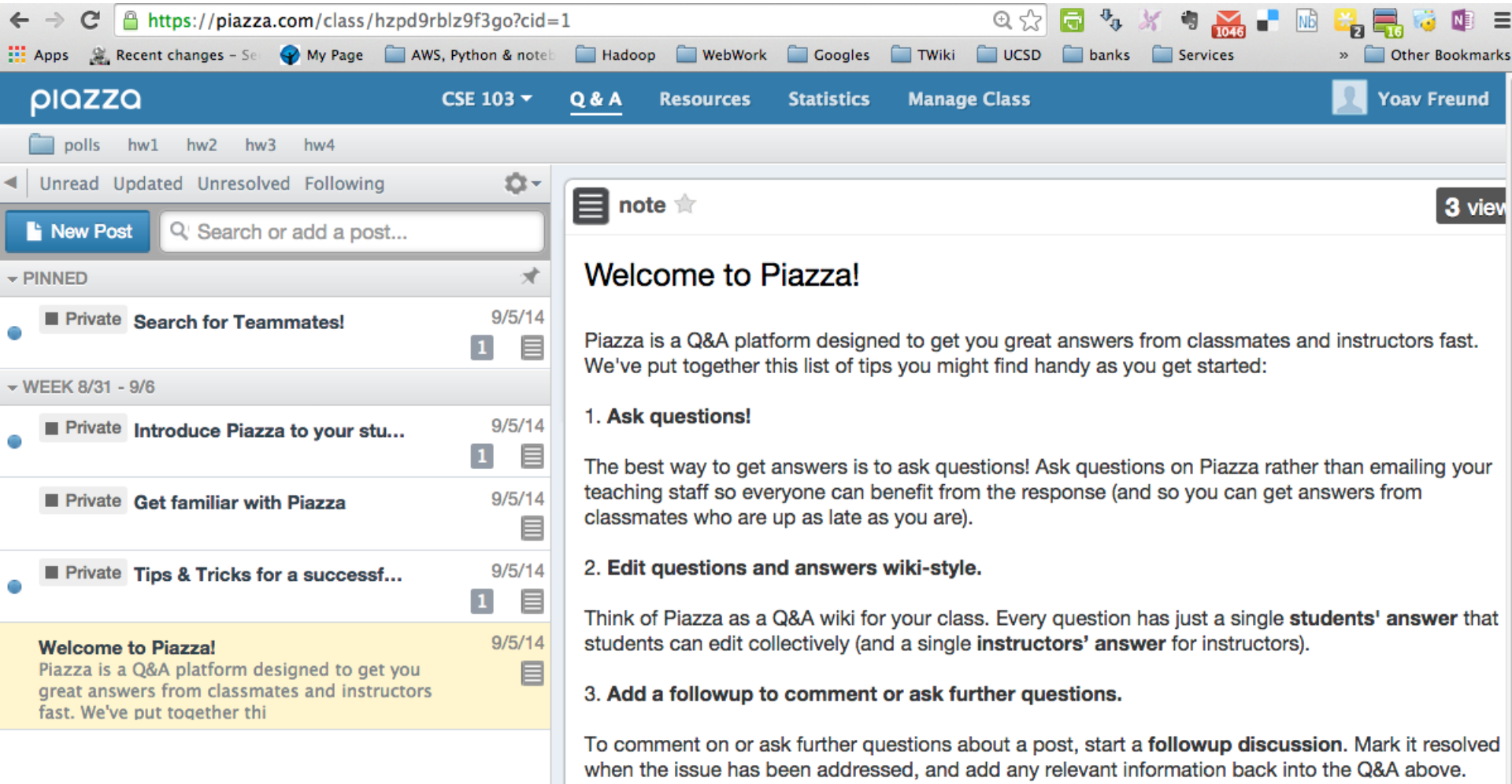
| | 0-30 days | | 0-365 days | |
|---|---|---|---|---|
| | stroke | no event | stroke | no event |
| treatment | 33 | 191 | 45 | 179 |
| control | 13 | 214 | 28 | 199 |
| Total | 46 | 405 | 73 | 378 |

Table 1.2: Descriptive statistics for the stent study.

- Within 30 days:
  - proportion of stroke in treated patients: 33/224=0.15=15%
  - Proportion of stroke in control patients: 13/227=0.06=6%
- Within 1 year:
  - proportion of stroke in treated patients: 45/224=0.2=20%
  - Proportion of stroke in control patients: 28/227=0.12=12%
- Clearly, we cannot say that using the stent helps.
- Can we say that using a stent hurts?
  - Not necessarily.
  - The numbers might be a random fluctuation.
  - There can be other factors, such as the procedure for inserting the stent.
  - The chosen patients might not be representative.
  - …

# Piazza

- **Main purpose:** discussion of Webwork assignment
- Search and read previous postings before
- Use it for everything! Any problem/question/idea/complaint
- 3 lowest grades



# For next class

- Make sure you have accounts on:
  - Open-edx (ID@ucsd.edu, password=PID)
  - Piazza
- Read chapter 1 of the class notes, comment in places that you don't understand.
- Post your questions on Piazza
- See you on wed!