Basic Combinatorics

Sets

- Explicit : A={1,4,2}
- Implicit: A={i|i is an odd number}
- Intersection: $x \in A \cap B \;\; if \;\; x \in A \; and \; x \in B$
- Union: $x \in A \cup B \;\; if \;\; x \in A \; or \; x \in B$

Products of sets

- Taking all possible combinations.

Style

Color

Size

XL

L

M

S

XS

Product Set={ (, ,),(, ,),…,}

Size of Product Set=

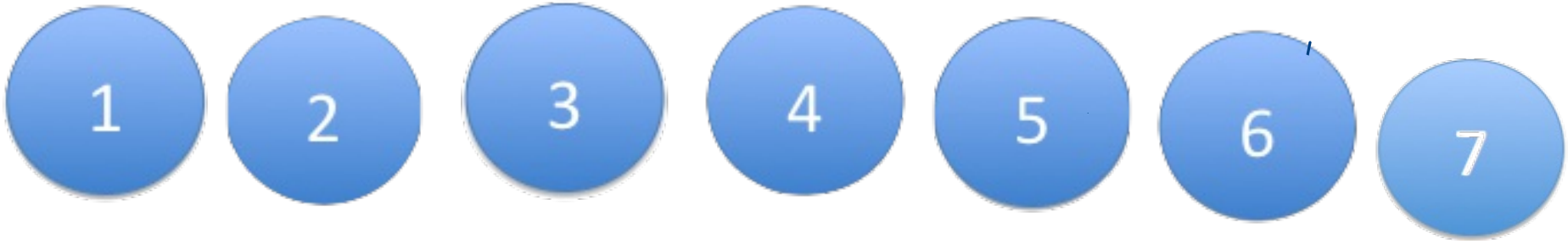
Raising a set to a power

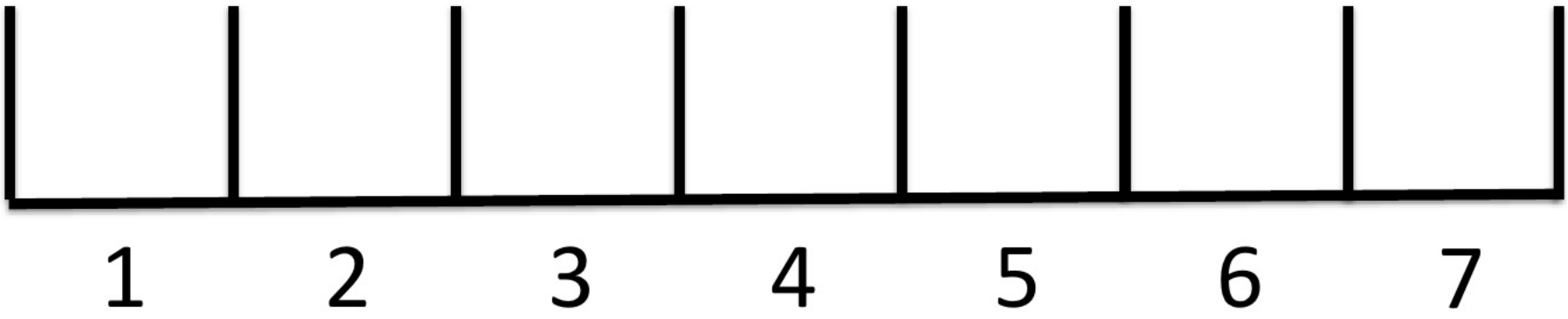
- The set of all binary sequences of length 7:
  - 0000000, 0000001, 0000010,…
  - 1111101, 1111110, 1111111
- Using product notation:
  - $\{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\} = \{0,1\}^7$
- Size:
  - $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7 = 128$

The Factorial Function

How many ways are there to order n different objects?

How many ways are there to order 7 different objects?





The Factorial function

- The number of possible ways to put n different objects into n different slots is

$$n * (n-1) * (n-2) * \cdots * 2 * 1 \doteq n!$$

- We say “n factorial”

Permutations

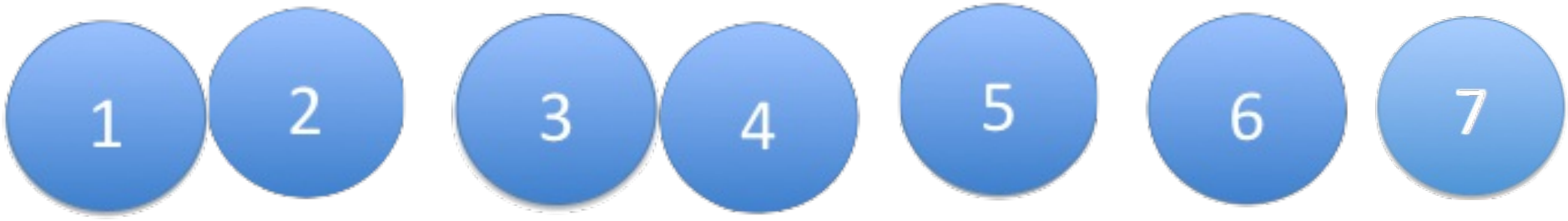
How many ways are there

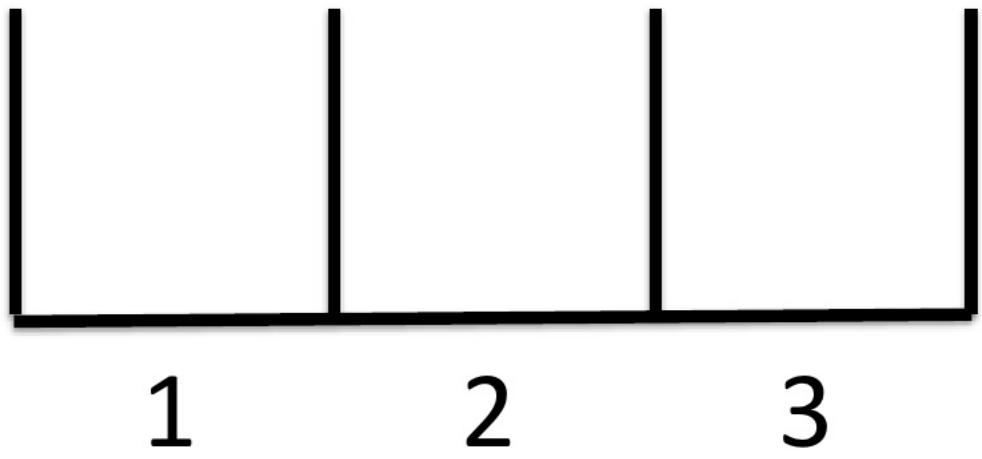
To pick k out of n elements

When the order matters

How many ways are there to pick 3 out of 7 elements

When the order matters





The Permutation Function

- The number of possible ways to put k<n different objects into n different slots is
- For n=7, k=3:

$$7 * 6 * 5 = \frac{7 * 6 * 5 * 4 * 3 * 2 * 1}{4 * 3 * 2 * 1} = \frac{7!}{(7-3)!}$$

- In general:

$$P(n,k) \doteq \frac{n!}{(n-k)!}$$

combinations

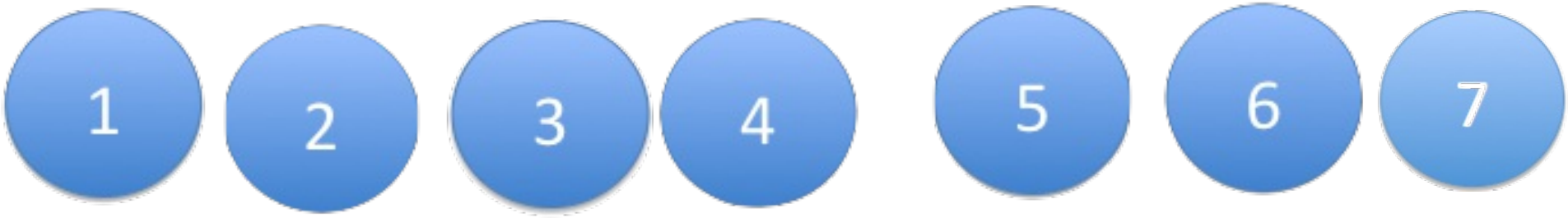
How many ways are there

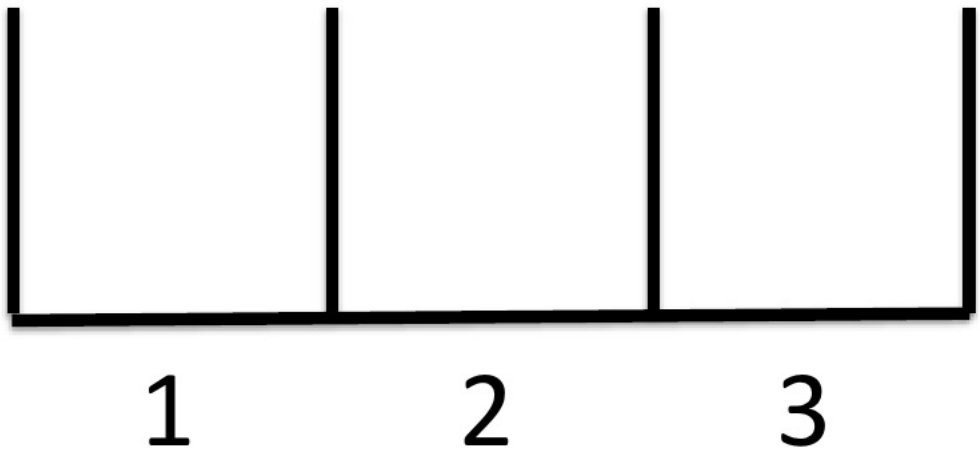
To pick k out of n elements

When the order does not matter

How many ways are there to pick 3 out of 7 elements

When the order does not matter





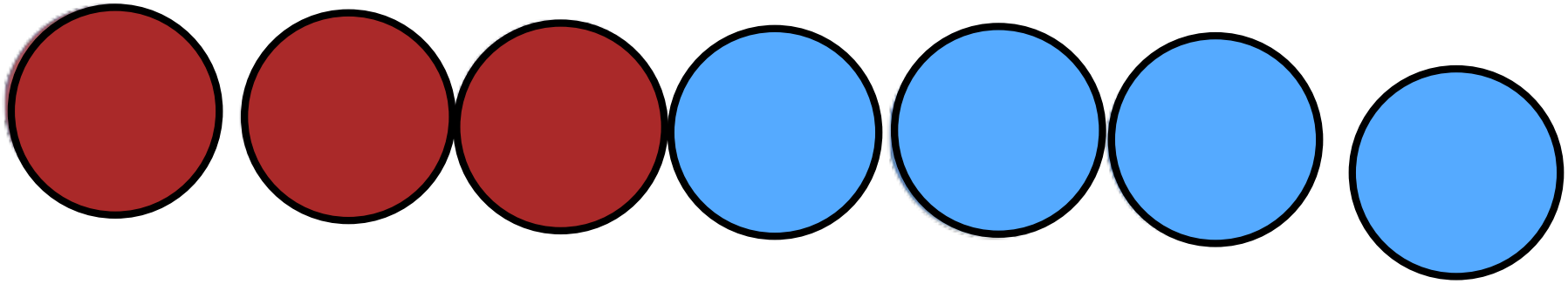
The Combinatorial function

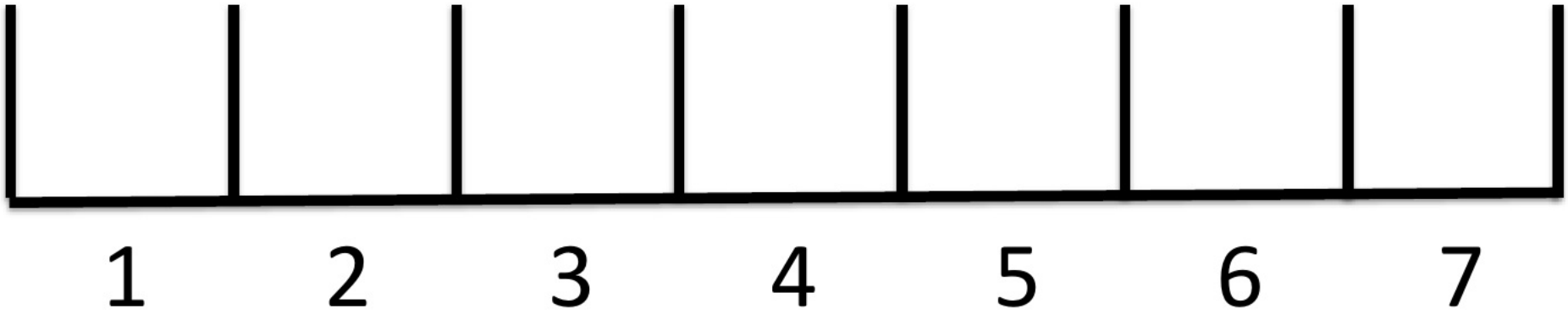
- The number of possible ways to place k identical objects into n different slots is

- $C(n,k) \doteq \binom{n}{k} \doteq \frac{P(n,k)}{k!} = \frac{n!}{k!(n-k)!}$

- We Say “n choose k”

How many ways are there to order 3 red and 4 blue balls?





this problem is identical to the previous one

How many different 3 digit numbers from the digits 1-9 ?

How many different 4 digit numbers from the digits 1-9 where all of the digits are different?

How many different 4 digit numbers from the digits 1-9 where all of the digits are different and the digits are in increasing order?

Suppose you choose 4 different digits from the set 1-9 and you place them in increasing order. What is the probability that the first digit is 3 ?

What is the size of the sample space?

What is the size of the event?

How many different words can be created by rearranging (all) the letters in the word MISSISSIPPI ?

M I S S I S S I P P I

Suppose the letters of the word MISSISSIPPI are put in a random order. What is the probability that the result is again MISSISSIPPI?

What is the size of the sample space?

M
I I I I
S S S S
P P

What is the size of the event?

A fair coin is flipped 11 times, what is the probability of 4 heads and 7 tails ?

What is the size of the sample space?

What is the size of the event?