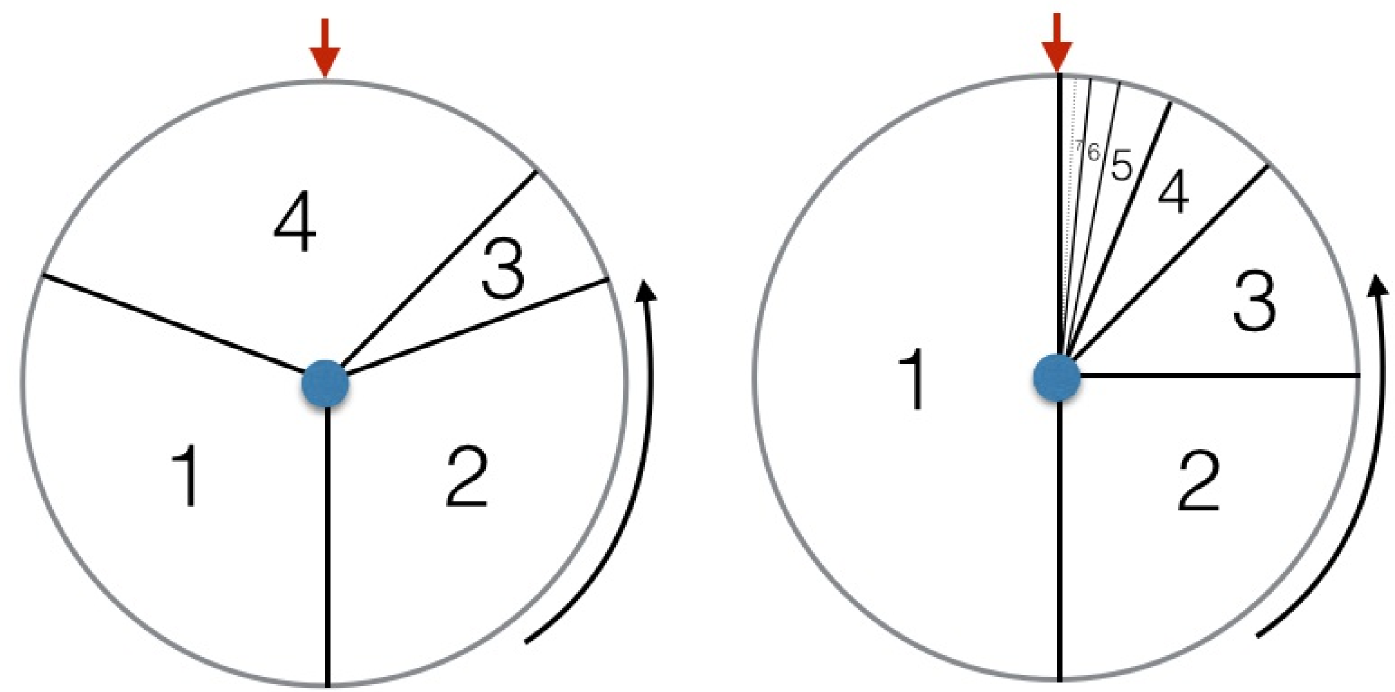
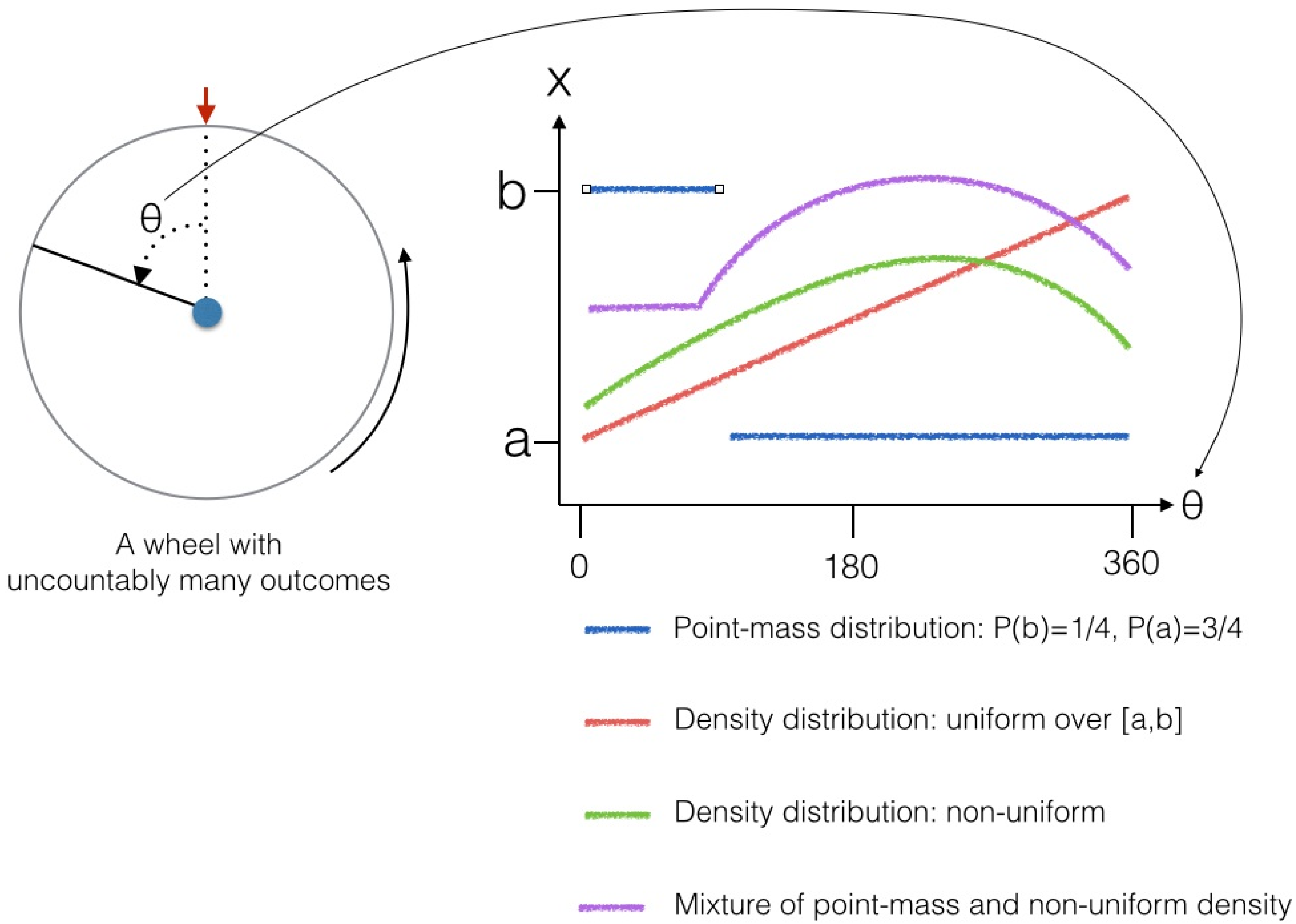
Random Variables, Expectation and Variance



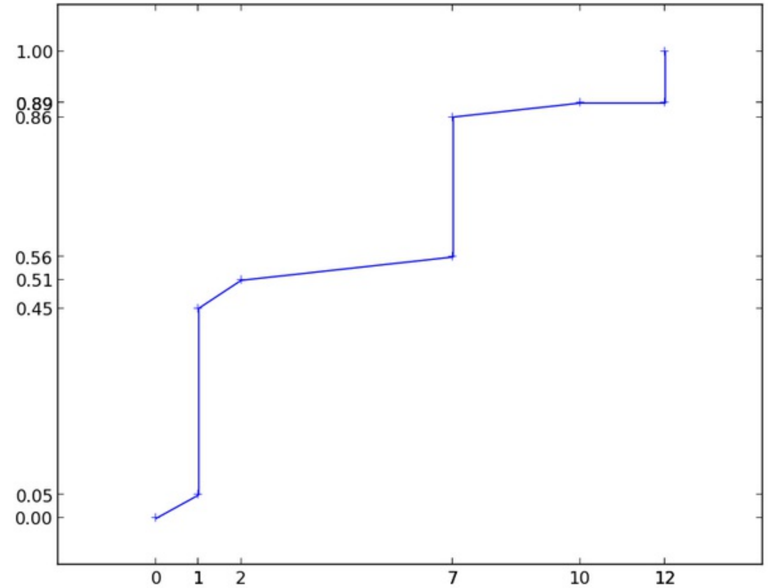
(a) A wheel with four outcomes

(b) A wheel with Infinitely many outcomes



θ
X
b
a
θ
0
180
360
A wheel with
uncountably many outcomes
Point-mass distribution: P(b)=1/4, P(a)=3/4
Density distribution: uniform over [a,b]
Density distribution: non-uniform
Mixture of point-mass and non-uniform density

(1 pt)
Given the following cumulative distribution:



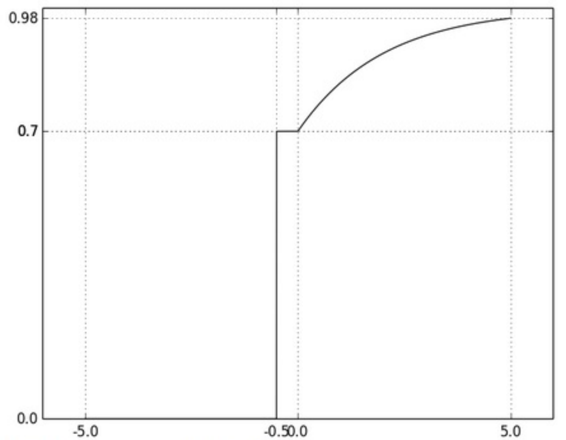
What are the uniform densities and mass distributions that could have been summed to produce this CDF?
Uniform distributions:

- Uniform on (0.00, ______) of probability density ______
- Uniform on (1.00, ______) of probability density ______

Point masses (ordered by the location):

- Point mass at ______ with probability ______
- Point mass at ______ with probability ______
- Point mass at ______ with probability ______

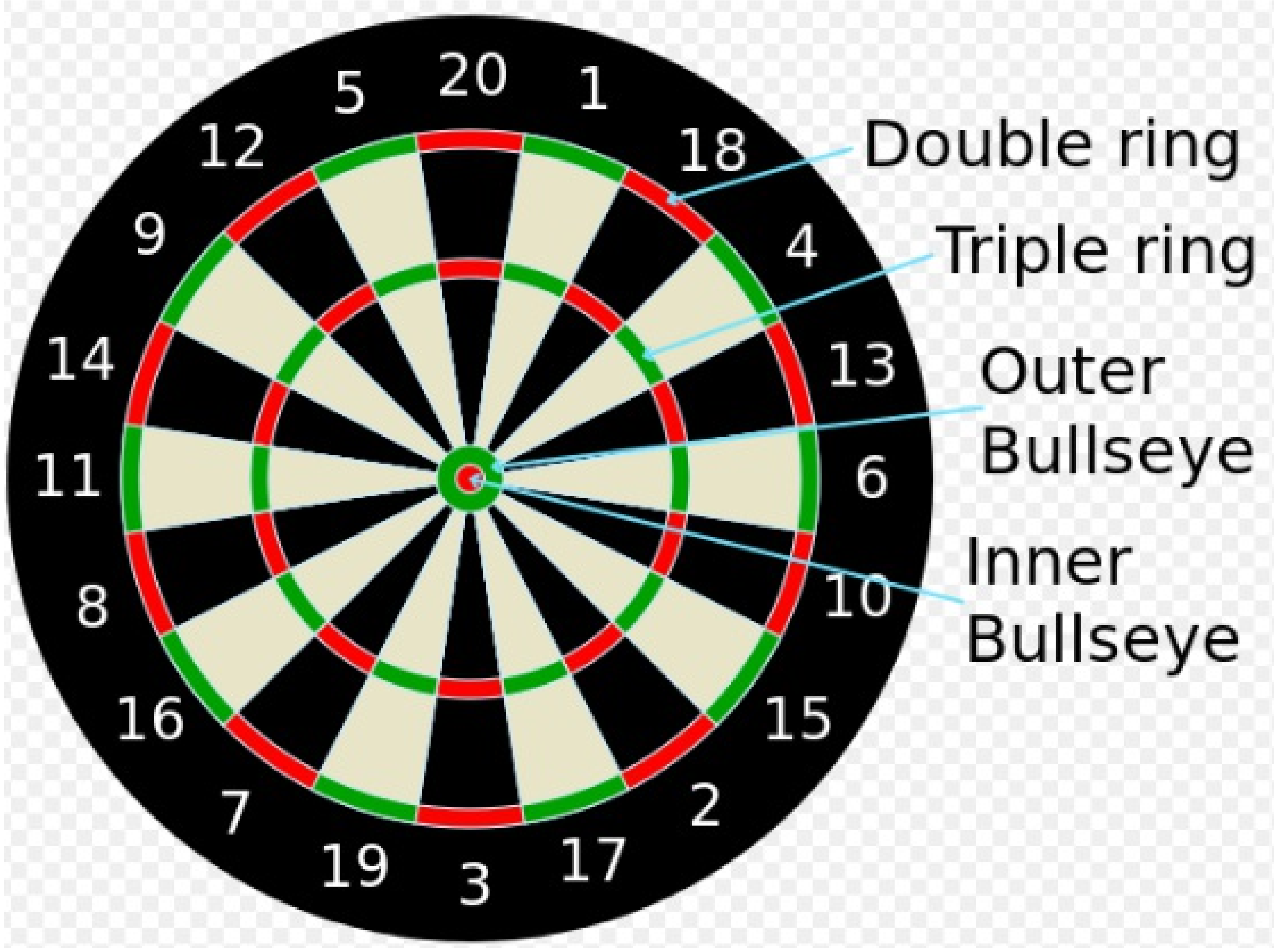
(1 pt)
Below is the CDF of a mixture distribution with **two** components.
One of the components is either a normal or an exponential distribution; the other is either a point mass or a uniform distribution.
All parameters of component distributions are small multiples of 0.5.
λ of exponential components and std of normal components take on value 0.5, 1 or 1.5.
Component weights take on multiples of 0.05 and they need to sum to one.



Identify the component distributions:

- The exponential component has λ of 0.5. Its component weight is
- Point mass on . Its component weight is

Densities over a 2D space

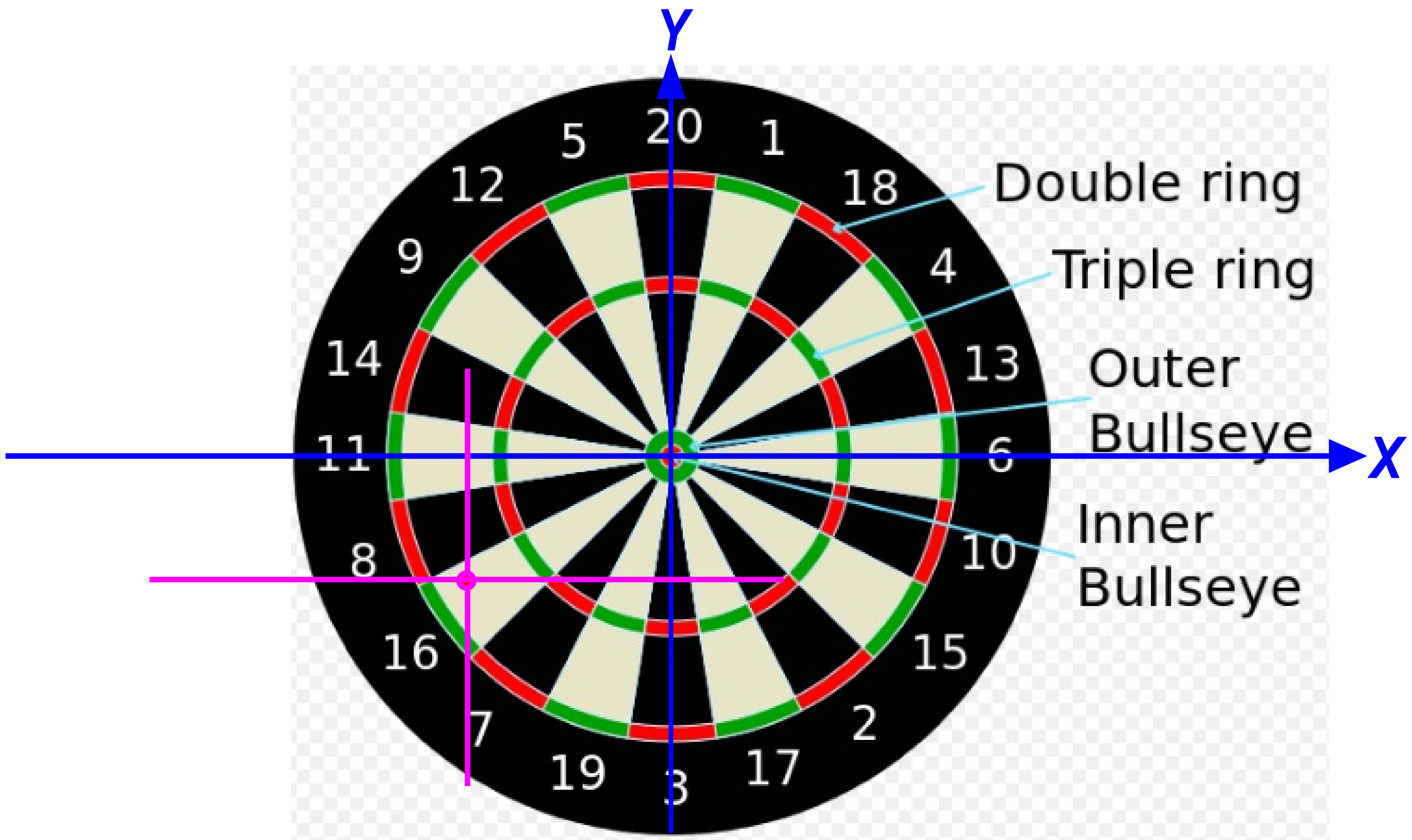


the sample space is the plane

x and y are mappings from the plane to R

Such mappings are called Random Variables

A natural assumption: the probability distribution of the location (x,y) that the dart falls is a density function that is highest at the bullseye and gets lower the further from the bullseye you get.



***Events** are sets.*
Events map outcomes to {True,False}
True = Outcome in set.
False = Outcome not in set.

Examples of Events:

1. ***Dart lands on Inner Bullseye***
2. ***Dart lands on double ring.***
3. ***Dart lands on the "14" section***

***Random variables** are functions (mappings) from Omega to R*
Examples of Random variables:

1. ***X position of dart***
2. ***Y position of dart***
3. ***Distance of dart from middle of target***
4. ***The number associated with the section in which the dart landed***

baseball acronyms
G Games Played
PA Plate Appearances
AB At Bat
R Runs Scored
H Hits
D ?
T ?
HR Home Runs
RBI Runs Batted In
BB Bases on Balls (walks)
SO Strikeouts
BA ?
OBP On base Percentage
SLG Slugging Percentage
OPS OBP+SLG

Player	G	PA	AB	R	H	D	T	HR	RBI	BB	SO	BA	OBP	SLG	OPS
Mike Napoli	17	64	49	13	16	4	0	6	14	15	16	.327	.484	.776	1.260
Josh Donaldson	20	88	72	17	28	8	0	5	15	14	11	.389	.500	.708	1.208
Hunter Pence	21	92	78	19	25	4	0	9	26	13	14	.321	.413	.718	1.131
Matt Carpenter	21	102	88	23	35	11	2	1	11	12	19	.398	.480	.602	1.083
Ryan Zimmerman	21	97	90	22	29	2	0	**11**	16	7	18	.322	.371	.711	1.082
Freddie Freeman	20	85	73	15	26	3	0	6	17	10	15	.356	.435	.644	1.079
Michael Cuddyer	16	67	62	8	26	4	0	3	14	4	10	.419	.448	.629	1.077
Adam Lind	18	60	55	11	16	2	0	7	17	5	11	.291	.350	.709	1.059
Andrew McCutchen	20	83	66	13	22	5	1	3	8	13	11	.333	.470	.576	1.046
Prince Fielder	20	86	79	11	30	7	0	4	14	6	13	.380	.419	.620	1.039
Shin-Soo Choo	18	85	60	15	18	3	0	4	10	21	12	.300	.488	.550	1.038
Paul Goldschmidt	21	92	80	12	27	6	2	4	19	11	19	.338	.424	.613	1.036
Moises Sierra	19	67	63	8	22	12	1	1	9	4	14	.349	.388	.619	1.007
Josmil Pinto	16	62	58	9	21	5	0	3	9	4	12	.362	.403	.603	1.007
Mike Trout	21	95	71	16	21	5	1	3	10	23	21	.296	.474	.521	.995
Yoenis Cespedes	19	80	77	12	26	2	1	6	19	2	19	.338	.363	.623	.986
Matt Holliday	20	92	76	14	28	6	0	2	20	14	13	.368	.457	.526	.983
David Ortiz	20	91	76	18	21	9	0	5	16	13	16	.276	.385	.592	.977
Chase Headley	17	67	56	8	15	2	0	5	9	10	12	.268	.388	.571	.959
Matt Adams	19	72	69	13	21	1	0	7	14	3	21	.304	.333	.623	.957
Joey Votto	20	94	73	12	22	3	0	4	9	20	17	.301	.447	.507	.954
Eric Hosmer	20	87	78	12	27	6	1	2	12	9	18	.346	.414	.526	.939
Wil Myers	20	84	78	10	24	9	0	4	10	6	18	.308	.357	.577	.934
Giancarlo Stanton	20	85	72	12	19	3	0	6	16	11	27	.264	.376	.556	.932
Desmond Jennings	21	83	68	7	19	6	1	3	13	13	16	.279	.398	.529	.927

Outcome space: ***all possible performances of baseball hitters for a month***
Outcome: ***The performance of a particular player***
Random variables: ***measures of performance: G, PA, AB ...***

Events: ***More than 8 home runs,***
OPS higher than 1.0, 1.1, 1.2, ...

Two random variables: X, Y are independent if and only if
any event conditioned on X
is independent of
any event conditioned on Y

Joint distribution of two independent random variables

	X=1	**X=2**	**X=10**	**P(Y=y)**
Y=-1	1/12	1/12	2/12	4/12= 1/3
Y=+1	2/12	2/12	4/12	8/12= 2/3
P(X=x)	3/12= 1/4	3/12= 1/4	6/12= 1/2	

Marginals

Joint distribution of two dependent random variables

	X=1	**X=2**	**X=10**	**P(Y=y)**
Y=-1	1/12	2/12	1/12	4/12= 1/3
Y=+1	2/12	1/12	5/12	8/12= 2/3
P(X=x)	3/12= 1/4	3/12= 1/4	6/12= 1/2	

Marginals

Expected Value

- Suppose X is a discrete random variable $P(X = a_i) = p_i$
  - The expected value of X is $E(X) = \sum_{i=1}^{n} p_i a_i$
- Suppose X is a continuous random variable with density f
  - The expected value of X is $E(X) = \int_{-\infty}^{+\infty} f(x) x dx$
- $E(X)$ is a property of the distribution, it is not a random variable.
- The average is a random variable:
  - $Average(x_1, x_2, \ldots, x_n) \doteq \frac{1}{n} \sum_{i=1}^{n} x_i$
- When n is large, the average tends to be close to the mean.

Example - Binary random variables:

Let $X_1, X_2, \ldots, X_{100}$

Be independent binary random variables: $P(X_i = 0) = P(X_i = 1) = \frac{1}{2}$

Let $S = \frac{1}{100}\sum_{i=1}^{100} X_i$ S is the ________, S is/is-not a random variable?

$E(X_i) = 0 \times \frac{1}{2} + 1 \times \frac{1}{2} = \frac{1}{2}$, $E(X_i)$ is/is-not a random variable?

What is $E(S)$?

Rules for expected value:

1. If a, b are constants and X is a random variable then

$$E(aX+b) = aE(X)+b$$

2. If X, Y are random variables (dependent or independent)

$$E(X+Y) = E(X)+E(Y)$$

—> what is $E(aX+bY+c) = ?$

3. If the distribution of the RV X is a mixture of two distributions:

$P = \mu P_1 + (1-\mu)P_2$ then

$$E_P(X) = \mu E_{P_1}(X) + (1-\mu)E_{P_2}(X)$$

So now, $S = \frac{1}{100}\sum_{i=1}^{100} X_i$, what is $E(S)$?

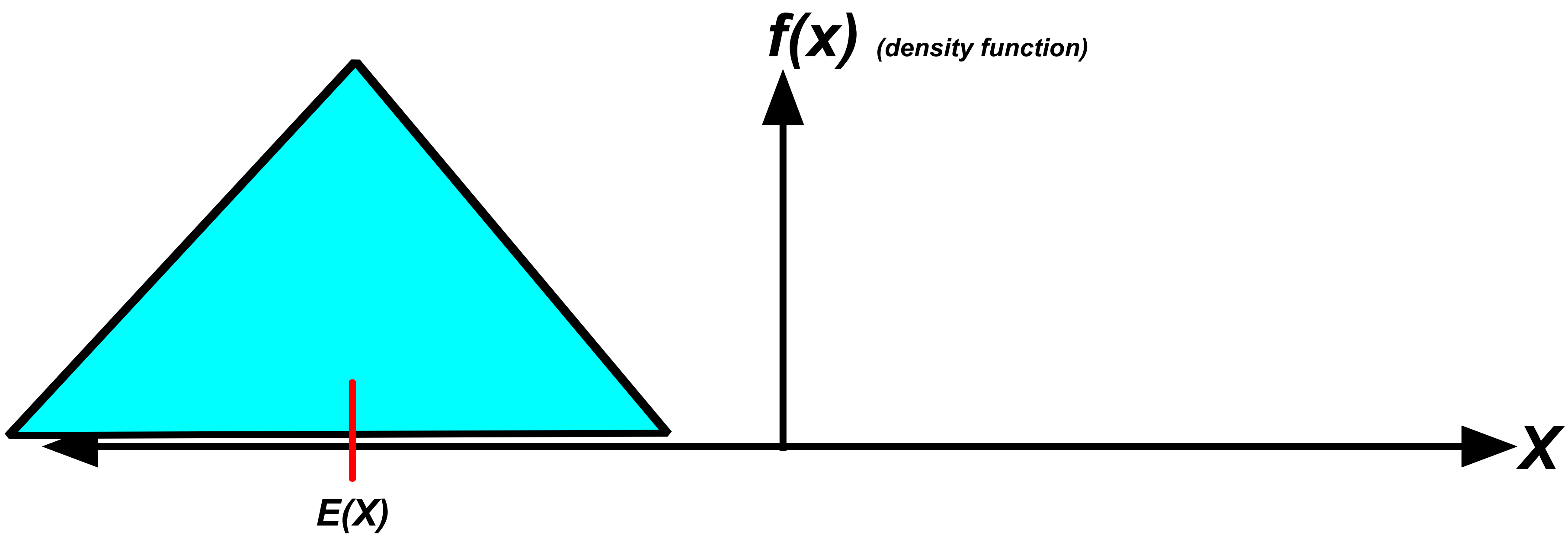
The mean is the center of mass of the distribution

If the distribution is symmetric around zero, then the mean is zero.
If the distribution is symmetric around a, then the mean is a.

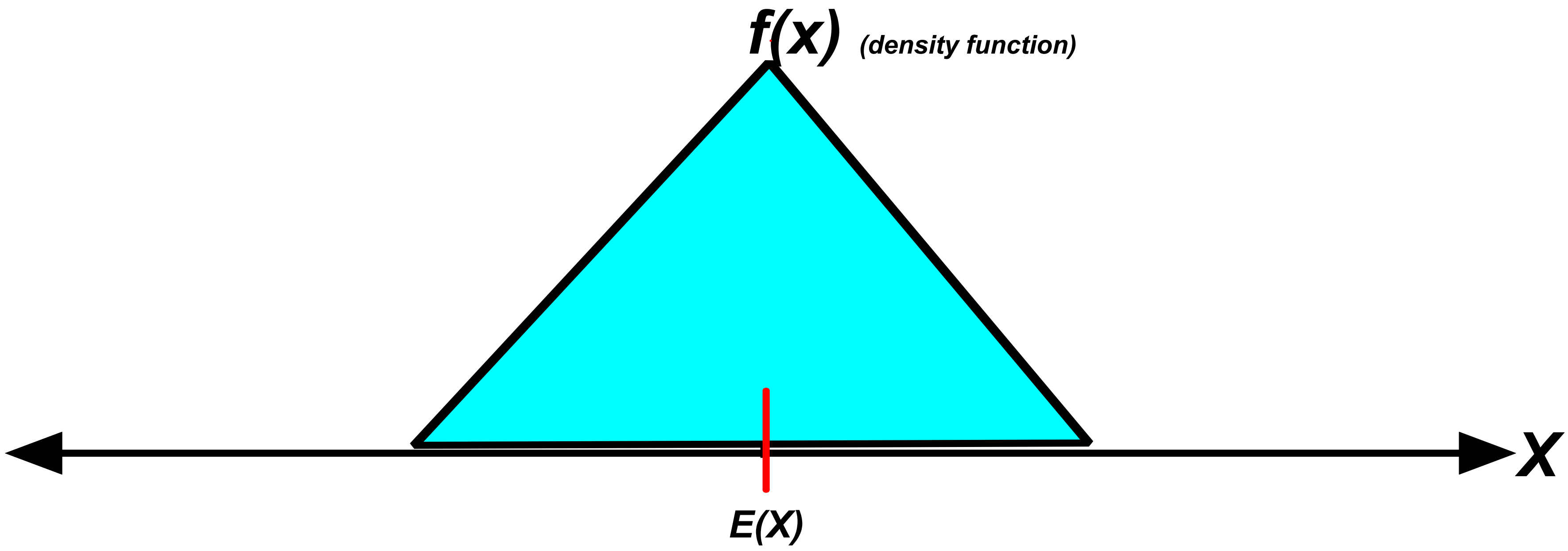
1. If a,b are constants and X is a random variable then

$$E(aX+b) = aE(X) + b$$

E(X) corresponds to the location. If we subtract the mean we have a distribution centered at zero: E(X-E(X))=E(X)-E(X)=0



The mean corresponds to the location of the "center" of the distribution.
How do we measure the "width" of the distribution?



Measuring the width of the distribution

Lets use $\mu \doteq E(X)$

We already know that $E(X-\mu)=0$

To find the width we could use $E\left(\left|X-\mu\right|\right)$

But it is much more convenient to use:

$$Var(X) \doteq E\left((X-\mu)^2\right)$$

Using the rules for expected value (remember that μ is a constant)

$$Var(X) \doteq E\left((X-\mu)^2\right) = E\left(X^2 - 2\mu X + \mu^2\right)$$

$$= E(X^2) - 2\mu E(X) + \mu^2 = E(X^2) - E(X)^2$$

Properties of the variance

1. If a, b are constants and X is a random variable then

$$Var(aX+b) = a^2 Var(X)$$

2. If X, Y are Independent Random Variables, then

$$Var(X+Y) = Var(X) + Var(Y)$$

3. If the distribution of the RV X is a mixture of two distributions:

$P = \mu P_1 + (1-\mu) P_2$ then..... (nothing)

Why do we need the std-dev?

For monday:

1. ***Start working on Week4 assignment.***
2. ***Start reading chapter 7.***