Spin-1/2 Operators and Their Eigenstates in Bra-Ket Notation

Spin Operators in Bra-Ket Form

Let $|+\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|-\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, and let $\hbar$ be Planck's constant. The spin operators for a spin-1/2 particle are:

S_z Operator

$$S_z = \frac{\hbar}{2} \left(|+\rangle \langle +| - |-\rangle \langle -| \right)$$

S_x Operator

$$S_x = \frac{\hbar}{2} \left(|+\rangle \langle -| + |-\rangle \langle +| \right)$$

S_y Operator

$$S_y = \frac{\hbar}{2i} \left(|+\rangle \langle -| - |-\rangle \langle +| \right)$$

Eigenstates of Spin Operators

Eigenstates of S_z

$$|+\rangle_z = |+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |-\rangle_z = |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Eigenstates of S_x

$$|+\rangle_x = \frac{1}{\sqrt{2}} \left(|+\rangle + |-\rangle \right), \quad |-\rangle_x = \frac{1}{\sqrt{2}} \left(|+\rangle - |-\rangle \right)$$

Eigenstates of S_y

$$|+\rangle_y = \frac{1}{\sqrt{2}} \left(|+\rangle + i\,|-\rangle \right), \quad |-\rangle_y = \frac{1}{\sqrt{2}} \left(|+\rangle - i\,|-\rangle \right)$$

Summary Table

Operator	$+\hbar/2$ Eigenstate	$-\hbar/2$ Eigenstate
S_z	$\lvert+\rangle$	$\lvert-\rangle$
S_x	$\frac{1}{\sqrt{2}}(\lvert+\rangle + \lvert-\rangle)$	$\frac{1}{\sqrt{2}}(\lvert+\rangle - \lvert-\rangle)$
S_y	$\frac{1}{\sqrt{2}}(\lvert+\rangle + i\,\lvert-\rangle)$	$\frac{1}{\sqrt{2}}(\lvert+\rangle - i\,\lvert-\rangle)$