Synchrony and stability in electric power systems (and Josephson Junction arrays)

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ECS14 - mpipks Dresden - 22.09.2014

FNSNF

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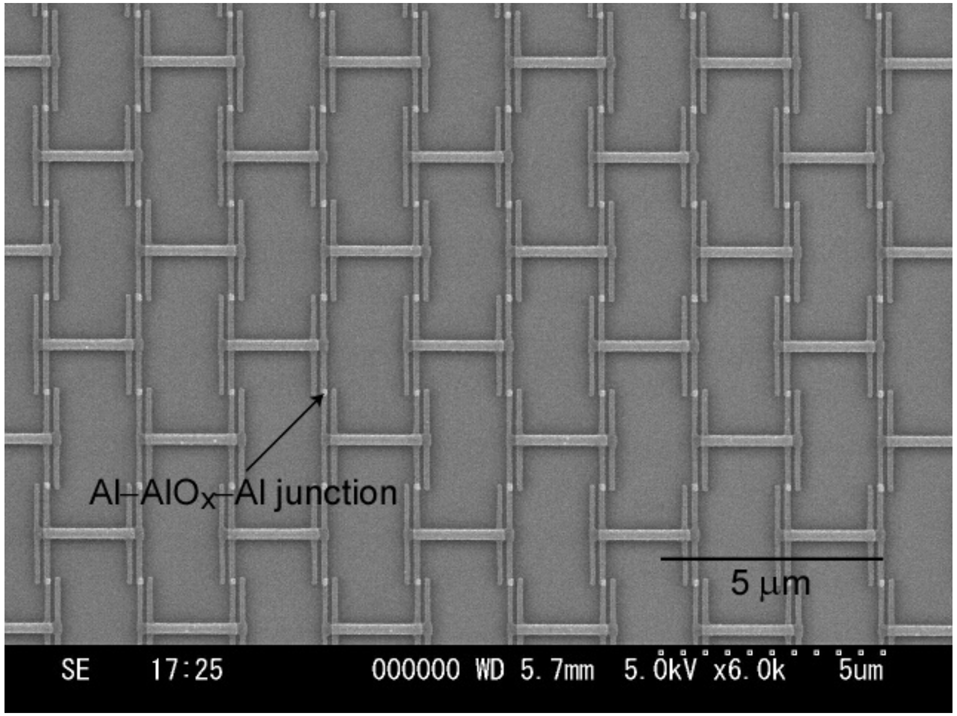
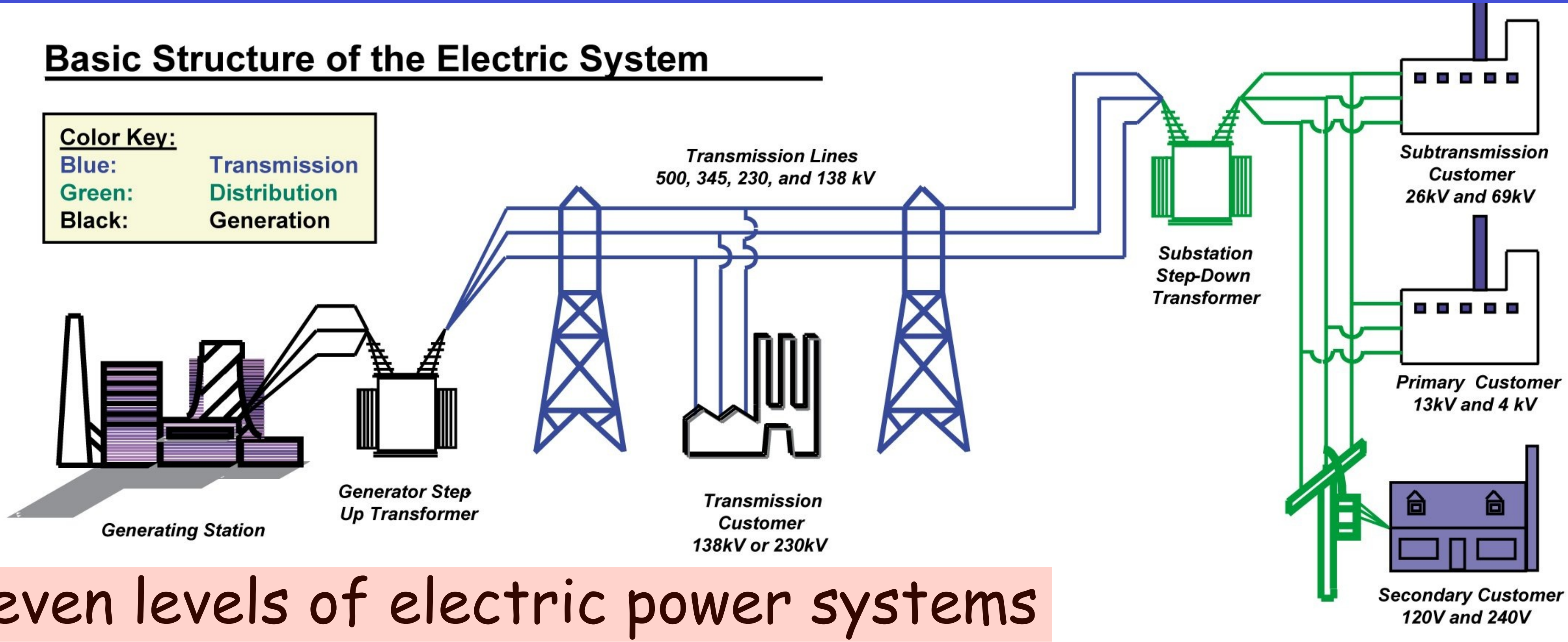
Josephson junction arrays vs. electric power systems ?

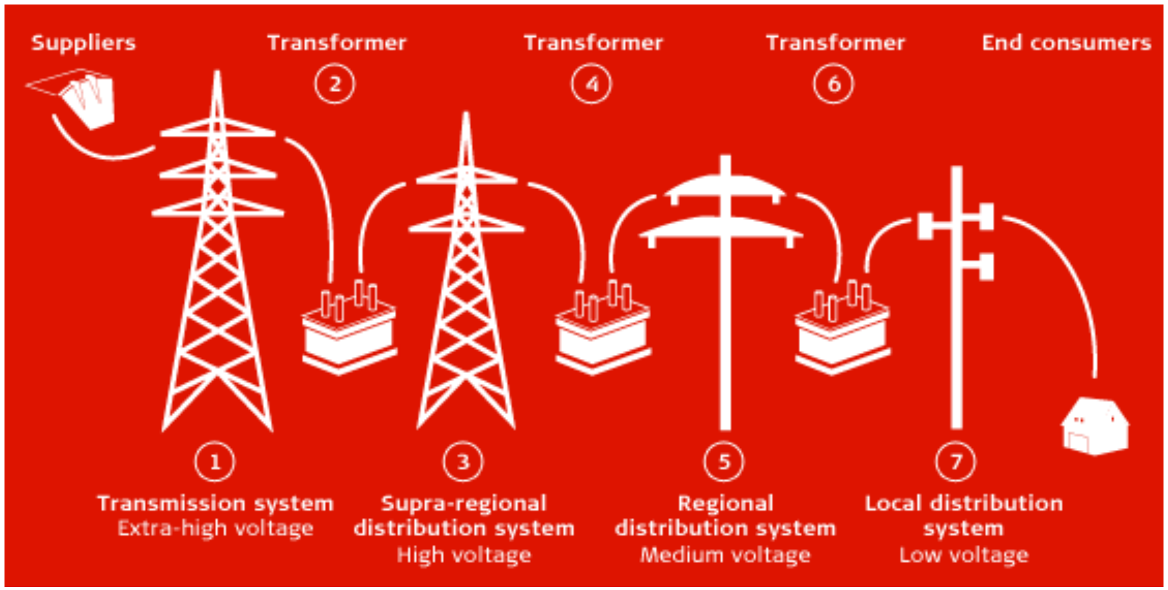


Takahide, Yagi, Kanda, Ootuka, and Kobayashi
Phys. Rev. Lett. 85, 1974 (2000)

What are electric power systems ?



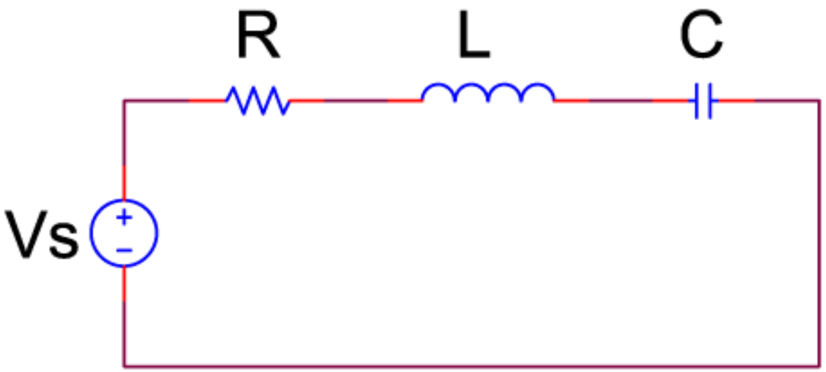
Seven levels of electric power systems



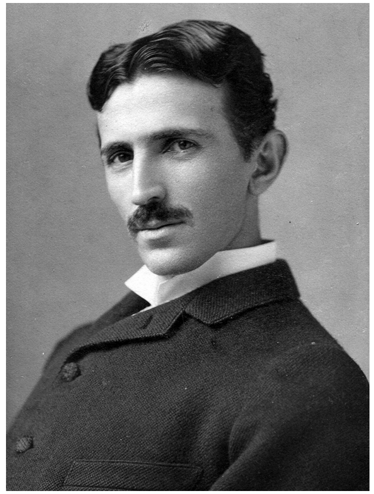
Power is ~conserved
i.e. write Eq. for power

What are electric power systems ?

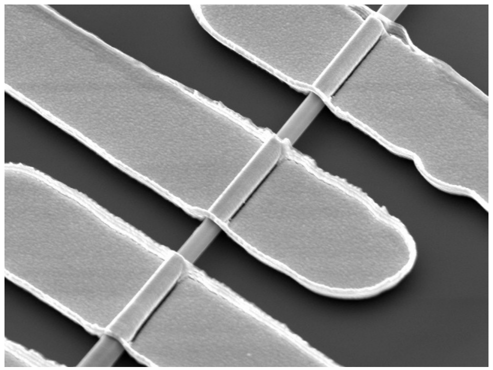
- AC electric current/voltages
  (minimize losses ~ high voltages,
  but then need transformers)
- current and voltage not in phase



$$u(t) = u_0 \exp[i\omega t]$$
$$i(t) = i_0 \exp[i(\omega t + \phi)]$$
$$\tan(\phi) = (\omega L - 1/\omega C)/R$$


N Tesla 1856-1943

- complex impedance $u(\omega) = Z(\omega)\, i(\omega) \qquad Z(\omega) = R + i\omega L - i/\omega C$
- inductance more important than resistance for large conductors



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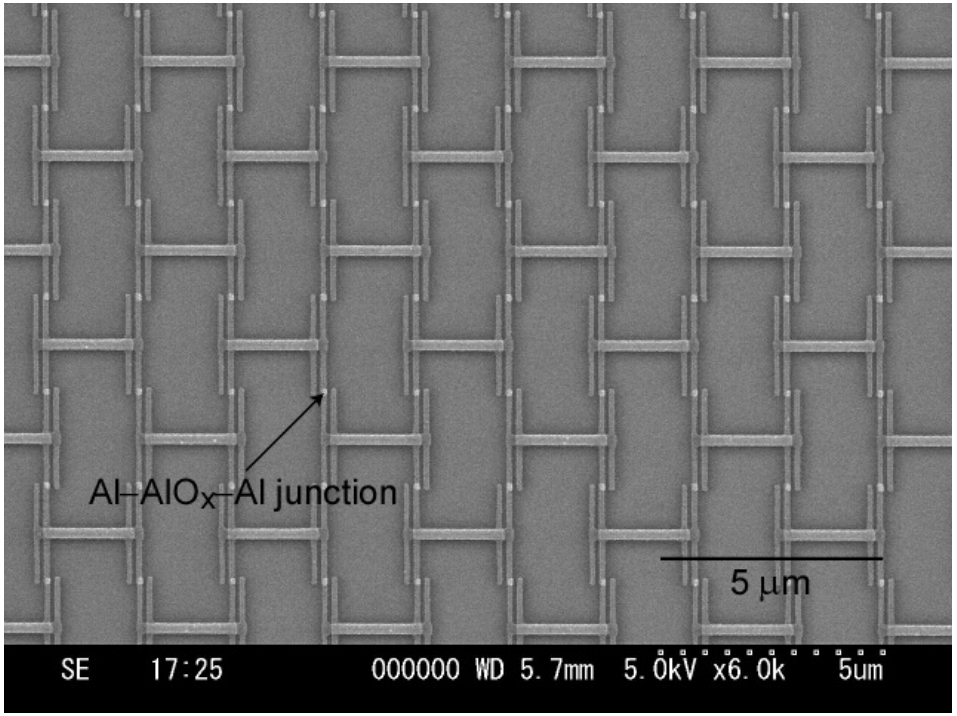
Josephson junction arrays vs. electric power systems !



Josephson current

$$I_{ij} = I_c \sin(\theta_j - \theta_i)$$

Transmitted power

$$P_{ij} \simeq |V_i V_j| K_{ij} \sin(\theta_j - \theta_i)$$

Power flow equations

- Three-phase system (AC)
- Symmetric (all phases with same amplitude+phase)
- Power grid/network with buses (nodes) and lines
- Buses characterized by voltage, current and relative phase
- Active and reactive power injected/extracted

$$P_i = \sum_j |V_i V_j| \left[G_{ij} \cos(\theta_j - \theta_i) + K_{ij} \sin(\theta_j - \theta_i) \right]$$

$$Q_i = \sum_j |V_i V_j| \left[G_{ij} \sin(\theta_j - \theta_i) - K_{ij} \cos(\theta_j - \theta_i) \right]$$

Susceptance
between i and j

Voltages
at buses i and j

Conductance
between i and j

Phases
at buses i and j

Power flow equations

(power is conserved upon voltage transformation)

$$P_i = \sum_j |V_i V_j| \left[G_{ij} \cos(\theta_j - \theta_i) + K_{ij} \sin(\theta_j - \theta_i) \right]$$

$$Q_i = \sum_j |V_i V_j| \left[G_{ij} \sin(\theta_j - \theta_i) - K_{ij} \cos(\theta_j - \theta_i) \right]$$

- These equations *must* be obeyed at any time for steady-state operation of the electric power network
- Voltages (+-), G's and K's are fixed

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Power flow equations - linear regime

- inductance more important for large conductors

$$P_i \simeq \sum_j |V_i V_j| K_{ij} \sin(\theta_j - \theta_i)$$

- linear regime

$$P_i \simeq \sum_j |V_i V_j| K_{ij} (\theta_j - \theta_i)$$

- i.e. analogous to Landauer-Büttiker

$$I_i = \frac{2e^2}{h} \sum_j G_{ij} (V_j - V_i)$$

Note: current conservation

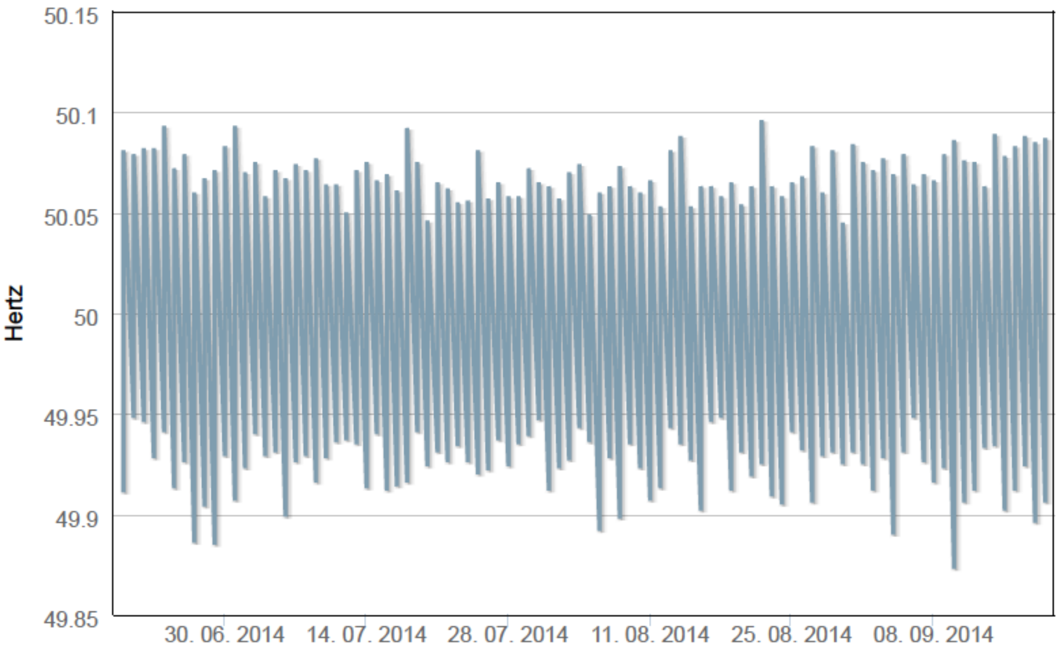
$$\sum_j K_{ij} = 0$$

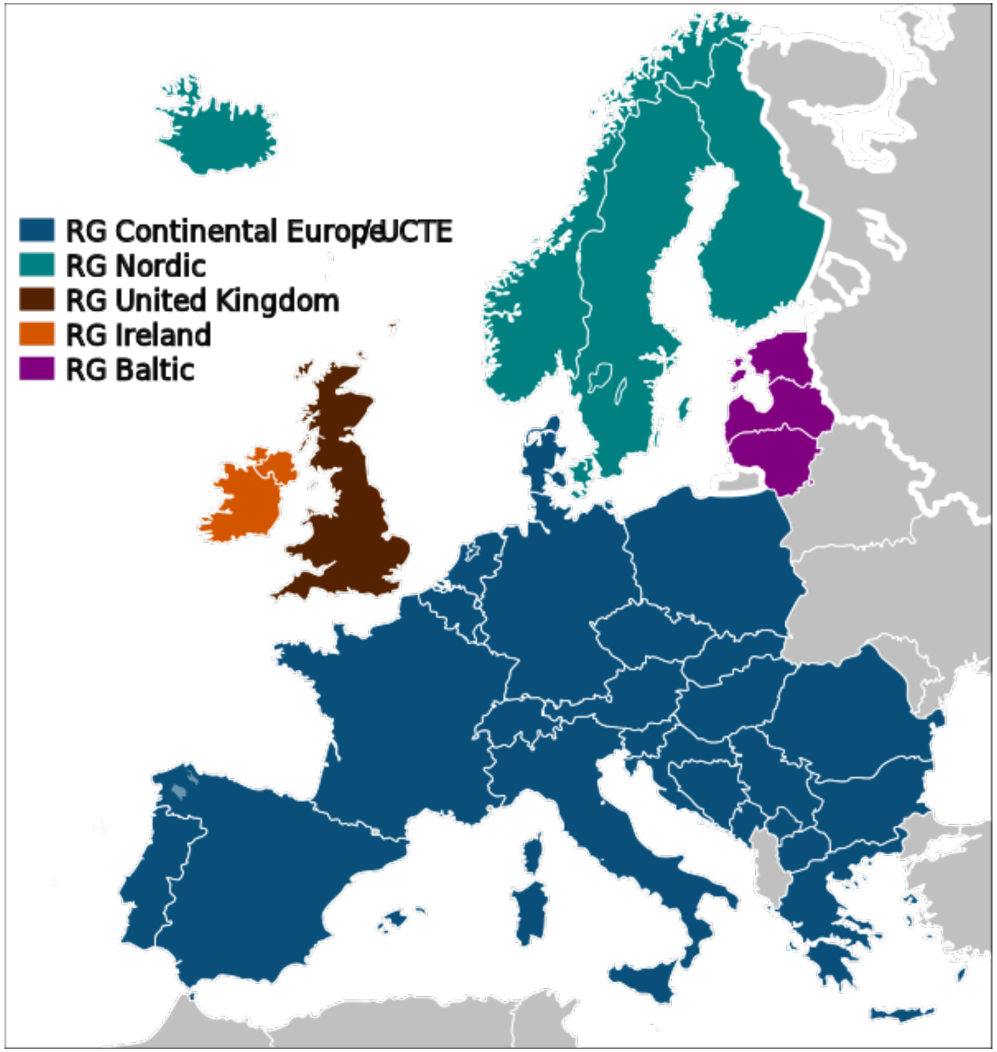
$$\sum_j G_{ij} = 0$$

Note (important) :

$$\boxed{\sum_i P_i = \sum_{ij} G_{ij} \cos(\theta_j - \theta_i)}$$

Steady-state operation : synchrony over 1000's of kms



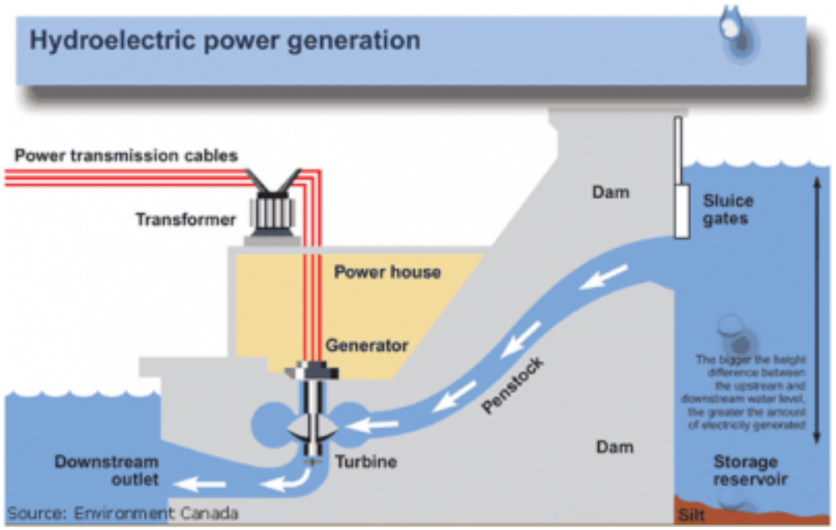


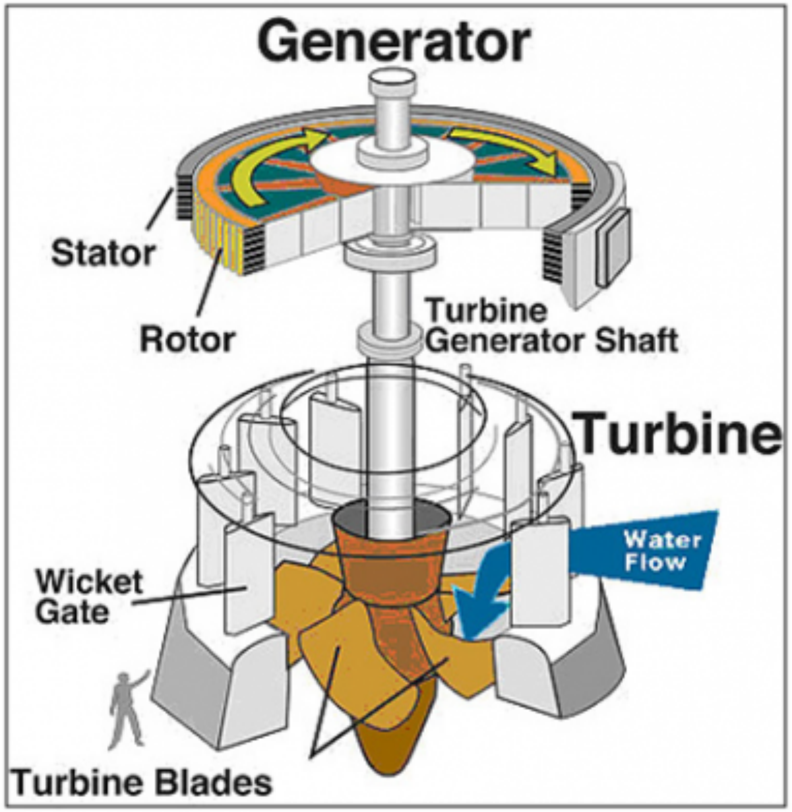
Loss of synchrony : blackouts

Italy blackout, sep 28 2003

Northeast blackout, aug 14 2003

Time-evolution of frequency : swing equations





- Electricity production with rotating machines
- Potential, chemical, nuclear or thermal energy converted into mechanical energy (rotation)
- Mechanical power converted into electric power
- Time-dependence : think power instead of energy
- Balance between power in, power out and energy change in rotator : *SWING EQUATIONS*

Time-evolution of frequency : swing equations

- Power balance

$$\frac{dW_i}{dt} + P_i^{(d)} = P_i^{(m)} - P_i^{(g)}$$

Change in KE of rotator

Damping power (losses from friction)

Power input

Electric power output

- Swing equation for angles

$$\alpha \frac{d^2\theta_i}{dt^2} + \frac{d\theta_i}{dt} = P_i - \sum_j K_{ij} \sin(\theta_j - \theta_i)$$

From inertia of rotator

Damping / friction

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Coupled oscillators and synchronization

$$\text{Neglect } \alpha \frac{d^2\theta_i}{dt^2} + \frac{d\theta_i}{dt} = P_i - \sum_j K_{ij} \sin(\theta_j - \theta_i)$$

frequency of oscillator #i

free frequency of oscillator #i

coupling between i and j (periodic in angle)

C Huygens 1629-1695

Ubiquitous model :

- biology (heart; brain; crickets; fireflies)
- physics (lasers; Josephson junctions)
- engineering (power systems)

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Coupled oscillators and synchronization

$$\frac{d\theta_i}{dt} = P_i - \sum_j K_{ij} \sin(\theta_j - \theta_i)$$

frequency of oscillator #i

free frequency of oscillator #i

coupling between i and j (periodic in angle)

Different types of spontaneous synchrony

- phase/angle synchrony
- frequency synchrony

C Huygens 1629-1695

Gauge invariance : physicists do not care about phase/angle synchrony… or do they ?

Coupled oscillators and synchronization

$$\frac{d\theta_i}{dt} = P_i - \sum_j K_{ij} \sin(\theta_j - \theta_i)$$

- Winfree (1967) : do mean field theory
- Kuramoto (1984)

(i) K_{ij}=K/N

i.e. all oscillators (nodes; buses) coupled to each other with same coupling

(ii) Introduce order parameter

$$\boxed{r(t) = \left| \frac{1}{N} \sum_j \exp[i\theta_j(t)] \right| = |r(t)| \exp[i\Psi(t)]}$$

Coupled oscillators and synchronization

$$\frac{d\theta_i}{dt} = P_i - \sum_j K_{ij} \sin(\theta_j - \theta_i)$$

- Kuramoto model $K_{ij}=K/N$

mean field justified, i.e.

$$\boxed{\frac{d\theta_i}{dt} = P_i - Kr\sin(\Psi - \theta_i)}$$

$$\boxed{r(t) = \left|\frac{1}{N}\sum_j \exp[i\theta_j(t)]\right| = |r(t)|\exp[i\Psi(t)]}$$

Coupled oscillators and synchronization

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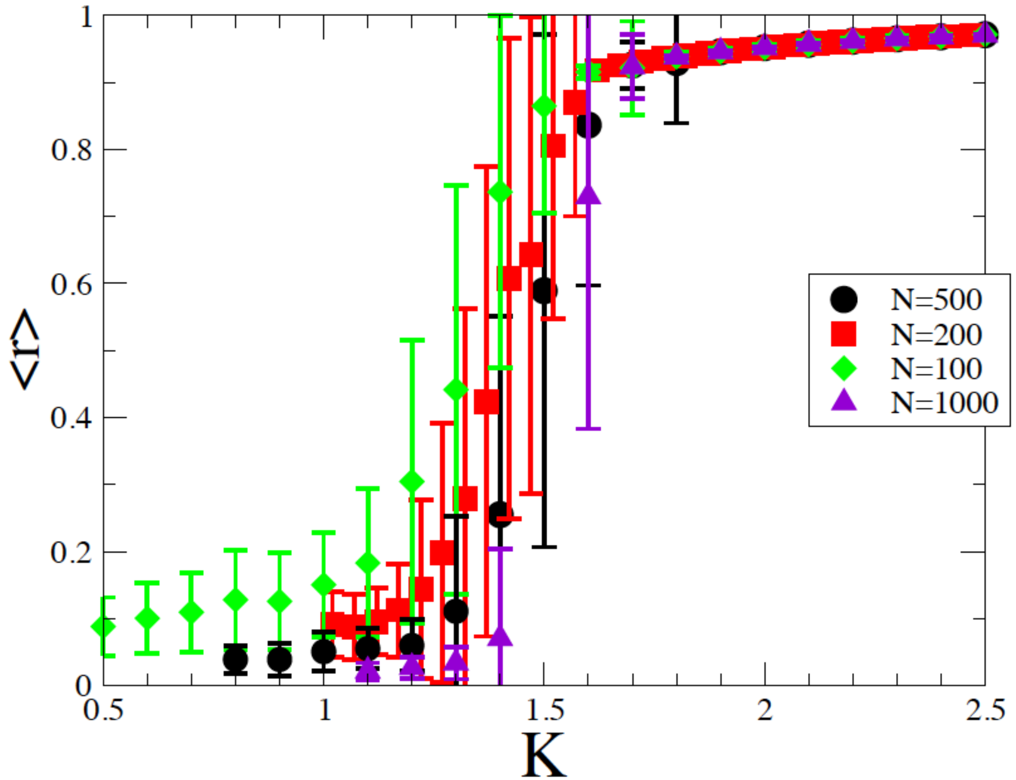

Theoretical threshold for spontaneous synchrony: $K_c=4/\pi$ (Kuramoto)

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Coupled oscillators and synchronization

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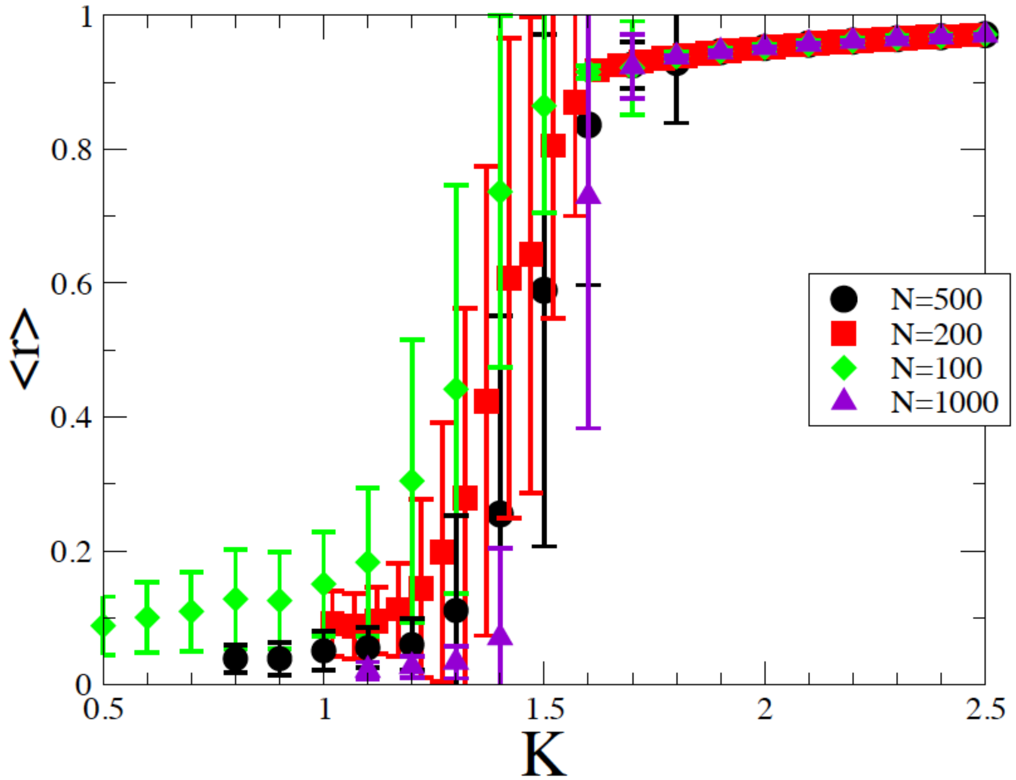
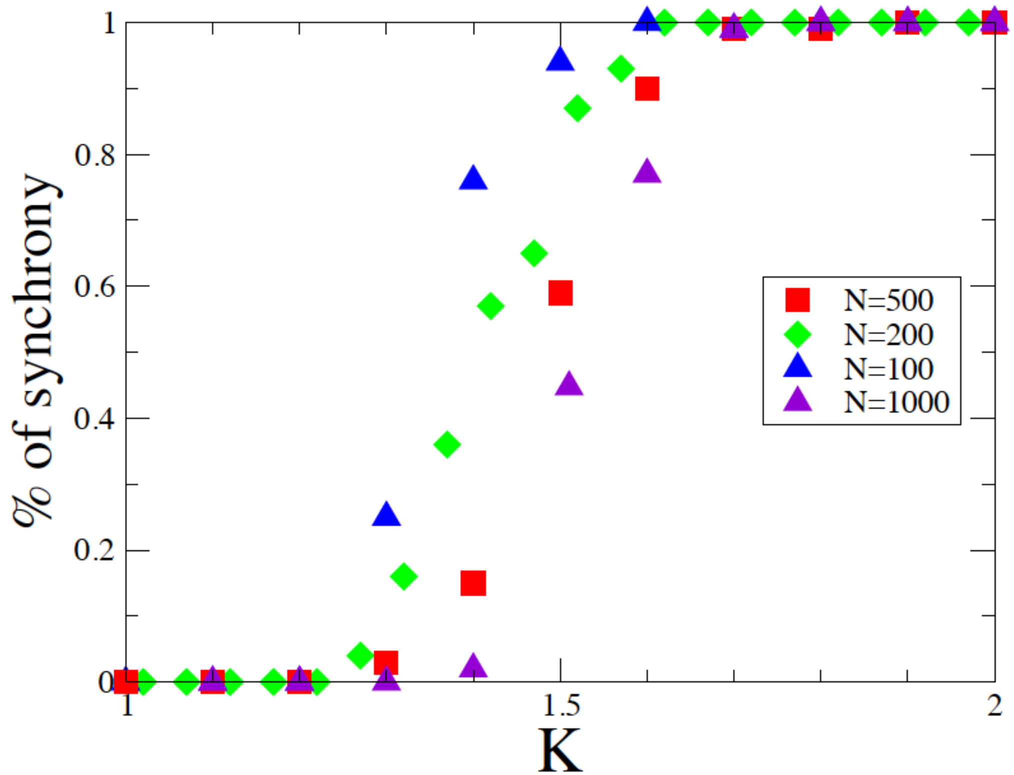
- Kuramoto model $K_{ij}=K/N$

order parameter vs. frequency locking !





Coupled oscillators and synchronization

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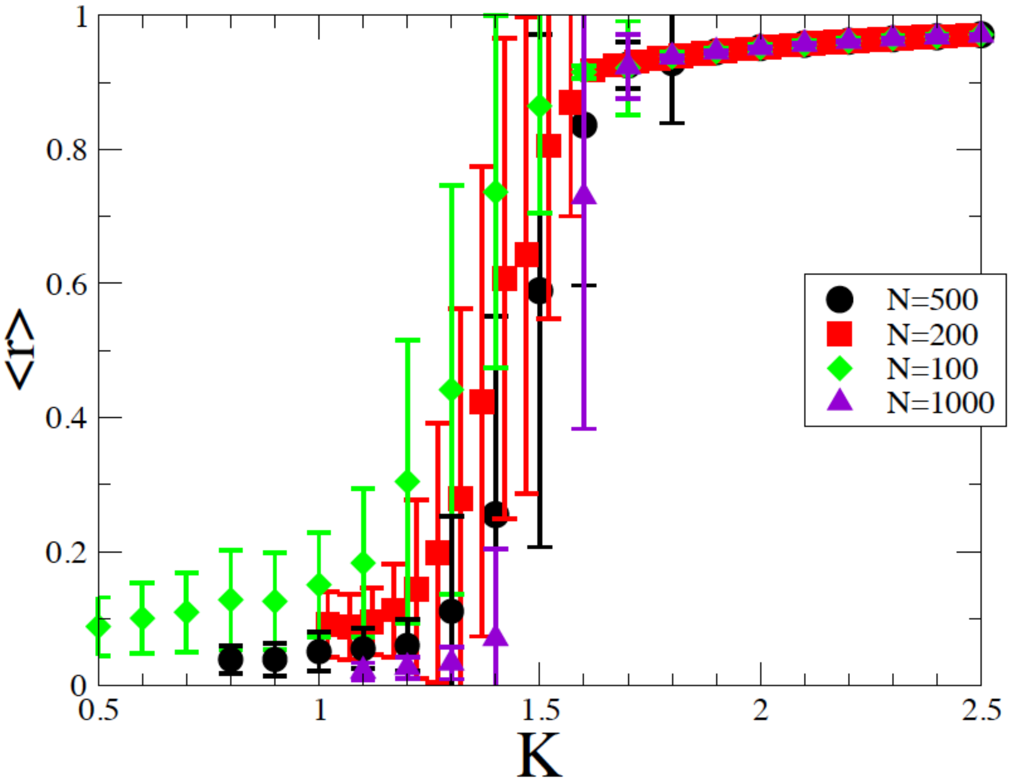

- phase synchrony for r > 0.8
- phase synchrony and frequency synchrony

? WHY ?

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Coupled oscillators, synchronization and stability

$$\alpha \frac{d^2\theta_i}{dt^2} + \frac{d\theta_i}{dt} = P_i - \sum_j K_{ij} \sin(\theta_j - \theta_i)$$

- suspose a synchronized solution exists
- what is its stability under angle perturbation ?

A.: linearize the dynamics about that solution

$$\frac{d\delta\theta_i}{dt} = -\sum_j K_{ij} \cos[\theta_j^{(0)} - \theta_i^{(0)}](\delta\theta_j - \delta\theta_i)$$

Coupled oscillators, synchronization and stability

$$\frac{d\theta_i}{dt} = P_i - \sum_j K_{ij} \sin(\theta_j - \theta_i)$$

- suppose a synchronized solution exists
- what is its stability ?

A.: linearize the dynamics about that solution

$$\frac{d\delta\vec{\theta}}{dt} = -\underline{\kappa}\big(\vec{\theta}^{(0)}\big)\ \delta\vec{\theta}$$

stability/instability vs. eigenvalues of $\underline{\kappa}\big(\vec{\theta}^{(0)}\big)$

Properties of the stability matrix

$$\boxed{\frac{d\delta\vec{\theta}}{dt} = -\underline{\kappa}(\vec{\theta}^{(0)})\ \delta\vec{\theta}}$$

Depends on network and synchronous solution

$$\kappa_{ij} = \delta_{ij} \sum_k K_{ik} \cos(\theta_k^{(0)} - \theta_i^{(0)}) - K_{ij}(1-\delta_{ij}) \cos(\theta_j^{(0)} - \theta_i^{(0)})$$

- one e-value = 0 (~current conservation/gauge invariance)
- all e-values positive if, for all i,j $\cos(\theta_j - \theta_i) \geq 0$
- if, for all i,j $\cos(\theta_j - \theta_i) = 1$

e-values	=0	1-fold
	=KN	(N-1)-fold

- if, for all i,j $\cos(\theta_j - \theta_i) = 1$ and connectivity C > N

e-values	=0	1-fold
	=KC	(N-1)-fold

Properties of the stability matrix

$$\boxed{\frac{d\delta\vec{\theta}}{dt} = -\underline{\kappa}(\vec{\theta}^{(0)})\ \delta\vec{\theta}}$$

Depends on network and synchronous solution

- Qualitatively :

1. no negative e-value = stability for $\cos(\theta_j - \theta_i) \geq 0$
2. smaller positive e-values = less (but still) stable
   for lesser connectivity
3. smaller line capacity/coupling requires larger angle differences to
   carry fixed power current ~ less stable
4. when angles differ by more than 90º, instability sets in
   and synchronous solutions cease to exist

   CONNECTION BETWEEN ANGLE AND FREQUENCY SYNCHRONY

Properties of the stability matrix : stability regime

General strategy :

(i) investigate the properties of the stability matrix
vs. the distribution of angles

(ii) understand the distribution of angles in synchrony
vs. properties of the network/grid

Note: order parameter at synchrony
vs. variance of angle distributions

$$\boxed{r_\infty \simeq [1 - \mathrm{var}(\theta)/2]}$$

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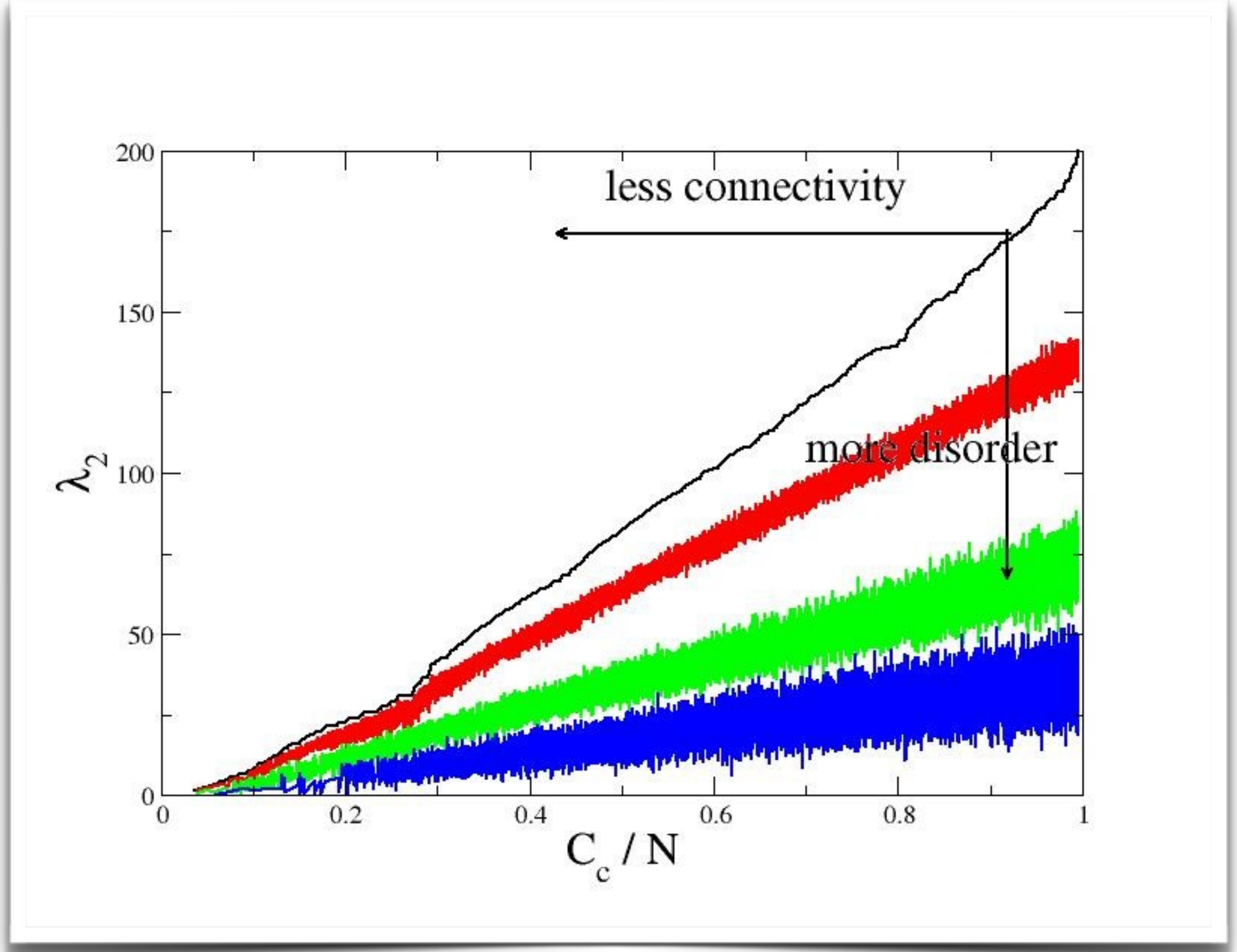
Properties of the stability matrix : stability regime

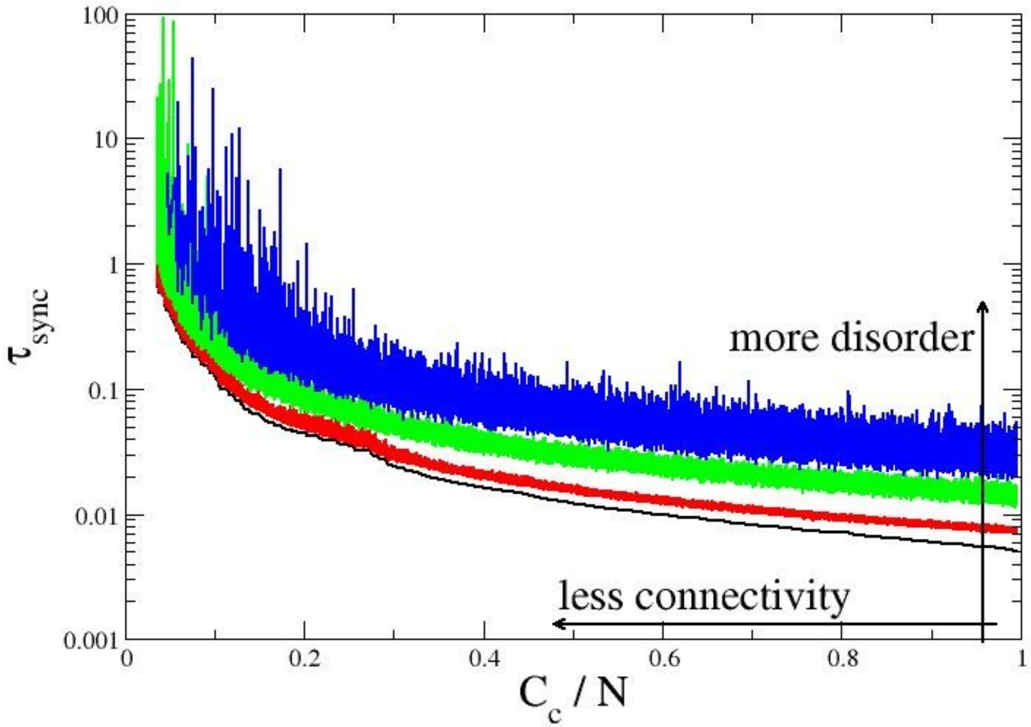
- Degree of stability determined by second smallest (positive) e-value



$$\kappa_{ij} = \delta_{ij} \sum_k K_{ik} \cos(\theta_k^{(0)} - \theta_i^{(0)}) - K_{ij}(1 - \delta_{ij}) \cos(\theta_j^{(0)} - \theta_i^{(0)})$$

Properties of the stability matrix : stability regime

- Re-synchronization time (after perturbation) $\tau_{\text{sync}} = \lambda_2^{-1}$



- Re-synchronization time increases with

  -increasing disorder (variance of angles)

  -decreasing network connectivity

Properties of the stability matrix : stability regime

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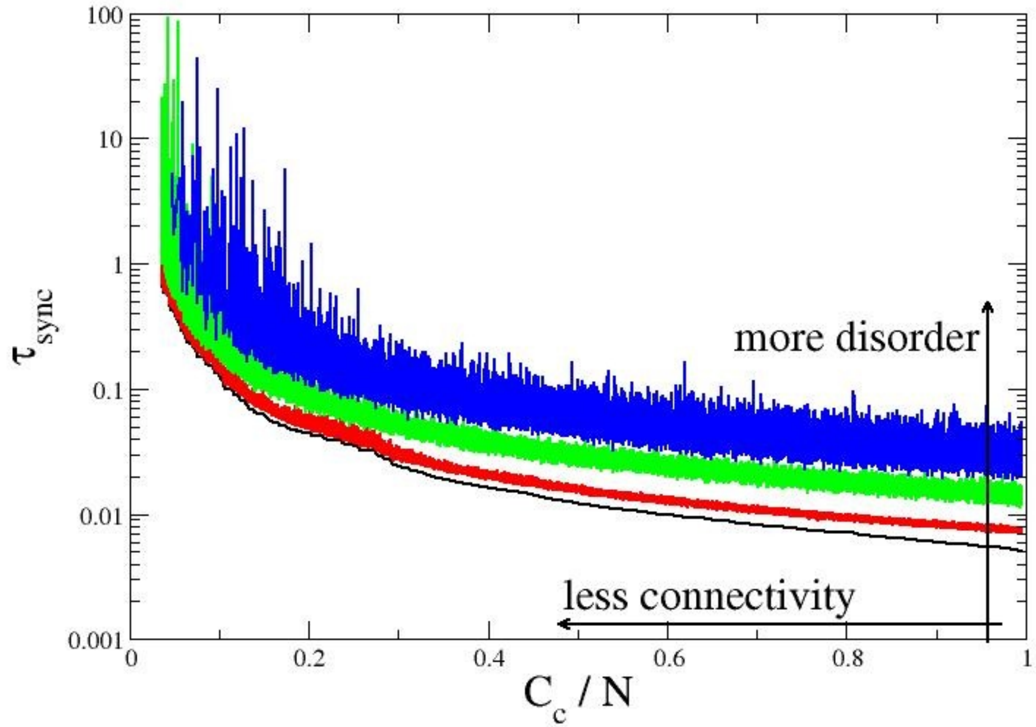

- Re-synchronization time fluctuations increase with

  -increasing disorder (variance of angles)

  -decreasing network connectivity

Strategy

- The goal is to investigate stability / resynchronisation via a random matrix theory of the stability matrix
- Why *random* ?
  (Angles are determined by network, frequencies, couplings…)
- Does this make any sense at all vs. true synchrony problem ?

Strategy

- The goal is to investigate stability / resynchronisation via a random matrix theory of the stability matrix
- Why *random* ?
  (Angles are determined by network, frequencies, couplings…)
- Does this make any sense at all vs. true synchrony problem ?

Back to original problem : damped, coupled oscillators

$$\alpha \frac{d^2\theta_i}{dt^2} + \frac{d\theta_i}{dt} = P_i - \sum_j K_{ij} \sin(\theta_j - \theta_i)$$

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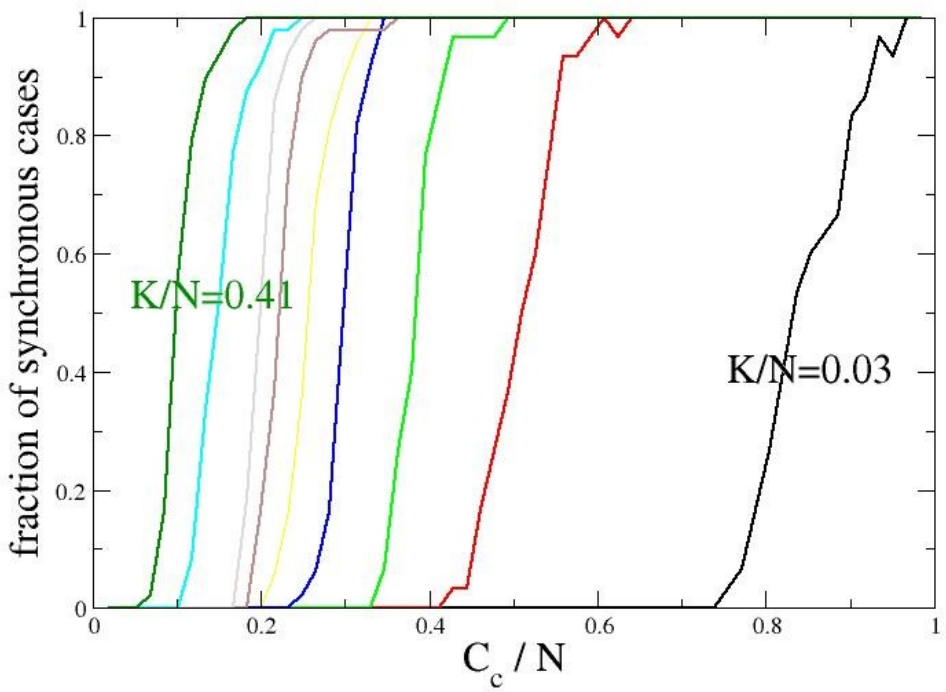
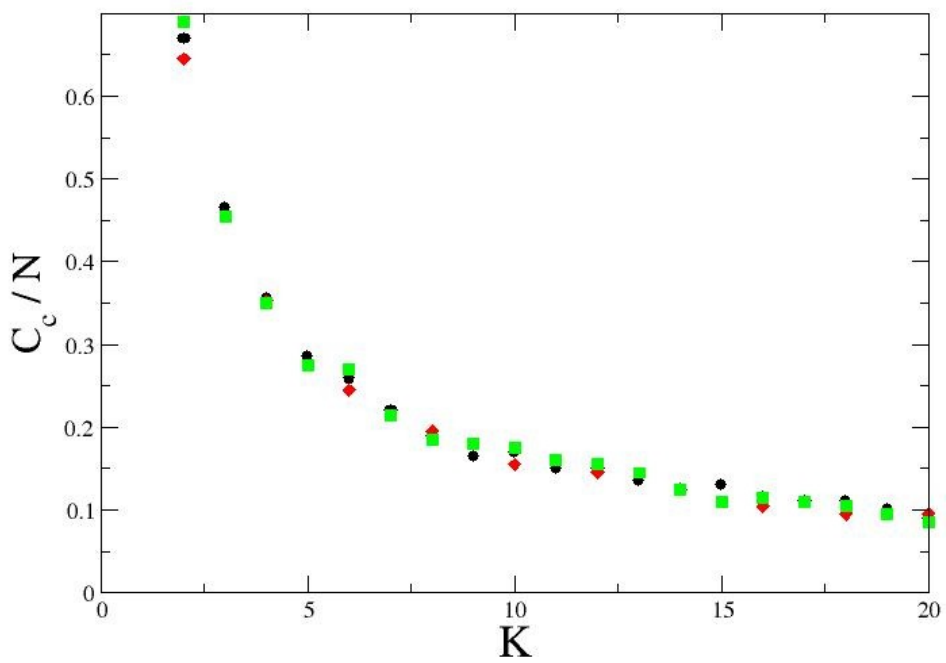
Some results

Harder to get spontaneous synchrony with

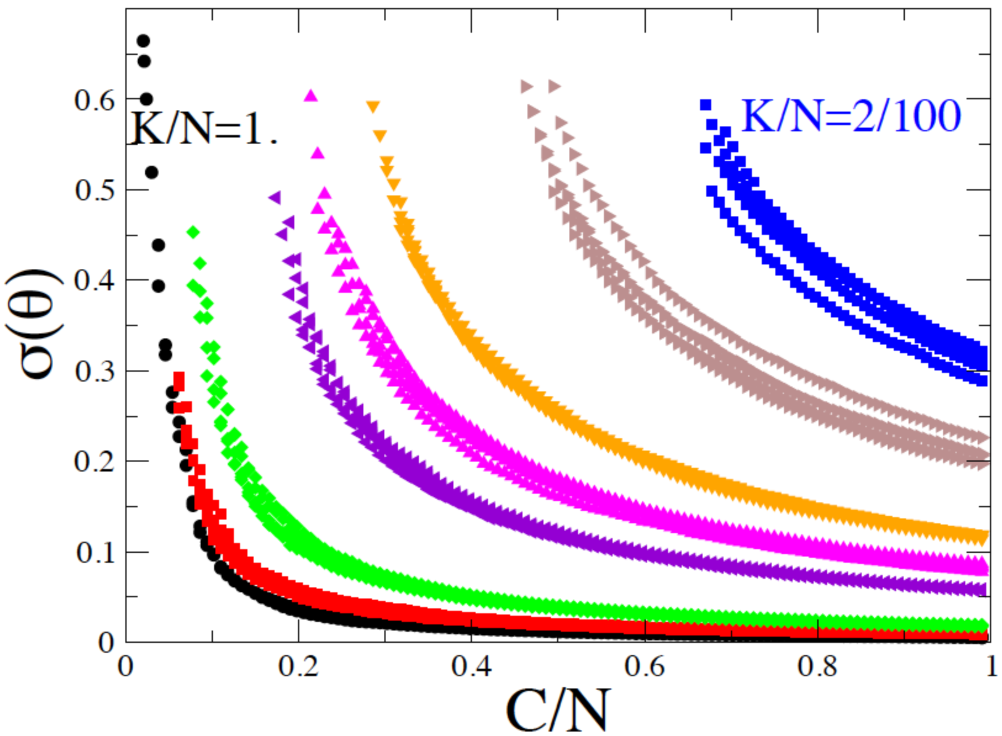
-smaller coupling (line capacity)
-less couplings (smaller connectivity)

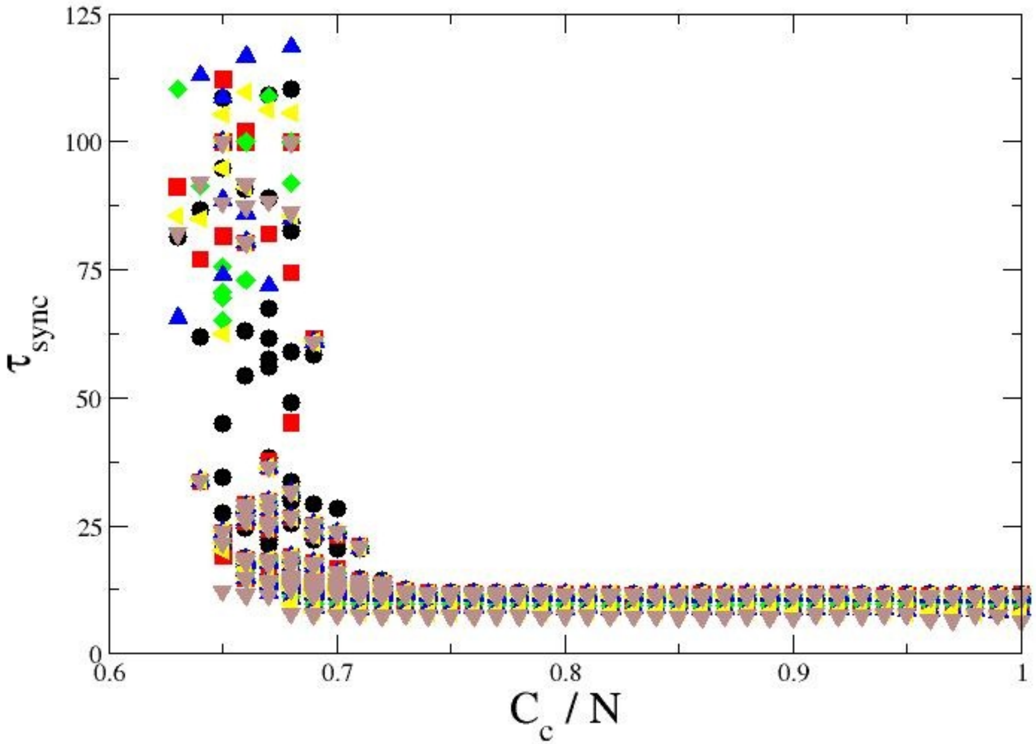




Some results

All this seems to be related to broadening of angle distribution at synchrony and the associated increa in stability time !!





Conclusion

New route for understanding synchrony in coupled oscillators systems via distribution of angles at synchron

Towards a random matrix theory of electric power syste

Dedicated to

Quantum transport

M Büttiker

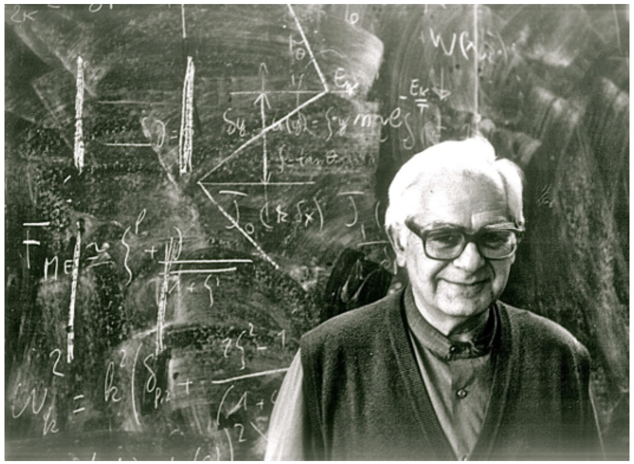
Quantum echoes

P Levstein

Semiclassical methods

M Gutzwiller

Random matrix theory



O Bohigas