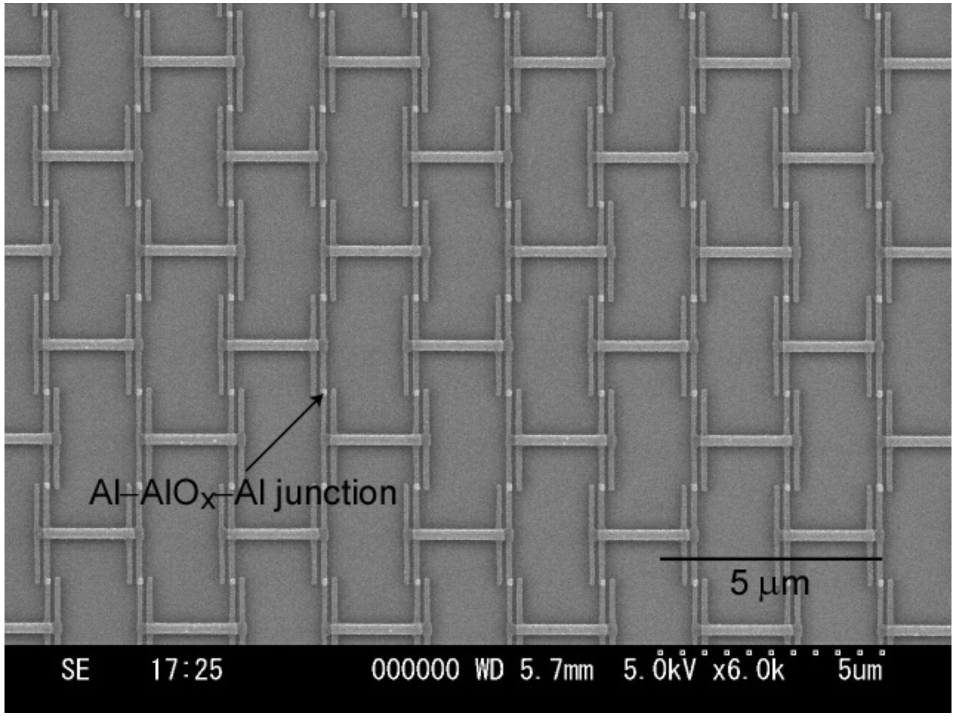
HIGH-VOLTAGE ELECTRIC POWER TRANSPORT

A PHYSICICIST'S PERSPECTIVE

Philippe Jacquod

Konstanz - 27.10.2015

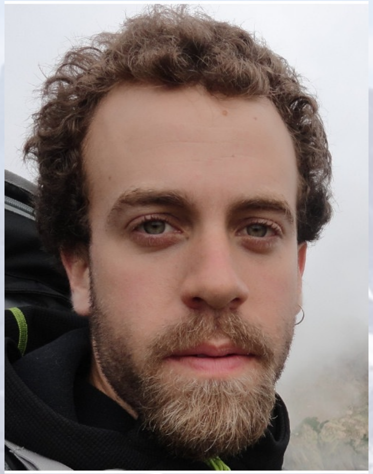
Superconducting vs. high voltage electric transport !



Takahide, Yagi, Kanda, Ootuka, and Kobayashi
Phys. Rev. Lett. 85, 1974 (2000)

The team

Tommaso Coletta, postdoc



Robin Delabays, grad student

Inanc Adagideli (Sabanci)

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- Electric power systems
- Steady-state : power flow equations
- Transient stability : swing equations
- Loop vs. parallel flows in high-voltage AC transport
- Vortex currents

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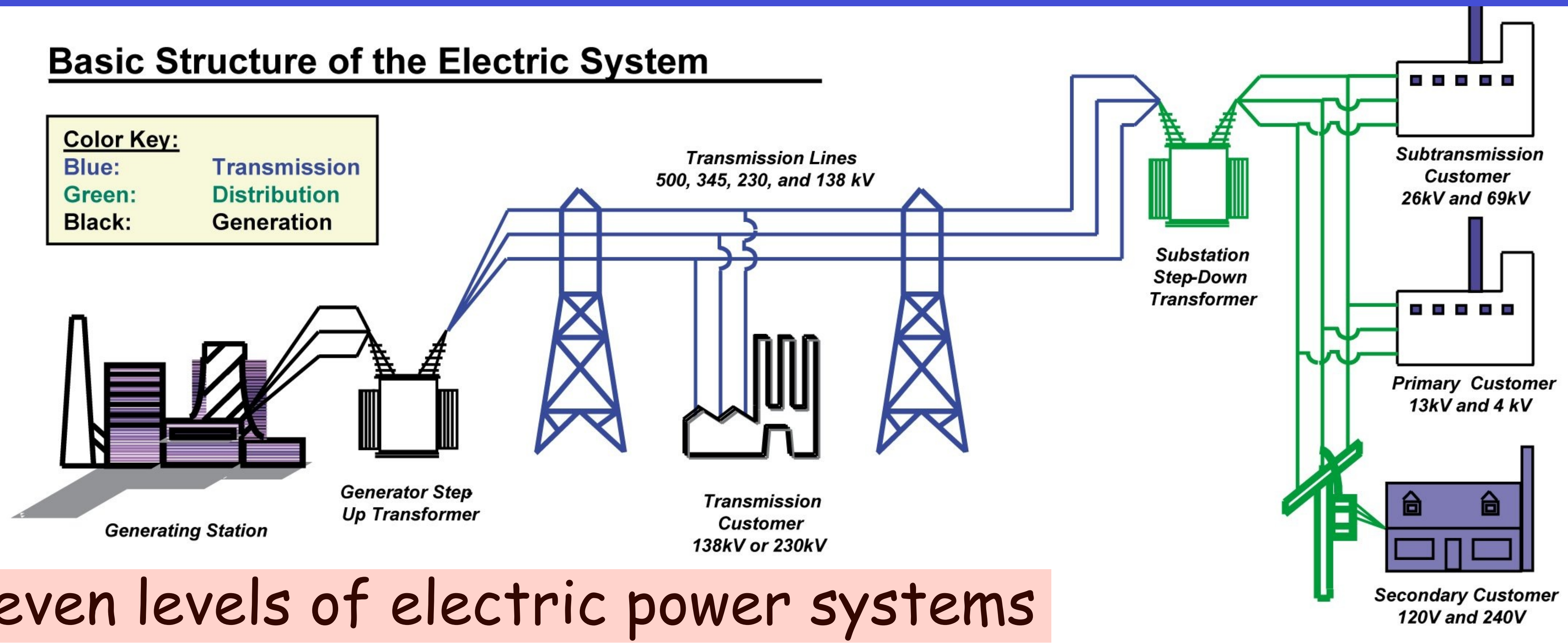
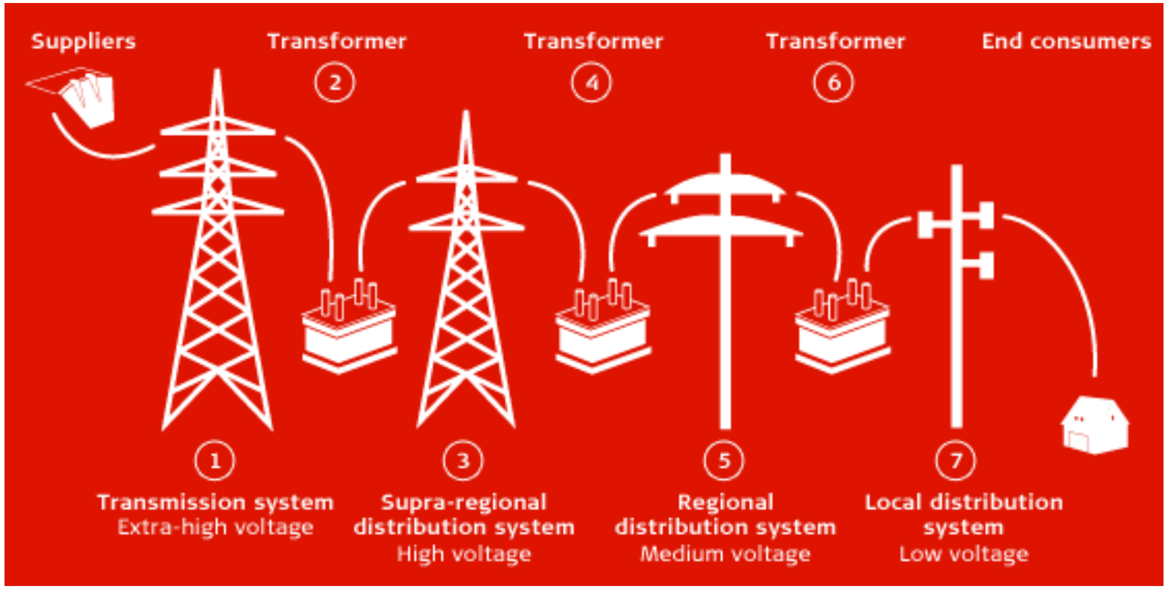
What are electric power systems ?



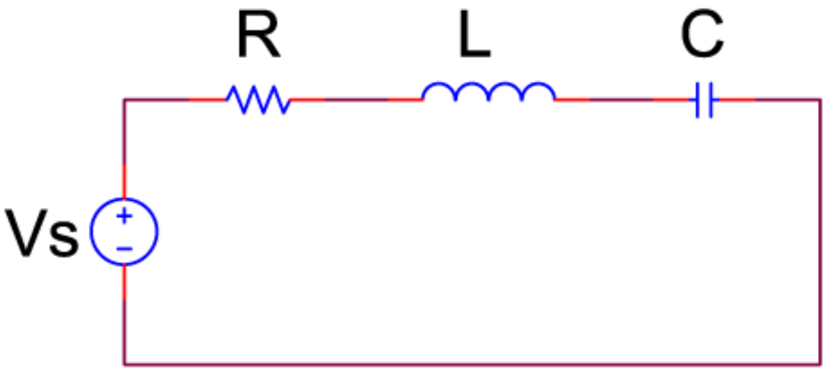
Seven levels of electric power systems



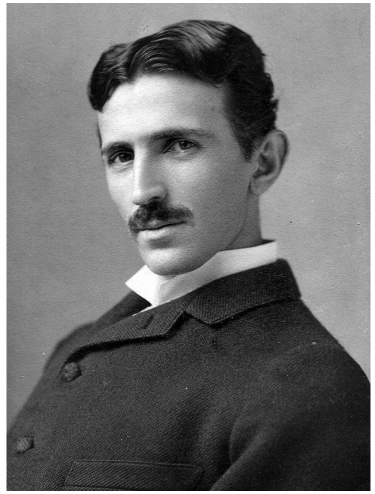
Power :
*conserved (~)
*control parameter
"write Eq. for power"

What are electric power systems ?

- AC electric current/voltages
  (minimize losses ~ high voltages,
  but then need transformers)
- current and voltage not in phase

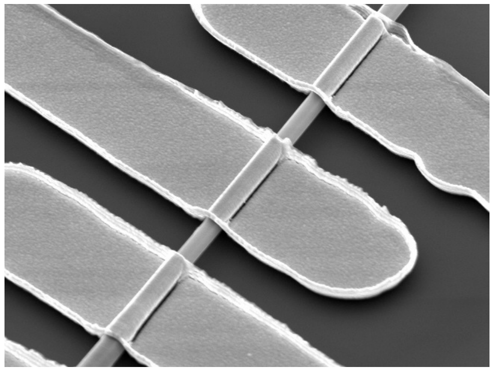


$$u(t) = u_0 \exp[i\omega t]$$
$$i(t) = i_0 \exp[i(\omega t + \phi)]$$
$$\tan(\phi) = (\omega L - 1/\omega C)/R$$



N Tesla 1856-1943

- complex impedance $u(\omega) = Z(\omega)\, i(\omega) \qquad Z(\omega) = R + i\omega L - i/\omega C$
- inductance more important than resistance for large conductors



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AC transport

- linear relation between currents and voltages

$$\boxed{I_i = \sum_j Y_{ij} V_j}$$

Y_{ij} : admittance matrix

$$\sum_i Y_{ij} = 0 \qquad \sum_j Y_{ij} = 0$$

aka gauge invariance
and particle conservation
(neglecting losses)

- Complex power : S(t) = u(t) x i*(t)

- Active power vs. reactive power

Active power	vs.	reactive power
P=Re(S)		Q=Im(S)
finite time-average		zero time-average
"truly transmitted" (injected and consumed)		"oscillating in the circuit"

Power flow equations
(power is conserved upon voltage transformation)

$$P_i = \sum_j |V_i V_j| \left[G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \right]$$

$$Q_i = \sum_j |V_i V_j| \left[G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \right]$$

Voltages at buses i and j

Conductance between i and j

Phases at buses i and j

Susceptance between i and j

- These equations *must* be obeyed at any time for steady-state operation of the electric power network

Power flow equations

- Admittance dominated by its imaginary part for large conductors

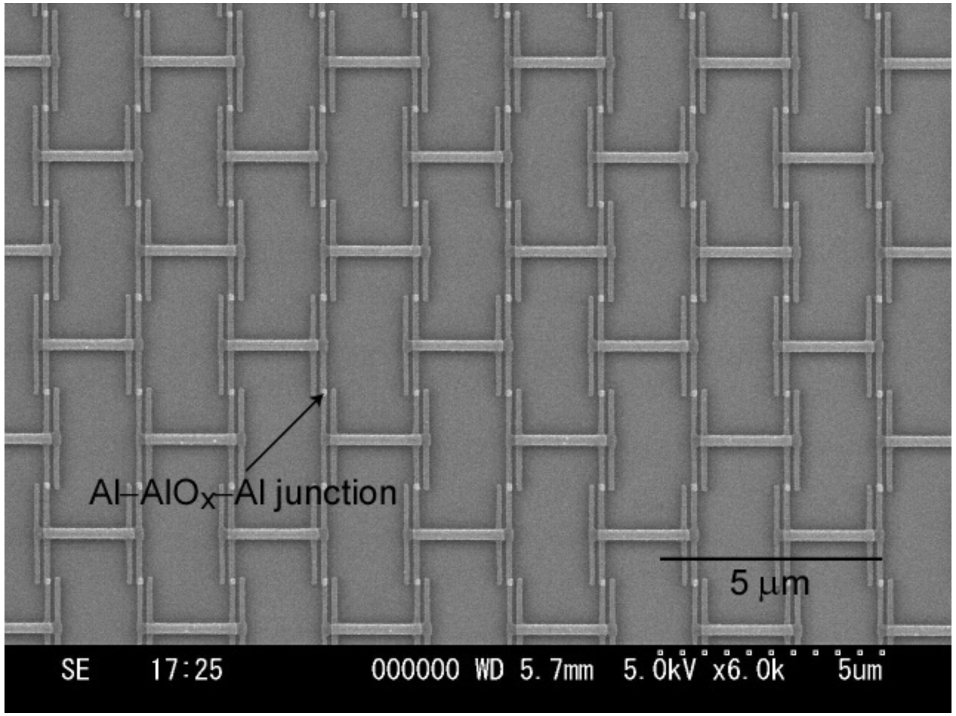
→ neglect conductance

$$P_i \simeq \sum_j |V_i V_j| B_{ij} \sin(\theta_i - \theta_j)$$

$$Q_i \simeq -\sum_j |V_i V_j| B_{ij} \cos(\theta_i - \theta_j)$$

- No conductance ~no voltage drop
  - $|V_iV_j|\ B_{ij} \rightarrow B_{ij}$
  - consider P only

Josephson junction arrays vs. electric power systems !



Josephson current

$$I_{ij} = I_c \sin(\theta_i - \theta_j)$$

Transmitted power

$$P_{ij} \simeq B_{ij} \sin(\theta_i - \theta_j)$$

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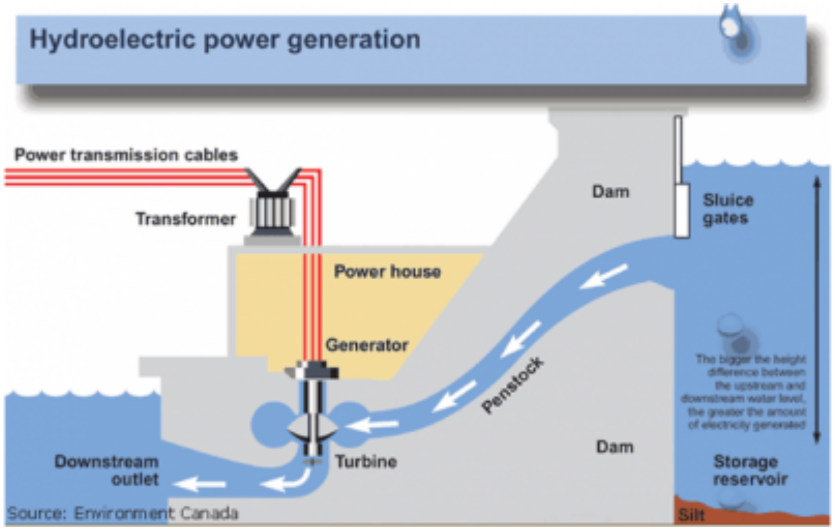
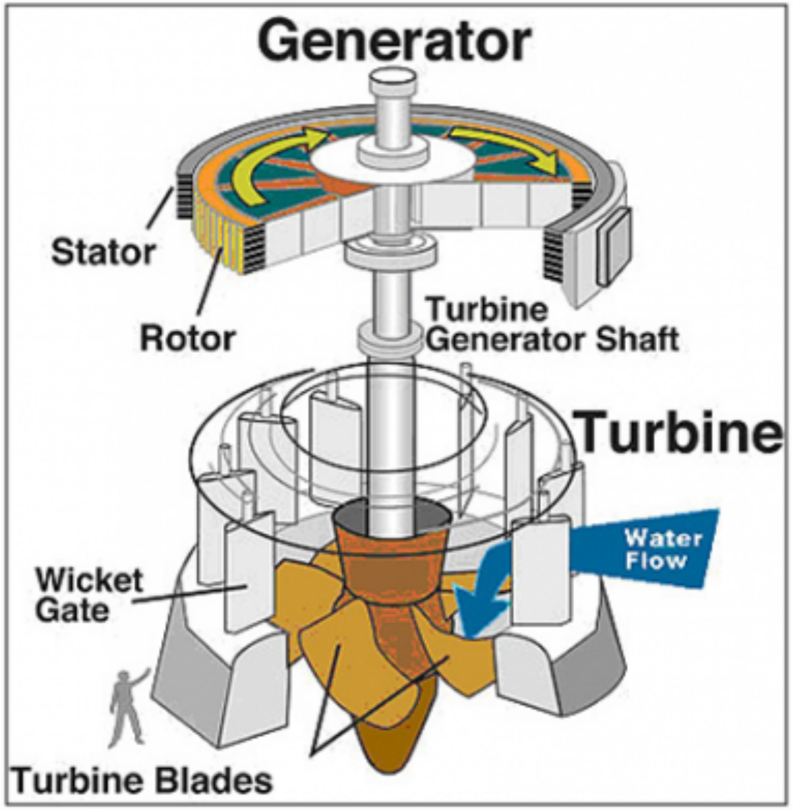
Time-evolution of frequency : swing equations





- Electricity production with rotating machines
- Potential, chemical, nuclear or thermal energy converted into mechanical energy (rotation)
- Mechanical power converted into electric power
- Time-dependence : think power instead of energy
- Balance between power in, power out and energy change in rotator : *SWING EQUATIONS*

Time-evolution of frequency : swing equations

- Power balance

$$\frac{dW_i}{dt} + P_i^{(d)} = P_i^{(m)} - P_i^{(g)}$$

Change in KE of rotator

Damping power (losses from friction)

Power input

Electric power output

- Swing equation for angles (in rotating frame @ 50Hz)

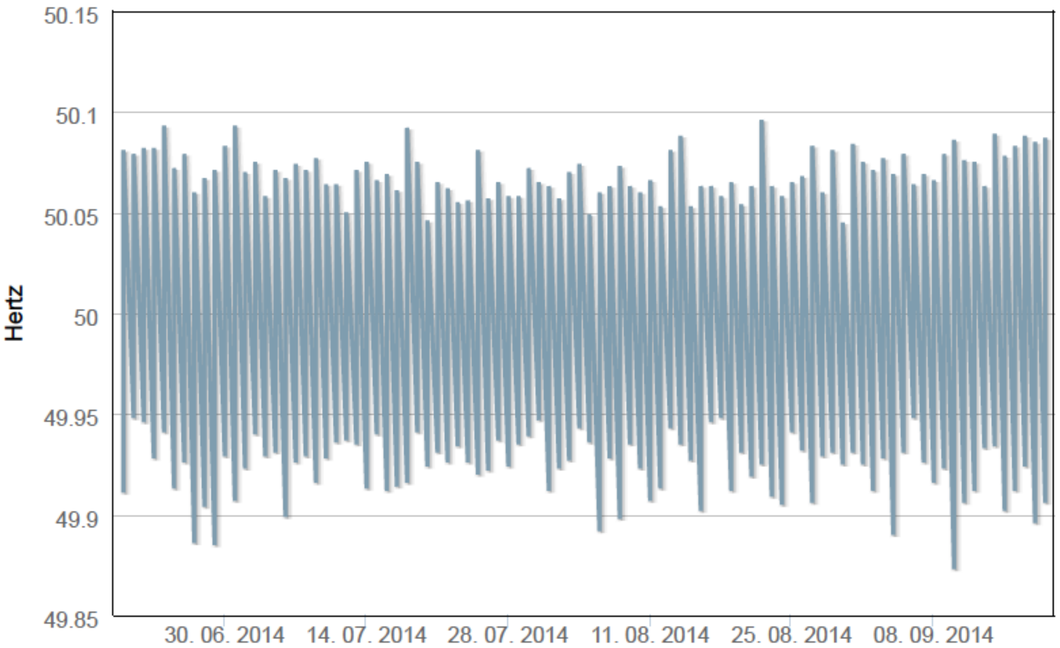
$$M\frac{d^2\theta_i}{dt^2} + D\frac{d\theta_i}{dt} = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$

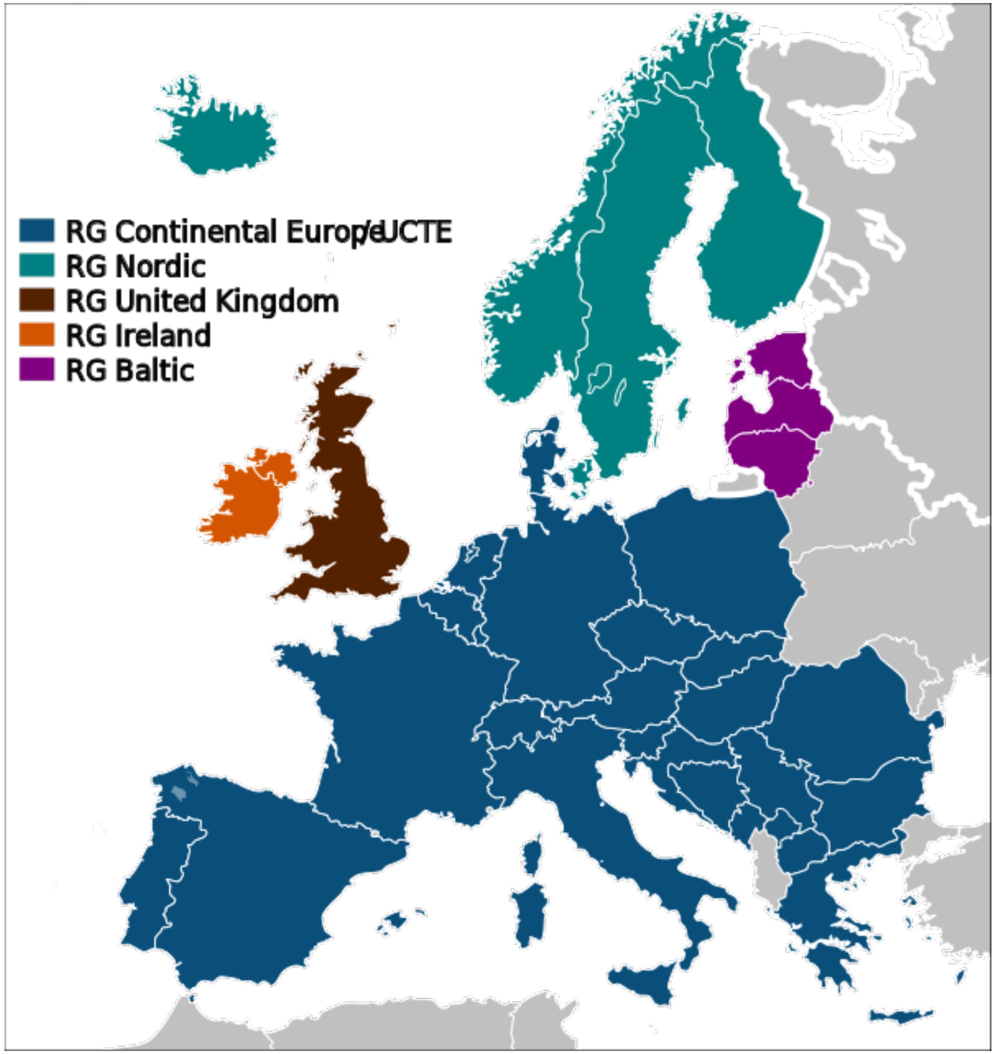
Inertia of rotator

Damping / friction

- Solutions of power-flow eqs. = steady-state solutions of swing eqs.
  ~synchronous solution (angles rotate in unison)

Steady-state operation : synchrony over 1000's of kms





Loss of synchrony : blackouts

Italy blackout, sep 28 2003

Northeast blackout, aug 14 2003

From the swing equations to linear stability

$$M\frac{d^2\theta_i}{dt^2} + D\frac{d\theta_i}{dt} = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$

- suppose a synchronized solution exists
- what is its stability under angle perturbation ?

A.: *linearize the dynamics about that solution
*perturbed condition goes back exponentially fast to the initial solution if the stability matrix

$$\mathcal{M}_{ij} = -\delta_{ij}\sum_k B_{ik}\cos(\theta_i^{(0)} - \theta_k^{(0)}) + B_{ij}(1-\delta_{ij})\cos(\theta_i^{(0)} - \theta_j^{(0)})$$

is negative semidefinite

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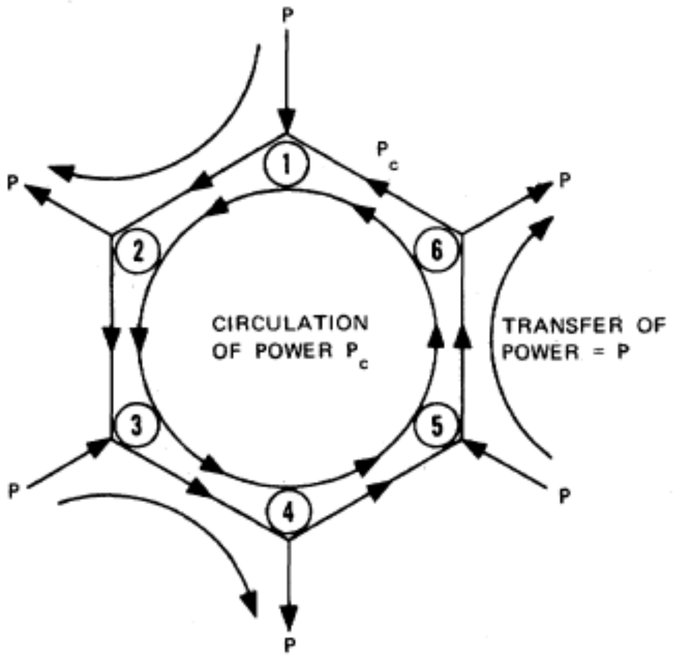
How many stable solutions to the power flow equations ?

Korsak '72 :
Different solutions vs. loop currents



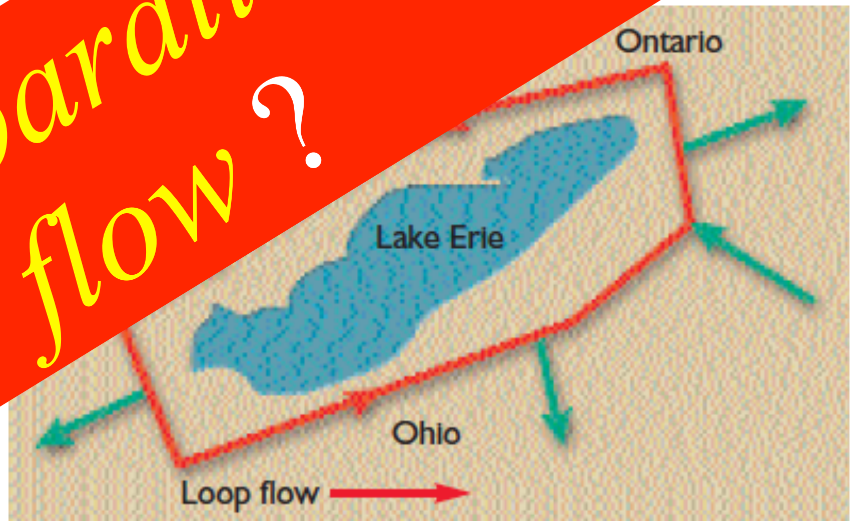
10. CONCLUSIONS

From the example networks above, it is evident that stable load flows are not necessarily unique, and that power flows, following a transient, for example, could possibly lock into a stable configuration in which power circulates in one or more loops of the network, with great increase in losses as a result. Also, such power circulation would be undesirable from the standpoint of system security because of the fact that a relatively small transient may knock the system's "energy" function H "over the hill," so to speak, and "unlock" the circulation of power, causing the system to seek the next lower stable energy minimum, with the result that much bigger transient surges of power and voltage may result than in the initial triggering transient.

See also: Janssens and Kamagate '03
Ochab and Gora '10

Parallel flows vs. loop flows

"*Electric power does not follow a specified path but divides am... ansmission routes based on Kirchhoff's laws and network conditions a...* This pattern results in a phenomenon called **circulating power of w... types : loop flow and parallel flow.**"



How can one tell parallel flow from loop flow ?

"Where t... large geographical obstacles, such as the Rocky Mount... Great Lakes in the East, *loop flows* around the obstacle... *an drive as much as 1 GW of power in a circle*, taking up transmissi... pacity *without delivering power to consumers.*"

Casazza, Electrical World (1998)
Lerner, The industrial physicist (2003)

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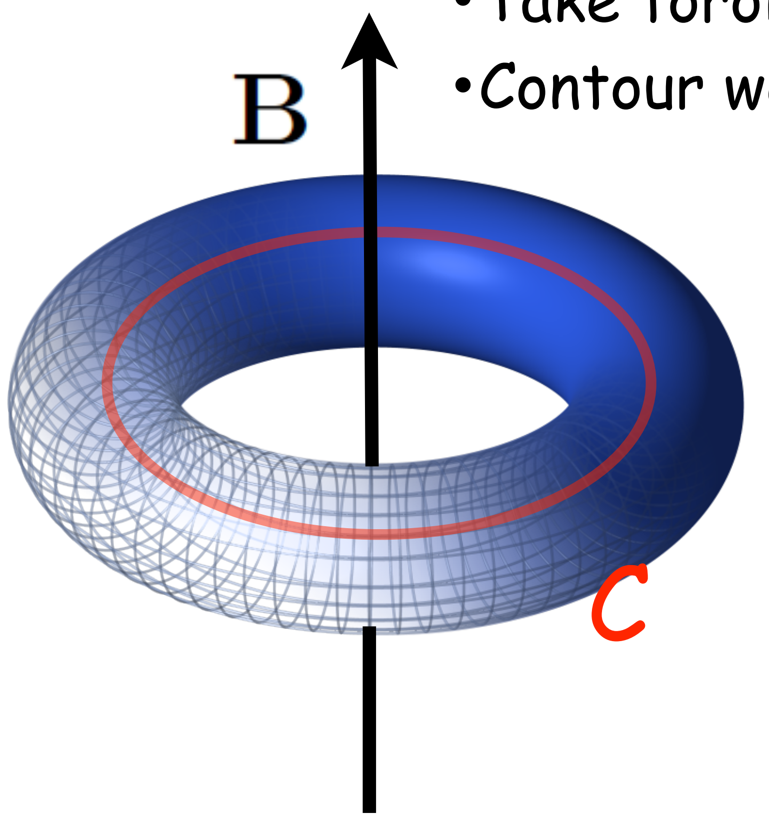
Topological quantum number : flux quantization with SC

- Landau theory of superconductivity - macroscopic wavefunction

$$\psi(\mathbf{x}) = |\psi(\mathbf{x})| e^{i\phi(\mathbf{x})}$$

- Gauge-invariant current $\mathbf{J}_s = \frac{e\hbar}{2m} n_s \left(\nabla\phi - \frac{2e}{\hbar}\mathbf{A} \right)$



- Take toroidal SC pierced by B-field
- Contour well inside SC : Meissner effect

$$\mathbf{B}\big|_C = \mathbf{J}_s\big|_C \equiv 0$$

-> $\oint_C \mathbf{A}d\mathbf{l} = \int\int_{\partial C} \mathbf{B}d\mathbf{f} = \varphi$

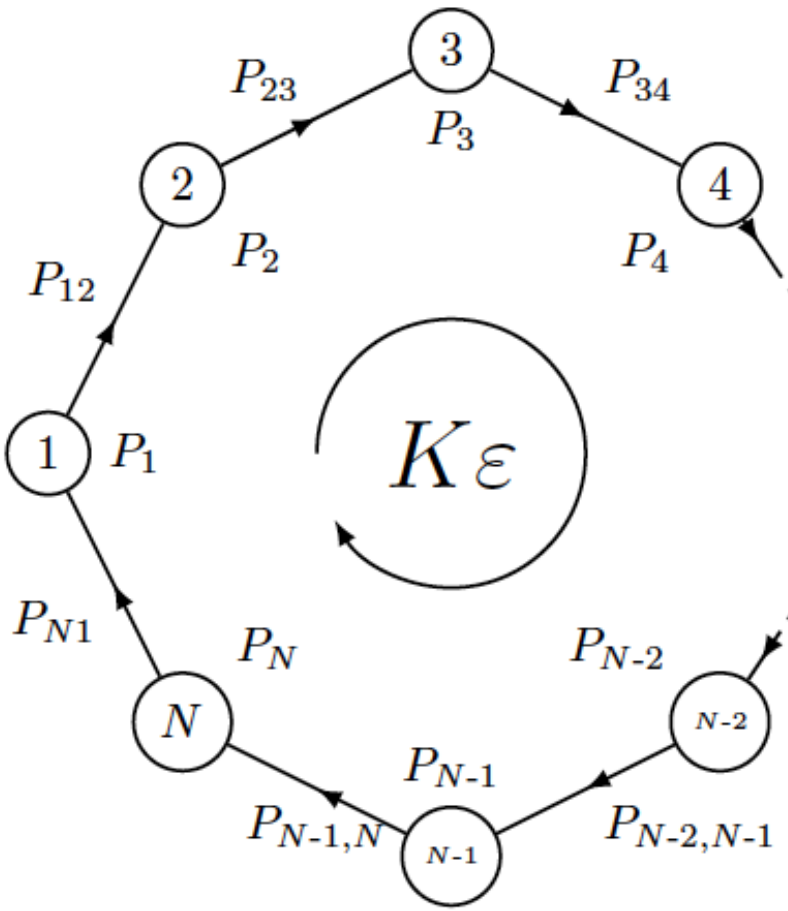
-> $\oint_C \mathbf{A}d\mathbf{l} = \frac{\hbar}{2e}(\phi_+ - \phi_-) = m\frac{h}{2e}$

-> flux quantization (winding number) $\boxed{\varphi = m\varphi_0}$

Exps.: Deaver and Fairbanks '61 (Sn cylinders); Gough et al. '87 (high Tc)

Power flow solutions with different winding numbers

$$P_i = \sum_j P_{ij} = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$



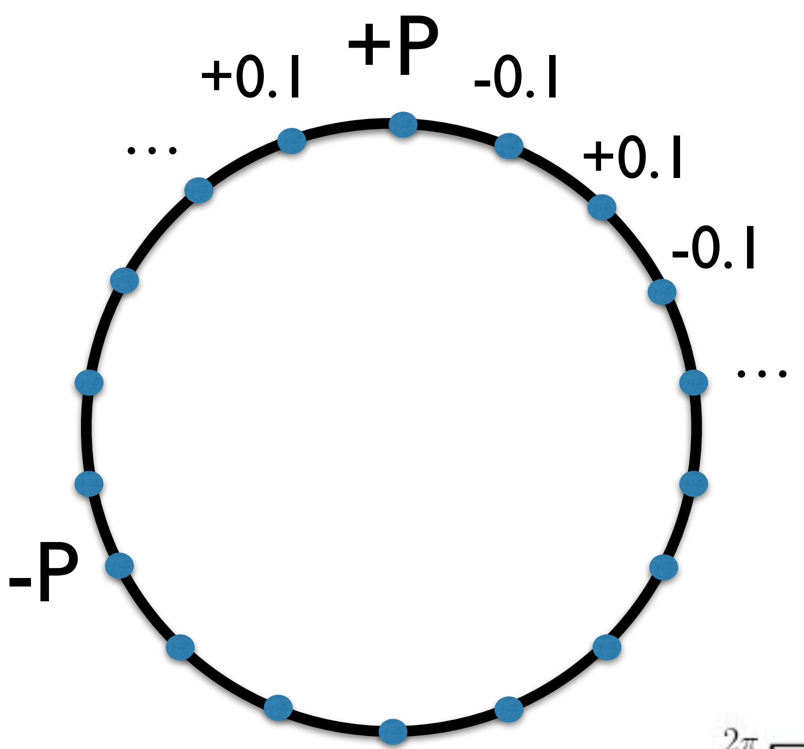
- P_i : power injection (consumption) at node i
- P_{ij} : power flow from i to j
- $B_{ij}=K$: line capacity (i.e. $P_{ij} < K$)
- $K\varepsilon$: loop flow ($0<\varepsilon<1$)
- θ_i : angle at node i

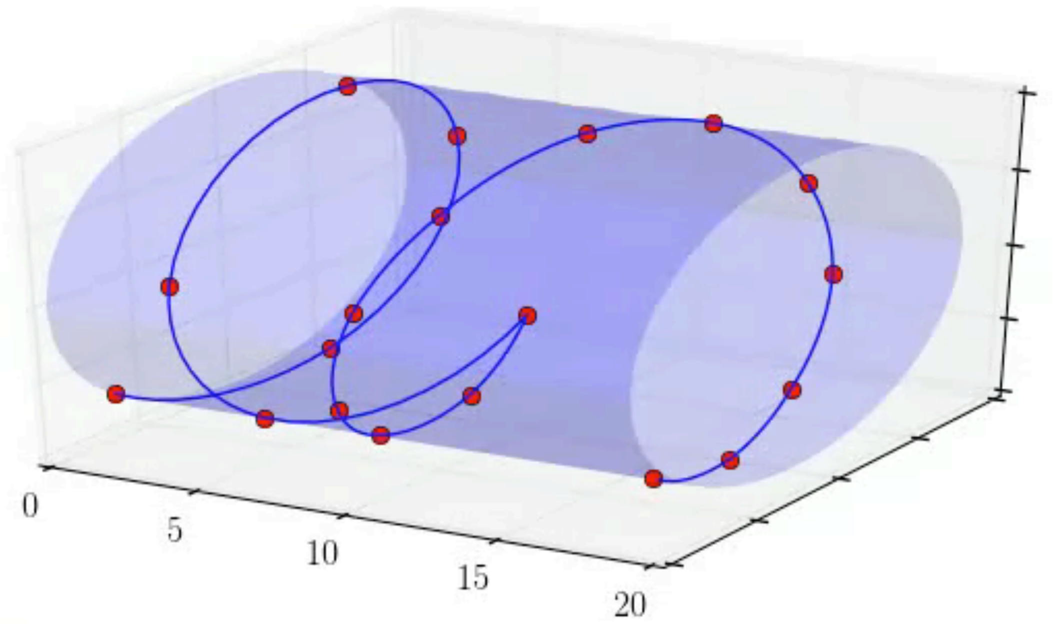
Define winding number :

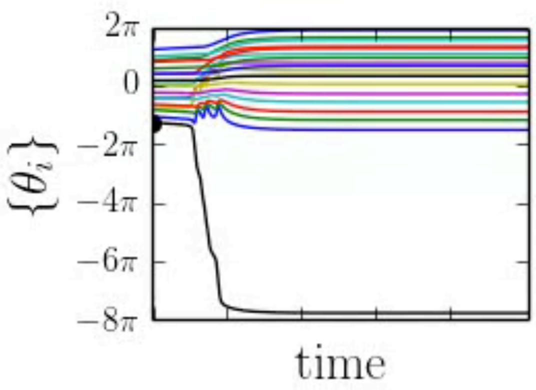
$$m=\Sigma_i|\theta_{i+1}-\theta_i|/2\pi$$

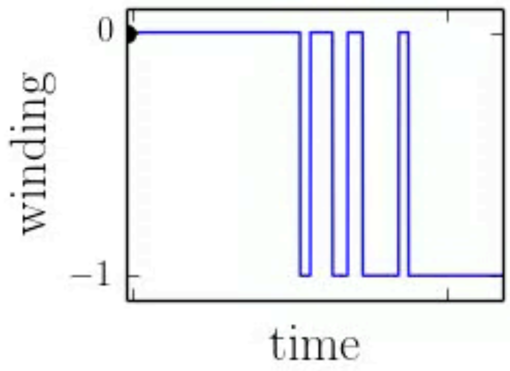
|...| : inside $[-\pi,\pi]$

Power flow solutions with different winding numbers









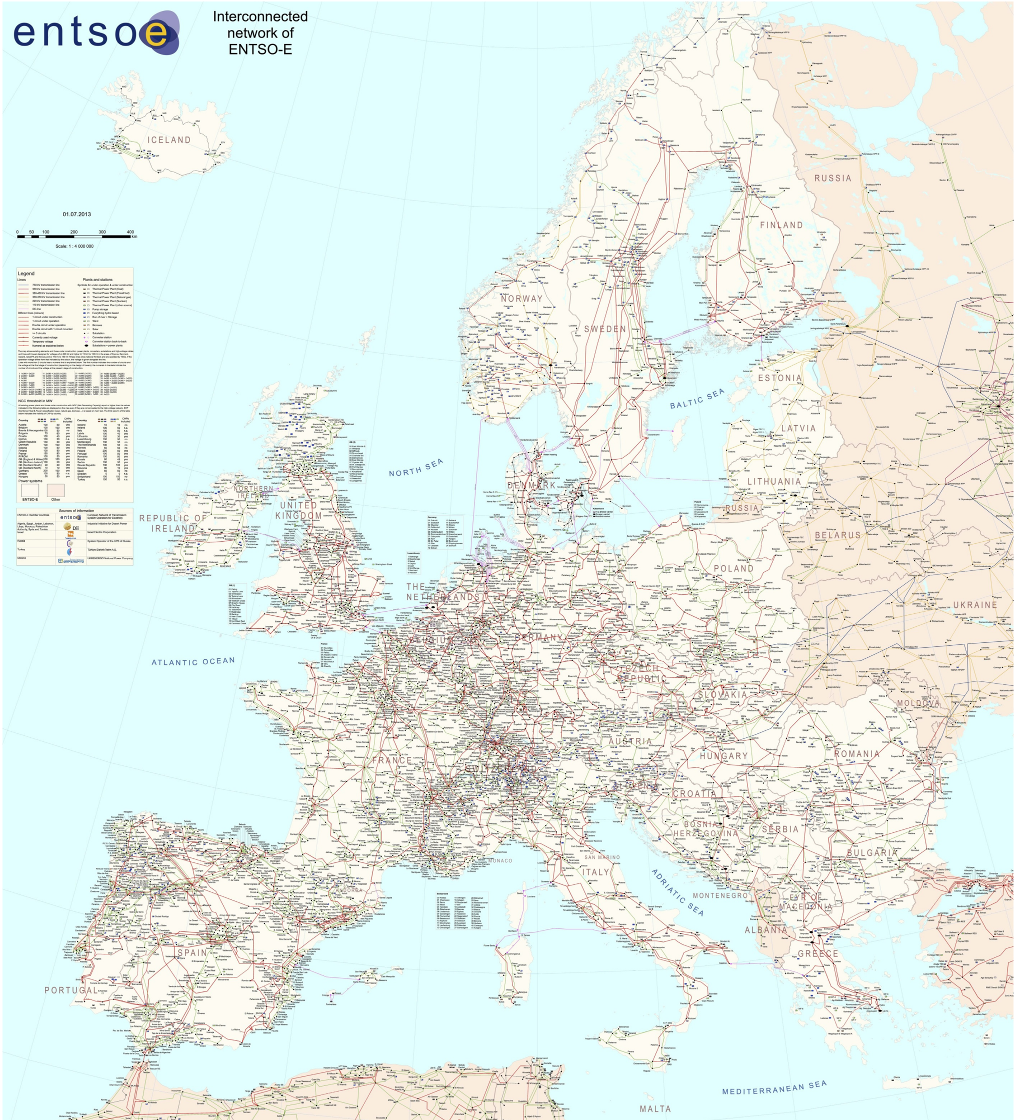
How many stable solutions to the power flow equations ?

Delabays, Coletta and PJ '15 :

Thm: Different solutions to the following power-flow problem

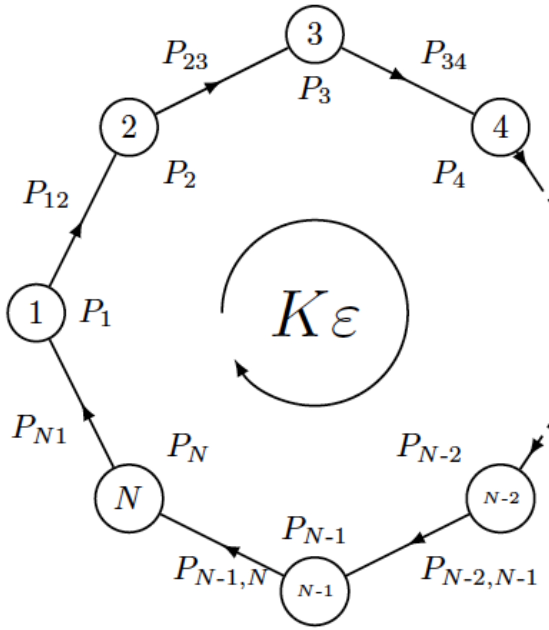
$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

may differ by loop current(s) only
i.e. by winding number on some of
the loops of the network



Single-loop problem

Delabays, Coletta and PJ '15 :



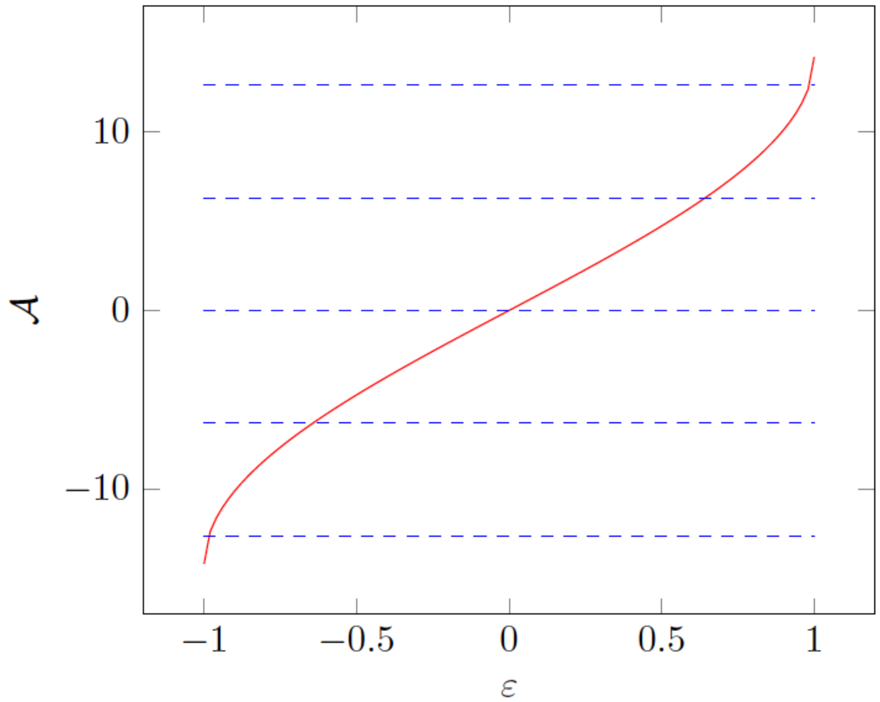
(i) For a single loop with N nodes, the number of solutions is algebraically bound from above:

$$\mathcal{N}(K) \leq 2\,\mathrm{Int}[N/4] + 1$$

(ii) The number of solutions decreases monotonously with the line capacity

Sketch of proof :

$$2\pi m = \sum_i |\theta_{i+1} - \theta_i| = \sum_i \arcsin(\epsilon + P^*_{i,i+1}/K)$$



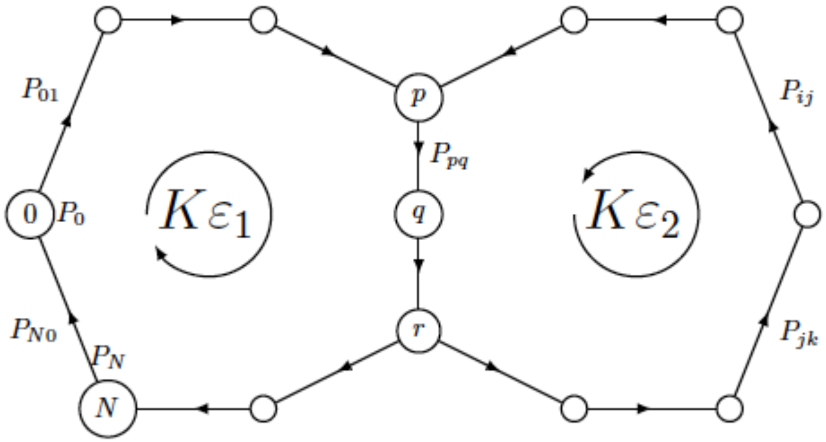
Delabays, Coletta and PJ '15 :

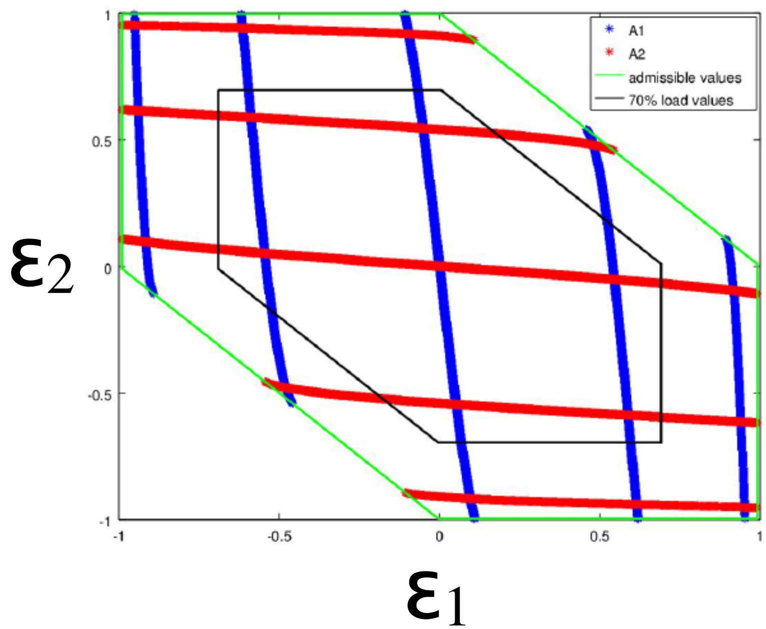
(i) For a multi-loop network, the number of solutions is algebraically bound from above:

$$\mathcal{N}(K) \leq \prod_k \Big(2\mathrm{Int}[N_k] + 1\Big)$$

(ii) The number of solutions decreases monotonously with the line capacity

Remark : pairs of loops with one edge in common behave differently from those with several common edges (work in progress)

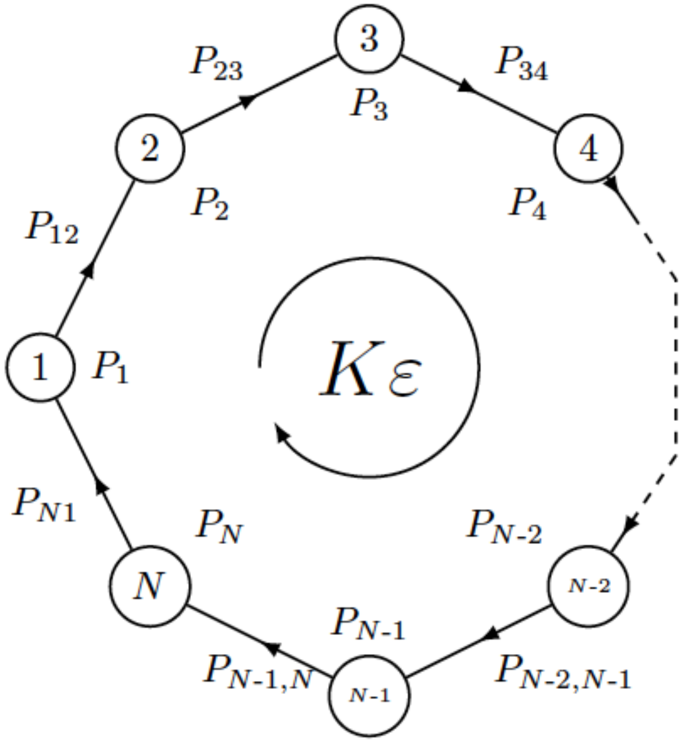




Summary so far

- Different solutions differ by their winding number around one or the other loop in the network

- We define vortex flows (no longer loop flow) as arising from finite winding number



Define winding number :

$$m=\Sigma_i|\theta_{i+1}-\theta_i|/2\pi$$

$|\ldots|$: inside $[-\pi,\pi]$

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Generating vortex current with finite m

VOLUME 87, NUMBER 21 PHYSICAL REVIEW LETTERS 19 NOVEMBER 2001

Quantum Phase Slips in Superconducting Nanowires

C. N. Lau, N. Markovic, M. Bockrath, A. Bezryadin,* and M. Tinkham

SC-insulator transition in thin films

VOLUME 89, NUMBER 9 PHYSICAL REVIEW LETTERS 26 AUGUST 2002

Persistent Current in Superconducting Nanorings

K. A. Matveev,[1] A. I. Larkin,[2,3] and L. I. Glazman[2]

Tunneling between two sols. with different winding #

PHYSICAL REVIEW B **87**, 174513 (2013)

Quantum phase slips in Josephson junction rings

G. Rastelli,[1,2] I. M. Pop,[3,4] and F. W. J. Hekking[1]

Study of a single loop

At given magnetic flux, each QPS can be described as the tunneling of the phase difference of a single junction of almost 2π, accompanied by a small harmonic displacement of the phase difference of the other $N-1$ junctions.

Generating vortex current with finite m

For the Josephson problem :

$$H = \frac{1}{2}\sum_{n,m=0}^{N-1} \hat{Q}_n \bar{\bar{C}}_{nm}^{-1} \hat{Q}_m - E_J \sum_{n=0}^{N-1} \cos\left(\hat{\varphi}_{n+1} - \hat{\varphi}_n + \frac{2\pi \Phi_B}{N\Phi_0}\right)$$

perturbation

classical Hamiltonian
defines winding number

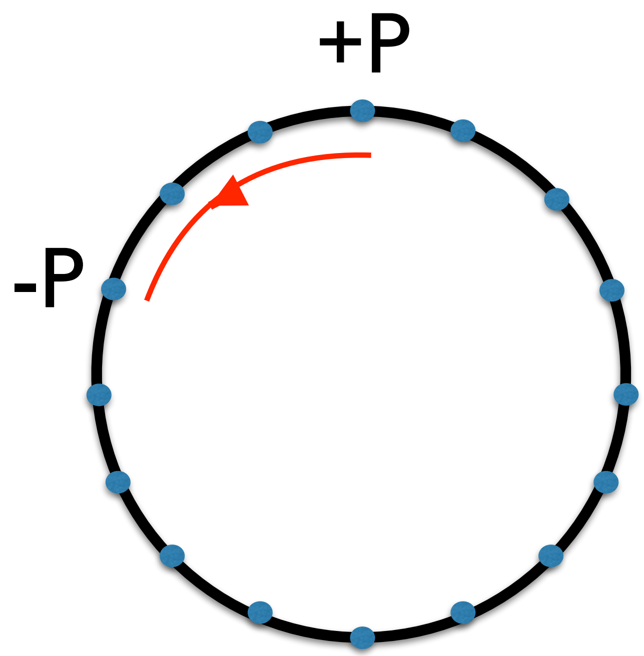
For the power-flow problem :

(i) Tripping and reclosure of a line

Electrical Power and Energy Systems 25 (2003) 591–597

Loop flows in a ring AC power system

Noël Janssens[a,b,*], Amadou Kamagate[c]



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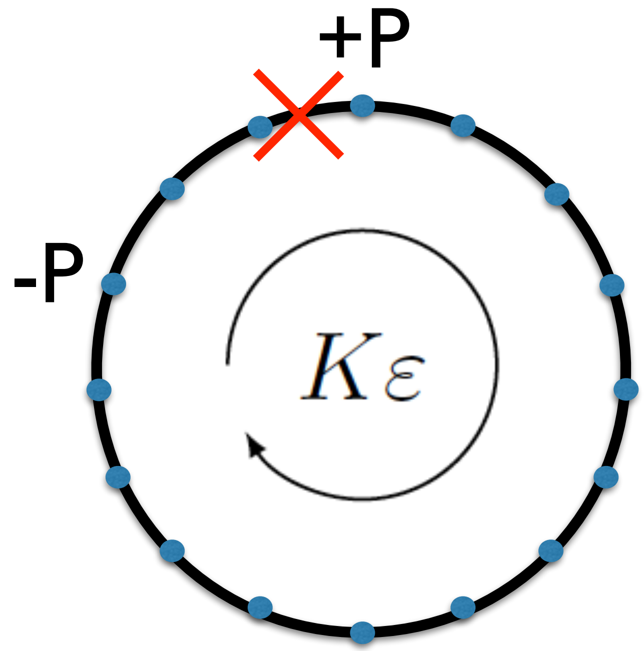

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Loop flows in a ring AC power system

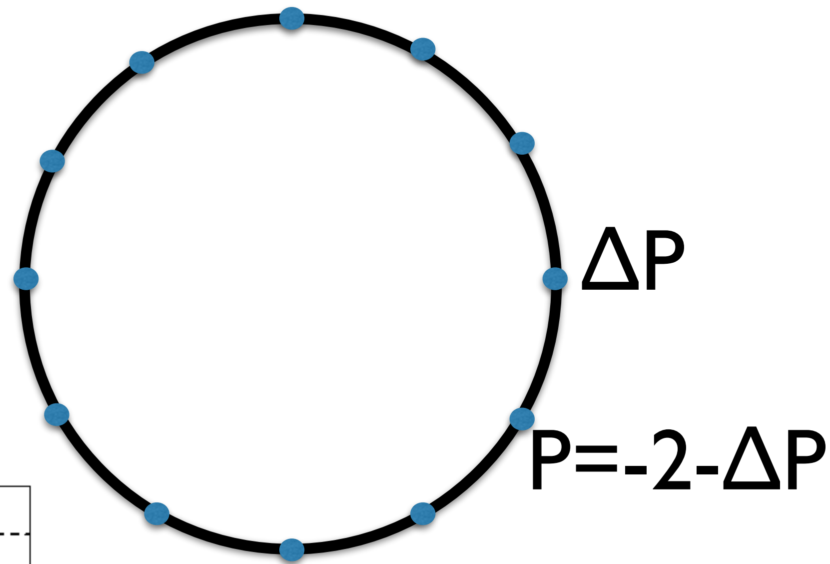
Noël Janssens[a,b,*], Amadou Kamagate[c]

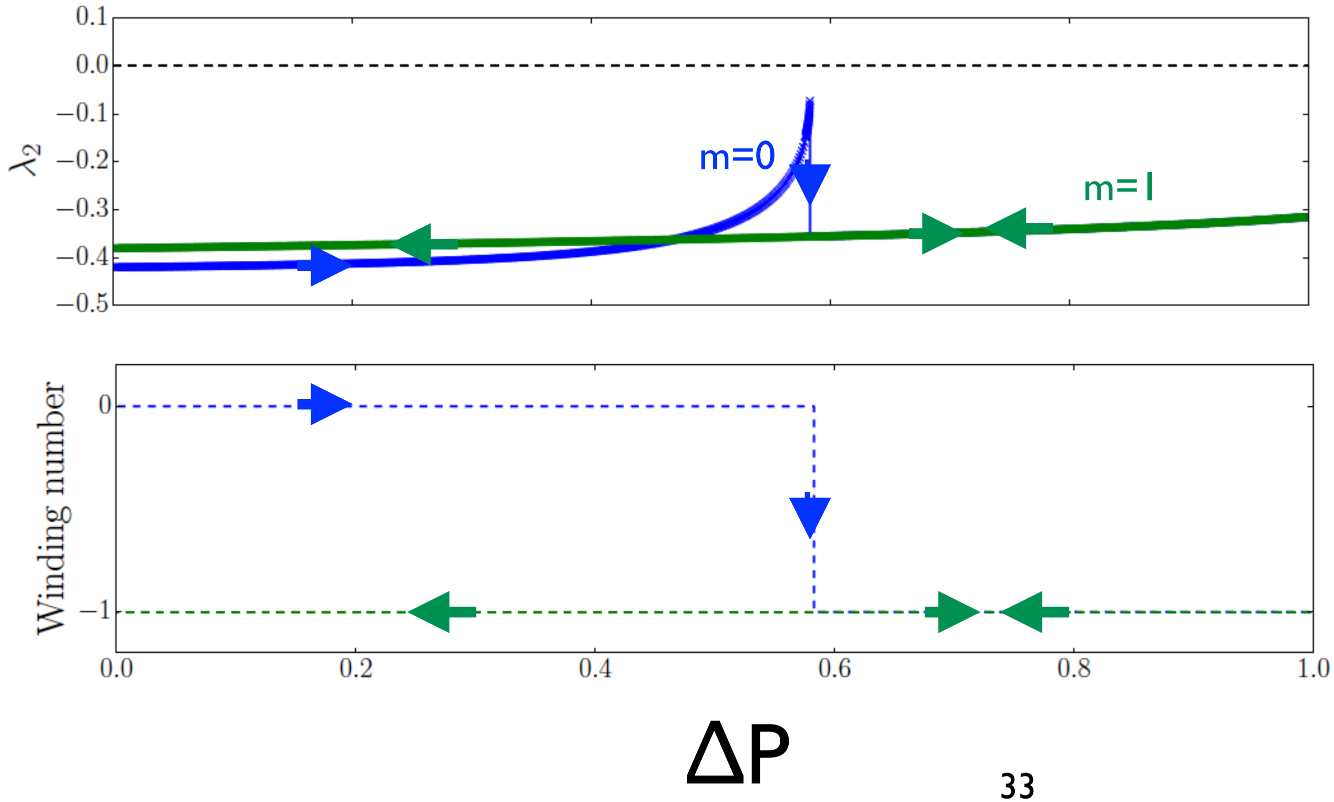
Generating vortex current with finite m

For the power-flow problem :

(ii) Increasing load on an already loaded line

Delabays, Coletta, Adagideli and PJ '15 :





Increasing ΔP :
*m=0 loses stability at ΔP=0.58
*transient to m=1

Decreasing ΔP :
*stay on m=1
(topological protection)

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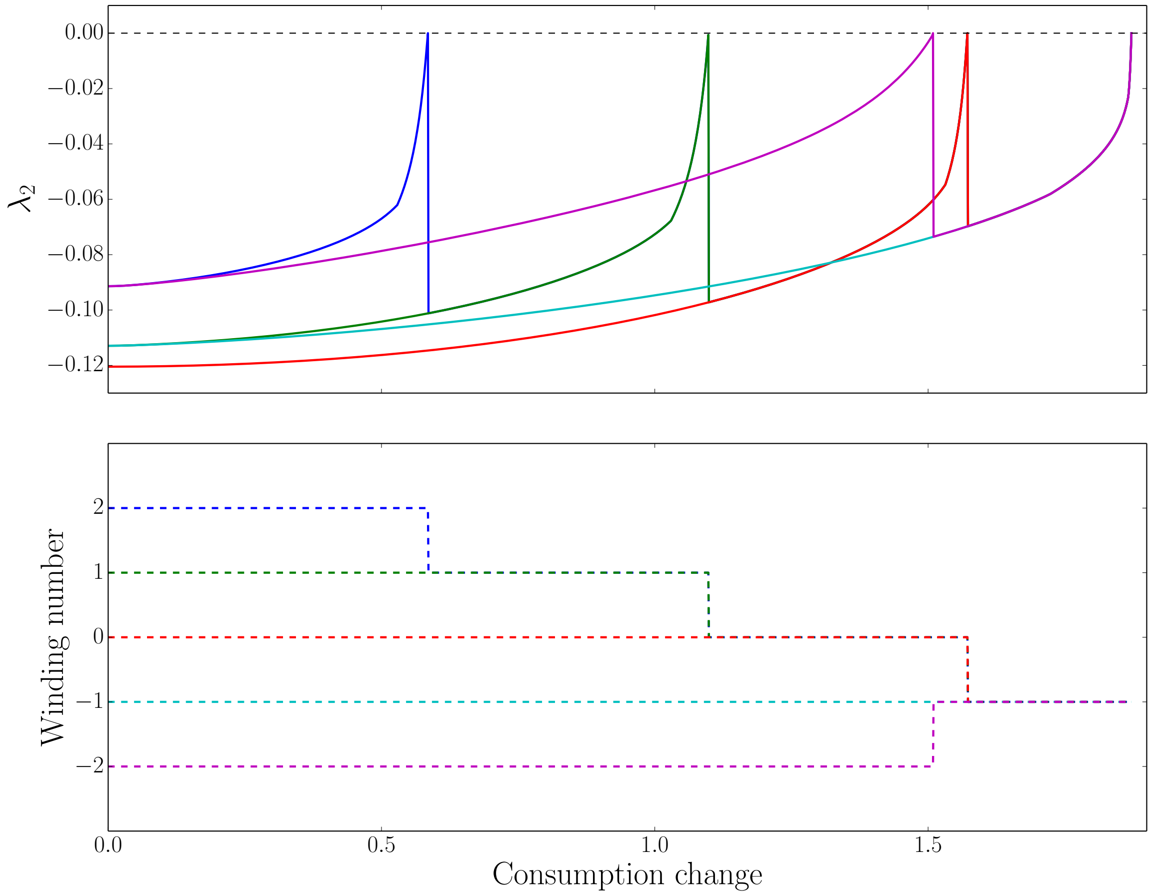
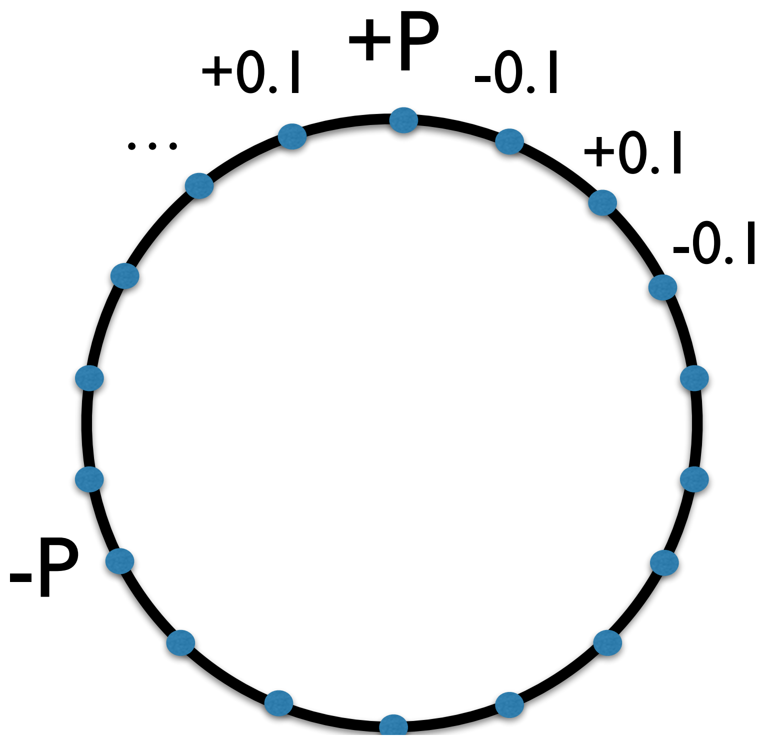
Generating vortex current with finite m

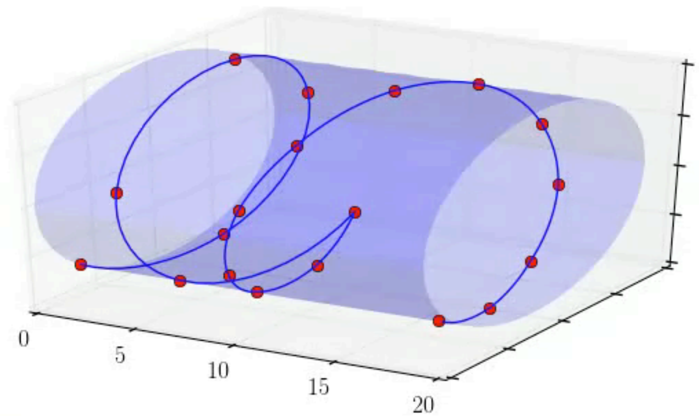
For the power-flow problem :

(iii) Commensurability

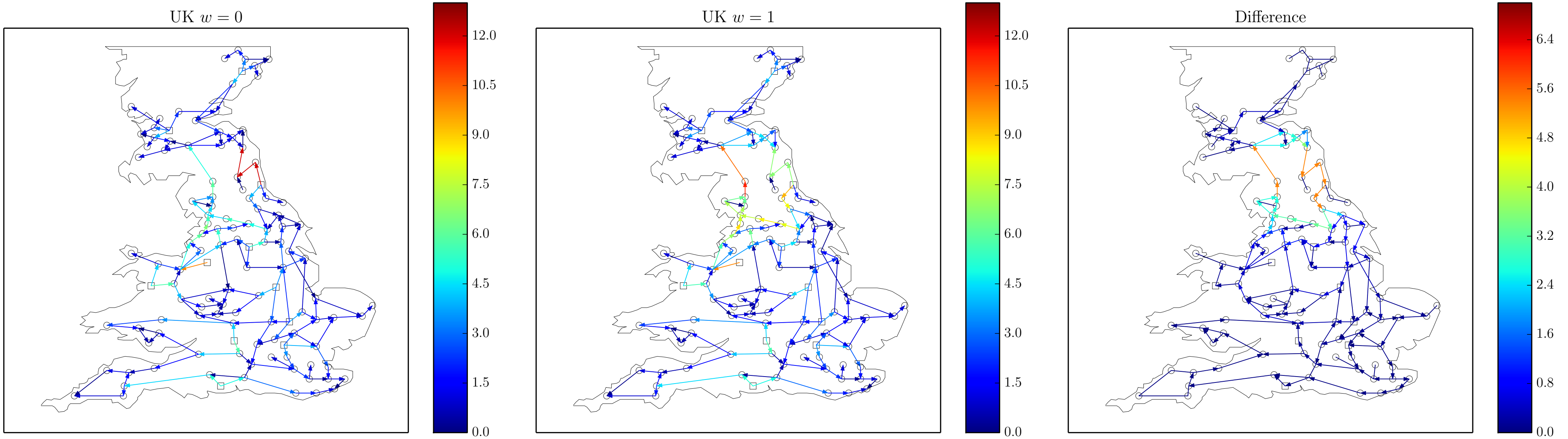
Delabays, Coletta and PJ '15 :

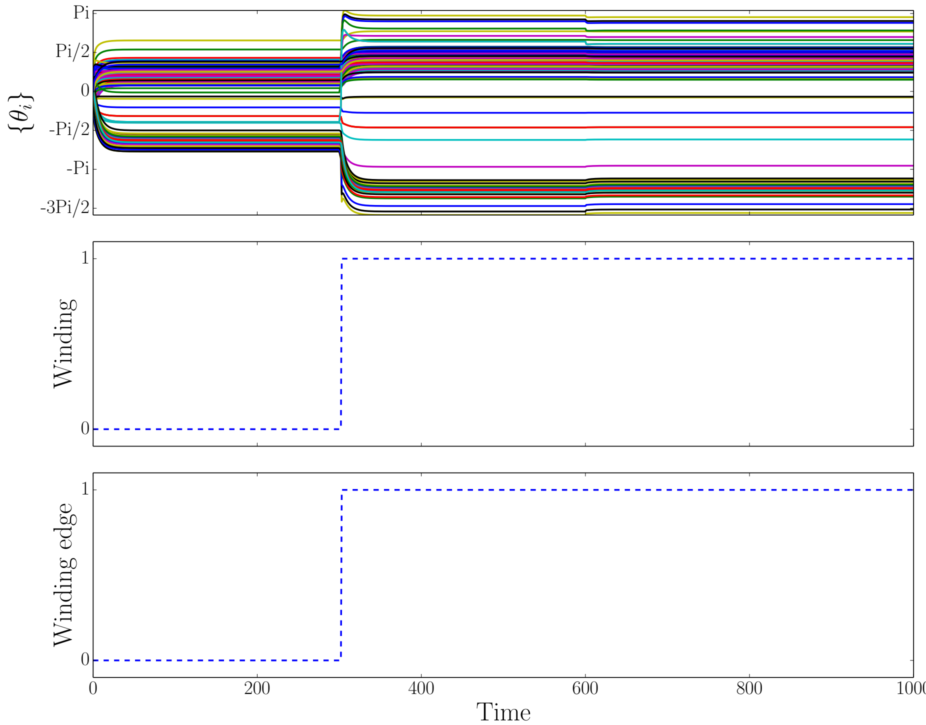






Generating vortex currents in realistic networks

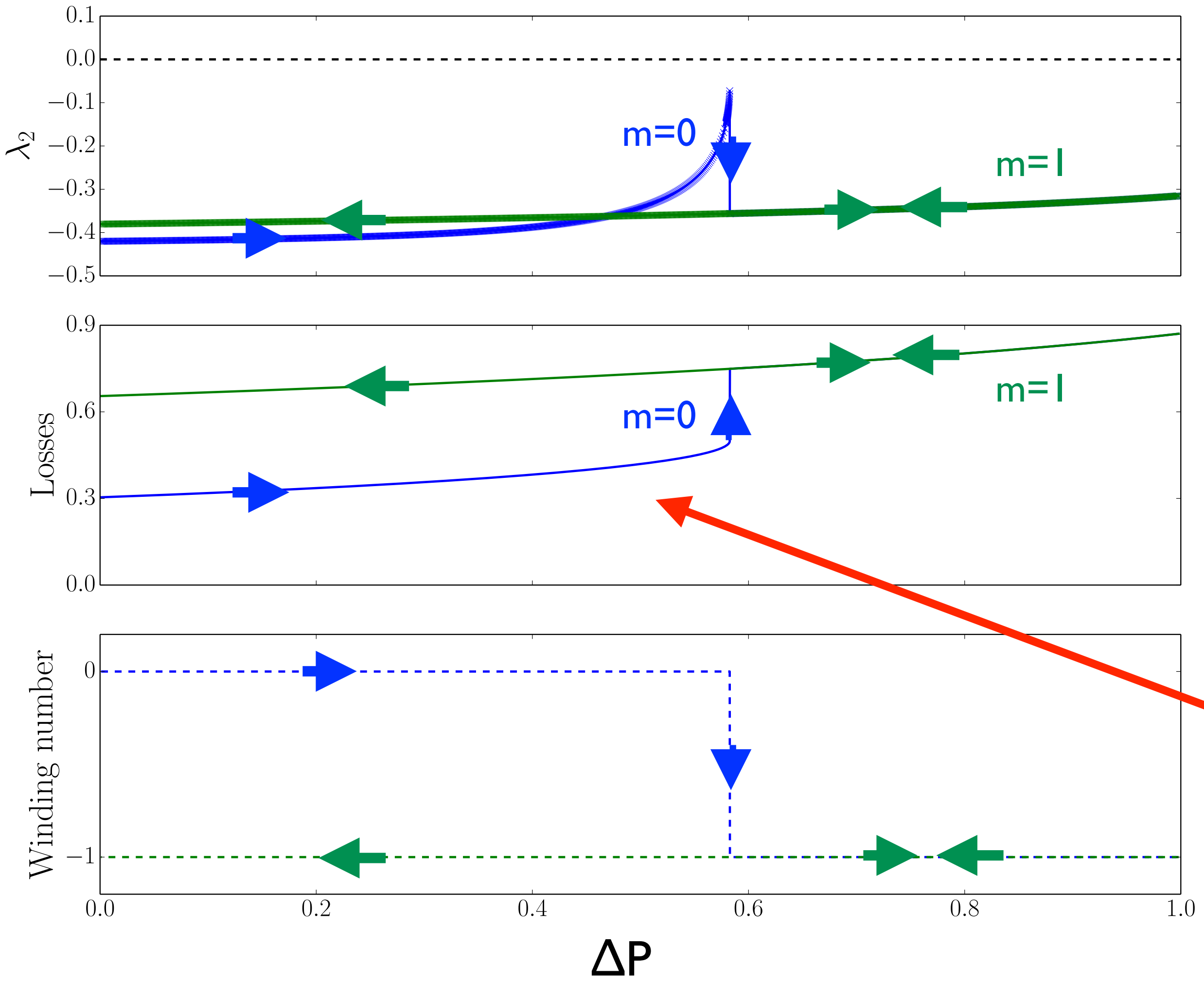


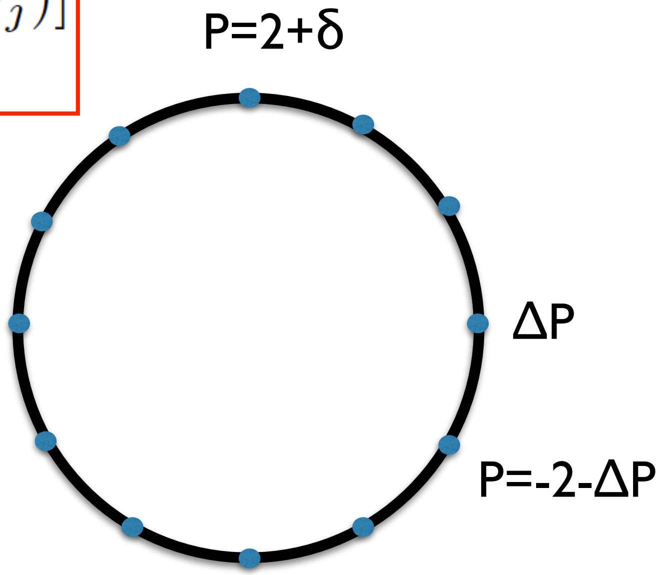


Vortex currents vs. dissipation

$$\boxed{P_i = \sum_j |V_i V_j| \left[G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \right]}$$

finite conductance = dissipation
(note $\Sigma P_i \neq 0$)





Increasing ΔP :
*m=0 loses stability at ΔP=0.58
*transient to m=1

Decreasing ΔP :
*stay on m=1
Despite higher losses !!!
"Topological protection"

Conclusions

* Strong similarities between high voltage electric transport and superconducting (Josephson) transport - VORTEX CURRENTS

* Need to go beyond our approximations, i.e.
  - finite G
  - voltage drops
  - active and reactive powers

  ...and see how vortex currents persist

* Loss of m=0 stability vs. Kosterlitz-Thouless (vortex-antivortex pairs?)