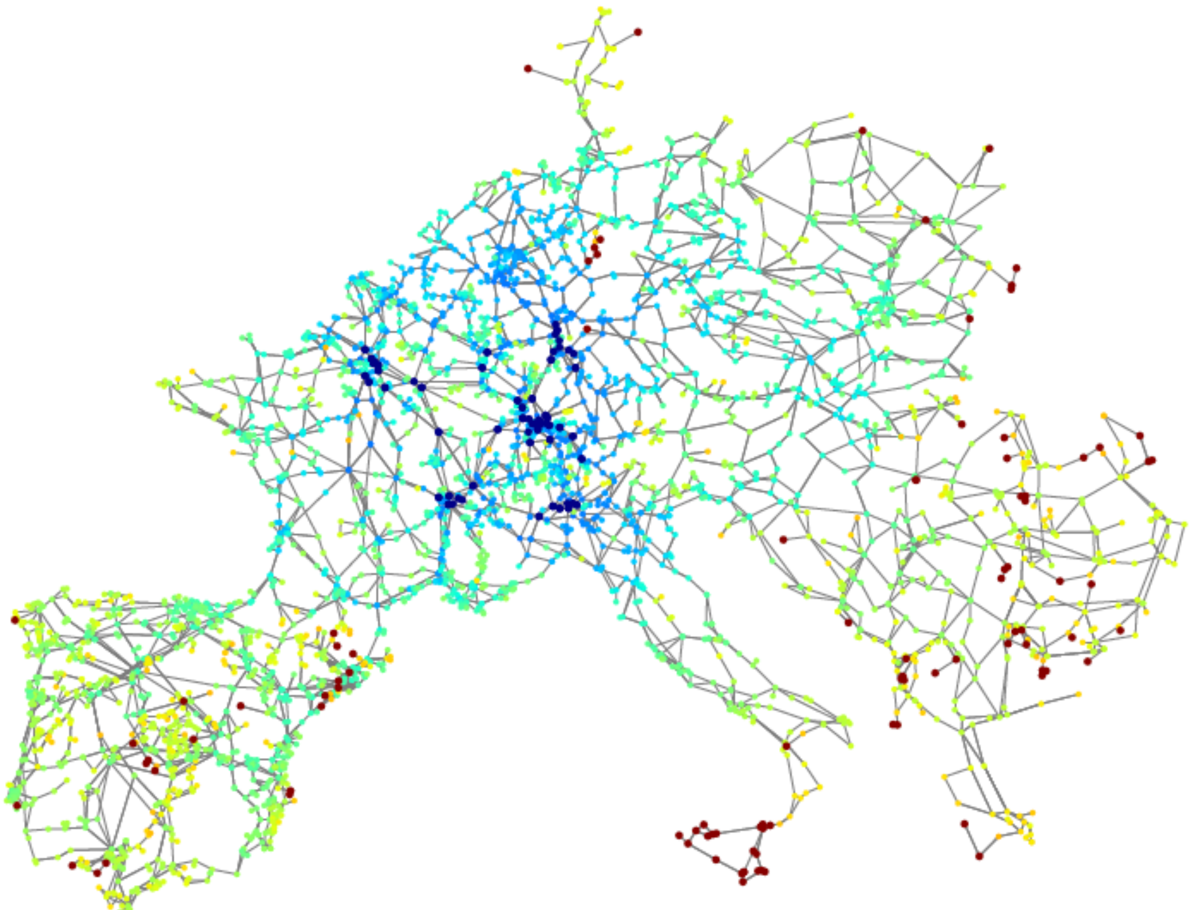
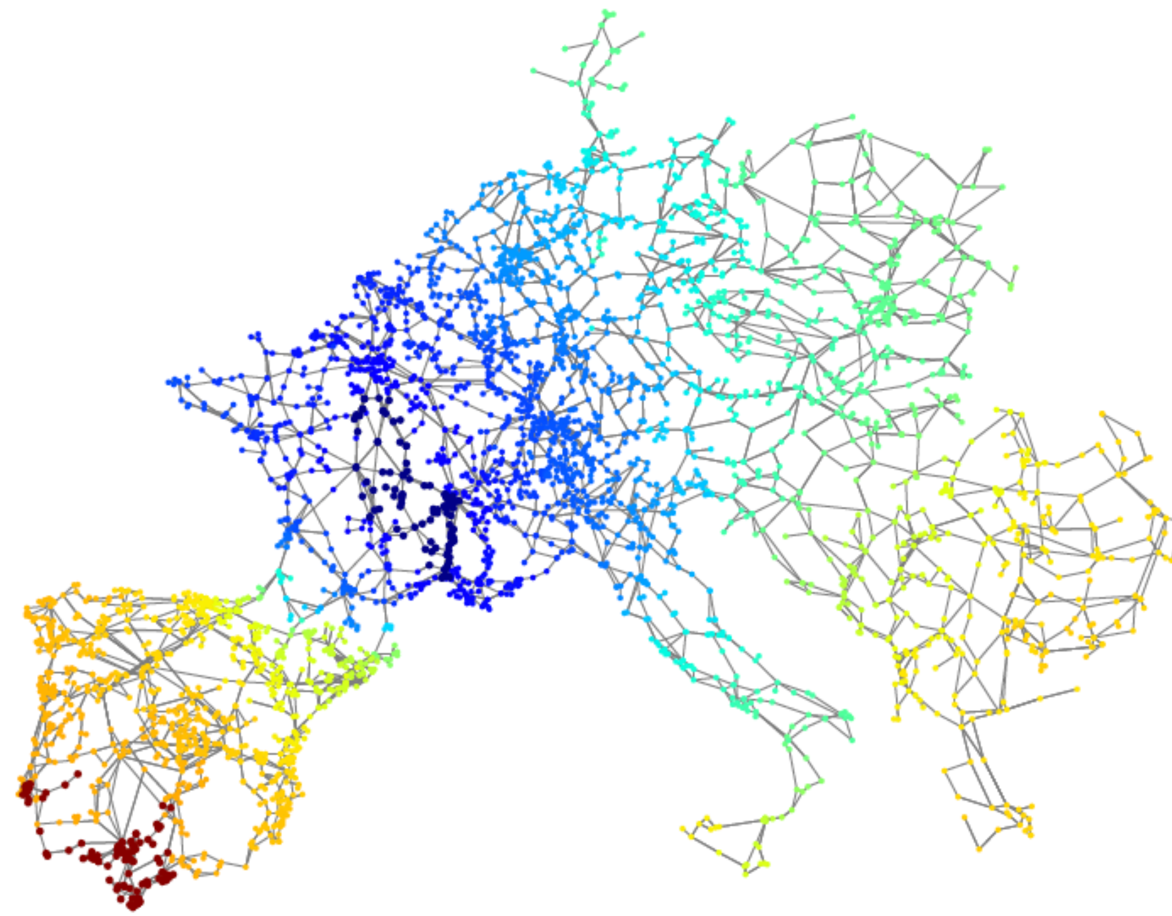
Global robustness vs. local vulnerabilities in network-coupled dynamical systems





Philippe Jacquod

DQMP/GVA - 6.4.2018

The questions of interest

Given a set of dynamical systems with couplings between them defined on a certain graph :

The questions of interest

Given a set of dynamical systems with couplings between them defined on a certain graph :

1. Is the network-coupled system globally robust against disturbances ?

The questions of interest

Given a set of dynamical systems with couplings between them defined on a certain graph :

1. Is the network-coupled system globally robust against disturbances ?

2. Is the network-coupled system robust against specific, targeted disturbances ?

3. What specific disturbance(s) would lead to the worst response of the system ?

4. Where should these specific disturbances act to lead to the worst response ?

The questions of interest

Given a set of dynamical systems with couplings between them defined on a certain graph :

1. Is the network-coupled system globally robust against disturbances ?

2. Is the network-coupled system robust against specific, targeted disturbances ?

3. What specific disturbance(s) would lead to the worst response of the system ?

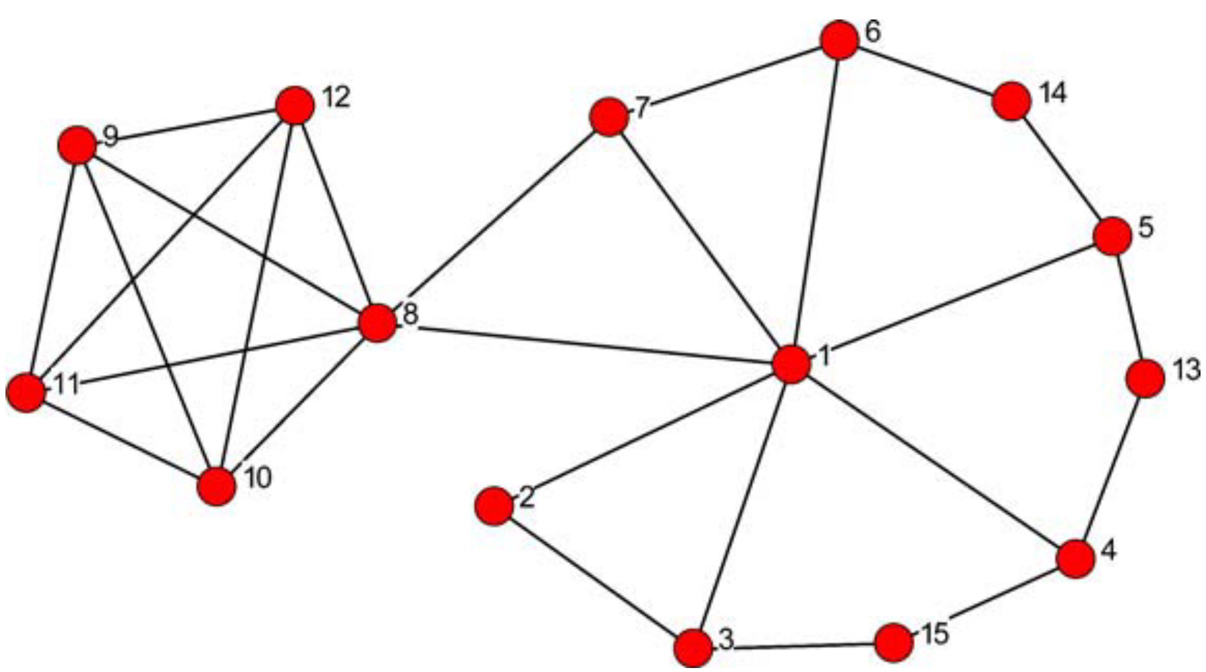
4. Where should these specific disturbances act to lead to the worst response ?

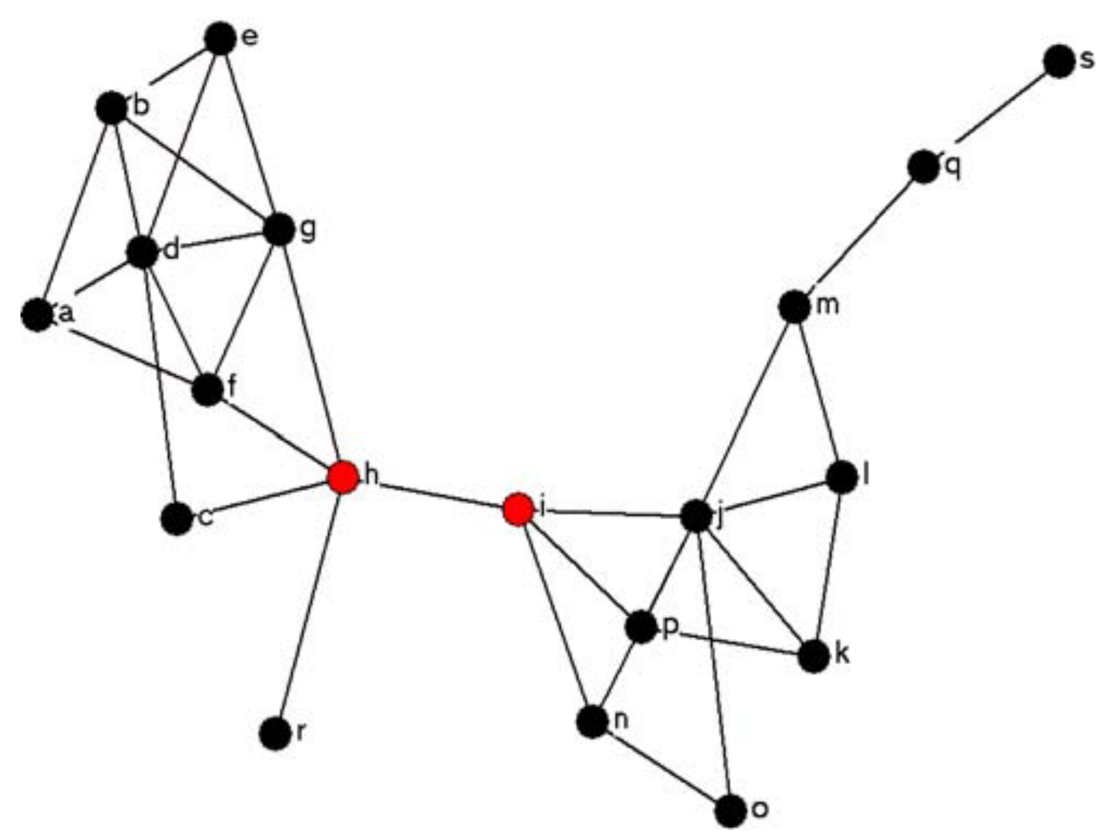
Global robustness vs. local vulnerabilities

The key player problem

1. Given a network, find the node which, if removed, would maximally disrupt communication among the remaining nodes.

2. Given a network, find the node that is maximally connected to all other nodes.

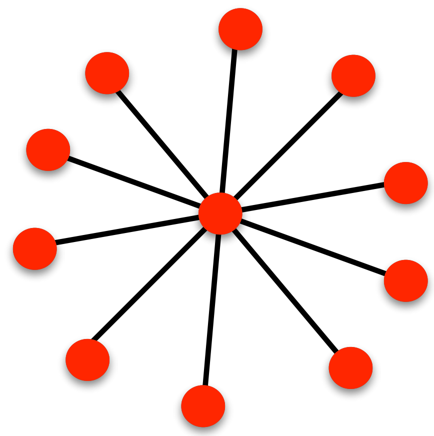




Borgatti :"The key player problem" (2003); Comput Math Organiz Theor (2006)

The key player problem : an answer

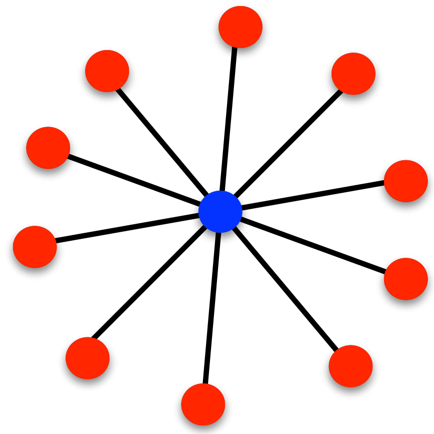
Star graph :



Which node should one remove to maximally disrupt communication among the remaining nodes ?

The key player problem : an answer

Star graph :

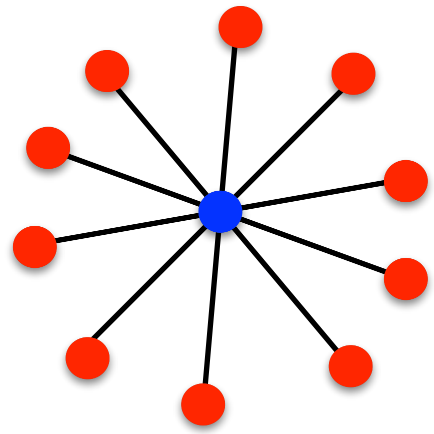


Which node should one remove to maximally disrupt communication among the remaining nodes ?

A: the central one, obviously…

The key player problem : an answer

Star graph :

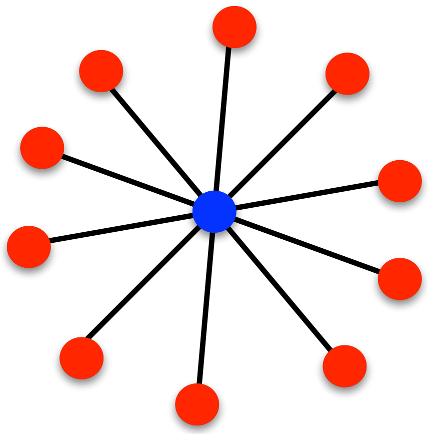


Which node should one remove to maximally disrupt communication among the remaining nodes ?
A: the central one, obviously…

…but then the central node is the node

The key player problem : an answer

Star graph :



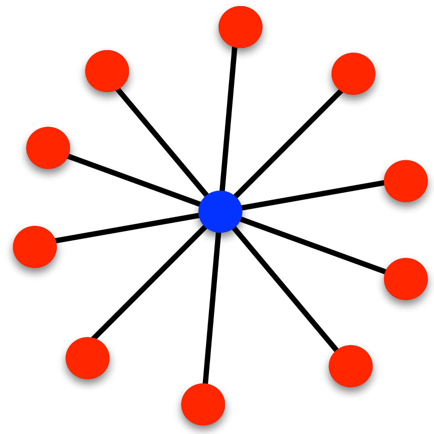
Which node should one remove to maximally disrupt communication among the remaining nodes ?
A: the central one, obviously…

…but then the central node is the node

*with largest degree

The key player problem : an answer

Star graph :



Which node should one remove to maximally disrupt communication among the remaining nodes ?

A: the central one, obviously…

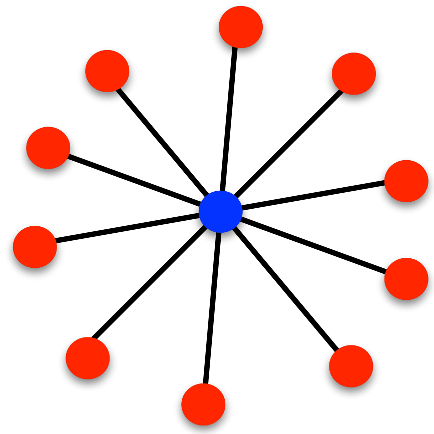
…but then the central node is the node

*with largest degree

*that is closest to all other nodes

The key player problem : an answer

Star graph :



Which node should one remove to maximally disrupt communication among the remaining nodes ?
A: the central one, obviously…

…but then the central node is the node

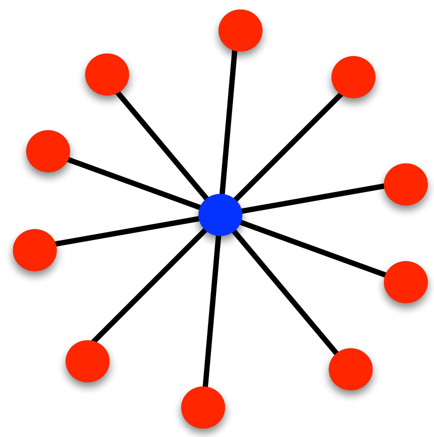
*with largest degree

*that is closest to all other nodes

*through which most shortest paths go

The key player problem : an answer

Star graph :



Which node should one remove to maximally disrupt communication among the remaining nodes ?

A: the central one, obviously…

…but then the central node is the node

*with largest degree

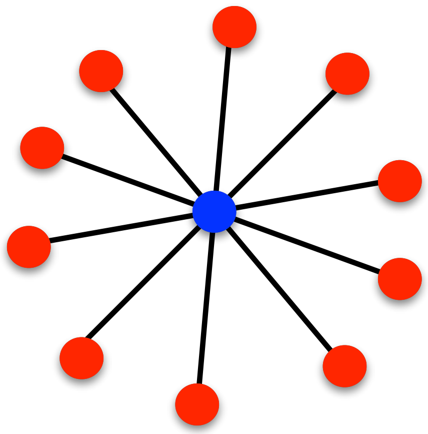
*that is closest to all other nodes

*through which most shortest paths go

*that maximizes the dominant eigenvector of the graph matrix

The key player problem : an answer

Star graph :



Which node should one remove to maximally disrupt communication among the remaining nodes ?
A: the central one, obviously…

…but then the central node is the node

*with largest degree

*that is closest to all other nodes

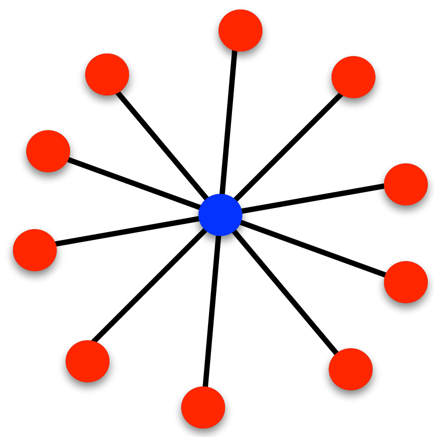
*through which most shortest paths go

*that maximizes the dominant eigenvector of the graph matrix

*with the highest probability in the stationary distribution of the natural random walk on the graph.

The key player problem : an answer

Star graph :



Which node should one remove to maximally disrupt communication among the remaining nodes ?
A: the central one, obviously…

…but then the central node is the node

*with largest degree

*that is closest to all other nodes

*through which most shortest paths go

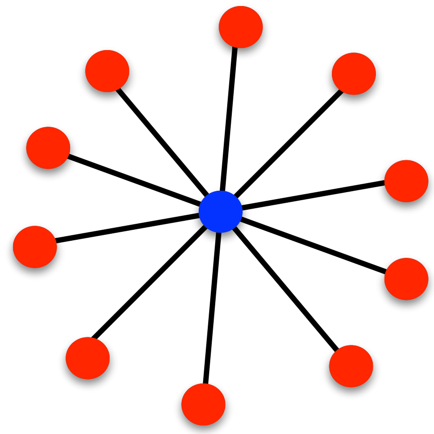
*that maximizes the dominant eigenvector of the graph matrix

*with the highest probability in the stationary distribution of the natural random walk on the graph.

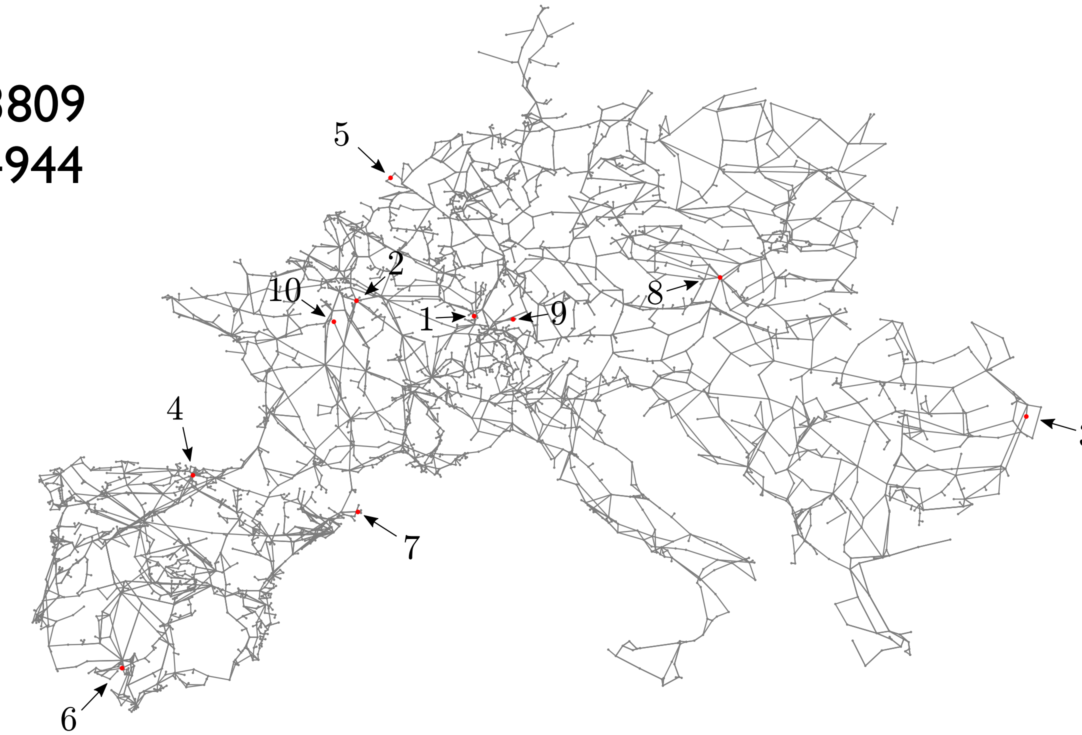
Which one of these property makes the central node the key player ?

The key player problem : an answer

Star graph :



#nodes : 3809
#edges : 4944



Which node should one re
communication among
A: the central

...but then the central node is the node

*with largest degree

*that is closest to all other nodes

*through which most shortest paths go

*that maximizes the dominant eigenvector of the graph matrix

*with the highest probability in the stationary
distribution of the natural random walk on the graph.

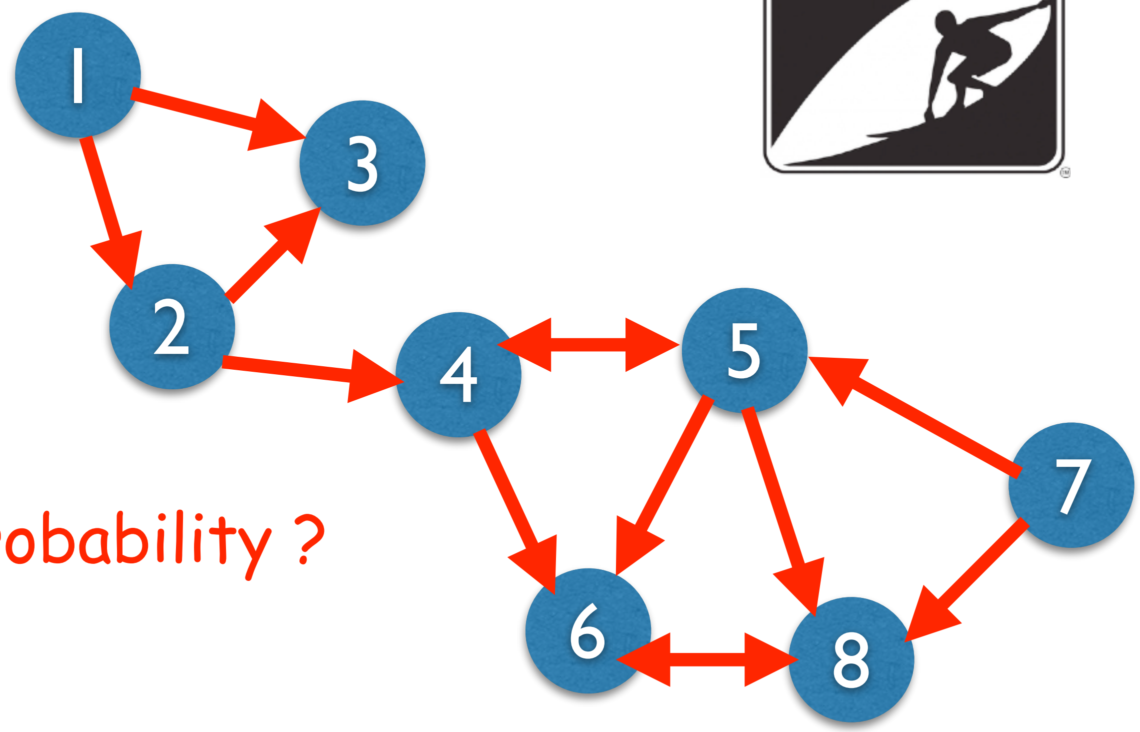
Which one of these property makes the central node the key player ?

The key player problem : Google's answer

From Markov to PageRank:

"Internet surfing as random walk
on a directed network"

? Where do you end up with the highest probability ?



Brin and Page : Comput. Netw. ISDN Syst. (1998)

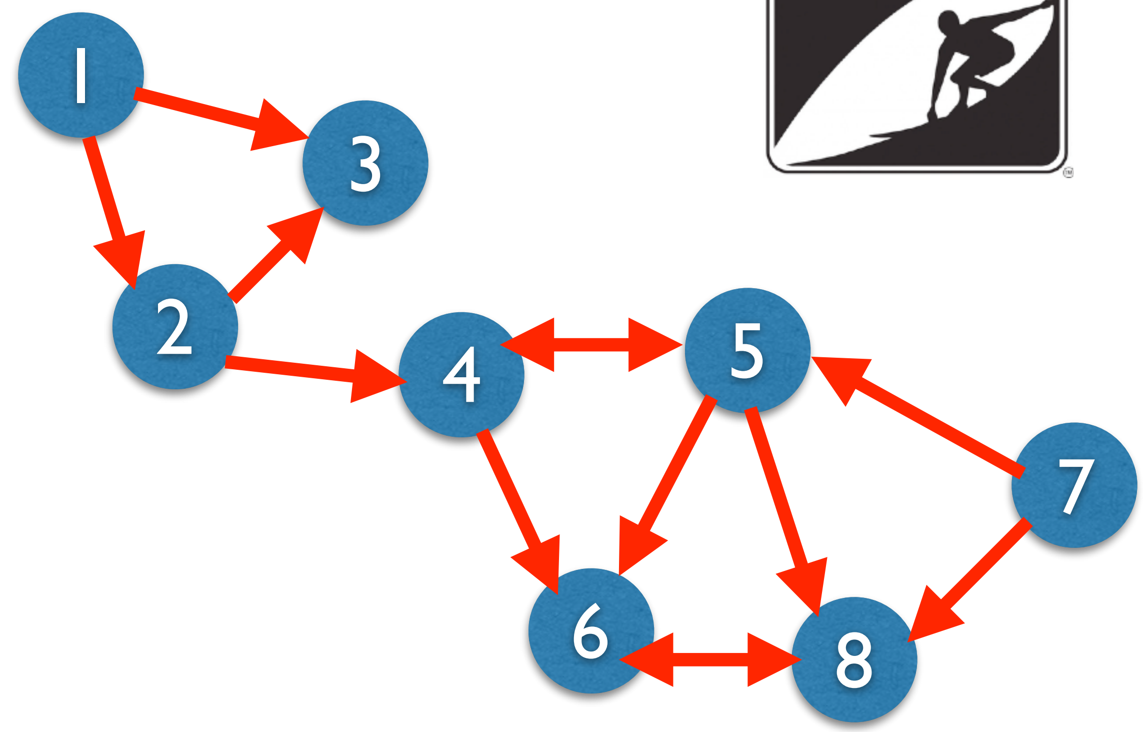
The key player problem : Google's answer

From Markov to PageRank:

"Internet surfing as random walk on a directed network"

(i) define the adjacency matrix

$$A_{ij} = \begin{cases} 1 \ \text{if} \ j \to i \\ 0 \ \text{if} \ j \nrightarrow i \end{cases}$$

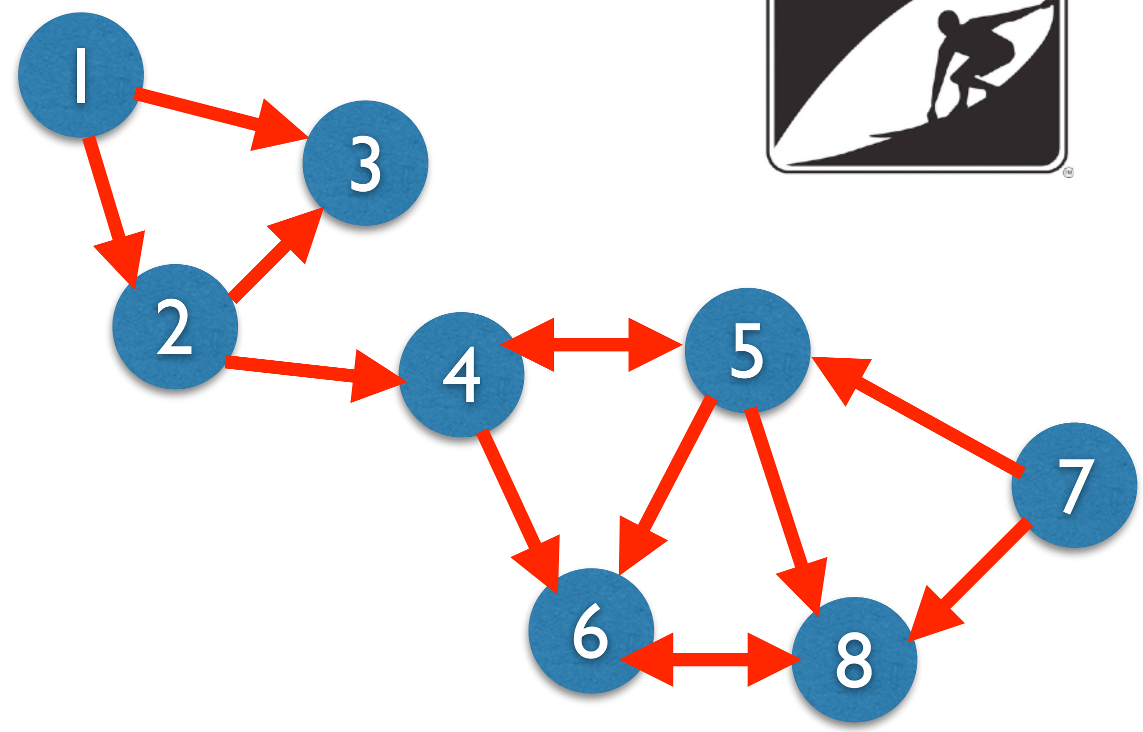


Brin and Page : Comput. Netw. ISDN Syst. (1998)

The key player problem : Google's answer

From Markov to PageRank:

"Internet surfing as random walk
on a directed network"



(i) define the adjacency matrix

$$A_{ij} = \begin{cases} 1 \ \text{if} \ \ j \to i \\ 0 \ \text{if} \ \ j \nrightarrow i \end{cases}$$

(ii) define the stochastic (Markov process) matrix

$$S_{ij} = \begin{cases} A_{ij}/\sum_k A_{kj} \ \ \text{if} \ \ \sum_k A_{kj} \neq 0 \\ 1/N \ \ \text{otherwise} \end{cases}$$

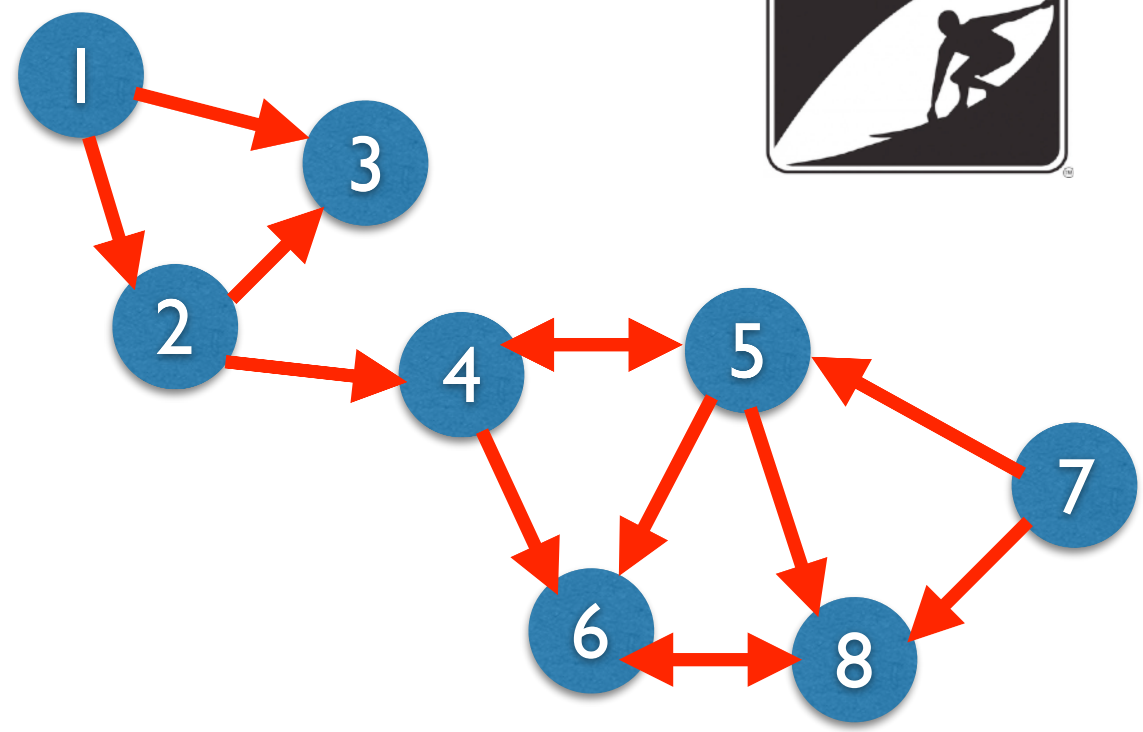
$$\mathbf{S} = \begin{pmatrix} 0 & 0 & 1/8 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1/8 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 1/8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 1/8 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 1/8 & 1/2 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1/8 & 1/2 & 1/3 & 0 & 0 & 1 \\ 0 & 0 & 1/8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/8 & 0 & 1/3 & 1 & 1/2 & 0 \end{pmatrix}$$

Brin and Page : Comput. Netw. ISDN Syst. (1998)

The key player problem : Google's answer

From Markov to PageRank:

"Internet surfing as random walk on a directed network"



(i) define the adjacency matrix

$$A_{ij} = \begin{cases} 1 \text{ if } j \to i \\ 0 \text{ if } j \nrightarrow i \end{cases}$$

(ii) define the stochastic (Markov process) matrix

$$S_{ij} = \begin{cases} A_{ij}/\sum_k A_{kj} \text{ if } \sum_k A_{kj} \neq 0 \\ 1/N \text{ otherwise} \end{cases}$$

(iii) define the Google matrix

$$G_{ij} = \alpha S_{ij} + (1-\alpha)/N$$

$$\mathbf{G} = \begin{pmatrix} 1/40 & 1/40 & 1/8 & 1/40 & 1/40 & 1/40 & 1/40 & 1/40 \\ 17/40 & 1/40 & 1/8 & 1/40 & 1/40 & 1/40 & 1/40 & 1/40 \\ 17/40 & 17/40 & 1/8 & 1/40 & 1/40 & 1/40 & 1/40 & 1/40 \\ 1/40 & 17/40 & 1/8 & 1/40 & 7/24 & 1/40 & 1/40 & 1/40 \\ 1/40 & 1/40 & 1/8 & 17/40 & 1/40 & 1/40 & 17/40 & 1/40 \\ 1/40 & 1/40 & 1/8 & 17/40 & 7/24 & 1/40 & 1/40 & 33/40 \\ 1/40 & 1/40 & 1/8 & 1/40 & 1/40 & 1/40 & 1/40 & 1/40 \\ 1/40 & 1/40 & 1/8 & 1/40 & 7/24 & 33/40 & 17/40 & 1/40 \end{pmatrix}$$

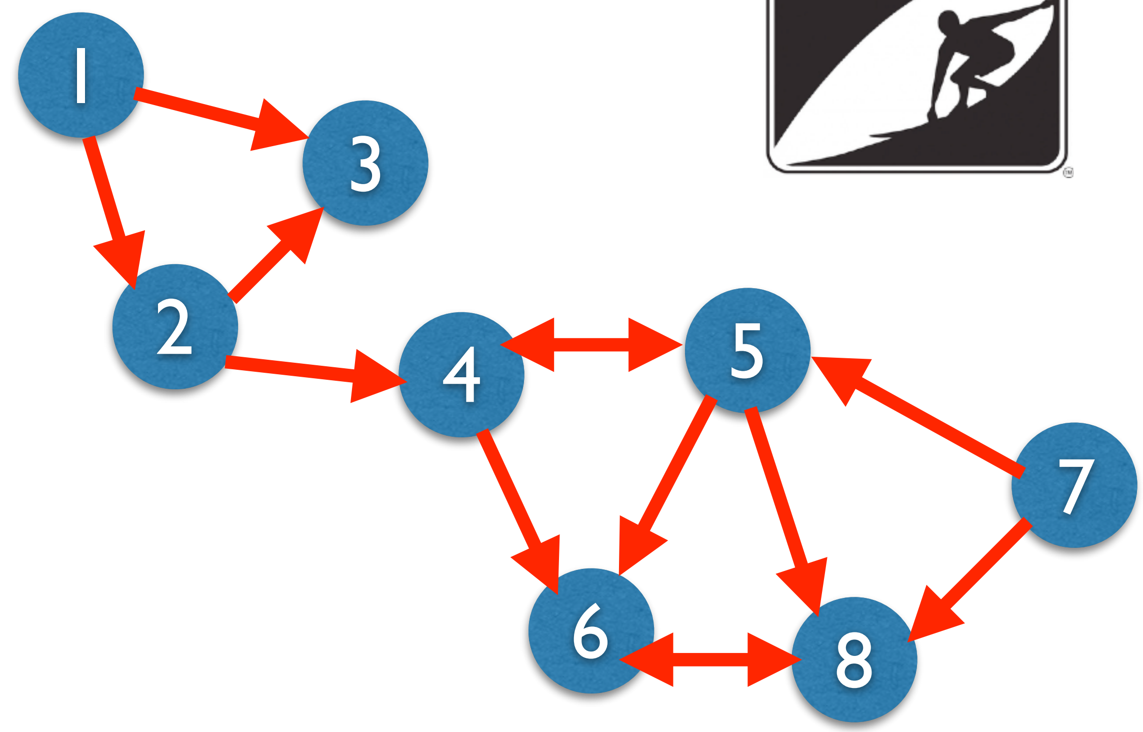
$\alpha = 0.8$

Brin and Page : Comput. Netw. ISDN Syst. (1998)

The key player problem : Google's answer

From Markov to PageRank:

"Internet surfing as random walk on a directed network"



Properties of the Google matrix

$$G_{ij} = \alpha S_{ij} + (1 - \alpha) N$$

Only real, positive coefficients

-> one of its eigenvectors has (Perron-Frobenius)

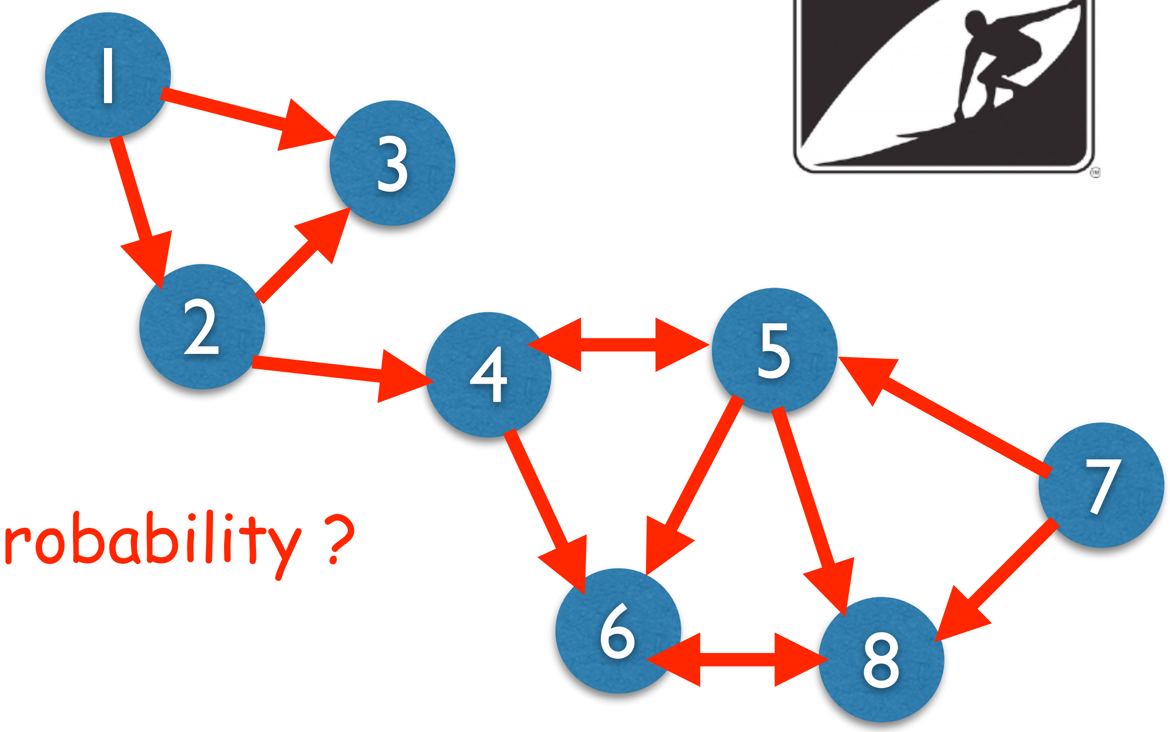
(1) a real, positive eigenvalue with largest absolute value

(2) all its components real and positive

Brin and Page : Comput. Netw. ISDN Syst. (1998)

The key player problem : Google's answer

From Markov to PageRank:



"Internet surfing as random walk
on a directed network"

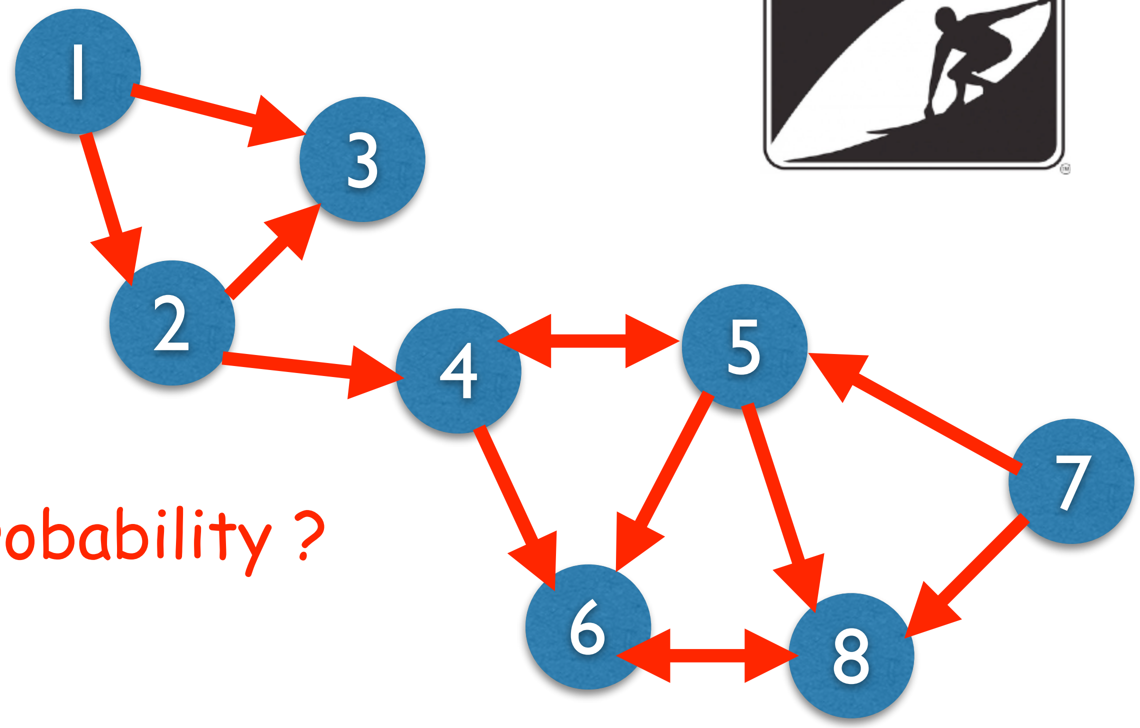
? Where do you end up with the highest probability ?

A: on the node with largest component of the Perron-Frobenius mode !

Brin and Page : Comput. Netw. ISDN Syst. (1998)

The key player problem : Google's answer

From Markov to PageRank:



"Internet surfing as random walk
on a directed network"

? Where do you end up with the highest probability ?

A: on the node with largest component of the Perron-Frobenius mode !

-> PageRank = ranking nodes on a network with the
components of the Perron-Frobenius mode

Brin and Page : Comput. Netw. ISDN Syst. (1998)

A bit of literature

Centralities have been introduced to solve the “key player” problem
*vs. graph/network matrix (geodesic, betweenness, Bonacich, Katz, PageRank…)

A bit of literature

Centralities have been introduced to solve the "key player" problem
*vs. graph/network matrix (geodesic, betweenness, Bonacich, Katz, PageRank…)

Comput Math Organiz Theor (2006) 12: 21–34
DOI 10.1007/s10588-006-7084-x

Identifying sets of key players in a social network

Stephen P. Borgatti

A bit of literature

Centralities have been introduced to solve the “key player” problem
*vs. graph/network matrix (geodesic, betweenness, Bonacich, Katz, PageRank…)

Comput Math Organiz Theor (2006) 12: 21–34
DOI 10.1007/s10588-006-7084-x

Identifying sets of key players in a social network

Stephen P. Borgatti

Econometrica, Vol. 74, No. 5 (September, 2006), 1403–1417

WHO'S WHO IN NETWORKS. WANTED: THE KEY PLAYER

BY CORALIO BALLESTER, ANTONI CALVÓ-ARMENGOL, AND YVES ZENOU[1]

A bit of literature

Centralities have been introduced to solve the "key player" problem
*vs. graph/network matrix (geodesic, betweenness, Bonacich, Katz, PageRank…)

Comput Math Organiz Theor (2006) 12: 21–34
DOI 10.1007/s10588-006-7084-x

Identifying sets of key players in a social network

Stephen P. Borgatti

Econometrica, Vol. 74, No. 5 (September, 2006), 1403–1417

WHO'S WHO IN NETWORKS. WANTED: THE KEY PLAYER

BY CORALIO BALLESTER, ANTONI CALVÓ-ARMENGOL, AND YVES ZENOU[1]

Internet Mathematics Vol. 10: 222–262

Axioms for Centrality

Paolo Boldi and Sebastiano Vigna

A bit of literature

Centralities have been introduced to solve the “key player” problem
*vs. graph/network matrix (geodesic, betweenness, Bonacich, Katz, PageRank…)

Comput Math Organiz Theor (2006) 12: 21–34
DOI 10.1007/s10588-006-7084-x

Identifying sets of key players in a social network

Stephen P. Borgatti

Econometrica, Vol. 74, No. 5 (September, 2006), 1403–1417

WHO’S WHO IN NETWORKS. WANTED: THE KEY PLAYER

BY CORALIO BALLESTER, ANTONI CALVÓ-ARMENGOL, AND YVES ZENOU[1]

Internet Mathematics Vol. 10: 222–262

Axioms for Centrality

Paolo Boldi and Sebastiano Vigna

…leading to purely graph-theoretic approaches (rather successful)

A bit of literature

Centralities have been introduced to solve the "key player" problem
*vs. graph/network matrix (geodesic, betweenness, Bonacich, Katz, PageRank…)

Comput Math Organiz Theor (2006) 12: 21–34
DOI 10.1007/s10588-006-7084-x

Identifying sets of key players in a social network

Stephen P. Borgatti

Econometrica, Vol. 74, No. 5 (September, 2006), 1403–1417

WHO'S WHO IN NETWORKS. WANTED: THE KEY PLAYER

BY CORALIO BALLESTER, ANTONI CALVÓ-ARMENGOL, AND YVES ZENOU[1]

Internet Mathematics Vol. 10: 222–262

Axioms for Centrality

Paolo Boldi and Sebastiano Vigna

…leading to purely graph-theoretic approaches (rather successful)

Collective dynamics of 'small-world' networks

Duncan J. Watts* & Steven H. Strogatz

Department of Theoretical and Applied Mechanics, Kimball Hall, Cornell University, Ithaca, New York 14853, USA

A bit of literature

Centralities have been introduced to solve the “key player” problem
*vs. graph/network matrix (geodesic, betweenness, Bonacich, Katz, PageRank…)

Comput Math Organiz Theor (2006) 12: 21–34
DOI 10.1007/s10588-006-7084-x

Identifying sets of key players in a social network

Stephen P. Borgatti

Econometrica, Vol. 74, No. 5 (September, 2006), 1403–1417

WHO’S WHO IN NETWORKS. WANTED: THE KEY PLAYER

BY CORALIO BALLESTER, ANTONI CALVÓ-ARMENGOL, AND YVES ZENOU[1]

Internet Mathematics Vol. 10: 222–262

Axioms for Centrality

Paolo Boldi and Sebastiano Vigna

…leading to purely graph-theoretic approaches (rather successful)

Collective dynamics of ‘small-world’ networks

Duncan J. Watts* **& Steven H. Strogatz**
Department of Theoretical and Applied Mechanics, Kimball Hall, Cornell University, Ithaca, New York 14853, USA

Navigation in a small world

It is easier to find short chains between points in some networks than others.

A bit of literature

Centralities have been introduced to solve the “key player” problem
*vs. graph/network matrix (geodesic, betweenness, Bonacich, Katz, PageRank…)

Comput Math Organiz Theor (2006) 12: 21–34
DOI 10.1007/s10588-006-7084-x

Identifying sets of key players in a social network

Stephen P. Borgatti

Econometrica, Vol. 74, No. 5 (September, 2006), 1403–1417

WHO’S WHO IN NETWORKS. WANTED: THE KEY PLAYER

BY CORALIO BALLESTER, ANTONI CALVÓ-ARMENGOL, AND YVES ZENOU[1]

Internet Mathematics Vol. 10: 222–262

Axioms for Centrality

Paolo Boldi and Sebastiano Vigna

…leading to purely graph-theoretic approaches (rather successful)

Collective dynamics of ‘small-world’ networks

Duncan J. Watts* **& Steven H. Strogatz**
Department of Theoretical and Applied Mechanics, Kimball Hall, Cornell University, Ithaca, New York 14853, USA

PRL **102,** 238703 (2009) PHYSICAL REVIEW LETTERS week ending 12 JUNE 2009

Extended Navigability of Small World Networks: Exact Results and New Insights

Cécile Caretta Cartozo and Paolo De Los Rios

Navigation in a small world

It is easier to find short chains between points in some networks than others.

A bit of literature

Centralities have been introduced to solve the "key player" problem
*vs. graph/network matrix (geodesic, betweenness, Bonacich, Katz, PageRank…)

Comput Math Organiz Theor (2006) 12: 21–34
DOI 10.1007/s10588-006-7084-x

Identifying sets of key players in a social network

Stephen P. Borgatti

Econometrica, Vol. 74, No. 5 (September, 2006), 1403–1417

WHO'S WHO IN NETWORKS. WANTED: THE KEY PLAYER

BY CORALIO BALLESTER, ANTONI CALVÓ-ARMENGOL, AND YVES ZENOU[1]

Internet Mathematics Vol. 10: 222–262

Axioms for Centrality

Paolo Boldi and Sebastiano Vigna

…leading to purely graph-theoretic approaches (rather successful)

Collective dynamics of 'small-world' networks

Duncan J. Watts* **& Steven H. Strogatz**
Department of Theoretical and Applied Mechanics, Kimball Hall, Cornell University, Ithaca, New York 14853, USA

Annu. Rev. Sociol. 2001. 27:415–44

BIRDS OF A FEATHER: Homophily in Social Networks

Miller McPherson[1], Lynn Smith-Lovin[1], and James M Cook[2]

PRL **102,** 238703 (2009) PHYSICAL REVIEW LETTERS week ending 12 JUNE 2009

Extended Navigability of Small World Networks: Exact Results and New Insights

Cécile Caretta Cartozo and Paolo De Los Rios

Navigation in a small world

It is easier to find short chains between points in some networks than others.

A bit of literature

Centralities have been introduced to solve the "key player" problem
*vs. graph/network matrix (geodesic, betweenness, Bonacich, Katz, PageRank…)

Comput Math Organiz Theor (2006) 12: 21–34
DOI 10.1007/s10588-006-7084-x

Identifying sets of key players in a social network

Stephen P. Borgatti

Econometrica, Vol. 74, No. 5 (September, 2006), 1403–1417

WHO'S WHO IN NETWORKS. WANTED: THE KEY PLAYER

BY CORALIO BALLESTER, ANTONI CALVÓ-ARMENGOL, AND YVES ZENOU[1]

Internet Mathematics Vol. 10: 222–262

Axioms for Centrality

Paolo Boldi and Sebastiano Vigna

…leading to purely graph-theoretic approaches (rather successful)

Collective dynamics of 'small-world' networks

Duncan J. Watts* & Steven H. Strogatz

Department of Theoretical and Applied Mechanics, Kimball Hall, Cornell University, Ithaca, New York 14853, USA

THE ROYAL SOCIETY

doi 10.1098/rspb.2001.1767

Complexity and fragility in ecological networks

Ricard V. Solé[1,2*] and José M. Montoya[1,3]

Annu. Rev. Sociol. 2001. 27:415–44
Copyright © 2001 by Annual Reviews. All rights reserved

BIRDS OF A FEATHER: Homophily in Social Networks

Miller McPherson[1], Lynn Smith-Lovin[1], and James M Cook[2]

PRL **102,** 238703 (2009) PHYSICAL REVIEW LETTERS week ending 12 JUNE 2009

Extended Navigability of Small World Networks: Exact Results and New Insights

Cécile Caretta Cartozo and Paolo De Los Rios

Navigation in a small world

It is easier to find short chains between points in some networks than others.

A bit of literature

Centralities have been introduced to solve the “key player” problem
*vs. graph/network matrix (geodesic, betweenness, Bonacich, Katz, PageRank…)

Comput Math Organiz Theor (2006) 12: 21–34
DOI 10.1007/s10588-006-7084-x

Identifying sets of key players in a social network

Stephen P. Borgatti

Econometrica, Vol. 74, No. 5 (September, 2006), 1403–1417

WHO’S WHO IN NETWORKS. WANTED: THE KEY PLAYER

BY CORALIO BALLESTER, ANTONI CALVÓ-ARMENGOL, AND YVES ZENOU[1]

Internet Mathematics Vol. 10: 222–262

Axioms for Centrality

Paolo Boldi and Sebastiano Vigna

…leading to purely graph-theoretic approaches (rather successful)

Collective dynamics of ‘small-world’ networks

Duncan J. Watts* **& Steven H. Strogatz**
Department of Theoretical and Applied Mechanics, Kim...
Cornell University, Ithaca, New York 14853, USA

THE ROYAL SOCIETY
doi 10.1098/rspb.2001.1767

Complexity and fragility in ecological networks

Curvature of co-links uncovers hidden thematic layers in the World Wide Web

Jean-Pierre Eckmann*† and Elisha Moses‡
*Département de Physique Théorique and Section de Mathématiques, Université de Genève, 32 Boulevard D'Yvoy, CH-1211 Genève 4, Switzerland; and ‡Department of Physics of Complex Systems, Weizmann Institute of Science, Rehovot 76100, Israel

REVIEW LETTERS week ending 12 JUNE 2009

...rld Networks: Exact Results and New Insights

Cécile Caretta Cartozo and Paolo De Los Rios

BIRDS OF A FEATHER: Homophily in Social Networks

Miller McPherson[1], Lynn Smith-Lovin[1], and James M Cook[2]

Navigation in a small world

It is easier to find short chains between points in some networks than others.

A bit of literature

CHAOS **20**, 033122 (2010)

Do topological models provide good information about electricity infrastructure vulnerability?

Paul Hines,[1,a)] Eduardo Cotilla-Sanchez,[1,b)] and Seth Blumsack[2,c)]
[1]*School of Engineering, University of Vermont, Burlington, Vermont 05405, USA*
[2]*Department of Energy and Mineral Engineering, Pennsylvania State University, University Park, Pennsylvania 16802, USA*

(Received 7 April 2010; accepted 24 August 2010; published online 28 September 2010)

In order to identify the extent to which results from topological graph models are useful for modeling vulnerability in electricity infrastructure, we measure the susceptibility of power networks to random failures and directed attacks using three measures of vulnerability: characteristic path lengths, connectivity loss, and blackout sizes. The first two are purely topological metrics. The blackout size calculation results from a model of cascading failure in power networks. Testing the response of 40 areas within the Eastern U.S. power grid and a standard IEEE test case to a variety of attack/failure vectors indicates that directed attacks result in larger failures using all three vulnerability measures, but the attack-vectors that appear to cause the most damage depend on the measure chosen. While the topological metrics and the power grid model show some similar trends, the vulnerability metrics for individual simulations show only a mild correlation. We conclude that evaluating vulnerability in power networks using purely topological metrics can be misleading.
 [doi:10.1063/1.3489887]

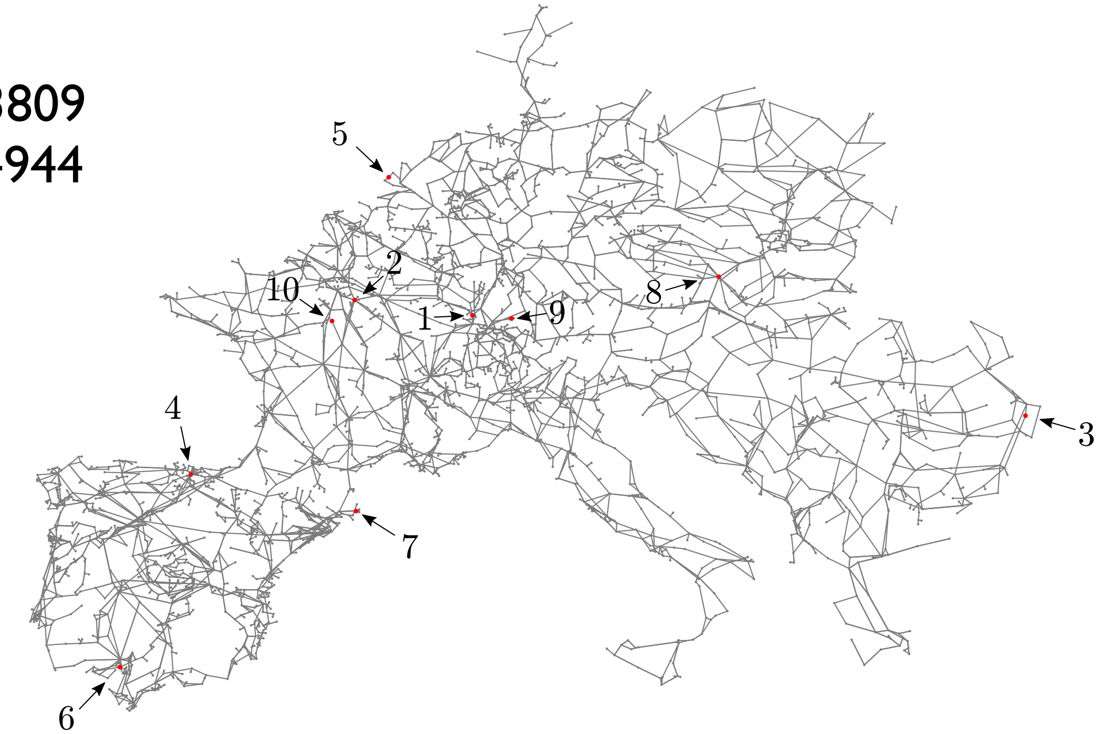
The key player problem : deterministically coupled systems

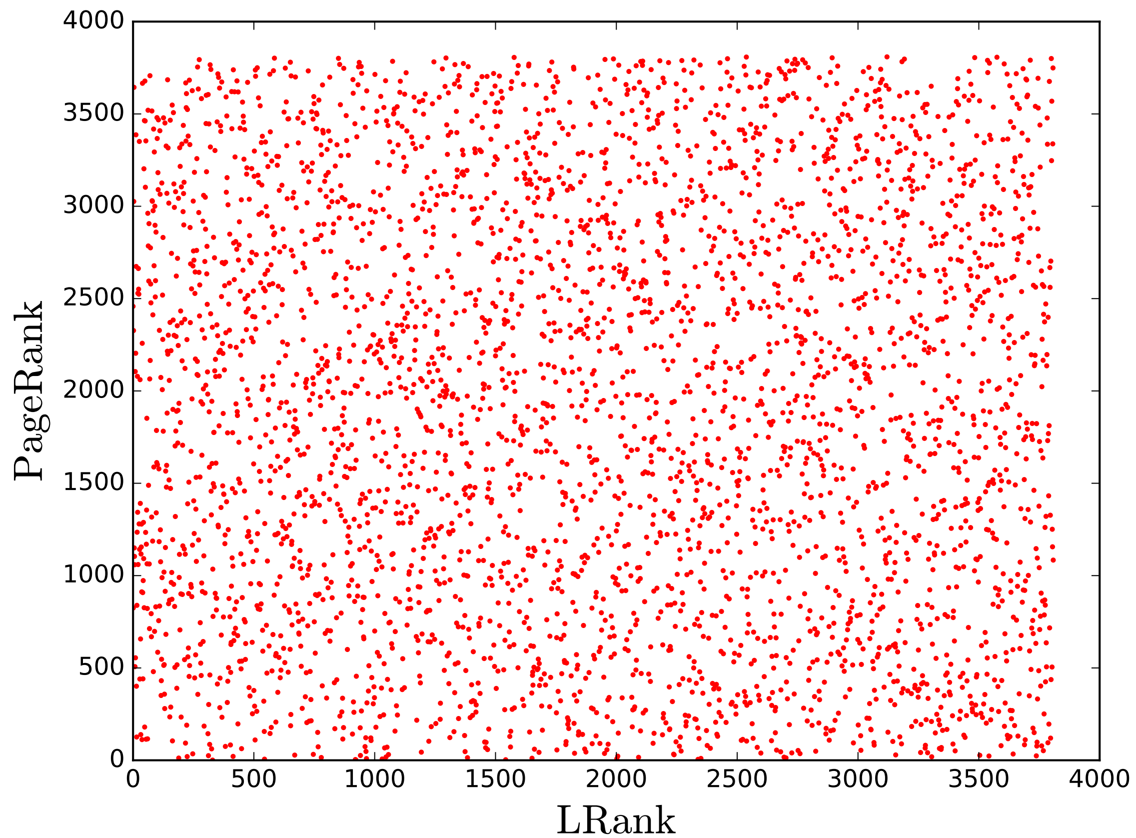
node #	$C_i^{(1)}$	C_i^{geo}	Degree	Katz	PageRank	$\mathcal{P}_2$
1	22.15	7.82	6	1.03195	1327	4.7×10^{-4}
2	22.07	7.68	5	1.03062	196	4.7×10^{-4}
3	18.59	3.54	5	1.03162	1041	6.6×10^{-4}
4	16.33	6.58	10	1.00103	1740	1.1×10^{-3}
5	16.32	5.56	2	1.03127	3470	1.1×10^{-3}
6	12.74	3.98	6	1.00067	3408	2×10^{-3}
7	10.77	6.58	3	1.00085	1076	2.5×10^{-3}
8	10.77	2.91	2	1.00016	2403	2.7×10^{-3}
9	9.66	3.53	2	1.00035	1532	3×10^{-3}
10	8.11	4.65	1	1.00001	3367	4×10^{-3}

Resistance distance centrality a.k.a. LRank

Numerically computed performance measure

#nodes : 3809
#edges : 4944





Tyloo, Pagnier and PJ : to be published (2018)

The key player problem : deterministically coupled systems

node #	$C_i^{(1)}$	C_i^{geo}	Degree	Katz	PageRank	$\mathcal{P}_2$
1	22.15	7.82	6	1.03195	1327	4.7×10^{-4}
2	22.07	7.68	5	1.03062	196	4.7×10^{-4}
3	18.59	3.54	5	1.03162	1041	6.6×10^{-4}
4	16					
5	16					
6	12					
7	10					
8	10					
9	9					
10	8					

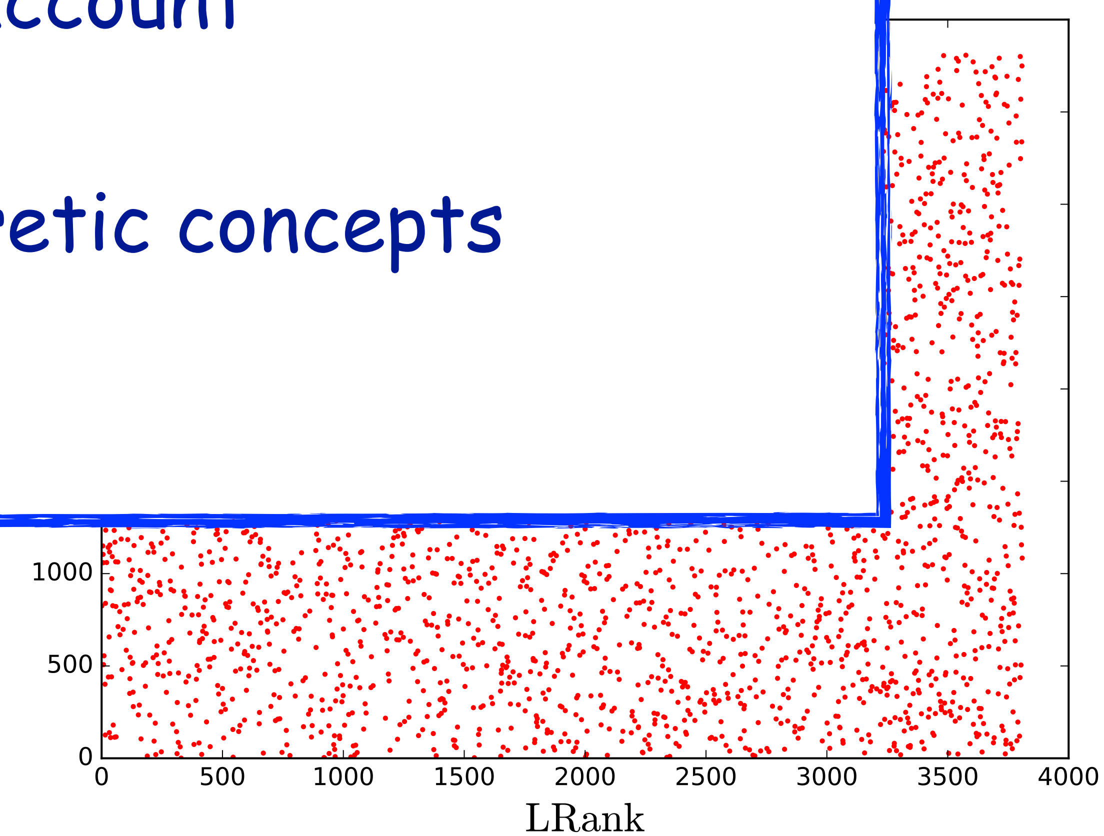
#nodes : 3809
#edges : 4944

Take-home message

- Global robustness assessment as well as the key player problem / nodal ranking needs to take details of the dynamics into account
- Can be done using graph theoretic concepts
- But not the usual ones…

Resista
ce
a.k.a. LRank

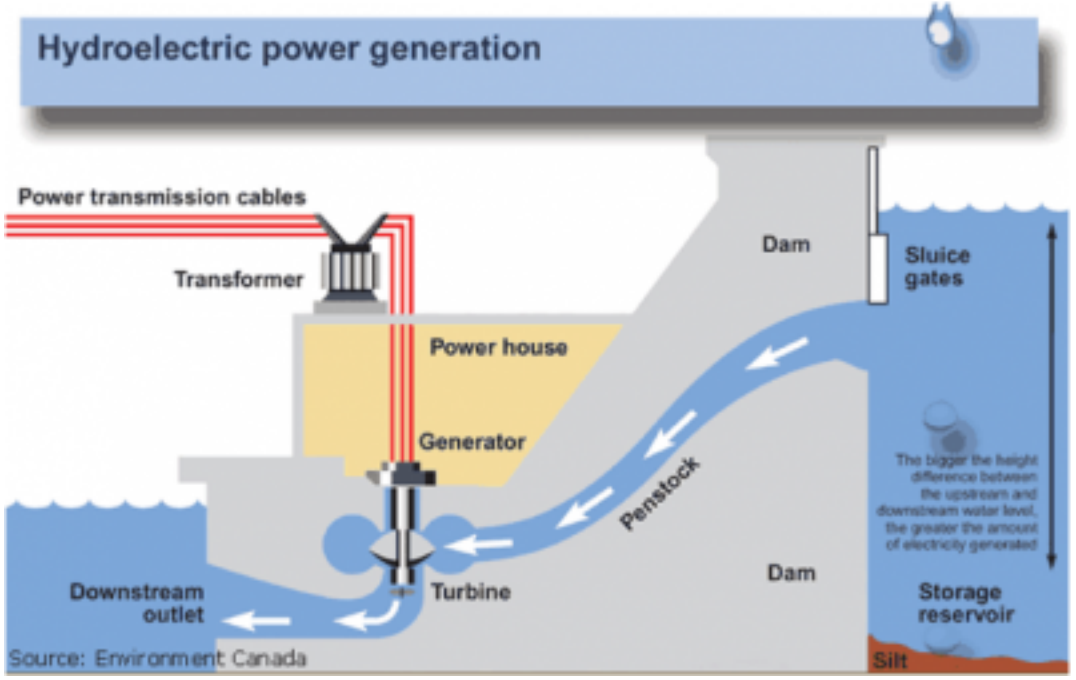
Numerically computed
performance measure

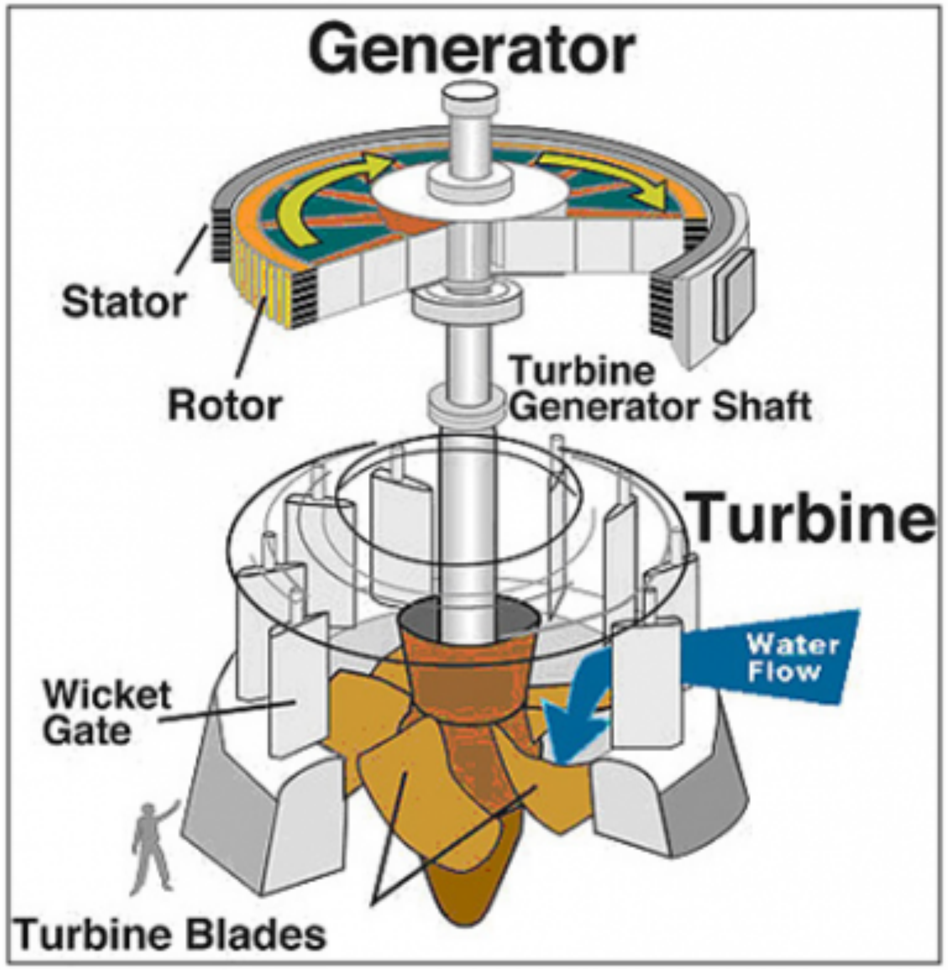


Tyloo, Pagnier and PJ : to be published (2018)

A bit of electric power engineering

- Electricity production with rotating machines
- Potential, chemical, nuclear or thermal energy converted into mechanical energy (rotation)
- Mechanical energy converted into electric energy
- Time-dependence : think power instead of energy
- Balance between power in, power out and energy change in rotator : **SWING EQUATIONS**





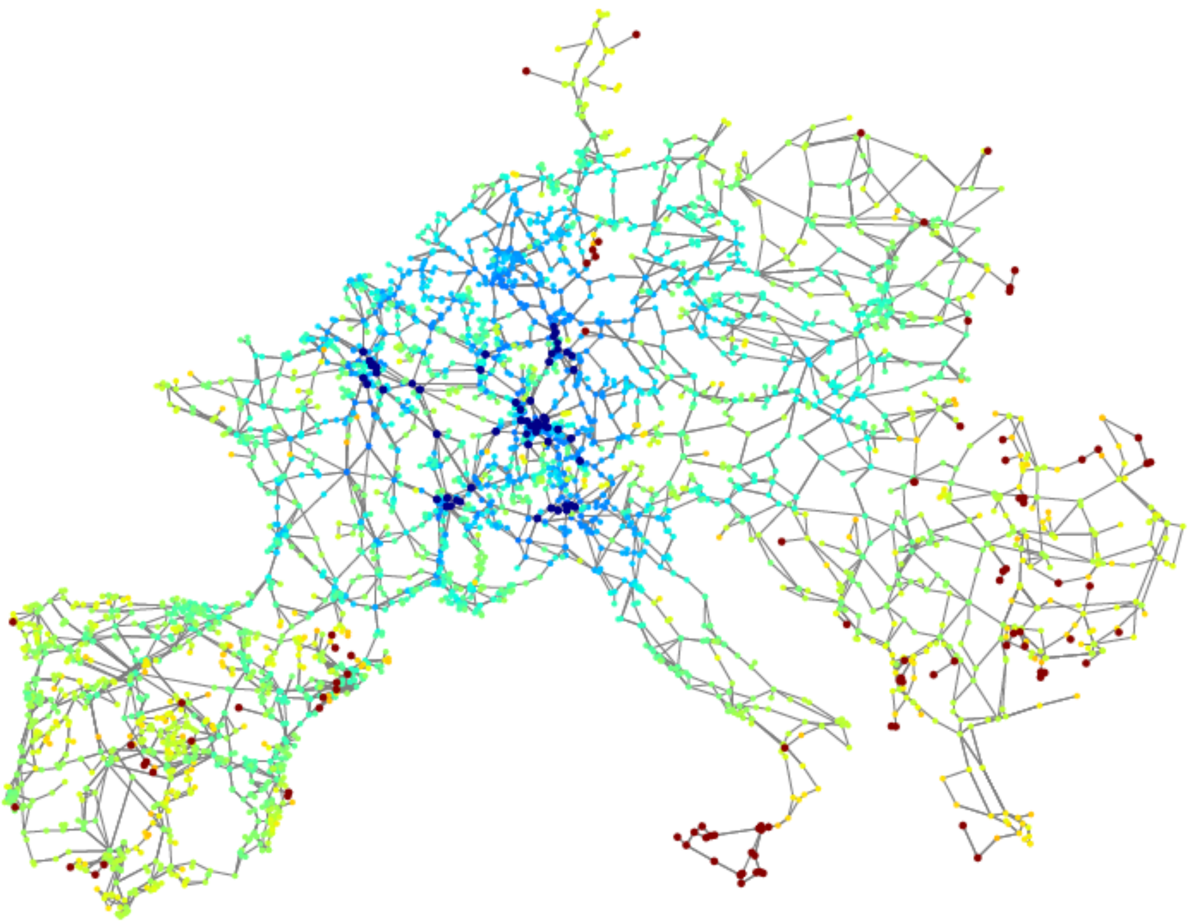
A bit of electric power engineering

Dynamics: swing Eqs. (neglect voltage variations from now on)

$$I_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum [B_{ij}\sin(\theta_i - \theta_j) + G_{ij}\cos(\theta_i - \theta_j)]$$

- i : node/bus index
- θ_i : voltage angle
- P>0 : production
- P<0 : consumption
- I : inertia ~ rot. kinetic energy
- D : damping ~ control
- Admittance : Y = g + i b;

  $G = g\,V_0$ $B = b\,V_0$



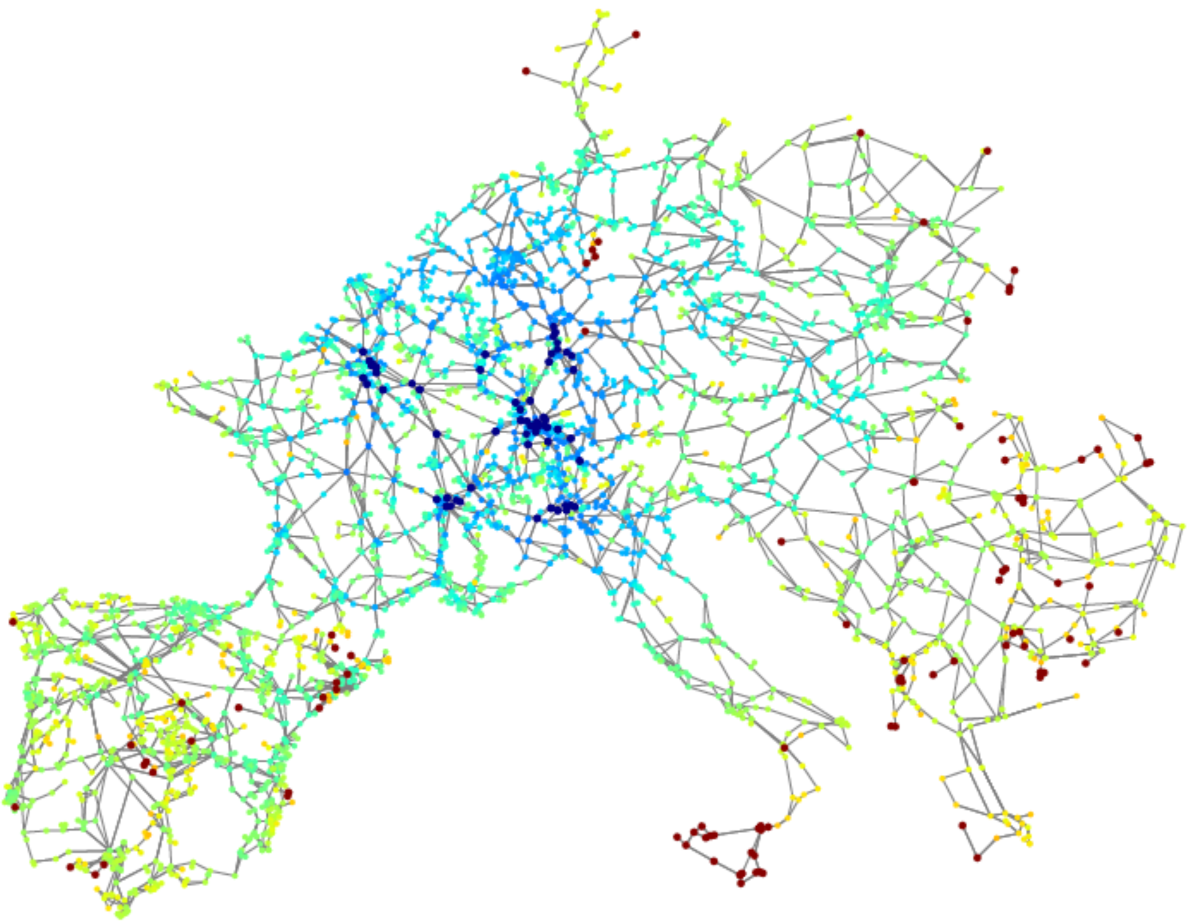
A bit of electric power engineering

Dynamics: swing Eqs. (neglect voltage variations from now on)

$$I_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum \left[B_{ij}\sin(\theta_i - \theta_j) + G_{ij}\cos(\theta_i - \theta_j)\right]$$

- i : node/bus index
- θ_i : voltage angle
- P>0 : production
- P<0 : consumption
- I : inertia ~ rot. kinetic energy
- D : damping ~ control
- Admittance : Y = g + i b;

  G=g V_0 B=b V_0



High to very high voltage approximation
G/B < 0.1 -> neglect G

$$I_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum B_{ij}\sin(\theta_i - \theta_j)$$

A bit of electric power engineering

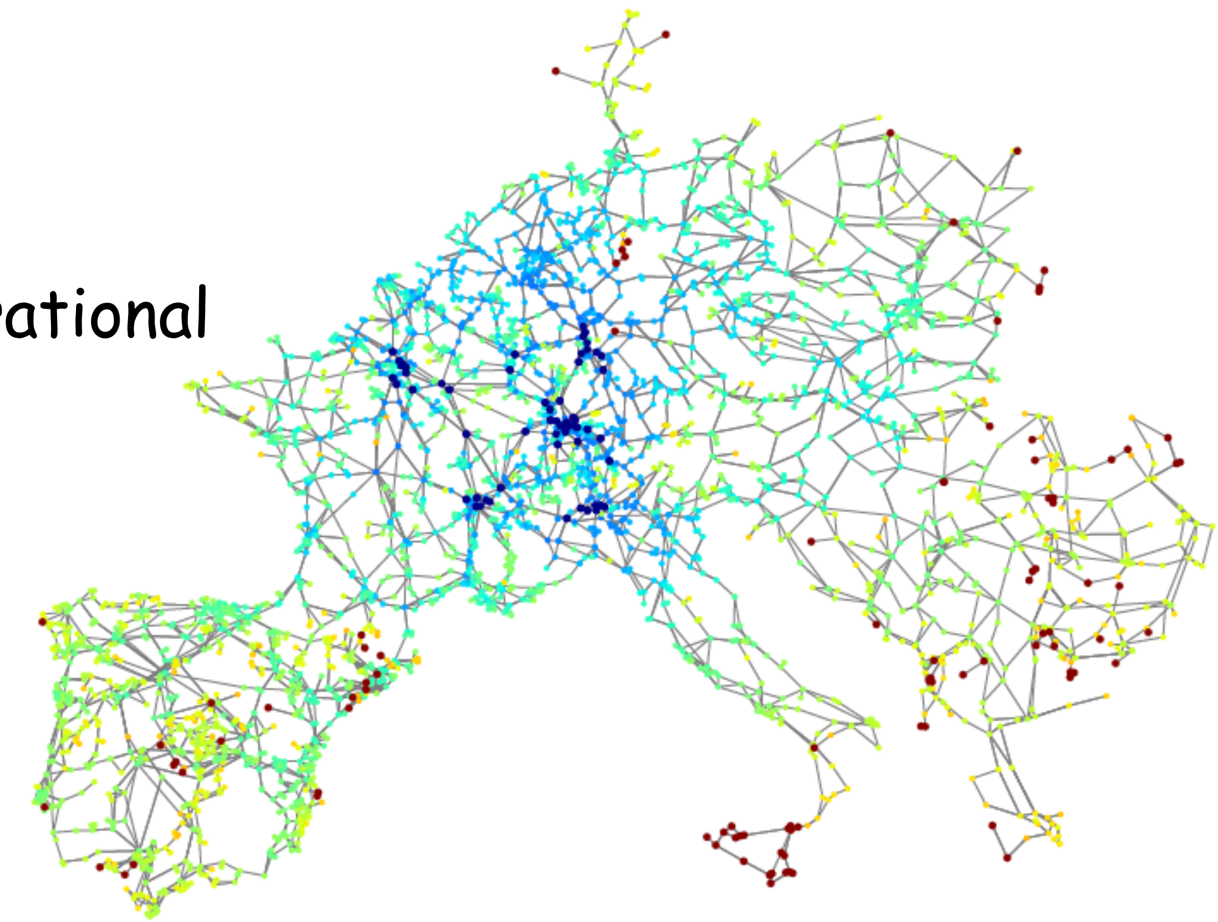
$$I_i \ddot{\theta} + D_i \dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j) \quad (*)$$

We are interested in

a) the synchronous fixed-points of (*) - operational states of the power grid

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

b) their stability - under specific or average disturbances



The program

$$I_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum B_{ij}\sin(\theta_i - \theta_j)$$

1) find fixed-point solutions
2) define perturbations (realistic or as a test source)
3) evaluate the transient induced by the perturbation
4) introduce a measure of performance based on the transient excursion

The program

$$I_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum B_{ij}\sin(\theta_i - \theta_j)$$

1) find fixed-point solutions $P_i = \sum_j B_{ij}\sin(\theta_i - \theta_j)$

2) define perturbations (realistic or as a test source)

3) evaluate the transient induced by the perturbation

4) introduce a measure of performance based on the transient excursion

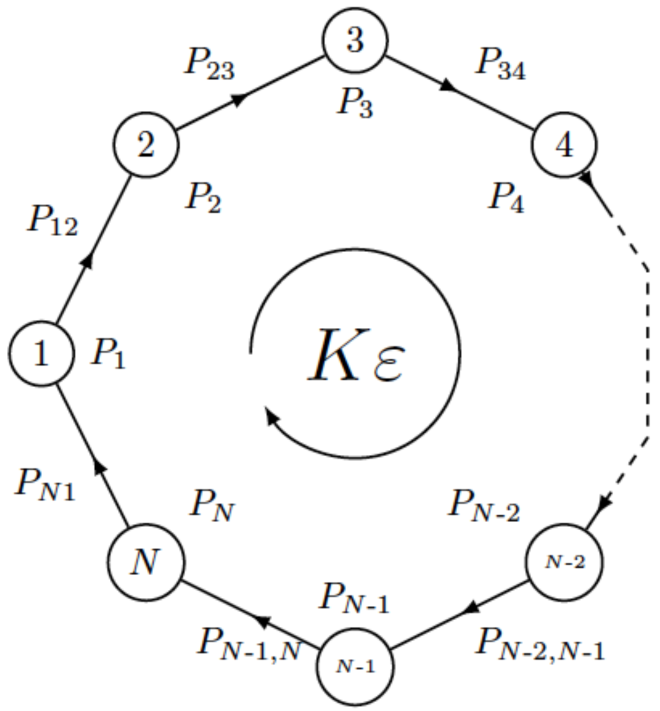
Circulating loop flows

*Thm: Different solutions to the following power-flow problem

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

may differ only by circulating loop current(s) in any network

Dörfler, Chertkov, Bullo, PNAS '13; Delabays, Coletta and PJ, JMP '16



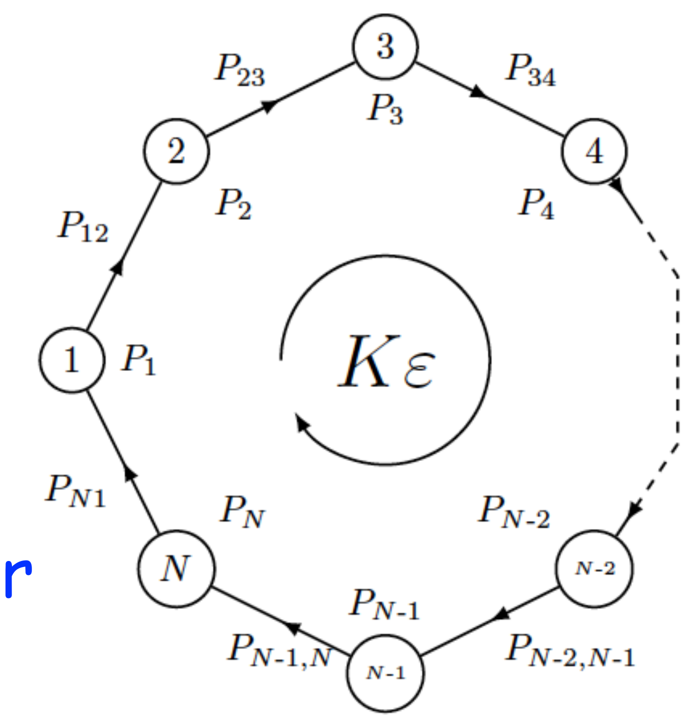
Circulating loop flows

*Thm: Different solutions to the following power-flow problem

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

may differ only by circulating loop current(s) in any network

Dörfler, Chertkov, Bullo, PNAS '13; Delabays, Coletta and PJ, JMP '16



*Voltage angle uniquely defined

→ $q = \sum_i |\theta_{i+1} - \theta_i|_{2\pi} / 2\pi \in \mathbb{Z}$ ~topological winding number

→ discretization of these loop currents ~vortex flows

Janssens and Kamagate '03

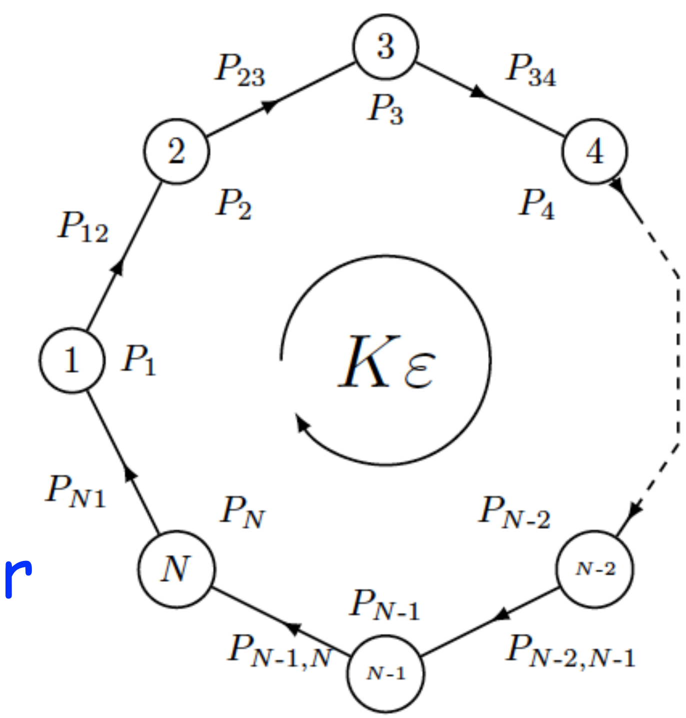
Circulating loop flows

*Thm: Different solutions to the following power-flow problem

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

may differ only by circulating loop current(s) in any network

Dörfler, Chertkov, Bullo, PNAS '13; Delabays, Coletta and PJ, JMP '16



*Voltage angle uniquely defined

→ $q = \sum_i |\theta_{i+1} - \theta_i|_{2\pi} / 2\pi \quad \in \mathbb{Z}$ ~topological winding number

→ discretization of these loop currents ~vortex flows

Janssens and Kamagate '03

→ number of stable solutions ~ number of possible vortex flows

Delabays, Coletta and PJ, JMP '16, JMP '17; Coletta, Delabays, Adagideli and PJ, NJP '16

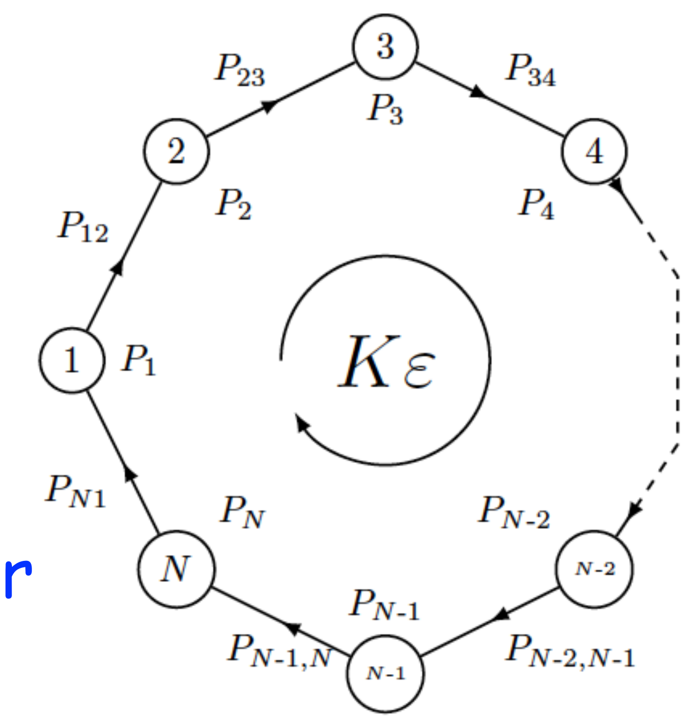
Circulating loop flows

*Thm: Different solutions to the following power-flow problem

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

may differ only by circulating loop current(s) in any network

Dörfler, Chertkov, Bullo, PNAS '13; Delabays, Coletta and PJ, JMP '16



*Voltage angle uniquely defined

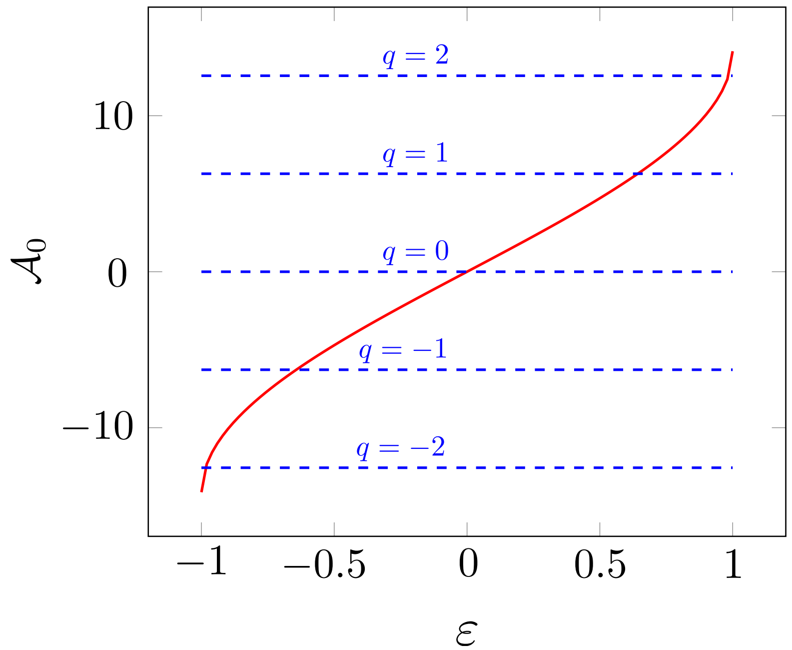
→ $q = \sum_i |\theta_{i+1} - \theta_i|_{2\pi} / 2\pi \quad \in \mathbb{Z}$ ~topological winding number

→ discretization of these loop currents ~vortex flows

Janssens and Kamagate '03

→ number of stable solutions ~ number of possible vortex flows

Delabays, Coletta and PJ, JMP '16, JMP '17; Coletta, Delabays, Adagideli and PJ, NJP '16



Single cycle

$$\mathcal{N} \leq 2\,\mathrm{Int}\,[n/4] + 1$$

Multi-cycle planar graph (conjecture)

$$\mathcal{N} \leq \prod_{k=1}^{c} \left[2 \cdot \mathrm{Int}\left((n_k + n_k')/4\right) + 1\right]$$

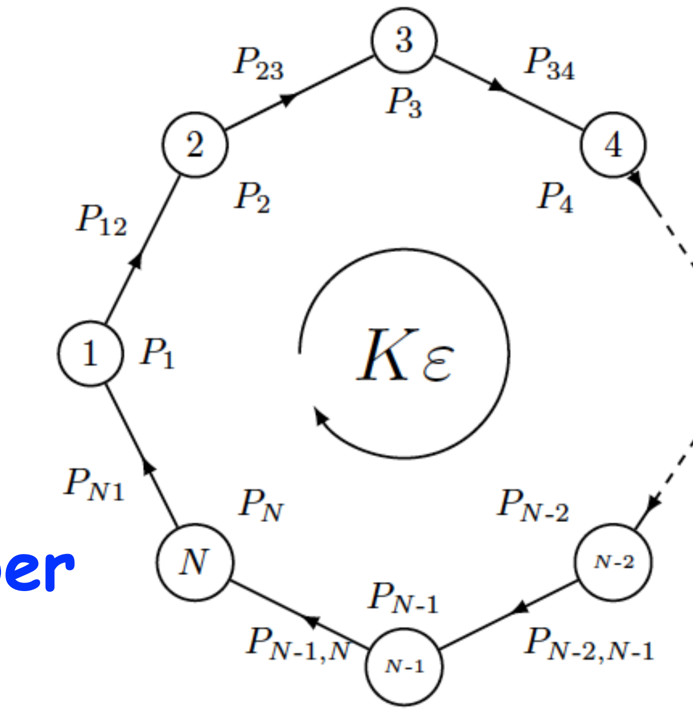
Circulating loop flows

*Thm: Different solutions to the following power-flow problem

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

may differ only by circulating loop current(s) **in any network**

Dörfler, Chertkov, Bullo, PNAS '13; Delabays, Coletta and PJ, JMP '16



*Voltage angle uniquely defined

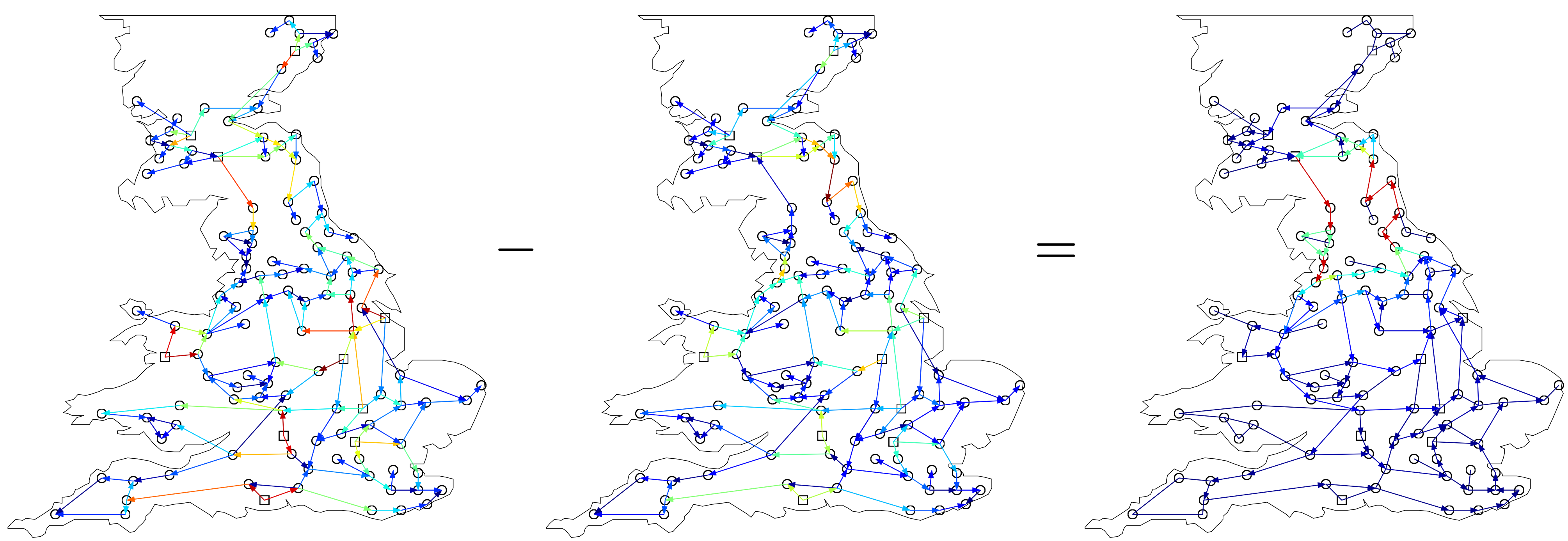
→ $q=\sum_i|\theta_{i+1}-\theta_i|_{2\pi}/2\pi \in \mathbb{Z}$ **~topological winding number**

→ discretization of these loop currents **~vortex flows**

Janssens and Kamagate '03

→ number of stable solutions ~ number of possible vortex flows

Delabays, Coletta and PJ, JMP '16, JMP '17; Coletta, Delabays, Adagideli and PJ, NJP '16



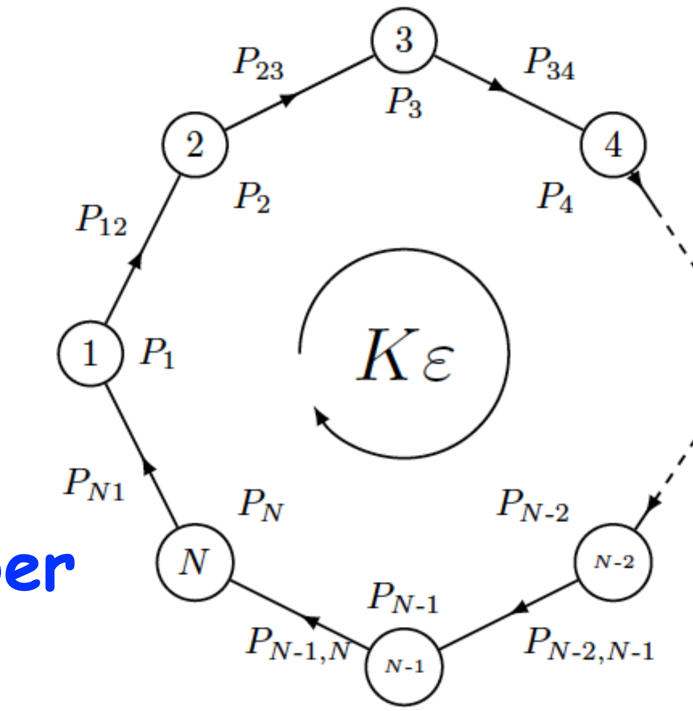
Circulating loop flows

*Thm: Different solutions to the following power-flow problem

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

may differ only by circulating loop current(s) **in any network**

Dörfler, Chertkov, Bullo, PNAS ’13; Delabays, Coletta and PJ, JMP ’16



*Voltage angle uniquely defined

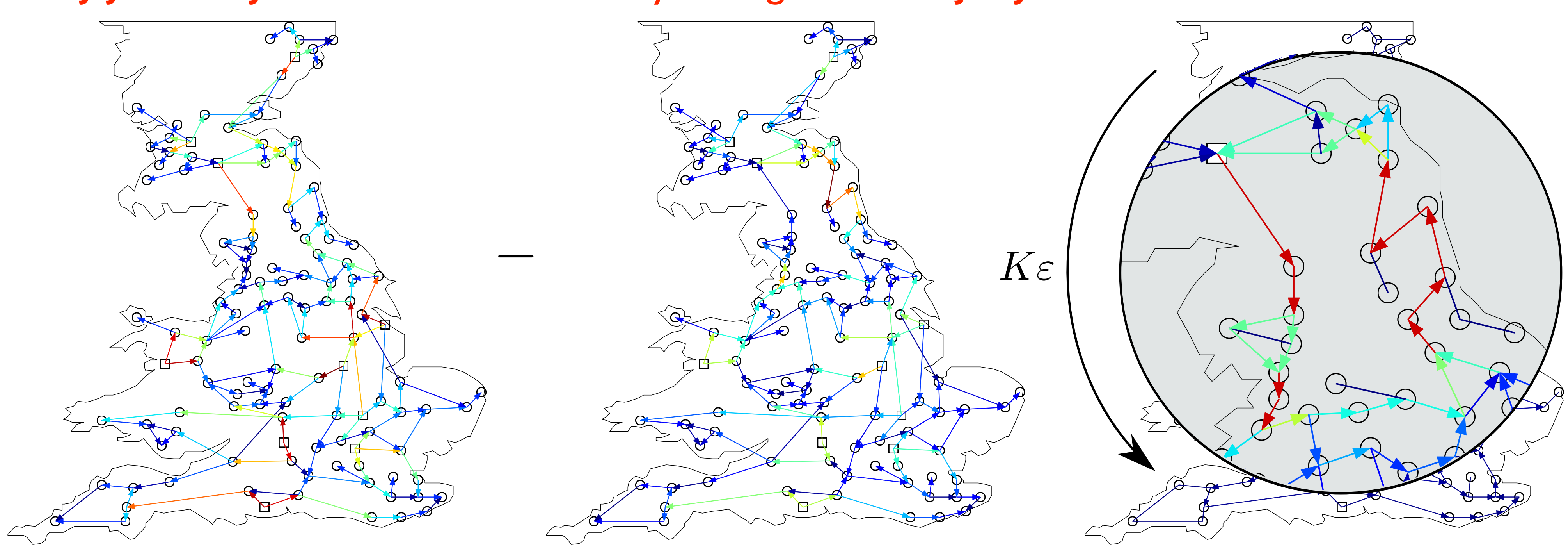
→ $q=\Sigma_i|\theta_{i+1}-\theta_i|_{2\pi}/2\pi \quad \in \mathbb{Z}$ **~topological winding number**

→ discretization of these loop currents **~vortex flows**

Janssens and Kamagate ‘03

→ number of stable solutions ~ number of possible vortex flows

Delabays, Coletta and PJ, JMP ’16, JMP ’17; Coletta, Delabays, Adagideli and PJ, NJP ‘16



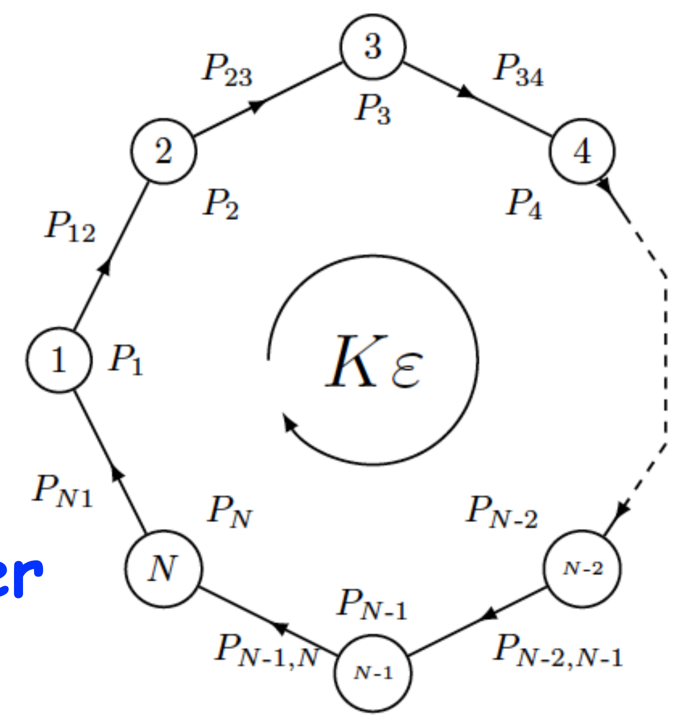
Circulating loop flows

*Thm: Different solutions to the following power-flow problem

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

may differ only by circulating loop current(s) **in any network**

Dörfler, Chertkov, Bullo, PNAS '13; Delabays, Coletta and PJ, JMP '16



*Voltage angle uniquely defined

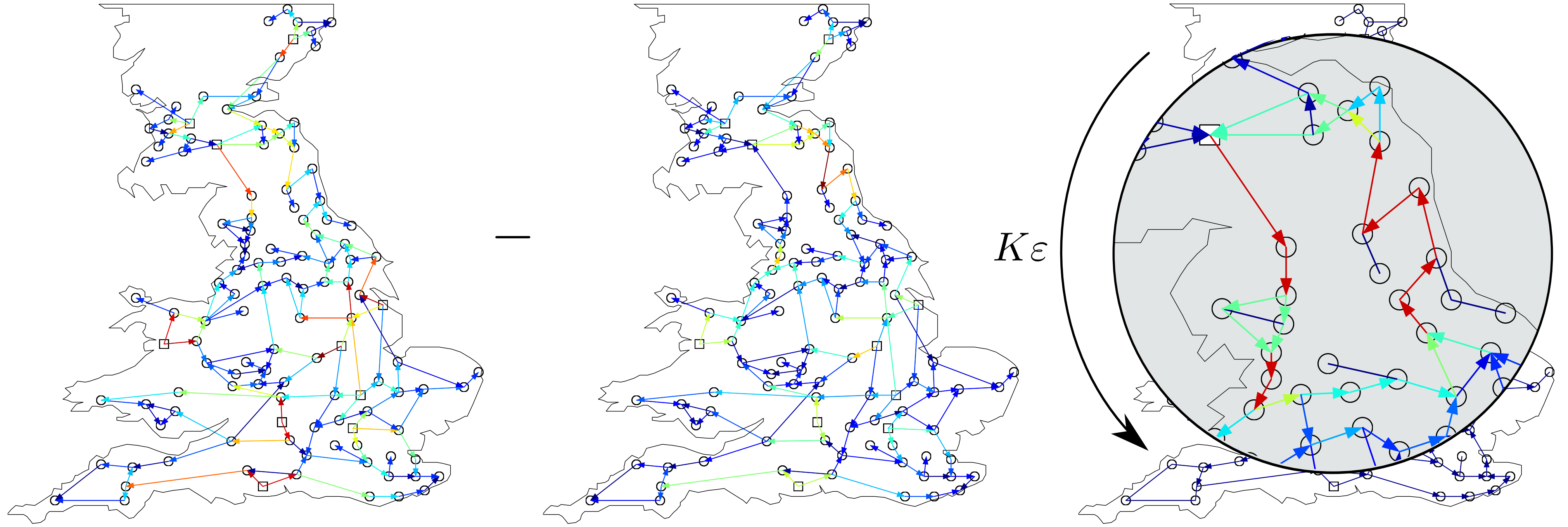
→ $q = \sum_i |\theta_{i+1} - \theta_i|_{2\pi} / 2\pi \quad \in \mathbb{Z}$ **~topological winding number**

→ discretization of these loop currents **~vortex flows**

Janssens and Kamagate '03

→ number of stable solutions ~ number of possible vortex flows

Delabays, Coletta and PJ, JMP '16, JMP '17; Coletta, Delabays, Adagideli and PJ, NJP '16



Numerically finding circulating loop flows

Algorithm to determine different solutions numerically

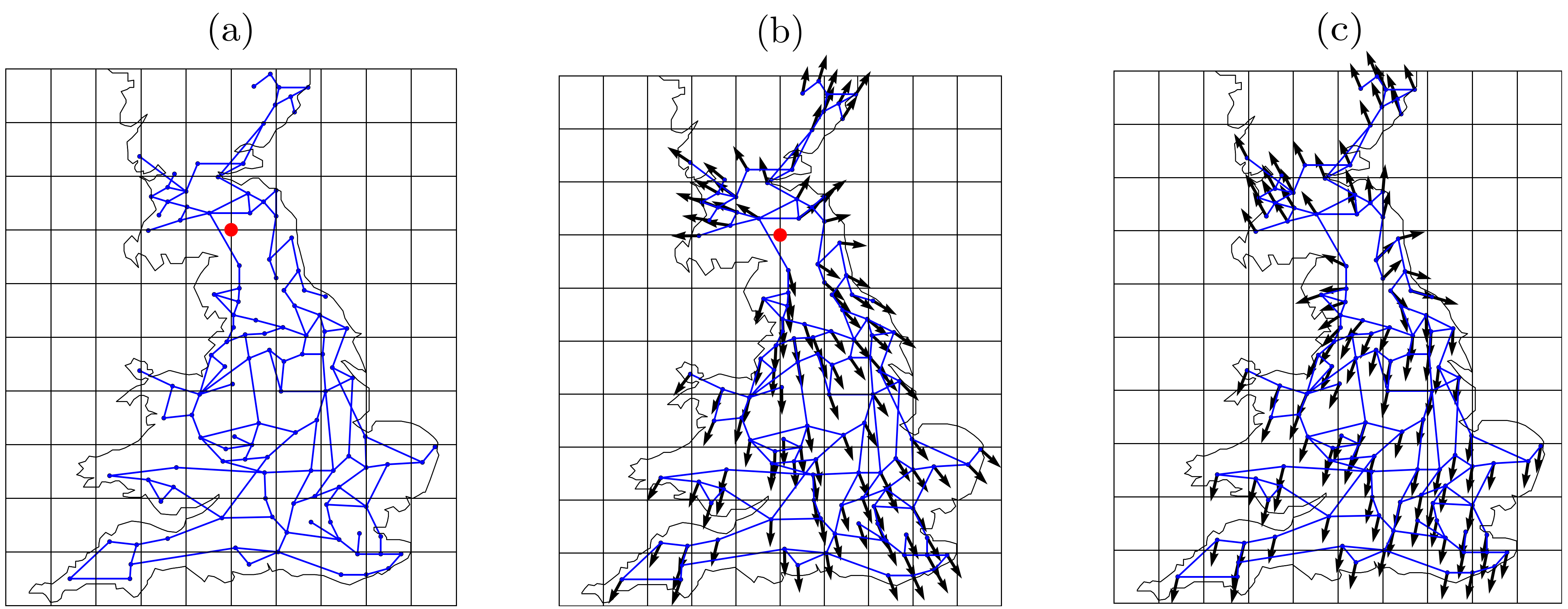
(i) use iterative method (Runge-Kutta on swing; Newton-Raphson on power flow…)

(ii) construct vortex-carrying initial state

$$\theta_i = q \arctan\left(\frac{y_i - y_0}{x_i - x_0}\right)$$

(iii) iterate and see where it converges

(iv) pick another vortex-carrying initial state



Delabays, Tyloo and PJ, Chaos '17

The program

$$I_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum B_{ij}\sin(\theta_i - \theta_j)$$

1) find fixed-point solutions

2) define perturbations (realistic or as a test source)

3) evaluate the transient induced by the perturbation

4) introduce a measure of performance based on the transient excursion

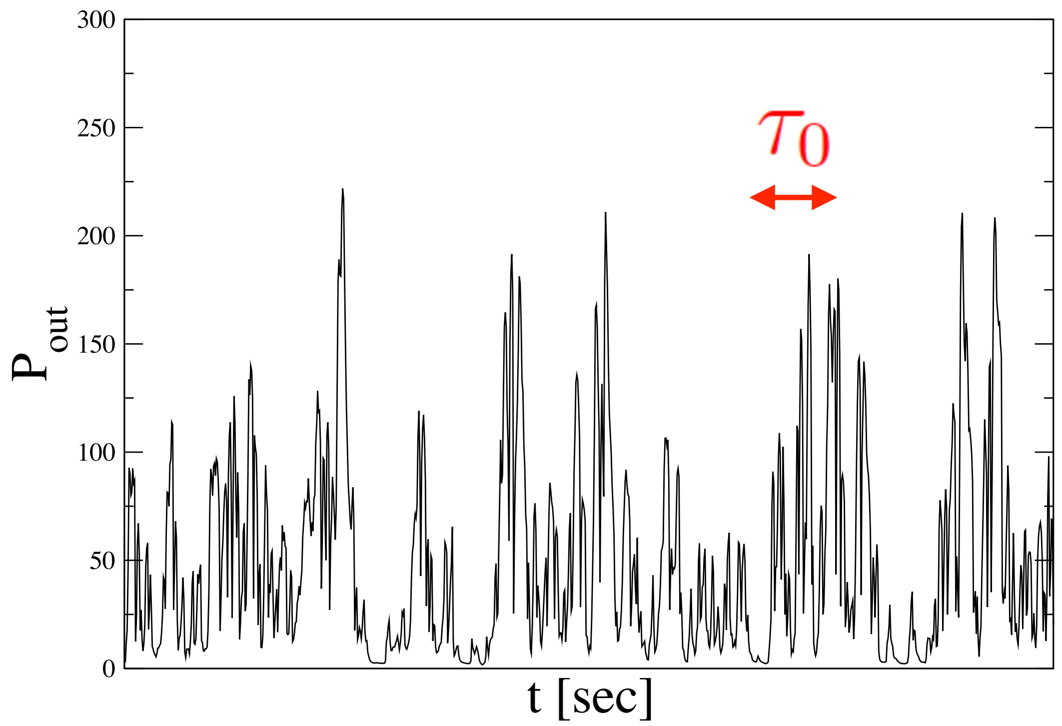
Perturbation #1 : Nodal noise

$$I_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum B_{ij}\sin(\theta_i - \theta_j)$$

$$P_i(t) = P_i^{(0)} + \delta P_i(t)$$

$$\langle \delta P_i(t) \rangle = 0$$

$$\langle \delta P_i(t_1)\delta P_j(t_2) \rangle = \delta P_0^2 \delta_{ij} e^{-|t_1 - t_2|/\tau_0}$$



- No spatial correlation
- Characteristic time τ_0

Tyloo, Coletta and PJ, PRL '18; Tyloo, Pagnier and PJ, in preparation

Perturbation #2 : line fault

$$I_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum B_{ij}\sin(\theta_i - \theta_j)$$

1) linearize the dynamics

$$I_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum L_{ij}\theta_j \qquad L_{ij} = \begin{cases} -B_{ij} & \text{if } \; j \neq i \\ \sum_k B_{ik} & \text{if } \; j = i \end{cases}$$

2) rank-1 perturbation of the Laplacian matrix L

$$\boldsymbol{L} \rightarrow \boldsymbol{L} - \delta(t)\tau B_{\alpha\beta}\, \boldsymbol{e}_{(\alpha,\beta)}\, \boldsymbol{e}_{(\alpha,\beta)}^{\top} \qquad \boldsymbol{e}_{(\alpha,\beta)} = (0, 0, ..., \underset{\substack{\uparrow \\ \alpha}}{1}, 0, ..., \underset{\substack{\uparrow \\ \beta}}{-1}, ...0)$$

Coletta and PJ, submitted to IEEE TCNS '18

The program

$$I_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum B_{ij}\sin(\theta_i - \theta_j)$$

1) find fixed-point solutions

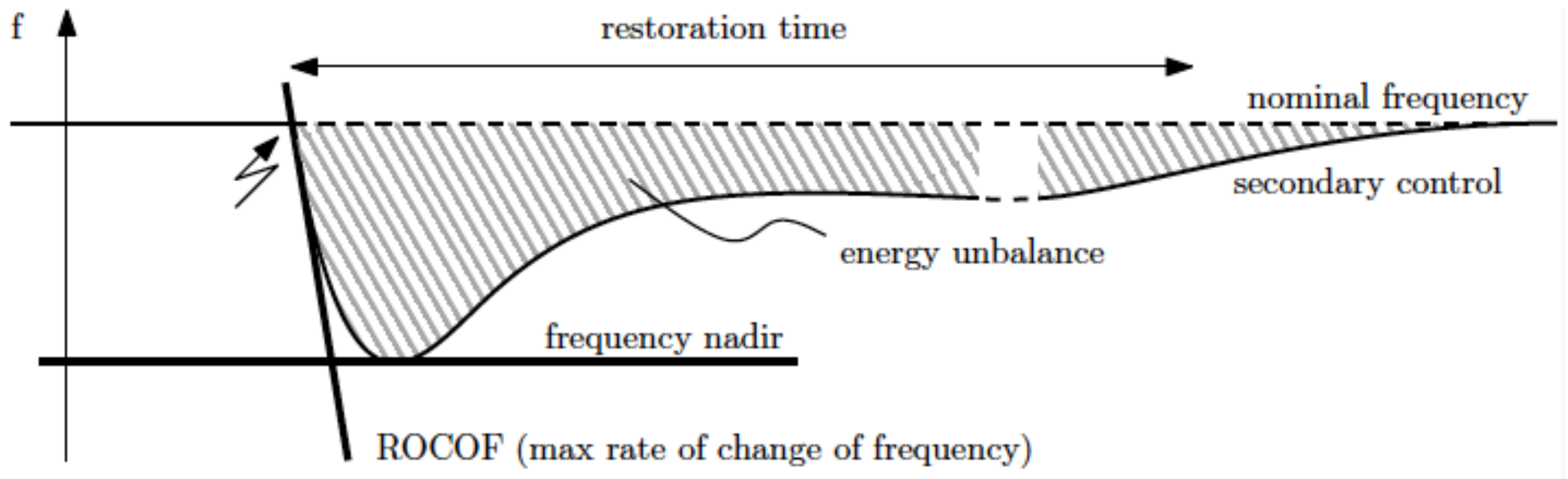
2) define perturbations (realistic or as a test source)

3) evaluate the transient induced by the perturbation

4) introduce a measure of performance based on the transient excursion

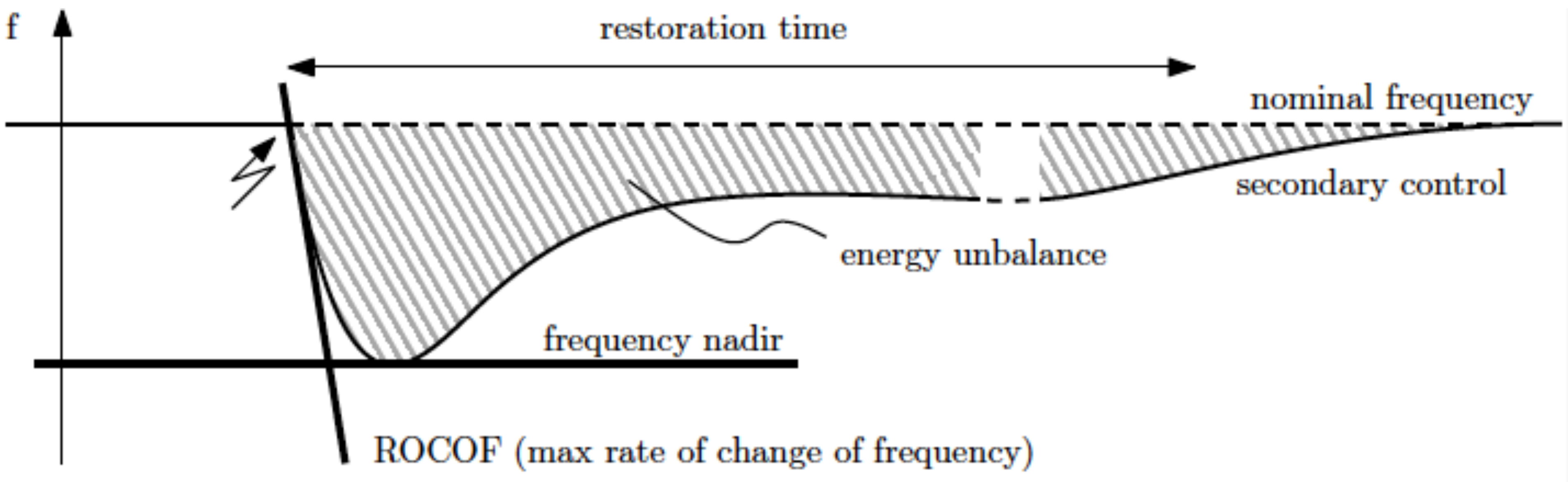
The program

$$I_i \ddot{\theta} + D_i \dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j)$$



The program

$$I_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum B_{ij}\sin(\theta_i - \theta_j)$$



$$\mathcal{P}_1(T) = \int_0^T \mathrm{d}t\, \delta\boldsymbol{\theta}^2(t)$$

$$\mathcal{P}_2(T) = \int_0^T \mathrm{d}t\, \delta\dot{\boldsymbol{\theta}}^2(t)$$

Take limit $T \to \infty$ whenever possible

Power grid with fluctuating feed-in

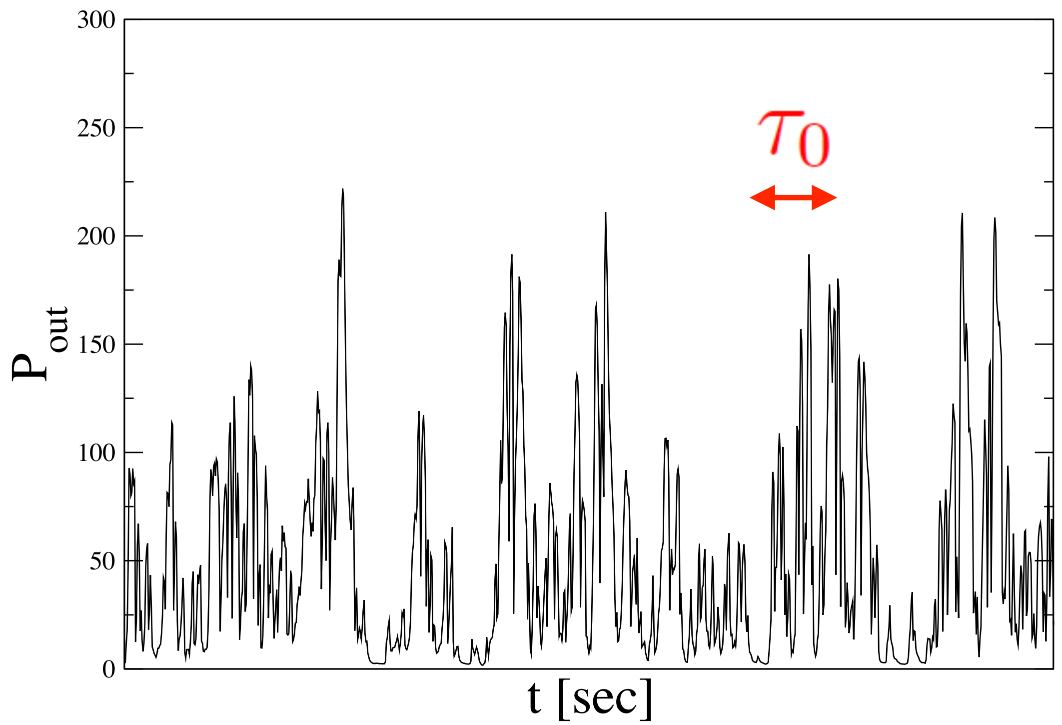
Angle dynamics (simplified; stay tuned)

$$\dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j)$$

$$P_i(t) = P_i^{(0)} + \delta P_i(t)$$

$$\theta_i(t) = \theta_i^{(0)} + \delta \theta_i(t)$$

$$P_i^{(0)} = \sum B_{ij} \sin(\theta_i^{(0)} - \theta_j^{(0)})$$

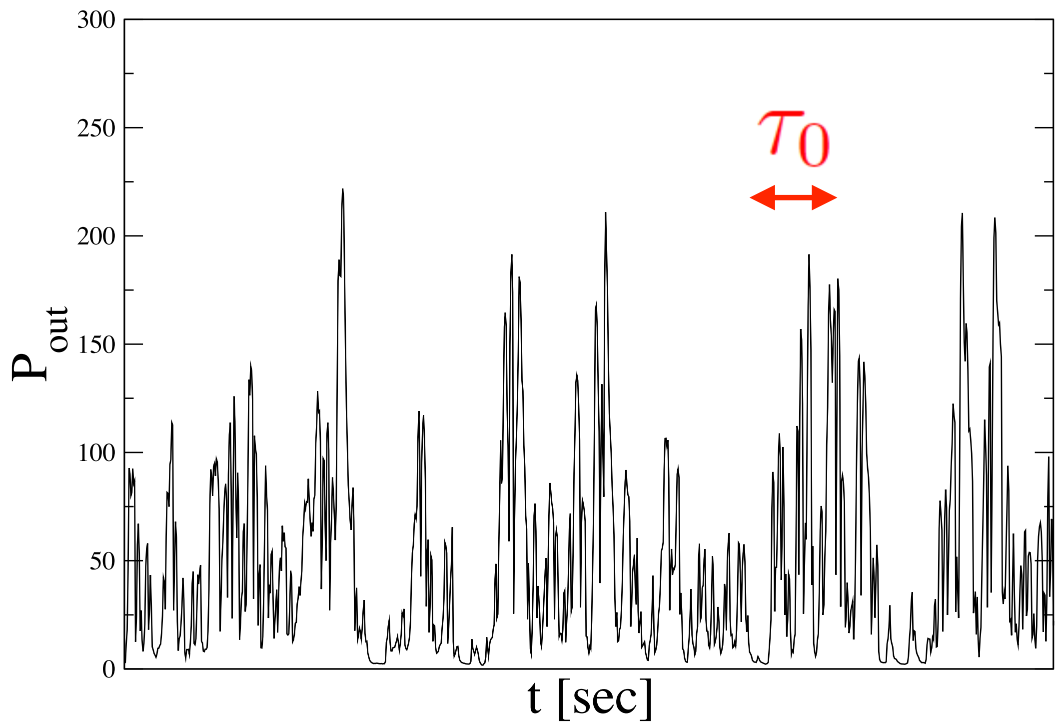


Power grid with fluctuating feed-in

Angle dynamics (simplified; stay tuned)

$$\dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j)$$

$$P_i(t) = P_i^{(0)} + \delta P_i(t)$$
$$\theta_i(t) = \theta_i^{(0)} + \delta\theta_i(t)$$
$$P_i^{(0)} = \sum B_{ij} \sin(\theta_i^{(0)} - \theta_j^{(0)})$$



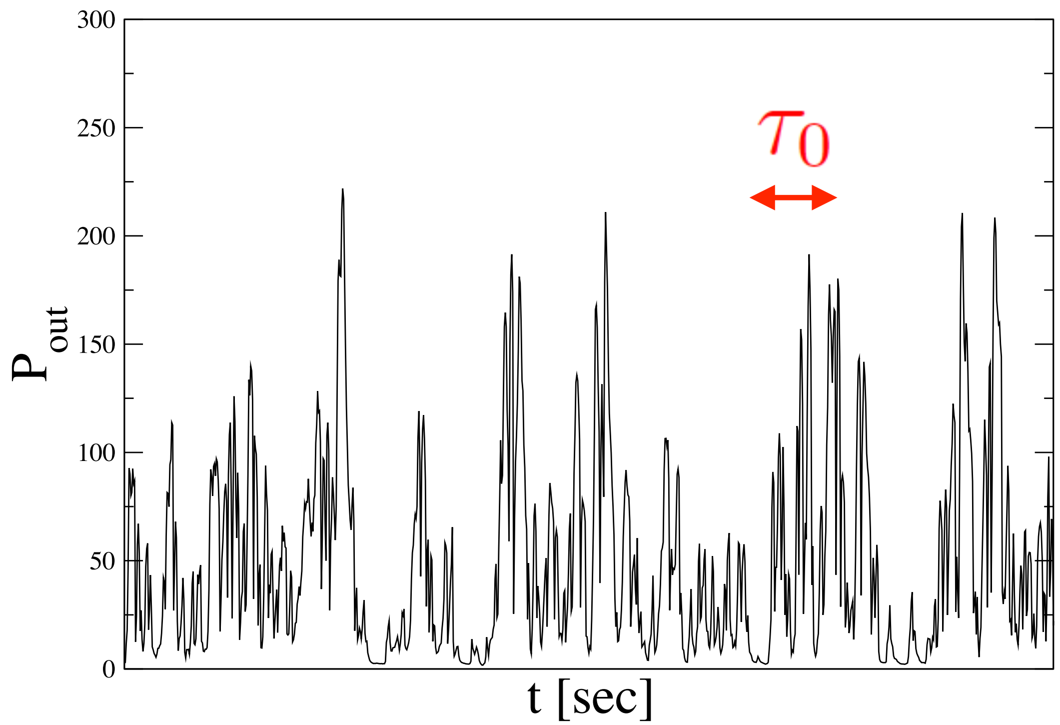
Can one characterize $\delta\theta_i(t)$ given $\delta P_i(t)$?

Power grid with fluctuating feed-in

Angle dynamics (simplified; stay tuned)

$$\dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j)$$

$$P_i(t) = P_i^{(0)} + \delta P_i(t)$$
$$\theta_i(t) = \theta_i^{(0)} + \delta\theta_i(t)$$
$$P_i^{(0)} = \sum B_{ij} \sin(\theta_i^{(0)} - \theta_j^{(0)})$$



Can one characterize $\delta\theta_i(t)$ given $\delta P_i(t)$?

A: (i) linearize the dynamics about a fixed-point solution

$$\delta\dot{\vec{\theta}} = \delta\vec{P} + \mathcal{M}\delta\vec{\theta}$$

(ii) expand angles over eigenmodes of stability matrix

$$\mathcal{M}\vec{\phi}_\beta = \lambda_\beta\vec{\phi}_\beta$$

→ *get equation for coefficients of expansion !*

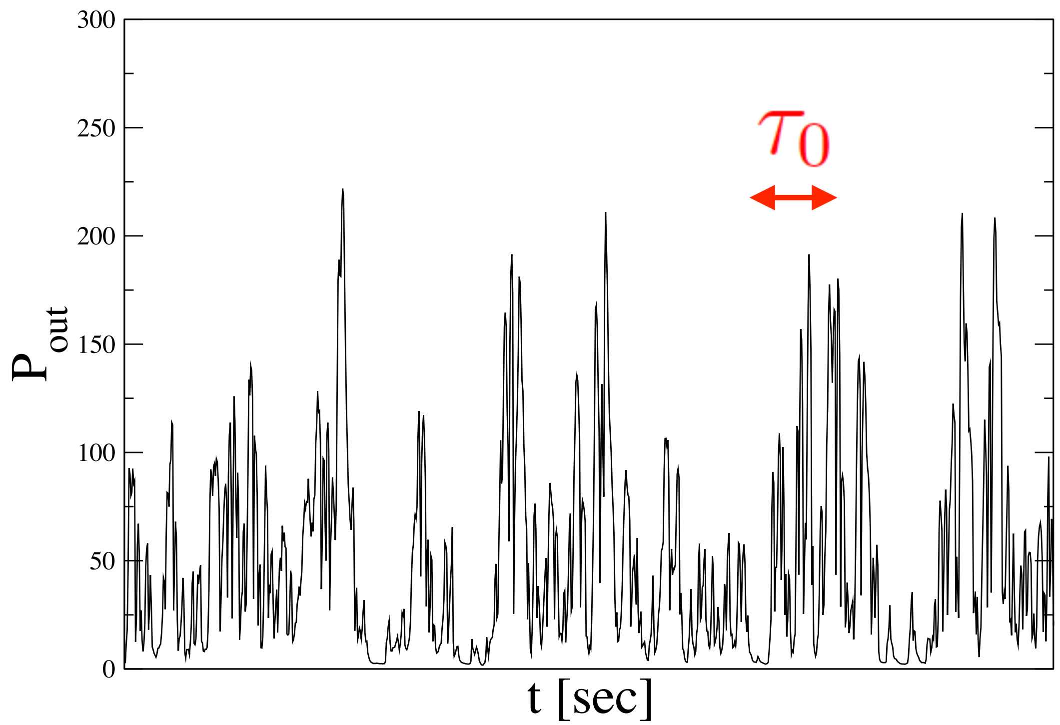
$$\delta\vec{\theta}(t) = \sum_\beta c_\beta(t)\vec{\phi}_\beta$$

Power grid with fluctuating feed-in

Angle dynamics (simplified; stay tuned)

$$\dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j)$$

$$\boxed{\begin{aligned} P_i(t) &= P_i^{(0)} + \delta P_i(t) \\ \theta_i(t) &= \theta_i^{(0)} + \delta\theta_i(t) \\ P_i^{(0)} &= \sum B_{ij} \sin(\theta_i^{(0)} - \theta_j^{(0)}) \end{aligned}}$$



Can one characterize $\delta\theta_i(t)$ given $\delta P_i(t)$?

A: (i) linearize the dynamics about a fixed-point solution $\quad \delta\dot{\vec{\theta}} = \delta\vec{P} + \mathcal{M}\delta\vec{\theta}$

(ii) expand angles over eigenmodes of stability matrix $\quad \mathcal{M}\vec{\phi}_\beta = \lambda_\beta\vec{\phi}_\beta$

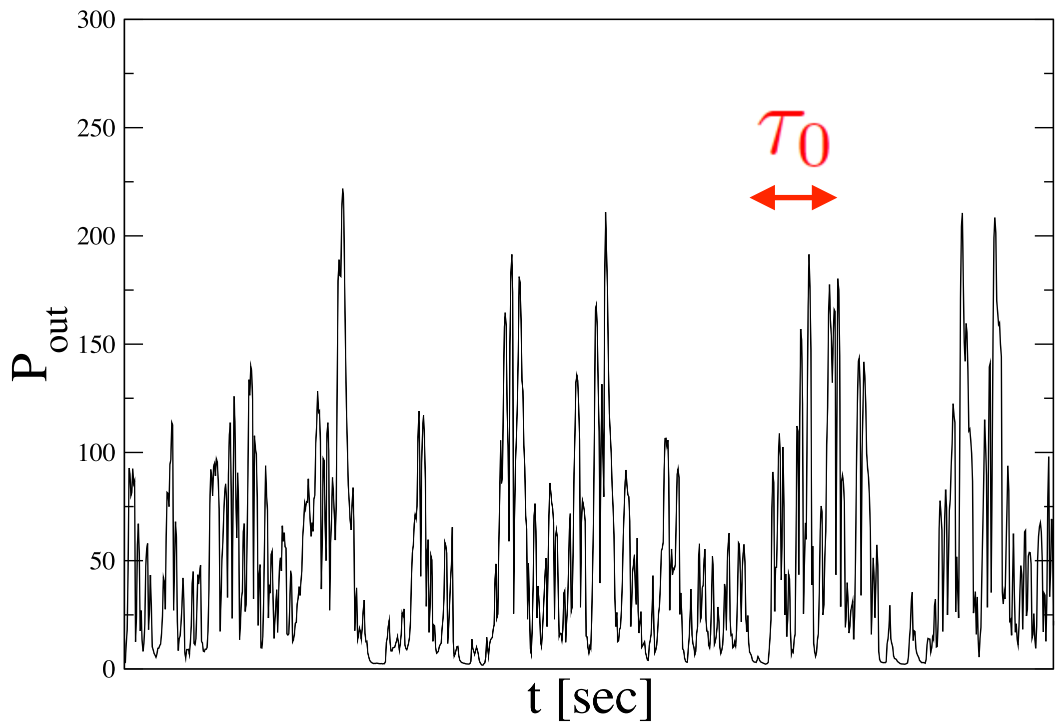
→ *get equation for coefficients of expansion !* $\quad \delta\vec{\theta}(t) = \sum_\beta c_\beta(t)\vec{\phi}_\beta$

Power grid with fluctuating feed-in

Angle dynamics (simplified; stay tuned)

$$\dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j)$$

$$P_i(t) = P_i^{(0)} + \delta P_i(t)$$
$$\theta_i(t) = \theta_i^{(0)} + \delta\theta_i(t)$$
$$P_i^{(0)} = \sum B_{ij} \sin(\theta_i^{(0)} - \theta_j^{(0)})$$



Can one characterize $\delta\theta_i(t)$ given $\delta P_i(t)$?

A: (i) linearize the dynamics about a fixed-point solution

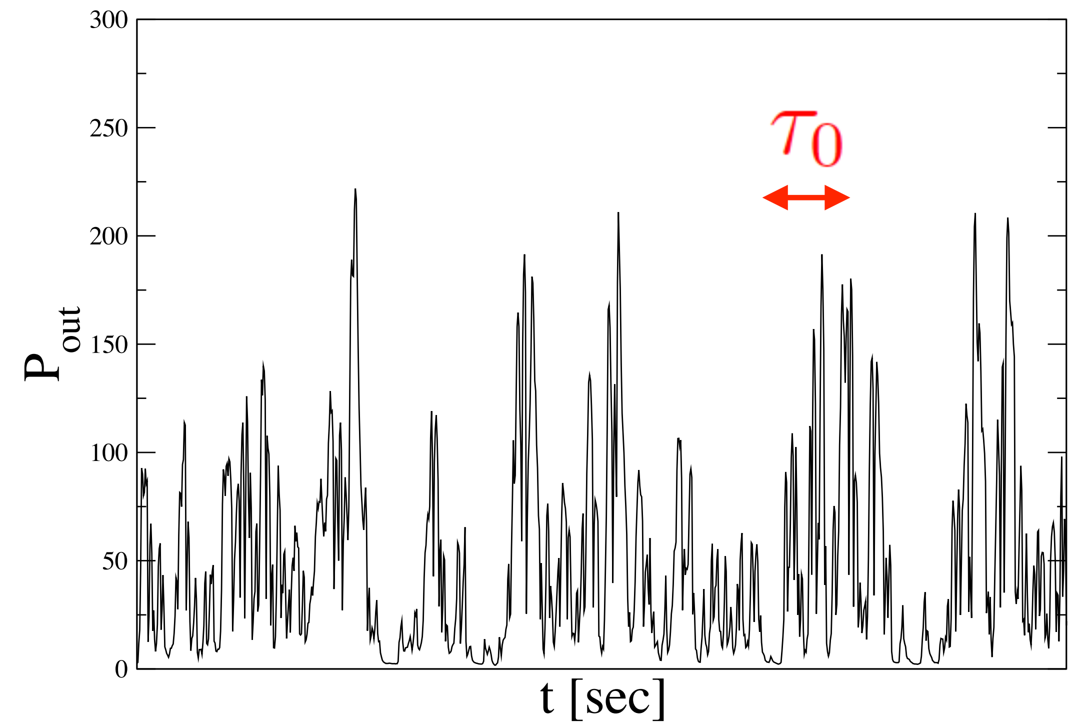
$$\rightarrow \quad \delta\dot{\theta}_i = \delta P_i - \sum B_{ij} \cos(\theta_i^{(0)} - \theta_j^{(0)}) \, (\delta\theta_i - \delta\theta_j)$$

Power grid with fluctuating feed-in

Angle dynamics (simplified; stay tuned)

$$\dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j)$$

$$\boxed{\begin{aligned} P_i(t) &= P_i^{(0)} + \delta P_i(t) \\ \theta_i(t) &= \theta_i^{(0)} + \delta\theta_i(t) \\ P_i^{(0)} &= \sum B_{ij} \sin(\theta_i^{(0)} - \theta_j^{(0)}) \end{aligned}}$$



equation for coefficients of expansion $\delta\vec{\theta}(t) = \sum_{\beta} c_\beta(t)\vec{\phi}_\beta$

Langevin equation : $$\dot{c}_\alpha(t) = \lambda_\alpha c_\alpha(t) + \delta\vec{P}(t) \cdot \vec{\phi}_\alpha$$

gives exponential decay of deviation (usual)

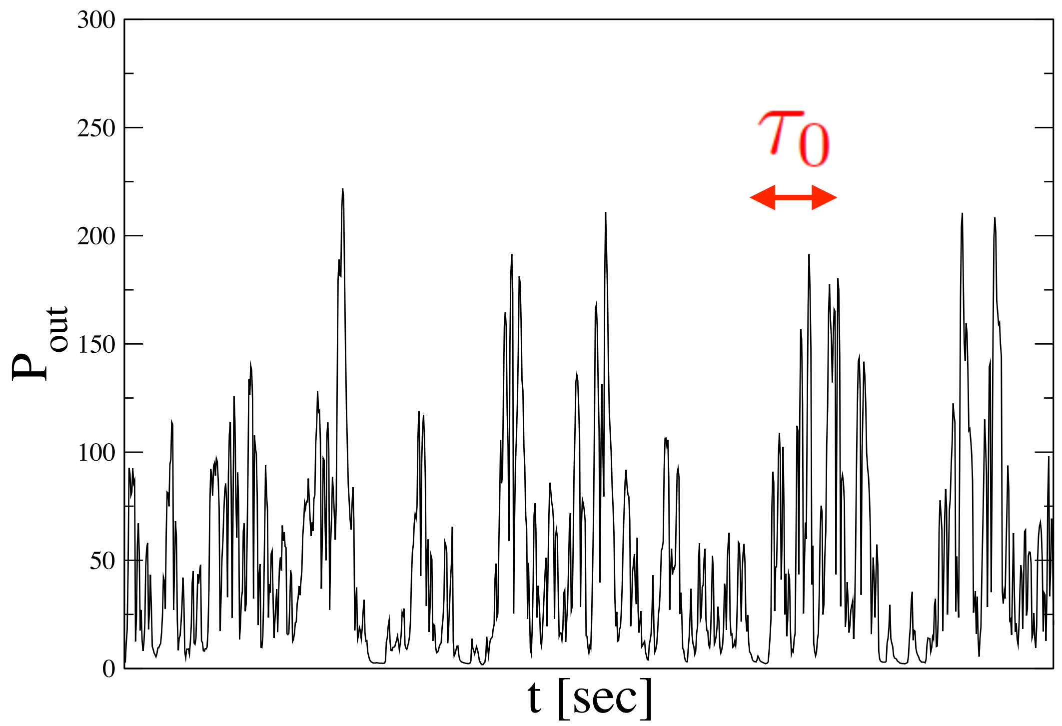
fluctuations about exponential decay

Power grid with fluctuating feed-in

Angle dynamics (simplified; stay tuned)

$$\dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j)$$

$$\boxed{\begin{aligned} P_i(t) &= P_i^{(0)} + \delta P_i(t) \\ \theta_i(t) &= \theta_i^{(0)} + \delta\theta_i(t) \\ P_i^{(0)} &= \sum B_{ij} \sin(\theta_i^{(0)} - \theta_j^{(0)}) \end{aligned}}$$



equation for coefficients of expansion $\delta\vec{\theta}(t) = \sum_\beta c_\beta(t)\vec{\phi}_\beta$

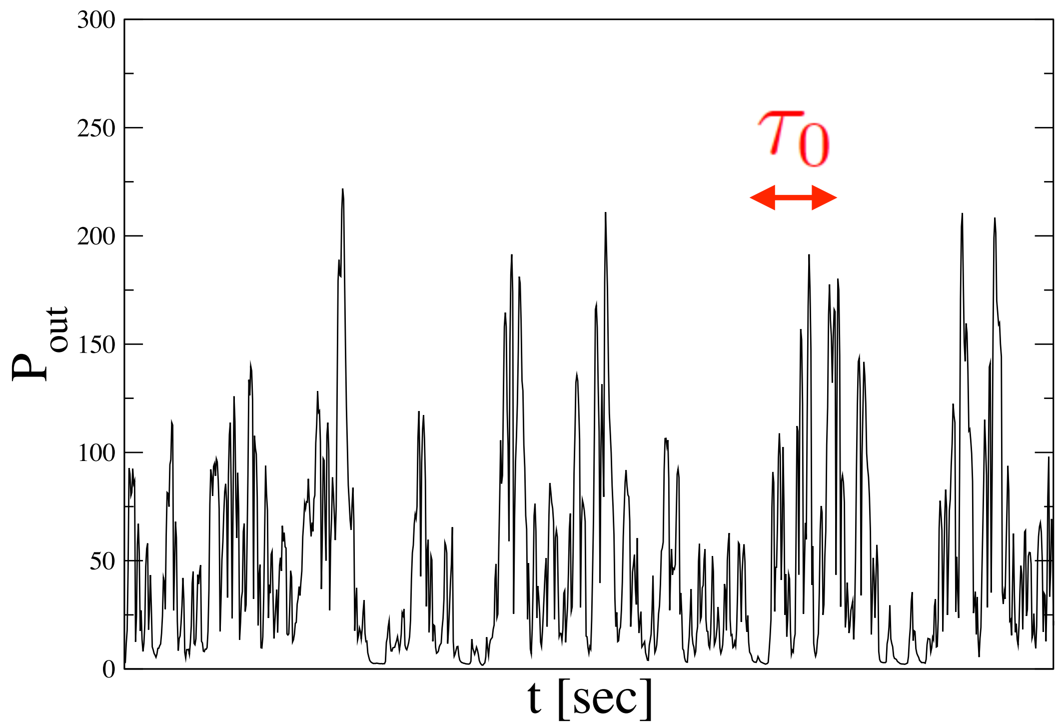
$$\boxed{c_\alpha(t) = e^{\lambda_\alpha t} c_\alpha(0) + e^{\lambda_\alpha t} \int_0^t e^{-\lambda_\alpha t'}\, \delta\vec{P}(t') \cdot \vec{\phi}_\alpha \mathrm{d}t'}$$

Power grid with fluctuating feed-in

Angle dynamics (simplified; stay tuned)

$$\dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j)$$

$$\boxed{\begin{aligned} P_i(t) &= P_i^{(0)} + \delta P_i(t) \\ \theta_i(t) &= \theta_i^{(0)} + \delta\theta_i(t) \\ P_i^{(0)} &= \sum B_{ij} \sin(\theta_i^{(0)} - \theta_j^{(0)}) \end{aligned}}$$



Example of fluctuations $\langle \delta P_i(t) \rangle = 0$ $\langle \delta P_i(t_1) \delta P_j(t_2) \rangle = \delta P_0^2 \delta_{ij} e^{-|t_1 - t_2|/\tau_0}$

- No spatial correlation
- Characteristic time τ_0

$$\longrightarrow \langle c_\alpha(t) \rangle = 0$$

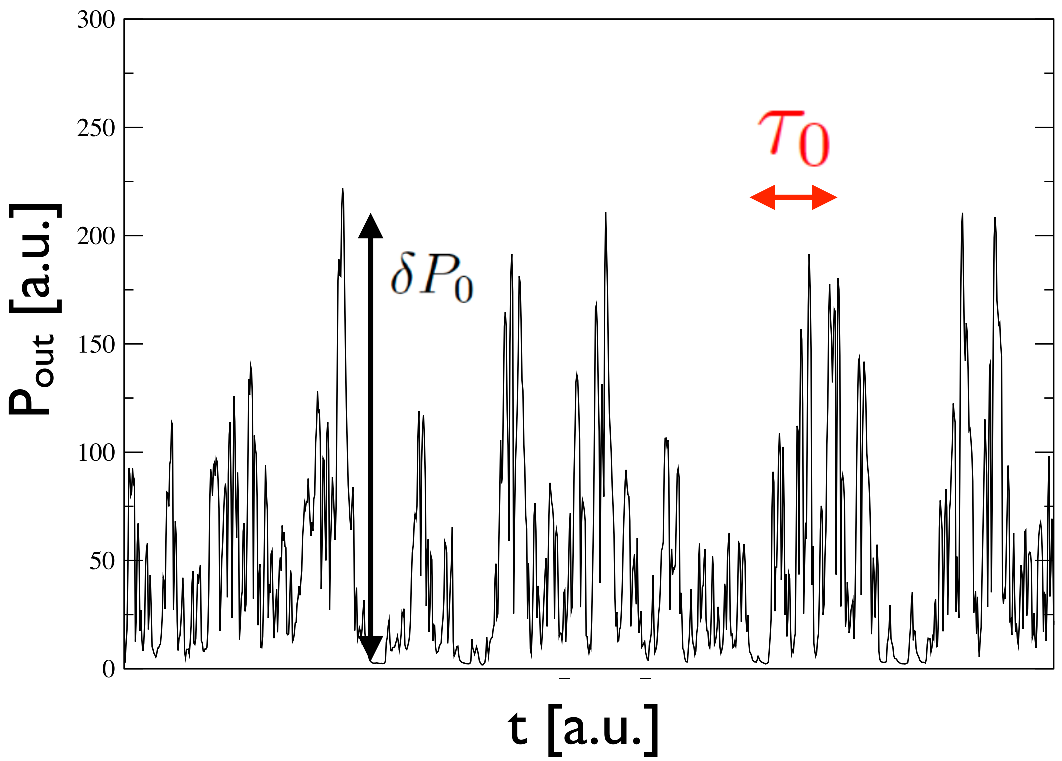
$$\longrightarrow \langle c_\alpha^2(t) \rangle = e^{2\lambda_\alpha t} \iint_0^t e^{-\lambda_\alpha(t_1+t_2)} \langle (\delta\vec{P}(t_1) \cdot \vec{\phi}_\alpha)(\delta\vec{P}(t_2) \cdot \vec{\phi}_\alpha) \rangle \mathrm{d}t_1 \mathrm{d}t_2$$

Power grid with fluctuating feed-in

Angle dynamics (simplified; stay tuned)

$$\dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j)$$

$$\boxed{\begin{aligned} P_i(t) &= P_i^{(0)} + \delta P_i(t) \\ \theta_i(t) &= \theta_i^{(0)} + \delta\theta_i(t) \\ P_i^{(0)} &= \sum B_{ij} \sin(\theta_i^{(0)} - \theta_j^{(0)}) \end{aligned}}$$



Example of fluctuations

$$\langle \delta P_i(t) \rangle = 0$$

$$\langle \delta P_i(t_1) \delta P_j(t_2) \rangle = \delta P_0^2 \delta_{ij} e^{-|t_1 - t_2|/\tau_0}$$

- No spatial correlation
- Characteristic time τ_0

$$\boxed{\begin{aligned} &\langle \delta\theta_i(t) \rangle = 0 \\ &\lim_{t\to\infty} \langle \delta\boldsymbol{\theta}^2(t) \rangle = \delta P_0^2\, \tau_0 \sum_\alpha \frac{\sum_{\text{noisy } i} |\phi_{i,\alpha}|^2}{\lambda_\alpha(\lambda_\alpha \tau_0 - 1)} \\ &\lim_{t \ll \lambda_\alpha^{-1}, \chi^{-1}} \langle \delta\vec{\theta}(t)^2 \rangle = \delta P_0^2 \sum_{\text{noisy } i} |\phi_{i,\alpha}|^2\, t^2 \end{aligned}}$$

Results

component of mode of Laplacian on perturbation site k

$$\mathcal{P}_1 = \delta P_0^2\, T \sum_{\alpha \geq 2} \frac{\phi_{\alpha,k}^2 (I/D + \tau_0)}{\lambda_\alpha(\lambda_\alpha \tau_0 + D + I\tau_0^{-1})}$$

$$\mathcal{P}_2 = \delta P_0^2\, T \sum_{\alpha \geq 2} \frac{\phi_{\alpha,k}^2}{D(\lambda_\alpha \tau_o + D + I\tau_0^{-1})}$$

Next : (1) vary time scale τ_0 from shortest to largest

(2) average vs. specific perturbation location

$$\langle \delta P_i(t) \rangle = 0$$
$$\langle \delta P_i(t_1) \delta P_j(t_2) \rangle = \delta P_0^2 \delta_{ij} e^{-|t_1 - t_2|/\tau_0}$$

$$\mathcal{P}_1(T) = \int_0^T \mathrm{d}t\, \delta\boldsymbol{\theta}^2(t)$$
$$\mathcal{P}_2(T) = \int_0^T \mathrm{d}t\, \delta\dot{\boldsymbol{\theta}}^2(t)$$

Tyloo, Pagnier and PJ, in preparation

Results : (i) vs. correlation time

τ_0 is shortest time scale

$$\mathcal{P}_1 \cong \frac{\delta P_0^2 \tau_0}{D} \sum_{\alpha \geq 2} \frac{\phi_{\alpha,k}^2}{\lambda_\alpha} \qquad \mathcal{P}_2 \cong \frac{\delta P_0^2 \tau_0}{DI} \frac{(n-1)}{n}$$

τ_0 is longest time scale

$$\mathcal{P}_1 \cong \delta P_0^2 \sum_{\alpha \geq 2} \frac{\phi_{\alpha,k}^2}{\lambda_\alpha^2} \qquad \mathcal{P}_2 \cong \frac{\delta P_0^2}{D\tau_0} \sum_{\alpha \geq 2} \frac{\phi_{\alpha,k}^2}{\lambda_\alpha}$$

$$\mathcal{P}_1(T) = \int_0^T \mathrm{d}t \, \delta\boldsymbol{\theta}^2(t)$$

$$\mathcal{P}_2(T) = \int_0^T \mathrm{d}t \, \delta\dot{\boldsymbol{\theta}}^2(t)$$

Tyloo, Pagnier and PJ, in preparation

Results : (i) vs. correlation time

τ_0 is shortest time scale

$$\mathcal{P}_1 \cong \frac{\delta P_0^2 \tau_0}{D} \sum_{\alpha \geq 2} \frac{\phi_{\alpha,k}^2}{\lambda_\alpha} \qquad \mathcal{P}_2 \cong \frac{\delta P_0^2 \tau_0}{DI} \frac{(n-1)}{n}$$

τ_0 is longest time scale

$$\mathcal{P}_1 \cong \delta P_0^2 \sum_{\alpha \geq 2} \frac{\phi_{\alpha,k}^2}{\lambda_\alpha^2} \qquad \mathcal{P}_2 \cong \frac{\delta P_0^2}{D\tau_0} \sum_{\alpha \geq 2} \frac{\phi_{\alpha,k}^2}{\lambda_\alpha}$$

They all depend on

$$\sum_{\alpha \geq 2} \frac{\phi_{\alpha,k}^2}{\lambda_\alpha^p} \quad !!!$$

Tyloo, Pagnier and PJ, in preparation

Results : (ii) global robustness vs. Kirchhoff indices

PHYSICAL REVIEW LETTERS 120, 084101 (2018)

Robustness of Synchrony in Complex Networks and Generalized Kirchhoff Indices

M. Tyloo,[1,2] T. Coletta,[1] and Ph. Jacquod[1]
[1]*School of Engineering, University of Applied Sciences of Western Switzerland HES-SO, CH-1951 Sion, Switzerland*
[2]*Institute of Physics, EPF Lausanne, CH-1015 Lausanne, Switzerland*

Take average over perturbation location $\overline{\phi^2_{\alpha k}} = n^{-1} \sum_k \phi^2_{\alpha k} = n^{-1}$

Introduce "generalized Kirchhoff indices" $Kf_p = n \sum_{\alpha \geq 2} \lambda_\alpha^{-p}$

τ_0 is shortest time scale

$$\overline{\mathcal{P}}_1 \cong \frac{\delta P_0^2 \tau_0}{D n^2} Kf_1 \qquad \mathcal{P}_2 \cong \frac{\delta P_0^2 \tau_0}{DI} \frac{(n-1)}{n}$$

τ_0 is longest time scale

$$\overline{\mathcal{P}}_1 \cong \frac{\delta P_0^2}{n^2} Kf_2 \qquad \overline{\mathcal{P}}_2 \cong \frac{\delta P_0^2}{n^2 \tau_0} Kf_1$$

Kirchhof index Kf1 : Klein and Randic, JMC '93.

Results : (ii) global robustness vs. Kirchhoff indices

PHYSICAL REVIEW LETTERS 120, 084101 (2018)

Robustness of Synchrony in Complex Networks and Generalized Kirchhoff Indices

M. Tyloo,[1,2] T. Coletta,[1] and Ph. Jacquod[1]
[1]*School of Engineering, University of Applied Sciences of Western Switzerland HES-SO, CH-1951 Sion, Switzerland*
[2]*Institute of Physics, EPF Lausanne, CH-1015 Lausanne, Switzerland*

Take average over perturbation location $\overline{\phi_{\alpha k}^2} = n^{-1}\sum_k \phi_{\alpha k}^2 = n^{-1}$

Intr

Take-home message #1

- Global robustness assessment using Kirchhoff indices

τ_0 is shortest time scale

$$\overline{\mathcal{P}}_1 \cong \frac{\delta P_0^2 \tau_0}{D n^2} Kf_1 \qquad \mathcal{P}_2 \cong \frac{\delta P_0^2 \tau_0}{D I} \frac{(n-1)}{n}$$

τ_0 is longest time scale

$$\overline{\mathcal{P}}_1 \cong \frac{\delta P_0^2}{n^2} Kf_2 \qquad \overline{\mathcal{P}}_2 \cong \frac{\delta P_0^2}{n^2 \tau_0} Kf_1$$

Kirchhof index Kf1 : Klein and Randic, JMC ’93.

Results : (iii) specific / local vulnerabilities

Resistance distance vs. Laplacian matrix (its pseudoinverse)

$$\Omega_{ik} = \mathbb{L}^{\dagger}_{ii} + \mathbb{L}^{\dagger}_{kk} - 2\mathbb{L}^{\dagger}_{ik} = \sum_{\alpha \geq 2} \frac{(\phi_{\alpha,i} - \phi_{\alpha,k})^2}{\lambda_\alpha}$$

~effective resistance between i and k, for equivalent network of resistors

$$\sum_{\alpha \geq 2} \frac{\phi^2_{\alpha,k}}{\lambda_\alpha} = \sum_i \Omega_{ik} - \frac{Kf_1}{n^2}$$

Tyloo, Pagnier and PJ, in preparation

Results : (iii) specific / local vulnerabilities

Resistance distance vs. Laplacian matrix (its pseudoinverse)

$$\Omega_{ik} = \mathbb{L}^{\dagger}_{ii} + \mathbb{L}^{\dagger}_{kk} - 2\mathbb{L}^{\dagger}_{ik} = \sum_{\alpha \geq 2} \frac{(\phi_{\alpha,i} - \phi_{\alpha,k})^2}{\lambda_\alpha}$$

~effective resistance between i and k, for equivalent network of resistors

$$\sum_{\alpha \geq 2} \frac{\phi^2_{\alpha,k}}{\lambda_\alpha} = \sum_i \Omega_{ik} - \frac{Kf_1}{n^2}$$

Tyloo, Pagnier and PJ, in preparation

Results : (iii) specific / local vulnerabilities

Resistance distance vs. Laplacian matrix (its pseudoinverse)

$$\Omega_{ik} = \mathbb{L}^{\dagger}_{ii} + \mathbb{L}^{\dagger}_{kk} - 2\mathbb{L}^{\dagger}_{ik} = \sum_{\alpha \geq 2} \frac{(\phi_{\alpha,i} - \phi_{\alpha,k})^2}{\lambda_\alpha}$$

~effective resistance between i and k, for equivalent network of resistors

$$\longrightarrow \sum_{\alpha \geq 2} \frac{\phi^2_{\alpha,k}}{\lambda_\alpha} = \sum_i \Omega_{ik} - \frac{Kf_1}{n^2}$$

~inverse resistive centrality

$$C_i^{(1)} = \left[n^{-1} \sum_j \Omega_{ij} \right]^{-1} = \left[\sum_{\alpha \geq 2} \frac{\phi^2_{\alpha,i}}{\lambda_\alpha} + n^{-2} Kf_1 \right]^{-1}$$

Tyloo, Pagnier and PJ, in preparation

Results : (iii) specific / local vulnerabilities

Resistance distance vs. Laplacian matrix (its pseudoinverse)

$$\Omega_{ik} = \mathbb{L}^{\dagger}_{ii} + \mathbb{L}^{\dagger}_{kk} - 2\mathbb{L}^{\dagger}_{ik} = \sum_{\alpha \geq 2} \frac{(\phi_{\alpha,i} - \phi_{\alpha,k})^2}{\lambda_\alpha}$$

~effective resistance between i and k, for equivalent network of resistors

$$\sum_{\alpha \geq 2} \frac{\phi^2_{\alpha,k}}{\lambda_\alpha} = \sum_i \Omega_{ik} - \frac{Kf_1}{n^2}$$

~inverse resistive centrality

$$C_i^{(1)} = \left[n^{-1} \sum_j \Omega_{ij} \right]^{-1} = \left[\sum_{\alpha \geq 2} \frac{\phi^2_{\alpha,i}}{\lambda_\alpha} + n^{-2} Kf_1 \right]^{-1}$$

~inverse resistive centrality
for squared Laplacian

$$C_i^{(2)} = \left[\sum_{\alpha \geq 2} \frac{\phi^2_{\alpha,i}}{\lambda^2_\alpha} + n^{-2} Kf_2 \right]^{-1}$$

Tyloo, Pagnier and PJ, in preparation

Results : (iii) specific / local vulnerabilities

Resistance distance vs. Laplacian matrix (its pseudoinverse)

$$\Omega_{ik} = \mathbb{L}^{\dagger}_{ii} + \mathbb{L}^{\dagger}_{kk} - 2\mathbb{L}^{\dagger}_{ik} = \sum_{\alpha\geq 2} \frac{(\phi_{\alpha,i} - \phi_{\alpha,k})^2}{\lambda_\alpha}$$

~effective resistance between i and k, for equivalent network of resistors

$$\sum_{\alpha\geq 2} \frac{\phi^2_{\alpha,k}}{\lambda_\alpha} = \sum_i \Omega_{ik} - \frac{Kf}{n^2}$$

τ_0 is shortest time scale

$$\mathcal{P}_1 = \frac{\delta P_0^2 \tau_0}{D} \left(C_k^{(1)-1} - n^{-2} K f_1 \right) \qquad \mathcal{P}_2 \cong \frac{\delta P_0^2 \tau_0}{DI} \frac{(n-1)}{n}$$

τ_0 is longest time scale

$$\mathcal{P}_1 = \delta P_0^2 \left(C_k^{(2)^{-1}} - n^{-2} K f_2 \right) \qquad \mathcal{P}_2 \cong \frac{\delta P_0^2}{D\tau_0} \left(C_k^{(1)-1} - n^{-2} K f_1 \right)$$

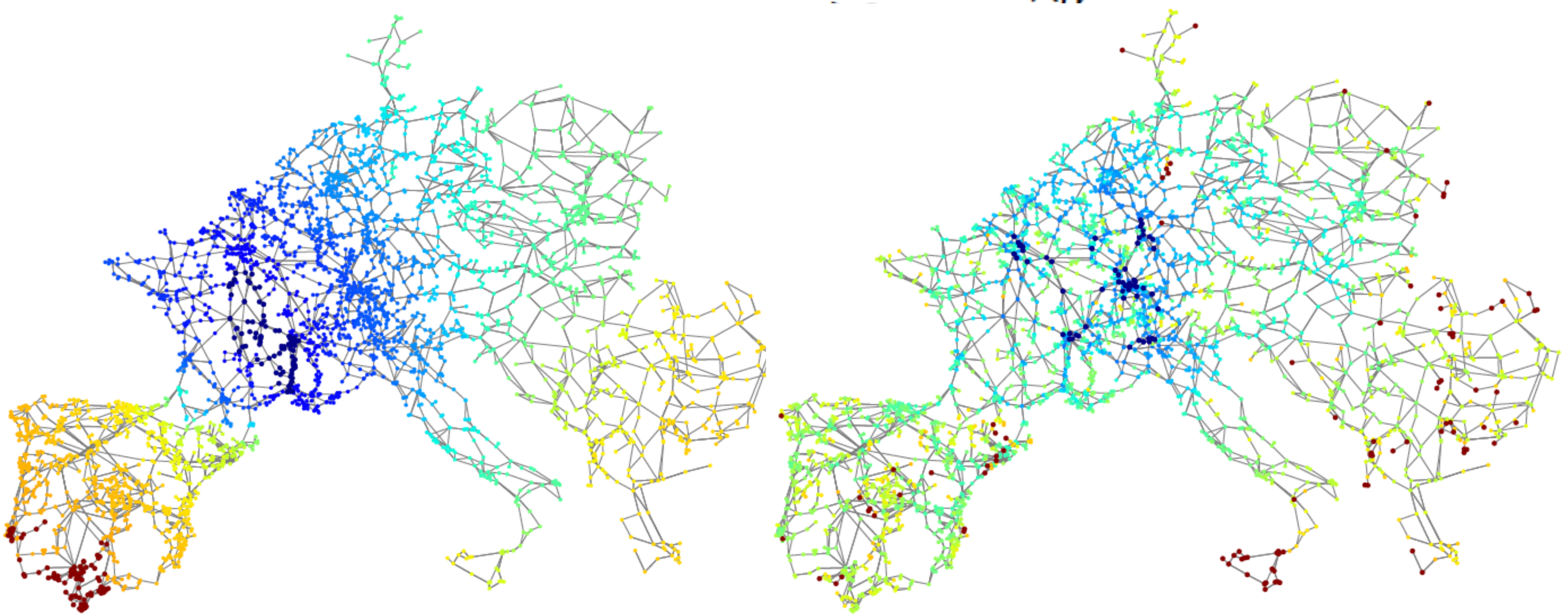
Tyloo, Pagnier and PJ, in preparation

Results : (iii) specific / local vulnerabilities

Resistance distance vs. Laplacian matrix (its pseudoinverse)

$$\Omega_{ik} = \mathbb{L}^{\dagger}_{ii} + \mathbb{L}^{\dagger}_{kk} - 2\mathbb{L}^{\dagger}_{ik} = \sum \frac{(\phi_{\alpha,i} - \phi_{\alpha,k})^2}{\lambda_\alpha}$$

~ef



τ_0 is longest time scale

$$\mathcal{P}_1 = \delta P_0^2 \left({C_k^{(2)}}^{-1} - n^{-2} K f_2 \right) \qquad \mathcal{P}_2 \cong \frac{\delta P_0^2}{D\tau_0} \left(C_k^{(1)-1} - n^{-2} K f_1 \right)$$

Tyloo, Pagnier and PJ, in preparation

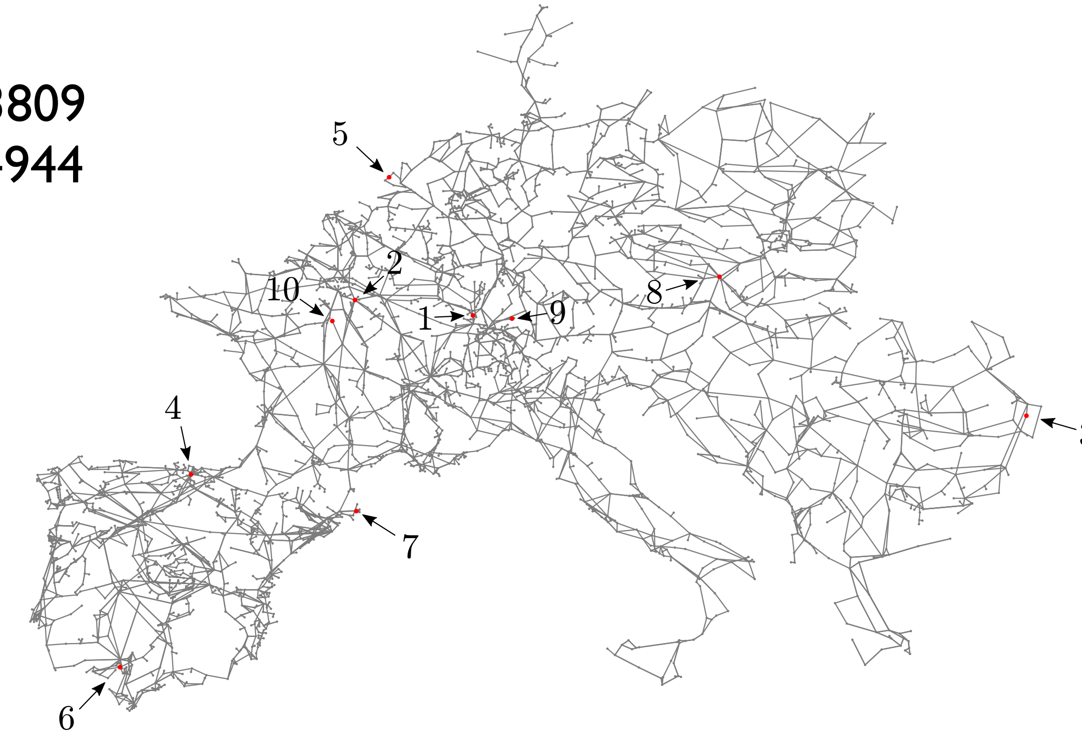
The key player problem : deterministically coupled systems

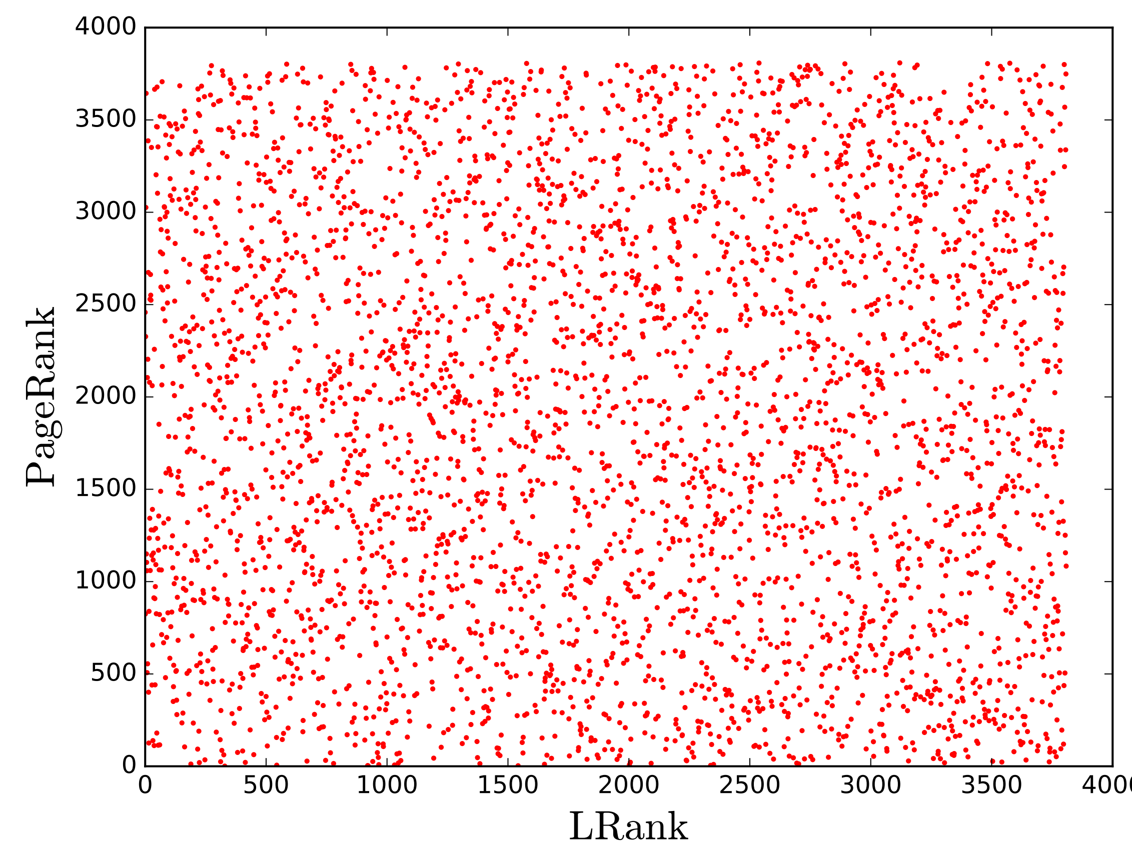
node #	$C_i^{(1)}$	C_i^{geo}	Degree	Katz	PageRank	$\mathcal{P}_2$
1	22.15	7.82	6	1.03195	1327	4.7×10^{-4}
2	22.07	7.68	5	1.03062	196	4.7×10^{-4}
3	18.59	3.54	5	1.03162	1041	6.6×10^{-4}
4	16.33	6.58	10	1.00103	1740	1.1×10^{-3}
5	16.32	5.56	2	1.03127	3470	1.1×10^{-3}
6	12.74	3.98	6	1.00067	3408	2×10^{-3}
7	10.77	6.58	3	1.00085	1076	2.5×10^{-3}
8	10.77	2.91	2	1.00016	2403	2.7×10^{-3}
9	9.66	3.53	2	1.00035	1532	3×10^{-3}
10	8.11	4.65	1	1.00001	3367	4×10^{-3}

Resistance distance centrality a.k.a. LRank

Numerically computed performance measure

#nodes : 3809
#edges : 4944



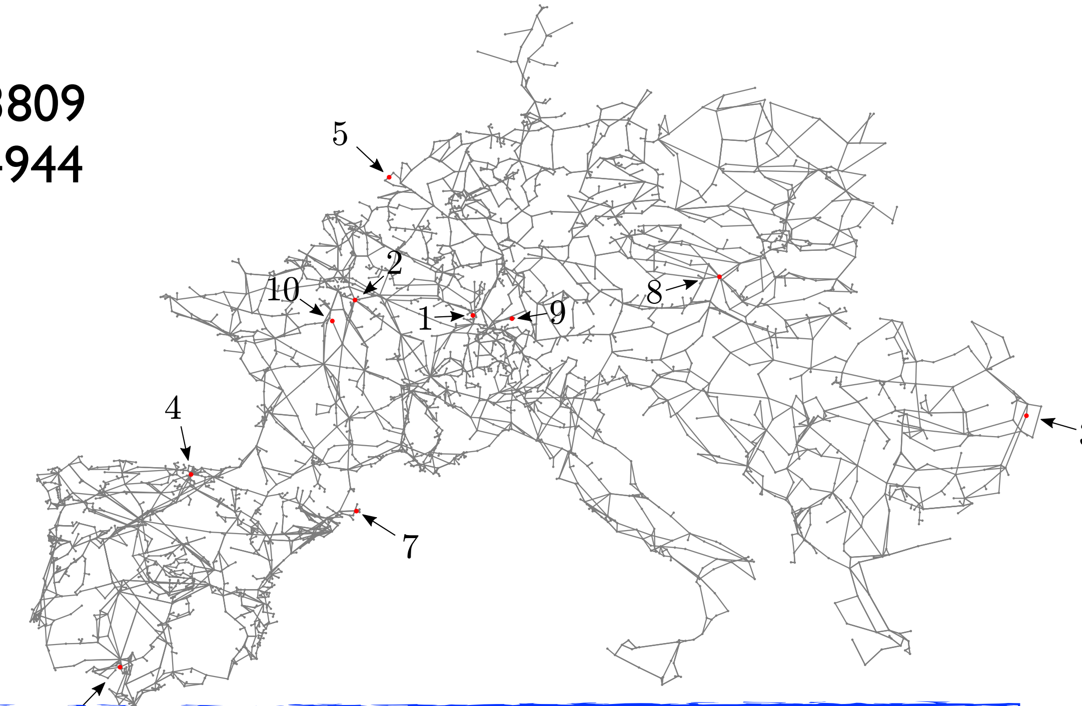


Tyloo, Pagnier and PJ : to be published (2018)

The key player problem : deterministically coupled systems

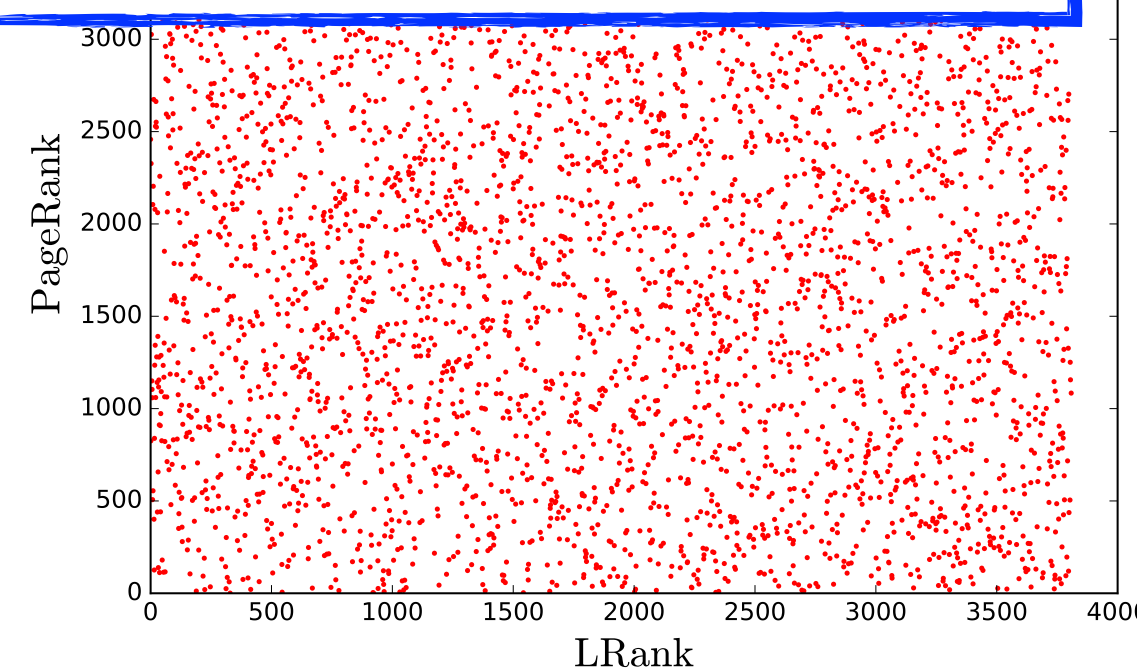
node #	$C_i^{(1)}$	C_i^{geo}	Degree	Katz	PageRank	$\mathcal{P}_2$
1	22.15	7.82	6	1.03195	1327	4.7×10^{-4}
2	22.07	7.68	5	1.03062	196	4.7×10^{-4}
3	18.59	3.54	5	1.03162	1041	6.6×10^{-4}
4	16.33	6.58	10	1.00103	1740	1.1×10^{-3}
5	16.32	5.56	2	1.03127	3470	1.1×10^{-3}
6	12.74	3.98	6	1.00067	3408	2×10^{-3}
7	10.77	6.58	3	1.00085	1076	2.5×10^{-3}
8	1[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9	9[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	8[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

#nodes : 3809
#edges : 4944



Take-home message #2

- Local vulnerabilities ranked with resistive centralities



Resistance distance centrality a.k.a. LRank

Numerically computed performance measure

Tyloo, Pagnier and PJ : to be published (2018)

Results : (v) Line fault

$$I_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum B_{ij}\sin(\theta_i - \theta_j)$$

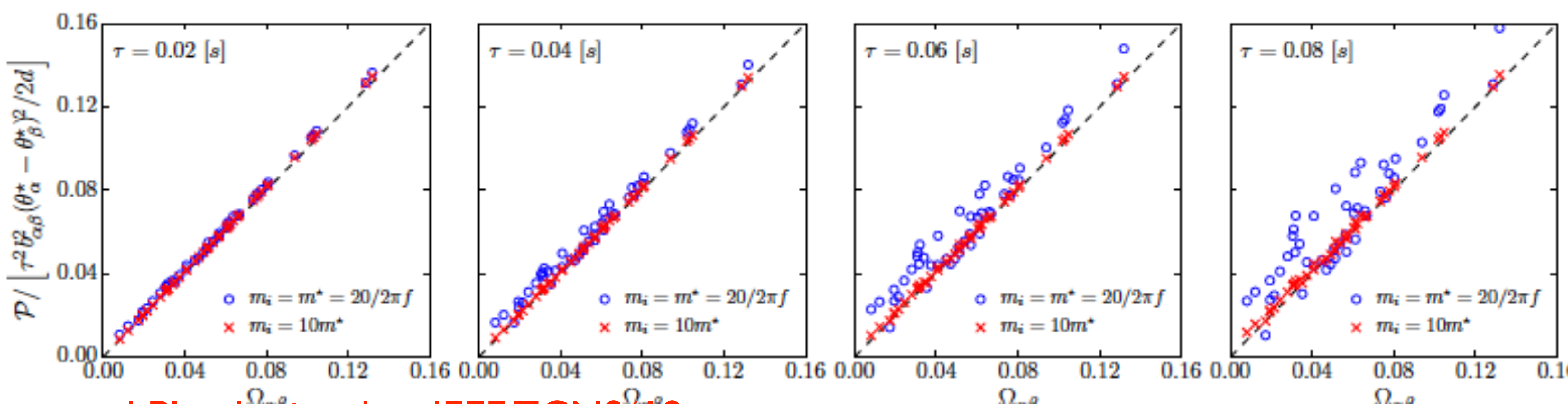
$$\boldsymbol{L} \to \boldsymbol{L} - \delta(t)\tau B_{\alpha\beta}\,\boldsymbol{e}_{(\alpha,\beta)}\,\boldsymbol{e}^{\top}_{(\alpha,\beta)}$$

$$\boldsymbol{e}_{(\alpha,\beta)} = (0, 0, ..., 1, 0, ..., -1, ...0)$$

α β

$$\boxed{\mathcal{P}_1 = \frac{P^2_{\alpha,\beta}\,\tau^2}{2D}\Omega_{\alpha\beta}}$$

with the original load on the faulted line $P_{\alpha,\beta} = B_{\alpha\beta}(\theta^*_\alpha - \theta^*_\beta)$



Coletta and PJ, submitted to IEEE TCNS '18

Results : (v) Line fault

$$I_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum B_{ij}\sin(\theta_i - \theta_j)$$

$$\boldsymbol{L} \rightarrow \boldsymbol{L} - \delta(t)\tau B_{\alpha\beta}\, \boldsymbol{e}_{(\alpha,\beta)}\, \boldsymbol{e}^{\top}_{(\alpha,\beta)}$$

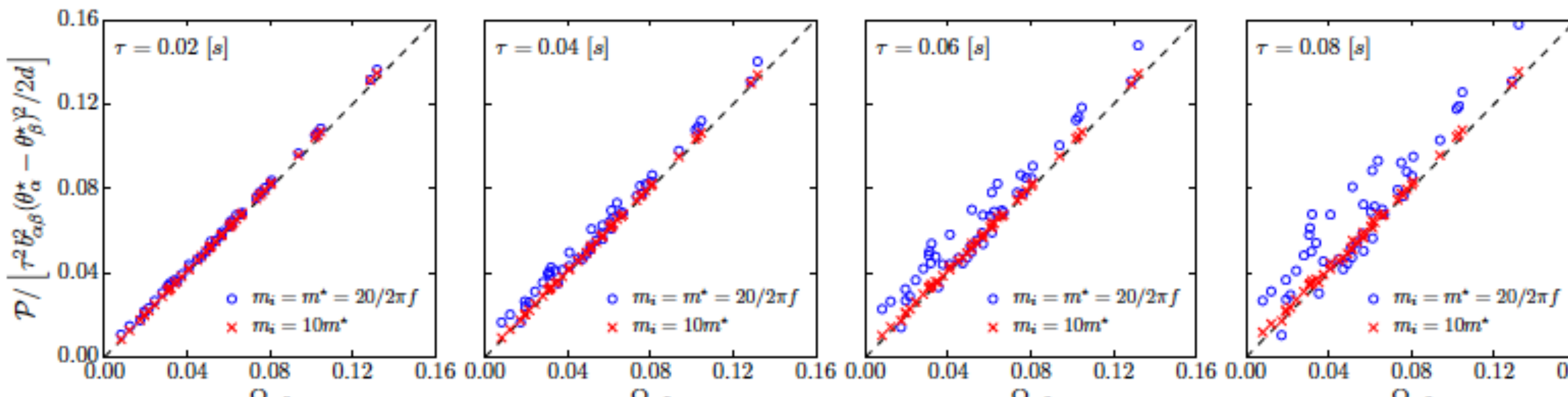
$$\boldsymbol{e}_{(\alpha,\beta)} = (0, 0, ..., \underset{\alpha}{1}, 0, ..., \underset{\beta}{-1}, ...0)$$

$$\boxed{\mathcal{P}_1 = \frac{P^2_{\alpha,\beta}\,\tau^2}{2D}\Omega_{\alpha\beta}}$$

with

Take-home message #3

- Local line vulnerabilities ranked with resistive distances



Coletta and PJ, submitted to IEEE TCNS '18

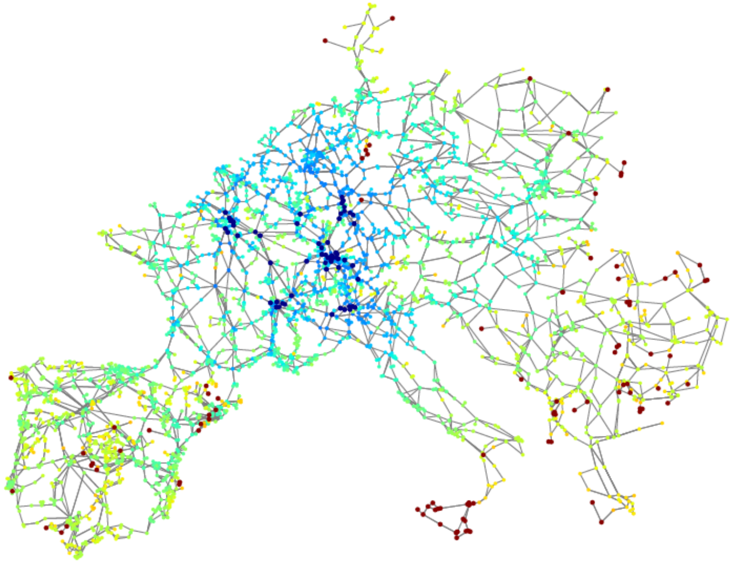
Conclusion

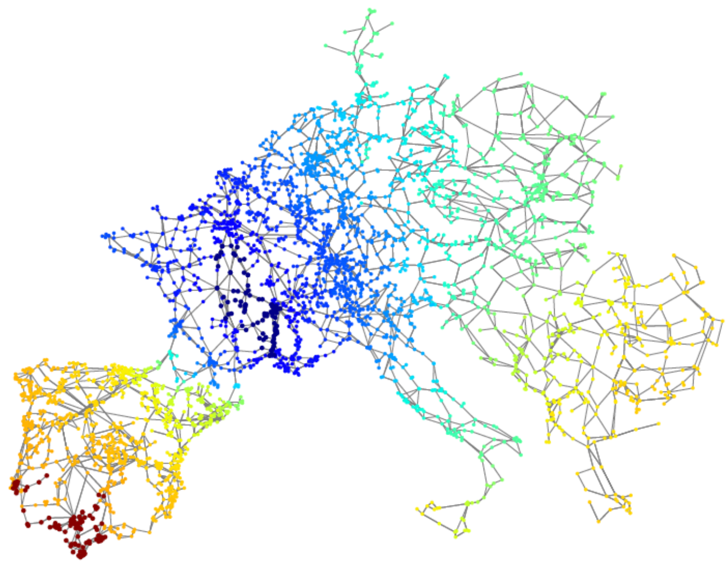
Robustness assessment and local vulnerability ranking / key player problem in deterministic, network-coupled dynamical systems

$$\dot{\mathbf{x}} = \mathbf{P} - \mathbb{M}\mathbf{x} \qquad\qquad \mathbb{I}\ddot{\mathbf{x}} + \mathbb{D}\dot{\mathbf{x}} = \mathbf{P} - \mathbb{M}\mathbf{x}$$

Look at distances, centralities, indices related to the matrix M !

Impact :
planning of electric power grids
real-time assessment of grid stability

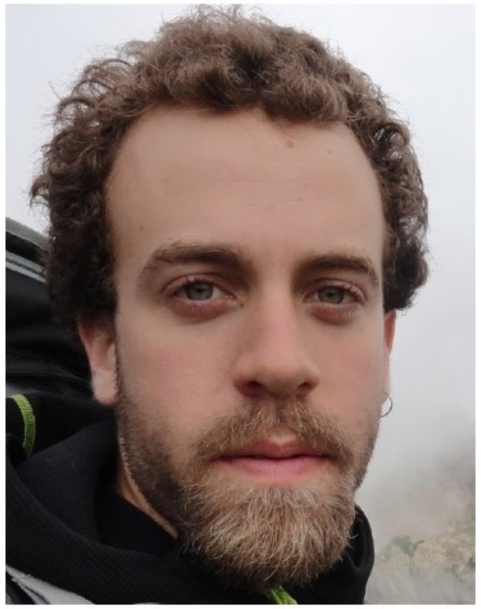




The team

Tommaso Coletta, postdoc
(now with Cargil)

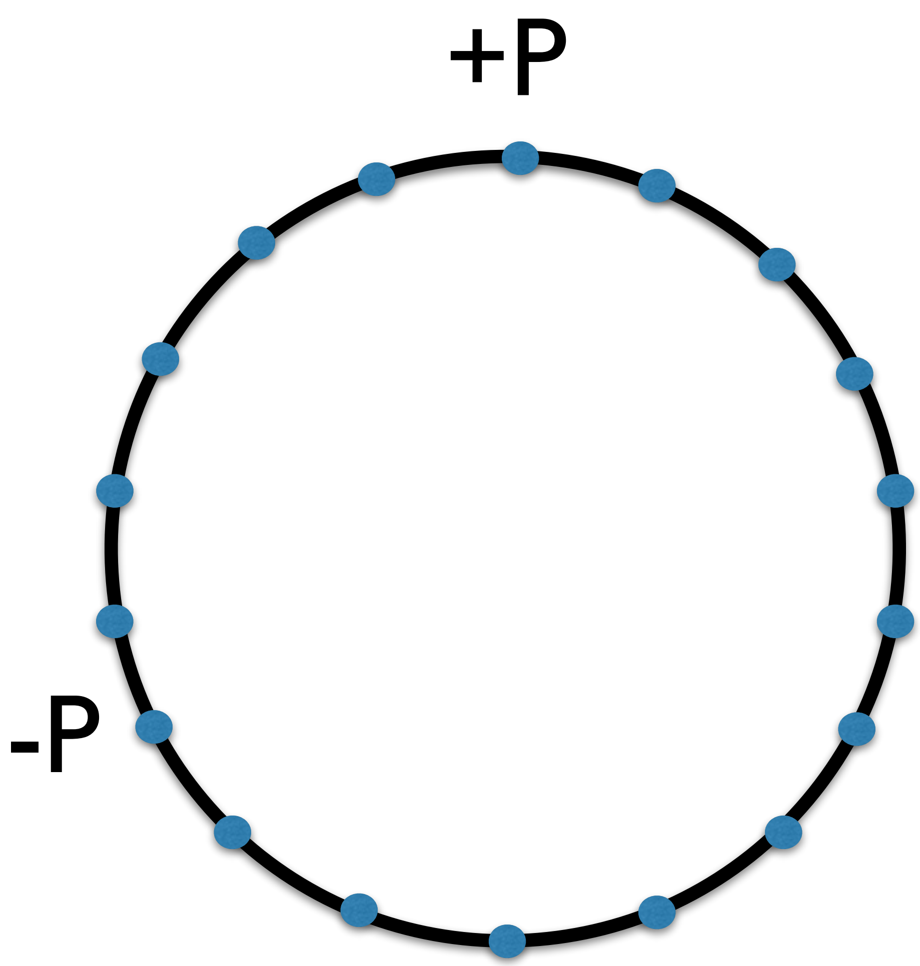
Robin Delabays, PhD student
(now postdoc)

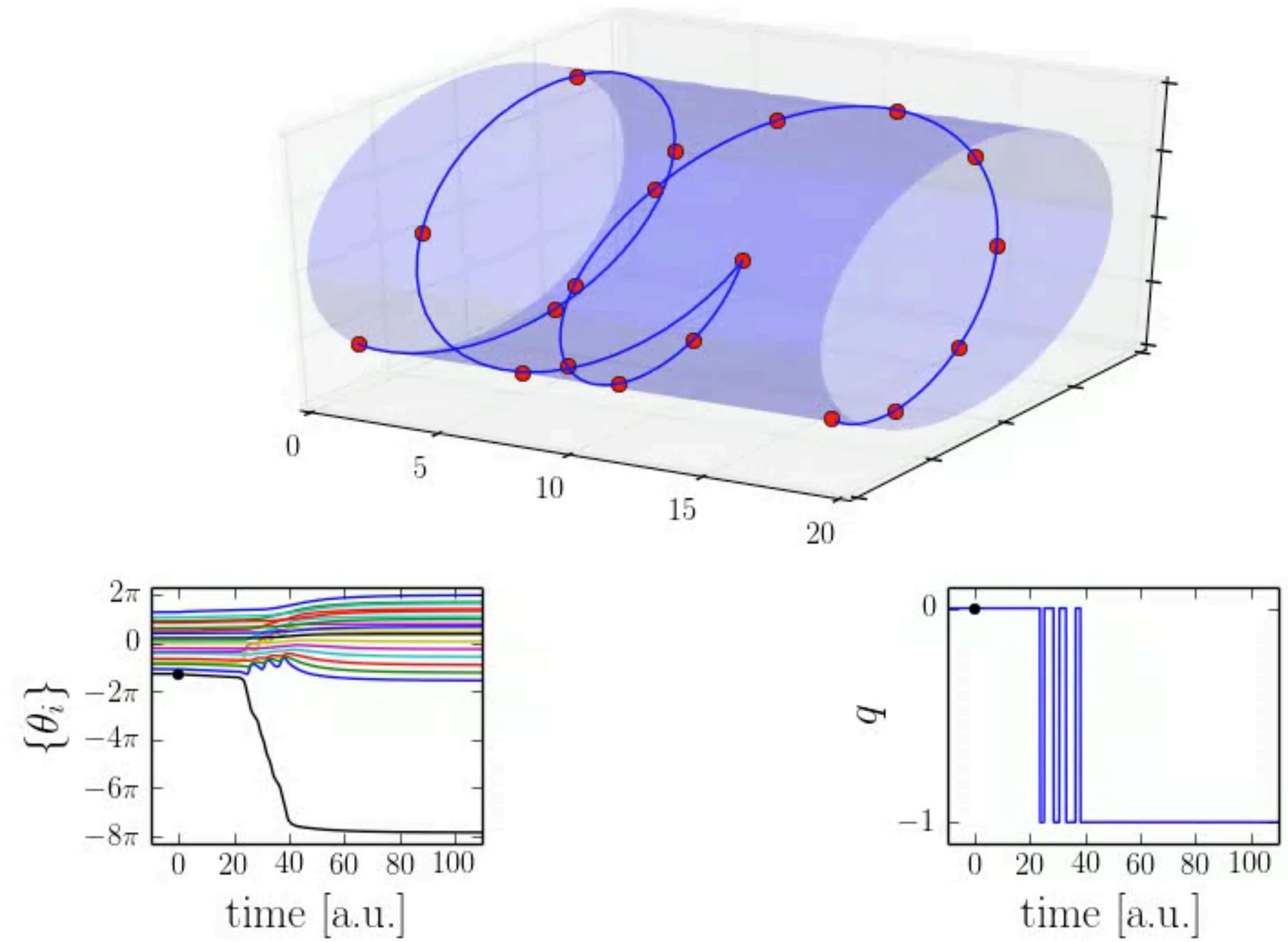


Laurent Pagnier, PhD student

Melvyn Tyloo, PhD student

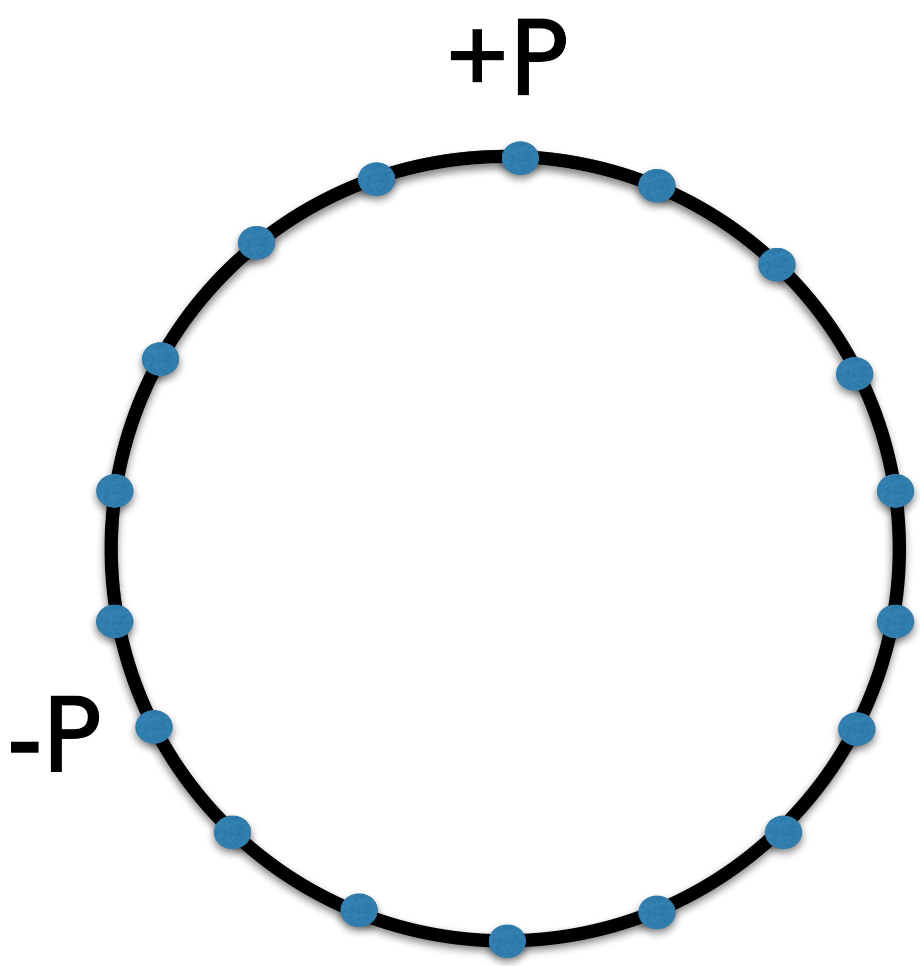
Dynamical generation of vortex flows

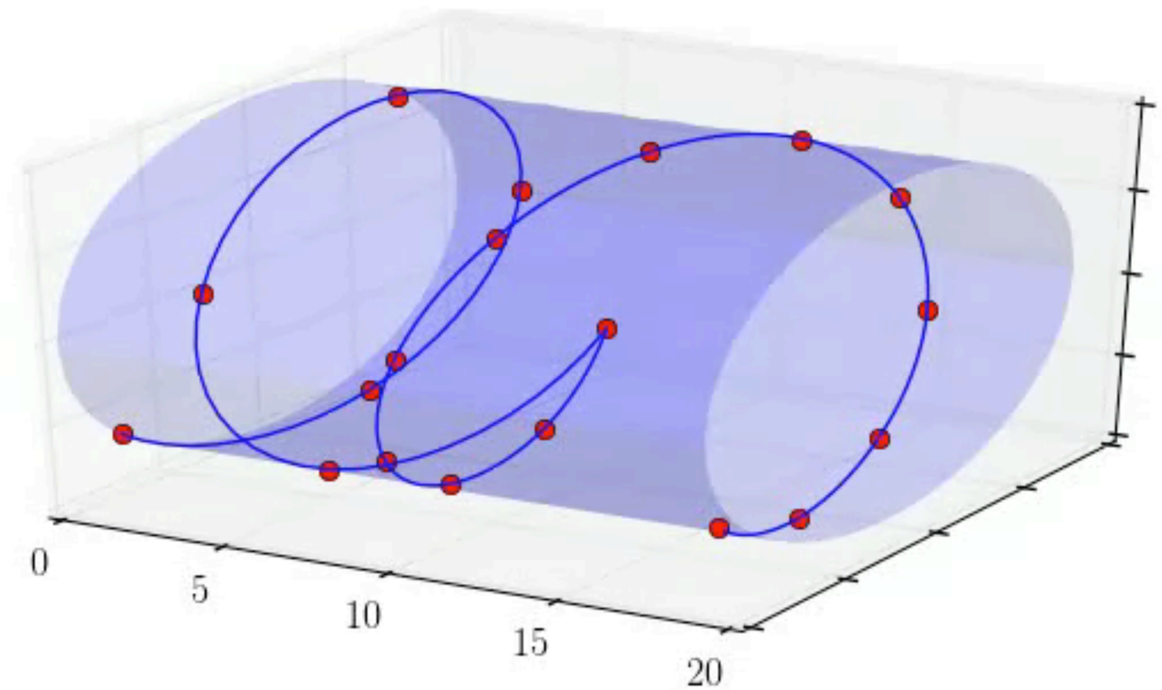


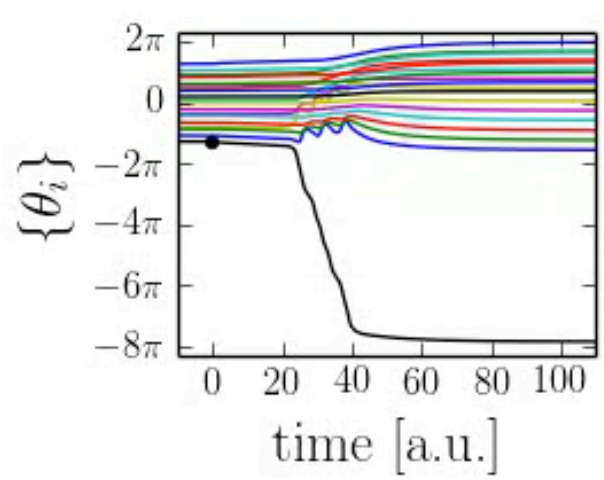


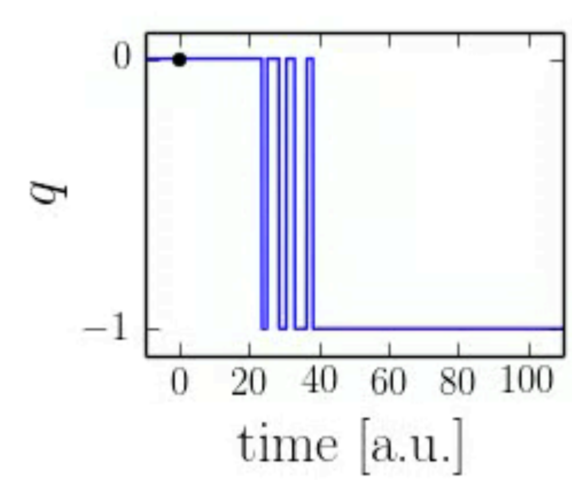
Similar to quantum phase slips in JJ arrays
Lau, Markovic, Bockrath, Bezryadin and Tinkham '01
Matveev, Larkin and Glazman '02

Dynamical generation of vortex flows









Similar to quantum phase slips in JJ arrays
Lau, Markovic, Bockrath, Bezryadin and Tinkham '01
Matveev, Larkin and Glazman '02