Loop Flows in High Voltage AC Power Grids

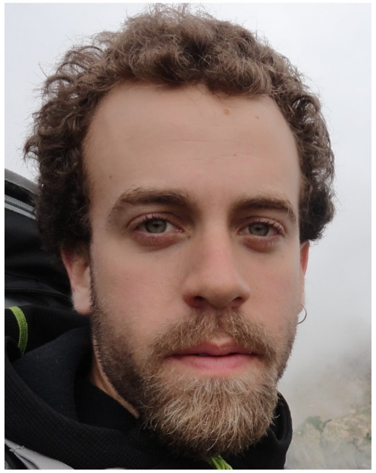
(A Physicist's perspective)

Philippe Jacquod

Coletta and PJ, Phys Rev E 93, 032222 (2016)
Delabays, Coletta, and PJ, J Math Phys 57, 032701 (2016)
Coletta, Delabays, Adagideli and PJ, New J Phys (2016)
Delabays, Coletta, and PJ, arXiv:1609.02359, submitted to J Math Phys

The team

Tommaso Coletta, postdoc



Robin Delabays, PhD student

Inanc Adagideli (Sabanci)

Vortices, topological protection and all that jazz

David Thouless

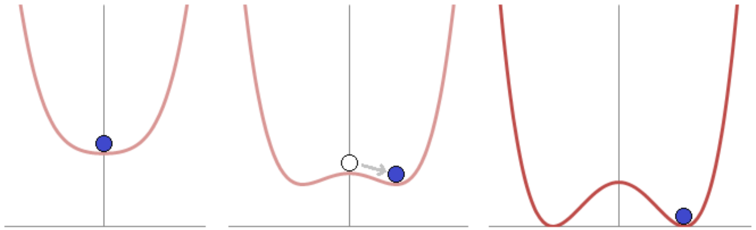
Duncan Haldane

Michael Kosterlitz

The Nobel Prize in Physics 2016 was divided, one half awarded to David J. Thouless, the other half jointly to F. Duncan M. Haldane and J. Michael Kosterlitz "for theoretical discoveries of topological phase transitions and topological phases of matter".

Symmetries and classification of states of matter

- Phase transition / spontaneous symmetry breaking
- Phase with broken symmetry : order parameter
- Classification with symmetry of order parameter



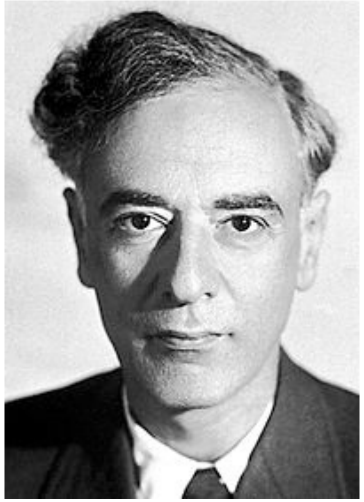


Table 1. Order parameters for phase transitions in various systems.

System	Transition	Order parameter
Liquid-gas	Condensation/evaporation	Density difference $\Delta\rho = \rho_{\text{liquid}} - \rho_{\text{gas}}$
Binary liquid mixture	Unmixing	Composition difference $\Delta c = c^{(2)}_{\text{coex}} - c^{(1)}_{\text{coex}}$
Nematic liquid	Orientational ordering	$\frac{1}{2}\langle 3\cos^2\theta - 1\rangle$
Quantum liquid	Normal fluid$\leftrightarrow$superfluid	$\langle\psi\rangle$, ψ = wavefunction
Liquid-solid	Melting/crystallisation	ρ_G, G = reciprocal lattice vector
Magnetic solid	Ferromagnetic (T_c)	Spontaneous magnetisation $\boldsymbol{M}$
	Antiferromagnetic (T_N)	Sublattice magnetisation $\boldsymbol{M}_s$
Solid binary mixture	Unmixing	$\Delta c = c^{(2)}_{\text{coex}} - c^{(1)}_{\text{coex}}$
AB	Sublattice ordering	$\psi = (\Delta c^{\text{II}} - \Delta c^{\text{I}})/2$
Dielectric solid	Ferroelectric (T_c)	Polarisation $\boldsymbol{P}$
	Antiferroelectric (T_N)	Sublattice polarisation $\boldsymbol{P}_s$
Molecular crystal	Orientational ordering	$Y_{lm}(\theta, \phi)$

Table from K Binder, Rep Prog Phys '87

The Kosterlitz-Thouless phase transition

David Mermin Herbert Wagner

Mermin and Wagner (thm) :

"There is no spontaneous breaking of a continuous symmetry at finite temperature in dimension d <= 2"

Continuous : order parameter must have 2 or more components
Dimension : d=2 is the critical dimension

What happens for d=2, 2-component order parameter ?

The Kosterlitz-Thouless phase transition

Somewhere between low and high temperature :

KT phase transition vs. dissociation of vortex-antivortex pairs

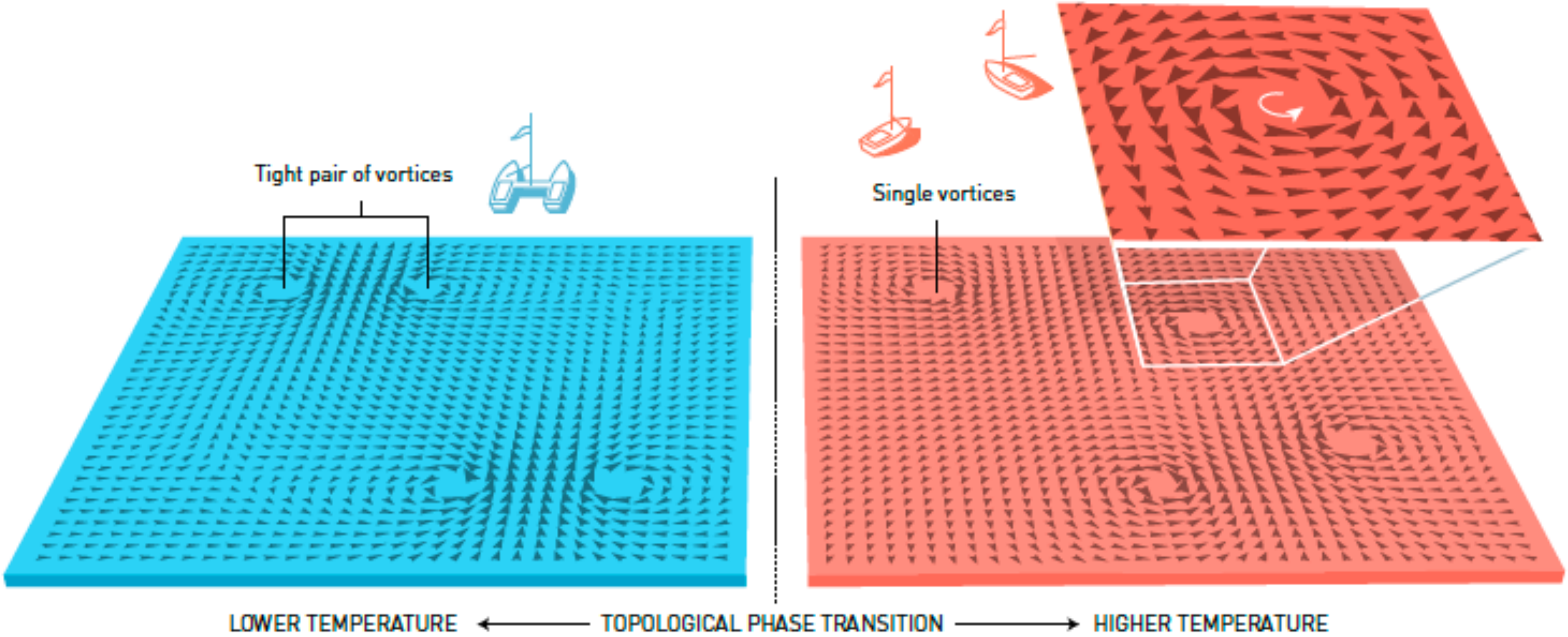
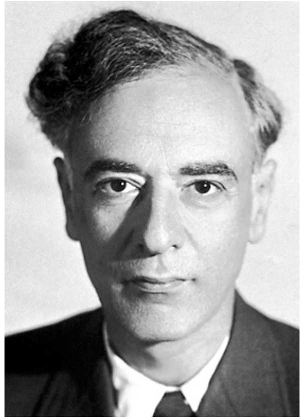


Illustration: ©Johan Jarnestad/The Royal Swedish Academy of Sciences

Vortices in superfluids and quantization of circulation



L Landau
1908-1968

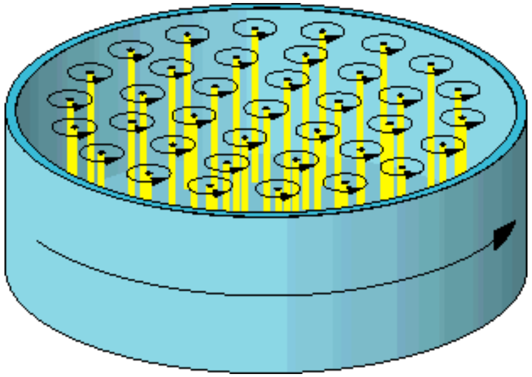
- Landau theory of superfluidity - complex order parameter

$$\psi(\mathbf{x}) = |\psi(\mathbf{x})| e^{i\phi(\mathbf{x})}$$

- Superfluid current

$$\boxed{\mathbf{J}_{\mathrm{SF}} = \frac{\hbar|\Psi|^2}{M_{\mathrm{SF}}} \nabla\phi}$$

- Vortex : topological defect with SF -> 0 in center
- Contour around that center



-> quantization of circulation

$$\oint \mathbf{J}_{\mathrm{SF}} \mathrm{d}\mathbf{l} = \frac{\hbar|\Psi|^2}{M_{\mathrm{SF}}} (\phi_+ - \phi_-) = \frac{\hbar|\Psi|^2}{M_{\mathrm{SF}}} 2\pi m \qquad m \in Z$$

Th. : Onsager '49, Feynman '55

The problem

"Electric power does not follow a specified path but divide … sion routes based on Kirchhoff's laws and network conditi… ern results in a phenomenon called circulating power … loop flow and parallel flow."

Loop flows are analogous to circulating currents around vortices in superfluids!

"W… geographical obstacles, such as the Rocky … Lakes in the East, loop flows around the ob… e as much as 1 GW of power in a circle, *taking up tran… without delivering power to consumers.*"

Casazza, Electr… orld (1998)
Lerner, The industrial physicist (2003)

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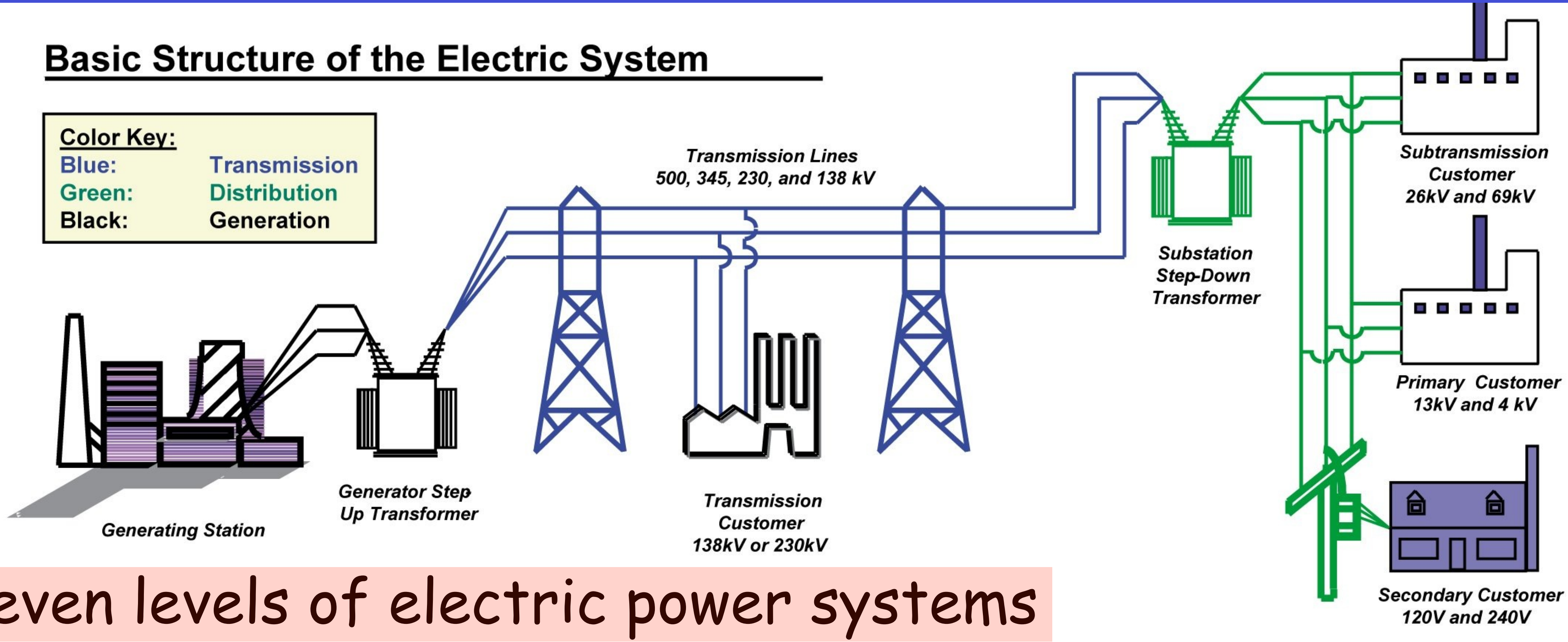
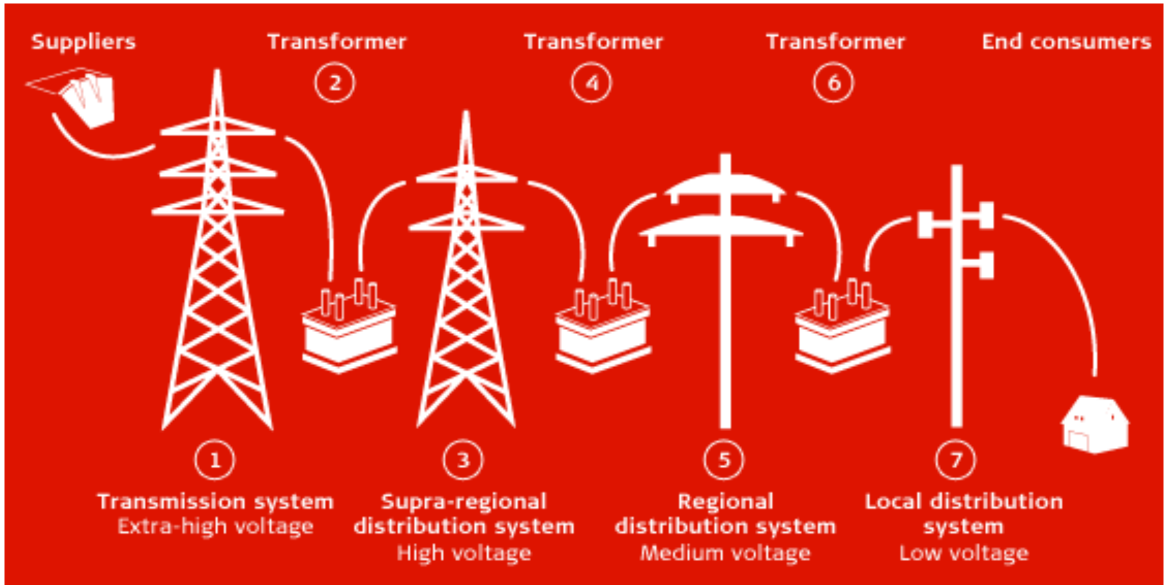
What are electric power systems ?



Seven levels of electric power systems



Power :
*conserved from one level to another (~)
*control parameter
"write Eq. for power"

Power flow equations

$$P_i = \sum_j |V_i V_j| \left[G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)\right]$$

$$Q_i = \sum_j |V_i V_j| \left[G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)\right]$$

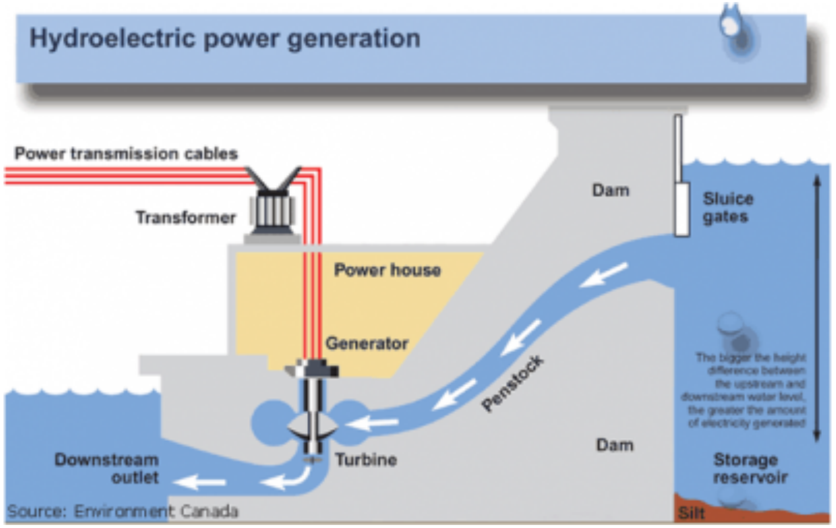
Voltages
at buses i and j

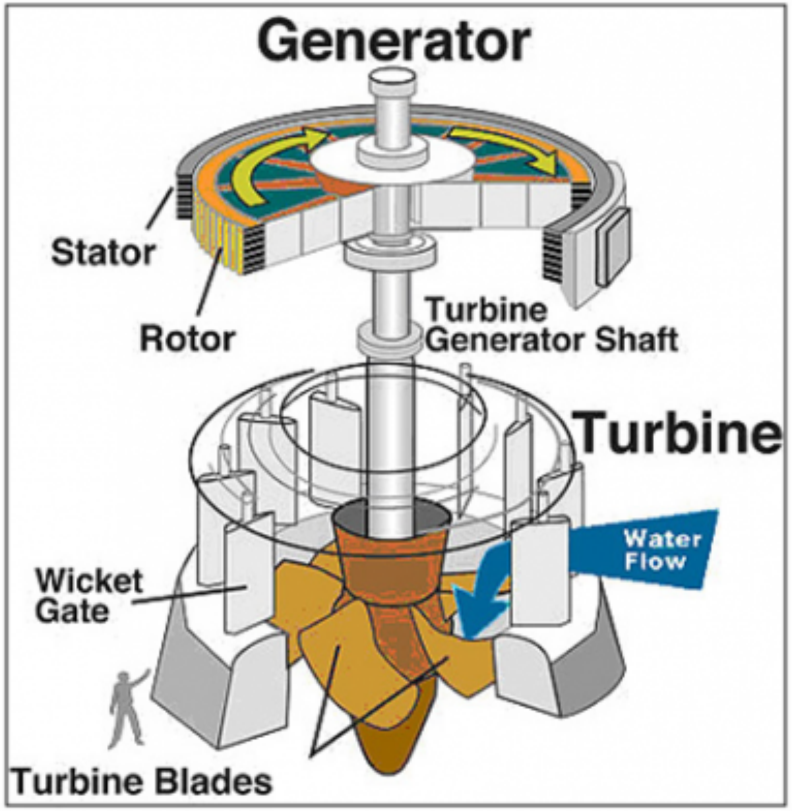
Conductance
matrix

Phases
at buses i and j

Susceptance
matrix

Time-evolution of frequency : swing equations





- Electricity production with rotating machines
- Potential, chemical, nuclear or thermal energy converted into kinetic energy (rotation)
- Mechanical energy converted into electric energy
- Time-dependence : think power instead of energy
- Balance between power in, power out and energy change in rotator : *SWING EQUATIONS*

Hes·so// VALAIS WALLIS

Time-evolution of frequency : swing equations

- Power balance

$$\frac{dW_i}{dt} + P_i^{(d)} = P_i^{(m)} - P_i^{(g)}$$

Change in KE of rotator

Damping power (losses from friction)

Power input

Electric power output

- Swing equation for angles (in rotating frame @ 50Hz)

$$M_i \frac{d^2\theta_i}{dt^2} + D_i \frac{d\theta_i}{dt} = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

Inertia of rotator

Damping / friction

- Solutions of power-flow eqs. = steady-state solutions of swing eqs. ~synchronous solution (angles rotate in unison)

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From the swing equations to linear stability

$$M_i \frac{d^2\theta_i}{dt^2} + D_i \frac{d\theta_i}{dt} = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

- suppose a synchronized solution exists
- what is its stability under angle perturbation ?

A.: *linearize the dynamics about that solution
*perturbed condition goes back exponentially fast to the initial solution if the stability matrix

$$\mathcal{M}_{ij} = -\delta_{ij} \sum_k B_{ik} \cos(\theta_i^{(0)} - \theta_k^{(0)}) + B_{ij}(1 - \delta_{ij}) \cos(\theta_i^{(0)} - \theta_j^{(0)})$$

is negative semidefinite, ***regardless of inertia !***

Coletta and PJ (2016); Manik et al. (2014)

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Approximated power flow equations : (1) lossless line

- Admittance dominated by its imaginary part
  for large conductors ~ high voltage
  G/B < 0.1 for 200kV and more

neglect conductance

$$P_i \simeq \sum_j |V_i V_j| B_{ij} \sin(\theta_i - \theta_j)$$

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

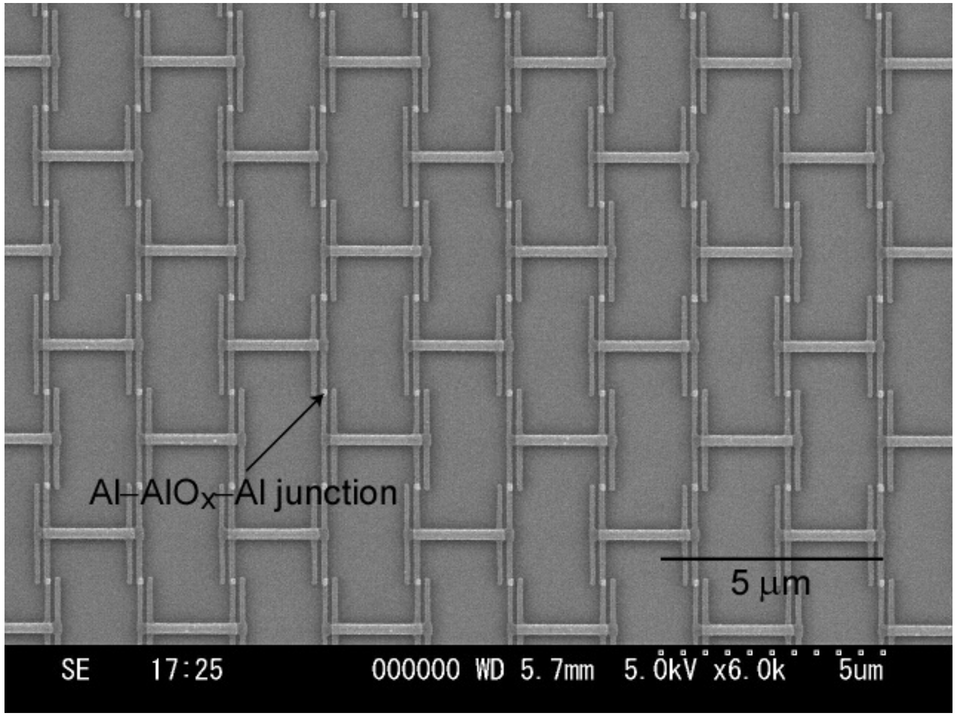
* consider P only

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

- No loss = balance of power

$$\sum_i P_i = 0$$

Josephson junction arrays vs. electric power systems !



Takahide, Yagi, Kanda, Ootuka, and Kobayashi
Phys. Rev. Lett. 85, 1974 (2000)

Josephson current

$$I_{ij} = I_c \sin(\theta_i - \theta_j)$$

Transmitted power

$$P_{ij} \simeq B_{ij} \sin(\theta_i - \theta_j)$$

First question

- Consider the power flow problem in the lossless line approximation

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

- It is entirely defined by

*the graph/network/grid (admittance matrix B_{ij})
*the set of power injections/consumptions $\{P_i\}$

Question : how many different solutions are there ?

First answer : an infinite # of them, since $\{\theta_i + C\}$ is also a solution for any constant C

Define "different" as differing by more than C

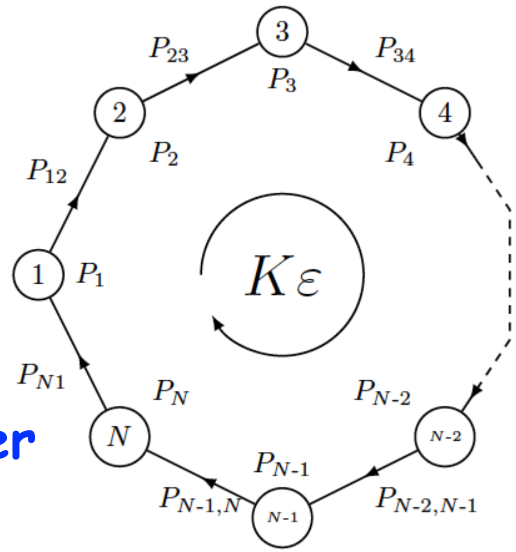
Circulating loop flows

*Thm: Different solutions to the following power-flow problem

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

may differ only by circulating loop current(s) **in any network**

Dörfler, Chertkov, Bullo, PNAS '13; Delabays, Coletta and PJ, JMP '16



*Voltage angle uniquely defined

→ $q=\Sigma_i||\theta_{i+1}-\theta_i||/2\pi \in \mathbb{Z}$ **~topological winding number**

→ "quantization" of these loop currents **~vortex flows**

Janssens and Kamagate '03

Circulating loop flows

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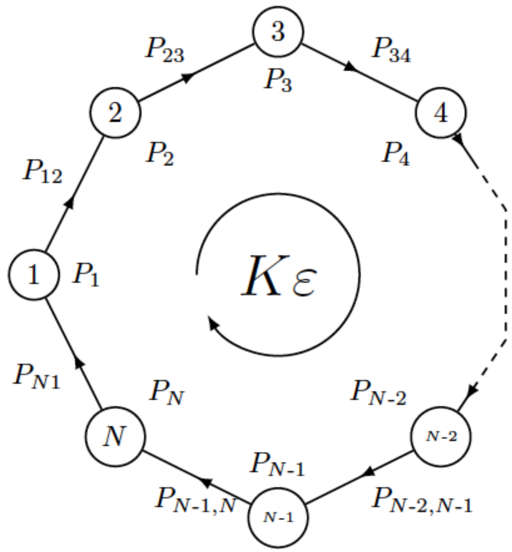

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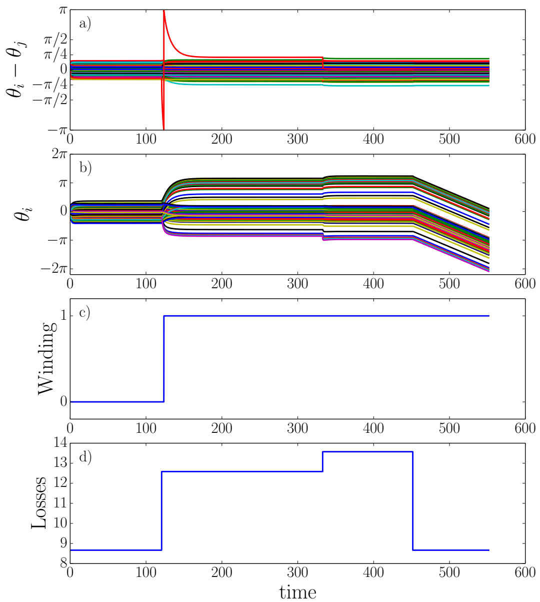

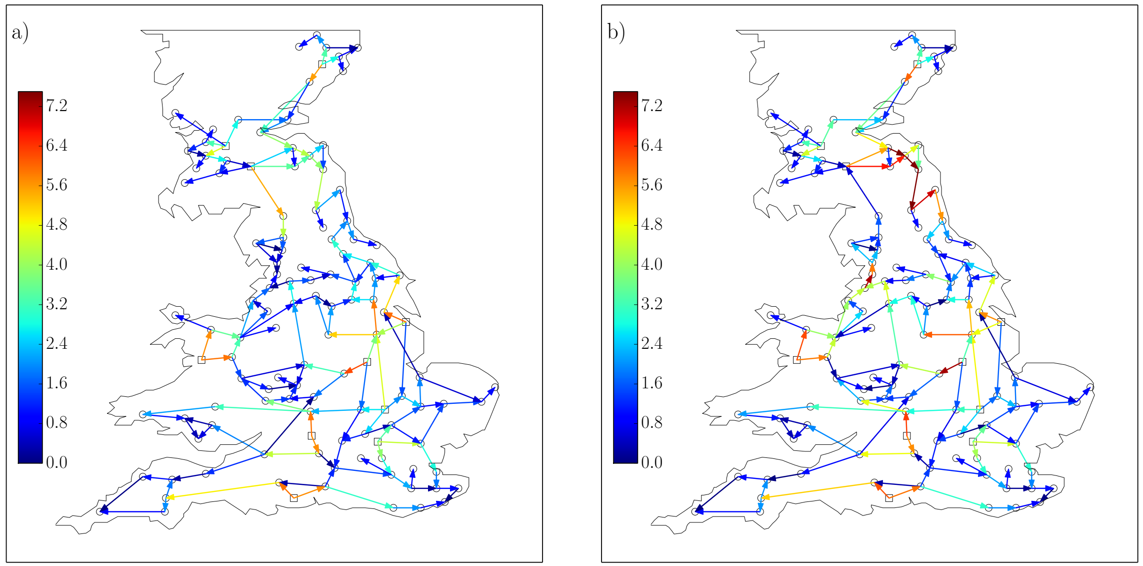

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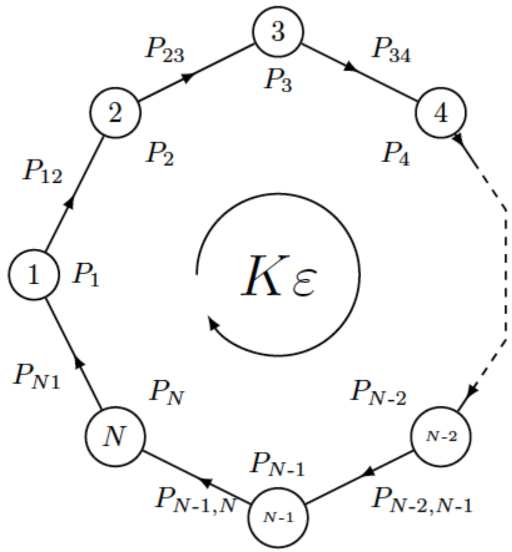

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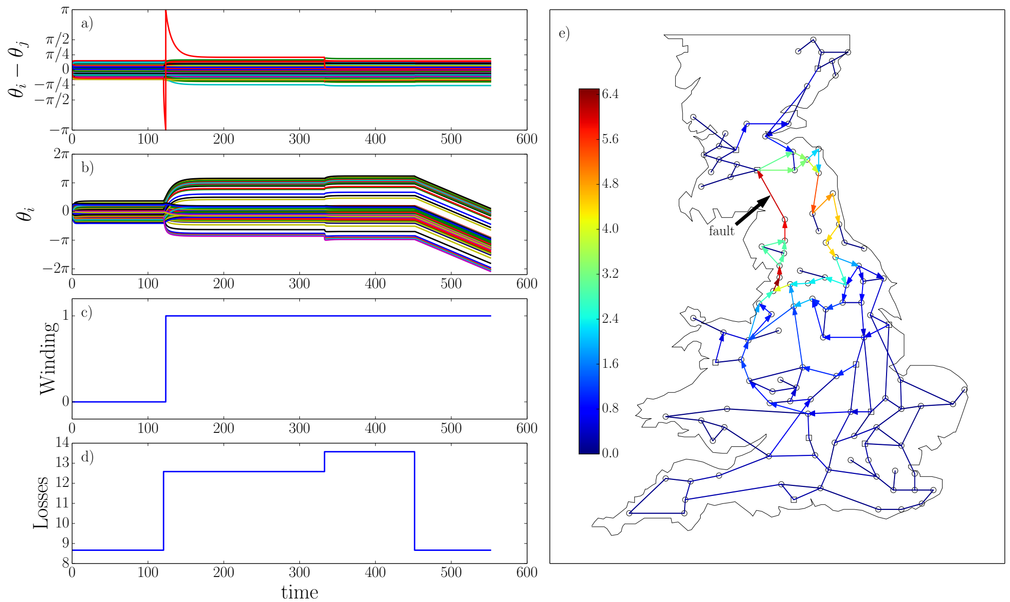


Stable solutions to the power flow problem

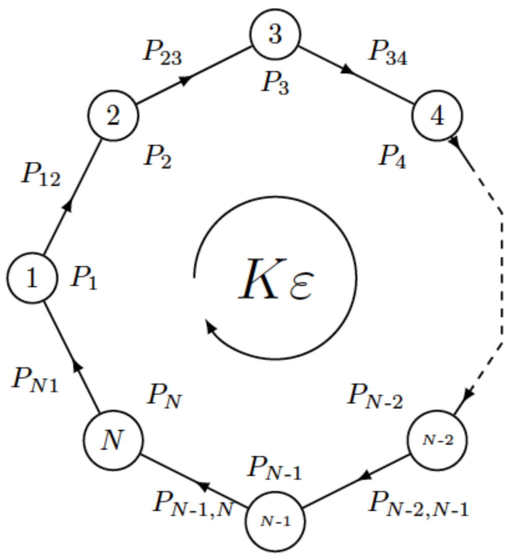
- Consider the power flow problem in the lossless line approximation

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j) = \sum_{j\sim i} P_{ij}$$

$$\frac{d\theta_i}{dt} = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

Question : how many different stable solutions are there ?

Answer #1 : take single-cycle network



Flow between i and i+1 $\quad P_{i,i+1} = P^*_{i,i+1} + K\varepsilon$

$$P^*_{i,i+1} := \sum_{j=1}^{i} P_j$$

"Quantization" condition $\quad q := (2\pi)^{-1} \sum_{i=1}^{n} \Delta_{i,i+1} \in \mathbb{Z}$

With two different possibilities $\quad \Delta_{i,i+1} = a_i(\varepsilon)$

$$a_i(\varepsilon) = \begin{cases} \arcsin\left(\varepsilon + P^*_{i,i+1}/K\right) & \Longrightarrow \quad \Delta_{i,i+1} \in [-\pi/2, \pi/2]\,, \\ \pi - \arcsin\left(\varepsilon + P^*_{i,i+1}/K\right) & \Longrightarrow \quad \Delta_{i,i+1} \in (-\pi, -\pi/2) \cup (\pi/2, \pi] \end{cases}$$

Delabays, Coletta and PJ, JMP '16

Stable solutions to the power flow problem

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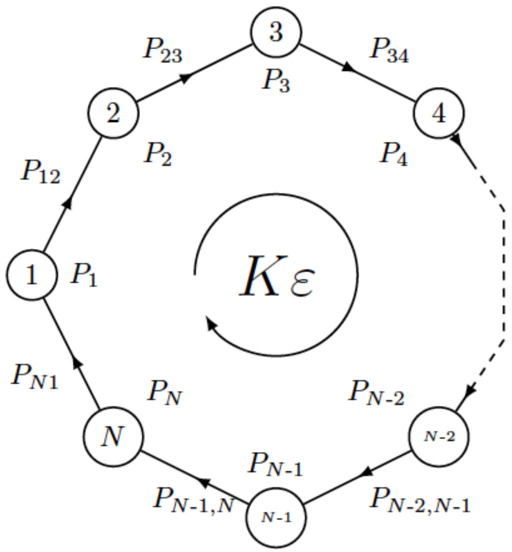

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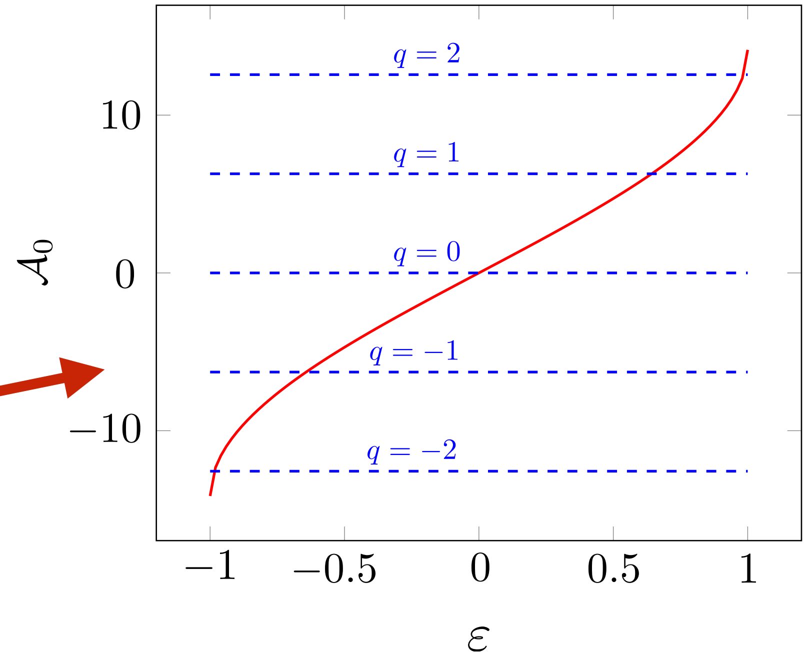
a. Take only first possibility (always stable)

$$\mathcal{A}_0(K,\varepsilon) := \sum_{i=1}^{n} \Delta_{i,i+1} = \sum_{i=1}^{n} \arcsin\left(\varepsilon + P^*_{i,i+1}/K\right) = 2\pi q$$

-count number of intersections



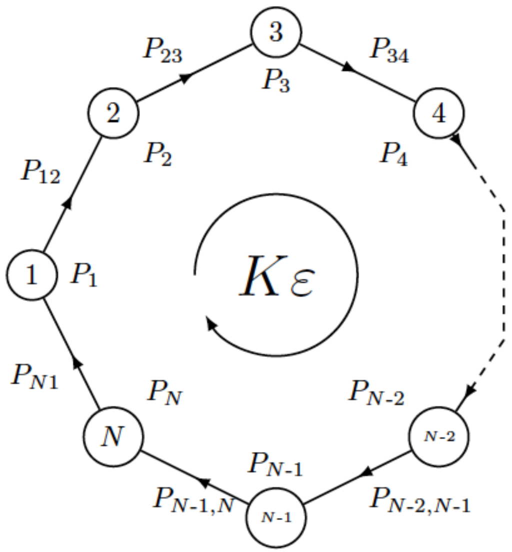
$\rightarrow \quad \mathcal{N} \leq 2\,\mathrm{Int}\,[n/4] + 1$

Delabays, Coletta and PJ, JMP '16

Stable solutions to the power flow problem

Question : how many different stable solutions are there ?

Answer #1 : take single-cycle network



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b. Consider second possibility (not always stable)

One can show : (i) with at most one angle difference > π/2 is sol. stable

(ii) > π/2 possible only at finite K=B (not at infinite capacity)

(iii) "> π/2" solution emerges from "< π/2" solution as one reduces K=B

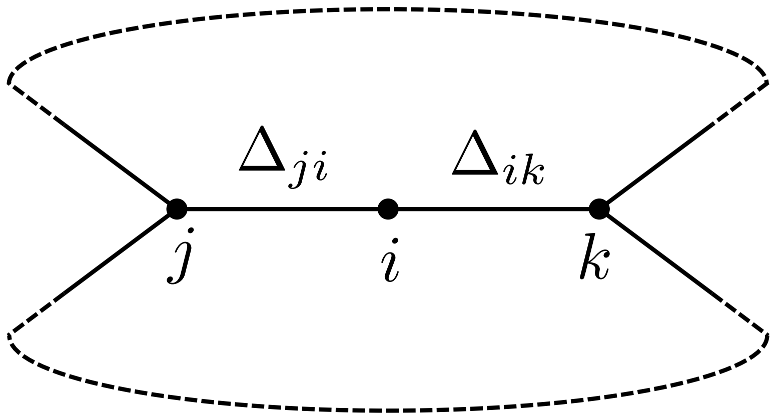
$$\longrightarrow \quad \boxed{N \leq 2\,\mathrm{Int}\,[n/4] + 1}$$

Delabays, Coletta and PJ, JMP '16

Stable solutions to the power flow problem

Question : how many different stable solutions are there ?

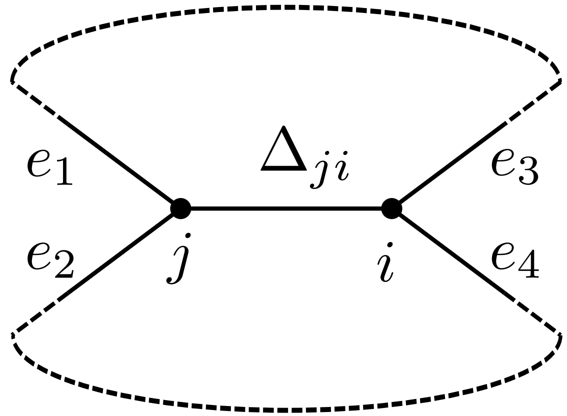
Answer #2 (partial) : meshed planar (i.e. multi-cycle) network



Loops with two or more edges in common

> π/2 possible only at finite K=B (not at infinite capacity)

$$\mathcal{N} \leq \prod_{k=1}^{c} \left[2 \cdot \mathrm{Int}\left(n_k/4\right) + 1\right]$$



Loops with one edge in common

> π/2 possible only at any K=B (also at infinite capacity!)

$$\mathcal{N} \leq \prod_{k=1}^{c} \left[2 \cdot \mathrm{Int}\left((n_k + n_k')/4\right) + 1\right]$$

?? conjecture ??

Delabays, Coletta and PJ, JMP '16

Vortex flows in AC power grids vs. superconductivity

- Vortex/circulation creation in SC via B-field

Can one create vortex flows in AC power grids ?
How ?

- Superconductors are dissipationless
- AC power grids are dissipative ~ they have a finite resistance

Do vortex flows survive the presence of dissipation ?

Hes·so// VALAIS WALLIS

Vortex flows in AC power grids vs. superconductivity

- Vortex/circulation creation in SC via B-field

Can one create vortex flows in AC power grids ? How ?

Three mechanisms : *dynamical phase slip
****line tripping***
****line tripping and reclosure***

- Superconductors are dissipationless
- AC power grids are dissipative ~ they have a finite resistance

Do vortex flows survive the presence of dissipation ?

YES ! They are robust against moderate amounts of dissipation

Coletta, Delabays, Adagideli and PJ, NJP '16

Approximated power flow equations : (2) losses to leading order

- Keep decoupling between P and Q

put back conductance

$$P_i = \sum_j [B_{ij} \sin(\theta_i - \theta_j) + G_{ij} \cos(\theta_i - \theta_j)]$$

- Losses are positive

$$\sum_i P_i = \sum_{ij} G_{ij} \cos(\theta_i - \theta_j)$$

$$\sum_i P_i > 0$$

Remark (important) :
$\{P_i\}$ and $\{\theta_i\}$ need to be self-consistently determined

Generation of vortex flow by line tripping

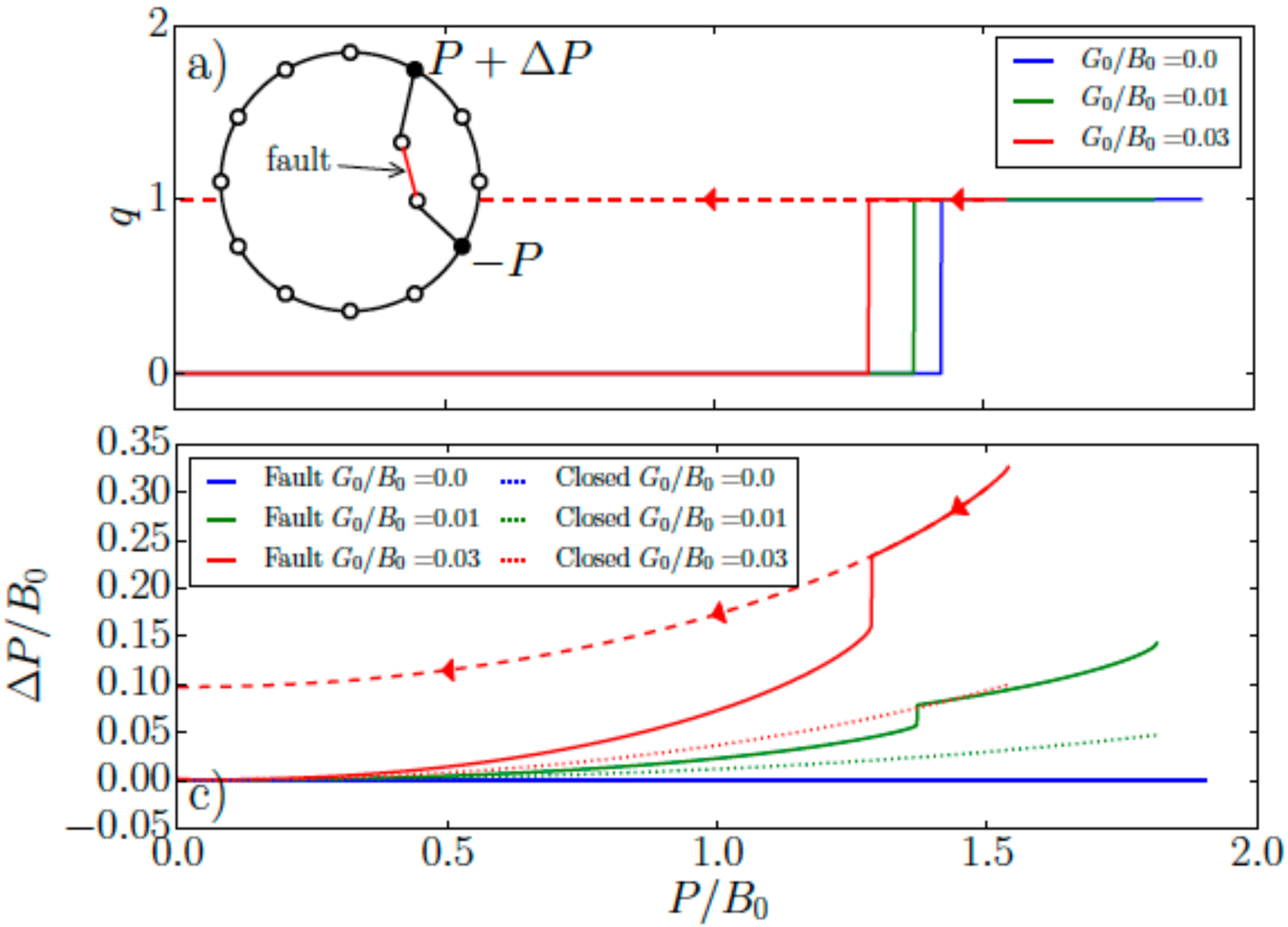
*Power grids are meshed - path redundancy
*Power is still supplied after one line trips
*Consider line tripping in an asymmetric, three-path circuit
*all winding numbers vanish initially

Power redistribution can lead to vortex flow with $q=\Sigma_i|\theta_{i+1}-\theta_i|/2\pi > 0$

Line tripping at $P/B_0 > 1.3$

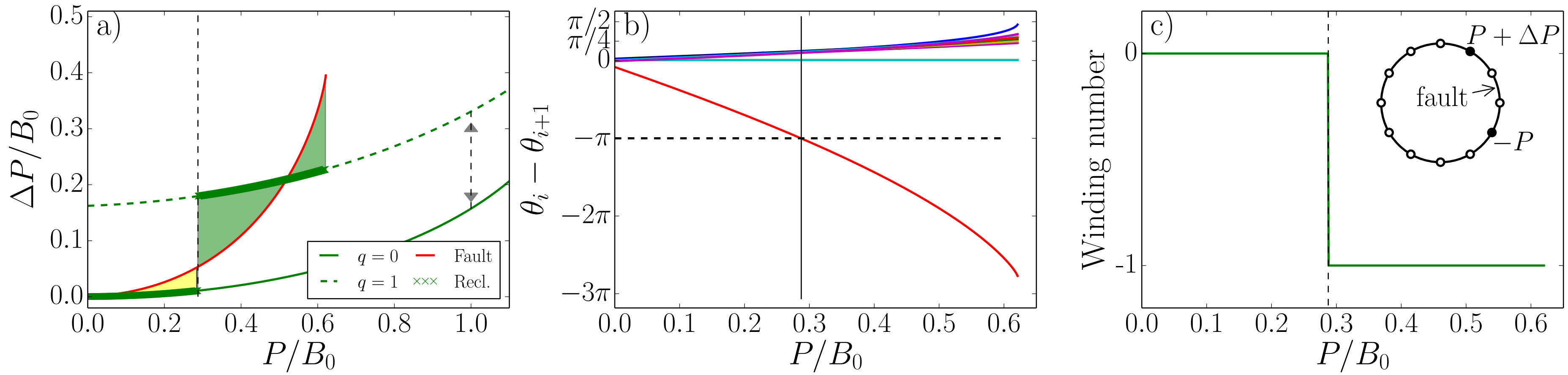
→ q=1

Vortex state characterized by
-hysteresis
 = **topological protection**
-higher losses



Coletta, Delabays, Adagideli and PJ '16

Generation of vortex flow by line tripping and reclosure



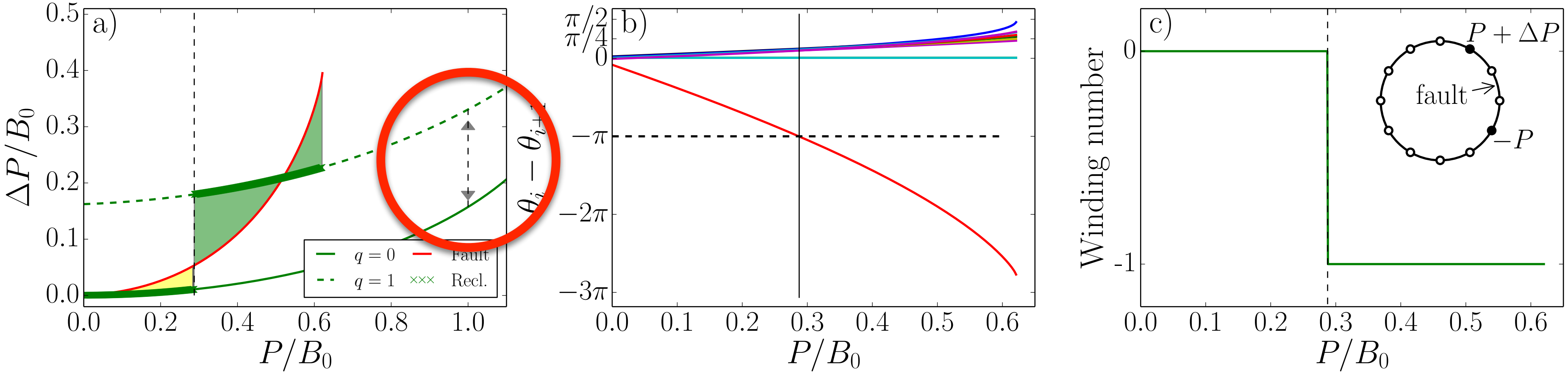
- Single-loop network
- One producer, one consumer
- One line trips, is later reclosed

Vortex formation for $P/B_0 > 0.26$

Vortex formation for $|\theta_{i+1}-\theta_i|>\pi$ (two ends of faulted line)

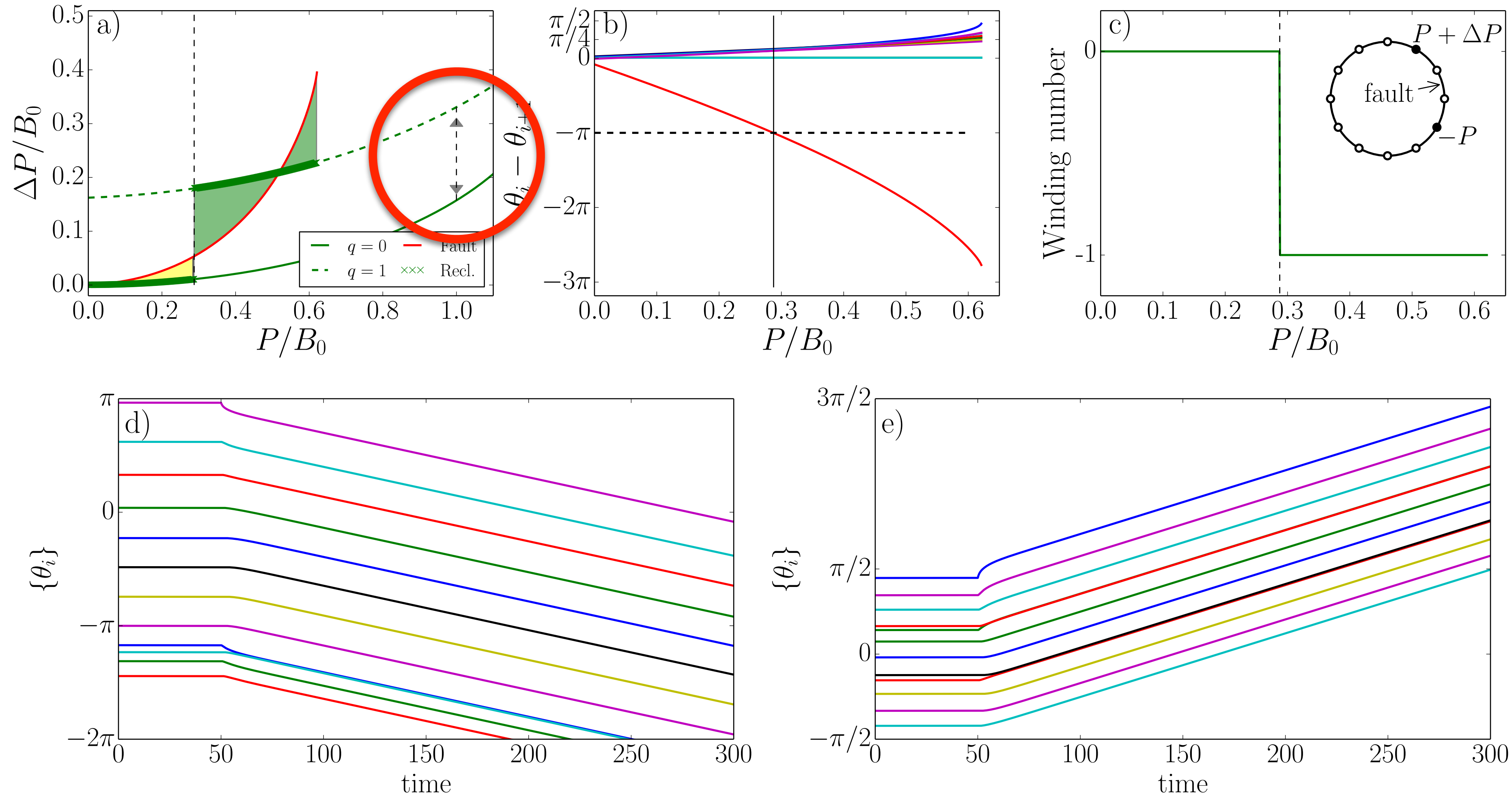
Coletta, Delabays, Adagideli and PJ '16; Janssens and Kamagate '03

Generation of vortex flow by line tripping and reclosure



Try to kill / create vortex by adapting ΔP ?

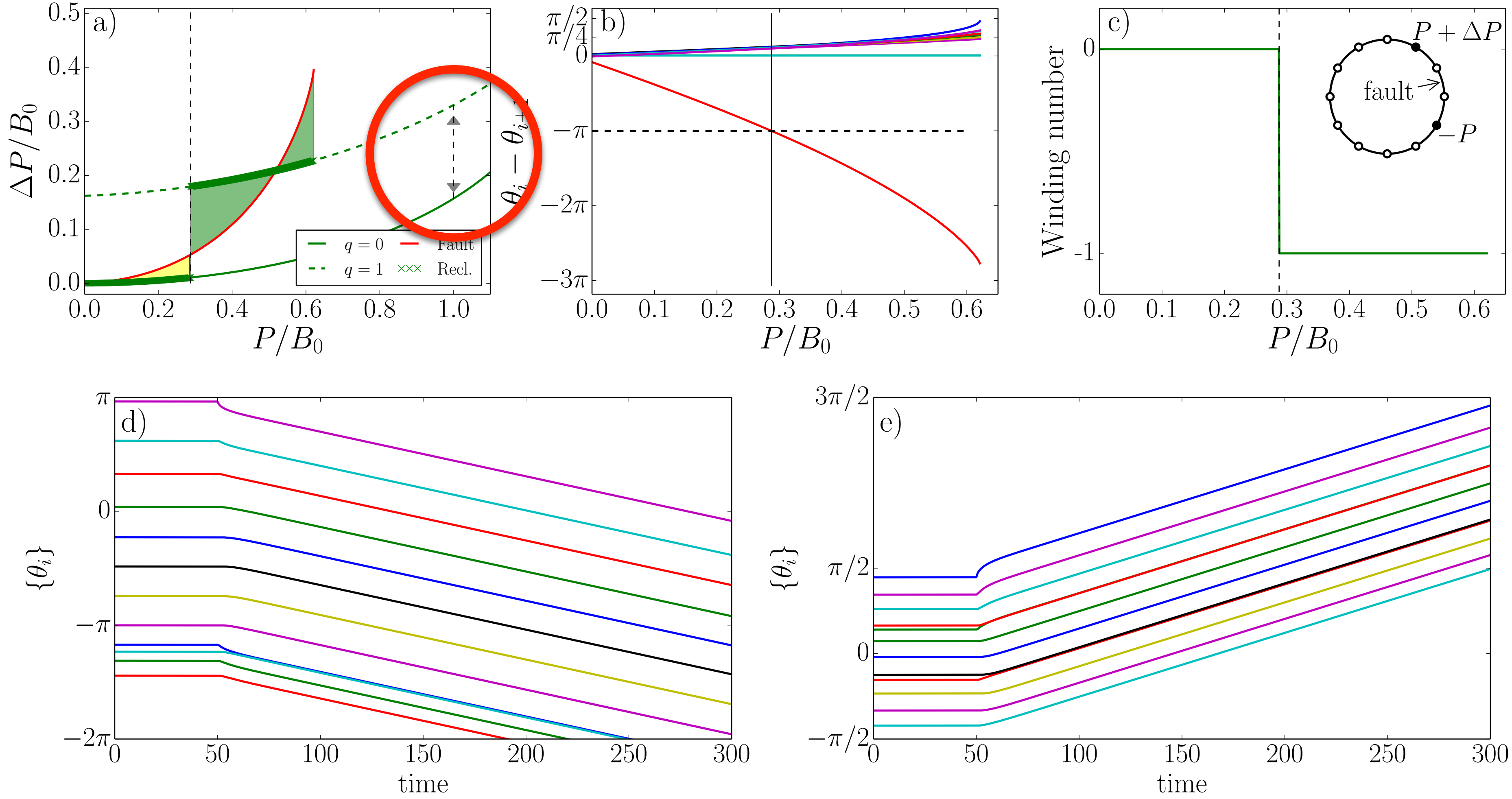
Generation of vortex flow by line tripping and reclosure



!! Cannot kill nor create vortex by adapting ΔP !!

Instead one changes the grid's frequency

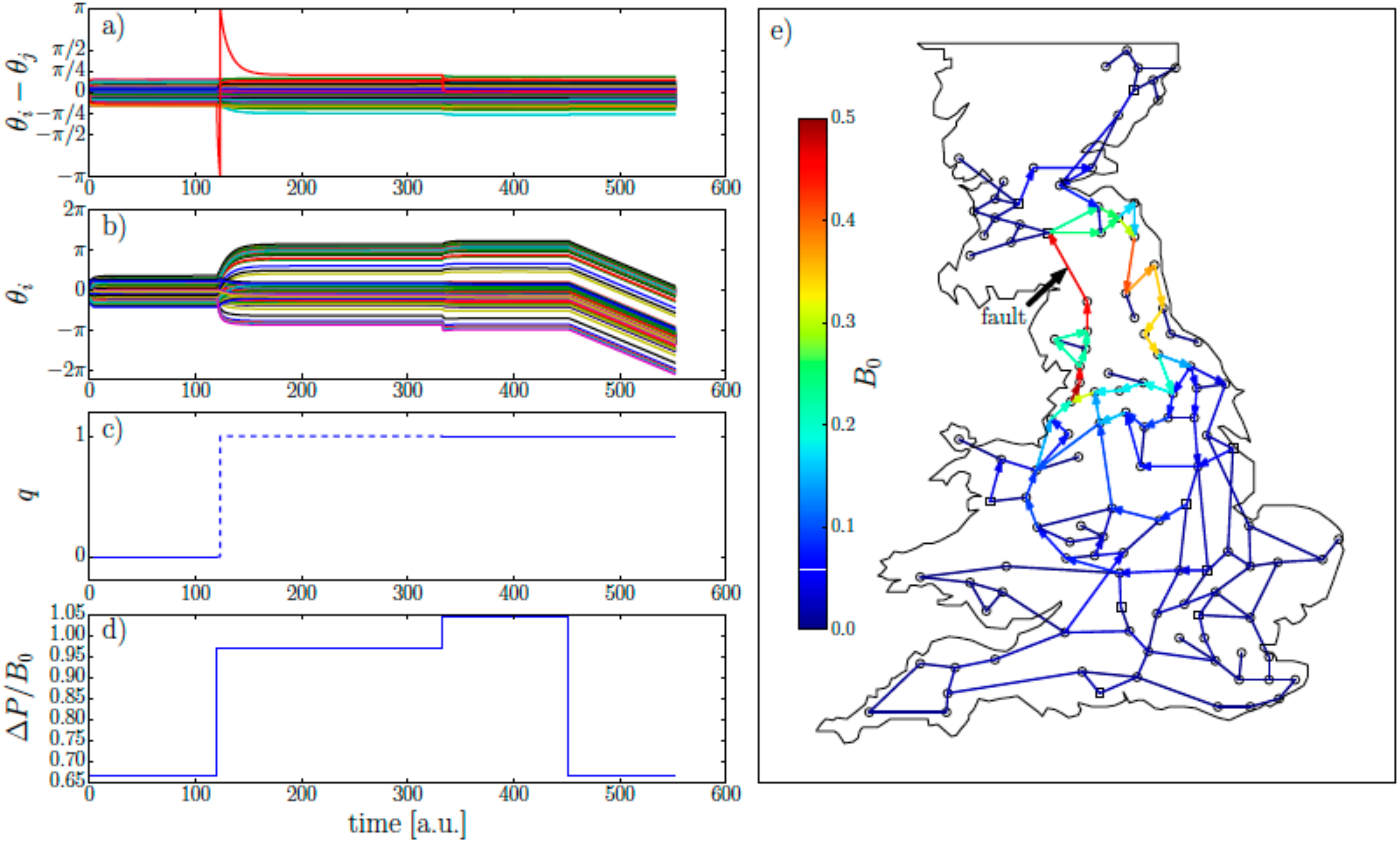
Generation of vortex flow by line tripping and reclosure



‼ Topological protection ‼

Coletta, Delabays, Adagideli and PJ, NJP '16

Generation of vortex flow by line tripping and reclosure



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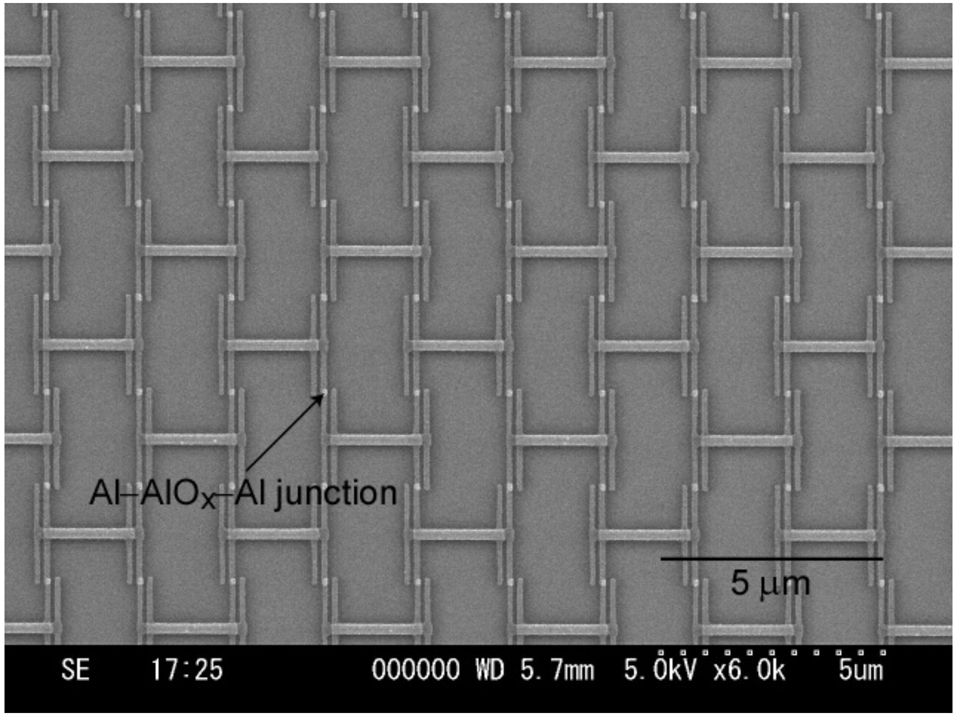
Take-home message

Profound, unexpected similarities between Josephson junction arrays and high voltage AC power grids !



dissipationless quantum fluid ⟷ *!!!!* ⟷ *dissipative classical system*

Superconductivity vs. AC electric power systems !

	Superconductor	high voltage AC power grid
State	$\Psi(\mathbf{x}) = \lvert\Psi(\mathbf{x})\rvert e^{i\theta(\mathbf{x})}$	$V_i = \lvert V_i\rvert e^{i\theta_i}$
Current / power flow	$I_{ij} = I_c \sin(\theta_i - \theta_j)$ DC Josephson current	$P_{ij} \simeq B_{ij} \sin(\theta_i - \theta_j)$ Power flow; lossless line approx.
winding # $q=\Sigma_i\lvert\theta_{i+1}-\theta_i\rvert/2\pi$	Flux quantization Persistent currents	Circulating loop flows

Hes·so// VALAIS WALLIS

Thank you !

Coletta and PJ, Phys Rev E 93, 032222 (2016)

Delabays, Coletta, and PJ, J Math Phys 57, 032701 (2016)

Coletta, Delabays, Adagideli and PJ, New J Phys ’16 (to appear)

Delabays, Coletta, and PJ, arXiv:1609.02359, submitted to J Math Phys

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Generation of vortex flow by line tripping and reclosure

*Lyapunov function - defines basins of attraction for different solutions

$$\mathcal{V}(\{\theta_i\}) = -\sum_l P_l \theta_l - \sum_{\langle l,m \rangle} B_0 \cos(\theta_l - \theta_m)$$

van Hemmen and Wreskinski '93

*Steady-state solutions have $\nabla \mathcal{V} = 0$

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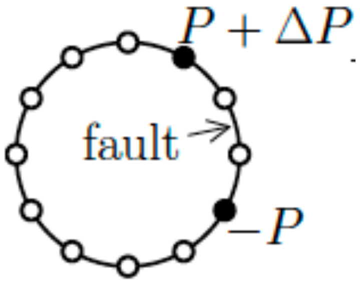
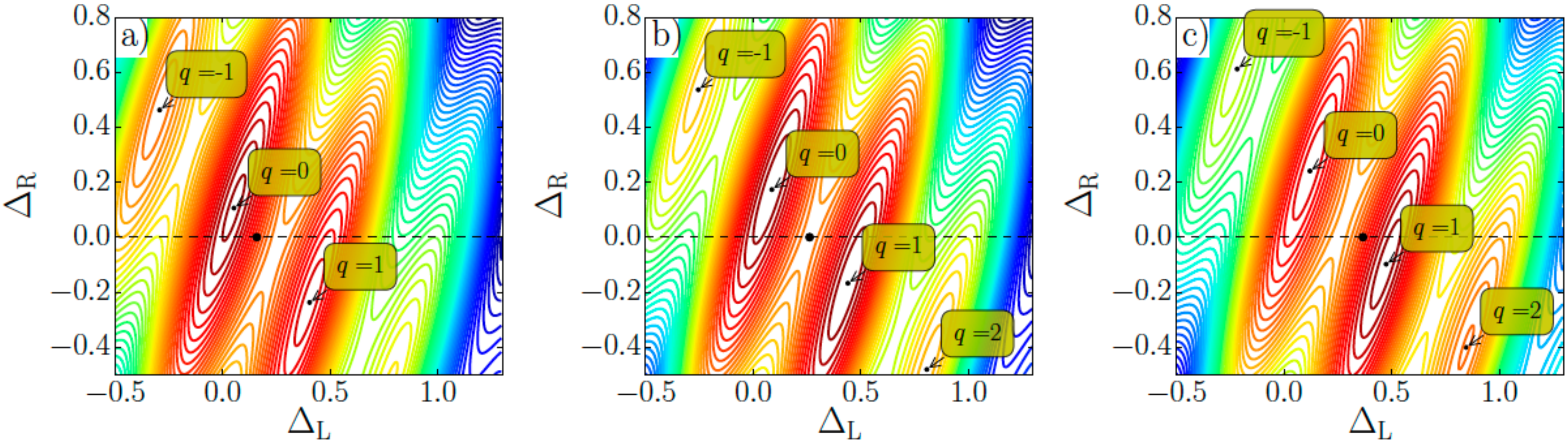


van Hemmen and Wreskinski '93

*In our case

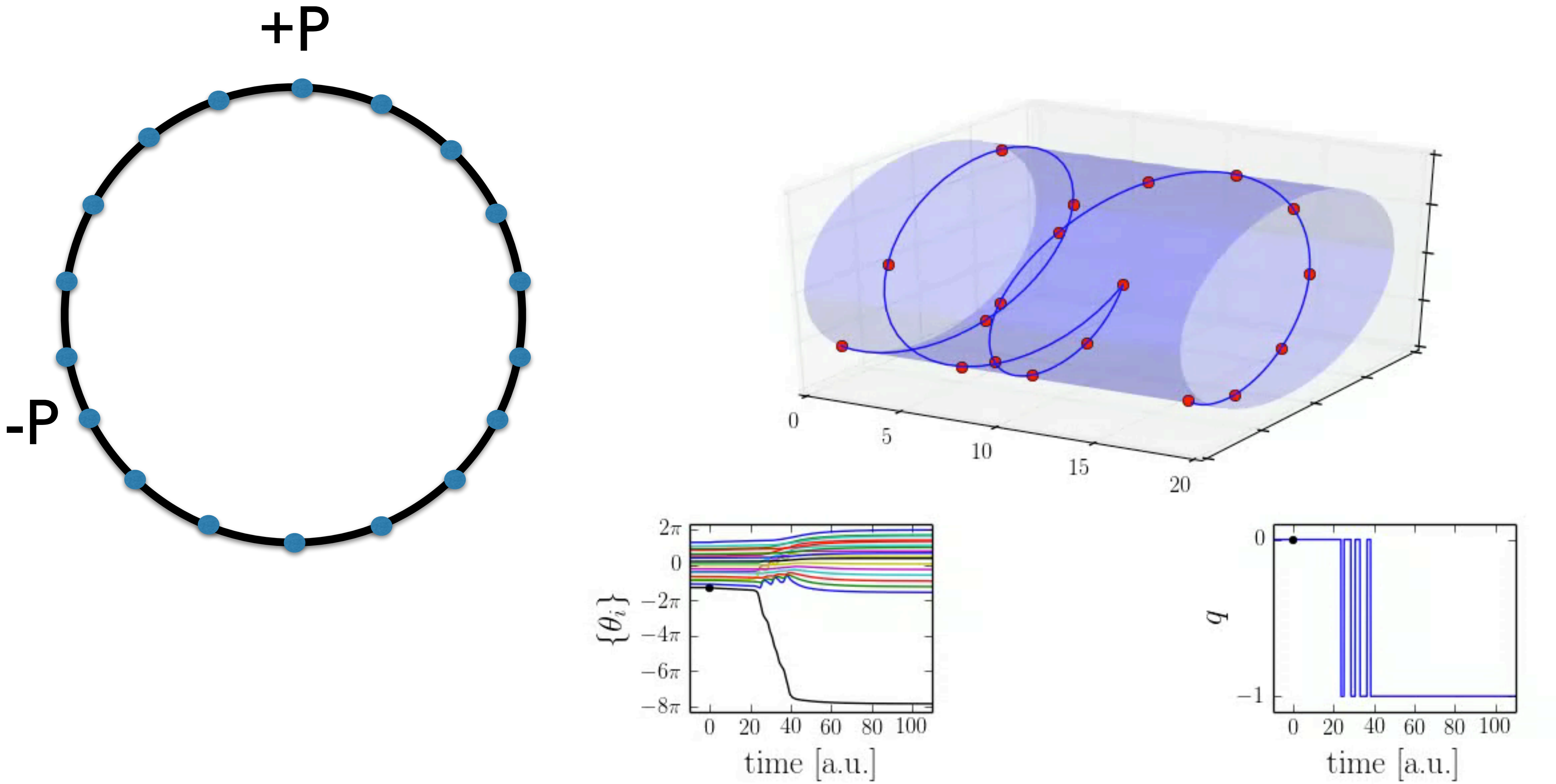
$$\mathcal{V}(\Delta_{\rm L}, \Delta_{\rm R}) = -N_{\rm L} P \Delta_{\rm L} - N_{\rm L} B_0 \cos\Delta_{\rm L} - (N_{\rm R}-1) B_0 \cos\Delta_{\rm R} - B_0 \cos(N_{\rm L}\Delta_{\rm L} - (N_{\rm R}-1)\Delta_{\rm R})$$



$P \approx 0.159 B_0$ $\quad P = B_0 \sin(\pi/12) \approx 0.259 B_0$ $\quad P \approx 0.359 B_0$

Coletta, Delabays, Adagideli and PJ, NJP '16

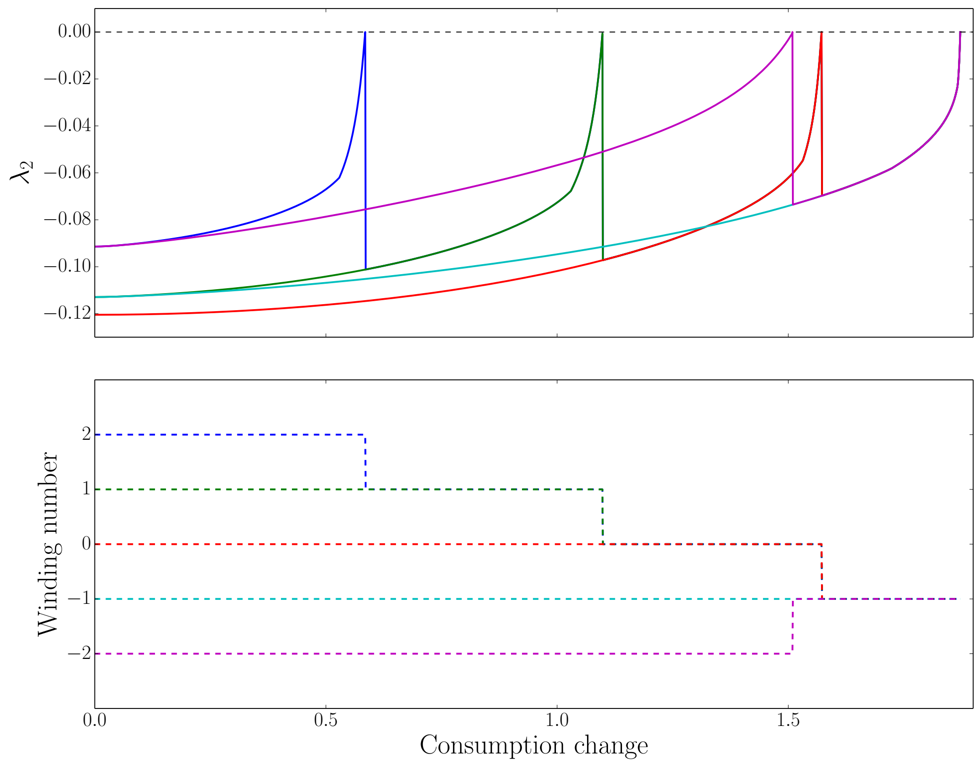
Dynamical generation of vortex flows

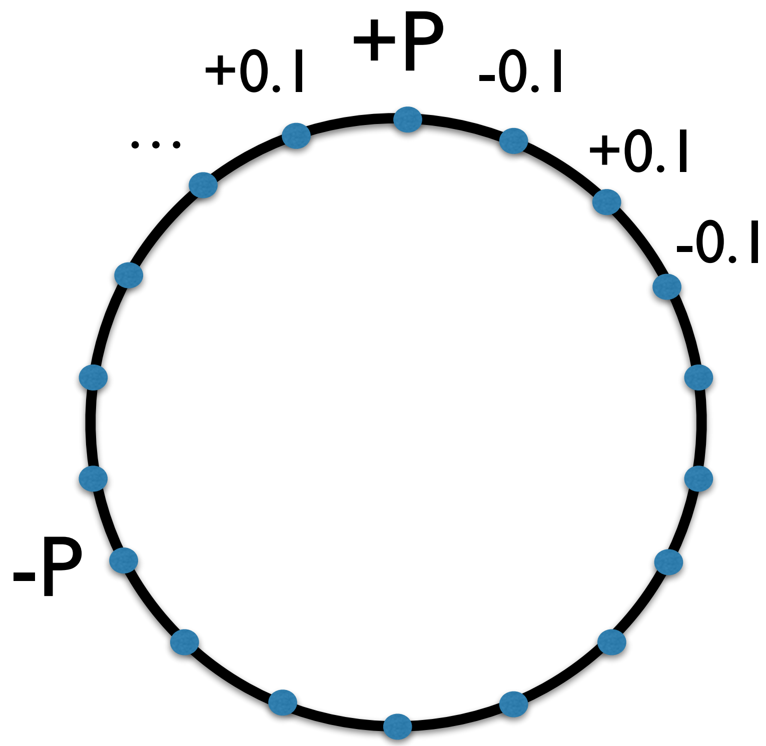


Similar to quantum phase slips in JJ arrays
Lau, Markovic, Bockrath, Bezryadin and Tinkham '01
Matveev, Larkin and Glazman '02

Dynamical generation of vortex flows

Loss of stability of q=0 solution





Coletta, Delabays, Adagideli and PJ, NJP '16

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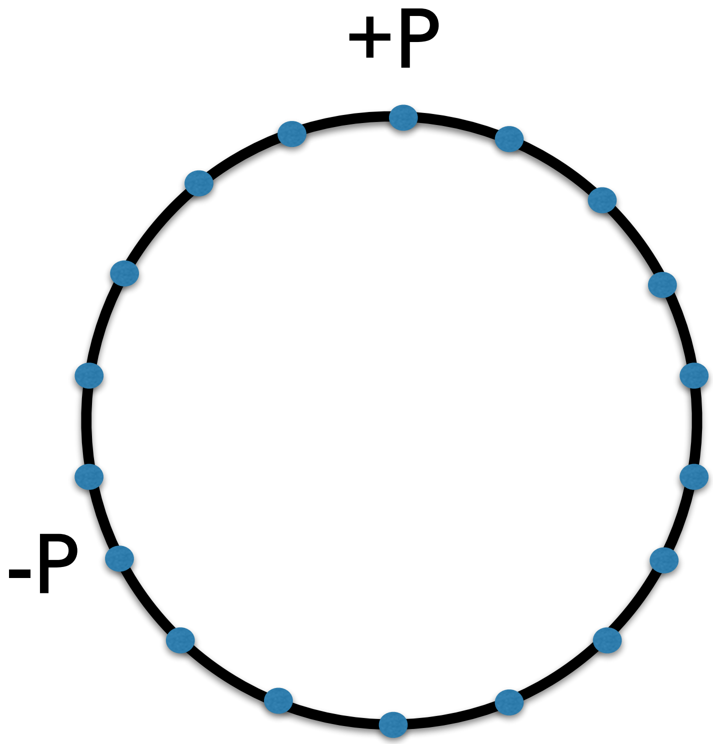

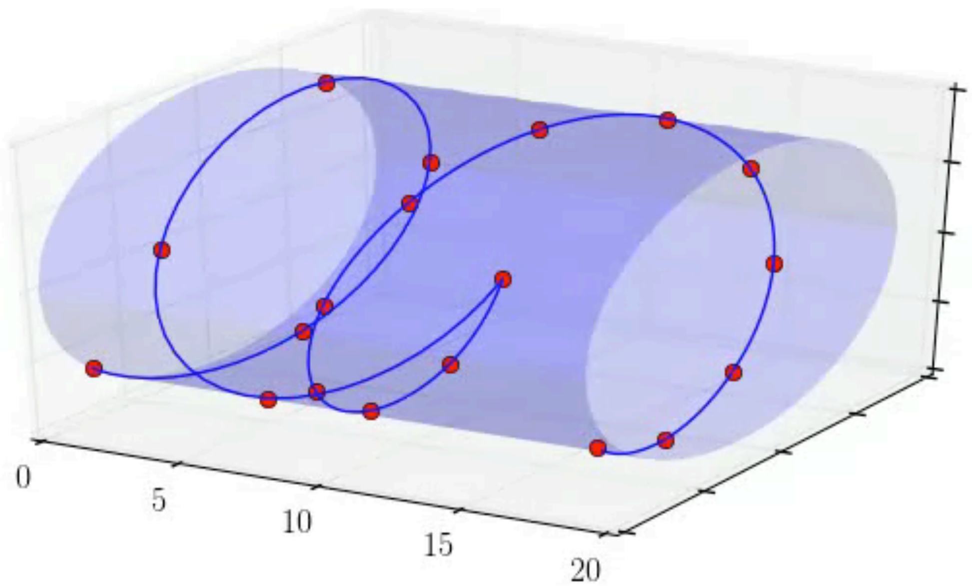

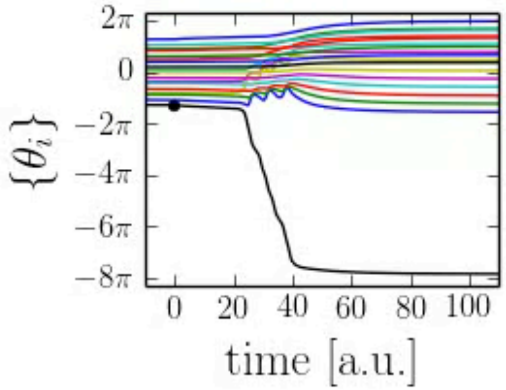

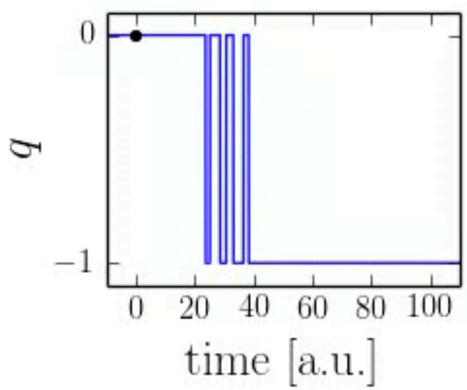

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