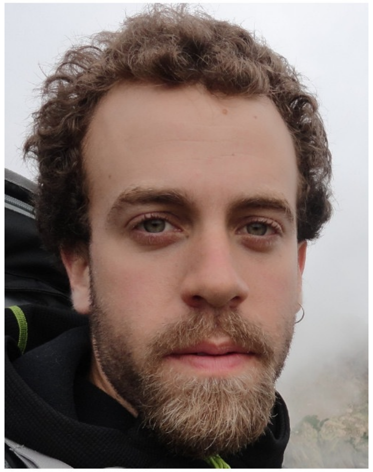
Topologically Protected Vortex Flows in High Voltage AC Power Grids

Philippe Jacquod
CONDYNET - Jacobs Universität Bremen - 24.06.2016

Delabays, Coletta, Adagideli and PJ, arXiv:1605.07925
Delabays, Coletta and PJ, J Math Phys 57, 032701 (2016)
Coletta and PJ, Phys Rev E 93, 032222 (2016)

The team

Tommaso Coletta, postdoc

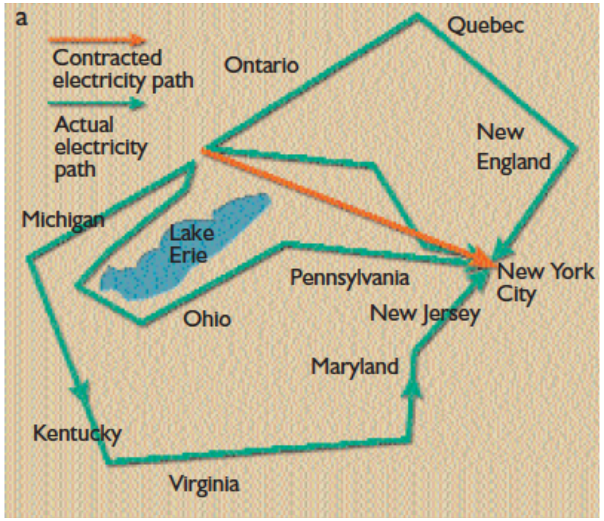


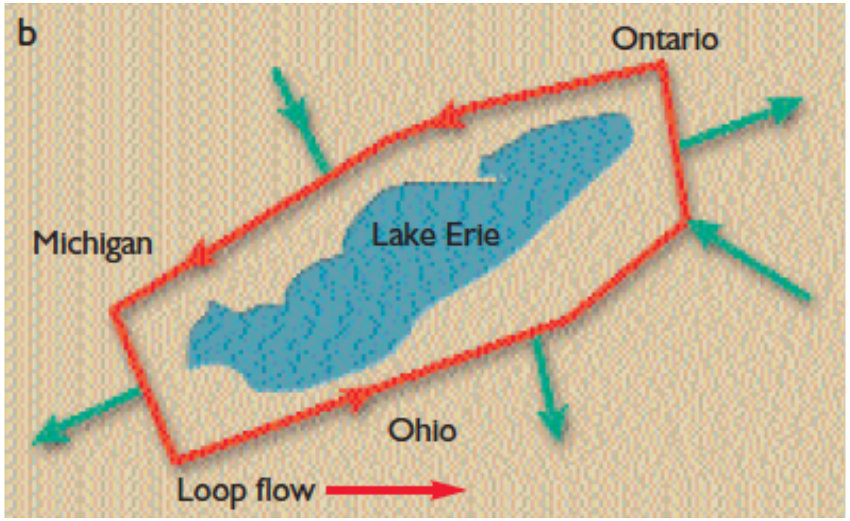
Robin Delabays, PhD student

Inanc Adagideli (Sabanci)

The problem

"Electric power does not follow a specified path but divides among transmission routes based on Kirchhoff's laws and network conditions at the time. This pattern results in a phenomenon called circulating power of which there are two types : loop flow and parallel flow."





"Where the network jogs around large geographical obstacles, such as the Rocky Mountains in the West or the Great Lakes in the East, loop flows around the obstacle are set up that can drive as much as 1 GW of power in a circle, *taking up transmission line capacity without delivering power to consumers.*"

Casazza, Electrical World (1998)
Lerner, The industrial physicist (2003)

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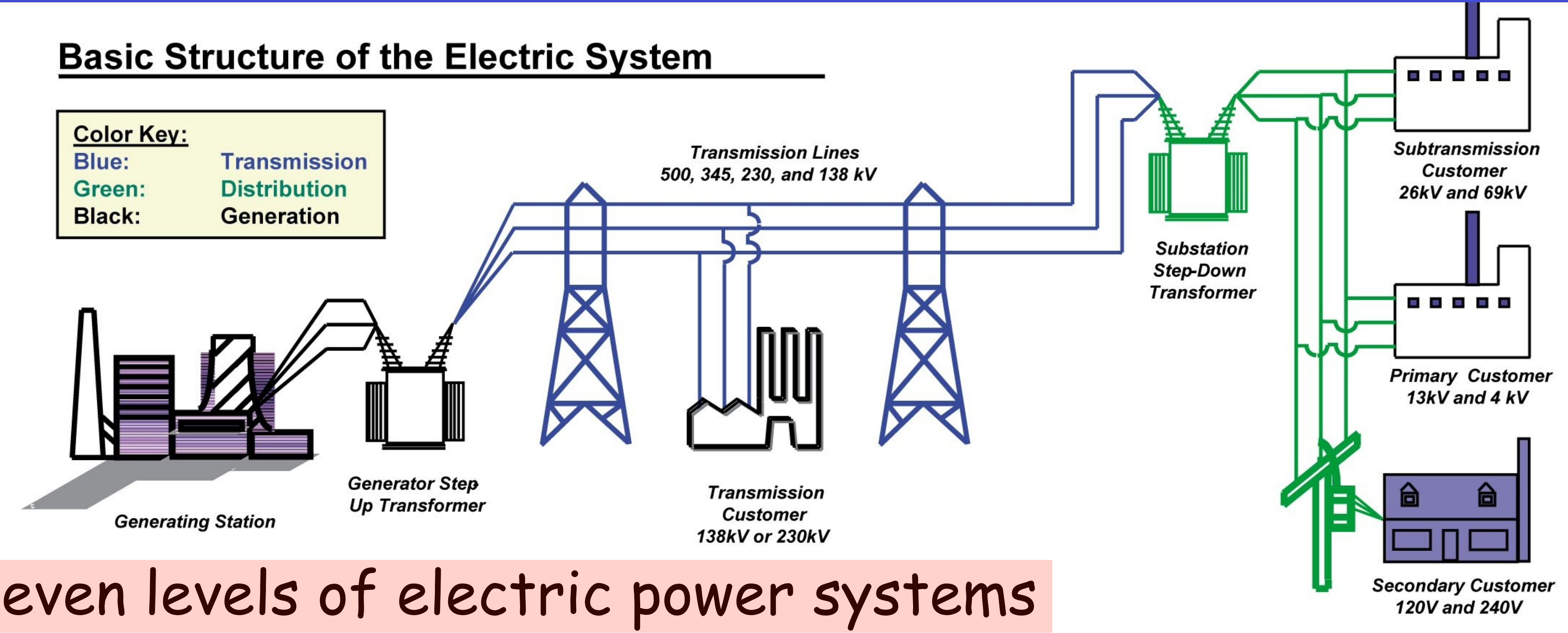
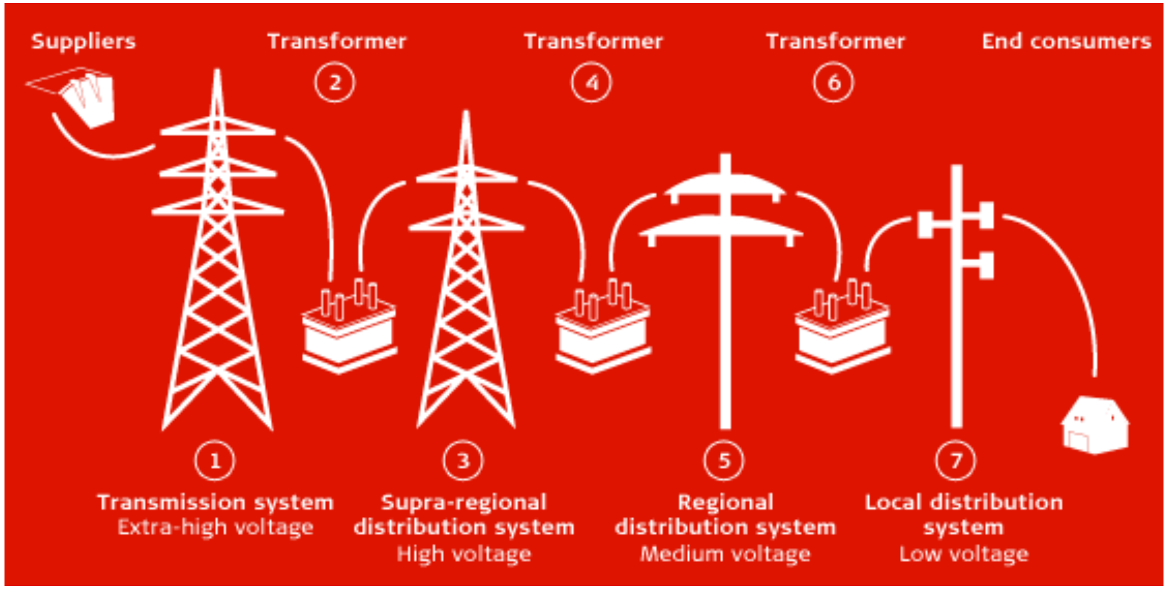
Hes·so// VALAIS WALLIS

What are electric power systems ?



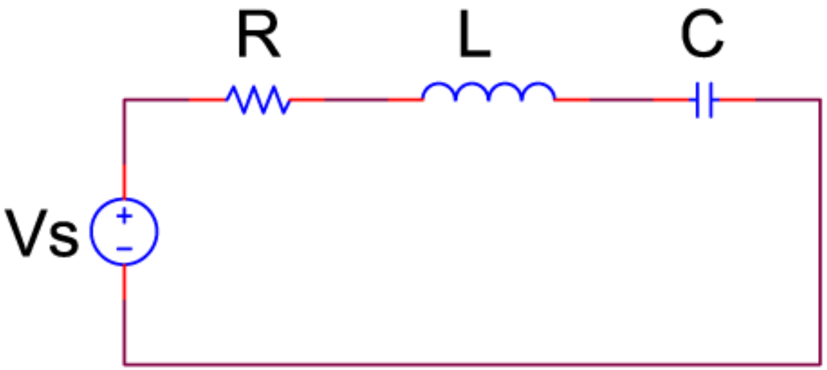
Seven levels of electric power systems



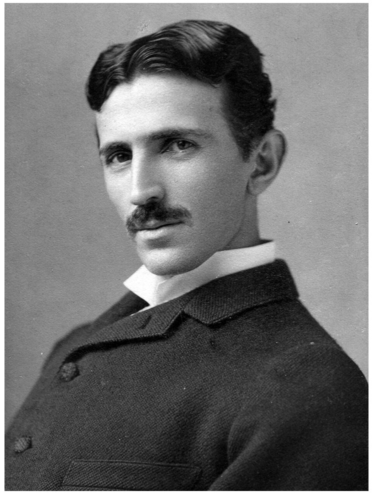
Power :
*conserved from one level to another (~)
*control parameter
"write Eq. for power"

What are electric power systems ?

- AC electric current/voltages
  (minimize losses ~ high voltages,
  but then need transformers)
- current and voltage not in phase



$$u(t) = u_0 \exp[i\omega t]$$
$$i(t) = i_0 \exp[i(\omega t + \phi)]$$
$$\tan(\phi) = (\omega L - 1/\omega C)/R$$



N Tesla 1856-1943

- complex impedance $\quad u(\omega) = Z(\omega)\, i(\omega) \qquad Z(\omega) = R + i\omega L - i/\omega C$
- inductance more important than resistance for large conductors

Steady-state AC transport

- linear relation between currents and voltages

$$\boxed{I_i = \sum_j Y_{ij} V_j}$$

Y_{ij} : admittance matrix
Kirchhoff's current law

- Complex power : $S(t) = u(t) \times i^*(t)$

- Active power vs. reactive power

Active power	reactive power
P=Re(S)	Q=Im(S)
finite time-average	zero time-average
"truly transmitted" (injected and consumed)	"oscillating in the circuit"

Power flow equations

(power is conserved upon voltage transformation)

$$P_i = \sum_j |V_i V_j| \left[G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \right]$$

$$Q_i = \sum_j |V_i V_j| \left[G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \right]$$

Voltages at buses i and j

Conductance matrix

Phases at buses i and j

Susceptance matrix

Approximated power flow equations : (1) lossless line

- Admittance dominated by its imaginary part
  for large conductors ~ high voltage
  G/B < 0.1 for 200kV and more

neglect conductance

$$P_i \simeq \sum_j |V_i V_j| B_{ij} \sin(\theta_i - \theta_j)$$

$$Q_i \simeq -\sum_j |V_i V_j| B_{ij} \cos(\theta_i - \theta_j)$$

- No conductance ~no voltage drop

consider constant voltage

* decoupling between P and Q
* consider P only

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

- No loss = balance of power

$$\sum_i P_i = 0$$

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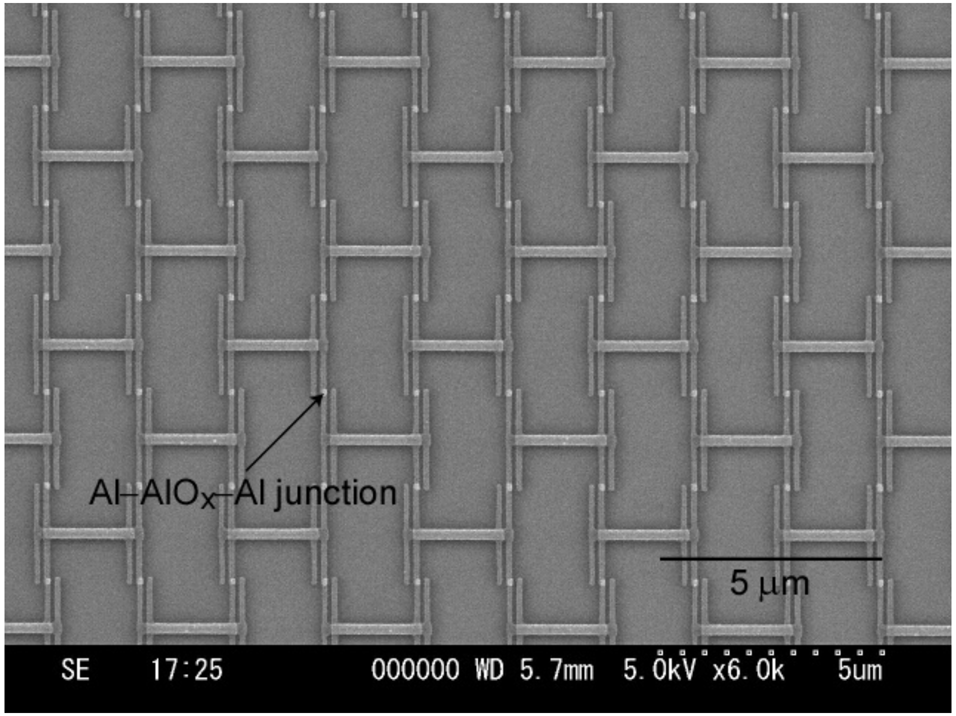
Josephson junction arrays vs. electric power systems !



Takahide, Yagi, Kanda, Ootuka, and Kobayashi
Phys. Rev. Lett. 85, 1974 (2000)

Josephson current

$$I_{ij} = I_c \sin(\theta_i - \theta_j)$$

Transmitted power

$$P_{ij} \simeq B_{ij} \sin(\theta_i - \theta_j)$$

First question

- Consider the power flow problem in the lossless line approximation

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

- It is entirely defined by

*the graph/network/grid (admittance matrix B_{ij})
*the set of power injections/consumptions $\{P_i\}$

Question : how many different solutions are there ?

First answer : an infinite # of them, since $\{\theta_i + C\}$ is also a solution for any constant C

Define "different" as differing by more than C

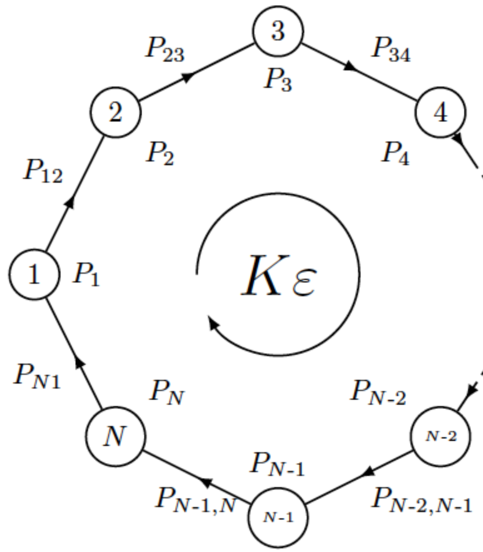
Circulating loop flows

*Thm: Different solutions to the following power-flow problem

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

may differ only by circulating loop current(s) **in any network**

Dörfler, Chertkov, Bullo, PNAS '13; Delabays, Coletta and PJ, JMP '16



*Voltage angle uniquely defined

→ $q=\Sigma_i|\theta_{i+1}-\theta_i|/2\pi \in \mathbb{Z}$ **~topological winding number**

→ "quantization" of these loop currents **~vortex flows**

Janssens and Kamagate '03

Hes·so// VALAIS WALLIS

Circulating loop flows

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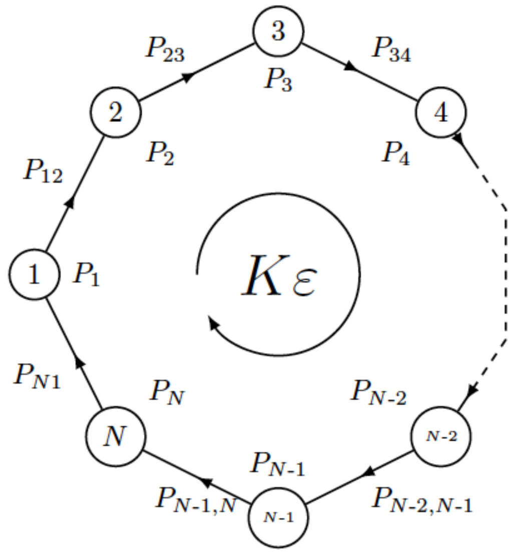

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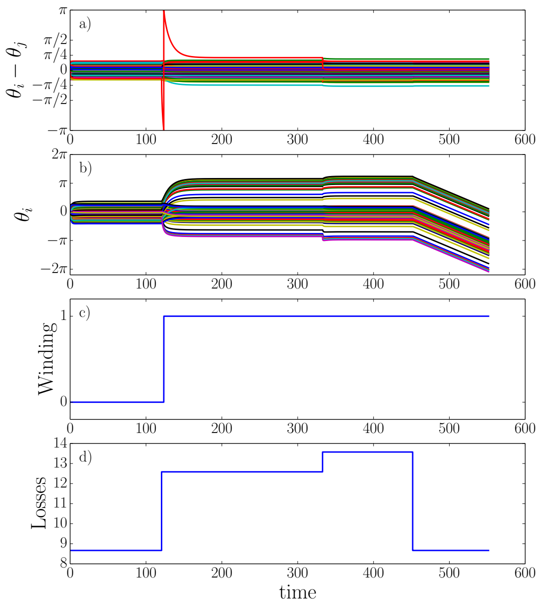

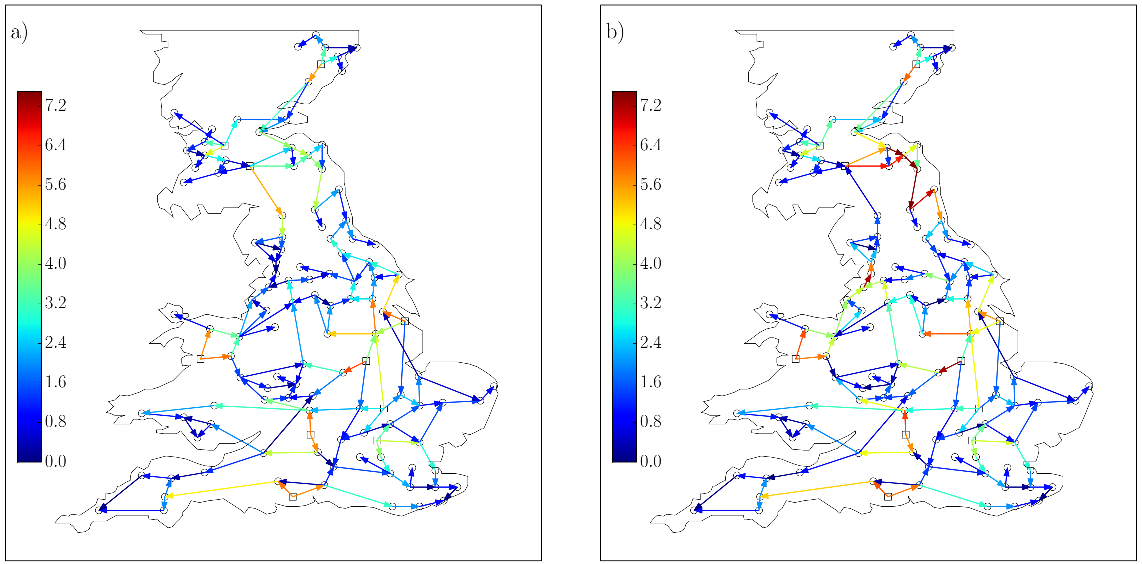

Circulating loop flows

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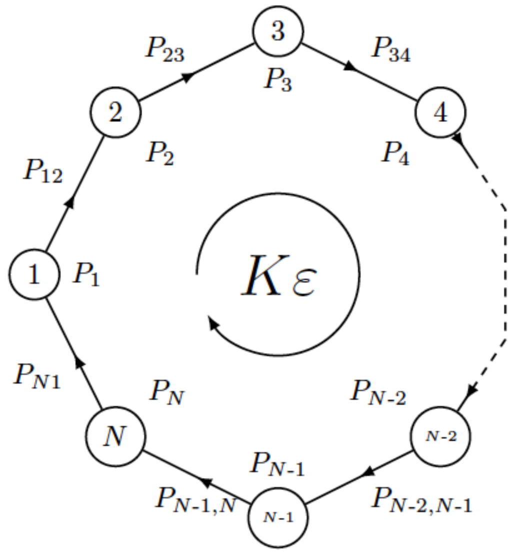

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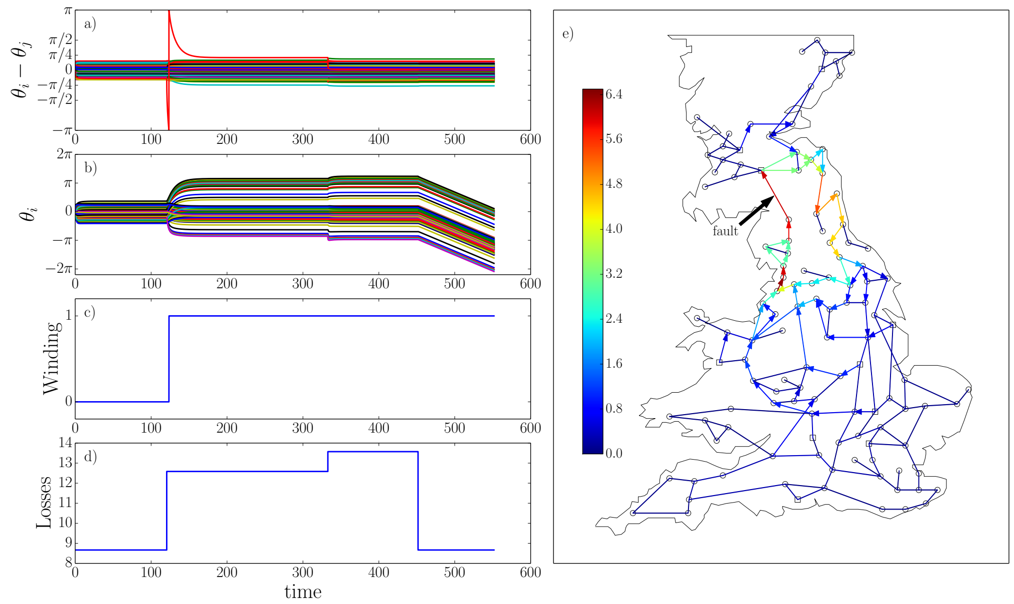
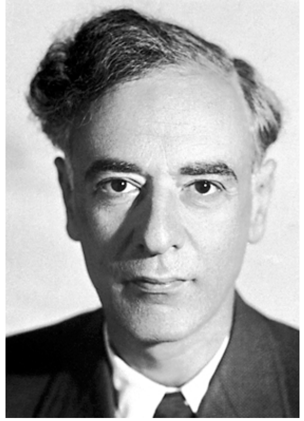


Vortices in superfluids and quantization of circulation



L Landau
1908-1968

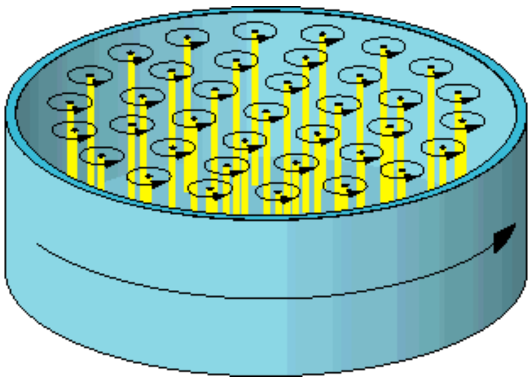
- Landau theory of superfluidity - macroscopic wavefunction

$$\psi(\mathbf{x}) = |\psi(\mathbf{x})| e^{i\phi(\mathbf{x})}$$

- Superfluid current

$$\boxed{\mathbf{J}_{\mathrm{SF}} = \frac{\hbar|\Psi|^2}{M_{\mathrm{SF}}}\nabla\phi}$$

- Vortex : topological defect with SF -> 0 in center
- Contour around that center



-> quantization of circulation

$$\oint \mathbf{J}_{\mathrm{SF}} \mathrm{d}\mathbf{l} = \frac{\hbar|\Psi|^2}{M_{\mathrm{SF}}}(\phi_+ - \phi_-) = \frac{\hbar|\Psi|^2}{M_{\mathrm{SF}}} 2\pi m \qquad m \in Z$$

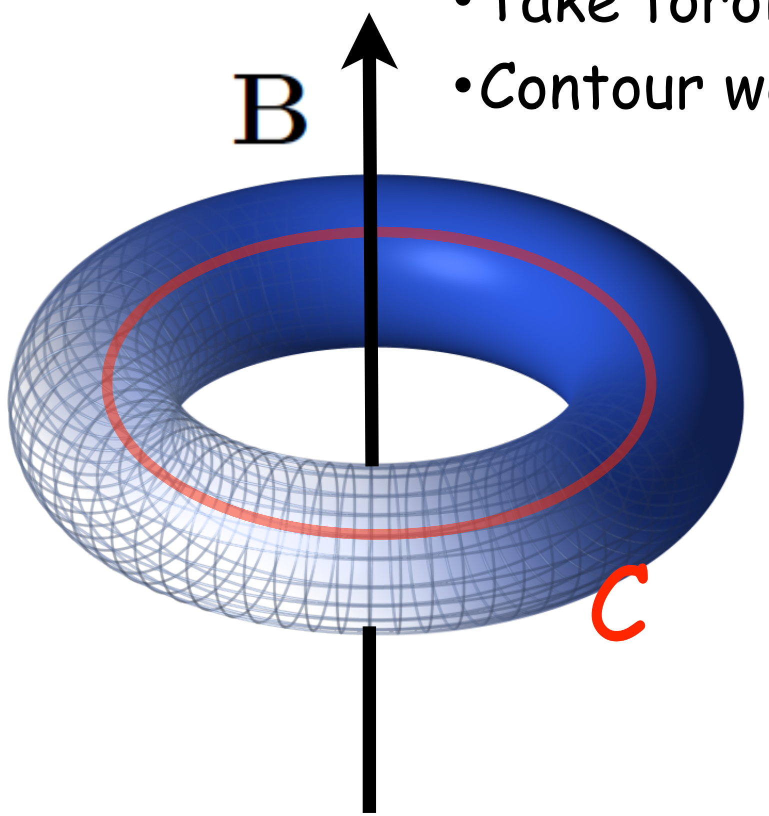
Th. : Onsager '49, Feynman '55

Topological number : flux quantization with SC

•Landau theory of superconductivity - macroscopic wavefunction

$$\psi(\mathbf{x}) = |\psi(\mathbf{x})|e^{i\phi(\mathbf{x})}$$

•Gauge-invariant current $$\mathbf{J}_s = \frac{e\hbar}{2m} n_s \left(\nabla\phi - \frac{2e}{\hbar}\mathbf{A} \right)$$



•Take toroidal SC pierced by B-field

•Contour well inside SC : Meissner effect

$$\mathbf{B}\big|_C = \mathbf{J}_s\big|_C \equiv 0$$

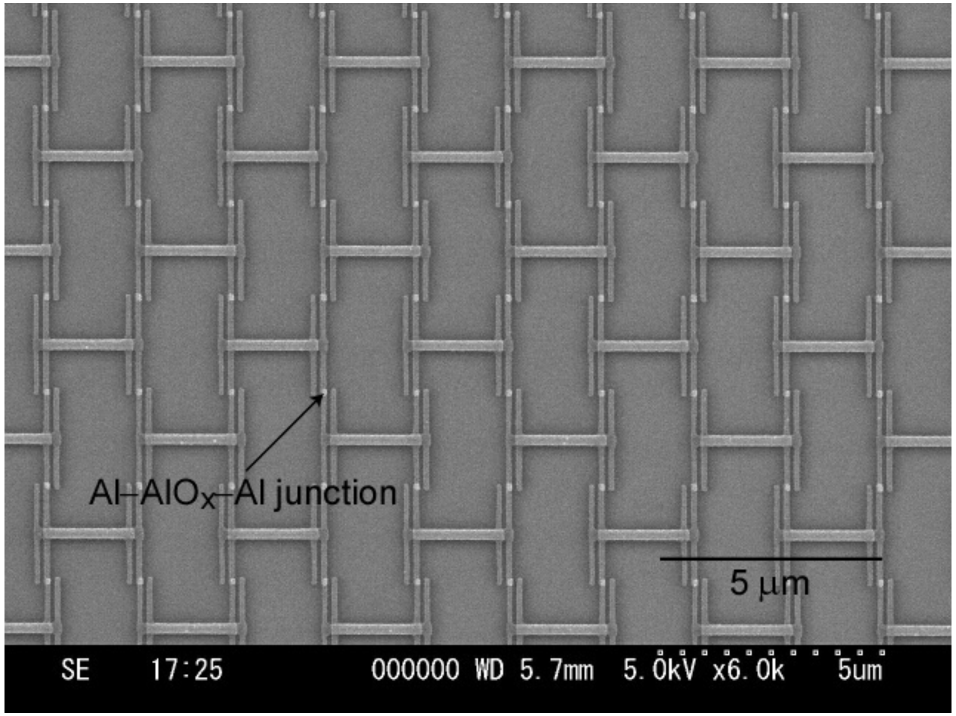
-> $$\oint_C \mathbf{A}d\mathbf{l} = \int\int_{\partial C} \mathbf{B}d\mathbf{f} = \varphi$$

-> $$\oint_C \mathbf{A}d\mathbf{l} = \frac{\hbar}{2e}(\phi_+ - \phi_-) = m\frac{h}{2e}$$

-> flux quantization $$\boxed{\varphi = m\varphi_0}$$

Exps.: Deaver and Fairbanks '61 (Sn cylinders); Gough et al. '87 (high Tc)

Josephson junction arrays vs. electric power systems !



Josephson current

$$I_{ij} = I_c \sin(\theta_i - \theta_j)$$

dissipationless quantum fluid

????

Transmitted power

$$P_{ij} \simeq B_{ij} \sin(\theta_i - \theta_j)$$

dissipative classical system

Approximated power flow equations : (2) losses to leading order

- Keep decoupling between P and Q

put back conductance

$$P_i = \sum_j [B_{ij} \sin(\theta_i - \theta_j) + G_{ij} \cos(\theta_i - \theta_j)]$$

- Losses are positive

$$\sum_i P_i = \sum_{ij} G_{ij} \cos(\theta_i - \theta_j)$$

$$\sum_i P_i > 0$$

Remark (important) :
$\{P_i\}$ and $\{\theta_i\}$ need to be self-consistently determined

Vortex flows in AC power grids vs. superconductivity

- Vortex/circulation creation in SC via B-field

Can one create vortex flows in AC power grids ?
How ?

- Superconductors are dissipationless
- AC power grids are dissipative ~ they have a finite resistance

Do vortex flows survive the presence of dissipation ?

Vortex flows in AC power grids vs. superconductivity

- Vortex/circulation creation in SC via B-field

Can one create vortex flows in AC power grids ?
How ?

*Three mechanisms : *dynamical phase slip*
**line tripping*
**line tripping and reclosure*

- Superconductors are dissipationless
- AC power grids are dissipative ~ they have a finite resistance

Do vortex flows survive the presence of dissipation ?

YES ! They are robust against moderate amounts of dissipation

Generation of vortex flow by line tripping

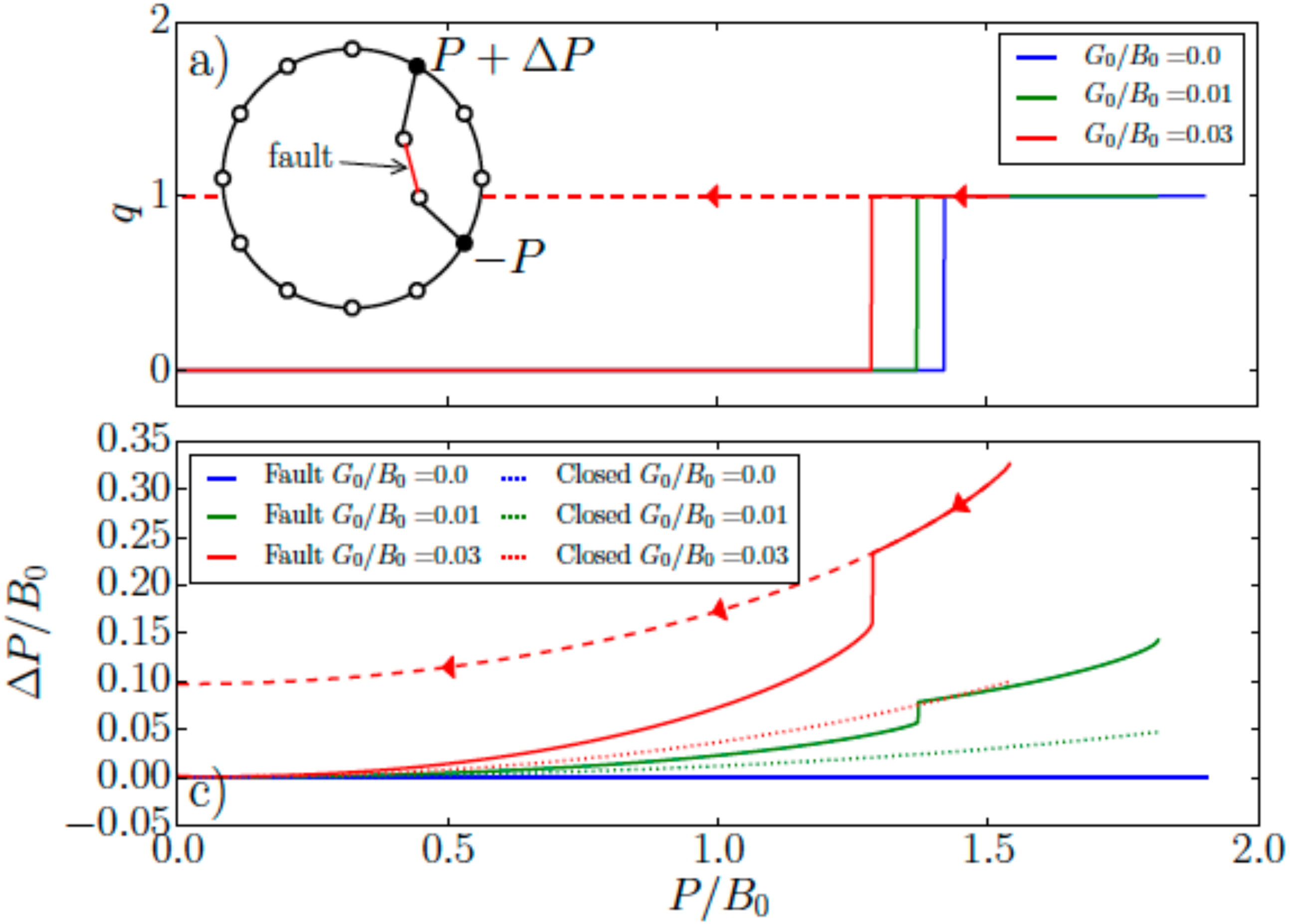
*Power grids are meshed - path redundancy
*Power is still supplied after one line trips
*Consider line tripping in an asymmetric, three-path circuit
*all winding numbers vanish initially

Power redistribution can lead to vortex flow with $q=\Sigma_i|\theta_{i+1}-\theta_i|/2\pi > 0$

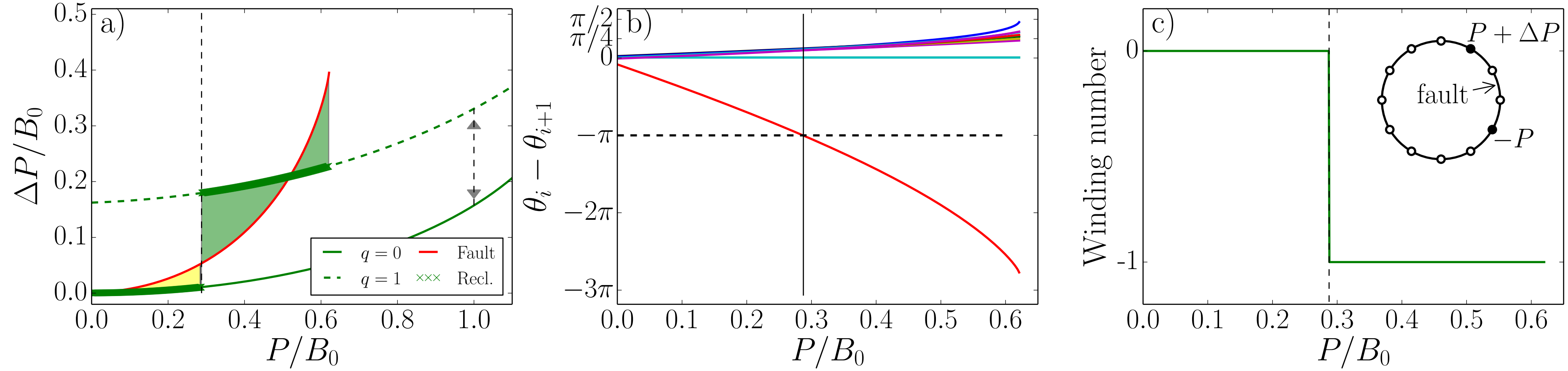
Line tripping at $P/B_0 > 1.3$

→ q=1

Vortex state characterized by
-hysteresis
  = **topological protection**
-higher losses



Generation of vortex flow by line tripping and reclosure



- Single-loop network
- One producer, one consumer
- One line trips, is later reclosed

Vortex formation for $P/B_0 > 0.26$

Vortex formation for $|\theta_{i+1}-\theta_i|>\pi$ (two ends of faulted line)

Generation of vortex flow by line tripping and reclosure

*Lyapunov function - defines basins of attraction for different solutions

$$\mathcal{V}(\{\theta_i\}) = -\sum_l P_l \theta_l - \sum_{\langle l,m \rangle} B_0 \cos(\theta_l - \theta_m)$$

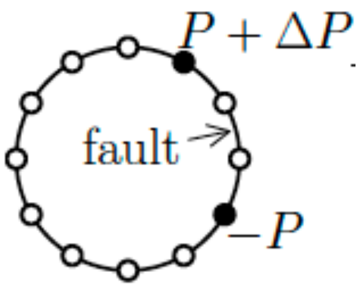
van Hemmen and Wreskinski '93

*Steady-state solutions have $\nabla \mathcal{V} = 0$

Generation of vortex flow by line tripping and reclosure

*Lyapunov function - defines basins of attraction for different solutions

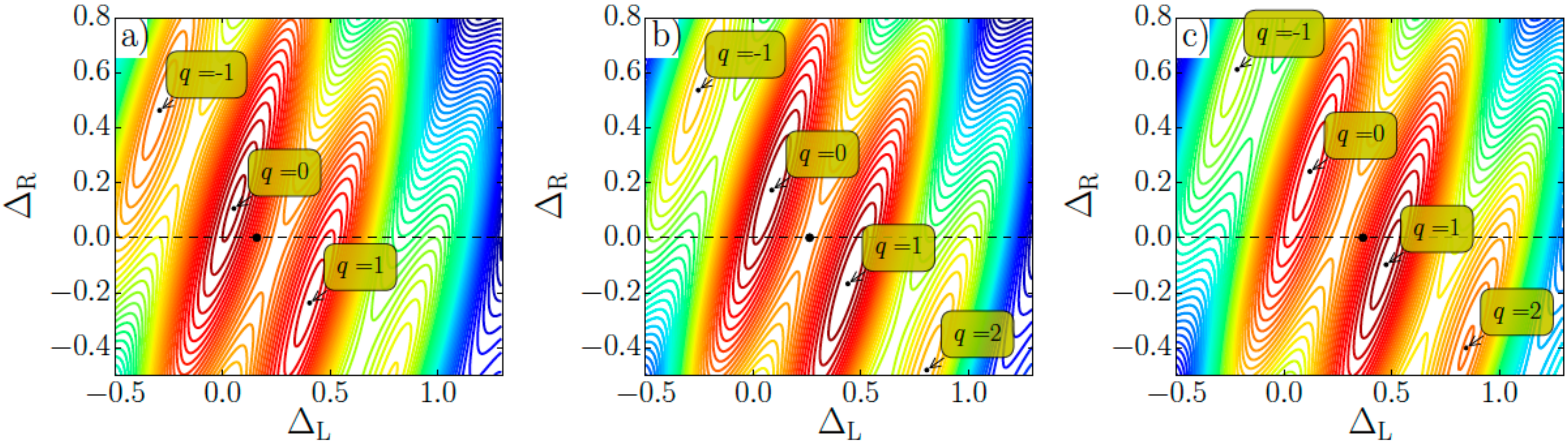
$$\mathcal{V}(\{\theta_i\}) = -\sum_l P_l\theta_l - \sum_{\langle l,m\rangle} B_0\cos(\theta_l - \theta_m)$$



van Hemmen and Wreskinski '93

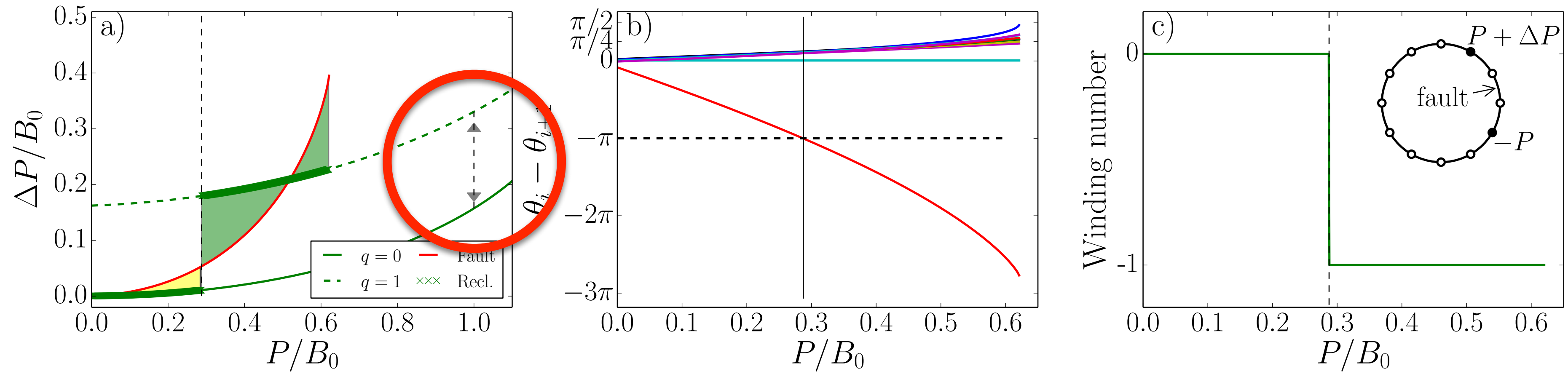
*In our case

$$\mathcal{V}(\Delta_\mathrm{L}, \Delta_\mathrm{R}) = -N_\mathrm{L}P\Delta_\mathrm{L} - N_\mathrm{L}B_0\cos\Delta_\mathrm{L} - (N_\mathrm{R}-1)B_0\cos\Delta_\mathrm{R} - B_0\cos(N_\mathrm{L}\Delta_\mathrm{L} - (N_\mathrm{R}-1)\Delta_\mathrm{R})$$



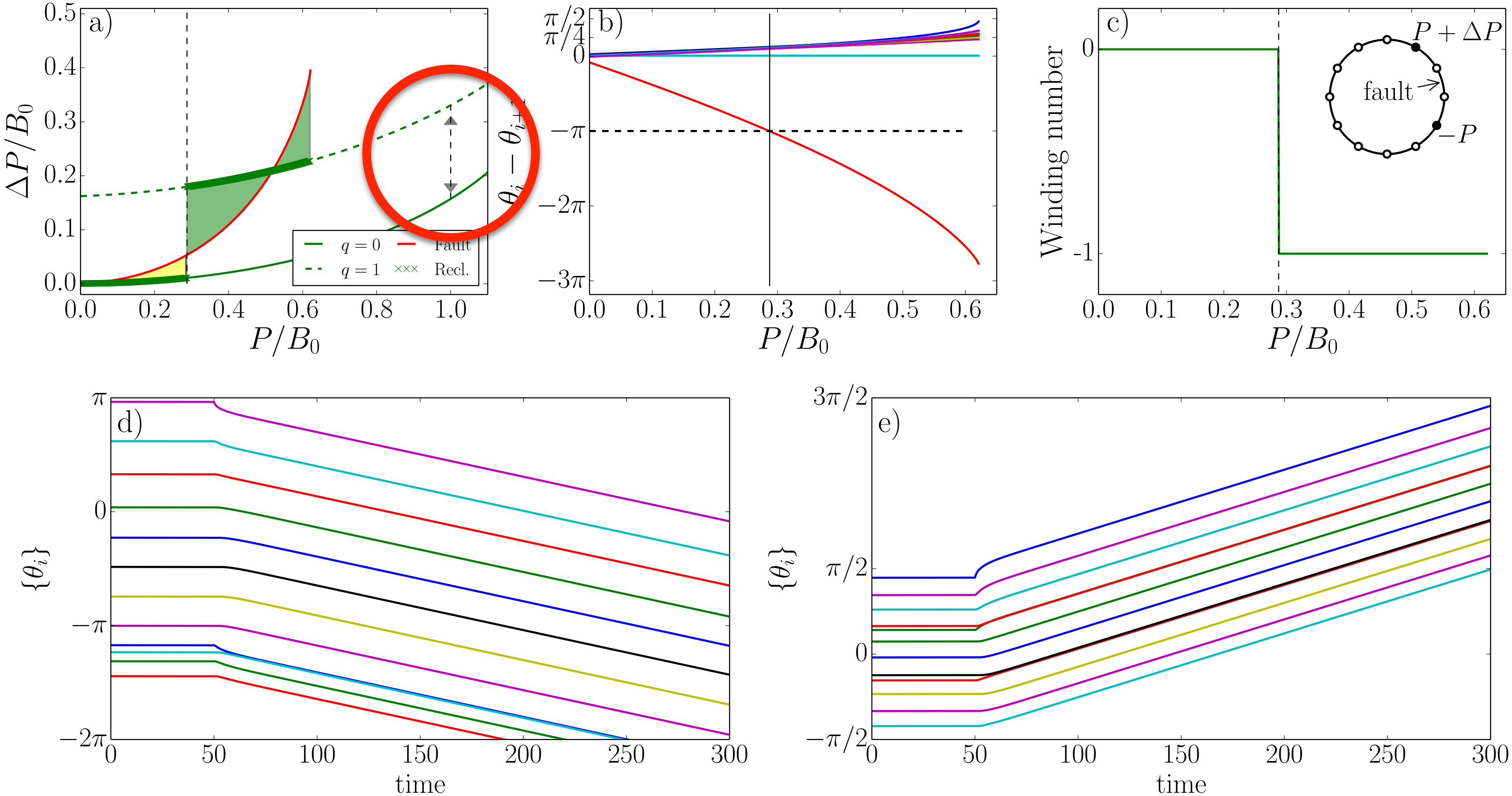
$P \approx 0.159B_0$ $\quad$ $P = B_0\sin(\pi/12) \approx 0.259B_0$ $\quad$ $P \approx 0.359B_0$

Generation of vortex flow by line tripping and reclosure



Try to kill / create vortex by adapting ΔP ?

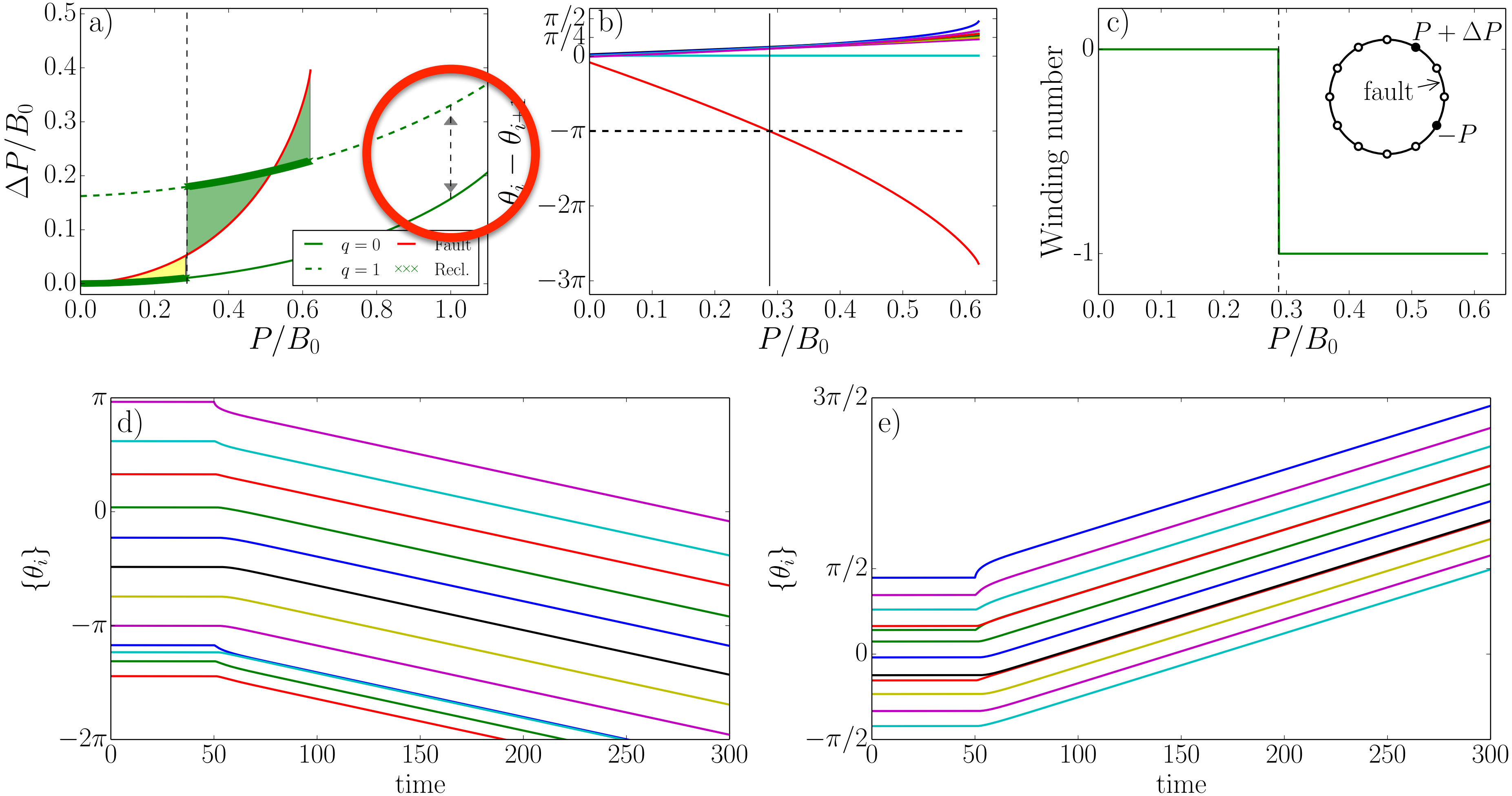
Generation of vortex flow by line tripping and reclosure



!! Cannot kill nor create vortex by adapting ΔP !!

Instead one changes the grid's frequency

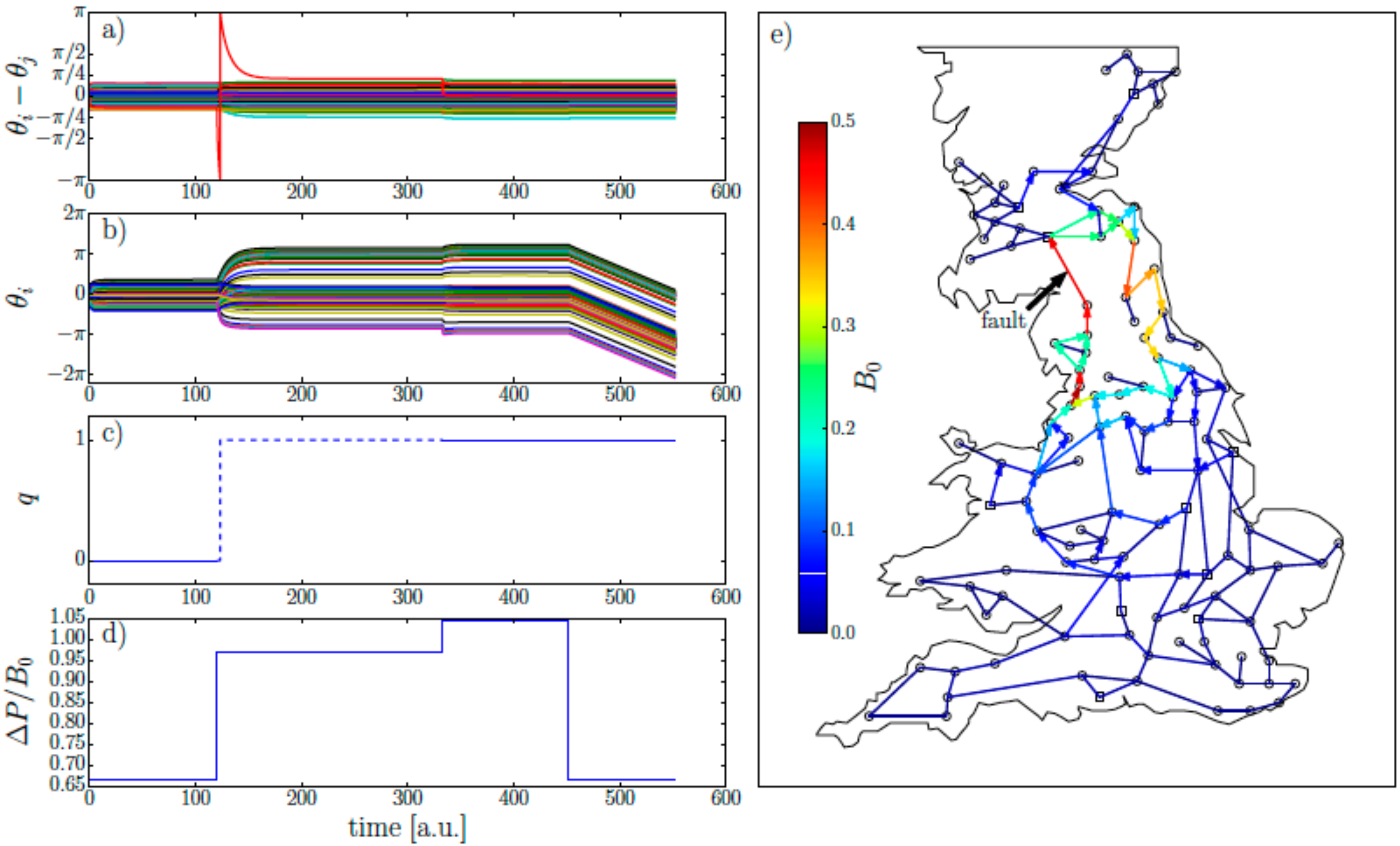
Generation of vortex flow by line tripping and reclosure



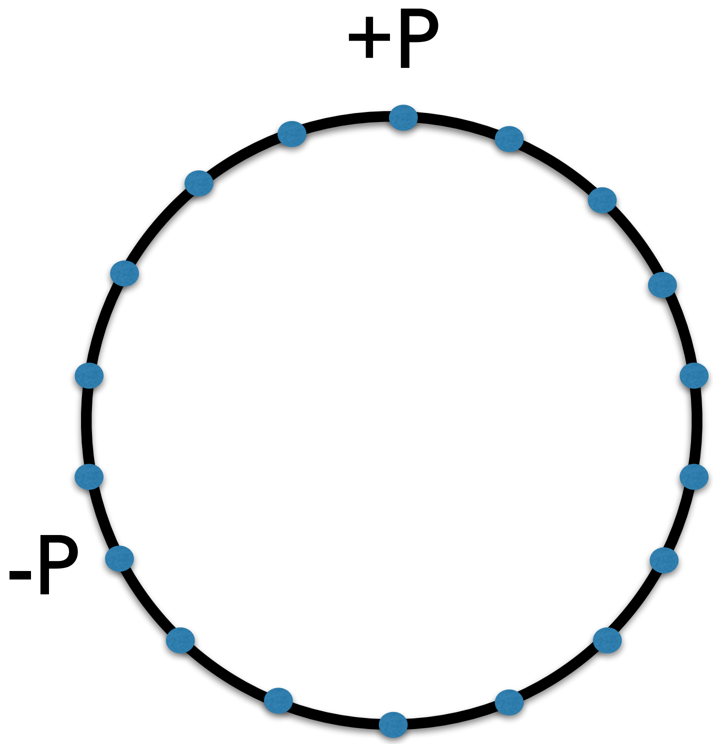
!! Cannot kill nor create vortex by adapting ΔP !!

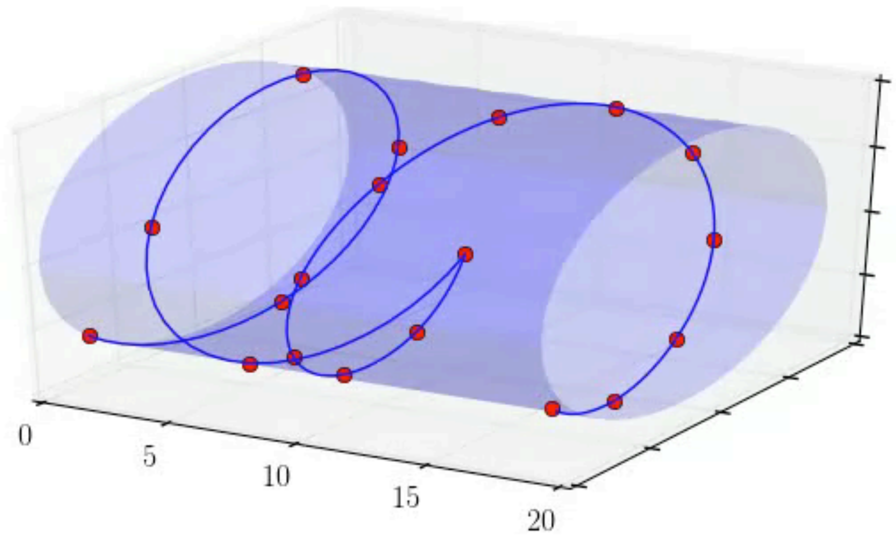
!! Topological protection !!

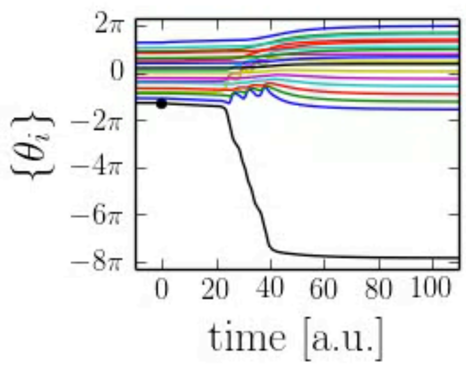
Generation of vortex flow by line tripping and reclosure

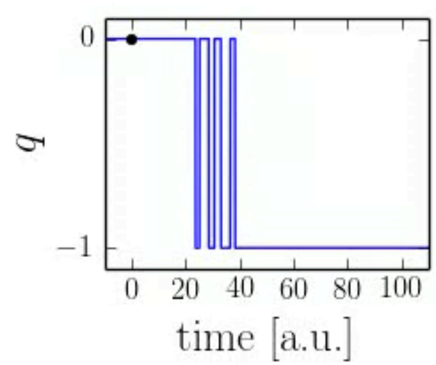


Dynamical generation of vortex flows





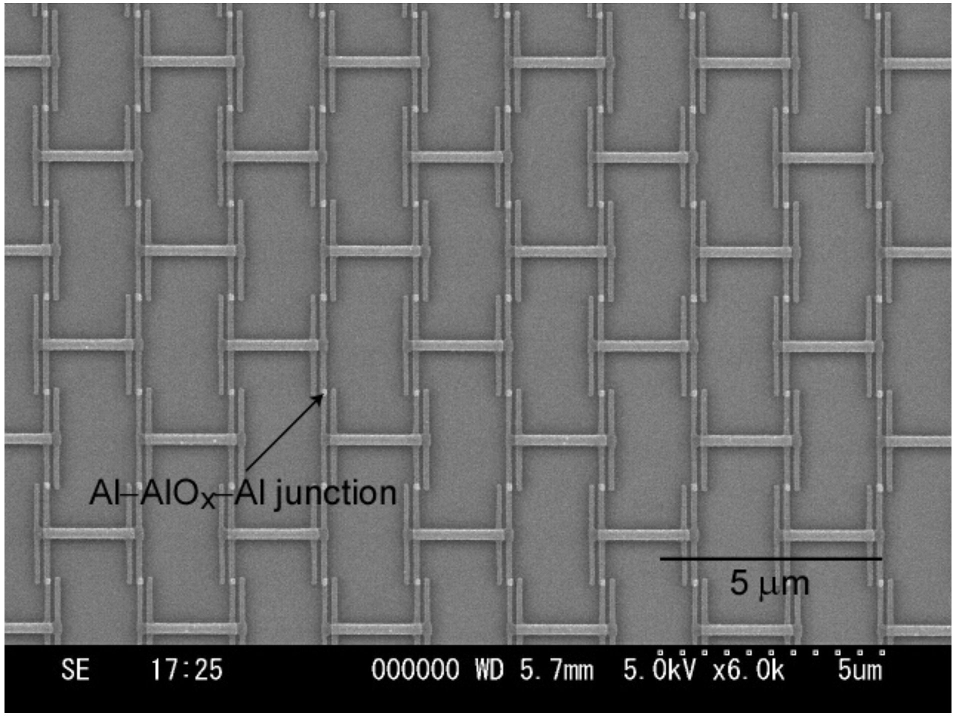




Similar to quantum phase slips in JJ arrays
Lau, Markovic, Bockrath, Bezryadin and Tinkham '01
Matveev, Larkin and Glazman '02

Take-home message

Profound, unexpected similarities between Josephson junction arrays and high voltage AC power grids !



dissipationless quantum fluid ⟷ !!!! *dissipative classical system*

Superconductivity vs. electric power systems !

	Superconductor	high voltage AC power grid
State	$\Psi(\mathbf{x}) = \lvert\Psi(\mathbf{x})\rvert e^{i\theta(\mathbf{x})}$	$V_i = \lvert V_i\rvert e^{i\theta_i}$
Current / power flow	$I_{ij} = I_c \sin(\theta_i - \theta_j)$ DC Josephson current	$P_{ij} \simeq B_{ij} \sin(\theta_i - \theta_j)$ Power flow; lossless line approx.
winding # $q=\Sigma_i\lvert\theta_{i+1}-\theta_i\rvert/2\pi$	Flux quantization Persistent currents	Circulating loop flows