Vortex Flows in High Voltage AC Power Grids

Philippe Jacquod

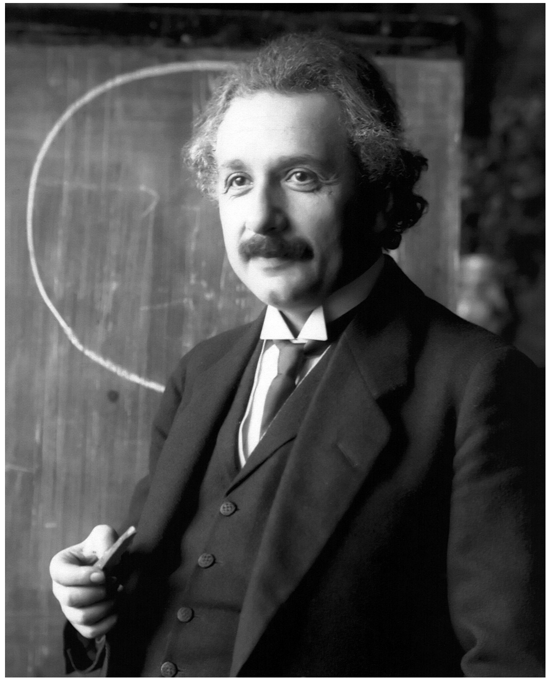
Coletta and PJ, Phys Rev E 93, 032222 (2016)
Delabays, Coletta, and PJ, J Math Phys 57, 032701 (2016)
Coletta, Delabays, Adagideli and PJ, New J Phys 18, 103042 (2016)
Delabays, Coletta, and PJ, arXiv:1609.02359, to appear in J Math Phys

"It is nice to know that the computer understands the problem.
But I would like to understand it too."

-E.P. Wigner

"Everything should be made as simple as possible, but not simpler."

-A. Einstein



Steady-State (balanced) AC transport : Power flow equations

$$P_i = \sum_j |V_i V_j| \left[G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)\right]$$

$$Q_i = \sum_j |V_i V_j| \left[G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)\right]$$

- i : node/bus index
- PV-buses : production
- PQ-buses : consumption
- 1 “slack-bus”

Set of coupled, transcendental (in particular nonlinear) equations

→ How many solutions are there ?

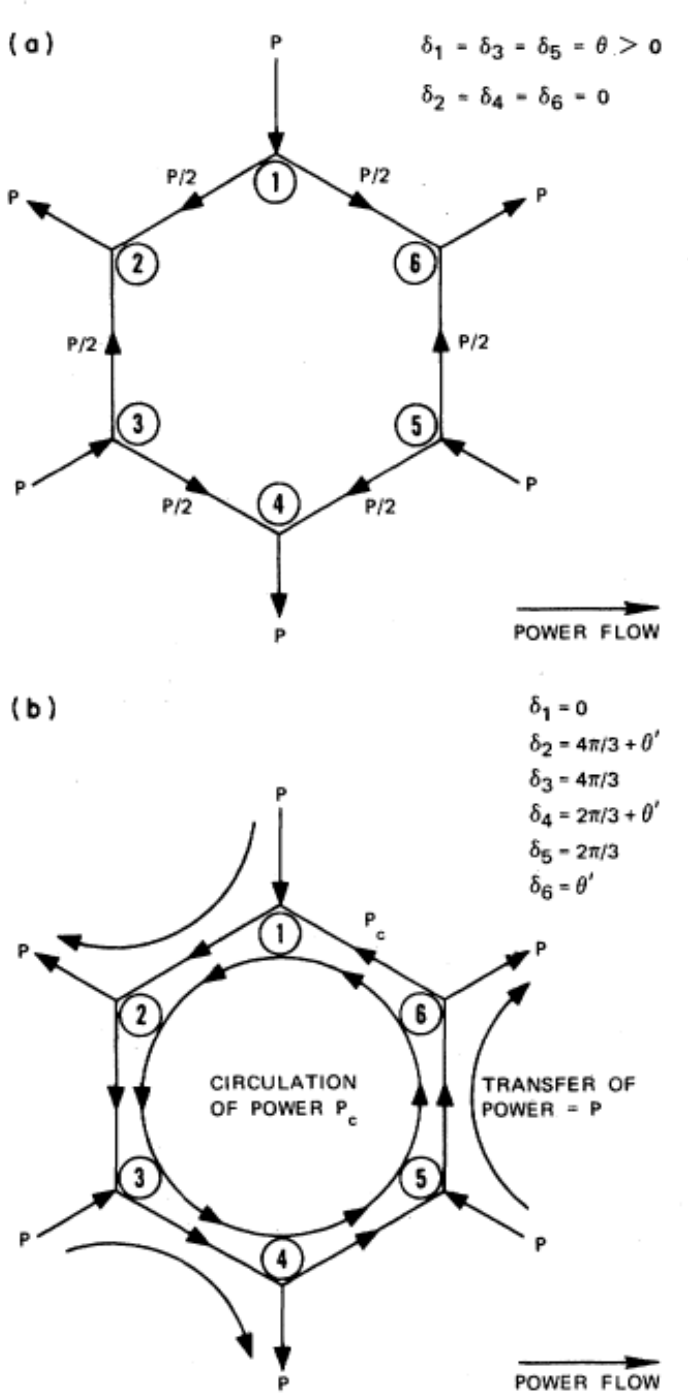
First answer : an infinite # of them, since $\{\theta_i + C\}$ is also a solution for any constant C

Define “different” as differing by more than C

Solution state entirely determined by $V_i = |V_i| e^{i\theta_i}$

~discrete complex function on the plane

How many stable power flow solutions ?



Korsak IEEE Trans. Power Appar. Syst. (1972)

Different solutions vs. loop currents

From the example networks above, it is evident that stable load flows are not necessarily unique, and that power flows, following a transient, for example, could possibly lock into a stable configuration in which power circulates in one or more loops of the network, with great increase in losses as a result.

Referee comment

Gerd Lüders (Yale University, New Haven, Conn.): This paper illustrates, through an example, that a power system can have more than one stable load flow solution or stable singular point. The study of the number of stable singular points is important:

1) for the operation of a power system, because one would be interested in operating the system on that stable singular point which would maximize its stability and minimize its losses, and

2) for applying Lyapunov's Method to transient stability studies of power systems, because this method requires a thorough knowledge of the singular points of a power system [9].

$$\sum^{\text{loop}} \delta_{ij} = \pm 2\pi k$$

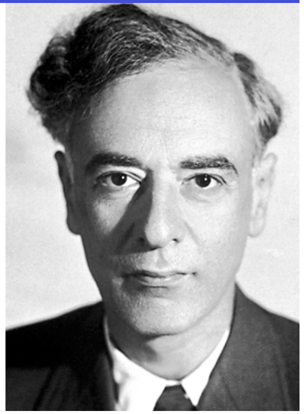
See also: Tavora and Smith '72, Janssens and Kamagate '03

Vortices in superfluids and superconductors

•Landau theory of superfluidity/superconductivity :

complex order parameter $$\psi(\mathbf{x}) = |\psi(\mathbf{x})|\, e^{i\theta(\mathbf{x})}$$

super-current $$\mathbf{J}(\mathbf{x}) = \frac{\hbar|\psi(\mathbf{x})|^2}{m_s}\nabla\theta(\mathbf{x})$$



L Landau
1908-1968

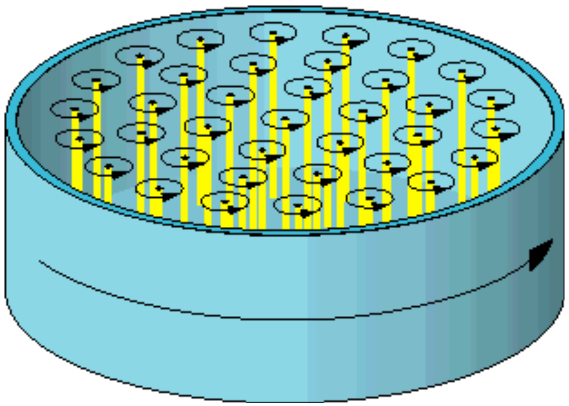
•Two cases :

(i) $|\psi(\mathbf{x})| > 0$ everywhere → $\oint_C \nabla\theta \cdot \mathrm{d}\mathbf{l} = 0$

(ii) $|\psi(\mathbf{x})| = 0$ somewhere → $\oint_C \nabla\theta \cdot \mathrm{d}\mathbf{l} = 2\pi q$

q=integer : "winding number"

~number of times phase goes around unit circle
as one travels around a loop in space



-> quantization of circulation ~vortex
(super-current around any loop)

$$\oint_C \mathbf{J}\cdot \mathrm{d}\mathbf{l} = \frac{\hbar|\psi(\mathbf{x})|^2}{m_s}(\theta_+ - \theta_-) = 2\pi q\frac{\hbar|\psi(\mathbf{x})|^2}{m_s}$$

Th. : Onsager '49, Feynman '55

From circulating loop flows to vortex flows

*Thm: Different solutions to the following power-flow problem

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

may differ only by circulating loop current(s) **in any network**

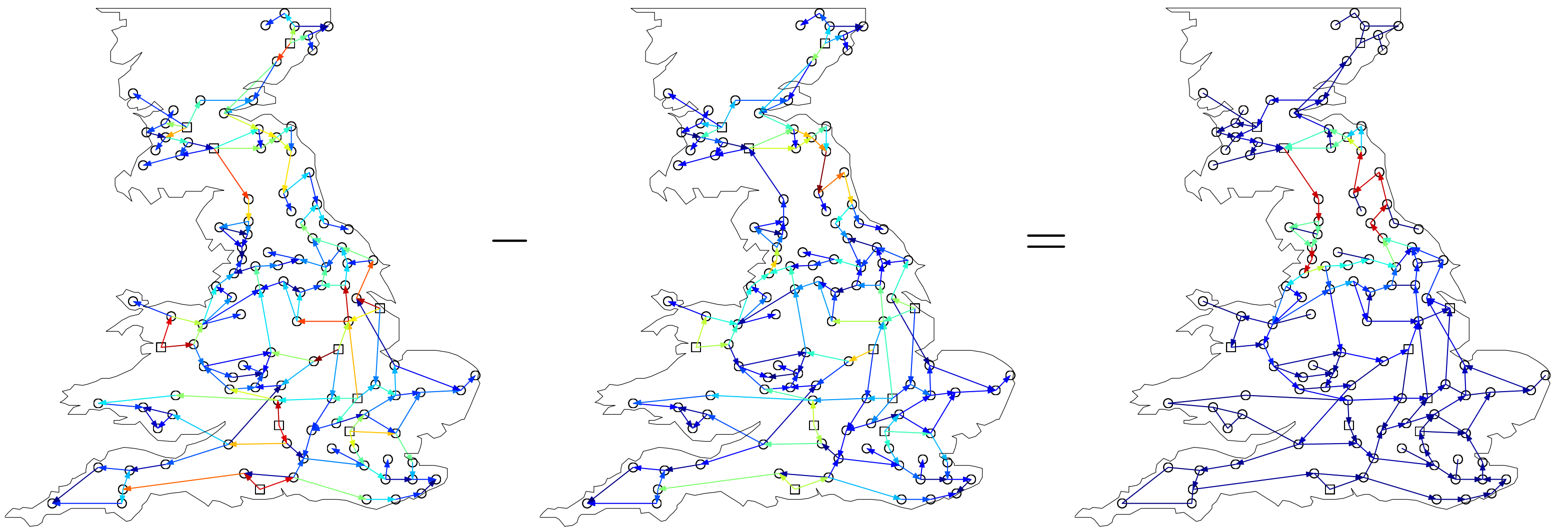
Dörfler, Chertkov, Bullo, PNAS '13; Delabays, Coletta and PJ, JMP '16

*Voltage angle uniquely defined

→ $q = \Sigma_i |\theta_{i+1} - \theta_i|_{2\pi} / 2\pi \quad \in \mathbb{Z}$ **~topological winding number**

→ discretization of these loop currents **~vortex flows**

Tavora and Smith '72, Janssens and Kamagate '03



Network : Witthaut and Timme '12

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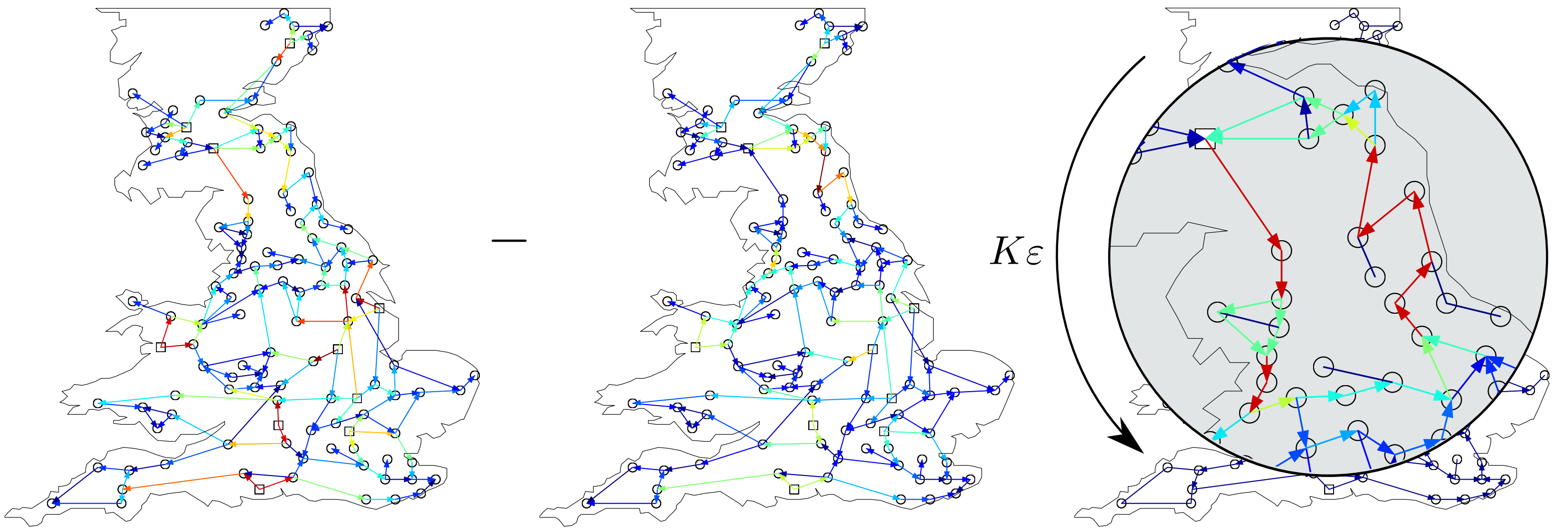
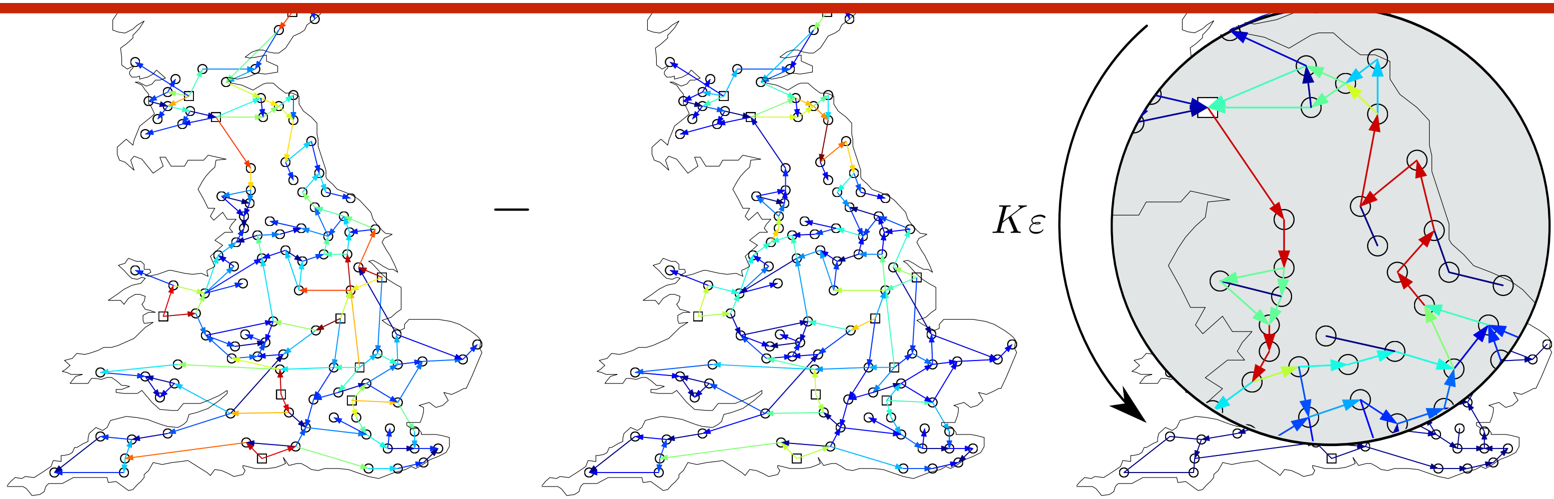


Network : Witthaut and Timme '12

Vortex flows

PF solutions with different vortex flows exist, but…

(i) how many of them are there ?
(ii) how can one identify them beforehand ?
how can one find them numerically ?
(iii) how are they generated ?



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Delabays, Coletta, and PJ, J Math Phys '16
Delabays, Coletta, and PJ, to appear in J Math Phys '17

Hes·so// VALAIS WALLIS

Stable solutions to the power flow problem

- Consider the power flow problem in the lossless line approximation

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j) = \sum_{j \sim i} P_{ij}$$

Question : how many different **stable** *solutions are there ?*

Check linear stability via swing equation

$$\frac{d\theta_i}{dt} = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

Stable solutions to the power flow problem

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If a solution is linearly stable with

$$\frac{d\theta_i}{dt} = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

then it is also linearly stable with

$$M\frac{d^2\theta_i}{dt^2} + D\frac{d\theta_i}{dt} = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

Manik et al. '14; Coletta and PJ '16

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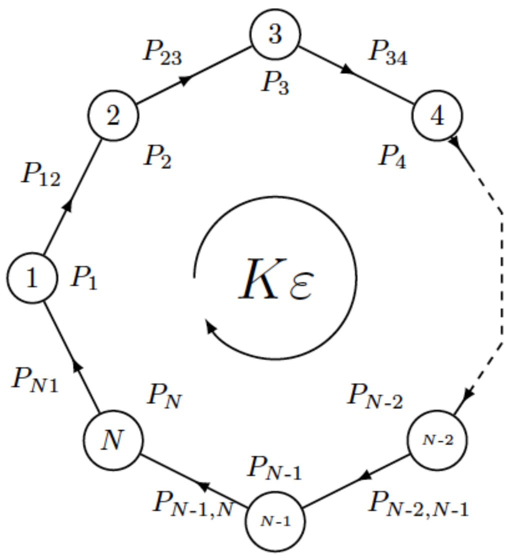
Question : how many different stable solutions are there ?

Answer #1 : single-cycle network



Flow between i and i+1 $\quad P_{i,i+1} = P^*_{i,i+1} + K\varepsilon$

Reference flow (arbitrary) $\quad P^*_{i,i+1} := \sum_{j=1}^{i} P_j$

"Quantization" condition $\quad q := (2\pi)^{-1} \sum_{i=1}^{n} \Delta_{i,i+1} \in \mathbb{Z}$

With two different possibilities $\quad \Delta_{i,i+1} = a_i(\varepsilon)$

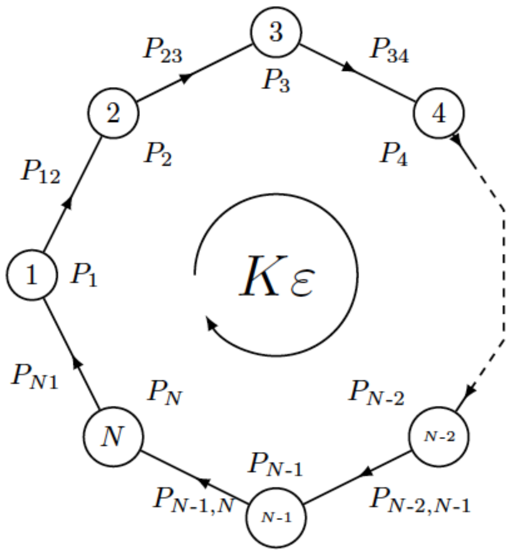
$$a_i(\varepsilon) = \begin{cases} \arcsin\left(\varepsilon + P^*_{i,i+1}/K\right) & \Longrightarrow \quad \Delta_{i,i+1} \in [-\pi/2, \pi/2]\,, \\ \pi - \arcsin\left(\varepsilon + P^*_{i,i+1}/K\right) & \Longrightarrow \quad \Delta_{i,i+1} \in (-\pi, -\pi/2) \cup (\pi/2, \pi] \end{cases}$$

Delabays, Coletta and PJ, JMP '16 (based on works by Rogge and Aeyels; Ochab and Gora; Taylor)

Stable solutions to the power flow problem

Question : how many different stable solutions are there ?

Answer #1 : take single-cycle network



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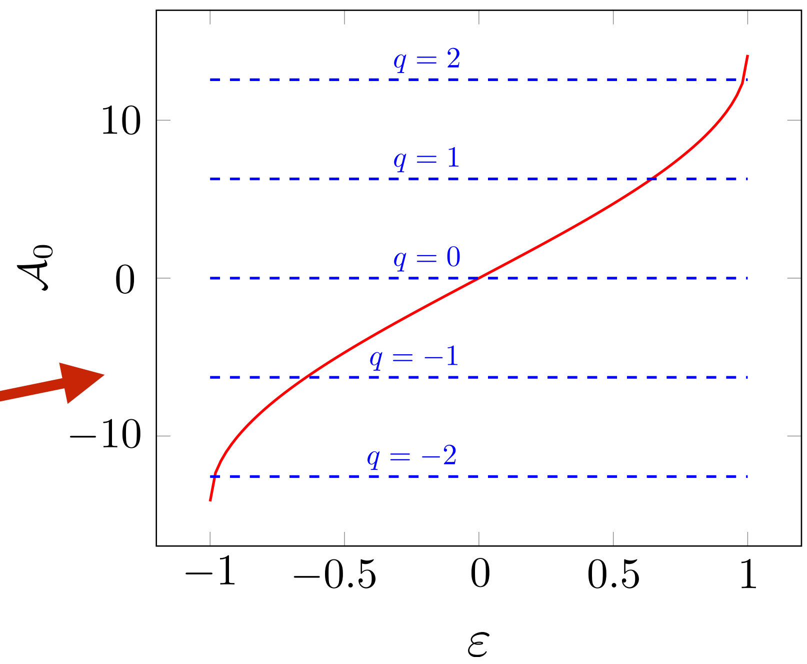
a. Take only first possibility (always stable)

$$\mathcal{A}_0(K,\varepsilon) := \sum_{i=1}^{n}\Delta_{i,i+1} = \sum_{i=1}^{n}\arcsin\left(\varepsilon + P^*_{i,i+1}/K\right) = 2\pi q$$

-count number of intersections

$\rightarrow$ $\mathcal{N} \leq 2\,\mathrm{Int}\,[n/4] + 1$



Delabays, Coletta and PJ, JMP '16

Stable solutions to the power flow problem

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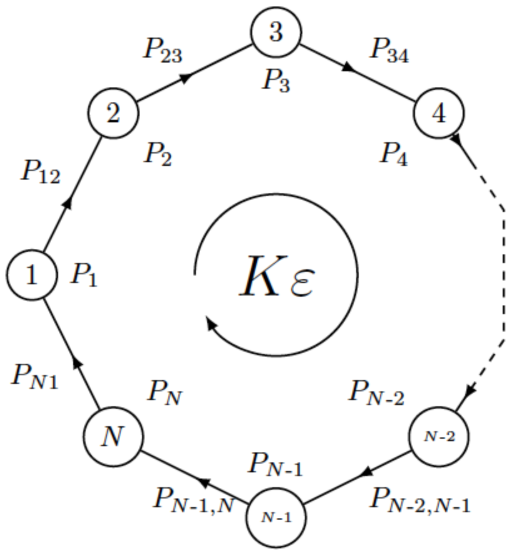

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b. Consider second possibility (not always stable)

One can show : (i) with at most one angle difference > π/2 is sol. stable

(ii) > π/2 possible only at finite B (not at infinite capacity)

(iii) "> π/2" solution emerges from "< π/2" solution as one reduces K=B

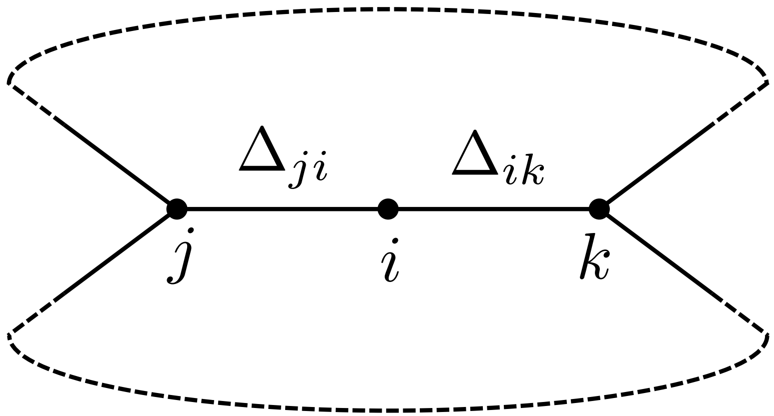
$$\longrightarrow \quad \boxed{N \le 2\,\mathrm{Int}\,[n/4] + 1}$$

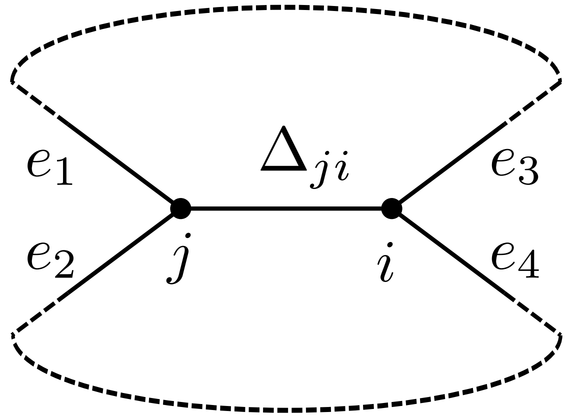
Delabays, Coletta and PJ, JMP '16

Stable solutions to the power flow problem

Question : how many different stable solutions are there ?

Answer #2 (partial) : meshed planar (i.e. multi-cycle) network





Loops with two or more edges in common

Loops with one edge in common

> π/2 possible only at finite K=B (not at infinite capacity)

$$\mathcal{N} \leq \prod_{k=1}^{c} \left[2 \cdot \mathrm{Int}\left(n_k/4\right) + 1\right]$$

> π/2 possible at any K=B (also at infinite capacity!)

$$\mathcal{N} \leq \prod_{k=1}^{c} \left[2 \cdot \mathrm{Int}\left((n_k + n_k')/4\right) + 1\right]$$

?? conjecture ??

Delabays, Coletta and PJ, to appear in JMP '17; see also Manik, Witthaut, Time arXiv:1611.09825

Vortex flows

PF solutions with vortex flows exist, but…

(ii) how can one identify them beforehand ?
how can one find them numerically ?

Numerically finding vortex flows

Algorithm to determine different solutions numerically

(i) use iterative method (RK on swing; NR on power flow…)

(ii) use different initial states

brute force : random initial state ?
…time-consuming to get states with finite q

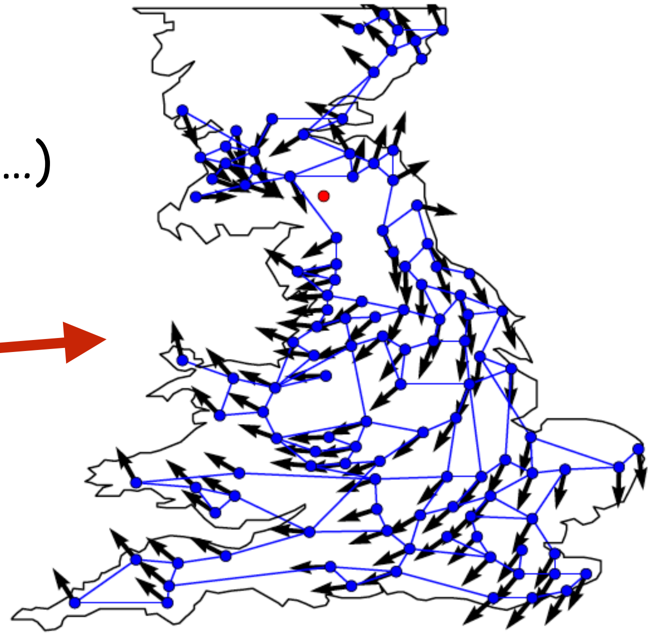
Numerically finding vortex flows

Algorithm to determine different solutions numerically

(i) use iterative method (RK on swing; NR on power flow…)

(ii) construct vortex-carrying initial state

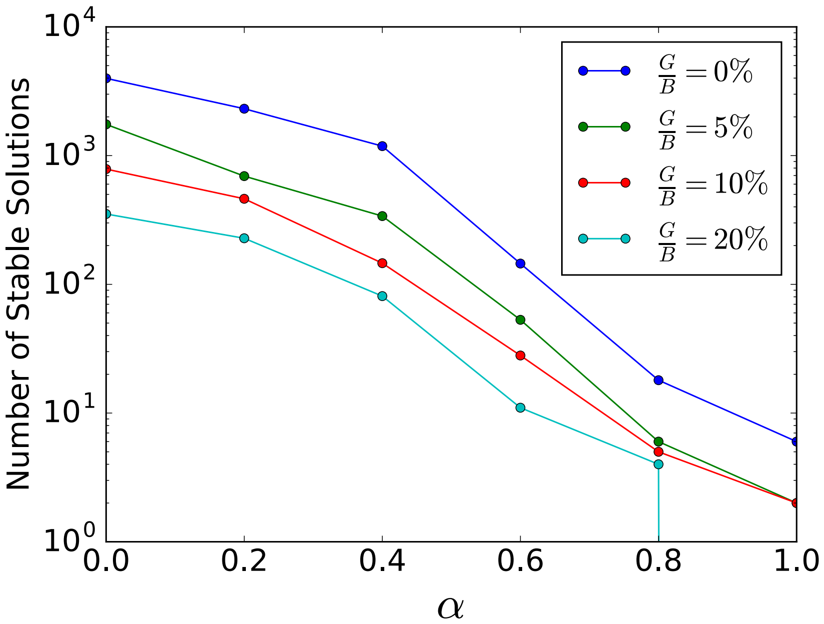
$$\theta_i = q \arctan\left(\frac{y_i - y_0}{x_i - x_0}\right)$$

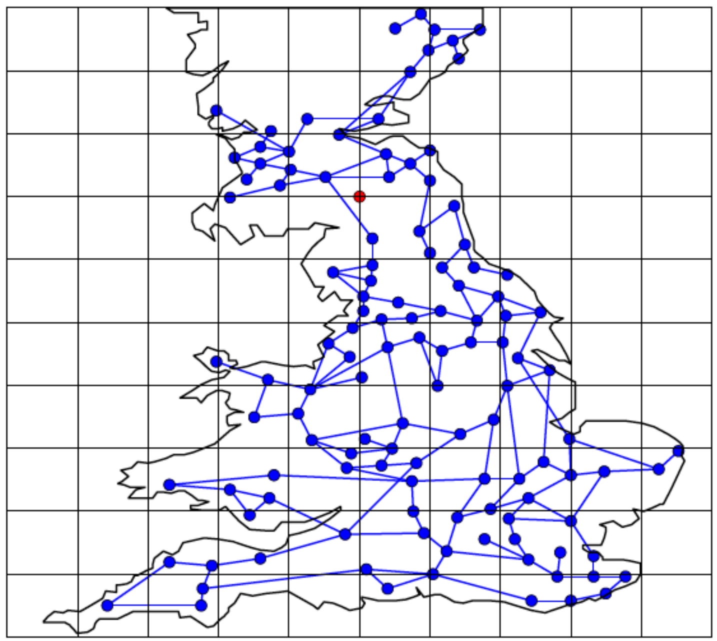


(iii) iterate and see where it converges

(iv) pick another vortex-carrying initial state

(v) re-iterate aso.





Tyloo and PJ, in preparation

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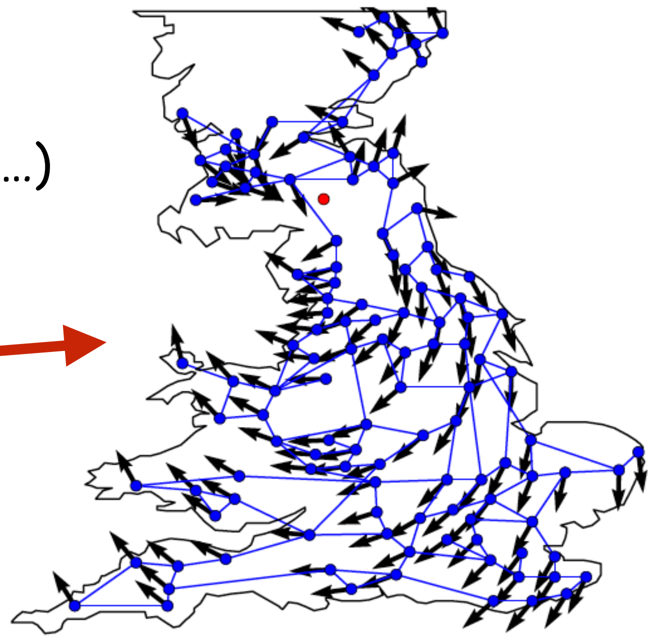

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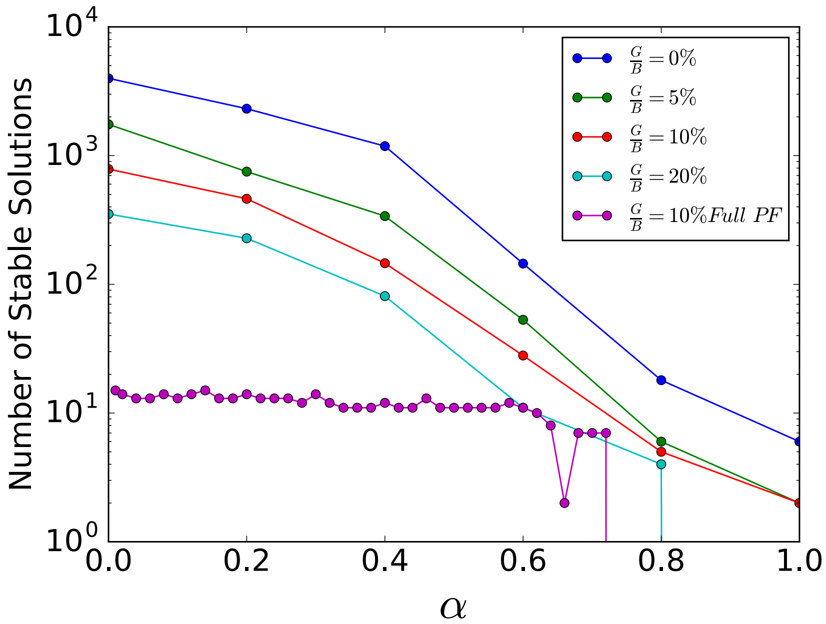
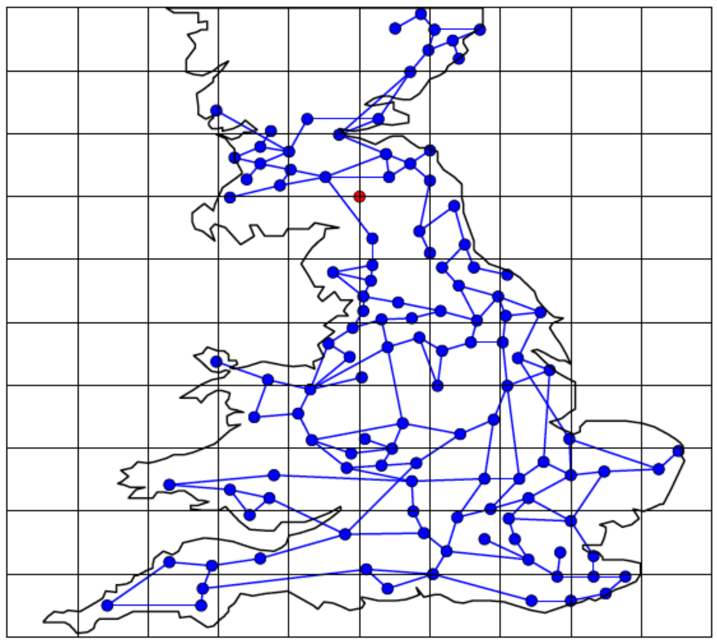




Tyloo and PJ, in preparation

Numerically finding vortex flows

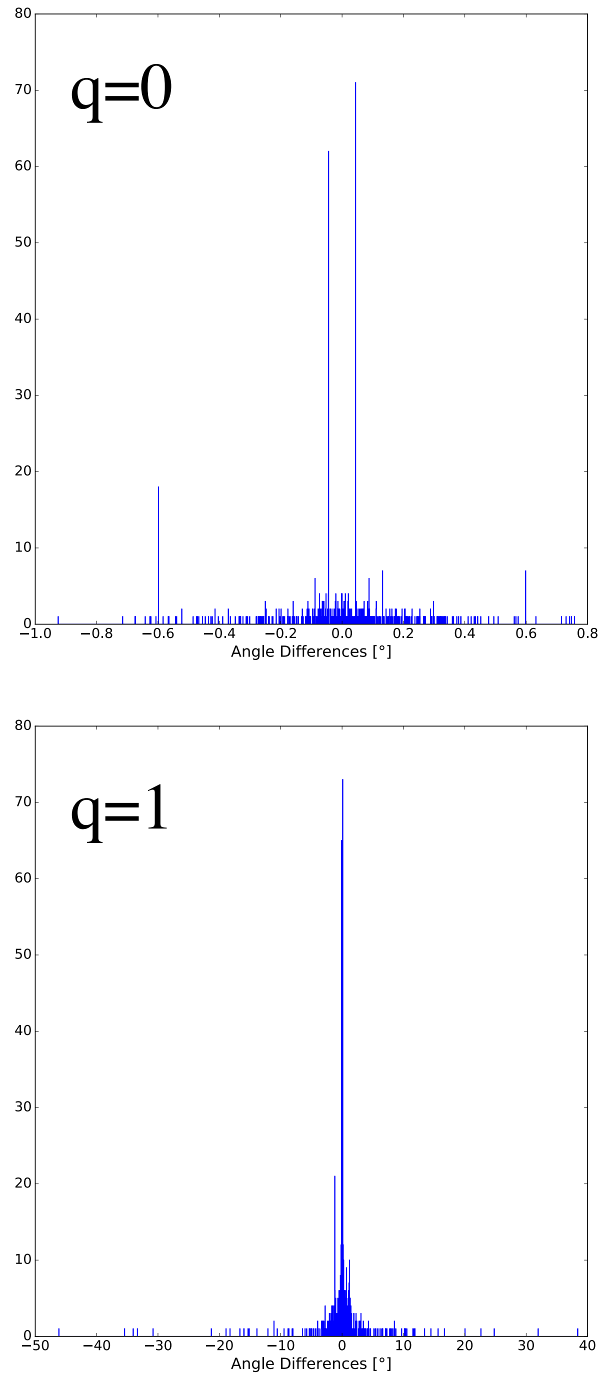
$$\theta_i = q \arctan\left(\frac{y_i - y_0}{x_i - x_0}\right)$$

$q \neq 0$ → larger angle differences

If such sols. occur at all in AC grids

(i) they are exceptional
(ii) they occur around big loops

…but how ?



Tyloo and PJ, in preparation

Vortex flows

PF solutions with vortex flows exist, but…

(iii) how do they appear ? how are they generated ?

Problem : topological protection

i.e. discrete jump $q \longrightarrow q' \neq q$ requires to unwind a large number of voltage angles

Coletta, Delabays, Adagideli and PJ, New J Phys. '16

Vortex flows

PF solutions with vortex flows exist, but…

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Solution :

(i) line tripping
(ii) line tripping and reclosing
(iii) dynamical phase slip (loss of stability of q state)
(iv) fluctuating productions (in progress)

Coletta, Delabays, Adagideli and PJ, New J Phys. '16

Generation of vortex flow by line tripping

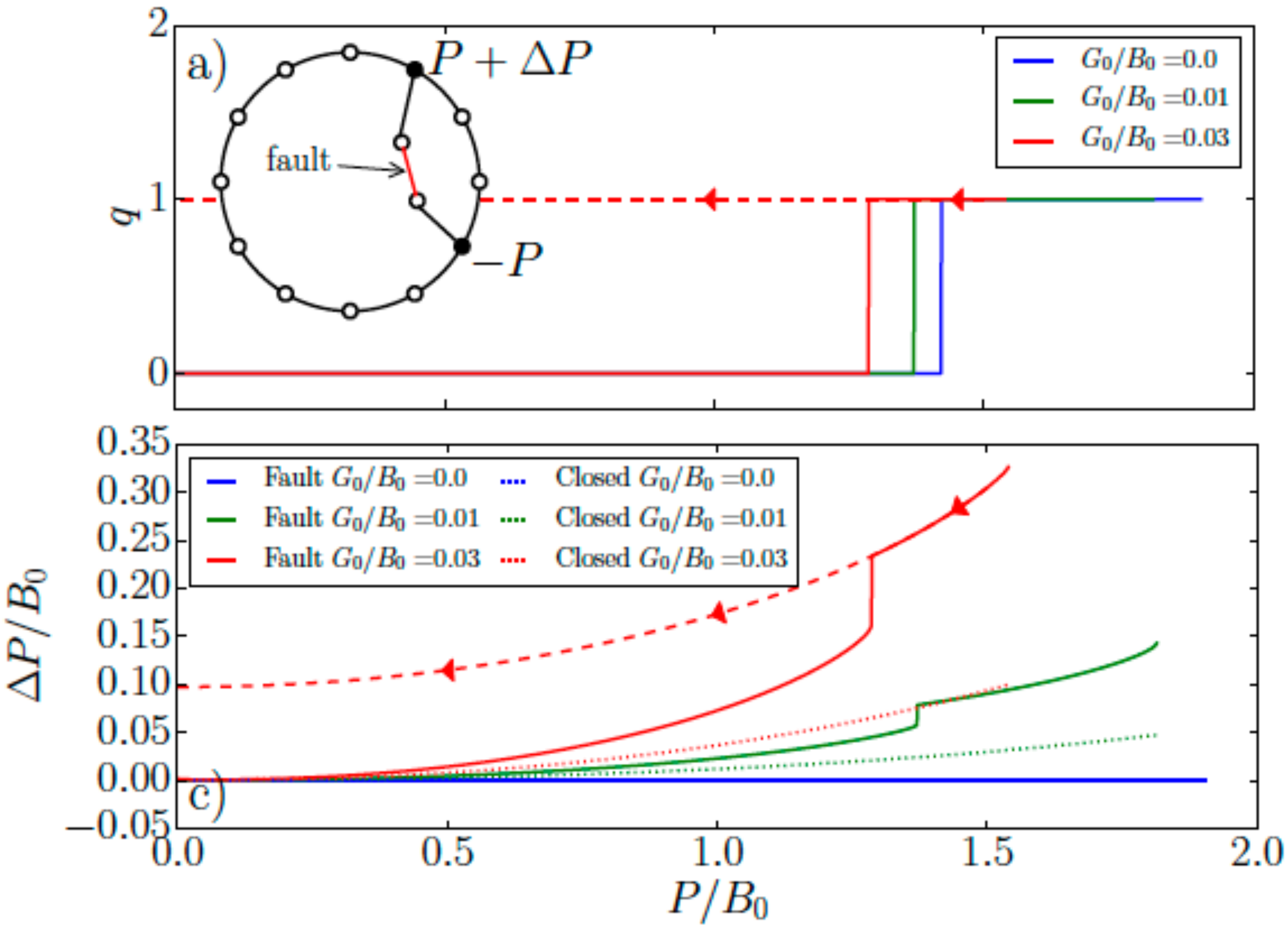
*Power grids are meshed - path redundancy
*Power is still supplied after one line trips
*Consider line tripping in an asymmetric, three-path circuit
*all winding numbers vanish initially

Power redistribution can lead to vortex flow with $q=\Sigma_i|\theta_{i+1}-\theta_i|/2\pi > 0$

Line tripping at $P/B_0 > 1.3$

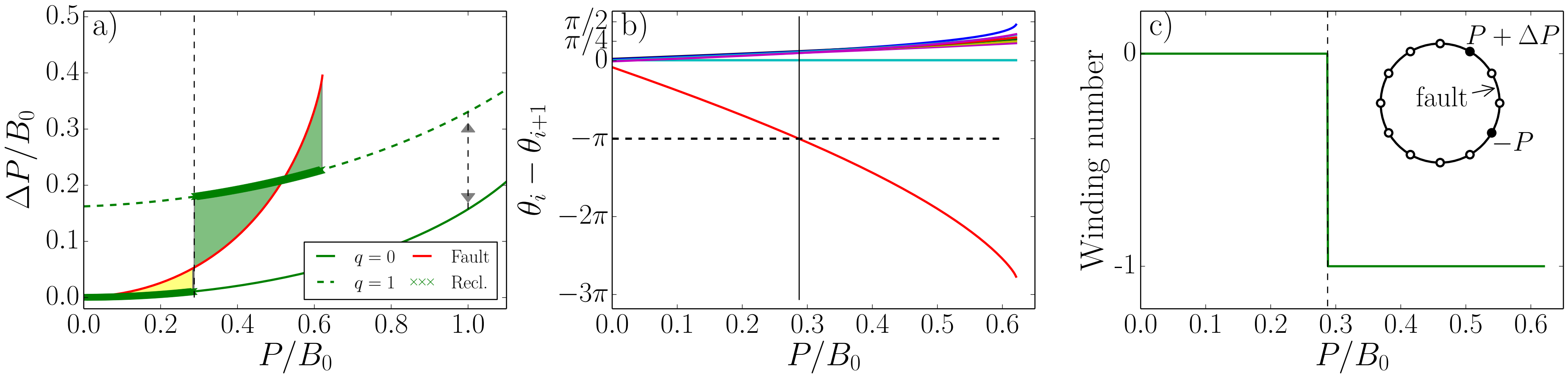
→ q=1

Vortex state characterized by
-hysteresis
= **topological protection**
-higher losses



Coletta, Delabays, Adagideli and PJ '16

Generation of vortex flow by line tripping and reclosure



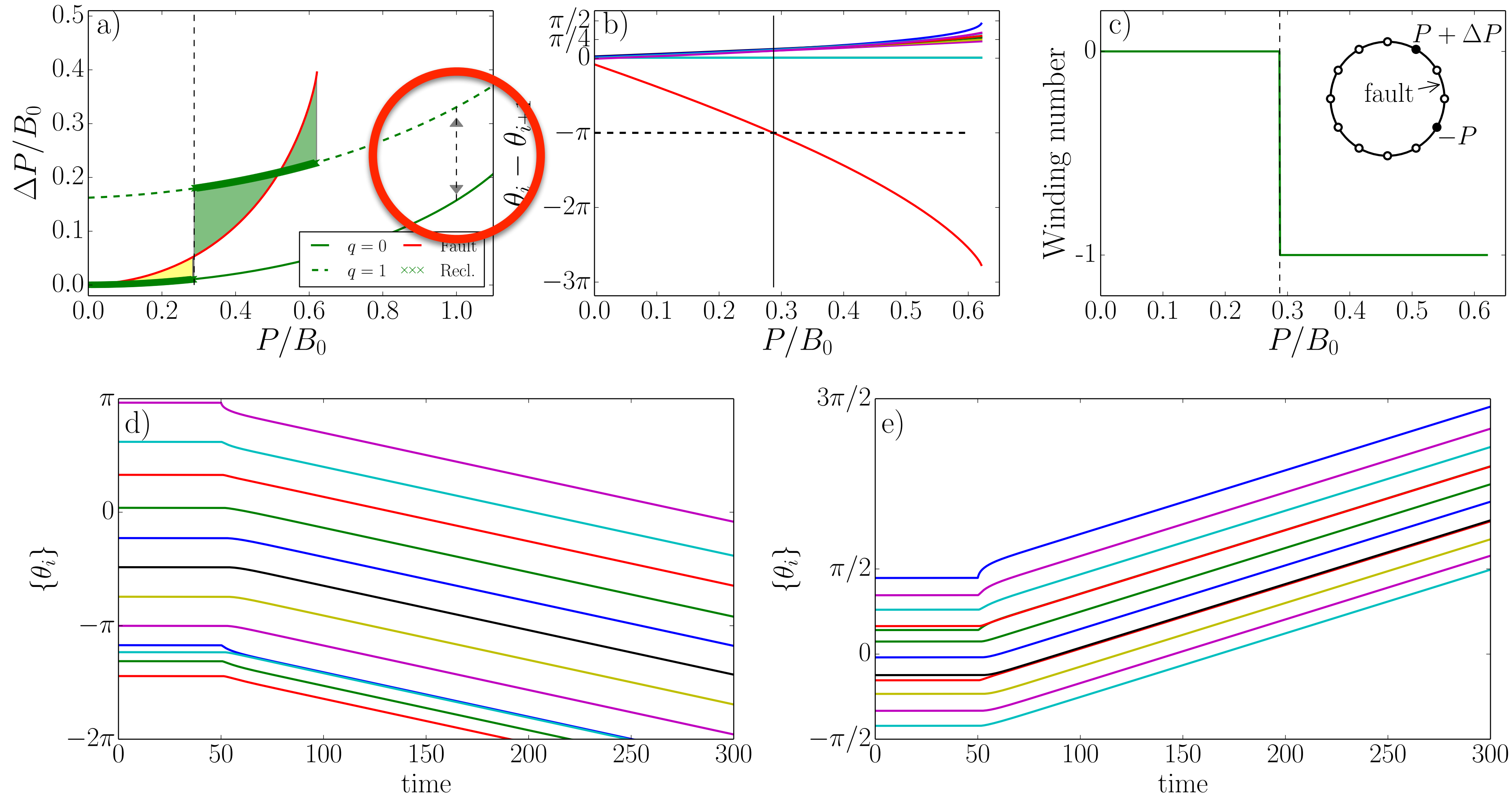
- Single-loop network
- One producer, one consumer
- One line trips, is later reclosed

Vortex formation for $P/B_0 > 0.26$

Vortex formation for $|\theta_{i+1}-\theta_i|>\pi$ (two ends of faulted line)

Coletta, Delabays, Adagideli and PJ '16; Janssens and Kamagate '03

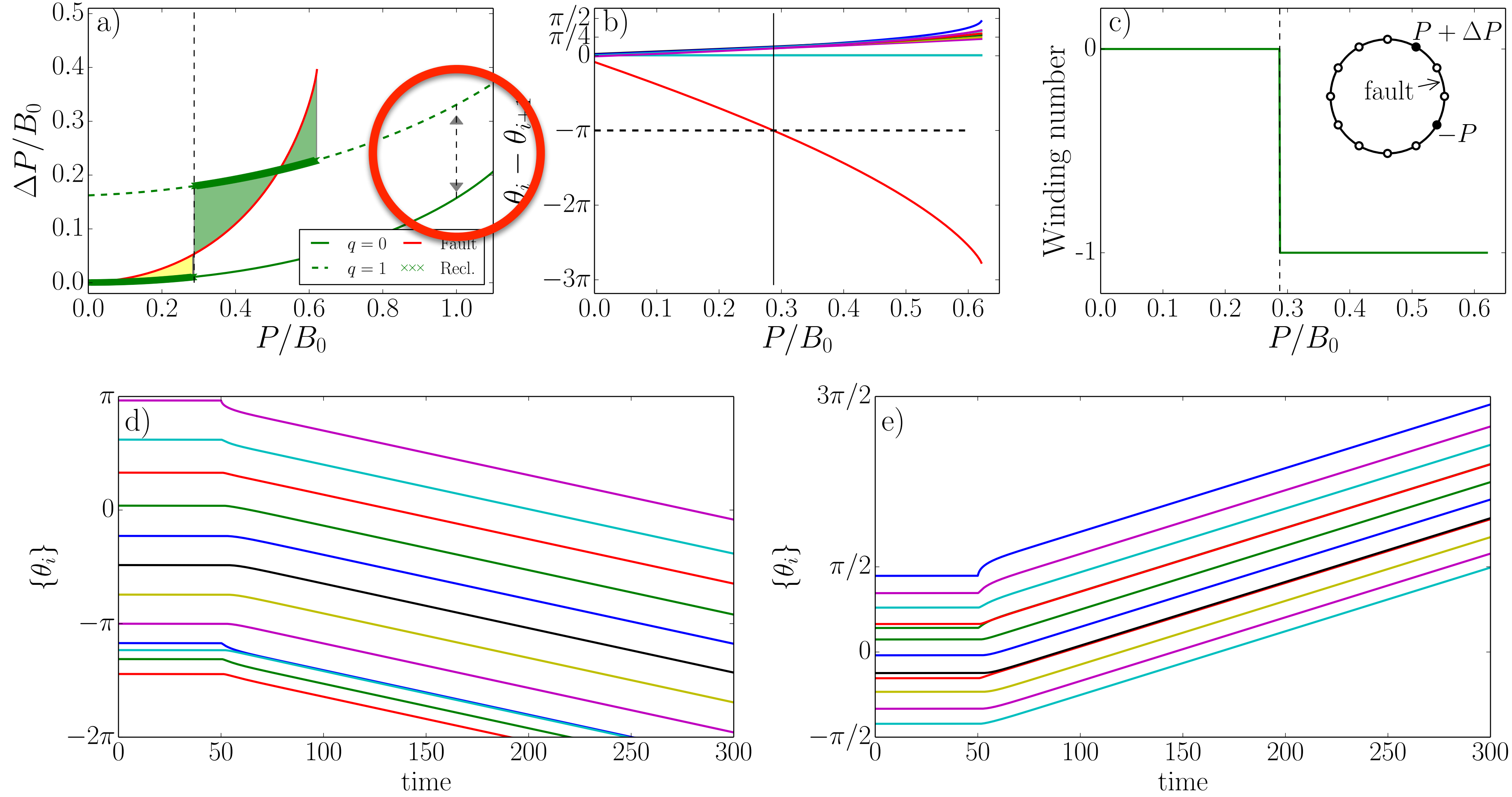
Generation of vortex flow by line tripping and reclosure



!! Cannot kill nor create vortex by adapting ΔP !!

Instead one changes the grid's frequency

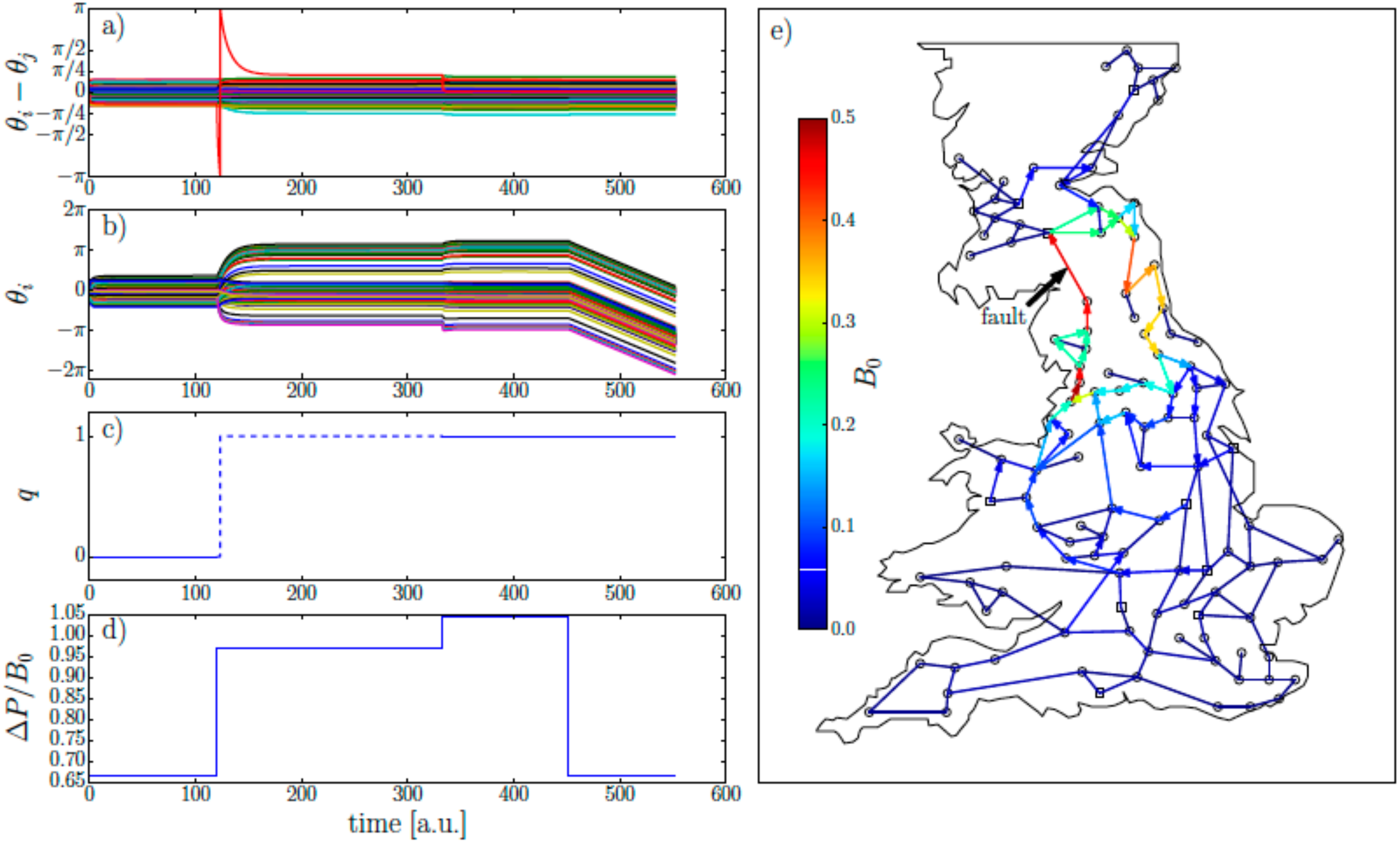
Generation of vortex flow by line tripping and reclosure



‼ Topological protection ‼

Coletta, Delabays, Adagideli and PJ, NJP '16

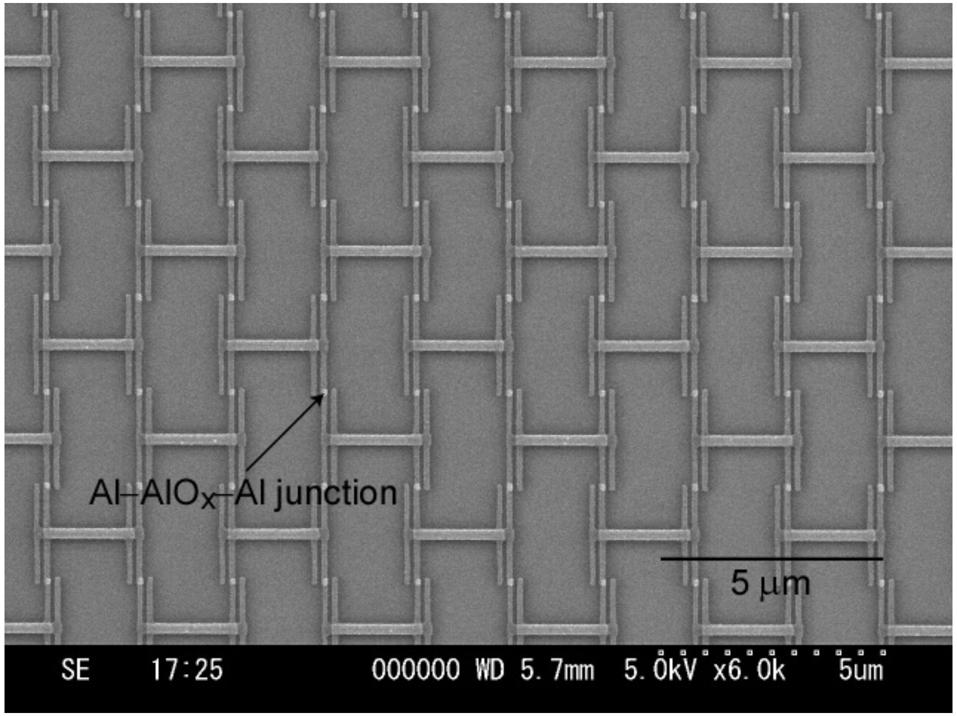
Generation of vortex flow by line tripping and reclosure



Coletta, Delabays, Adagideli and PJ, NJP '16

Take-home message (for physicists)

Profound, unexpected similarities between Josephson junction arrays and high voltage AC power grids !



dissipationless quantum fluid ⟷ !!!! ⟷ *dissipative classical system*

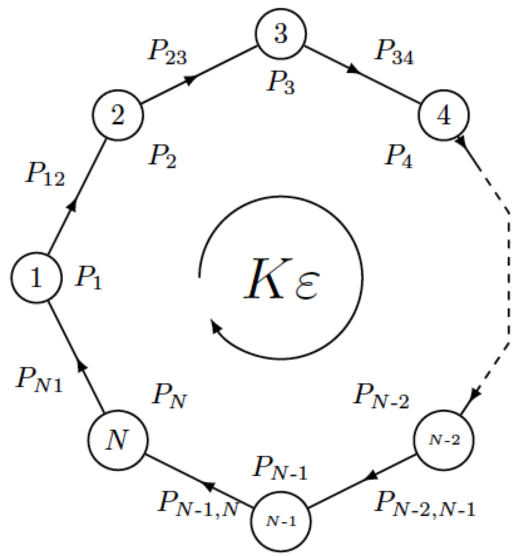
Vortex flows in AC power grids

- They exist in meshed network

- They are hard to generate but hard to get rid of
  (a.k.a. "topological protection")
  *generation mostly by topological changes

- They dissipate additional power

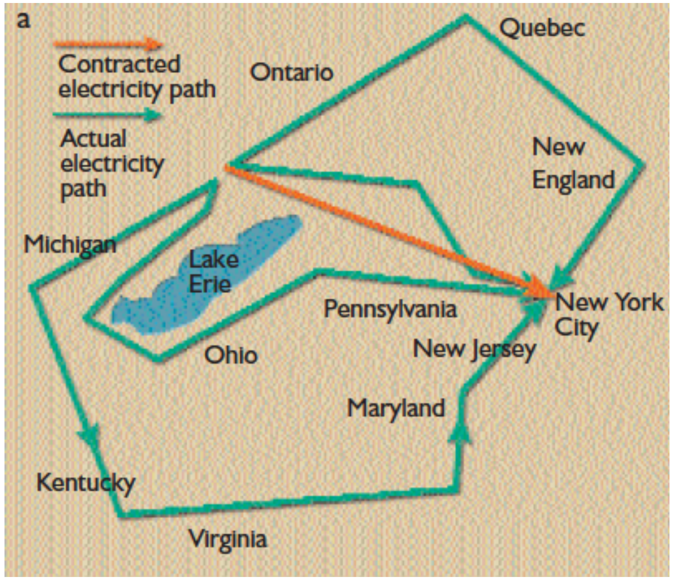
- They are exceptional occurences

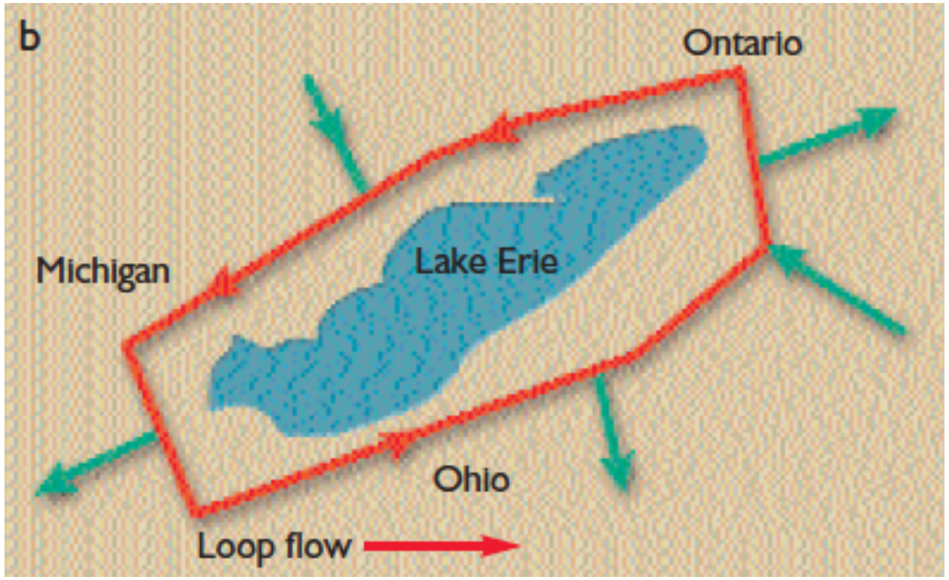


$$q := (2\pi)^{-1} \sum_{i=1}^{n} \Delta_{i,i+1} \in \mathbb{Z}$$

Loop flows

“Electric power does not follow a specified path but divides among transmission routes based on Kirchhoff’s laws and network conditions at the time. This pattern results in a phenomenon called circulating power of which there are two types : loop flow and parallel flow.”





“Where the network jogs around large geographical obstacles, such as the Rocky Mountains in the West or the Great Lakes in the East, loop flows around the obstacle are set up that can drive as much as 1 GW of power in a circle, *taking up transmission line capacity without delivering power to consumers.*”

Casazza, Electrical World (1998)
Lerner, The industrial physicist (2003)

Thank you !

Coletta and PJ, Phys Rev E 93, 032222 (2016)

Delabays, Coletta, and PJ, J Math Phys 57, 032701 (2016)

Coletta, Delabays, Adagideli and PJ, New J Phys 18, 103042 (2016)

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