High Voltage AC Power Grids
A (cond-mat) Physicist's perspective

Philippe Jacquod

Coletta and PJ, Phys Rev E 93, 032222 (2016)
Delabays, Coletta, and PJ, J Math Phys 57, 032701 (2016)
Coletta, Delabays, Adagideli and PJ, New J Phys (2016; to appear)
Delabays, Coletta, and PJ, arXiv:1609.02359, submitted to J Math Phys

FNSNF

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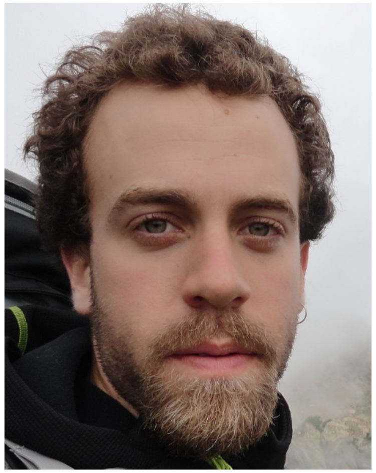
The team

Tommaso Coletta, postdoc



Robin Delabays, PhD student

Inanc Adagideli (Sabanci)

Vortices, topological protection and all that jazz

David J. Thouless

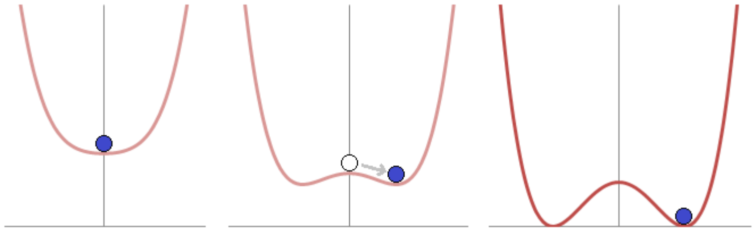
F. Duncan M. Haldane

J. Michael Kosterlitz

The Nobel Prize in Physics 2016 was divided, one half awarded to David J. Thouless, the other half jointly to F. Duncan M. Haldane and J. Michael Kosterlitz "for theoretical discoveries of topological phase transitions and topological phases of matter".

Symmetries and classification of states of matter

- Phase transition / spontaneous symmetry breaking
- Phase with broken symmetry : order parameter
- Classification with symmetry of order parameter



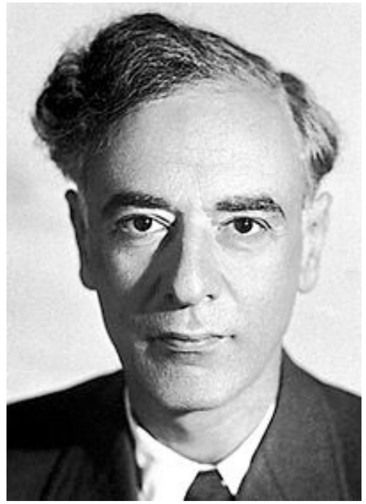


Table 1. Order parameters for phase transitions in various systems.

System	Transition	Order parameter
Liquid-gas	Condensation/evaporation	Density difference $\Delta\rho = \rho_{\text{liquid}} - \rho_{\text{gas}}$
Binary liquid mixture	Unmixing	Composition difference $\Delta c = c^{(2)}_{\text{coex}} - c^{(1)}_{\text{coex}}$
Nematic liquid	Orientational ordering	$\frac{1}{2}\langle 3\cos^2\theta - 1\rangle$
Quantum liquid	Normal fluid $\leftrightarrow$ superfluid	$\langle\psi\rangle$, ψ = wavefunction
Liquid-solid	Melting/crystallisation	ρ_G, $\boldsymbol{G}$ = reciprocal lattice vector
Magnetic solid	Ferromagnetic (T_c)	Spontaneous magnetisation $\boldsymbol{M}$
	Antiferromagnetic (T_N)	Sublattice magnetisation $\boldsymbol{M}_s$
Solid binary mixture	Unmixing	$\Delta c = c^{(2)}_{\text{coex}} - c^{(1)}_{\text{coex}}$
AB	Sublattice ordering	$\psi = (\Delta c^{\text{II}} - \Delta c^{\text{I}})/2$
Dielectric solid	Ferroelectric (T_c)	Polarisation $\boldsymbol{P}$
	Antiferroelectric (T_N)	Sublattice polarisation $\boldsymbol{P}_s$
Molecular crystal	Orientational ordering	$Y_{lm}(\theta, \phi)$

Table from K Binder, Rep Prog Phys '87

The Kosterlitz-Thouless phase transition

David Mermin   Herbert Wagner

Mermin and Wagner (thm) :

"There is no spontaneous breaking of a continuous symmetry at finite temperature in dimension d <= 2"

Continuous : order parameter must have 2 or more components

Dimension : d=2 is the critical dimension

What happens for d=2, 2-component order parameter ?

The Kosterlitz-Thouless phase transition

What happens for d=2, 2-component order parameter ?
aka 2D XY model

$$\mathcal{H}_{XY} = -J \sum_{\langle i;j \rangle} \mathbb{S}_i^x \mathbb{S}_j^x + \mathbb{S}_i^y \mathbb{S}_j^y \xrightarrow{\text{classical}} \mathcal{H}_{cl} = -J \sum_{\langle i;j \rangle} \cos(\theta_i - \theta_j)$$

Look at correlation function $G_2(\mathbf{r}) = \langle \vec{\mathbb{S}}_{\mathbf{r}} \vec{\mathbb{S}}_{\mathbf{0}} \rangle = \langle \exp[i(\theta_{\mathbf{r}} - \theta_0)] \rangle$

High temperature : exponential decay of correlations

$$G_2(\mathbf{r}) \propto \exp(-r/\xi) \qquad \xi = a/\ln(k_\mathrm{B} T/J)$$

The Kosterlitz-Thouless phase transition

What happens for d=2, 2-component order parameter ?
aka 2D XY model

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Look at correlation function $G_2(\mathbf{r}) = \langle \vec{\mathbb{S}}_{\mathbf{r}} \vec{\mathbb{S}}_{\mathbf{0}} \rangle = \langle \exp[i(\theta_{\mathbf{r}} - \theta_0)] \rangle$

Low temperature : power-law decay of correlations

$$G_2(\mathbf{r}) \propto r^{-\eta} \qquad \eta = k_{\mathrm{B}}T/2\pi J$$

Special kind of phase transition, i.e.
- no true long range order
- temperature-dependent critical exponent
- -> line of critical points

The Kosterlitz-Thouless phase transition

Somewhere between low and high temperature :

KT phase transition vs. dissociation of vortex-antivortex pairs

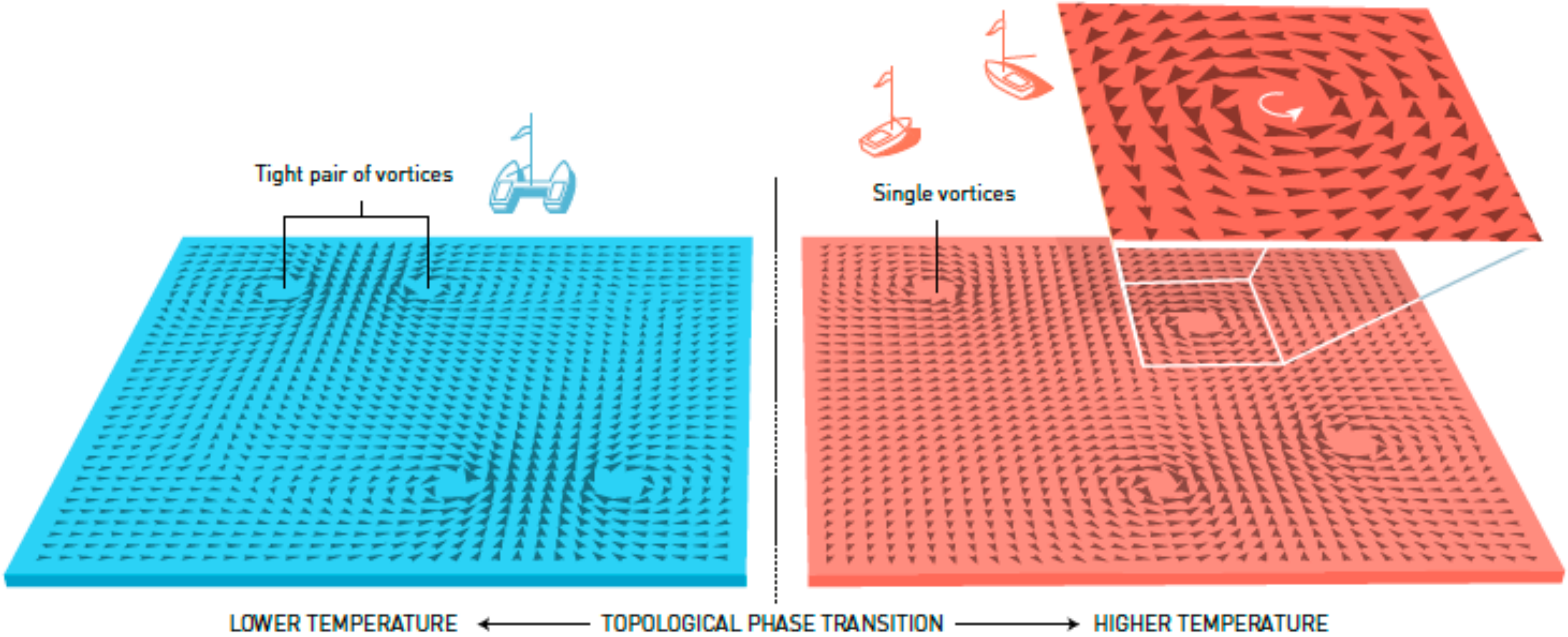
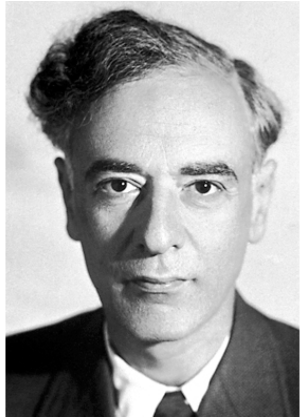


Illustration: ©Johan Jarnestad/The Royal Swedish Academy of Sciences

Vortices in superfluids and quantization of circulation



L Landau
1908-1968

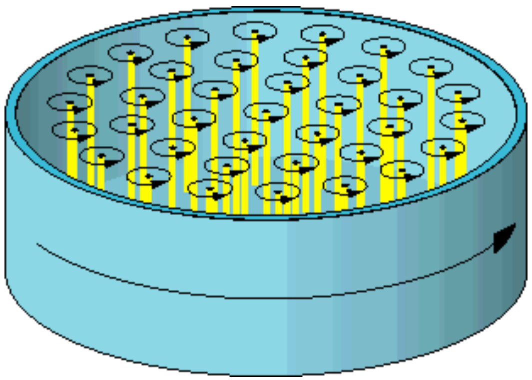
- Landau theory of superfluidity - macroscopic wavefunction

$$\psi(\mathbf{x}) = |\psi(\mathbf{x})| e^{i\phi(\mathbf{x})}$$

- Superfluid current

$$\boxed{\mathbf{J}_{\mathrm{SF}} = \frac{\hbar |\Psi|^2}{M_{\mathrm{SF}}} \nabla \phi}$$

- Vortex : topological defect with SF -> 0 in center
- Contour around that center



-> quantization of circulation

$$\oint \mathbf{J}_{\mathrm{SF}} \mathrm{d}\mathbf{l} = \frac{\hbar |\Psi|^2}{M_{\mathrm{SF}}} (\phi_+ - \phi_-) = \frac{\hbar |\Psi|^2}{M_{\mathrm{SF}}} 2\pi m \qquad m \in Z$$

Th. : Onsager '49, Feynman '55

The problem

"Electric power does not follow a specified path but divide[…]sion routes based on Kirchhoff's laws and network conditi[…]ern results in a phenomenon called circulating power[…] loop flow and parallel flow."

Loop flows are analogous to circulating currents around vortices in superfluids!

"W[…] geographical obstacles, such as the Rocky M[…] Lakes in the East, loop flows around the ob[…]e as much as 1 GW of power in a circle, *taking up tran[…] without delivering power to consumers.*"

Casazza, Electr[…]orld (1998)
Lerner, The industrial physicist (2003)

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- Electric power systems
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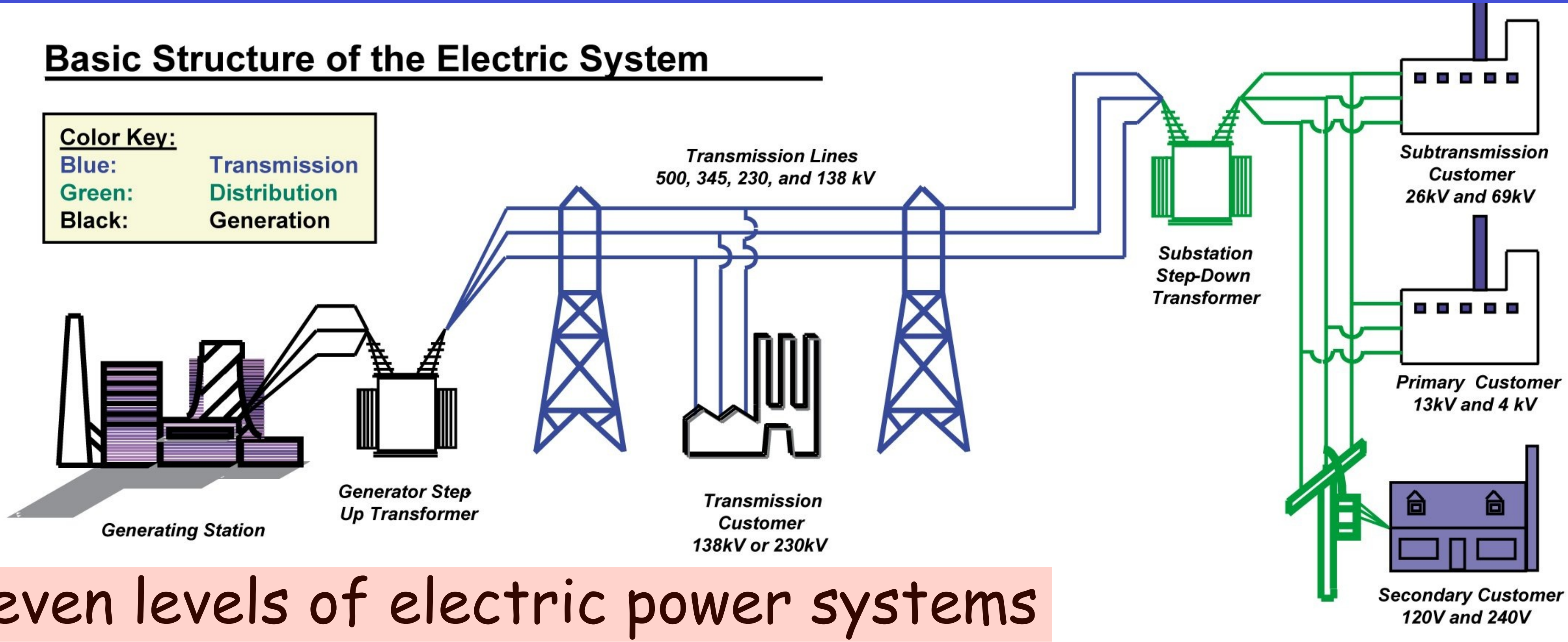
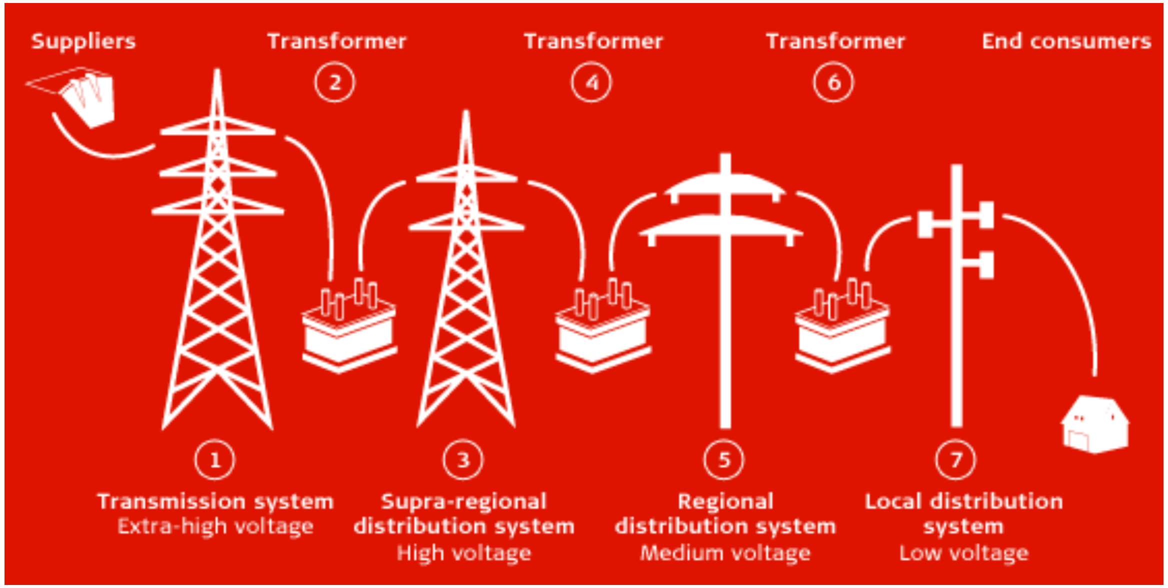
- Lossless line approximation and circulating power flows
- Vortex flows without and with dissipation
  - creation
  - topological protection

What are electric power systems ?



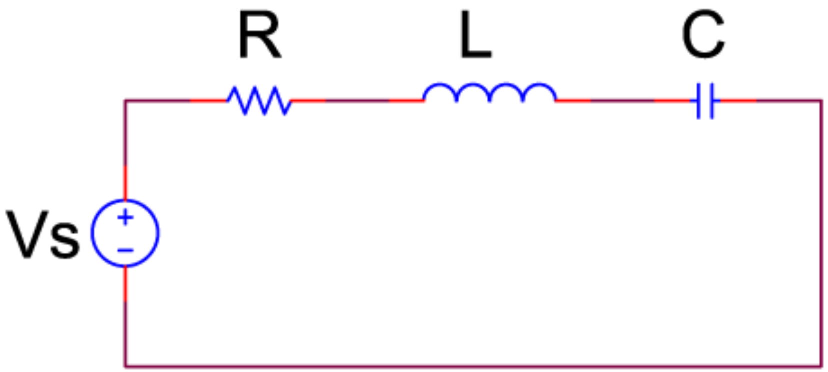
Seven levels of electric power systems



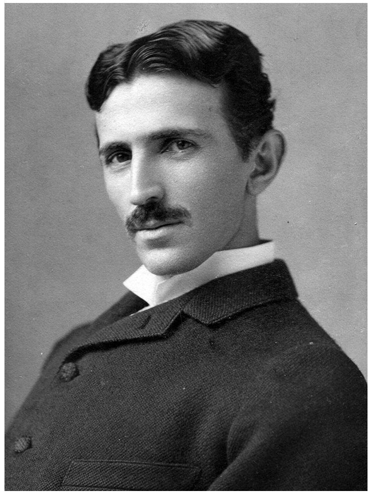
Power :
*conserved from one level to another (~)
*control parameter
"write Eq. for power"

What are electric power systems ?

- AC electric current/voltages
  (minimize losses ~ high voltages,
  but then need transformers)
- current and voltage not in phase



$$u(t) = u_0 \exp[i\omega t]$$
$$i(t) = i_0 \exp[i(\omega t + \phi)]$$
$$\tan(\phi) = (\omega L - 1/\omega C)/R$$



N Tesla 1856-1943

- complex impedance $u(\omega) = Z(\omega)\, i(\omega) \qquad Z(\omega) = R + i\omega L - i/\omega C$
- inductance more important than resistance for large conductors

Steady-state AC transport

- linear relation between currents and voltages

$$\boxed{I_i = \sum_j Y_{ij} V_j}$$

Y_{ij} : admittance matrix
Kirchhoff's current law

- Complex power : $S(t) = u(t) \times i^*(t)$
- Active power vs. reactive power

Active power	vs.	reactive power
P=Re(S)		Q=Im(S)
finite time-average		zero time-average
"truly transmitted" (injected and consumed)		"oscillating in the circuit"

Steady-State AC transport : Power flow equations

Power flow equations
(power is conserved upon voltage transformation)

$$P_i = \sum_j |V_i V_j| \left[G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \right]$$

$$Q_i = \sum_j |V_i V_j| \left[G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \right]$$

Voltages at buses i and j

Conductance matrix

Phases at buses i and j

Susceptance matrix

Approximated power flow equations : (1) lossless line

- Admittance dominated by its imaginary part
  for large conductors ~ high voltage
  G/B < 0.1 for 200kV and more

neglect conductance

$$P_i \simeq \sum_j |V_i V_j| B_{ij} \sin(\theta_i - \theta_j)$$

$$Q_i \simeq -\sum_j |V_i V_j| B_{ij} \cos(\theta_i - \theta_j)$$

- No conductance ~no voltage drop

consider constant voltage

* decoupling between P and Q
* consider P only

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

- No loss = balance of power

$$\sum_i P_i = 0$$

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$\theta_j)$

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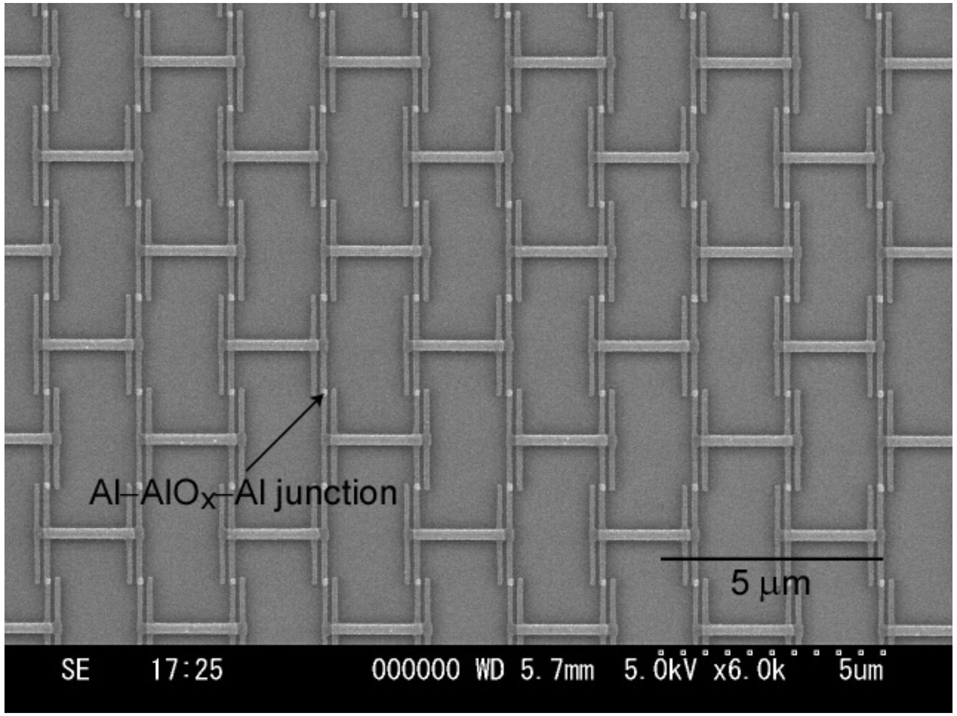
Josephson junction arrays vs. electric power systems !



Takahide, Yagi, Kanda, Ootuka, and Kobayashi
Phys. Rev. Lett. 85, 1974 (2000)

Josephson current

$$I_{ij} = I_c \sin(\theta_i - \theta_j)$$

Transmitted power

$$P_{ij} \simeq B_{ij} \sin(\theta_i - \theta_j)$$

First question

- Consider the power flow problem in the lossless line approximation

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

- It is entirely defined by

  *the graph/network/grid (admittance matrix B_{ij})
  *the set of power injections/consumptions $\{P_i\}$

Question : how many different solutions are there ?

First answer : an infinite # of them, since $\{\theta_i + C\}$ is also a solution for any constant C

Define "different" as differing by more than C

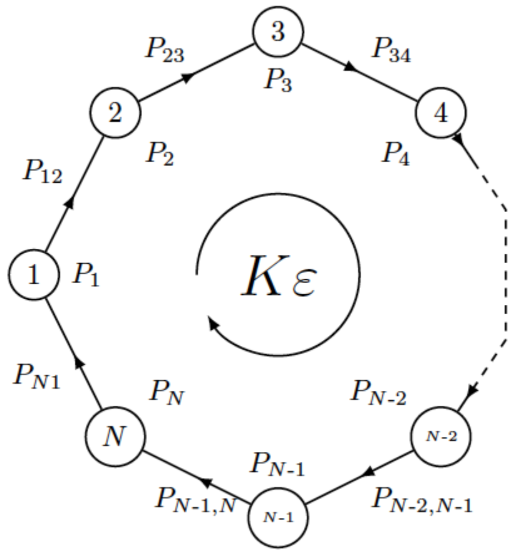
Circulating loop flows

*Thm: Different solutions to the following power-flow problem

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

may differ only by circulating loop current(s) **in any network**

Dörfler, Chertkov, Bullo, PNAS '13; Delabays, Coletta and PJ, JMP '16



*Voltage angle uniquely defined

→ $q=\Sigma_i|\theta_{i+1}-\theta_i|/2\pi \in \mathbb{Z}$ **~topological winding number**

→ "quantization" of these loop currents **~vortex flows**

Janssens and Kamagate '03

Circulating loop flows

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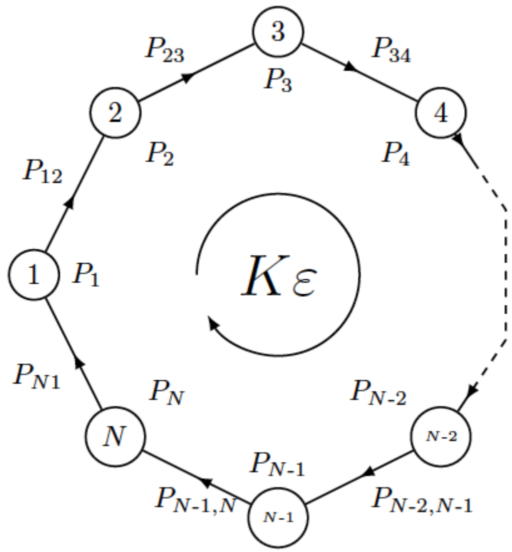

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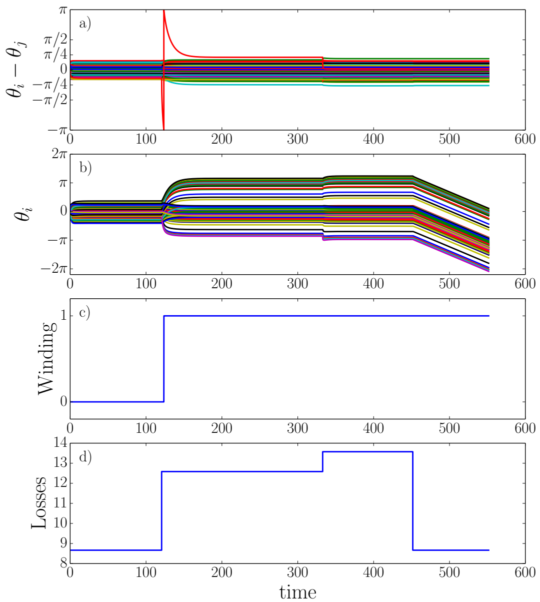

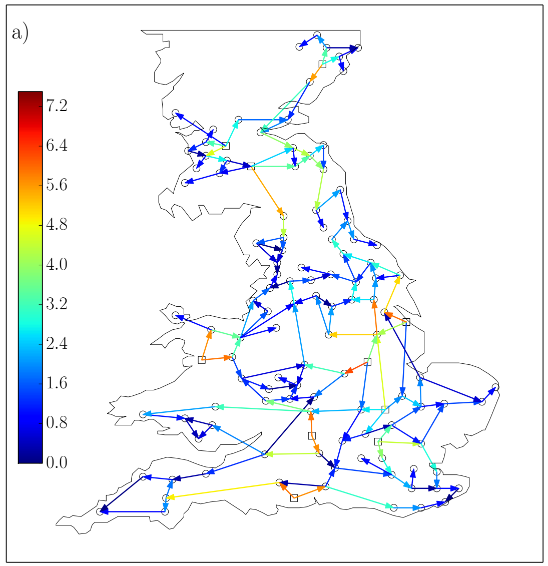

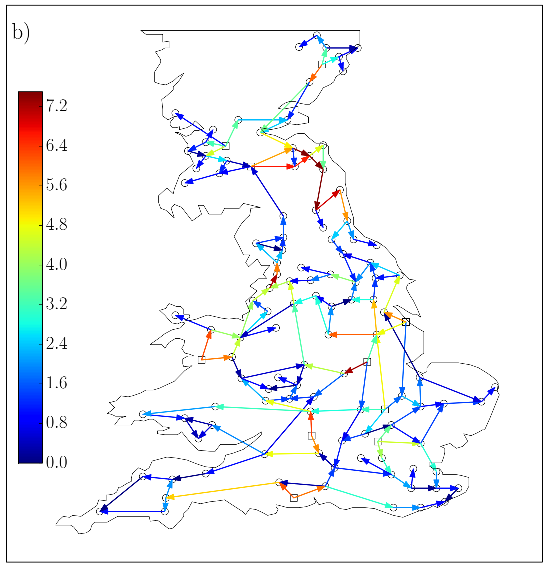

Circulating loop flows

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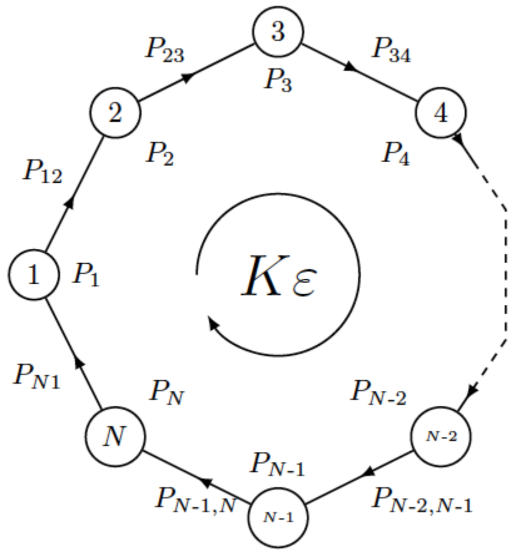

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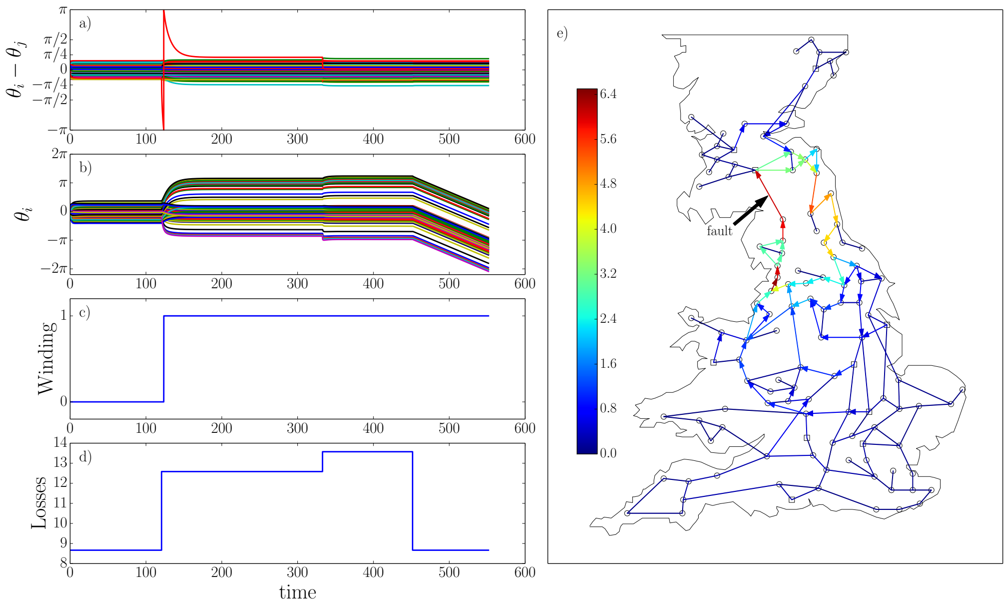


Vortex flows in AC power grids vs. superconductivity

- Vortex/circulation creation in SC via B-field

Can one create vortex flows in AC power grids ? How ?

- Superconductors are dissipationless
- AC power grids are dissipative ~ they have a finite resistance

Do vortex flows survive the presence of dissipation ?

Vortex flows in AC power grids vs. superconductivity

- Vortex/circulation creation in SC via B-field

Can one create vortex flows in AC power grids ? How ?

Three mechanisms : *dynamical phase slip
****line tripping***
****line tripping and reclosure***

- Superconductors are dissipationless
- AC power grids are dissipative ~ they have a finite resistance

Do vortex flows survive the presence of dissipation ?

YES ! They are robust against moderate amounts of dissipation

Coletta, Delabays, Adagideli and PJ, NJP '16

Approximated power flow equations : (2) losses to leading order

- Keep decoupling between P and Q

put back conductance

$$P_i = \sum_j [B_{ij} \sin(\theta_i - \theta_j) + G_{ij} \cos(\theta_i - \theta_j)]$$

- Losses are positive

$$\sum_i P_i = \sum_{ij} G_{ij} \cos(\theta_i - \theta_j)$$

$$\sum_i P_i > 0$$

Remark (important) :
$\{P_i\}$ and $\{\theta_i\}$ need to be self-consistently determined

Generation of vortex flow by line tripping

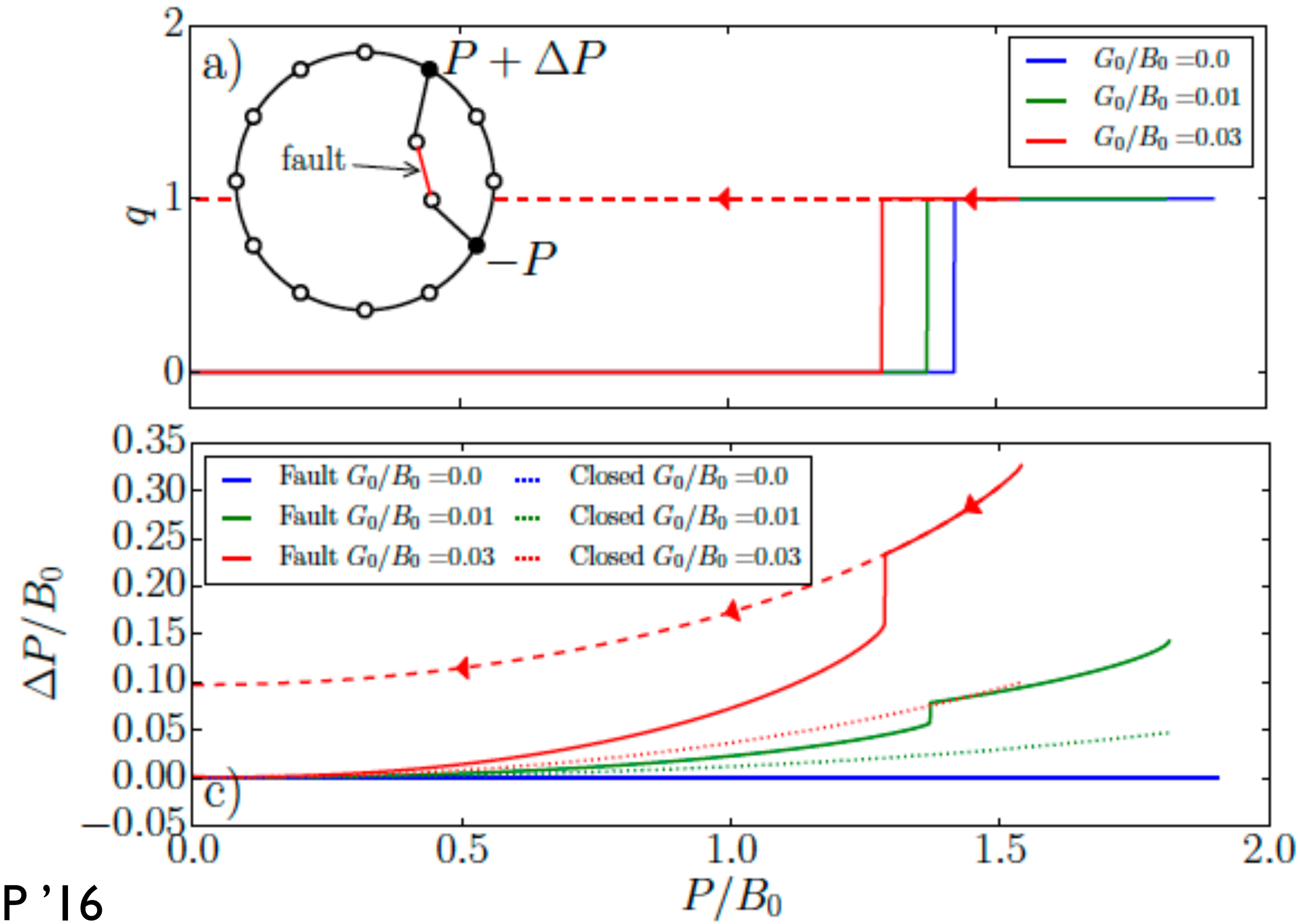
*Power grids are meshed - path redundancy
*Power is still supplied after one line trips
*Consider line tripping in an asymmetric, three-path circuit
*all winding numbers vanish initially

Power redistribution can lead to vortex flow with $q=\Sigma_i|\theta_{i+1}-\theta_i|/2\pi > 0$

Line tripping at $P/B_0 > 1.3$

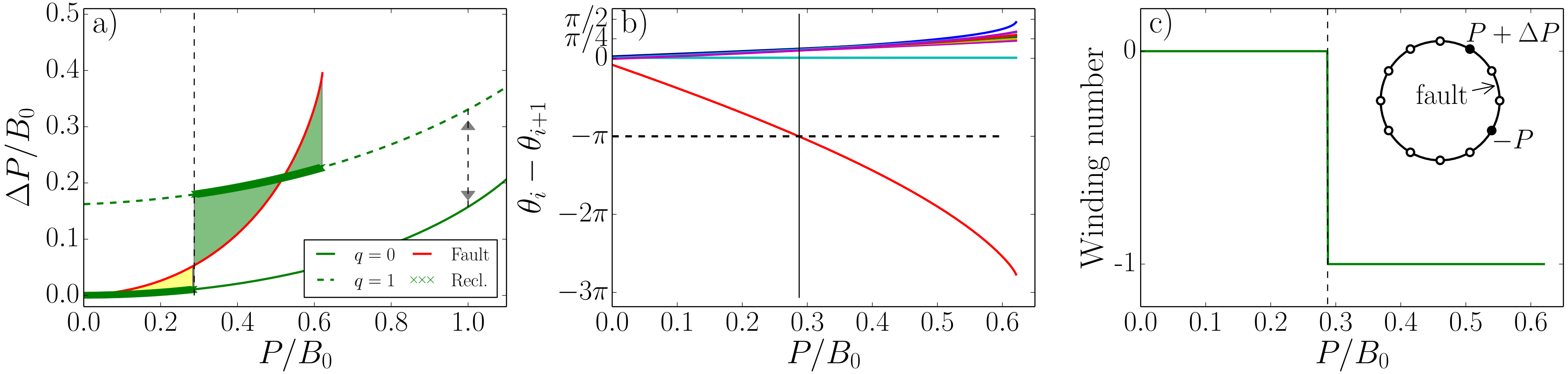
→ q=1

Vortex state characterized by
-hysteresis
= **topological protection**
-higher losses



Coletta, Delabays, Adagideli and PJ, NJP '16

Generation of vortex flow by line tripping and reclosure



- Single-loop network
- One producer, one consumer
- One line trips, is later reclosed

Vortex formation for $P/B_0 > 0.26$

Vortex formation for $|\theta_{i+1}-\theta_i|>\pi$ (two ends of faulted line)

Coletta, Delabays, Adagideli and PJ, NJP '16

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Generation of vortex flow by line tripping and reclosure

*Lyapunov function - defines basins of attraction for different solutions

$$\mathcal{V}(\{\theta_i\}) = -\sum_l P_l \theta_l - \sum_{\langle l,m \rangle} B_0 \cos(\theta_l - \theta_m)$$

van Hemmen and Wreskinski ‘93

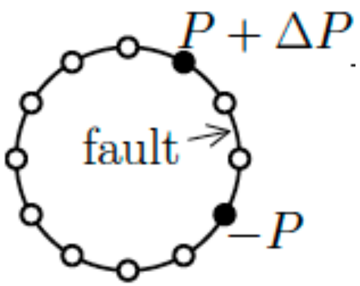
*Steady-state solutions have $\nabla \mathcal{V} = 0$

Generation of vortex flow by line tripping and reclosure

*Lyapunov function - defines basins of attraction for different solutions

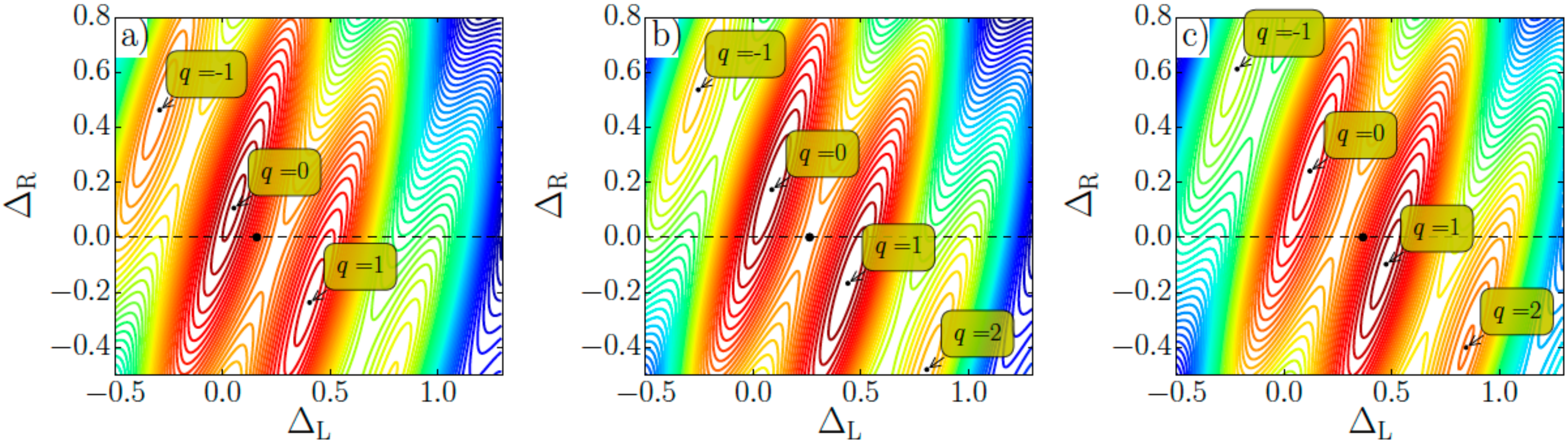
$$\mathcal{V}(\{\theta_i\}) = -\sum_l P_l \theta_l - \sum_{\langle l,m \rangle} B_0 \cos(\theta_l - \theta_m)$$

van Hemmen and Wreskinski '93



*In our case

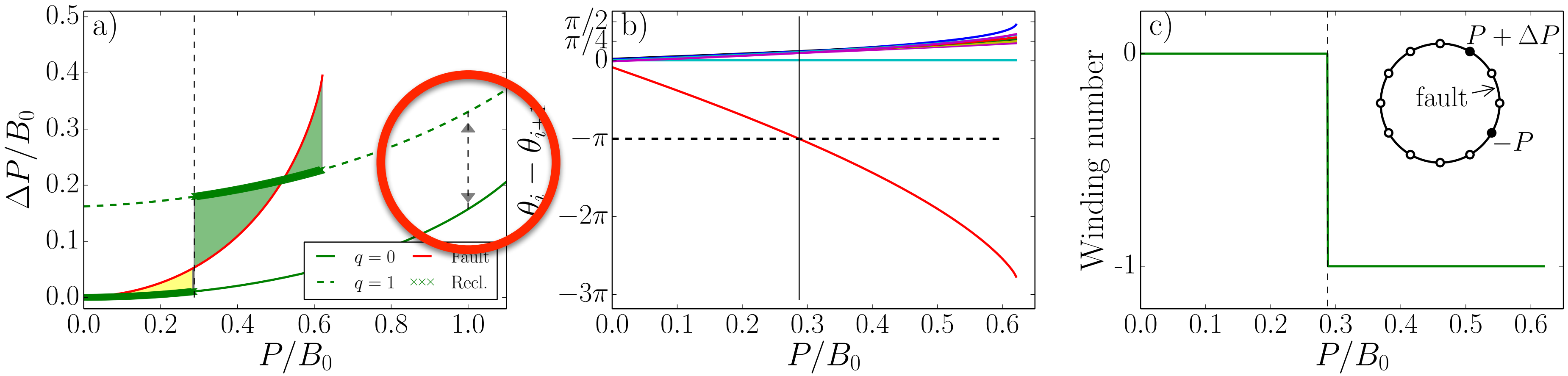
$$\mathcal{V}(\Delta_{\rm L}, \Delta_{\rm R}) = -N_{\rm L} P \Delta_{\rm L} - N_{\rm L} B_0 \cos \Delta_{\rm L} - (N_{\rm R}-1) B_0 \cos \Delta_{\rm R} - B_0 \cos(N_{\rm L}\Delta_{\rm L} - (N_{\rm R}-1)\Delta_{\rm R})$$



$P \approx 0.159 B_0$ $\quad$ $P = B_0 \sin(\pi/12) \approx 0.259 B_0$ $\quad$ $P \approx 0.359 B_0$

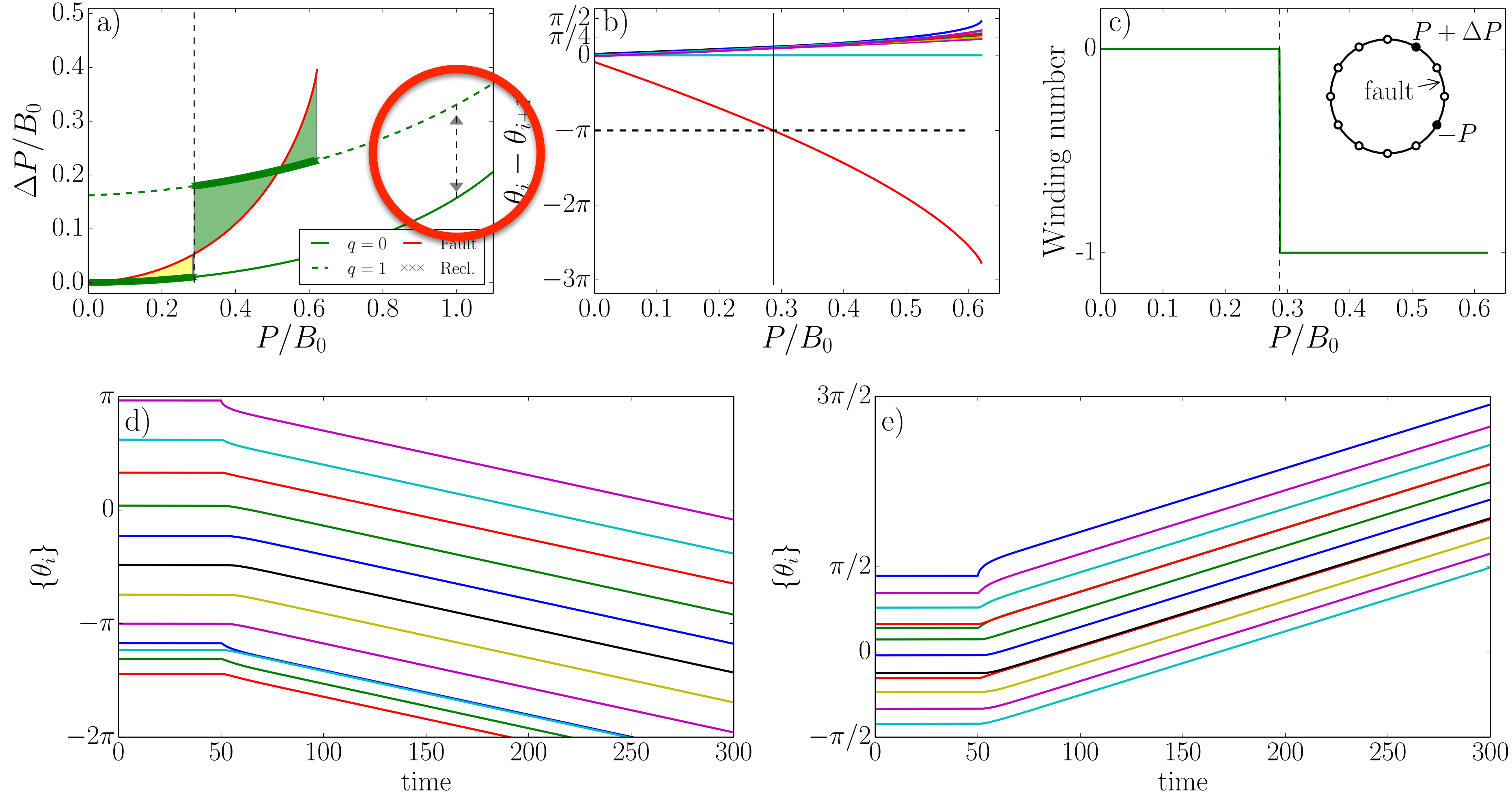
Coletta, Delabays, Adagideli and PJ, NJP '16

Generation of vortex flow by line tripping and reclosure



Try to kill / create vortex by adapting ΔP ?

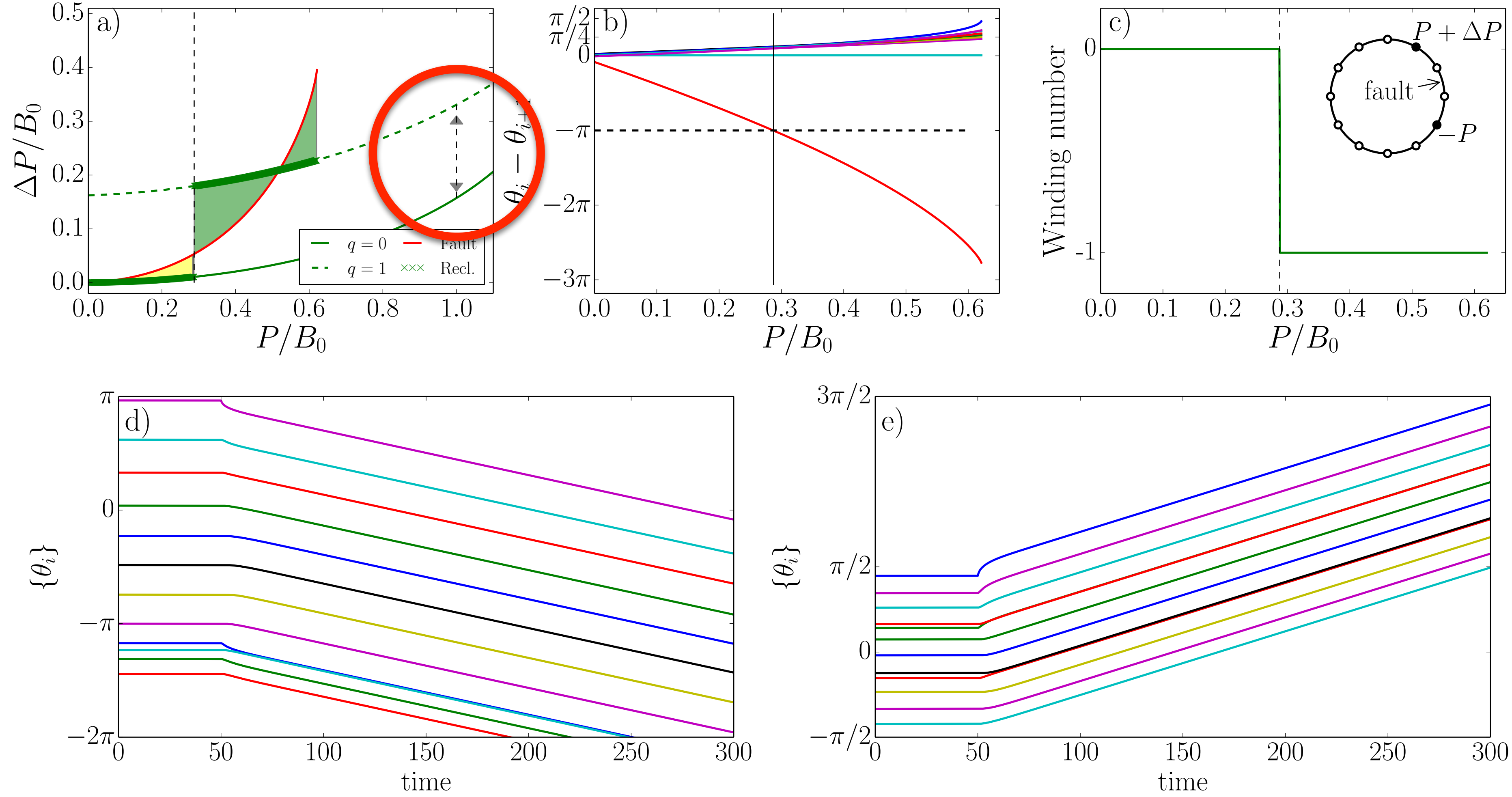
Generation of vortex flow by line tripping and reclosure



!! Cannot kill nor create vortex by adapting ΔP !!

Instead one changes the grid's frequency

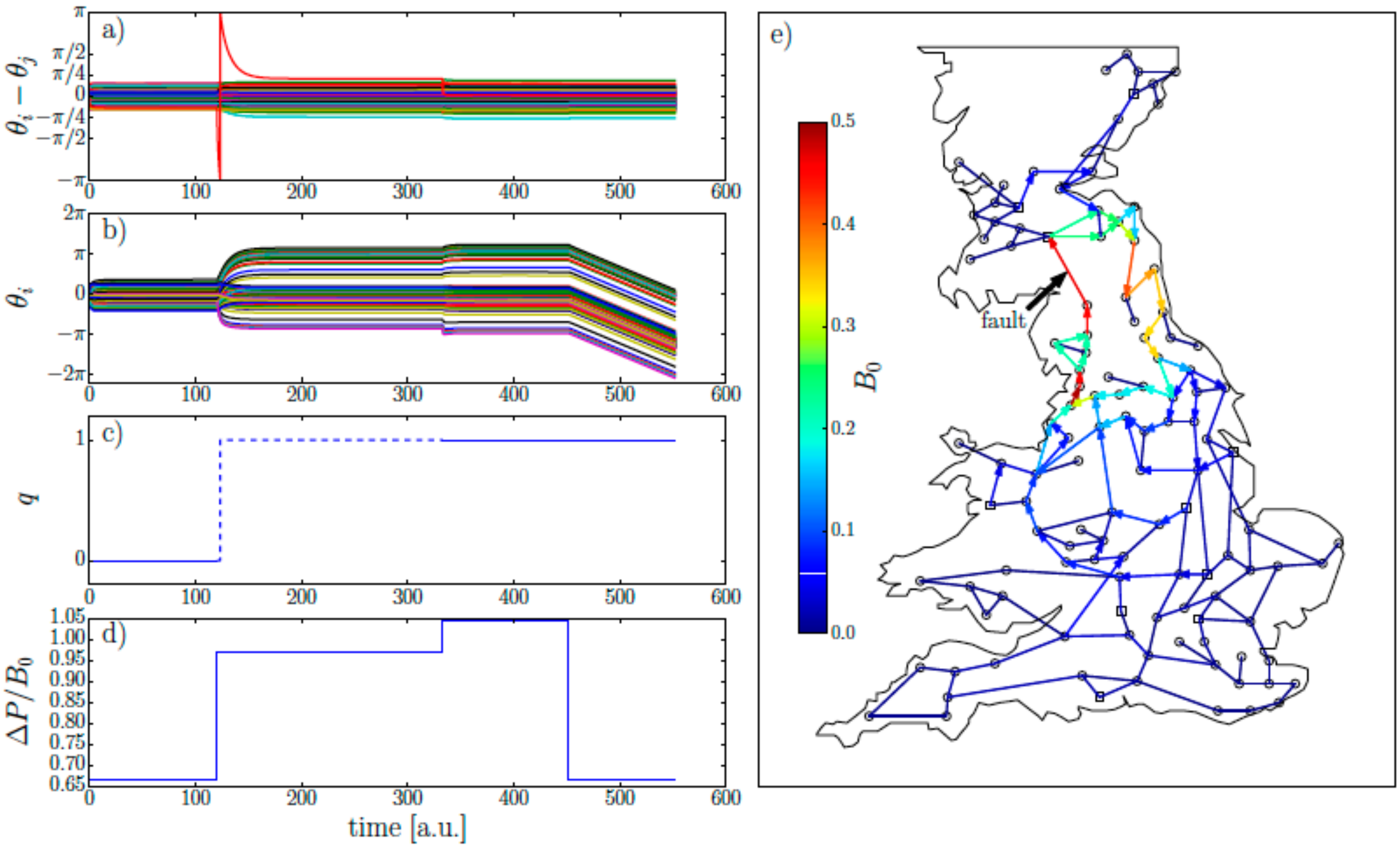
Generation of vortex flow by line tripping and reclosure



!! Topological protection !!

Coletta, Delabays, Adagideli and PJ, NJP '16

Generation of vortex flow by line tripping and reclosure



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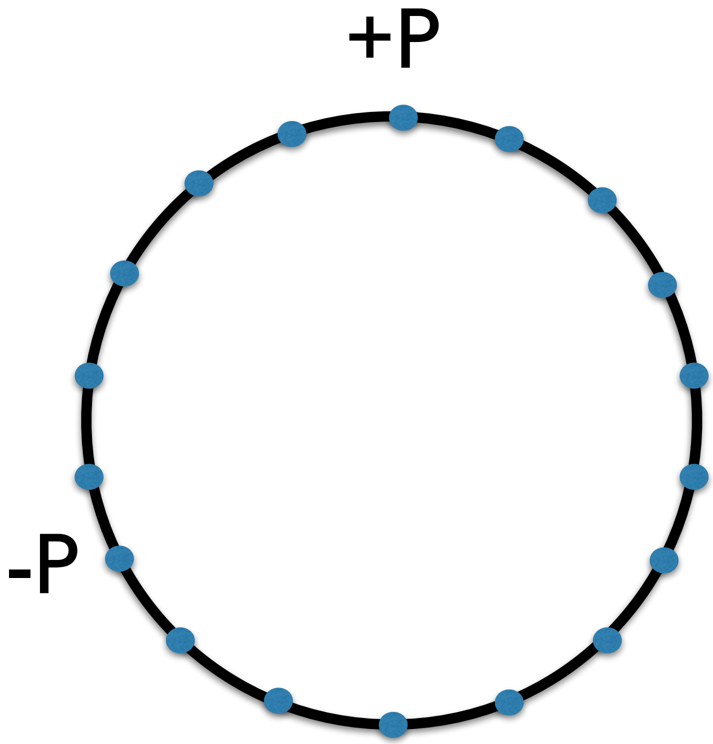
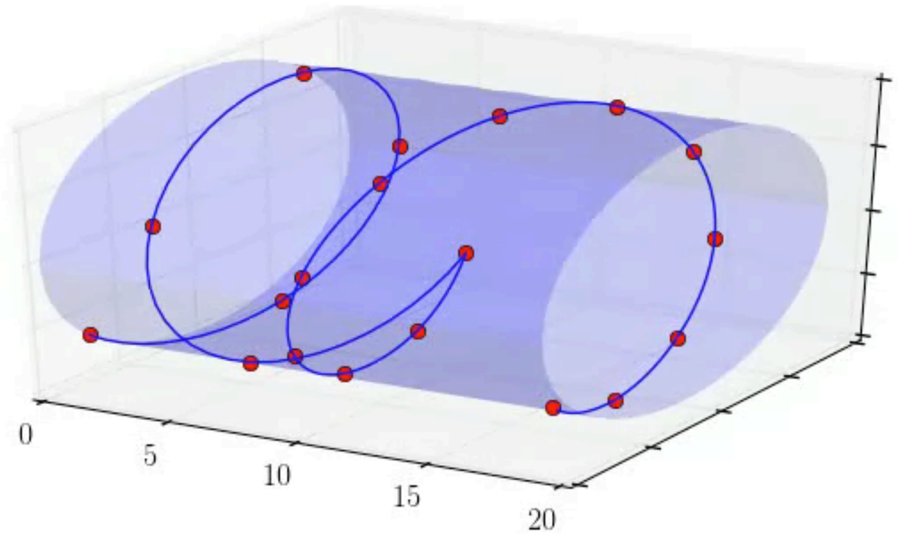
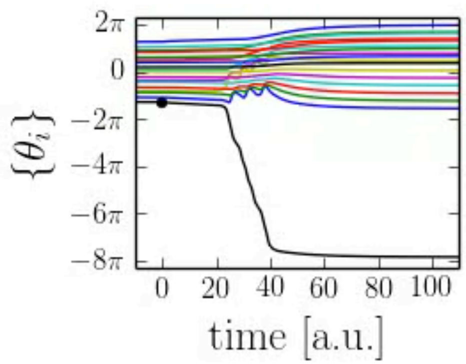
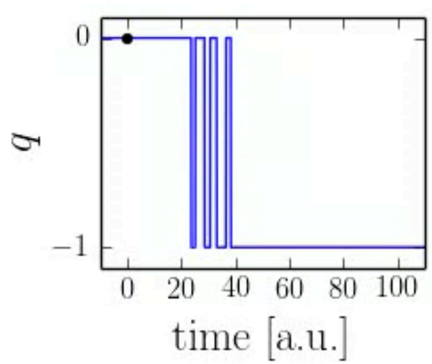
Dynamical generation of vortex flows



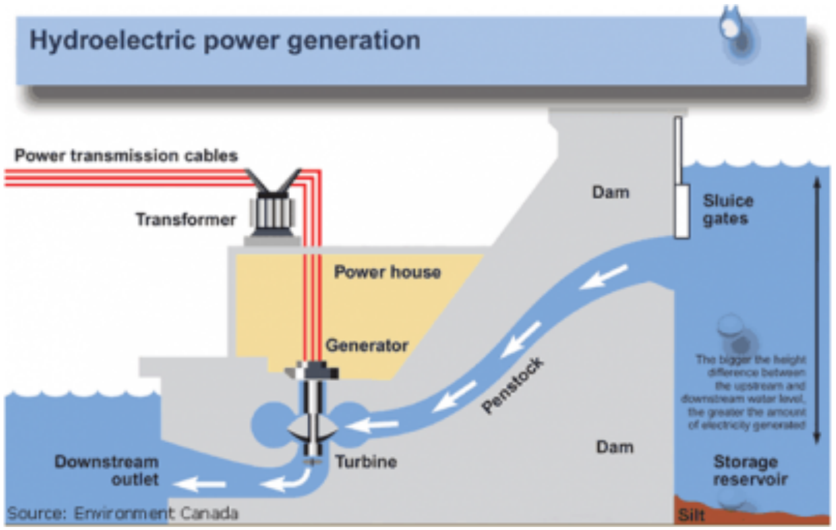


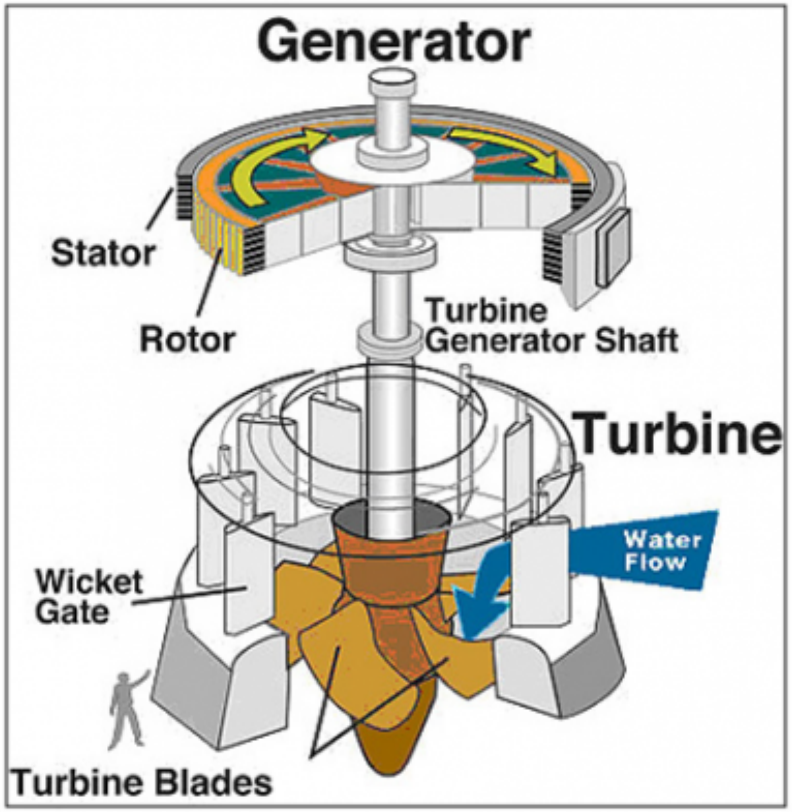




Similar to quantum phase slips in JJ arrays
Lau, Markovic, Bockrath, Bezryadin and Tinkham '01
Matveev, Larkin and Glazman '02

Time-evolution of frequency : swing equations





- Electricity production with rotating machines
- Potential, chemical, nuclear or thermal energy converted into mechanical energy (rotation)
- Mechanical power converted into electric power
- Time-dependence : think power instead of energy
- Balance between power in, power out and energy change in rotator : *SWING EQUATIONS*

Time-evolution of frequency : swing equations

- Power balance

$$\frac{dW_i}{dt} + P_i^{(d)} = P_i^{(m)} - P_i^{(g)}$$

Change in KE of rotator

Damping power (losses from friction)

Power input

Electric power output

- Swing equation for angles (in rotating frame @ 50Hz)

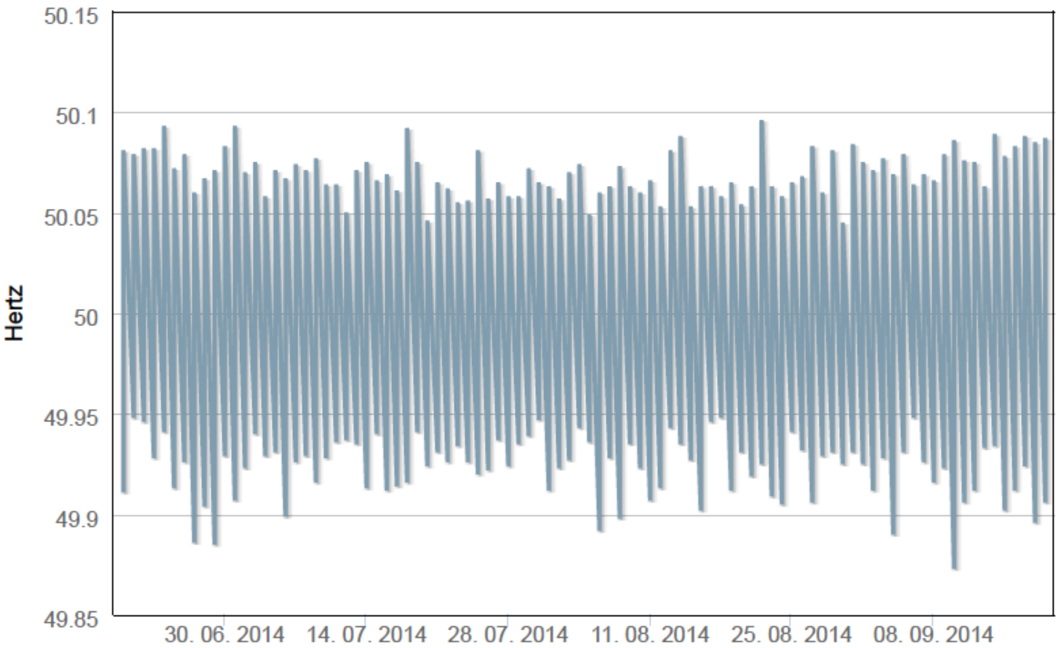
$$M\frac{d^2\theta_i}{dt^2} + D\frac{d\theta_i}{dt} = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$

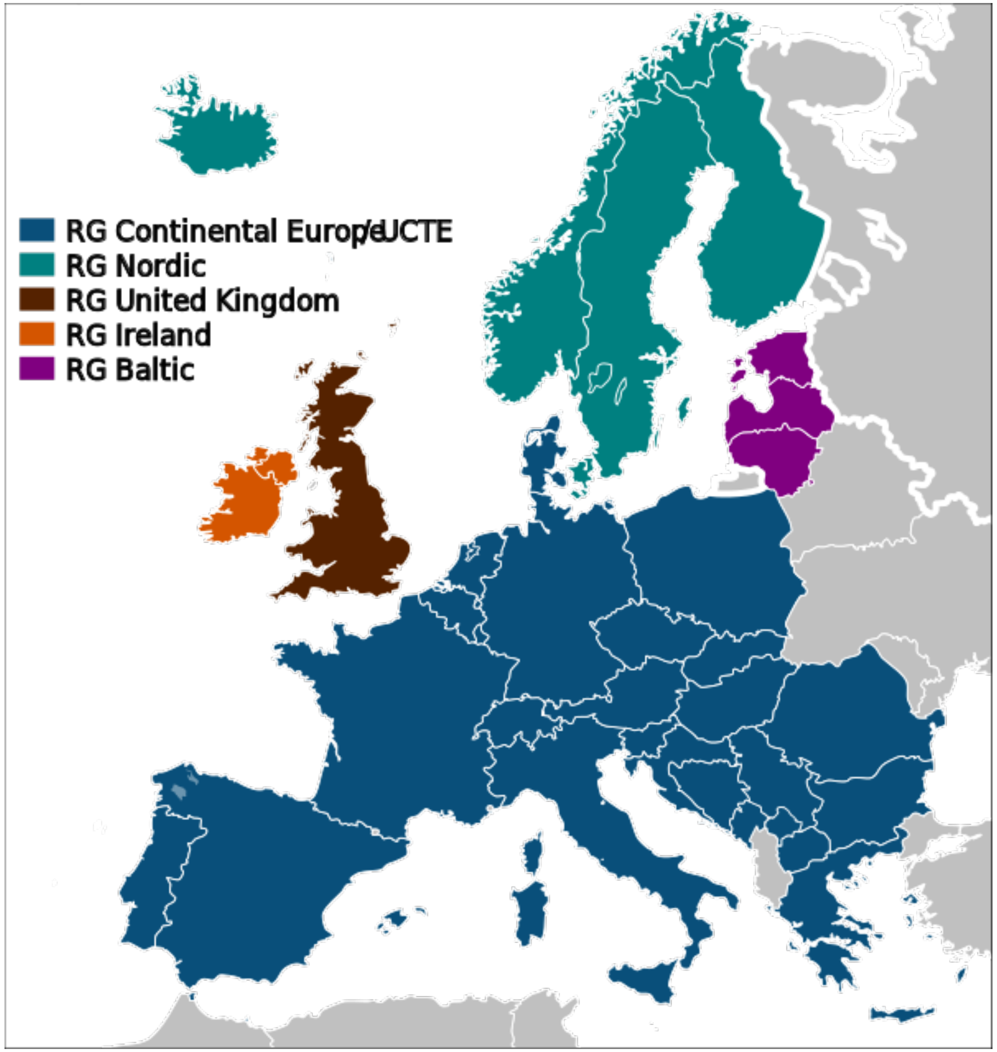
Inertia of rotator

Damping / friction

- Solutions of power-flow eqs. = steady-state solutions of swing eqs. ~synchronous solution (angles rotate in unison)

Steady-state operation : synchrony over 1000's of kms





Loss of synchrony : blackouts

Italy blackout, sep 28 2003

Northeast blackout, aug 14 2003

From the swing equations to linear stability

$$M\frac{d^2\theta_i}{dt^2} + D\frac{d\theta_i}{dt} = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$

- suppose a synchronized solution exists
- what is its stability under angle perturbation ?

A.: *linearize the dynamics about that solution

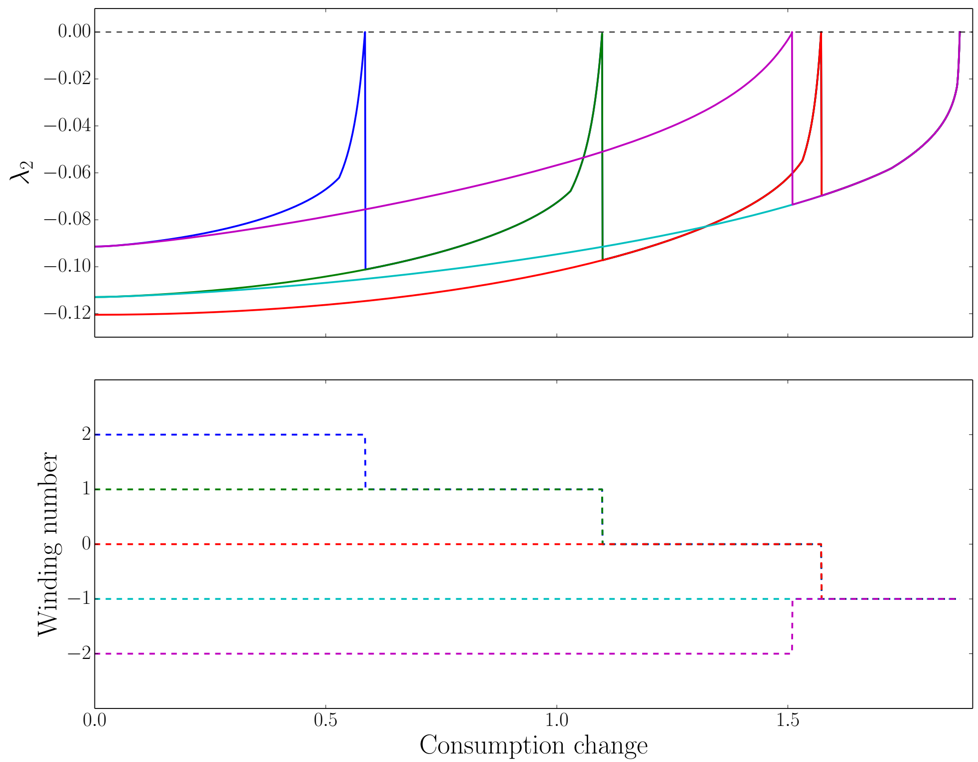
*perturbed condition goes back exponentially fast to the initial solution if the stability matrix

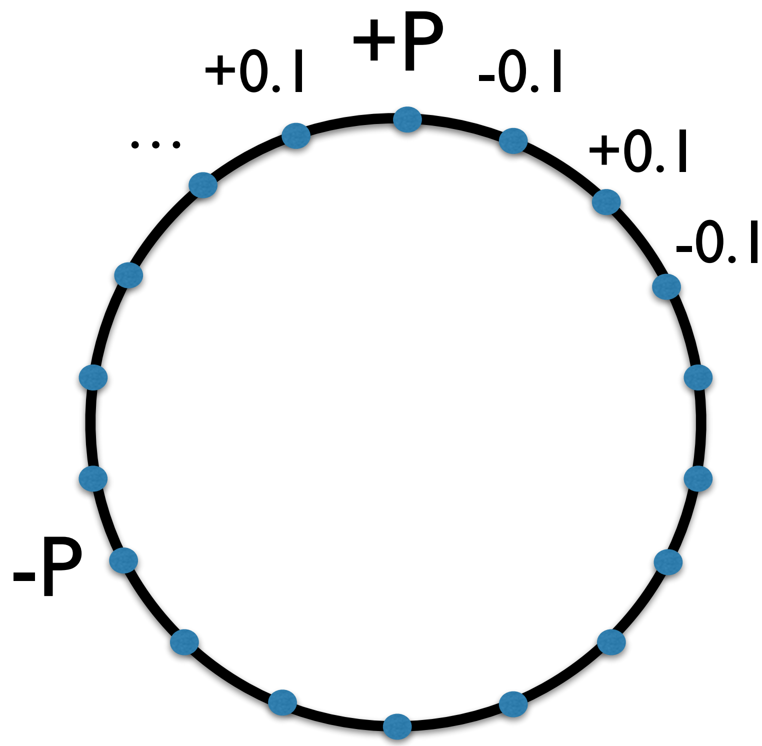
$$\mathcal{M}_{ij} = -\delta_{ij}\sum_k B_{ik}\cos(\theta_i^{(0)} - \theta_k^{(0)}) + B_{ij}(1-\delta_{ij})\cos(\theta_i^{(0)} - \theta_j^{(0)})$$

is negative semidefinite

Dynamical generation of vortex flows

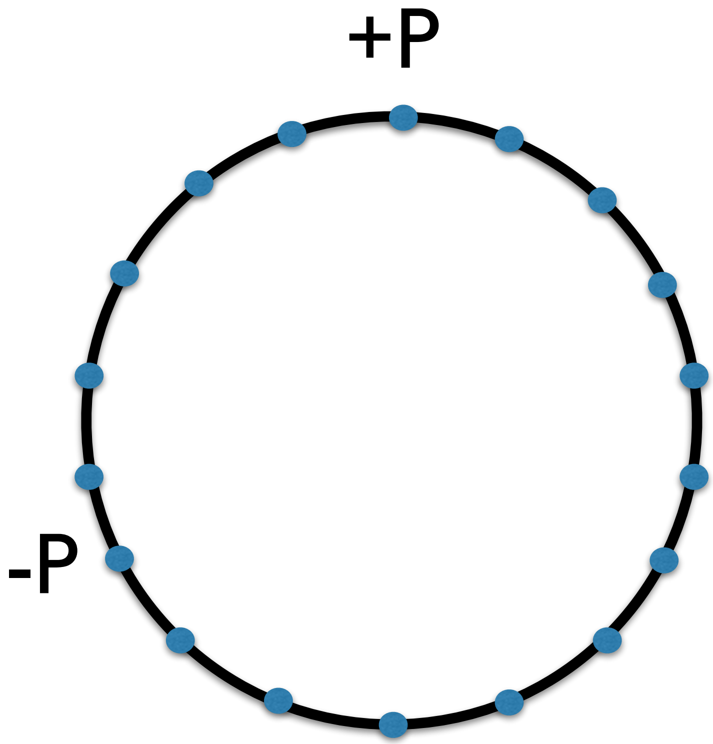
Loss of stability of q=0 solution

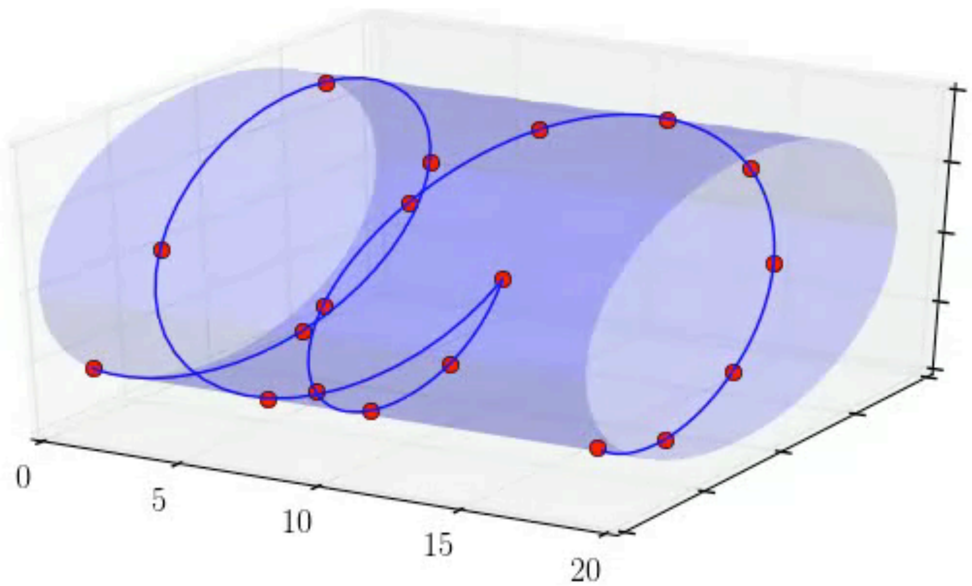


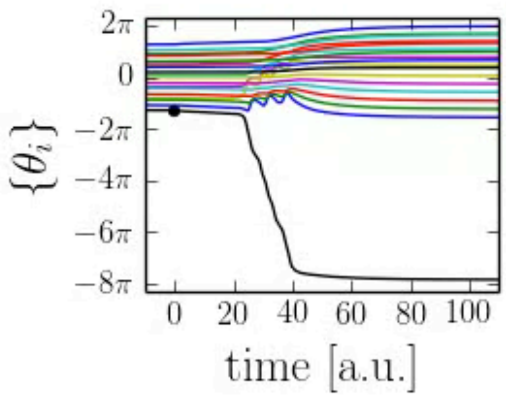


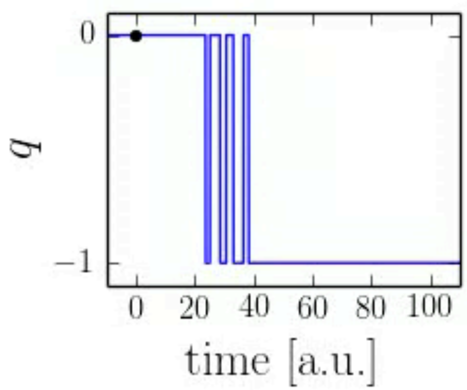
Coletta, Delabays, Adagideli and PJ, NJP ’16

Dynamical generation of vortex flows





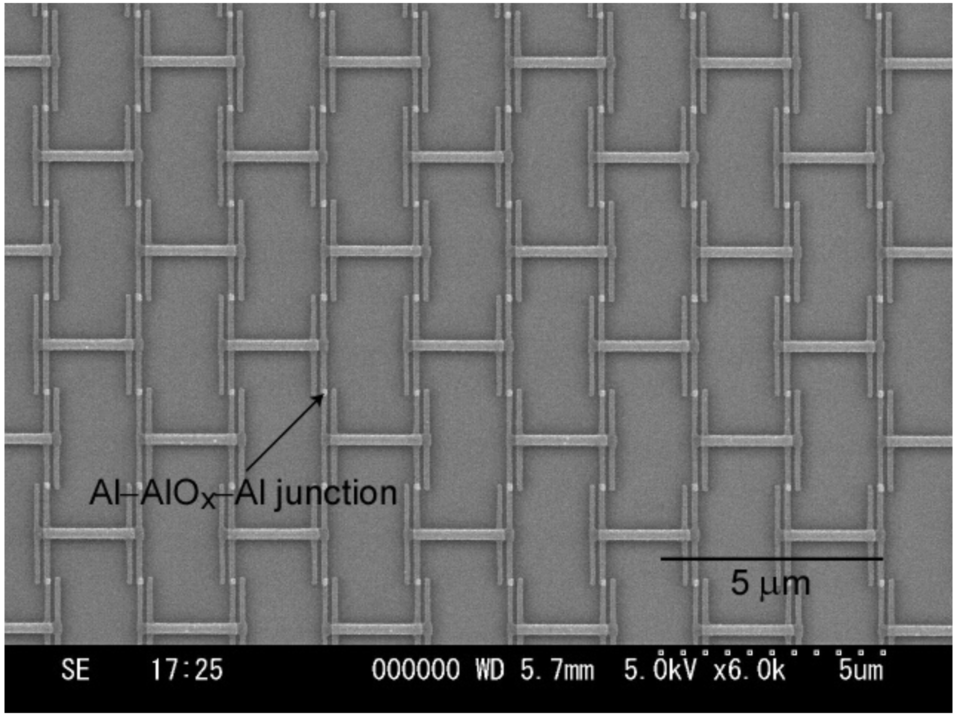




Similar to quantum phase slips in JJ arrays
Lau, Markovic, Bockrath, Bezryadin and Tinkham '01
Matveev, Larkin and Glazman '02

Take-home message

Profound, unexpected similarities between Josephson junction arrays and high voltage AC power grids !



dissipationless quantum fluid ⟷ !!!! *dissipative classical system*

Superconductivity vs. AC electric power systems !

	Superconductor	high voltage AC power grid
State	$\Psi(\mathbf{x}) = \|\Psi(\mathbf{x})\| e^{i\theta(\mathbf{x})}$	$V_i = \|V_i\| e^{i\theta_i}$
Current / power flow	$I_{ij} = I_c \sin(\theta_i - \theta_j)$ DC Josephson current	$P_{ij} \simeq B_{ij} \sin(\theta_i - \theta_j)$ Power flow; lossless line approx.
winding # $q=\Sigma_i\|\theta_{i+1}-\theta_i\|/2\pi$	Flux quantization Persistent currents	Circulating loop flows

Thank you !

Coletta and PJ, Phys Rev E 93, 032222 (2016)

Delabays, Coletta, and PJ, J Math Phys 57, 032701 (2016)

Coletta, Delabays, Adagideli and PJ, New J Phys '16 (to appear)

Delabays, Coletta, and PJ, arXiv:1609.02359, submitted to J Math Phys

Hes·so// VALAIS WALLIS