1 A take on percolation theory

The problem of percolation through a regular lattice can be approached from two directions: in the limit of *site percolation* only the decoration of the lattice sites is cnsidered. If two neighboring sites are occupied by the percolating species they are assigned to one *cluster* of sites. The system is percolating, if it exhibits at least one infinitely extended cluster. Let P_∞^{site} be the probability that such an inifinite cluster exists. Given the probability p of a lattice site being occupied by the percolating species there exists a critical concentration $p = p_{\mathrm{c}}^{\mathrm{site}}$, the site percolation threshold, below which P_∞ is exactly equal to zero.

The second limit for the description of percolation considers the *bonds* between sites. The same

1.1 Site percolation

For an infinite lattice with a concentration p of the conducting species, the *percolation probability* $P_\infty(p)$, i.e., the probability that an occupied site is part of a percolating (infinite) cluster is given by

$$P_\infty(p) = \begin{cases} \hat{B}_p \left(\frac{p}{p_{\mathrm{c}}} - 1\right)^{\beta_p} & \text{for} \quad p > p_{\mathrm{c}} \\ 0 & \text{else} \quad , \end{cases} \tag{1}$$

where the critical concentration p_{c} is the *site percolation threshold*. The prefactor $\hat{B}_p$ and the exponent β_p both depend on the lattice and the concentration.