MATH 281

Wednesday, February 22, 2023 1:55 PM

PROBABILITY

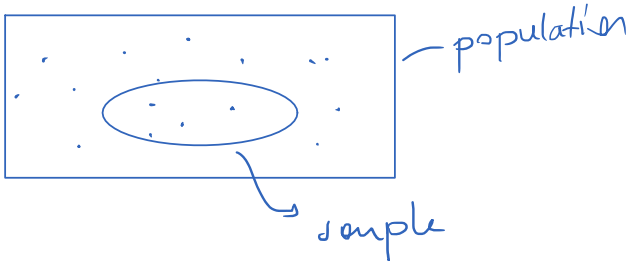
Probability is the study of randomness.

Randomness is the opposite of determinism.

A random phenomenon is one that can yield different outcomes in repeated experiment, even if we use exactly the same conditions in each experiment.

DEF: A population is a collection of individuals (units).

DEF: A sample is a collection of observations taken from a population. Namely, a smaller group drawn from population.



E.g. Population: All songs from the Eurovision song contest.

Sample: winning songs from the Eurovision song contest that were performed in English.

In Probability: the features of the population is known, we are guessing about the features of the sample.

In Statistics: the features of the sample is known, we are guessing about the features of the population.

y: (Probability): Assume that we have 4 green, 3 red apples in a basket. What is the probability that a chosen apple is red? $\frac{3}{7}$ → red / → total

y: (Statistics): Assume that we have a sample (1 green and 2 red apples). Find the frequency of red apples in a basket. $\frac{2}{3}$.

2.1. Sample Space

An experiment is any process that generates a set of data (observations or outcomes).

types of variables

- Qualitative (Categorical)
  - unordered → nominals
    - -head / tail
    - - Gender
    - - Hair color
  - ordered → ordinals
    - - AA, BB, ...
    - - Education level (Elementary, high school, ..)
- Quantitative (Numerical)
  - discrete
    - . The # of children in a family.
  - continuous
    - • Weight of a person.
    - • Amount of time.

DEF: A random experiment is an experiment subject to the uncertainty.

(It is unpredictable in a short time period but predictable in a long time period). eg: waiting time at a bus stop for the arrival of a bus.

eg: Toss a fair coin.
First toss: The result is unpredictable.
Toss the coin 200 times, our expectation will be 100 Heads (H) and 100 tails (T)

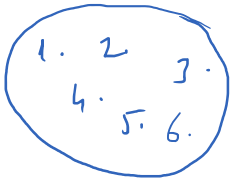
DEF: Each possible outcome of a random experiment is called a sample points. (elements of the population).

DEF: The collection of all possible outcomes of a random experiment is called a sample space (S or Ω).

ex: random experiment	sample points	sample space
tossing a coin	H, T	$\{H, T\}$.
tossing two coins	HH, HT, TH, TT	$\{HH, HT, TH, TT\}$.

Recall: Representation of the sets:

- by listing: $S = \{1, 2, 3, 4, 5, 6\}$.
- by Rule method: $S = \{x \in \mathbb{Z} \mid 1 \leq x \leq 6\}$.
- by Venn diagram:



Set Operations:

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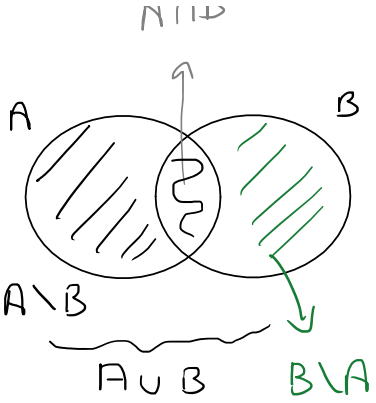
$A \cap B$

Union: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Intersection: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

Difference: $A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$.

Complement: A' or A^c $\quad A^c = \{x \mid x \notin A\}$



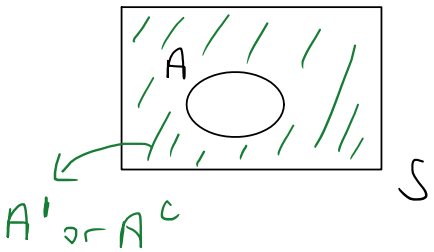
($\emptyset$: empty set)

- $A \cap \emptyset = \emptyset$
- $A \cup \emptyset = A$
- $A \cap A^c = \emptyset$
- $A \cup A' = S$
- $(A')' = A$

* $(A \cap B)' = A' \cup B'$

* $(A \cup B)' = A' \cap B'$

} De Morgan Rule.



2.2 Events

DEF: A subset of a sample space is called an event.

ex: Rolling a die: $S = \{1, 2, 3, 4, 5, 6\}$.

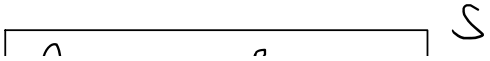
Let A be the event that the outcome is less than 4 $\Rightarrow A = \{1, 2, 3\}$.

Let B be the " " " " is an odd number $\Rightarrow B = \{1, 3, 5\}$.

$A^c = A' = \{4, 5, 6\}$, $\quad B' = \{2, 4, 6\}$. $\quad B \setminus A = \{5\}$.

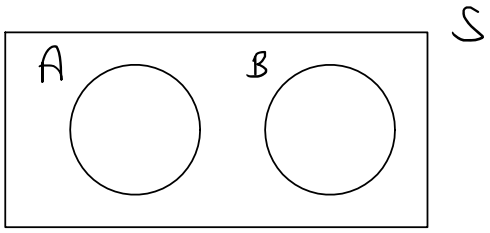
DEF: The events A and B are disjoint (mutually exclusive)

if $\boxed{A \cap B = \emptyset}$



if $A \cap B = \emptyset$

- $\emptyset$ refers to the event consisting of no outcomes. (null event).



A & B are disjoint.

ex: let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$.

since $A \cap B = \emptyset$ the sets A and B are mutually exclusive. (events).

Ex: If the experiment consists of tossing two dice, then the sample space consists of 36 points.

$$S = \{(i, j) \mid i, j = 1, 2, 3, 4, 5, 6\}.$$

where the outcome (i, j) is said to occur if i appears on the leftmost die and j on the other die.

Let E be the event that the sum of the dice equals 7.

$$E = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}.$$