Lecture 2:
Arrays & Linked Lists

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저작권 안내

(주)업스테이지가 제공하는 모든 교육 콘텐츠의 지식재산권은 운영 주체인 (주)업스테이지 또는 해당 저작물의 적법한 관리자에게 귀속되어 있습니다.

콘텐츠 일부 또는 전부를 복사, 복제, 판매, 재판매 공개, 공유 등을 할 수 없습니다.
유출될 경우 지식재산권 침해에 대한 책임을 부담할 수 있습니다.

유출에 해당하여 금지되는 행위의 예시는 다음과 같습니다.

- 콘텐츠를 재가공하여 온/오프라인으로 공개하는 행위
- 콘텐츠의 일부 또는 전부를 이용하여 인쇄물을 만드는 행위
- 콘텐츠의 전부 또는 일부를 녹취 또는 녹화하거나 녹취록을 작성하는 행위
- 콘텐츠의 전부 또는 일부를 스크린 캡쳐하거나 카메라로 촬영하는 행위
- 지인을 포함한 제3자에게 콘텐츠의 일부 또는 전부를 공유하는 행위
- 다른 정보와 결합하여 Upstage Education의 콘텐츠임을 알아볼 수 있는 저작물을 작성, 공개하는 행위
- 제공된 데이터의 일부 혹은 전부를 Upstage Education 프로젝트/실습 수행 이외의 목적으로 사용하는 행위

01

Arrays

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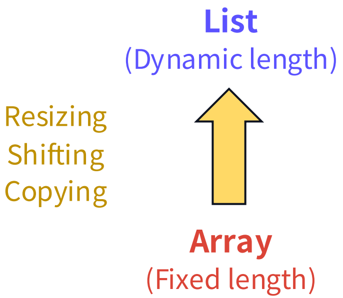
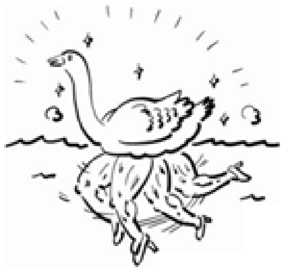
Arrays

- An **array** is an object comprising a numbered sequence of memory boxes
  - More fundamental data structure that Python lists are built on.
  - This is why we can easily access the *i*-th element of list A by using A[i].

- An array comprises
  - **Fixed** integer **length** (*N*) – should be set when initializing it
  - A sequence of *N* memory boxes (numbered 0 through *N* - 1)





[1]

[1] http://1moment.tistory.com

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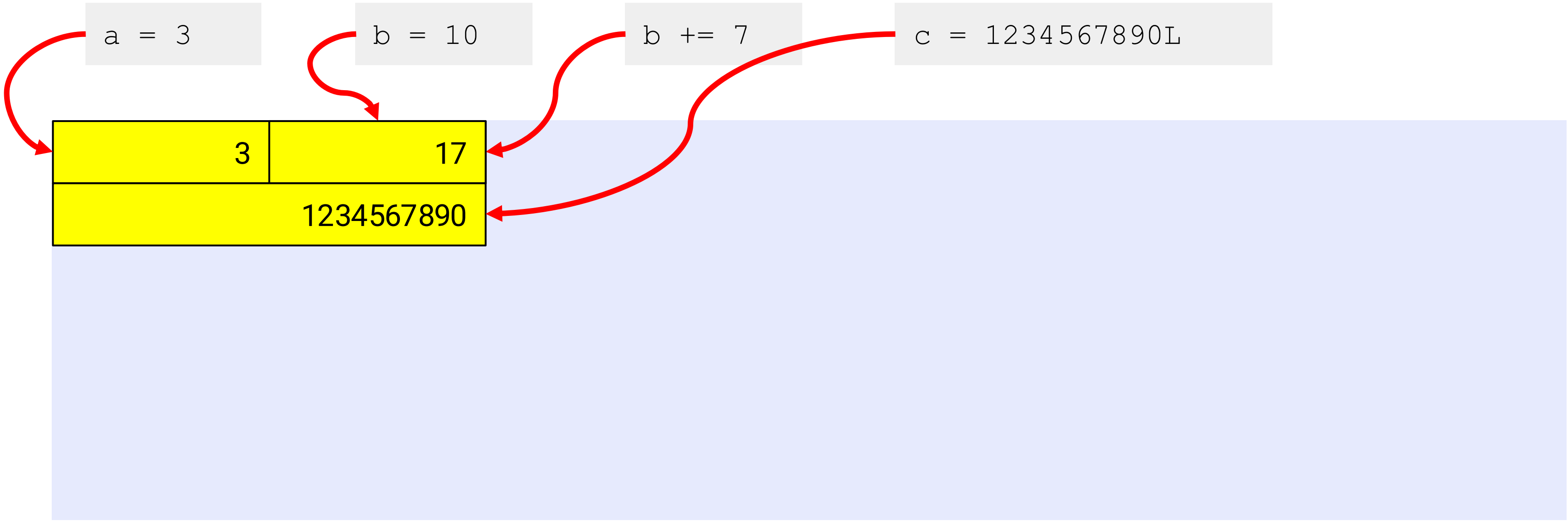
Internal Implementation: Memory

- Internally, all variables and constants we use in our program should be stored somewhere in **memory**.
  - For a single variable of a primitive type (int, float, ⋯), we know its size (how many bits are needed).



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Array Resizing

- Two problems of an array due to its fixed length
  - **Memory wastage**: if it contains only $n \ll N$ valid elements
  - **Memory shortage**: if it wants to contain more than N elements
- Array resizing: create another larger array and copy all the elements
  - L.append(3) when the current array is full.

L

Index	0	1	2	3	4	5	6	7	8	9
Value	-7	15	2	6	-1	5	4	10	-4	21

Index	0	1	2	3	4	5	6	7	8	9	10
Value	-7	15	2	6	-1	5	4	10	-4	21	3

Create
a new long array O(1)

Create(copy)
a new long array **$O(N)$**

Add
a new element O(1)

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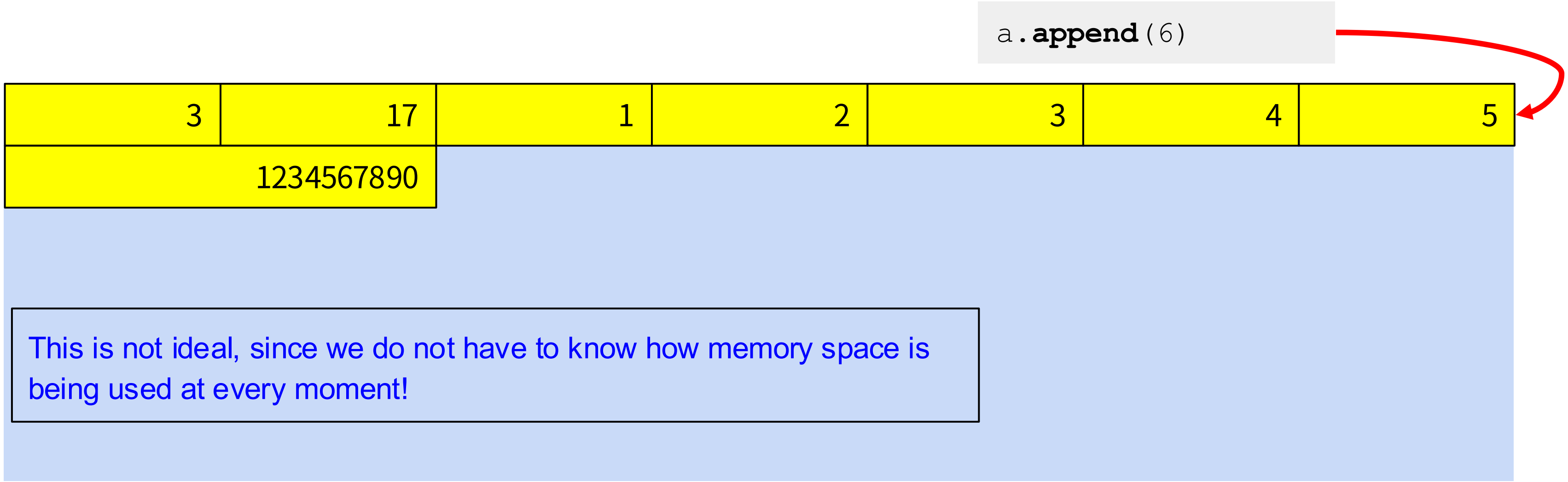
Internal Implementation: Memory

- Now, let's think about the time complexity!
  - **Theoretical time complexity** for `a.append(6)`? O(1)
  - **Actual time complexity** for `a.append(6)`, if there's no enough space after it? O(*N*)



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Array Resizing

- Array resizing is expensive: new memory boxes and copy operation
  - Increasing size by one every time is not efficient (too many resizing)
  - Increasing size too much at once is not efficient either (memory wastage)

- To resize fewer, Python list size grows as 0, 4, 8, 16, 25, 35, 46, 58, ···
  - Mild over-allocation proportional to the current size

- Anyway, is there any better way of organizing a collection of data to support append and pop easily?

02

Linked Lists

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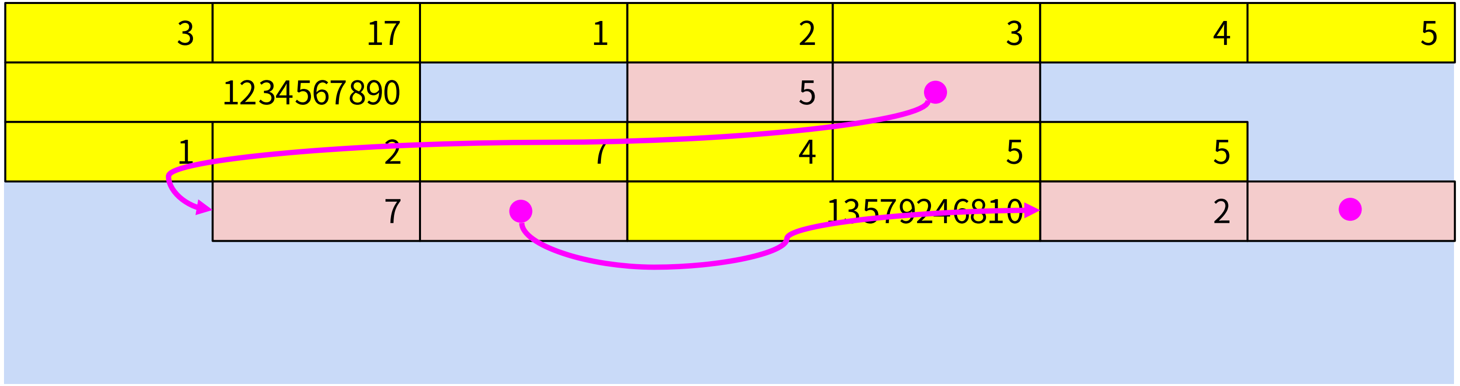
Linked Lists

- Main idea:
  - Allow each element in the list to be **scattered in the memory**.
  - Instead, each element **points to the next one**.

l.**append**(5)　　l.**append**(7)　　l.**append**(2)



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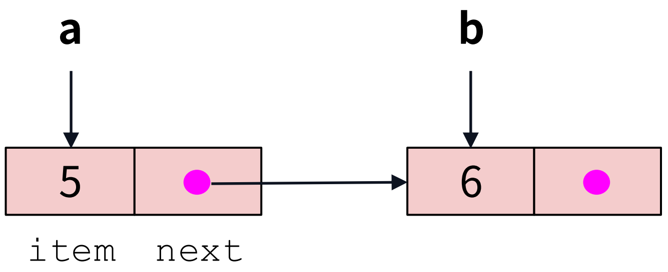
Linked Lists

- Class Node
  - Because we always need to store the **item** and the pointer to the **next** node, let's make this as a class!

```
class Node():
  def __init__(self, x):
    self.item = x
    self.next = None
```

```
a = Node(5)
b = Node(6)
a.next = b
```




```
print(a.item)
print(a.next.item)
```

5

6

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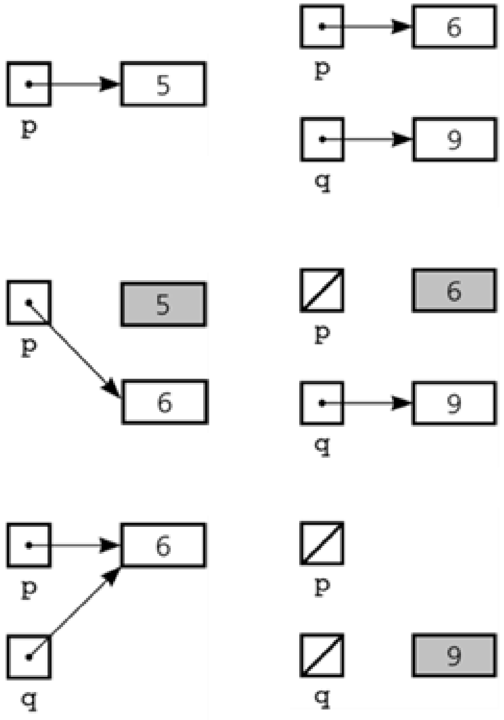
Review: Python Object Reference

	p.item	q.item
`p = Node(5)`	5	
`p = Node(6)`	6	
`q = p`	6	6
`q = Node(9)`	6	9
`p = None`	Error!	9
`q = p`	Error!	Error!



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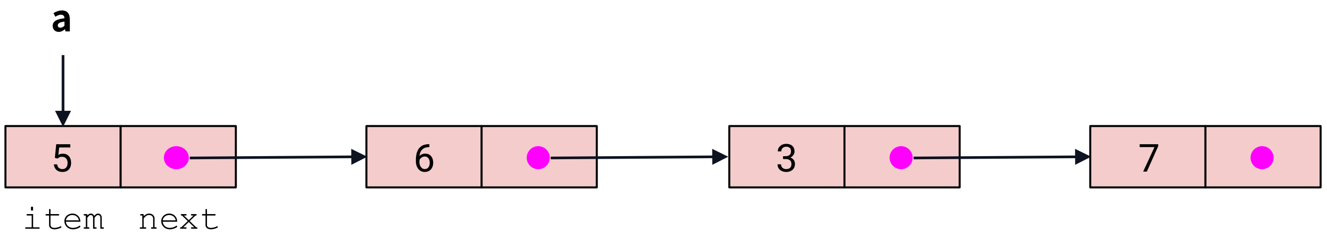
Singly Linked List

- Let's design the **singly linked list** data structure. What functionalities do we need?
  - **Creating** an empty list
  - **Adding / inserting** a new item
  - **Retrieving** an item
  - **Deleting / removing** an existing item

at position *i*



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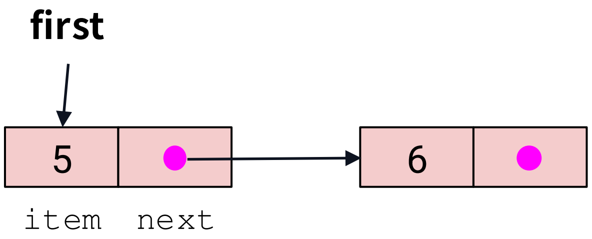
Singly Linked List

- Class `LinkedList`
  - We keep only the reference to the **first node**.
  - At creation, first node is `None`, having no element in the list.

```
class Node():
  def __init__(self, x):
    self.item = x
    self.next = None
```




```
class LinkedList():
  def __init__(self):
    self.first = None

  def insert(self, x, i):
    # insert x at [i]

  def get(self, i):
    # get item at [i]

  def delete(self, i):
    # delete item at [i]
```

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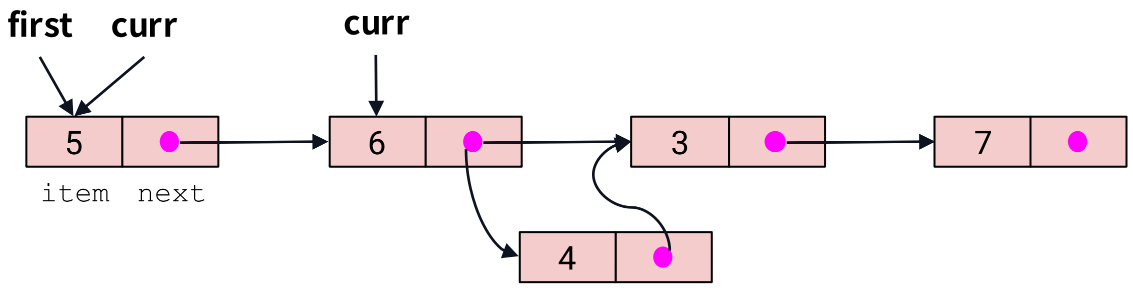
Inserting an Item at position *i*

- Step 1: Creating a new node with the given item.
- Step 2: Traverse to the (*i* - 1)-th position.
- Step 3: Set the new node's next as the original *i*-th node.
- Step 4: Update the (*i* - 1)-th node's next as the new node.

Example:
insert "4" at position 2.



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Inserting an Item at position *i*

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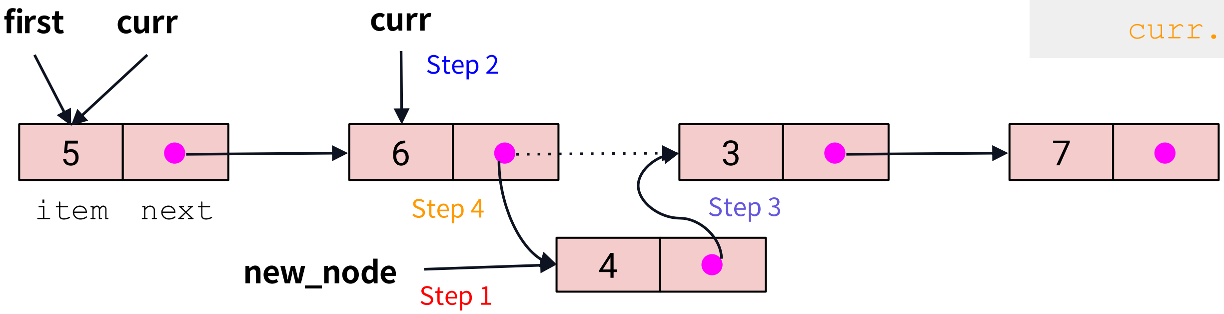
- Step 4: Update the (*i* - 1)-th node's next as the new node.

```
class Node():
  def __init__(self, x):
    self.item = x
    self.next = None
```




```
def insert(self, x, i):
  # insert x at [i]
  new_node = Node(x)

  pos = 0
  curr = self.first
  while pos < i - 1:
    curr = curr.next
    pos += 1

  new_node.next = curr.next

  curr.next = new_node
```

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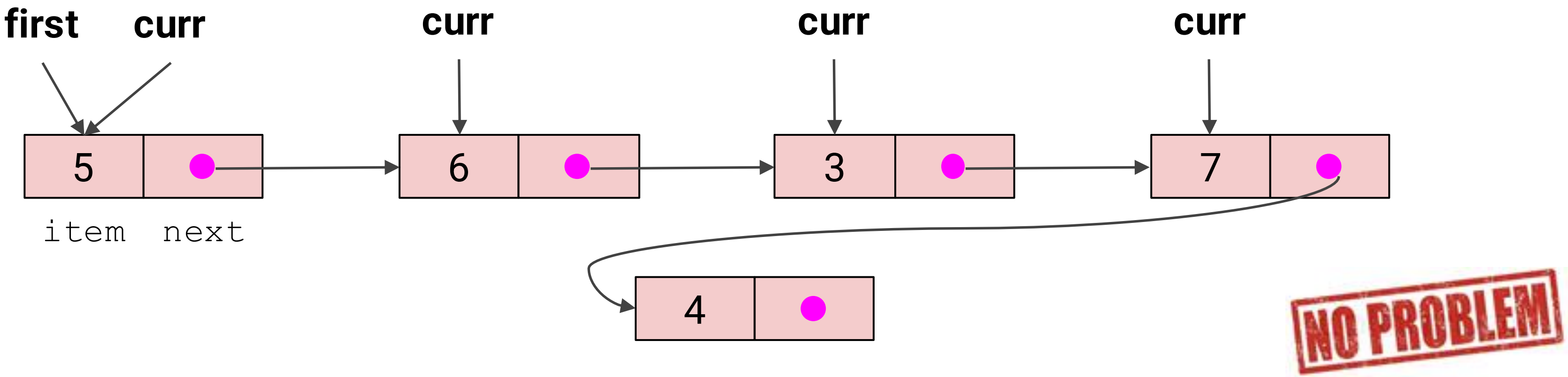
Does it work at the end?

- Step 1: Creating a new node with the given item.
- Step 2: Traverse to the (i - 1)-th position.
- Step 3: Set the new node's next as the original i-th node.
- Step 4: Update the (i - 1)-th node's next as the new node.

Example:
insert "4" at position **4.**



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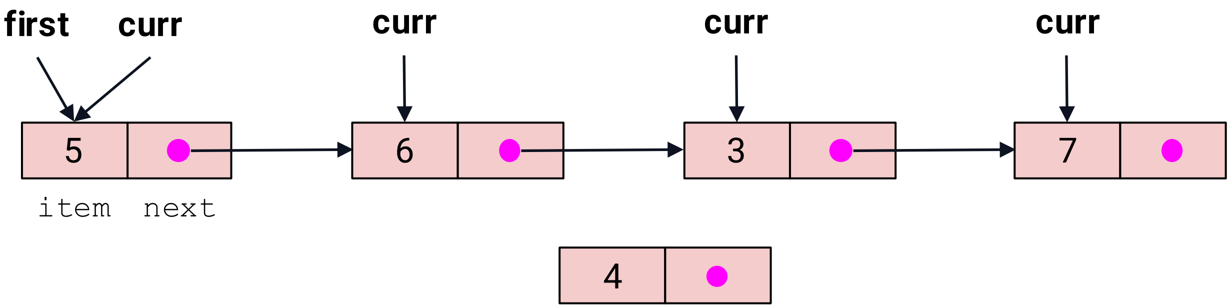
Does this work at the beginning?

- Step 1: Creating a new node with the given item.
- Step 2: Traverse to the (*i* - 1)-th position.
- We need special treatment when we insert at position 0!

Example:
insert “4” at position **0**.

i - 1 = -1 😨



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Inserting an Item at position *i*

- If inserting at the first position:
  - Step 1: Creating a new node with the given item.
  - Step 2: Set the new node's next as the original **first node**.
  - Step 3: **Update the first node** reference to the new node.
- else:
  - Step 1: Creating a new node with the given item.
  - Step 2: Traverse to the (*i* - 1)-th position.
  - Step 3: Set the new node's next as the original *i*-th node.
  - Step 4: Update the (*i* - 1)-th node's next as the new node.

Check: does this work when we insert the very first item (that is, does it work when `self.first = None`)?

```
def insert(self, x, i):
  # insert x at [i]
  if i == 0:
    new_node = Node(x)
    new_node.next = self.first
    self.first = new_node
  else:
    new_node = Node(x)

    pos = 0
    curr = self.first
    while pos < i - 1:
      curr = curr.next
      pos += 1

    new_node.next = curr.next

    curr.next = new_node
```

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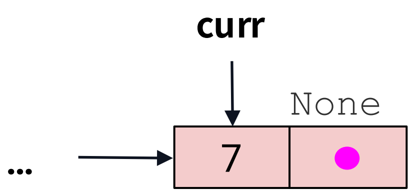
Inserting an Item at position *i*

Check: what happens with our code if *i* > last position?

It will crash here,

when it tries to access `None.next`




```
def insert(self, x, i):
  # insert x at [i]
  if i == 0:
    new_node = Node(x)
    new_node.next = self.first
    self.first = new_node
  else:
    new_node = Node(x)

    pos = 0
    curr = self.first
    while pos < i - 1:
      curr = curr.next
      pos += 1

    new_node.next = curr.next

    curr.next = new_node
```

We should prevent this, instead of letting the users to be responsible!

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Size Variable

- First try:
  - Let's add a check at the beginning, if *i* is within the valid range.
  - Valid range?

    From 0 to current length (item count)

- But, how do we know the size?
  - A naive way: traverse from the `first` until we meet `None`.
  - This is not efficient, since we need to traverse *N* items whenever we insert a new item, regardless of the target position. 😨
  - Any better way?

```
  def insert(self, x, i):
    # insert x at [i]
    if i > size: return
    if i == 0:
      new_node = Node(x)
      new_node.next =
self.first
      self.first = new_node
    else:
      new_node = Node(x)
    elif i <= size:
      pos = 0
      curr = self.first
      while pos < i - 1:
        curr = curr.next
        pos += 1

      new_node.next = curr.next
      curr.next = new_node
```

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Size Variable

- Solution: Let's keep the `size` variable in the class, and maintain it whenever we insert or delete an element.
- Time complexity?

O(1)

```
class LinkedList():
  def __init__(self):
    self.first = None
    self.size = 0

  def insert(self, x, i):
    # insert x at [i]

  def get(self, i):
    # get item at [i]

  def delete(self, i):
    # delete item at [i]
```

```
def insert(self, x, i):
  # insert x at [i]
  if i == 0:
    new_node = Node(x)
    new_node.next = self.first
    self.first = new_node
    self.size += 1
  elif i <= self.size:
    new_node = Node(x)

    pos = 0
    curr = self.first
    while pos < i - 1:
      curr = curr.next
      pos += 1

    new_node.next = curr.next
    curr.next = new_node
    self.size += 1
```

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Retrieving an Item at position *i – Homework*

- Basic logic
  - Step 1: Traverse to the *i*-th position.
  - Step 2: Return the item in the node.
- Any special cases to consider?
  - Check if your implementation works when
    - `i = 0`
    - `i > self.size`
    - `self.size = 0`
    - …

```
def get(self, i):
  # get item at [i]

  # TODO(students): implement!

  return ?
```

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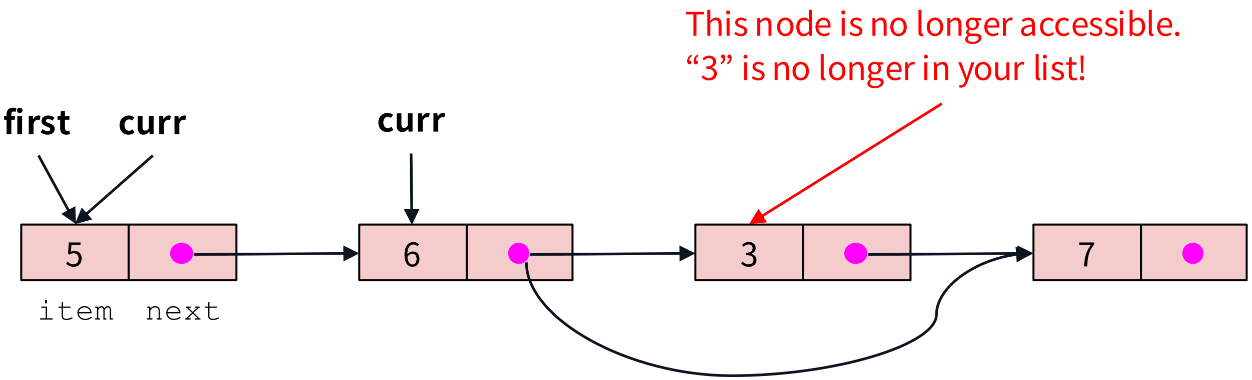
Deleting an Item at position *i*

- Step 1: Traverse to the (*i* - 1)-th position.
- Step 2: Set the (*i* - 1)-th node's next as the target's next.

Example:
delete item at position 2.



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Deleting an Item at position *i*

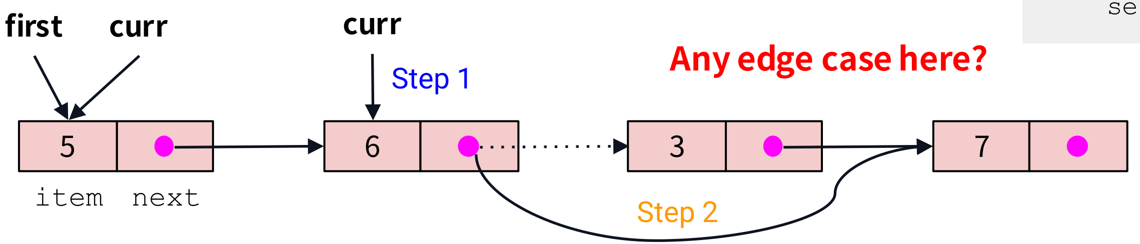
- Step 1: Traverse to the (*i* - 1)-th position.
- Step 2: Set the (*i* - 1)-th node's next as the target's next.

```
class Node():
  def __init__(self, x):
    self.item = x
    self.next = None
```

```
def delete(self, i):
  # delete item at [i]
  pos = 0
  curr = self.first
  while pos < i - 1:
    curr = curr.next
    pos += 1

  curr.next = curr.next.next

  self.size -= 1
```



Any edge case here?

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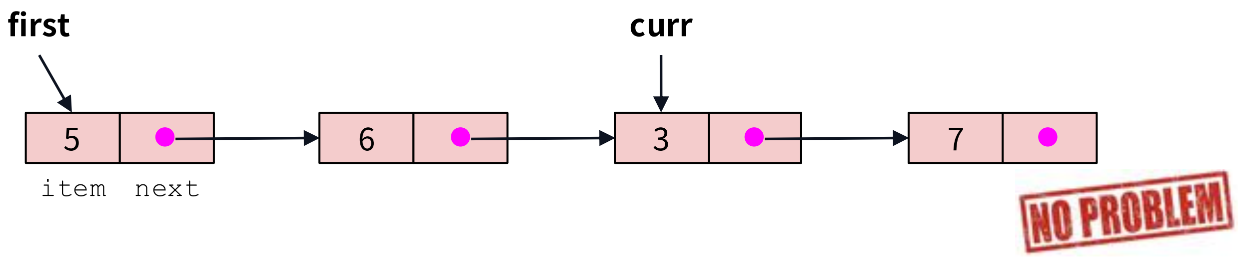
Does this work at the end?

- Step 1: Traverse to the (i - 1)-th position.
- Step 2: Set the (i - 1)-th node's next as the target's next.

Example:
delete item at position **3**.



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Does this work at the beginning?

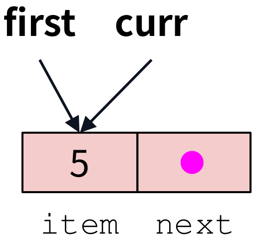
- Step 1: Traverse to the (*i* - 1)-th position.
- Step 2: Set the (*i* - 1)-th node's next as the target's next.

Example:
delete item at position **0**.

Again, we need special treatment when we delete the first one!

```
if i == 0:
  self.first = self.first.next
else:
  ...
```

i - 1 = -1



This condition is never satisfied.

```
def delete(self, i):
  # delete item at [i]
  pos = 0
  curr = self.first
  while pos < i - 1:
    curr = curr.next
    pos += 1

  curr.next = curr.next.next
  self.size -= 1
```

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Time Complexity

- Time complexity of Linked List?

Task	Worst case	Average case	Best case
Insertion	$O(N)$	$O(N)$	$O(1)$
Retrieval	$O(N)$	$O(N)$	$O(1)$
Deletion	$O(N)$	$O(N)$	$O(1)$

Happen when?

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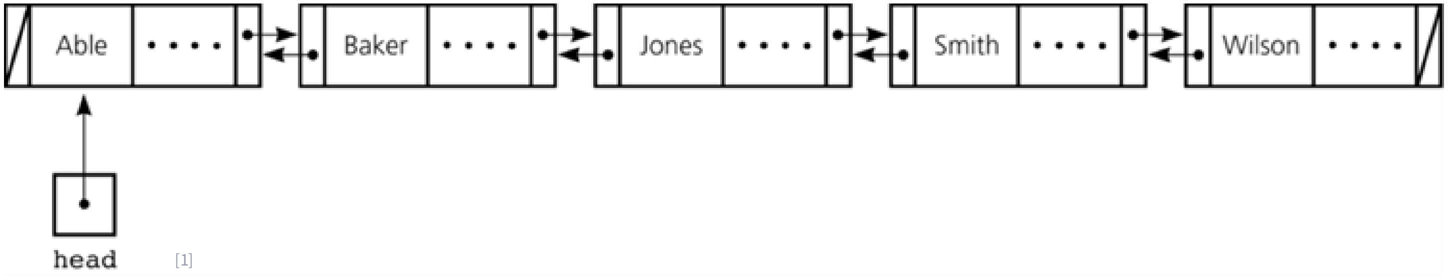
Doubly Linked List



[1]

- Sometimes it is useful to have ability to access *previous* items.
- No asymptotic benefit on complexity.

[1] https://maryash.github.io/235/projects/project_3/project_3.html

Comparison

- Array
  - **Consecutive** memory space is assigned.
  - Fixed length
  - **Random access** is supported in **O(1)**.
  - Suffers from **item shifting**.

- Linked list
  - **Scattered** in the memory space.
  - Additional space is needed for storing the next reference.
  - **No random access** allowed. (Linear traversal is required, taking $O(N)$.)
  - **No shifting** is needed even with size change.

03

Applications of Linked Lists

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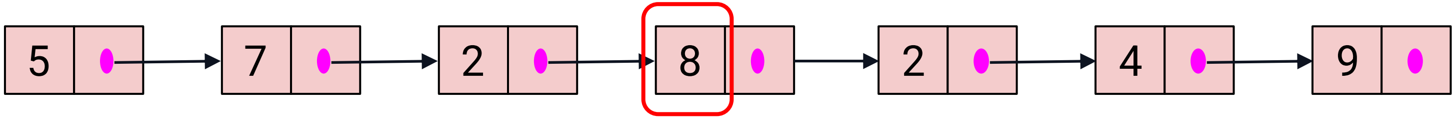
Application Questions

- Print the middle of a given linked list. (Assume list size is not maintained.)
- Brute-force solution
  - Traverse the entire list to count the number of elements.
  - Traverse half of them again.
  - $\rightarrow O(N)$



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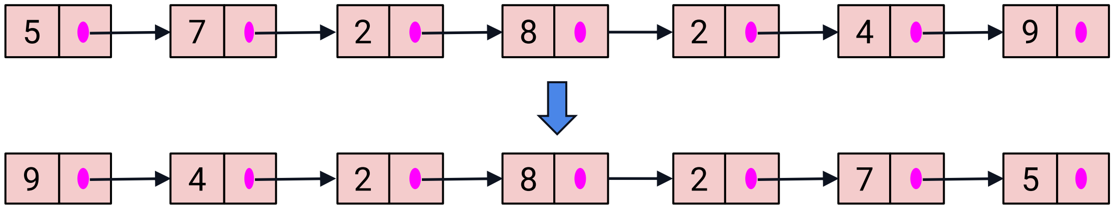
Application Questions

- Reverse a given linked list.



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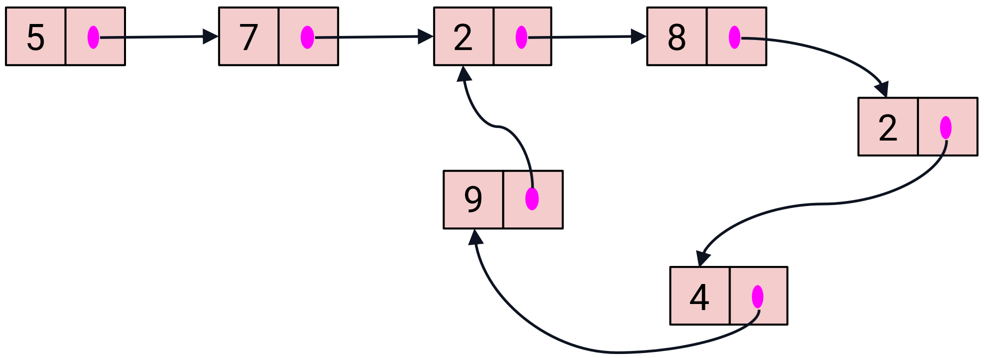
Application Questions

- Detect if there is a cycle in the given linked list.



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