Lecture 4: Hash Tables

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저작권 안내

(주)업스테이지가 제공하는 모든 교육 콘텐츠의 지식재산권은
운영 주체인 (주)업스테이지 또는 해당 저작물의 적법한 관리자에게 귀속되어 있습니다.

콘텐츠 일부 또는 전부를 복사, 복제, 판매, 재판매 공개, 공유 등을 할 수 없습니다.

유출될 경우 지식재산권 침해에 대한 책임을 부담할 수 있습니다.

유출에 해당하여 금지되는 행위의 예시는 다음과 같습니다.

- 콘텐츠를 재가공하여 온/오프라인으로 공개하는 행위
- 콘텐츠의 일부 또는 전부를 이용하여 인쇄물을 만드는 행위
- 콘텐츠의 전부 또는 일부를 녹취 또는 녹화하거나 녹취록을 작성하는 행위
- 콘텐츠의 전부 또는 일부를 스크린 캡쳐하거나 카메라로 촬영하는 행위
- 지인을 포함한 제3자에게 콘텐츠의 일부 또는 전부를 공유하는 행위
- 다른 정보와 결합하여 Upstage Education의 콘텐츠임을 알아볼 수 있는 저작물을 작성, 공개하는 행위
- 제공된 데이터의 일부 혹은 전부를 Upstage Education 프로젝트/실습 수행 이외의 목적으로 사용하는 행위

Data Structures So Far

	Insertion	Retrieval	Deletion	Restriction
Sorted Array	O(*M*)	O(log *M*)	O(*M*)	
Sorted Linked list	O(*M*)	O(*M*)	O(*M*)	
Stack	O(1)	O(1)	O(1)	LIFO
Queue	O(1)	O(1)	O(1)	FIFO
Hash Table	O(1)	O(1)	O(1)	

Needed for applications that need radically fast operations:

- 911 emergency calls and locating caller's address
- Airline information system
- Web search

Data Indexed Arrays

Data Indexed Arrays

- A regular list: [2, 5, 9, 10]
  - In this setting, we take O(*N*) to search a particular value.

Index	[0]	[1]	[2]	[3]
Value	2	5	9	10

- What if we represent the data in a **data-indexed array**?

Index	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
Value	F	F	**T**	F	F	**T**	F	F	F	**T**	**T**

Data Indexed Arrays

- A data-indexed array has **all possible data as its indices**!
  - Initially, all values of the array are **False** (*i.e.*, `di_array[x] = False` for all `x` in `di_array`), meaning the array is empty.
  - When we **insert** some value, the corresponding index becomes **True**.
  - Here, we assume that no duplicate keys are allowed.

```
di_array = DataIndexedArray()
di_array.insert(3)
di_array.insert(6)
di_array.delete(3)
```

Index	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
Value	F	F	F	T	F	F	T	F	F	F	F

Data Indexed Arrays

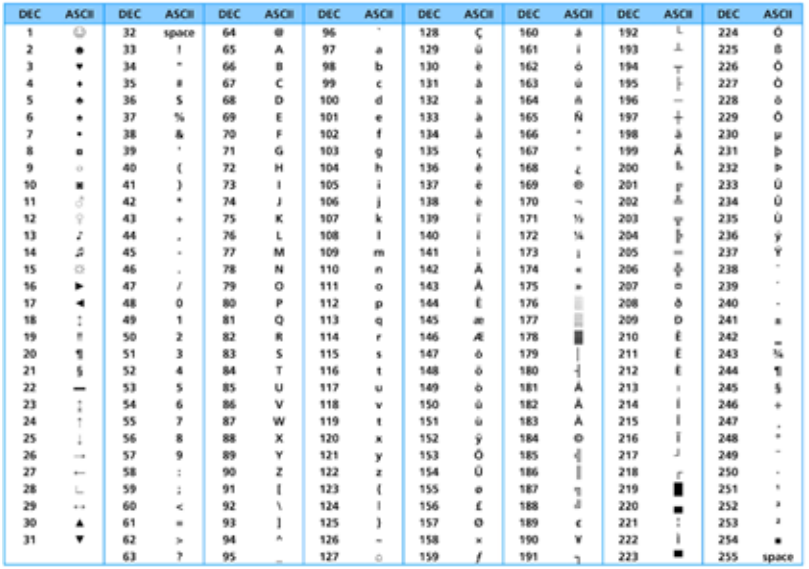
- Why is this good?
  - We can insert, delete, and retrieve values in **O(1)**!
- Any problem then?
  - We can't deal with values larger than 10 (the array size).
  - In other words, we need to use **very large memory space** if we'd like to deal with large number, even though the data size itself is small.
  - It is **not trivial how to deal with non-integer values**, like string or floating point numbers.

```
di_array.insert(21057381)
```

```
di_array.insert("ewha")
```

Strings in Data Indexed Arrays

- Recall that each character can be represented as an ASCII code value (0~255).
  - “ewha”:
    - e: 101
    - w: 87
    - h : 104
    - a: 97
  - “ewha”$_{256}$: $(101 \times 256^3) + (87 \times 256^2) + (104 \times 256) + 97$
- In this way, a string can be represented as a **unique integer**.



DEC	ASCII	DEC	ASCII	DEC	ASCII	DEC	ASCII	DEC	ASCII	DEC	ASCII	DEC	ASCII	DEC	ASCII
1	☺	32	space	64	@	96	`	128	Ç	160	á	192	└	224	Ó
2	☻	33	!	65	A	97	a	129	ü	161	í	193	┴	225	ß
3	♥	34	"	66	B	98	b	130	é	162	ó	194	┬	226	Ô
4	♦	35	#	67	C	99	c	131	â	163	ú	195	├	227	Ò
5	♣	36	$	68	D	100	d	132	ä	164	ñ	196	─	228	õ
6	♠	37	%	69	E	101	e	133	à	165	Ñ	197	┼	229	Õ
7	•	38	&	70	F	102	f	134	å	166	ª	198	ã	230	µ
8	◘	39	'	71	G	103	g	135	ç	167	º	199	Ã	231	þ
9	○	40	(	72	H	104	h	136	ê	168	¿	200	╚	232	Þ
10	◙	41	)	73	I	105	i	137	ë	169	®	201	╔	233	Ú
11	♂	42	*	74	J	106	j	138	è	170	¬	202	╩	234	Û
12	♀	43	+	75	K	107	k	139	ï	171	½	203	╦	235	Ù
13	♪	44	,	76	L	108	l	140	î	172	¼	204	╠	236	ý
14	♫	45	-	77	M	109	m	141	ì	173	¡	205	═	237	Ý
15	☼	46	.	78	N	110	n	142	Ä	174	«	206	╬	238	¯
16	►	47	/	79	O	111	o	143	Å	175	»	207	¤	239	´
17	◄	48	0	80	P	112	p	144	É	176	░	208	ð	240	
18	↕	49	1	81	Q	113	q	145	æ	177	▒	209	Ð	241	±
19	‼	50	2	82	R	114	r	146	Æ	178	▓	210	Ê	242	‗
20	¶	51	3	83	S	115	s	147	ô	179	│	211	Ë	243	¾
21	§	52	4	84	T	116	t	148	ö	180	┤	212	È	244	¶
22	▬	53	5	85	U	117	u	149	ò	181	Á	213	ı	245	§
23	↨	54	6	86	V	118	v	150	û	182	Â	214	Í	246	÷
24	↑	55	7	87	W	119	w	151	ù	183	À	215	Î	247	¸
25	↓	56	8	88	X	120	x	152	ÿ	184	©	216	Ï	248	°
26	→	57	9	89	Y	121	y	153	Ö	185	╣	217	┘	249	¨
27	←	58	:	90	Z	122	z	154	Ü	186	║	218	┌	250	·
28	∟	59	;	91	[	123	{	155	ø	187	╗	219	█	251	¹
29	↔	60	<	92	\	124	|	156	£	188	╝	220	▄	252	³
30	▲	61	=	93	]	125	}	157	Ø	189	¢	221	¦	253	²
31	▼	62	>	94	^	126	~	158	×	190	¥	222	▌	254	■
		63	?	95	_	127	⌂	159	ƒ	191	┐	223	▀	255	space

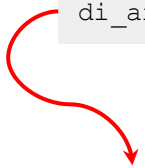
[1]

[1] https://nl.pinterest.com/pin/395261304802336725/

Large Numbers in Data Indexed Arrays

- But still, we can't avoid the problem of treating large numbers.
  - We may need some way to put **arbitrary integers** in a **reasonable size of memory**.

```
di_array.insert(21057381)
```



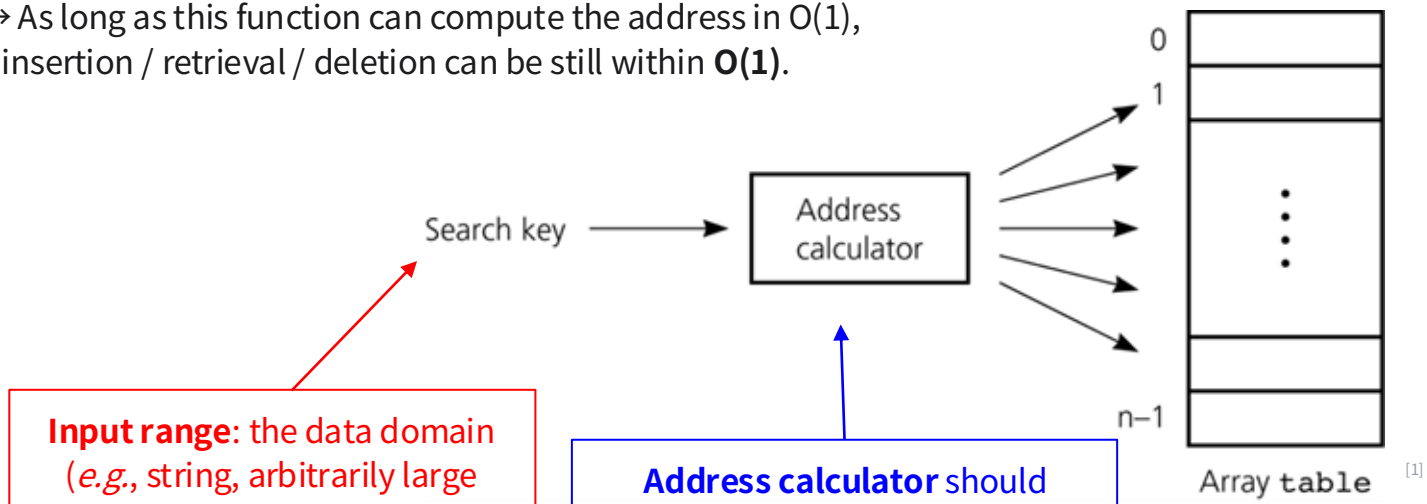
Index	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	…	[999]
Value	F	F	F	F	F	F	F	F	F	⋯	F

Hash Tables

Hash Table

Given a key, a function (called **hash function**) determines where to locate it.

→ As long as this function can compute the address in O(1), insertion / retrieval / deletion can be still within **O(1)**.



[1] https://www.slideshare.net/slideshow/hashing-in-data-structure-is-presented-in-these-slides/274382978

Hash Functions

- A good hash function:
  - Easy and **fast** to compute (in O(1))
  - **Scatter** the data **evenly** on the hash table.
- Selection digits
  - *h*(001364825) = 35
- Folding
  - *h*(001364825) = 001 + 364 + 825 = 1190
- Modulo arithmetic
  - *h*(x) = x `mod` table_size
  - *h*(1004) = 4 (if table_size = 100)
  - This is widely used, as it maps the integers **uniformly** over the internal array regardless of its size!

Hmm, then what should we do if two different keys coincidently map to the same address?

Modulo operator (%)
x % y is the remainder when we divide x by y.

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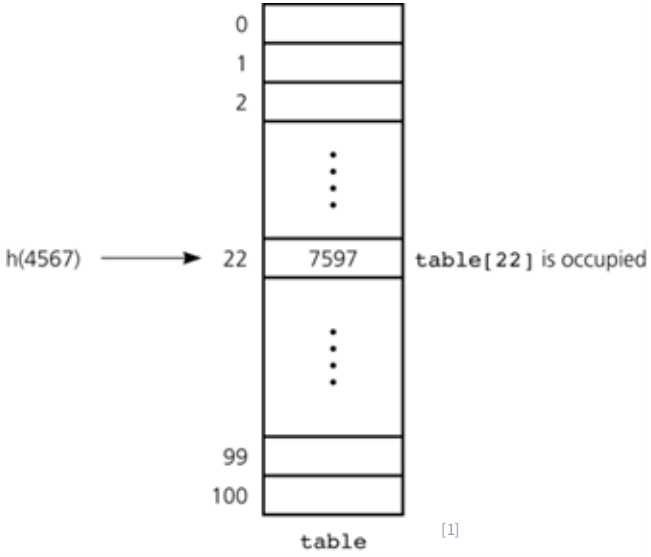
Collision

- Yes, unfortunately, it can happen with all hash functions we listed 😨.
  - **Collision**: the phenomenon that two keys are mapped into the same location in the hash table.
- Selection digits
  - $h(001364825) = h(741325385) = 35$
- Folding
  - $h(001364825) = h(825364001) = 1190$
- Modulo arithmetic
  - $h(2205) = h(2405) = 5$ if table_size = 100



[1]

[1] https://www.cs.colostate.edu/~cs165/Spring20/slides/16-Hashing.pdf

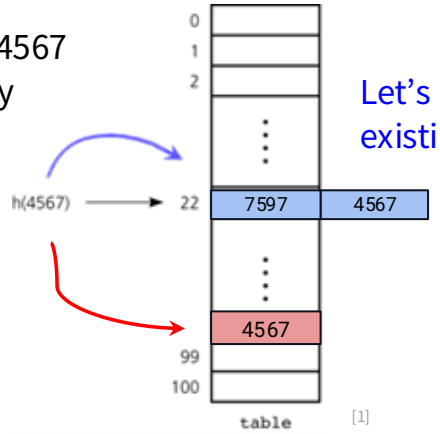
Collision Resolution

- Collision is unavoidable; so, we need a method to resolve it efficiently.
- There are two groups of approaches:
  - **Open addressing**: allowing the items to be located in another place if the collision happened.
  - **Separate chaining**: allowing more than one item to be located at a place.

Example: According to our hash function, 4567 needs to be located at [22], but it is already occupied 😨.

Open Addressing

Let's find another place to put it.



[1]

Separate chaining

Let's put it there together with the existing one(s).

[1] https://www.cs.colostate.edu/~cs165/Spring20/slides/16-Hashing.pdf

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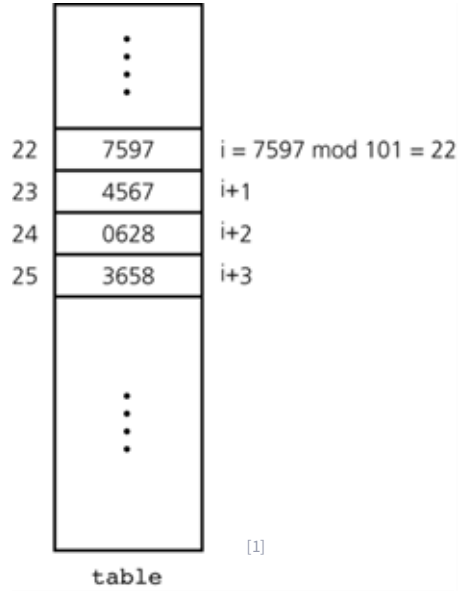
Open Addressing

- **Linear probing**: if the designated spot is occupied, try the right next index, until it finds an available one.

  $h_i(x) = (h_0(x) + i) \bmod \text{table\_size}$

- Causes **primary clustering**:
  - All keys mapped to [22], [23], [24], [25], ⋯ will suffer from collision.
  - If this cluster gets bigger, insertion / retrieval / deletion of keys corresponding to this region will take O(*N*) instead of O(1).
  - Gets significantly worse especially if consecutive numbers are sequentially added. (*e.g.*, 22 → 23 → 24, ⋯)



[1] https://www.cs.colostate.edu/~cs165/Spring20/slides/16-Hashing.pdf

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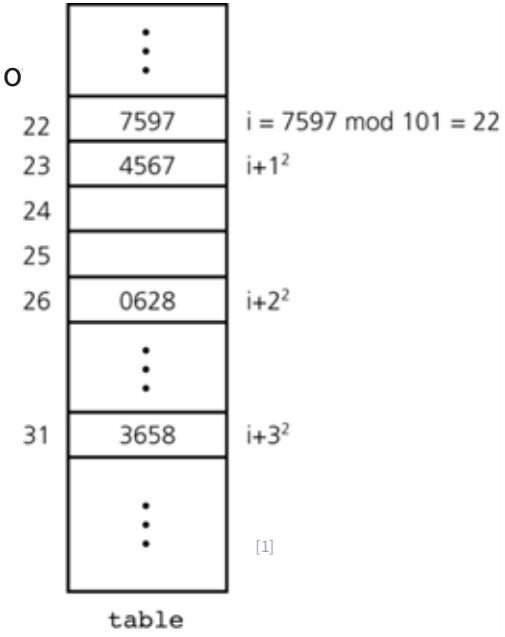
Open Addressing

- **Quadratic probing**: if the designated spot is occupied, try the next set o indices, quadratically increasing with the number of trials.

  $h_i(x) = (h_0(x) + i^2) \bmod \text{table\_size}$

- Solves **primary clustering**:
  - Items collided at [23] suffers less from significant collisions originally happened at [22].

- Still suffers from **secondary clustering**:
  - For items collided at the same index for the first time, they must always follow the same trace.



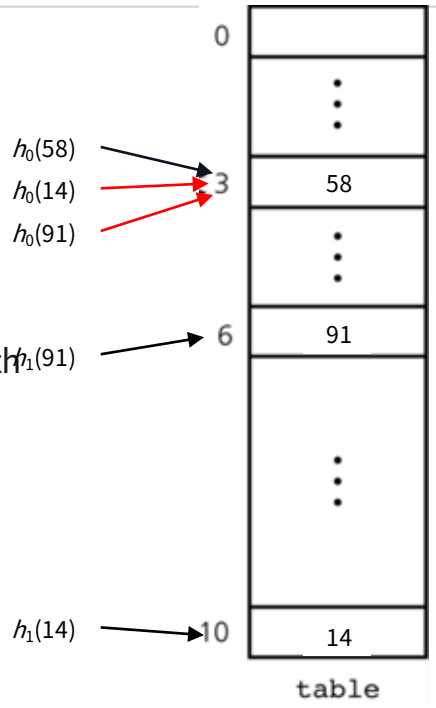
[1] https://www.cs.colostate.edu/~cs165/Spring20/slides/16-Hashing.pdf

Open Addressing

- **Double Hashing**: using two different hash functions to reduce collision probability.

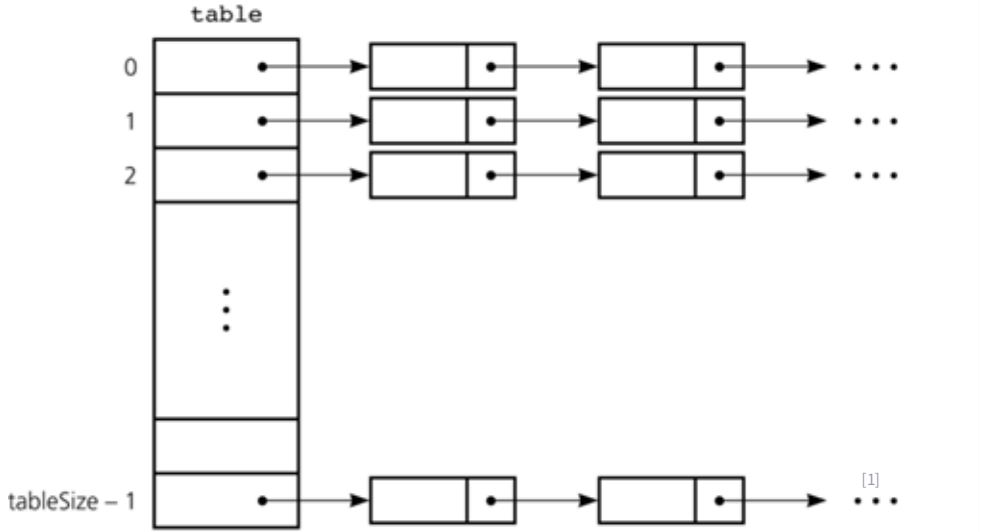
  $h_i(x) = (\alpha(x) + i * \beta(x)) \bmod$ table_size
  where $\alpha(x)$, $\beta(x)$: some hash functions

- Can avoid primary and secondary clustering, since we probe with different steps depending on the input.



Separate Chaining

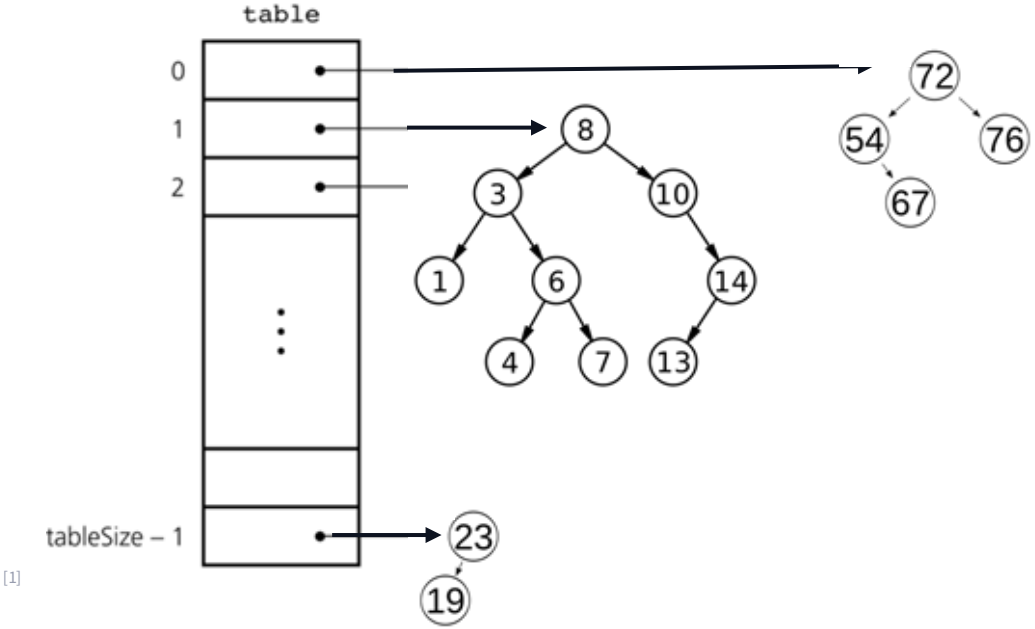
- Each location of the hash table is allowed to have more than one item.
  - A simplest example: a reference to a linked list.



[1] https://medium.com/@ramyjzh/data-structures-for-dummies-hash-tables-579ddd1a4389

Separate Chaining

- Actually, we can put any other data structure that support key-based search.



[1]

[1] https://medium.com/@ramyjzh/data-structures-for-dummies-hash-tables-579ddd1a4389

Performance of a Hash Table

- In general, having more items in the table reduces performance of a hash table.
  - This is unique to hash tables, unlike most other data structures!
- Let's define **load factor (α)** formally: N/M
  - N: the actual number of elements inserted
  - M: the number of indices (table_size)
- With **open addressing**, α is always between 0 and 1. Once it reaches to 1, we can't add more data because there is no more available space.
  - If α gets **close to 1**, there are probably lots of collisions, meaning **longer processing time**.
- With **separate chaining**, load factor can be > 1.
  - In this case, load factor means the average chain length per index, which is equivalent to the expected number of linear traversal to locate an item.
  - Obviously, higher load factor means slower data processing.

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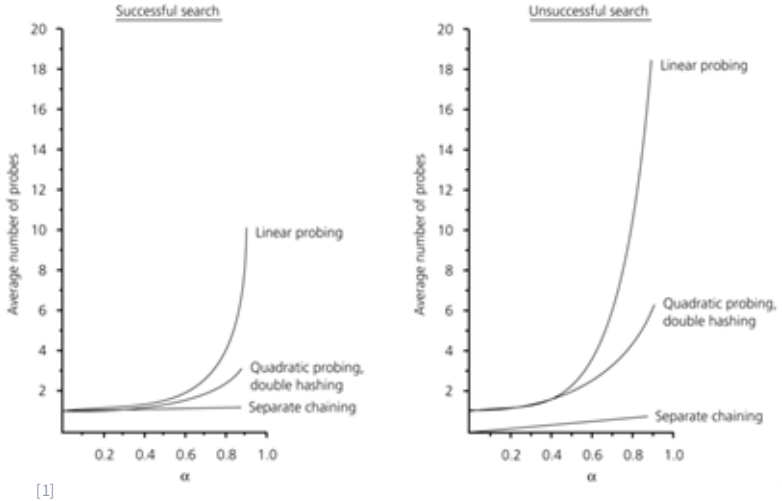
Efficiency of Hashing



[1]

When the load factor is low, hash table achieves ≈O(1), regardless of the collision resolution method.

[1] https://www.eecs.yorku.ca/course_archive/2003-04/F/2011/2011A/DatStr_152_HashTables.pdf

How to Decide the Table Size (M)

- If we know the data size (N) to be inserted, we can set $M = N/\alpha$, where α is the desired load factor.
  - With small α, we focus more on speed.
  - With large α, we care more about the memory space.
  - It is known that α **= 0.6~0.75** is a good balance between them.

- What if we don't know the data size?
  - We have no choice but to dynamically adapt.
  - Starting with some default size (say, 101), then (roughly) double it when the load factor becomes larger than some threshold (*e.g.*, 0.6~0.75).

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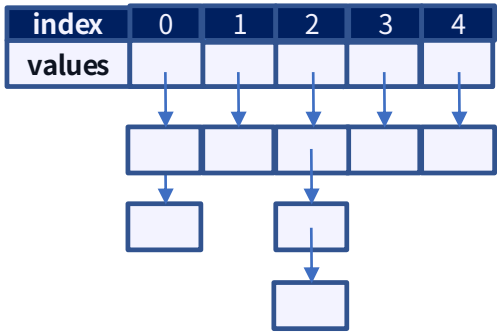
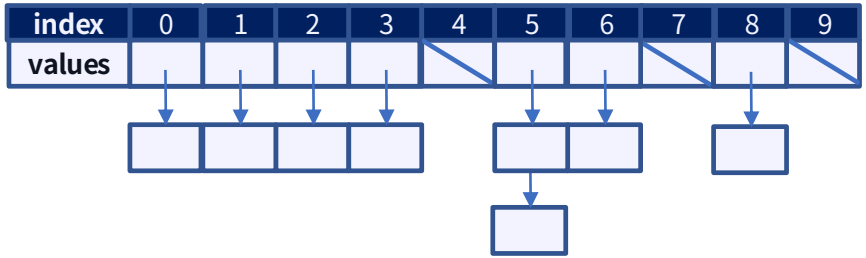
Resizing Example



N=0	N=1	N=2	N=3	N=4
M=5	M=5	M=5	M=5	M=5
α=0	α=0.2	α=0.4	α=0.6	α=0.8

N=5	N=6	N=7	N=8
M=5	M=5	M=5	M=5
α=1.0	α=1.2	α=1.4	α=1.6



Resizing!

Redistributing all items!

N=8
M=10
α=0.8

Time Complexity of Resizing

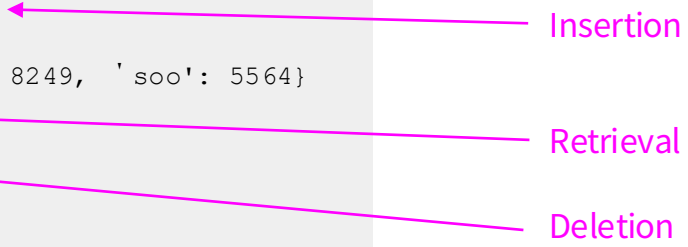
- Besides resizing, hash table achieves O(1) insertion, deletion, and retrieval.
- Resizing is obviously not free 😅
  - Resizing a hash table with N items requires **$O(N)$** time to redistribute all N items.
- A good news 😀: we don't always resize since one resizing operation doubles the number of indices.
  - The number of redistributing items while inserting N items:
    - $1 + 2 + 4 + 8 + \cdots + N = 2N - 1$
  - Overall, redistributing cost becomes $O((2N-1)/N)$ = **O(1) on average**!

Implementations: Dictionary vs Set in Python

Python Dictionary

- Python internally provides the hash table under the name of `Dictionary`.
  - **Key-value pairs** are stored in a hash table.
  - Key are unique and hashable.
  - Insertion, retrieval, deletion supported in **O(1)**.

```
>>> tel = {'jerry': 1086, 'jose': 8249}
>>> tel['soo'] = 4127
>>> tel
{'jerry': 1086, 'jose': 8249, 'soo': 5564}
>>> tel['jose']
8249
>>> del tel['jerry']
>>> tel['shawn'] = 8080
>>> tel
{'jose': 8249, 'soo': 5564, 'shawn': 8080}
>>> list(tel)
['jose', 'soo', 'shawn']
>>> sorted(tel)
['shawn', 'soo', 'jose']
>>> 'jinri' in tel
False
>>> 'jeongwoo' not in tel
True
```



Python Set

- If you don't have an associated `value`, you may use Python `Set`, instead of `Dictionary`.
  - **Only elements (keys)** are stored in the hash table.
  - Elements are unique and hashable.
  - Insertion, retrieval, deletion supported in **O(1)**.

```
>>> a = set('abracadabra')
>>> b = set('alacazam')
>>> a
{'a', 'r', 'b', 'c', 'd'}    Unique letters in a
>>> a - b
{'r', 'd', 'b'}  Set difference
>>> a | b
{'a', 'c', 'r', 'd', 'b', 'm', 'z', 'l'}    Set OR
>>> a & b
{'a', 'c'}  Set AND
>>> a ^ b
{'r', 'd', 'b', 'm', 'z', 'l'}    Set XOR
```

Applications of Hash Tables

Problem 1 - Palindrome

409. Longest Palindrome

Easy | Topics | Companies

Given a string `s` which consists of lowercase or uppercase letters, return the length of the **longest palindrome** that can be built with those letters.

Letters are **case sensitive**, for example, `"Aa"` is not considered a palindrome.

Example 1:

```
Input: s = "abccccdd"
Output: 7
Explanation: One longest palindrome that can be built is "dccaccd", whose length is 7.
```

Example 2:

```
Input: s = "a"
Output: 1
Explanation: The longest palindrome that can be built is "a", whose length is 1.
```

Problem 2 - Smallest Missing Integer

- Given a list of integers, return the smallest positive integer which is not in that list.

Examples:

- [7, 2, 3, 5, 4, 1] → 6
- [-1, 5, 2, 3, 9] → 1
- [17, 25, 4308, 1, 99] → 2

```
# TODO(students): implement this!
def smallest_missing_pos_int(self, list):
  for each item in the list:
    insert into a hash table (hastset)

  for i = 1, 2, 3, ...:
    if the hash table contains i: keep going
    else: return i
```

Note: think about how many times we should iterate here.

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Problem 3 – Two Sum

1. Two Sum

Solved

Easy | Topics | Companies | Hint

Given an array of integers `nums` and an integer `target`, return *indices of the two numbers such that they add up to* `target`.

You may assume that each input would have ***exactly* one solution**, and you may not use the *same* element twice.

You can return the answer in any order.

Example 1:

```
Input: nums = [2,7,11,15], target = 9
Output: [0,1]
Explanation: Because nums[0] + nums[1] == 9, we return [0, 1].
```

Example 2:

```
Input: nums = [3,2,4], target = 6
Output: [1,2]
```

Example 3:

```
Input: nums = [3,3], target = 6
Output: [0,1]
```

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Building intelligence for the future of work