Lecture 1:
Basic of Algorithm & Computational Complexity

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저작권 안내

(주)업스테이지가 제공하는 모든 교육 콘텐츠의 지식재산권은 운영 주체인 (주)업스테이지 또는 해당 저작물의 적법한 관리자에게 귀속되어 있습니다.

콘텐츠 일부 또는 전부를 복사, 복제, 판매, 재판매 공개, 공유 등을 할 수 없습니다.
유출될 경우 지식재산권 침해에 대한 책임을 부담할 수 있습니다.

유출에 해당하여 금지되는 행위의 예시는 다음과 같습니다.

- 콘텐츠를 재가공하여 온/오프라인으로 공개하는 행위
- 콘텐츠의 일부 또는 전부를 이용하여 인쇄물을 만드는 행위
- 콘텐츠의 전부 또는 일부를 녹취 또는 녹화하거나 녹취록을 작성하는 행위
- 콘텐츠의 전부 또는 일부를 스크린 캡쳐하거나 카메라로 촬영하는 행위
- 지인을 포함한 제3자에게 콘텐츠의 일부 또는 전부를 공유하는 행위
- 다른 정보와 결합하여 Upstage Education의 콘텐츠임을 알아볼 수 있는 저작물을 작성, 공개하는 행위
- 제공된 데이터의 일부 혹은 전부를 Upstage Education 프로젝트/실습 수행 이외의 목적으로 사용하는 행위

강사 소개

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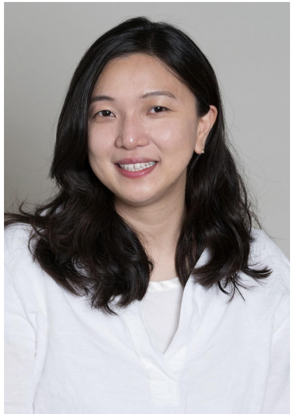
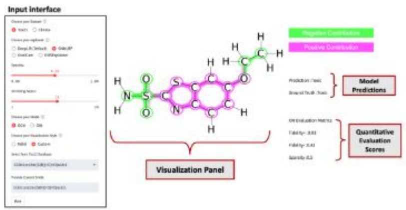
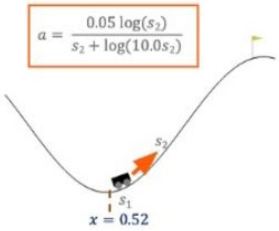


관심 연구 분야

- Explainable AI (XAI)
- NLP and Medical AI
- AI for Science
- RL based Optimization



Explainable AI



RL based Optimization

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Algorithms & Complexity

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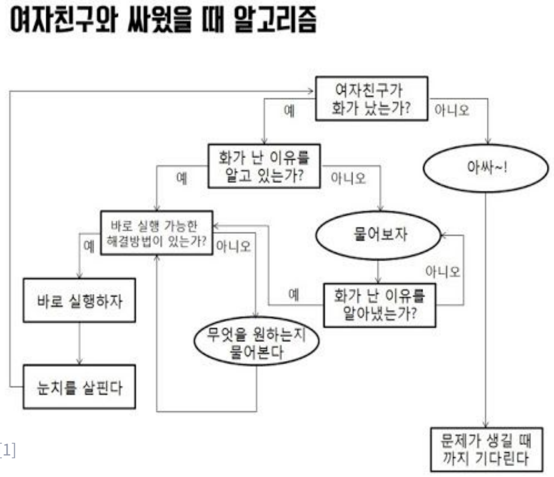
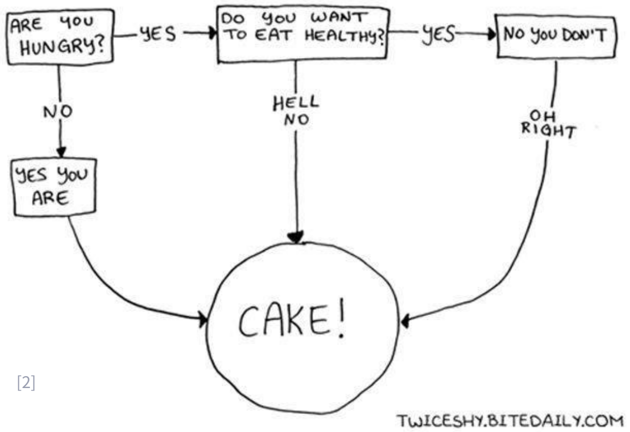
What is Algorithm?

- **Computational procedure** to solve a problem



[1]



[2]

[1] https://blog.naver.com/sumr2002/220359932991
[2] http://jokesandfun.de/infographic/the-cake-is-a-factflowchart/

Efficiency of an Algorithm

Do you like fast/efficient computer program, or slow one?

[1]

[2]

[3]

Of course, we always expect our computers to do their jobs **most efficiently**!

[1] https://www.vecteezy.com/photo/20621853-stressed-and-overworked-businessman
[2] https://stock.adobe.com/search?k=smashed+computer&asset_id=52541504
[3] https://www.istockphoto.com/kr/%EC%82%AC%EC%A7%84/%EC%97%AC%EC%9E%90-%EC%BB%B4%ED%93%A8%ED%84%B0-gm118986833-12293508

Computation Complexity

- Cost of algorithm = Sum of operation costs

- Model of computation specifies
  - What **operations** an algorithm is allowed to use
  - Cost (time, space) of each operation
- Execution costs
  - **Time** complexity of a program: how much **time**?
  - **Space** complexity of a program: how much **memory**?

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Measuring Time Complexity

- **Measure execution time** in seconds using a client program (*e.g.*, time module)
    - (+) Easy to measure
    - (+) Gives actual time
    - (-) Large amounts of time might be required.
    - (-) Results depend on lots of factors (machine, compiler, data…)

- Count the number of operations **in terms of input size** N
    - (+) Machine independent.
    - (+) Gives algorithm's **scalability.**
    - (-) Tedious to compute…
    - (-) Does not give actual time.

- Fortunately, we care only about asymptotic behavior (with a very large N – Big Data!)

Elementary School Algorithm

Example: Integer multiplication

- Input: two *N*-digit numbers *x*, *y*
- Output: product of *x* and *y*
- Primitive operations allowed:
  - Add 2 single-digit numbers
  - Multiply 2 single-digit numbers

$$
\begin{array}{r}
2\,3\,3 \\
5678 \\
\times 1234 \\
\hline
22712 \\
17034 \\
11356 \\
5678 \\
\hline
7006652
\end{array}
$$

Elementary School Algorithm

Example: Integer multiplication

- Input: two *N*-digit numbers *x*, *y*
- Output: product of *x* and *y*
- Primitive operations allowed:
  - Add 2 single-digit numbers
  - Multiply 2 single-digit numbers

How many primitive operations used?

```
      2 3 3
       5678
     × 1234
   --------
      22712
     17034
    11356
   5678
   --------
    7006652
```

For each row:

***N* multiplications**

(up to) ***N*-1 additions**

***N* rows**

In total,

***N*(2*N*-1) operations**

(up to) ***N*^{2} additions**

Total operations $\leq 3N^2$

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Software Engineer's Example

```
def linear_search(list, value):
  for i in range(len(list)):
    if list[i] == value:
      return i
  return -1
```

```
def selection_sort(list):
  for i in range(len(list)):
    smallest = i
    for j in range(i+1, len(list)):
      if list[j] < list[smallest]:
        smallest = j
    list[i], list[smallest] = list[smallest], 
list[i]
```

Let's denote `len(list)` as *N*:

Operation	Count
==	*1 to N*

Operation	Count
smallest = i	N
<	$(N^2 - N)/2$
smallest = j	0 to $(N^2 - N)/2$
swap	N

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Big O Notation

How to Characterize Time Complexity more formally and simply?

Simplification of Time Complexity

1. We care only about the worst-case performance!

← because we do not know what input data we will get in advance.

Operation	Count
smallest = i	N
<	$(N^2 - N)/2$
smallest = j	0 to $(N^2 - N)/2$
swap	N

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Simplification of Time Complexity

2. Focus only on a single operation with the highest order of growth (=most expensive).

- There may be multiple good choices. Then, just choose any of them.

Operation	Count
smallest = i	$\cancel{N}$
<	$\cancel{(N^2 - N)/2}$
smallest = j	$(N^2 - N)/2$
swap	$\cancel{N}$

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Simplification of Time Complexity

3. Remove lower order terms.

← They do not really matter, since the higher order one dominates.

Operation	Count
smallest = i	*ignored*
<	*ignored*
smallest = j	$(N^2 \cancel{- N})/2$
swap	*ignored*

Simplification of Time Complexity

4. Remove constants.

← We have already thrown away information at step 2. At this stage, constants are not meaningful.

Operation	Count
smallest = i	*ignored*
<	*ignored*
smallest = j	$(N^2)/\cancel{2}$
swap	*ignored*

Worst-case **order of growth**: N^2

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Formal Definition

- If a function $T(N)$ has its **order of growth** less than or equal to $f(N)$, we write this as $T(N) \in O(f(N))$, where O is called Big-O notation.

- More mathematically, $T(N) \in O(f(N))$ means that there exists a positive constant c such that $T(N) \leq c\,f(N)$ for all values of N greater than some positive N_0 (*i.e.*, large N).

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Example

- $T(N) = N^2 + 5N^5$
- $T(N) \in O(N^5)$?
  - That is, is there a positive constant c such that $N^2 + 5N^5 \le cN^5$ for large N?
  - Yes!
    - $N^2 + 5N^5 < N^5 + 5N^5 = 6N^5$
    - With $c = 6$, it holds.
- $T(N) \in O(N^7)$?
  - That is, is there a positive constant c such that $N^2 + 5N^5 \le cN^7$ for large N?
  - Yes!
    - $N^2 + 5N^5 < N^5 + 5N^5 = 6N^5 < 6N^7$
    - With $c = 6$, it holds.

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Example(cont'd)

- $T(N) = N^2 + 5N^5$
- $T(N) \in O(N^4)$?
  - That is, is there a positive constant c such that $N^2 + 5N^5 \leq cN^4$ for large N?
  - No 😕
    - Even if we set very large c, still for $N > c$, $5N^5 > N^5 > cN^4$.
- We are usually interested in the **tightest** order; in this example, $O(N^5)$.

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Exercise(cont'd)

- $T(N) = 2N^2 + 3N$ **$O(N^2)$**
- $T(N) = 1/N + 100$ **$O(1)$**
- $T(N) = 100\cos(N) + 50N^2$ **$O(N^2)$**
- $T(N) = \log N + 2N$ **$O(N)$**
- $T(N) = 2^N + N^2$ **$O(2^N)$**

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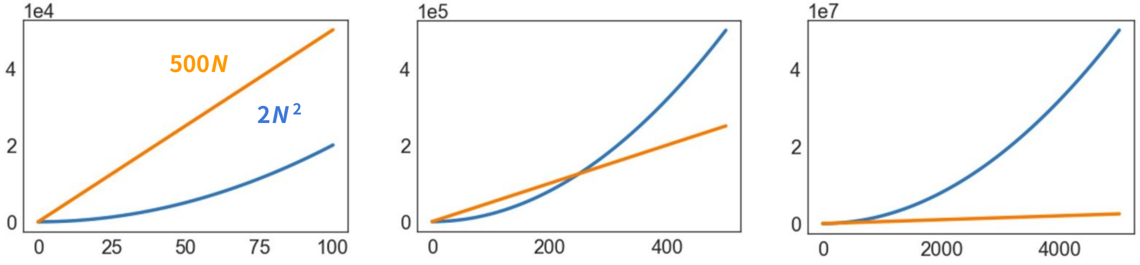
What is Important for Asymptotic Analysis?

- Compare the two algorithms below:
  - Algo1 requires $\mathbf{2N^2}$ operations, while
  - Algo2 requires $\mathbf{500N}$ operations.
  - Algo1 is faster than Algo2 for a small N, but becomes much slower for a very large N.
  - What is important? **How fast function is growing**!

- Order of growth:



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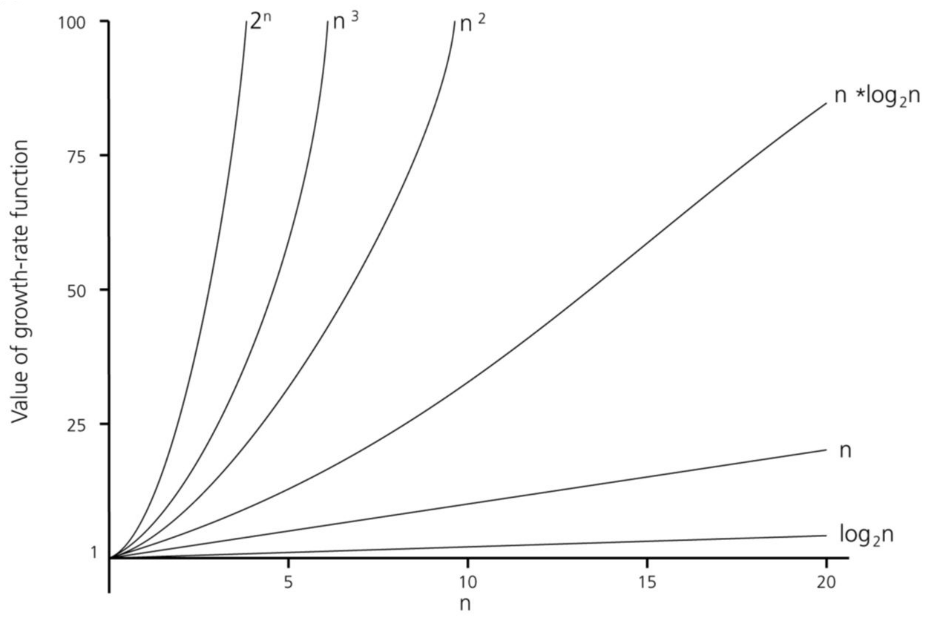
Asymptotic Analysis



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