Lecture 3: Stacks & Queues

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저작권 안내

(주)업스테이지가 제공하는 모든 교육 콘텐츠의 지식재산권은 운영 주체인 (주)업스테이지 또는 해당 저작물의 적법한 관리자에게 귀속되어 있습니다.

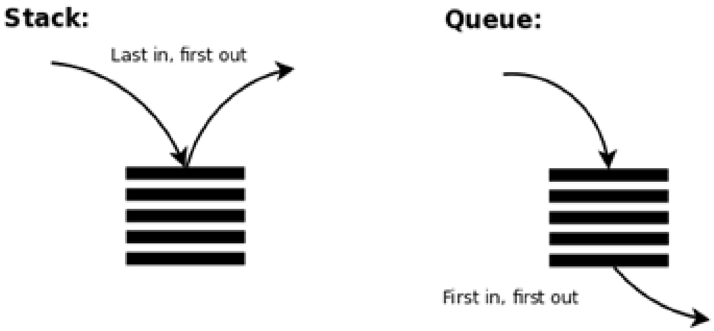
콘텐츠 일부 또는 전부를 복사, 복제, 판매, 재판매 공개, 공유 등을 할 수 없습니다.
유출될 경우 지식재산권 침해에 대한 책임을 부담할 수 있습니다.

유출에 해당하여 금지되는 행위의 예시는 다음과 같습니다.

- 콘텐츠를 재가공하여 온/오프라인으로 공개하는 행위
- 콘텐츠의 일부 또는 전부를 이용하여 인쇄물을 만드는 행위
- 콘텐츠의 전부 또는 일부를 녹취 또는 녹화하거나 녹취록을 작성하는 행위
- 콘텐츠의 전부 또는 일부를 스크린 캡쳐하거나 카메라로 촬영하는 행위
- 지인을 포함한 제3자에게 콘텐츠의 일부 또는 전부를 공유하는 행위
- 다른 정보와 결합하여 Upstage Education의 콘텐츠임을 알아볼 수 있는 저작물을 작성, 공개하는 행위
- 제공된 데이터의 일부 혹은 전부를 Upstage Education 프로젝트/실습 수행 이외의 목적으로 사용하는 행위

Today: Stacks & Queues

- Stack
  - Last In, First out (**LIFO**)
  - Access only to the most-recently added item.
- Queue
  - First In, First out (**FIFO**)
  - Access only to the item that was added earliest.



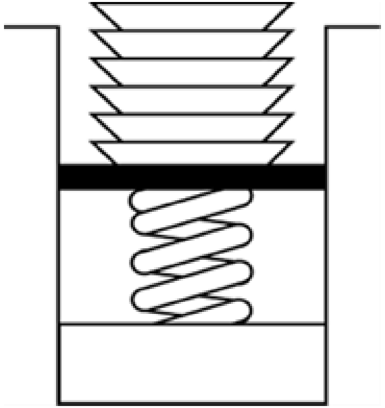
[1]

[1] https://gohighbrow.com/

01

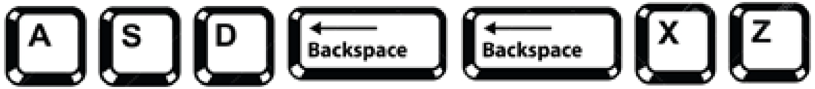
Stacks

Stack Examples



Stack of cafeteria dishes

[1]

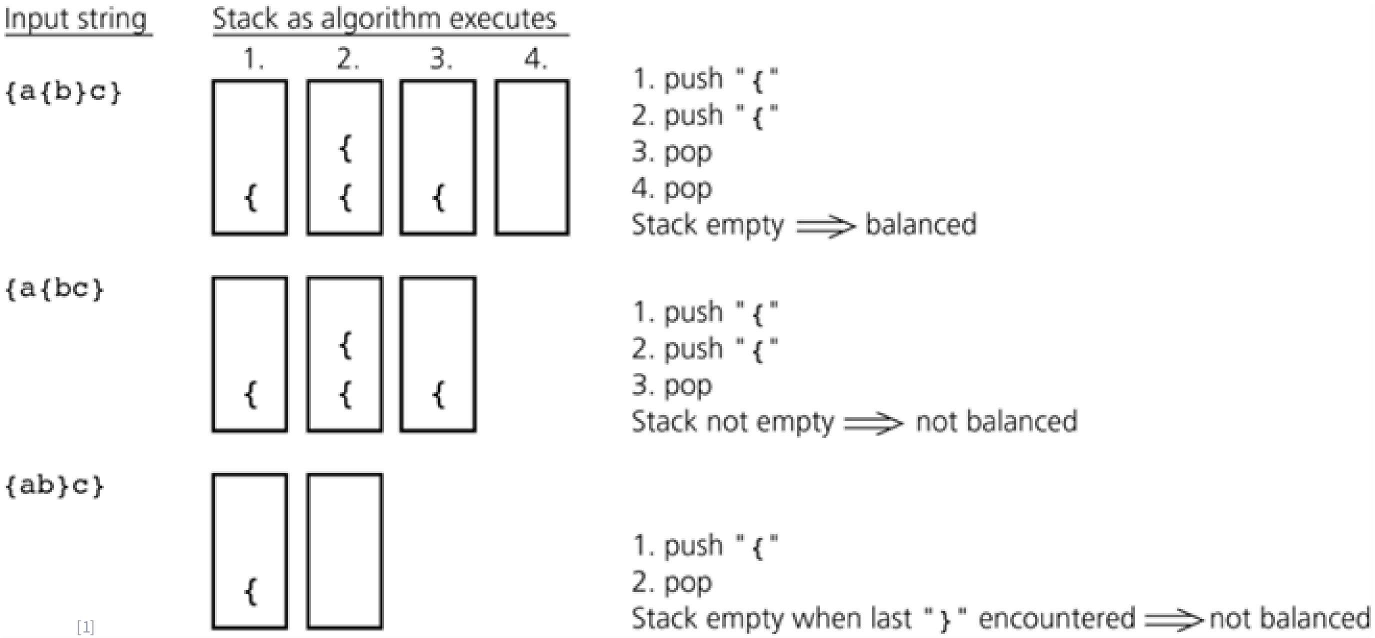


Backspacing with keyboard

[2]

[1] https://www.cs.vassar.edu/~cs125/lectures/lect9-Stacks/ch07.pdf
[2] https://kr.123rf.com/photo_15834863_키보드-키-버튼의-아이콘을-설정-문자-백-스페이스-삭제.html

Stack Example: Checking Balances of Braces



{a{b}c}

1. push "{"
2. push "{"
3. pop
4. pop
Stack empty ⟹ balanced

{a{bc}

1. push "{"
2. push "{"
3. pop
Stack not empty ⟹ not balanced

{ab}c}

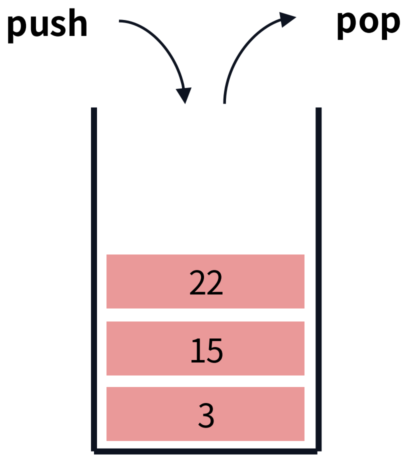
1. push "{"
2. pop
Stack empty when last "}" encountered ⟹ not balanced

[1]

[1] https://www.slideshare.net/slideshow/stack-10033613/10033613

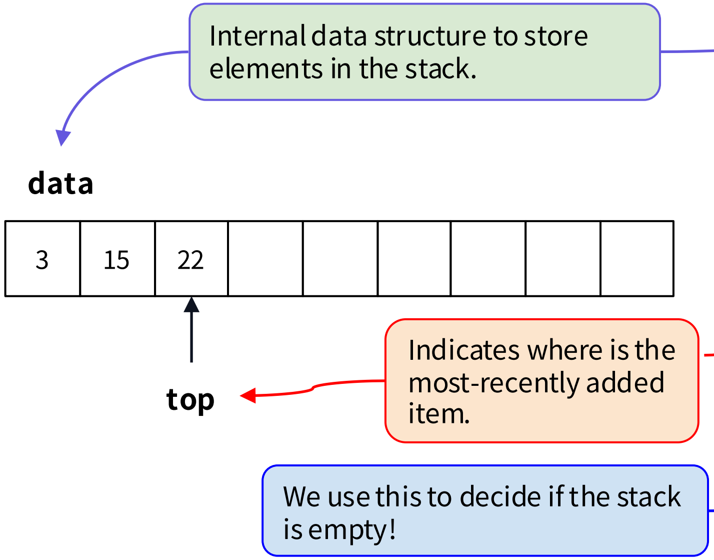
Stack Terminologies

- What functionality do we want to have with Class Stack?
  - Adding a new element (**push**)
  - Retrieving the most recent item (**peek**)
  - Deleting an item (**pop**)



Stack Class Design

- Array-based Implementation
  - We use Python `List` for simplicity here.



Internal data structure to store elements in the stack.

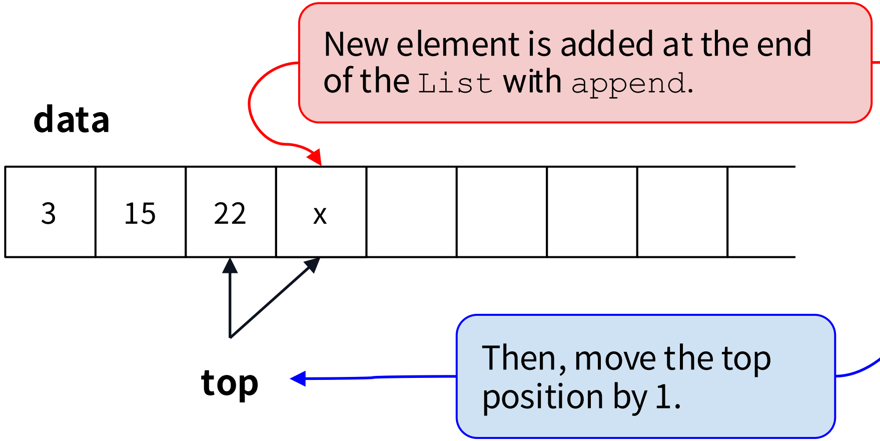
Indicates where is the most-recently added item.

We use this to decide if the stack is empty!

```
class Stack():
  def __init__(self):
    self.data = []
    self.top = -1

  def push(self, x):
    # insert x

  def peek(self):
    # get item

  def pop(self):
    # delete an item

  def is_empty(self):
    return (self.top == -1)
```

Stack Class Implementation

- Push
  - Do NOT specify where to insert.
  - The new element is added only at the top.



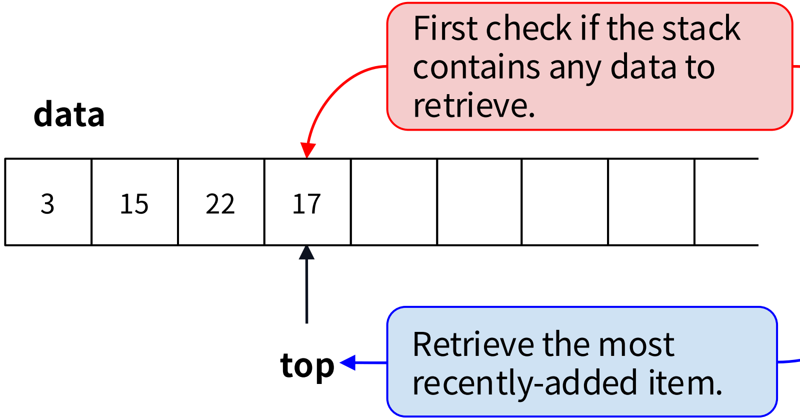
New element is added at the end of the `List` with `append`.

Then, move the top position by 1.

```
class Stack():
  def __init__(self):
    self.data = []
    self.top = -1

  def push(self, x):
    self.data.append(x)
    self.top += 1

  def peek(self):
    # get item

  def pop(self):
    # delete an item

  def is_empty(self):
    return (self.top == -1)
```

Stack Class Implementation

- Peek
  - Again, do NOT specify where to retrieve.
  - Stack always retrieves only the top element.



3	15	22	17					

First check if the stack contains any data to retrieve.

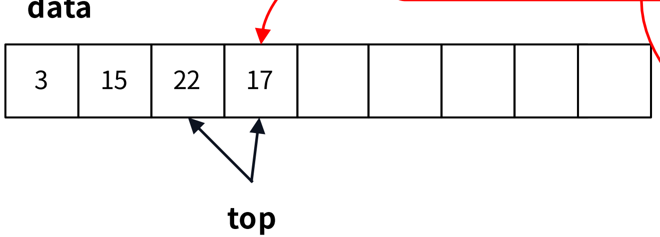
Retrieve the most recently-added item.

```
class Stack():
  def __init__(self):
    self.data = []
    self.top = -1

  def push(self, x):
    self.data.append(x)
    self.top += 1

  def peek(self):
    if not self.is_empty():
     return self.data[self.top]
    else: return None

  def pop(self):
    # delete an item

  def is_empty(self):
    return (self.top == -1)
```

Stack Class Implementation

- Pop
  - ○ Again, do NOT specify from where to delete.
  - ○ Stack always pops only the top element.

Remove the top item, then move the `top` pointer by 1!

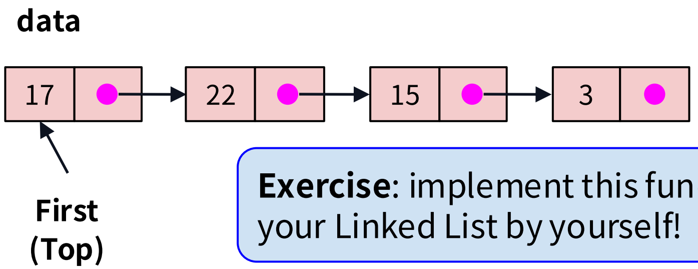


Note that with Python `list`, we may not explicitly need the variable `top`; instead, we may simply use `len(self.list)` to figure out the top position.

```
class Stack():
  def __init__(self):
    self.data = []
    self.top = -1

  def push(self, x):
    self.data.append(x)
    self.top += 1

  def peek(self):
    if not self.is_empty():
      return self.data[self.top]
    else: return None
  def pop(self):
    if not self.is_empty():
      del self.data[self.top]
      self.top -= 1
    else: return None
  def is_empty(self): (omitted)
```

Stack Class Design

- Reference-based Implementation
  - We may implement this by using a **Linked List**.
  - Recall that the singly linked list is accessed from the **first element**, sequentially.
    - We may naturally use the first element as the **top** element!
    - Thus, we don't have to maintain the top index.



Exercise: implement this function in your Linked List by yourself!

```
class Stack():
 def __init__(self):
  self.data = LinkedList()

 def push(self, x):
  # insert x

 def peek(self):
  # get item

 def pop(self):
  # delete an item

 def is_empty(self):
  return self.data.is_empty()
```

Stack Class Implementation

- How to implement push, peek, pop?
  - Use the functions of the Linked List!

```
class LinkedList():
  def __init__(self):
    self.first = None

  def insert(self, x, i):
    # insert x at [i]

  def get(self, i):
    # get item at [i]

  def delete(self, i):
    # delete item at [i]
```

```
class Stack():
  def __init__(self):
    self.data = LinkedList()

  def push(self, x):
    self.data.insert(x, 0)

  def peek(self):
    return self.data.get(0)

  def pop(self):
    self.data.delete(0)

  def is_empty(self):
    return self.data.is_empty()
```

Time Complexity

- Time complexity of Stack?

Task	Array-based	Reference-based
Insertion	O(1)	O(1)
Retrieval	O(1)	O(1)
Deletion	O(1)	O(1)

= Best cases only in linked list

Stack is more efficient than (more general) array or linked list, if the data and problem satisfy stack's condition!

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Applications of Stacks (Homework)

Application Questions

- Use stack(s) to check if a string with parentheses is well-formed.
  - "`(3+4)*(2+5)`" is well-formed.
  - "`((2*2)*3+1`" is not well-formed.
  - "`)(2+2`" is not well-formed.

- What if we have more than one types of parentheses?
  - "`{(2+1)*(3+2)-22}*7`" is well-formed.
  - "`{(7+2}*3)`" is not well-formed.

03

Queues

Queue Examples

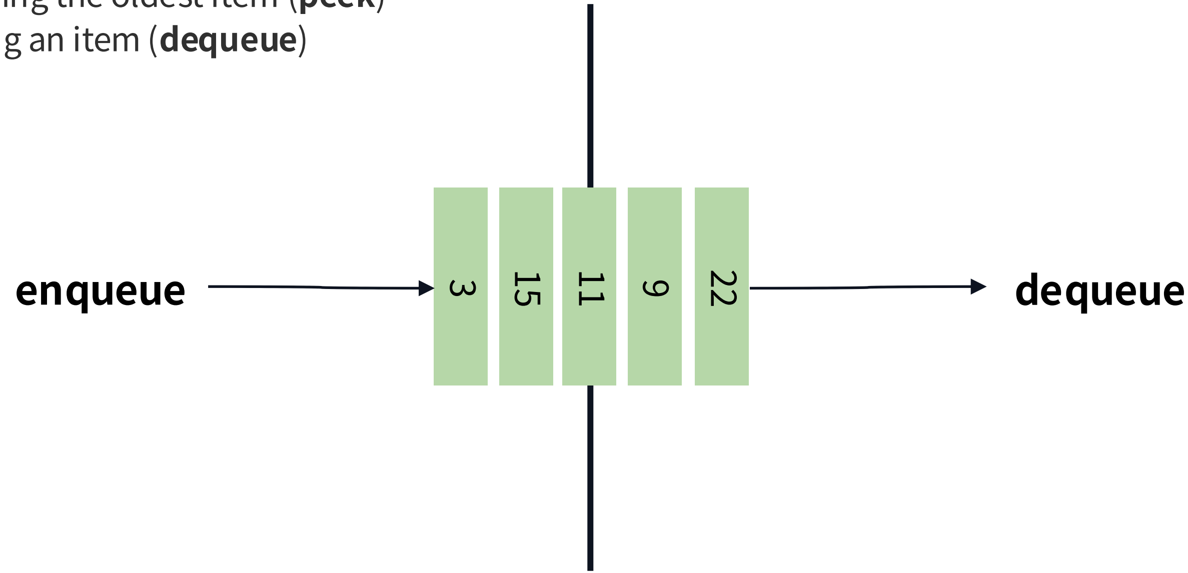
[1] Line of passengers at airport security

[2] Drink older milk first

[1] https://www.nbcnews.com/business/travel/tsa-replaces-head-security-airport-lines-keep-getting-longer-n579021
[2] https://brunch.co.kr/@myolivenote/1974

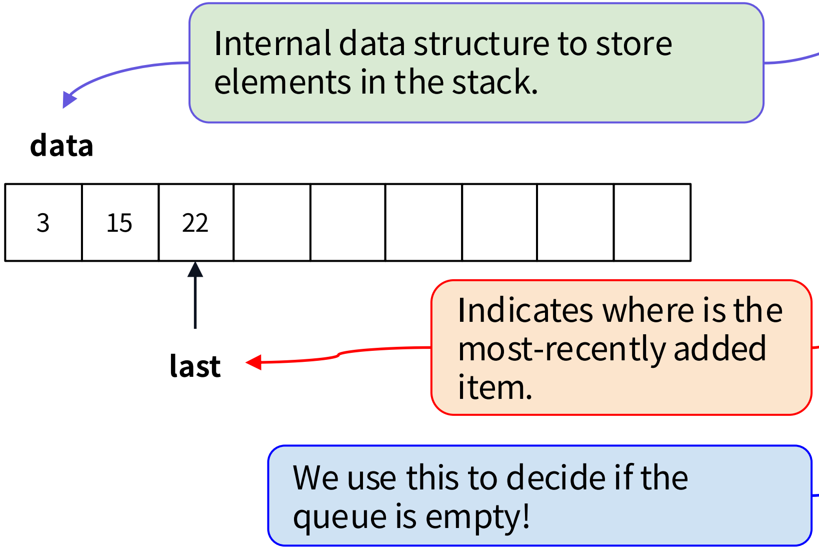
Queue Terminologies

- Similarly to the stack, queue also uses its own jargons:
    - Adding a new element (**enqueue**)
    - Retrieving the oldest item (**peek**)
    - Deleting an item (**dequeue**)



Queue Class Design

- Array-based Implementation
  - ○ We use Python `List` for simplicity here.



Internal data structure to store elements in the stack.

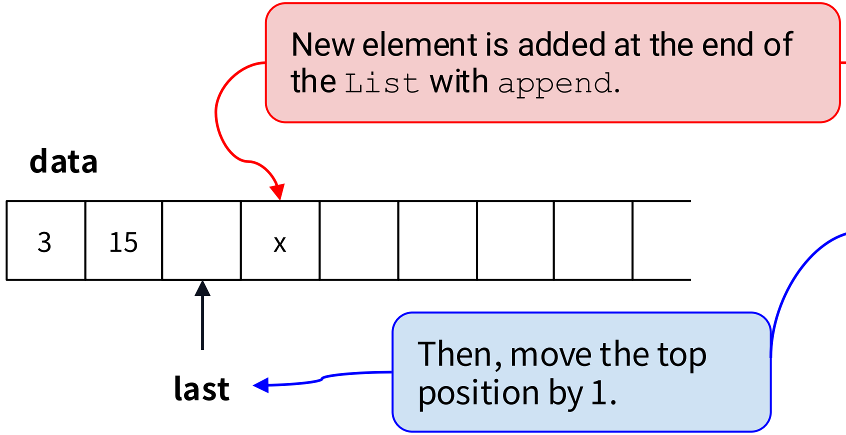
Indicates where is the most-recently added item.

We use this to decide if the queue is empty!

```
class Queue():
  def __init__(self):
    self.data = []
    self.last = -1

  def enqueue(self, x):
    # insert x

  def peek(self):
    # get item

  def dequeue(self):
    # delete an item

  def is_empty(self):
    return (self.last == -1)
```

Queue Class Implementation

- Enqueue
  - Insert always at the end (last).
  - Same as push in stack.



New element is added at the end of the `List` with `append`.

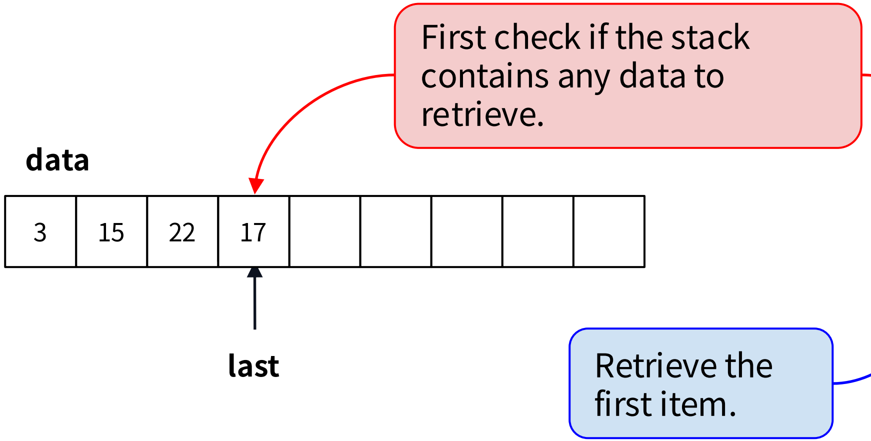
Then, move the top position by 1.

```
class Queue():
  def __init__(self):
    self.data = []
    self.last = -1

  def enqueue(self, x):
    self.data.append(x)
    self.last += 1

  def peek(self):
    # get item

  def dequeue(self):
    # delete an item

  def is_empty(self):
    return (self.last == -1)
```

Time complexity? **O(1)**

Queue Class Implementation

- Peek
  - We retrieve always the oldest item, which is located at the first.



First check if the stack contains any data to retrieve.

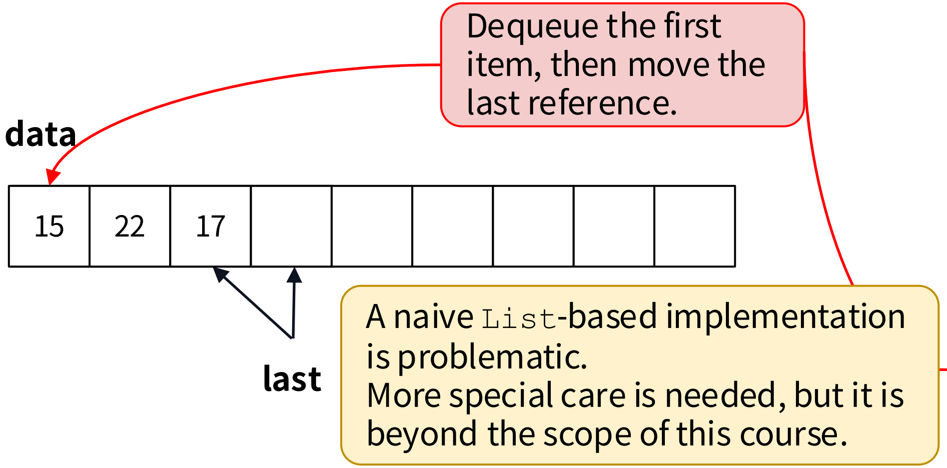
Retrieve the first item.

Time complexity? **O(1)**

```
class Queue():
  def __init__(self):
    self.data = []
    self.last = -1

  def enqueue(self, x):
    self.data.append(x)
    self.last += 1

  def peek(self):
    if not self.is_empty():
      return self.data[0]
    else: return None

  def dequeue(self):
    # delete an item

  def is_empty(self):
    return (self.last == -1)
```

Queue Class Implementation

- Dequeue
  - We dequeue always the oldest item, located at the first.



Dequeue the first item, then move the last reference.

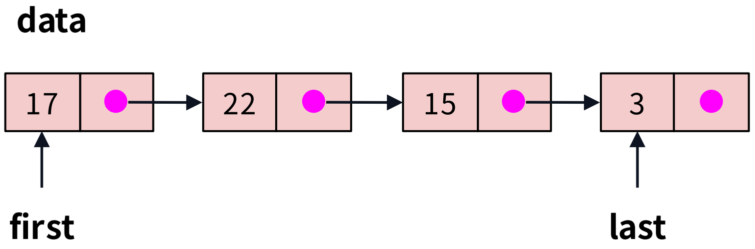
A naive `List`-based implementation is problematic.
More special care is needed, but it is beyond the scope of this course.

Time complexity? **O(*N*)** 😱

```
class Queue():
  def __init__(self):
    self.data = []
    self.last = -1

  def enqueue(self, x):
    self.data.append(x)
    self.last += 1

  def peek(self):
    if not self.is_empty():
      return self.data[0]
    else: return None

  def dequeue(self):
    if not self.is_empty():
      del self.data[0]
      self.last -= 1
  def is_empty(self): (omitted)
```

Queue Class Design

- Reference-based Implementation
  - Similarly to Stack, let's try **Linked List**.
  - As we enqueue and dequeue from different ends, we may keep references for both!
    - The beginning is naturally provided by the Linked List, so we only need to add the last reference.



```
class Queue():
  def __init__(self):
    self.data = LinkedList()
    self.last = None

  def enqueue(self, x):
    # insert x

  def peek(self):
    # get item

  def dequeue(self):
    # delete an item

  def is_empty(self):
    return self.data.is_empty()
```

Queue Class Implementation

- How to implement enqueue, peek, dequeue?
  - ○ Use the functions of the Linked List!

```
class LinkedList():
  def __init__(self):
    self.first = None

  def insert(self, x, i):
    # insert x at [i]

  def get(self, i):
    # get item at [i]

  def delete(self, i):
    # delete item at [i]
```

```
class Queue():
  def __init__(self):
    self.data = LinkedList()
    self.last = None

  def enqueue(self, x):
    # insert x

  def peek(self):
    return self.data.get(0)

  def dequeue(self):
    self.data.delete(0)

  def is_empty(self):
    return self.data.is_empty()
```

Queue Class Implementation

- How to implement enqueue, peek, dequeue?
  - ○ `Enqueue` is not as simple as others!
  - ○ First, we do not know the last index.
  - ○ Even though we maintain it, the `insert` of `LinkedList` will traverse the entire list, taking O(*N*) 😱.
  - ○ To avoid this, we need to take advantage of the `self.last` reference directly!

```
new_node = Node(x)
if self.last is None:
  self.data.first = new_node
else:
  self.last.next = new_node
self.last = new_node
```

```
class Queue():
  def __init__(self):
    self.data = LinkedList()
    self.last = None

  def enqueue(self, x):
    self.data.insert(x, ?)

  def peek(self):
    return self.data.get(0)

  def dequeue(self):
    self.data.delete(0)

  def is_empty(self):
    return
self.data.is_empty()
```

Time Complexity

- Time complexity of Queue?

Task	Array-based	Reference-based
Insertion	O(1)	O(1)
Retrieval	O(1)	O(1)
Deletion	O(1)	O(1)

We need special implementation to make deletion in O(1).

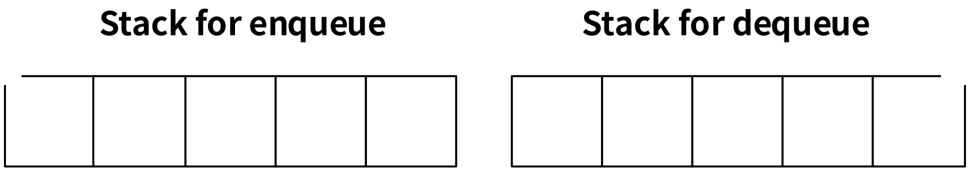
= Best cases only in linked list

04

Applications of Queues

Application Questions

- Implement Queue using two Stacks.
  - Main idea: use the first stack for enqueue, and the other for dequeue.
  - Whenever we get a dequeue request but the second stack is empty, pop all elements from the first stack and push them into the second stack.



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