UNIVERSITEIT VAN AMSTERDAM

COMPUTER VISION

ASSIGNMENT 3

HARRIS CORNER DETECTOR & OPTICAL FLOW

Ahmet Taskale
Student Number:12344087
ahmet.taskale@student.uva.nl

Andreea Teodora Patra
Student Number:13365169
andreea.patra@student.uva.nl

Jim Wagemans
Student Number: 11912286
jimwagemans@gmail.com

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1 Introduction

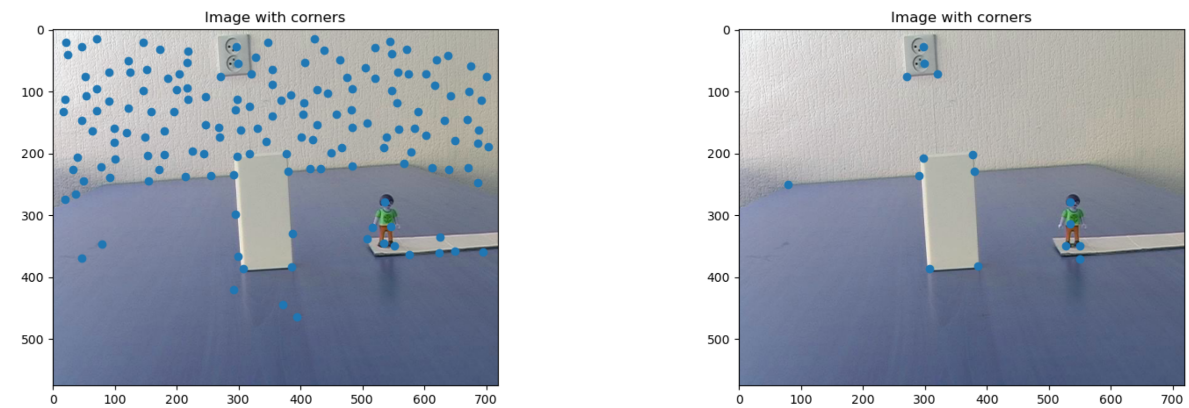
Monitor or feature tracking has many applications like facial detection, video surveillance, video games etc. The principle of motion tracking lies in recognizing a feature and monitoring its motion across multiple frames. The object of interest is monitored for spatial and temporal changes through a sequence of frames. Motion or object tracking starts with a suitable feature detection algorithm to detect features in an image. The second step is to compute the optical flow.

In this assignment, feature tracking algorithms like the Harris Corner Detector and the Shi-Tomasi Corner Detector will be summarized and discussed. Both methods are suitable for corner detection in an image, with minor differences between the two. In the second part of the assignment, the Lucas-Kanade algorithm will be implemented for the purpose of monitoring motion of features (corners) across frames by computing the optical flow of the image. Finally, combining the feature detection in the first part with the optical flow computation in the second part, feature tracking is applied to extract visual features like corners in order to monitor them across multiple frames.

2 Harris Corner Detector

Question 1

We implemented the algorithm as described in the assignment. We choose a window size of 15 in order to have in window size similar to the Lucas Kanade script. We first try to find a good value of sigma (for an image with values from 0 to 255). In figure 1 we see σ values 1 and 2. We see that $\sigma = 2$ smooths out the wall enough to not detect corners there.



(a) $\sigma = 1$ (b) $\sigma = 2$

Figure 1. Harris corner response for different sigma thresh hold 10

We next try to estimate a suitable value for the thresh hold. We used a thresh hold of 30. We wanted a thresh hold that would not pick up on fake corners in the wall while missing as few real corners as possible. We can see a comparison in figure 2

We are also interested in whether or not this is rotation invariant. We see this in figure 3 that (our implementation of) the corner response is mostly independent of rotation. This can be explained as follows. We are interested eigenvalues of the function

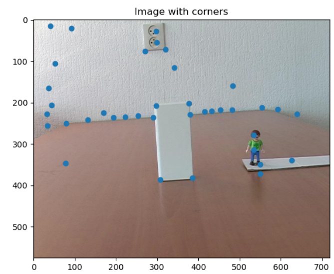
$$x \mapsto x^T Q x$$

If we rotate x using a rotation matrix R (which is orthogonal), we get the map

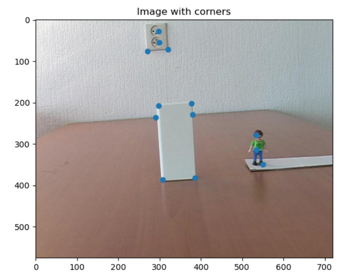
$$x \mapsto (Rx)^T Q R x = x^T (R^{-1} Q R) x$$

Since the matrices Q and $R^{-1}QR$ are similar, they have the same eigenvalues. Since the Harris response is $\lambda_1\lambda_2 - 0.04(\lambda_1+\lambda_2)^2$, the Harris response is rotation invariant. Our implementation is not completely rationally independent, this might be because of interpolation in rotational warping.

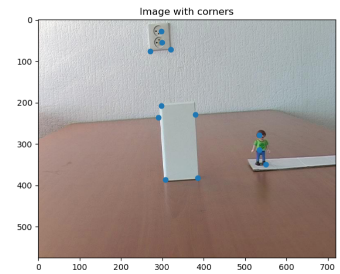
Question 2



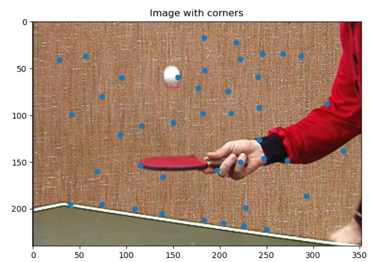
(a) thresh hold = 1



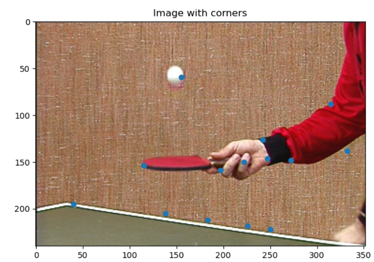
(b) thresh hold = 20



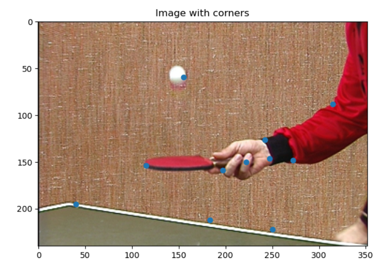
(c) thresh hold = 100



(d) thresh hold = 1

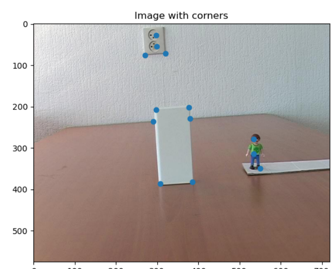


(e) thresh hold = 30

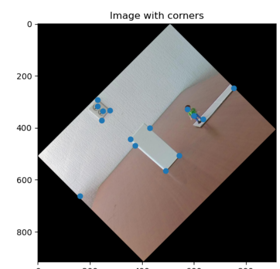


(f) thresh hold = 100

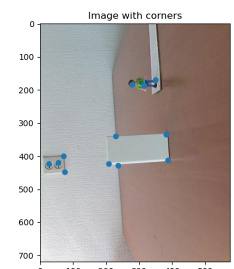
Figure 2. Harris corner response for different thresh hold $sigma = 2$



(a) no rotation



(b) rotation 45°



(c) rotation 90°

Figure 3. Rotated corner response

2.1

As mentioned above, the Harris Corner Detector defines cornerness H(x, y) by H = $\lambda_1\lambda_2 - 0.04(\lambda_1 + \lambda_2)^2$. Depending on the value of H, windows are classified as flat regions, edges or corners. A large value of H classifies the window as a corner, whereas a negative value classifies the windows as an edge. This is usually the case when one eigenvalue has a large value and the other one is small. Both eigenvalues being small indicates a flat region. Similar to the Harris Corner Detection, the definition of cornerness defined by Shi and Tomasi also has a mathematical approach to corner detection in an image. The difference lies in H, where the equation of Shi and Tomasi takes the minimum of the two eigenvalues. The equation can be defined as: H = min(λ_1, λ_2). In this case, if H is greater than a given threshold, the window is classified as a corner.

2.2

Translation invariance: A shift in derivatives and window function does not change the resulting outcome, since the eigenvalues remain the same. Therefore, the Shi-Tomasi method is translation invariant.

Rotation invariance: Similar to the answer given above, despite some rotation of if the image, it's shape and therefore the eigenvalues still remain the same. Thus, corner detection of the Shi-Tomasi method is invariant to rotation.

Scale invariance: The Shi-Tomasi method is not invariant to scaling, because the eigenvalues change. A simple example is shown below in Figure 4.

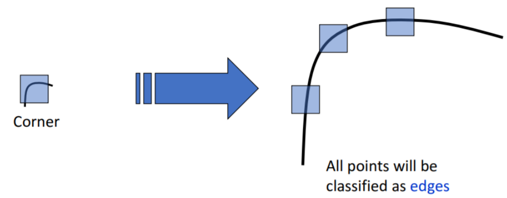


Figure 4. Demonstration of the Shi-Tomasi scale invariance

What is classified as a corner with a smaller scale, will be classified as edges with a larger scale. Therefore, the Shi-Tomasi corner detection method is not scale invariant.

2.3

a: Considering the Shi-Thomasi equation mentioned above, the cornerness value should be low when both eigenvalues are near 0.

b: Since the equation picks out the minimum eigenvalue, the cornerness value should be small and near zero in this case.

c: When both eigenvalues are big, naturally the cornerness value should be big as well. However, the H is defined as the minimum value of the two big eigenvalues.

3 Optical flow - Lucas Kanade algorithm

Question 1
The Lucas-Kanade algorithm is implemented for two sets of images and the resulting outputs are as follows:

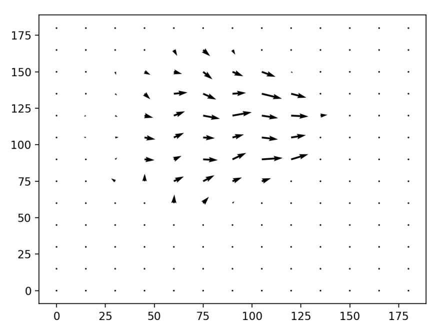


Figure 5. The optical flow for 'sphere' images

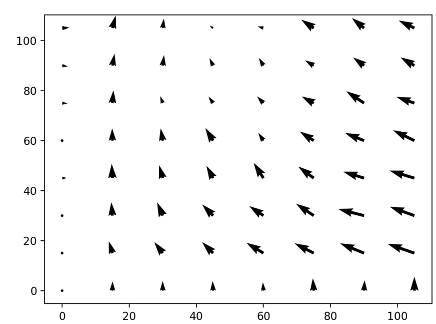


Figure 6. The optical flow for 'synth' images

For both of the images, a value of 0.5 was selected for the Gaussian Filter which solves the problem of noise that can be generated across frames.

Question 2
1. The Lucas-Kanade algorithm operates at a local scale as it assumes that the flow is smooth locally. The optical flow equation is calculated on neighbourhood of pixels which in our example are represented by 15x15 non-overlapping regions. Moreover, it assumes that the equation holds for all pixels within that specific window. On the other hand, the Horn-Schunck method is global which introduces a constraint of smoothness ($|| \nabla u ||^2 + || \nabla v ||^2$). Moreover, the following global function is sought to be minimised for the change in image brightness: $E = \int\int [I_x u + I_y v + I_t]^2 + \alpha^2 \, || \nabla u ||^2 + || \nabla v ||^2 \; dxdy$

From the implementation of both of the optical flow equations, we can conclude that Lucas-Kanade focuses on neighbourhoods of pixels, whilst Horn-Schunck uses the global image as reference.

2. For Lucas-Kanade, $A^T A$ needs to be invertible for the system of equations to be solvable. However, the matrix A is composed of the gradients of the image in the x direction and y direction and having pixels in flat regions, would mean that the elements of this matrix are going to be close to zero. As a result, the matrix would be ill-conditioned and the system unsolvable. On the other hand, in Horn-Schunck, by having a smoothing constraint, the velocity information of the flat regions is filled from the boundaries, by averaging the neighbouring velocity estimates.

4 Feature tracking

Question 1
In this section, we implement the algorithm for Feature Tracking using the Harris Corner Detection and the Lucas-Kanade algorithm. For the 'person_toy' images we have selected a value of 0.7 for the standard deviation of the Gaussian Filter and for the 'pingpong' images a value of 2. The Gaussian filter was used as a way of dealing with the noise that different frames can have with respect to the reference frame. We first detect the features using Harris Corner Detection on the reference frame and then, iteratively, we track them across multiples frames using the optical flow equation. In the animations provided, we can observe that Lucas-Kanade encounters challenges when following the fast moving ball across the frames.
Question 2
The main disadvantage of feature matching compared to optical flow based methods is that it is, in general, harder to calculate where the feature matches are spawn in the frames. On the other hand, algorithms like Lucas-Kanade, can be highly accurate by computing dense motion fields with respect to the shared motion of the feaures in the image. However, the challenges of Lucas-Kanade still need to be acknowledged, such as, large motion, strong illumination changes and changes of the appearance of the objects. Another advantage of optical flow is the computation complexity compared to feature matching.

5 Conclusion

The goal of this assignment was to create functions to implement the Harris Corner Detector, the Lucas-Kanade algorithm and put the latter two into practice by implementing a feature tracking algorithm.

Starting off with the Harris Corner Detector, results showed that for the given images values of σ = 2 and threshold = 30 were optimal for corner detection. Furthermore, it is demonstrated with rotated images that the Harris Corner method is rotation invariant.

Secondly, it was stated that the similar corner detection method, defined by Shi and Tomasi, differs in the calculation of the cornerness H. The Shi-Tomasi method takes the minimum of the two eigenvalues, where the window is classified as a corner provided that H is greater than the chosen threshold. Moreover, the Shi-Tomasi method is not invariant to scaling, since the eigenvalues in this case do not remain the same.

It was stated that the Lucas-Kande method operates at a local scale, since it assumes that the flow is smooth locally, while focusing on neighbouring pixels. The Horn-Schunck method is a global operator which introduces a constraint of smoothness and uses the global image as a reference.

Finally, the resulting videos with the feature tracking algorithm and the chosen set of parameters showed that the Lucas-Kanade encounters difficulties with the fast moving ball.