ACTUS: The algorithmic representation of financial contracts
VERSION v1.1-5ba27bb-2020-06-08

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About this document

This document provides the technical specifications of the Algorithmic Contract Types Unified Standards (ACTUS). It is developed, maintained, and released by the ACTUS Financial Research Foundation and provided by the same to the ACTUS Users Association under the terms of the open source license with which the document is published from time to time.

Versions

This document is versioned according to the following pattern: [major].[minor]-[revision]-[date] where [major] and [minor] are integers marking major and minor release, [revision] indicates the current revision in form of the respective git commit hash (short form), and [date] gives the respective date of the revision. Releases are recorded in the following table.

Date	Version	Description
2018-11-01	1.0-RC	First draft version of the technical specifications covering the "initial" 18 contracts.
2019-10-23	1.0	Stable release streamlined with dictionary. Specifically, this release includes following improvements to the draft version: • Fixed various naming conventions • Aligned state variable names with dictionary • Added `ContractStructure` and "Composition"-section • Added `settlementCurrency` attribute and updated `POF` accordingly • Added `exerciseDate` and `exerciseAmount` terms and states for contracts with contingent payments • Removed default convention and updated `POF` accordingly • Moved Taxonomy, Event, State, Contract Role definitions to dictionary • Fixed various bugs and inconsistencies
2020-06-08	1.1	Minor udpates and fixes for alignment with dictionary-v1.2

Acknowledgements

We would like to acknowledge all members of the *ACTUS Users Association* who contribute a lot of their time and expertise to the development, review, and testing of the ACTUS standards, in general, and this document, in particular. Without their valuable contributions the ACTUS standards would not exist in the form as they currently do.

Contents

1. Introduction

Financial contracts are legal agreements between two (or more) counterparties on the exchange of future cash flows. Such legal agreements are defined unambiguously by means of a set of contractual terms and logic. As a result, financial contracts can be described mathematically and represented digitally as machine readable algorithms. The benefits of representing financial contracts digitally are manifold; Traditionally, transaction processing has been a field in which tremendous efficiency gains could be realized by the introduction of *machines* and machine readable contracts. Or, financial analytics by nature of the domain builds on the availability of computable representations of these agreements where for reasons of tractability often times analytical approximations are used. Recently, the rise of distributed ledger and blockchain technologies and the various use cases for *smart contracts* has opened up new possibilities for *natively digital* financial contracts.

In general, the exchange of cash flows between counterparties follows certain patterns. A typical cash flow exchange pattern is a *bullet loan* contract where principal is exchanged initially followed by cyclical interest payments and the principal is paid back (in a lump sum) at maturity of the contract. While the principal payments are fixed a variety of flavours exist for how the cyclical interest payments are determined and/or paid. As an example, interest payments may be due monthly, annually or according to arbitrary periods, they may be determined based on fixed or variable rates, different year fraction calculation methods may be used or there might be no interest due at all. Another popular pattern is that of *amortizing loans* for which, as opposed to bullet loans, principal may be paid out and paid back in portions of fixed or variable amounts and according to cyclical or custom schedules. Other types of financial contracts include but are not limited to *shares*, *forwards*, *options*, *swaps*, *credit enhancements*, *repurchase agreements*, *securitization*, etc. By focusing on the main distinguishing features, ACTUS describes the vast majority of all financial contracts with a set of about 32 generalized cash flow exchange patterns or Contract Types (CTs), respectively. The ACTUS taxonomy (`https://github.com/actusfrf/actus-dictionary/blob/master/actus-dictionary-taxonomy.json`) provides a classification system organizing financial contracts according to their distinguishing cash flow patterns. Apart from this classification system the taxonomy also includes a description of and real-world instruments covered for each contract.

On the other hand, the legal agreements in financial contracts represent purely deterministic logic or the *mechanics of finance*, in other words. That is, a financial contract defines a fixed set of rules and conditions under which, given any external variables, the cash flow obligations can be determined unambiguously. For instance, in a *fixed rate loan* the cash flow obligations are defined explicitly. At the same time, a *variable rate loan* defines explicitly the rules under which the variable rate is fixed going forward such that the cash flow obligations can be derived unambiguously going forward. The same holds true for *derivative contracts* where the cash flow obligations arise given some underlying *reference instrument*. Similarly, for analytical purposes, given some assumption of the evolution of this reference instrument the cash flow obligations *conditioned on* this assumption can be derived unabiguously.

The properties of financial contracts described above build the foundation for a standardized, deterministic algorithmic description of the cash flow obligations arising from such agreements. Thereby, this description is technology agnostic and supports all use cases necessary for this very standard to be used throughout all finance functions from front office to back office and covering pricing, deal origination, transaction processing, as well as analytics, in general, and liquidity projections, valuation, P&L calculations and projections, and risk measurement and aggregation, in particular. Furthermore, this standard builds a formidable basis for distributed ledger-powered, natively digital *financial state machines* or *smart contracts*, in other words.

In this document, we provide the technical specification of the ACTUS standards or the mathematical description of financial contracts, in other words. We start by providing some basic notations used throughout the document followed by an introduction of the generic functions upon which financial contracts build. We continue in the following sections with an introduction of some additional foundational concepts *Composition*, and *Risk Factor Observer* and *Child Contract Observer*. Finally, we define the various ACTUS contracts in the last section.

2. Notations

2.1. **Contract Attributes.** Contract Attributes (attributes) represent the legal contractual terms that define the exchange of cash-flows of a financial contract. These attributes are defined and described in the ACTUS dictionary (`https://github.com/actusfrf/actus-dictionary/blob/master/actus-dictionary-terms.json`). Throughout this document attributes are referenced by their short name according to the dictionary. Further, vector-type attributes may be indexed with a subscript indicating that a specific vector-element is referenced.

Example 1 (Contract Attribute)**.** *The ACTUS attribute Initial Exchange Date is referenced in short form* `IED`*.*

Example 2 (Element of Vector-Type Attribute)**.** *The ACTUS attribute Array Cycle Anchor Date of Principal Redemption is a vector-type attribute and referenced as* `ARPRANX`*. The i-th element of the vector is represented by* $\texttt{ARPRANX}_i$*.*

2.2. **∅-Operator.** The ∅-operator is used to indicate that a certain property is undefined or, in other words, that no value has been assigned to the respective property. In particular, for optional contract attributes it means that the attribute is not defined and for schedule times (see section 3.1) it means that the respective schedule is empty, i.e. no schedule time defined.

Example 3 (Undefined Attribute)**.** $\texttt{IPANX} = \varnothing$ *indicates that attribute* `IPANX` *is undefined.*

Example 4 (Empty Schedule)**.** $\vec{t}^{IP} = \varnothing$ *means the same as* $\vec{t}^{IP} = \{\}$*, with* $\{\}$ *the empty set, and states that the IP schedule* $\vec{t}^{IP}$ *does not contain a schedule time.*

2.3. t_0**-Time.** t_0 represents `SD` of a contract and marks the time as per which the terms and implied state of a contract is represented. In general, from the contractual logic we are able to derive any contractual events and resulting states for any time $t > t_0$ but not for times $s < t_0$.

2.4. **State Variables.** State Variables (states) describe the state of a contract at a certain point in time t during its lifetime. Examples of such states are the (outstanding) Notional Principal, the applicable Nominal Interest Rate, or the current Contract Performance. The ACTUS dictionary (`https://github.com/actusfrf/actus-dictionary/blob/master/actus-dictionary-states.json`) defines all states and provides further information on their data type, format, etc.
In general, states represent certain terms of a contract that change along the contract lifetime according to either scheduled events or unscheduled events. Therefore, states representing a contractual term carry the exact same names as their term-counterpart.
States are written in their short form representation with first letter capitalized, printed in bold, and indexed with time.

Example 5 (State Variables)**.** *$\boldsymbol{Nt}_t$ refers to the state Notional Principal observed at time t.*

2.5. **Contract Events.** A Contract Event (event) e_t^k refers to any contractually scheduled or unscheduled event at a certain time t and of a certain type k. Contract events mark specific points in time during the lifetime of a contract at which a cash flow is being exchanged (see section 2.7) or the states of the contract are being updated (see section 2.6). The dictionary lists and describes all the event types k defined by the ACTUS standards (`https://github.com/actusfrf/actus-dictionary/blob/master/actus-dictionary-event-types.json`).
Throughout this document event types k are written in the short form as defined in the dictionary.
As an event always has an associated event time t and payoff $c \in \mathbb{R}$ we define two operators allowing to retrieve these quantities for any single event e_t^k or set of events $\{e_t^k, e_s^j, ...\}$ as follows;

$$\tau(x) = \begin{cases} t & \text{if} \quad x = e_t^k \\ \{t, s, ...\} & \text{else if} \quad x = \{e_t^k, e_s^j, ...\} \end{cases}$$

$$f(x) = \begin{cases} c & \text{if} \quad x = e_t^k \\ \{c_1, c_2, ...\} & \text{else if} \quad x = \{e_t^k, e_s^j, ...\} \end{cases}$$

with $c_1 = f(e_t^k), c_2 = f(e_s^j), ...$

Example 6 (Contract Events)**.** *The Initial Exchange Date event with event time s is written as e_s^{IED} with $\tau(e_s^{IED}) = s$ and $f(e_s^{IED}) = c$ where for any contract `CT` $c = POF\_IED\_CT()$.*

2.6. **State Transition Functions.** State Transition Functions (STF) define the transition of states from a pre-event to a post-event state when a certain event e_t^k applies. Thereby, the pre-event and post-event times are indexed with t^- and t^+, respectively.
These functions are specific to a certain event and contract. STFs are written according to the following pattern STF_[event type]_[contract type]() where [event type] and [contract type] refer to the respective event type and contract to which the STF belongs.

Example 7 (State Transition Functions)**.** *The STF for an IP event and PAM contract is written as STF_IP_PAM() and maps e.g. state Accrued Interest from a pre-event state $\boldsymbol{Ipac}_{t^-}$ to post-event state $\boldsymbol{Ipac}_{t^+}$.*

2.7. **Payoff Functions.** Payoff Functions (POF) define how the cash flow $c \in \mathbb{R}$ for a certain event e_t^k is being derived from current states and from the contract terms. If necessary, the resulting cash flow can be indexed with the event time c_t. These functions are specific to a certain event and contract. POFs are written according to the following pattern POF_[event type]_[contract type]() where [event type] and [contract type] refer to the respective event and contract to which the STF belongs.

Example 8 (Payoff Functions)**.** *The POF for an IP event e_t^{IP} and PAM contract is written as POF_IP_PAM() with $f(e_t^{IP}) = POF\_IP\_PAM()$.*

2.8. **Date/Time.** ACTUS builds on the ISO 8601 date/time format. Hence, dates are generally expressed in the following format: [YYYY]-[MM]-[DD]T[hh]:[mm]:[ss]. Time zone information is currently not supported.
A special case is *midnight*. ISO 8601 recognizes both times 00:00:00 and 24:00:00 each referring to midnight. Yet, while 24:00:00 refers to the end of one day, 00:00:00 refers to the beginning of the following day. In ACTUS the interpretation is the same why the time period (measured in any time unit) between the two points in time will always be zero.
For brevity, we use the term *time* for a specific date-time variable.
A note on implementation: As many implementations of the ISO 8601 format do not support the 24:00:00 format we interpret the timestamp `23:59:59` as midnight.

2.9. **Event Sequence.** Contract Events of different types may occur at the same time, i.e. exactly the same point in time. In this case, the sequence of evaluating their STF and POF is decisive for the resulting cash flows and state transitions. Hence, we use an event sequence indicator that can be found for each event in the event-dictionary and implies the order of executing different events at the exact same time.

2.10. **Contract Lifetime.** The lifetime of an ACTUS contract is the time period of its existence from the perspective of the analyzing user. For every point in time during its lifetime, an ACTUS contract can be analyzed in terms of current state and future cash flows.

The lifetime of a contract starts with its `SD` and ends with $\min(MD, AMD, PR^*, STD, TD, t^{max})$.

Note that PR^* refers to the PR event of a maturity contract after which **Nt**=0.0 (i.e. at which the remaining outstanding principal is redeemed). Further, `MD`, `AMD`, and PR(**Nt**=0.0) in the definition above do only apply for maturity contracts but have to be considered infinity in all other cases. Similarly, `STD` only applies for certain contracts and is considered infinity for all others. Finally, t^{max} is a parameter that may be used to restrict the considered lifetime in an analysis. In particular, this parameter is used for contracts that do not have a *natural* end to their lifetime such as STK.

3. Utility Functions

3.1. Schedule. A schedule is a function S mapping times s, T with $s < T$ and cycle c onto a sequence $\vec{t}$ of cyclic times

$$S(s,c,T) = \vec{t} = \begin{cases} \{\} & \text{if} \quad s = \varnothing \wedge T = \varnothing \\ s & \text{else if} \quad T = \varnothing \\ (s,T) & \text{else if} \quad c = \varnothing \\ (s = t_1, ..., t_n = T) & \text{else} \end{cases}$$

with $t_i < t_{i+1}, i = 1, 2, ...$ While the schedule function can be used to create arbitrary sequences of times, it is usually used to generate sequences of cyclic events $\vec{t^k}$ of a certain type k, e.g. $k = IP$ for interest payment events (see dictionary for a list of all events https://github.com/actusfrf/actus-dictionary/blob/master/actus-dictionary-event.json) and the following build inputs to the function

- s $= k$`ANX` with k`ANX` attribute cycle anchor date of event type k
- c $= k$`CL` with k`CL` event type k's schedule cycle
- T is the schedule end date (in many cases the contract's maturity date)

Thereby, cycles k`CL` have format NPS where

- N is an integer
- P is a time period unit (D=Day, W=Week, M=Month, Q=Quarter, H=Half Year, Y=Year)
- S is a *stub* information (+=long last stub, −=short last stub)

and with the stub defined as follows

- if $t_{n-1} + c = T \vee S =$'-' then no stub correction applies
- else t_n is removed from the schedule

Further, the schedule function takes a fourth, optional boolean argument B, i.e. $S(s,c,T,B)$ indicating whether the schedule end date T belongs to the schedule or not. More specifically:

- $B = T$ indicates that T is part of the schedule
- $B = F$ means that T is not part of the schedule

The sequence of schedule times $\vec{t^k}$ may also be influenced by the `EOF` and `BDC` conventions and the full function syntax becomes $S(s,c,T,\texttt{EOMC},\texttt{BDC})$. Due to such effects the sequence of schedule times can be non-equidistant or, in other words, $t_i^k - t_{i-1}^k \neq t_j^k - t_{j-1}^k, i \neq j$.

Note that for brevity we will omit the `EOMC` and `BDC` function arguments throughout this document.

3.2. Array Schedule. Array Schedules are defined by vector-valued inputs $\vec{s} = (s_0, s_1, ..., s_m)$ and $\vec{c} = (c_0, c_1, ..., c_m)$ to the array schedule function

$$\vec{S}(\vec{s}, \vec{c}, T) = (S(s_0, c_0, s_1 - c_0), \\ S(s_1, c_1, s_2 - c_1), ..., S(s_m, c_m, T))$$

Hence, array schedules are a generalization for regular schedules which coincide for $m = 1$. In accordance with regular schedules `EOMC` and `BDC` conventions also apply here.

3.3. End Of Month Shift Convention. For schedules $\vec{t^k}$ starting at time s which marks the end of a month with 30 or less days, e.g. April 30, and with a cycle c being a multiple of 1M- attribute `EOM` defines whether the schedule times are to fall on the 30th of all months (same day) or the 31st (end of month).

More specifically, `EOM` has an effect on a schedule $\vec{t^k}$ only if:

- s is the last day of a month with less than 31 days (Feb, April etc.)
- c $= NPS$ with $P \in (M, Q, HorY)$

As per the DD `EOM` can take one of the following values:

- EOM (EndOfMonth): times $t_i, i = 1, 2, ..., n-1$ are moved to the end of the respective months
- SD (SameDay): times $t_i, i = 1, 2, ..., n-1$ remain unchanged except in February, where it will go to the last day if the day of month of time s is higher than the number of days of February

3.4. Business Day Shift Convention. In general, contract events are scheduled for business days only. Therefore, the `BDC` convention defines how scheduled times $t_i, i = 1, 2, ..., n-1$ are shifted in case they fall on a non-business day:

- NULL: No shift
- SCF: Shift/Calculate following: The event is shifted to the following non working day. Calculation of the event happens after the shift
- SCMF: Shift/Calculate modified following: The event is shifted to the following non working day. However, if the following day happens to fall into the next month, then take preceeding non-working day. Calculation of the event happens after the shift
- CSF: Calculate/Shift following: Same like SCF however calculation of the event happens before the shift
- CSMF: Calculate/Shift modified following: Same like SCMF however calculation of the event happens before the shift
- SCP: Shift/Calculate preceding: The event is shifted to the last preceding non working day. Calculation of the event happens after the shift
- SCMP: Shift/Calculate modified preceding: The event is shifted to the last preceding non working day. However, if the preceding day happens to fall into the previous month, then take next non-working day. Calculation of the event happens after the shift
- CSP: Calculate/Shift preceding: Same like SCP however calculation of the event happens before the shift
- CSMP: Calculate/Shift modified preceding: Same like SCMP however calculation of the event happens before the shift

3.5. Business Day Calendar. Whether a specific day is a business day (cf. previous section) is defined by attribute `CLDR`. Such conventions generally depend on regional official holiday calendars. The Business Day Function interface allows determining for some `CLDR` whether any time t is a business day or not

$$B : t \mapsto \{true, false\}$$

where *true* indicates that t is a business day and $false$ that it is a holiday.

Example 9. *Two standard `CLDR` implementations are the following*

- *NoHoliday (default): every calendar day is a business day*
- *MondayToFriday: all weekdays Monday, Tuesday, Wednesday, Thursday, and Friday are business days*

3.6. **Year Fraction Convention.** Interest income and other calculations are based on *per annum* interest rates. Therefore, the year-fraction function interface Y is used to calculate the *fraction of a year* between any two times s and t with $t > s$ for which e.g. an (per annum) interest rate applies according to some day count convention `DCC`

$$Y : s, t, \texttt{DCC} \mapsto \mathbb{R}$$

Note, the year fraction function interface only defines the structure of year fraction functions but not an actual implementation thereof, or the respective `DCC`, respectively. Therefore, any `DCC` can be implemented according to the interface above supporting user-defined year fraction functions.

For brevity we will omit the `DCC` function argument wherever this does not lead to confusion.

3.7. **Contract Role Sign Convention.** The two parties to a contract are defined through attributes `CRID` and `CPID`. The first is the party initially *creating* the contract and the second is the counterparty, respectively. Thereby, both `CRID`/`CPID` can take any *role* in the contract or, more specifically, they can be the lender or borrower in a loan (PAM), fixed receiver or payer in an interest rate swap (SWAPS), etc.

The *role* of the `CRID` is defined through attribute `CNTRL`. The *role* of `CPID` is derived as the *opposite* side to the contract. Apart from `CNTRL` the attributes are *neutral* to the *role* of `CRID` (or `CPID`).

On the other hand, contractual cash flows generated by the POFs and certain states are *role-sensitive.* That is, from the perspective of the `CRID` these quantities represent either claims or obligations. Contract Role Sign function R maps the `CNTRL` attribute into $+1$ indicating a claim or -1 indicating an obligation

$$R : \texttt{CNTRL} \rightarrow \{-1, +1\}$$

When multiplying with a cash flow x the Contract Role Sign function thereby defines the direction of that flow:

$x > 0$: x flows from `CPID` to `CRID`
$x < 0$: x flows from `CRID` to `CPID`

Table 1 defines the domain of the Contract Role Sign function, i.e. the range of attribute `CNTRL`, with meaning and Contract Role Sign to which the function maps.

Value	Meaning	R
RPA	Real position asset	+1
RPL	Real position liability	-1
LG	Long position	+1
ST	Short position	-1
BUY	Protection buyer	+1
SEL	Protection seller	-1
RFL	Receive first (or fixed) leg	+1
PFL	Pay first (or fixed) leg	-1
COL	Collateral instrument	+1
CNO	Close-out netting instrument	+1
GUA	The guarantor in a Guarantee	-1
OBL	The obligee in a Guarantee	+1
UDL	The underlying to a composed contract	+1
UDLP	The underlying to a composed contract with positive sign	+1
UDLM	The underlying to a composed contract with negative sign	-1

Table 1. Contract Role definitions.

3.8. **Annuity Amount Function.** In an *Annuity* contract (ANN) the annuity amount is paid regularly from the *borrower* to the *lender.* Thereby, the annuity amount is comprised of a principal repayment portion and an interest portion and and dimensioned such that the total nominal amount n at time t is fully repaid at maturity T of the annuity. The Annuity Amount function A computes the annuity amount as follows

$$A(s,T,n,a,r) = (n+a)\frac{\prod_{i=1}^{m-1} 1 + rY(t_i, t_{i+1},)}{1 + \sum_{i=1}^{m-1} \prod_{j=i}^{m-1} 1 + rY(t_j, t_{j+1})}$$

with a the accrued interest as per time s, r the actual interest rate, $t_i, i = 1, 2, ..., m$ the schedule times $\inf t, t \in \vec{t}^{PR} \wedge t > s$, m the number of times t_i, and $\vec{t}^{PR}$ the PR-event schedule times of the Annuity contract as described in section 7.5.

3.9. **Canonical Contract Payoff Function.** The canonical payoff of a contract x is defined as the sum of all future event payoffs evaluated under current risk factor conditions, or

$$F(x,t) = \sum_{c \in C} c$$

with $C = f(U^{ev}(x, t \mid \{O^{rf}(\texttt{i}, s) = O^{rf}(\texttt{i}, t) \forall i \wedge s > t\}))$.

3.10. **Settlement Currency FX Rate.** Sometimes financial contracts are settled in a different currency (i.e. the *settlement currency*) `CURS` than the denomination currency `CUR`. Hence, payoffs are multiplied by the respective fx-rate which is derived by the following

$$X^{\texttt{CURS}}_{\texttt{CUR}}(t) = \begin{cases} 1 & \text{if} \quad \texttt{CURS} == \varnothing \vee \texttt{CURS} == \texttt{CUR} \\ f(t) & \text{else} \end{cases}$$

with $f(t) = O^{rf}(\texttt{concat(CUR,'/',CURS)}, t)$.

4. Contract Composition

The payoff of *Combined Contracts*, see the taxonomy, is derived from certain quantities of child contracts (also called *underlying instruments* or simply *underlyers*). In general, such child contracts can be any ACTUS contract - Basic or Combined - as well as any number of contracts - a single contract or a set of contracts. Indeed, in reality this is what Option, Swap, Swaption, but also any kind of Asset/Mortgage/etc. backed securities represent; *a hierarchical composition of different contracts linked by means of functional reationships.* This compositional approach provides maximum flexibility and, hence, allows capturing any real world use case. We here refer to a *referenced* (i.e. of lower hierarchical level) contract as a *child contract* and to a referencing (i.e. of higher hierarchical level) contract as a *parent contract.*

The ACTUS dictionary defines attribute `CTST` which captures the child contract(s) as part of the parent contract's set of attributes. Thereby, attribute `CTST` is of type `ContractReference[]` with `ContractReference` a reference to a child contract (in JSON notation)

```
{
"object": ,
"type": ,
"role":
}
```

Consult the dictionary for more information (cf. `https://github.com/actusfrf/actus-dictionary/blob/6e84435c70e1325e114c931badaa7e80b01a9a0d/actus-dictionary.json#L776`.
We will use the following notation to query reference objects from the `CTST` attribute

$$\texttt{CTST}^{type}_{role}(i)$$

where *role* and *type* filter the reference objects in `CTST` according to the respective values of fields `type` and `role`. For brevity, we will omit the index parameter i which indicates that we address the first (and usually only) reference object queried.

Example 10 (Underlying MarketObject-reference)**.** *The MarketObject reference of a simple Underlying e.g. to an Option contract is referenced as* $CTST^{MarketObjectIdentifier}_{Underlying}(1)$ *or, in short form, as* $CTST^{MarketObjectIdentifier}_{Underlying}$.

Example 11 (FirstLeg Contract-reference)**.** *The Contract object representing the first leg e.g. to a Swaps contract is referenced as* $CTST^{Contract}_{FirstLeg}(1)$ *or, in short form, as* $CTST^{Contract}_{FirstLeg}$.

5. Risk Factor Observer

The payoff of financial contracts always depends on the context in which it is evaluated and which is comprised of the following dimensions; counterparties, markets, and behavioral factors. We refer to these as the *risk factors* to which financial contracts are exposed to. This indicates that these factors are source of uncertainty because financial contracts only reference the factors but their dynamics is outside the control of any contractual agreement. Thus, such factors have to be *observed* and their changing states accounted for when evaluating the payoff of financial contracts. Therefore, we consider a standardized interface $O^o(i, t, S, M)$ that allows for *observing*; (1) the state of a certain risk factor i at any time t if $o =$'rf'

$$O^{rf} : i, t, S, M \mapsto \mathbb{R}$$

and (2) contractual but non-scheduled events if $o =$'ev'

$$O^{ev} : i, k, t, S, M \mapsto \{e^k_t, e^k_s, ...\}$$

The parameters to the Risk Factor Observer interface are as follows:

- i : the identifier of the risk factor observed
- k : the type of events observed
- t : the time (post) which to observe the risk factor
- S : the inner states of the contract at time t
- M : the contract terms of the contract as per time t

Note that the observer interface only defines the structure of an actual observer function but not the actual implementation. Thus, the interface allows for user-defined implementations of observer functions allowing e.g. for representing arbitrary assumptions on the evolution of future risk factor states which is key for any type of forward-looking analysis.

Example 12 ('rf'-Observer)**.** *The market-driven 3-month USD-Libor reference rate used as the variable rate in a variable rate loan contract is observed at any time t through* $O^{rf}(\texttt{MarketObjectCodeRateReset}, t)$.

Example 13 ('rf'-Observer). *Unscheduled (pre-) repayments of outstanding notional in a mortgage contract is observed at any time t through* $O^{ev}(\texttt{CID}, \texttt{PR}, t)$.

For brevity we will omit the S and M function arguments wherever this does not lead to confusion.

6. Child Contract Observer

In order to evaluate the derived payoff of combined contracts, we consider a standardized interface U^o that allows for *observing* on the parent level; (1) all future events, w.r.t. time t, if $o =$'ev'

$$U^{ev} : i, t, a \mapsto \{e^k_v, e^l_w, ...\}$$

with $v, w > t$ and event types k, l according to the schedule of the child contract, (2) a certain state variable x if $o =$'sv'

$$U^{sv} : i, t, x, a \mapsto \mathbb{R},$$

or (3) a particular contract attribute x of the child contract if $o =$'ca'

$$U^{ca} : i, x \mapsto y$$

with y a variable of value type of the respective attribute as per DD.

The parameters to the Child Contract Observer interface are as follows:

- i : the identifier of the child contract *observed*
- t : for $o \in \{ev, sv\}$ the time for which the respective quantity should be evaluated
- x : for $o \in \{sv, ca\}$ the quantity to be evaluated

a : for $o \in \{ev,sv\}$ a set of contract attributes to which the evaluated quantity should be conditioned

Note that the observer interface only defines the structure of an actual observer function but not the actual implementation. Thus, the interface allows for user-defined implementations of observer functions allowing e.g. for using arbitrary data structures.

Example 14 ('ev'-Observer)**.** *The future events, w.r.t. time t, of the first leg (i.e. child contract with* `Role`=***FirstLeg****) of a SWAPS contract with* $\mathit{CNTRL} = PFL$ *(i.e. pay first leg) can be evaluated as* $U^{ev}(\mathit{CTST}^{Contract}_{FirstLeg}, t \mid \{\mathit{CNTRL} = RPL\})$.

Example 15 ('sv'-Observer)**.** *The current state, w.r.t. time t, of state variable* $\boldsymbol{Nt}$ *of the first leg (i.e. child contract with* `Role`=***FirstLeg****) of a SWAPS contract with* CNTRL=*RFL (i.e. receive first leg) can be evaluated as* $U^{sv}(\mathit{CTST}^{Contract}_{FirstLeg}, t, \boldsymbol{Nt} \mid \{\mathit{CNTRL} = RPA\})$.

Example 16 ('ca'-Observer)**.** *The contract attribute* ***MOC*** *of the child contract Child (i.e. child contract with* `Role`=***Underlying****) of an OPTNS contract can be evaluated as* $U^{ca}(\mathit{CTST}^{Contract}_{Underlying}, \mathit{MOC})$.

For brevity we will omit the x and a function arguments wherever this does not lead to confusion.

7. Contract Types

7.1. PAM: Principal At Maturity.

PAM: Contract Schedule

Event	Schedule	Comments
AD	$\vec{t}^{AD} = (t_0, t_1, ..., t_n)$	With $t_i, i = 1, 2, ...$ a custom input
IED	$t^{IED} = \texttt{IED}$	
MD	$t^{MD} = \mathbf{Md}_{t_0}$	
PP	$\vec{t}^{PP} = \begin{cases} \varnothing & \text{if } \texttt{PPEF} = \text{'N'} \\ (\vec{u}, \vec{v}) & \text{else} \end{cases}$ where $\vec{u} = S(s, \texttt{OPCL}, T^{MD})$ $\vec{v} = O^{ev}(\texttt{CID}, \texttt{PP}, t)$	with $s = \begin{cases} \varnothing & \text{if } \texttt{OPANX} = \varnothing \wedge \texttt{OPCL} = \varnothing \\ \texttt{IED} + \texttt{OPCL} & \text{else if } \texttt{OPANX} = \varnothing \\ \texttt{OPANX} & \text{else} \end{cases}$
PY	$\vec{t}^{PY} = \begin{cases} \varnothing & \text{if } \texttt{PYTP} = \text{'O'} \\ \vec{t}^{PP} & \text{else} \end{cases}$	
FP	$\vec{t}^{FP} = \begin{cases} \varnothing & \text{if } \texttt{FER} = \varnothing \vee \texttt{FER} = 0 \\ S(s, \texttt{FECL}, T^{MD}) & \text{else} \end{cases}$	with $s = \begin{cases} \varnothing & \text{if } \texttt{FEANX} = \varnothing \wedge \texttt{FECL} = \varnothing \\ \texttt{IED} + \texttt{FECL} & \text{else if } \texttt{FEANX} = \varnothing \\ \texttt{FEANX} & \text{else} \end{cases}$
PRD	$t^{PRD} = \texttt{PRD}$	
TD	$t^{TD} = \texttt{TD}$	
IP	$\vec{t}^{IP} = \begin{cases} \varnothing & \text{if } \texttt{IPNR} = \varnothing \\ S(s, \texttt{IPCL}, T^{MD}) & \text{else} \end{cases}$	with $s = \begin{cases} \varnothing & \text{if } \texttt{IPANX} = \varnothing \wedge \texttt{IPCL} = \varnothing \\ \texttt{IPCED} & \text{else if } \texttt{IPCED} \neq \varnothing \\ \texttt{IED} + \texttt{IPCL} & \text{else if } \texttt{IPANX} = \varnothing \\ \texttt{IPANX} & \text{else} \end{cases}$
IPCI	$\vec{t}^{IPCI} = \begin{cases} \varnothing & \text{if } \texttt{IPCED} = \varnothing \\ S(s, \texttt{IPCL}, \texttt{IPCED}) & \text{else} \end{cases}$	with $s = \begin{cases} \varnothing & \text{if } \texttt{IPANX} = \varnothing \wedge \texttt{IPCL} = \varnothing \\ \texttt{IED} + \texttt{IPCL} & \text{else if } \texttt{IPANX} = \varnothing \\ \texttt{IPANX} & \text{else} \end{cases}$
RR	$\vec{t}^{RR} = \begin{cases} \varnothing & \text{if } \texttt{RRANX} = \varnothing \wedge \texttt{RRCL} = \varnothing \\ \vec{t} \setminus t^{RRY} & \text{else if} \texttt{RRNXT} \neq \varnothing \\ \vec{t} & \text{else} \end{cases}$ where $\vec{t} = S(s, \texttt{RRCL}, T^{MD})$	with $s = \begin{cases} \texttt{IED} + \texttt{RRCL} & \text{if } \texttt{RRANX} = \varnothing \\ \texttt{RRANX} & \text{else} \end{cases}$ $t^{RRY} = \inf t \in \vec{t} \mid t > \texttt{SD}$
RRF	$t^{RRF} = \begin{cases} \varnothing & \text{if } \texttt{RRANX} = \varnothing \wedge \texttt{RRCL} = \varnothing \\ \inf t \in \vec{t} \mid t > \texttt{SD} & \text{else} \end{cases}$ where $\vec{t} = S(s, \texttt{RRCL}, T^{MD})$	with $s = \begin{cases} \texttt{IED} + \texttt{RRCL} & \text{if } \texttt{RRANX} = \varnothing \\ \texttt{RRANX} & \text{else} \end{cases}$
SC	$\vec{t}^{SC} = \begin{cases} \varnothing & \text{if } \texttt{SCEF} = \text{'000'} \\ S(s, \texttt{SCCL}, T^{MD}) & \text{else} \end{cases}$	with $s = \begin{cases} \varnothing & \text{if } \texttt{SCANX} = \varnothing \wedge \texttt{SCCL} = \varnothing \\ \texttt{IED} + \texttt{SCCL} & \text{else if } \texttt{SCANX} = \varnothing \\ \texttt{SCANX} & \text{else} \end{cases}$
CE	$\vec{t}^{CE} = t(e^k) \mid \mathbf{Prf}_{t^-} \neq \mathbf{Prf}_{t^+}, \forall k$	

PAM: State Variables Initialization

State	Initialization per t_0	Comments
Md	$\mathbf{Md}_{t_0} = \texttt{MD}$	
Nt	$\mathbf{Nt}_{t_0} = \begin{cases} 0.0 & \text{if } \texttt{IED} > t_0 \\ R(\texttt{CNTRL}) \times \texttt{NT} & \text{else} \end{cases}$	
Ipnr	$\mathbf{Ipnr}_{t_0} = \begin{cases} 0.0 & \text{if } \texttt{IED} > t_0 \\ \texttt{IPNR} & \text{else} \end{cases}$	

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State	Initialization per t_0	Comments
Ipac	$\mathbf{Ipac}_{t_0} = \begin{cases} 0.0 & \text{if } \ \texttt{IPNR} = \varnothing \\ \texttt{IPAC} & \text{else if } \ \texttt{IPAC} \neq \varnothing \\ Y(t^-, t_0) \times \mathbf{Nt}_{t_0} \times \mathbf{Ipnr}_{t_0} & \text{else} \end{cases}$	with $t^- = \sup t \in \vec{t}^{IP} \mid t < t_0$
Feac	$\mathbf{Feac}_{t_0} = \begin{cases} 0.0 & \text{if } \ \texttt{FER} = \varnothing \\ \texttt{FEAC} & \text{else if } \ \texttt{FEAC} \neq \varnothing \\ Y(t^{FP-}, t_0) \times \mathbf{Nt}_{t_0} \times \texttt{FER} & \text{else if } \ \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t_0)}{Y(t^{FP-},t^{FP+})} \times \texttt{FER} & \text{else} \end{cases}$	with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
Nsc	$\mathbf{Nsc}_{t_0} = \begin{cases} \texttt{SCIXSD} & \text{if } \ \texttt{SCEF} = \text{'[x]N[x]'} \\ 1.0 & \text{else} \end{cases}$	
Isc	$\mathbf{Isc}_{t_0} = \begin{cases} \texttt{SCIXSD} & \text{if } \ \texttt{SCEF} = \text{'I[x][x]'} \\ 1.0 & \text{else} \end{cases}$	
Prf	$\mathbf{Prf}_{t_0} = \texttt{PRF}$	
Sd	$\mathbf{Sd}_{t_0} = t_0$	

PAM: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	0.0	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t-1}, t)\mathbf{Ipnr}_{t^-}\mathbf{Nt}_{t^-}$ $\mathbf{Sd}_{t^+} = t$
IED	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})(-1)(\texttt{NT} + \texttt{PDIED})$	$\mathbf{Nt}_{t^+} = R(\texttt{CNTRL})\texttt{NT}$ $\mathbf{Ipnr}_{t^+} = \begin{cases} 0.0 & \text{if } \ \texttt{IPNR} = \varnothing \\ \texttt{IPNR} & \text{else} \end{cases}$ $\mathbf{Ipac}_{t^+} = \begin{cases} \texttt{IPAC} & \text{if } \ \texttt{IPAC} \neq \varnothing \\ y\mathbf{Nt}_{t^+}\mathbf{Ipnr}_{t^+} & \text{if } \ \texttt{IPANX} \neq \varnothing \wedge \texttt{IPANX} < t \\ 0.0 & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $y = Y(\texttt{IPANX}, t)$
MD	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)(\mathbf{Nsc}_{t^-}\mathbf{Nt}_{t^-} + \mathbf{Isc}_{t^-}\mathbf{Ipac}_{t^-} + \mathbf{Feac}_{t^-})$	$\mathbf{Nt}_{t^+} = 0.0$ $\mathbf{Ipac}_{t^+} = 0.0$ $\mathbf{Feac}_{t^+} = 0.0$ $\mathbf{Sd}_{t^+} = t$
PP	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)f(O^{ev}(\texttt{CID}, \texttt{PP}, t))$	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Nt}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \ \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})} R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Nt}_{t^+} = \mathbf{Nt}_{t^-} - f(O^{ev}(\texttt{CID}, \texttt{PP}, t))$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
PY	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})\texttt{PYRT}$ if $\texttt{PYTP} = \text{'A'}$ $c\texttt{PYRT}$ if $\texttt{PYTP} = \text{'N'}$ $c \max(0, \mathbf{Ipnr}_{t^-} - O^{rf}(\texttt{RRMO}, t))$ if $\texttt{PYTP} = \text{'I'}$ with $c = X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}$	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Nt}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \ \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})} R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$

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Event	Payoff Function	State Transition Function
FP	$R(\texttt{CNTRL})c$ if $\texttt{FEB} = \text{'A'}$ $cY(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-} + \mathbf{Feac}_{t^-}$ if $\texttt{FEB} = \text{'N'}$ with $c = X^{\texttt{CURS}}_{\texttt{CUR}}(t)\texttt{FER}$	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Nt}_{t^-}$ $\mathbf{Feac}_{t^+} = 0.0$ $\mathbf{Sd}_{t^+} = t$
PRD	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})(-1)(\texttt{PPRD} + \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Nt}_{t^-})$	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Nt}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
TD	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})(\texttt{PTD} + \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Nt}_{t^-})$	$\mathbf{Nt}_{t^+} = 0.0$ $\mathbf{Ipac}_{t^+} = 0.0$ $\mathbf{Feac}_{t^+} = 0.0$ $\mathbf{Ipnr}_{t^+} = 0.0$ $\mathbf{Sd}_{t^+} = t$
IP	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)\mathbf{Isc}_{t^-}(\mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Nt}_{t^-})$	$\mathbf{Ipac}_{t^+} = 0.0$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
IPCI	0.0	$\mathbf{Nt}_{t^+} = \mathbf{Nt}_{t^-} + \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}\mathbf{Ipnr}_{t^-}$ $\mathbf{Ipac}_{t^+} = 0.0$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
RR	0.0	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Nt}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Ipnr}_{t^+} = \min(\max(\mathbf{Ipnr}_{t^-} + \Delta r, \texttt{RRLF}), \texttt{RRLC})$ $\mathbf{Sd}_{t^+} = t$ with $\Delta r = \min(\max(O^{rf}(\texttt{RRMO}, t)\texttt{RRMLT} + \texttt{RRSP} - \mathbf{Ipnr}_{t^-}, \texttt{RRPF}), \texttt{RRPC})$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
RRF	0.0	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Nt}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Ipnr}_{t^+} = \texttt{RRNXT}$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$

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Event	Payoff Function	State Transition Function
SC	0.0	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Nt}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Nsc}_{t^+} = \begin{cases} \mathbf{Nsc}_{t^-} & \text{if} \quad \texttt{SCEF} = [x]0[x] \\ \frac{O^{rf}(\texttt{SCMO}, t) - \texttt{SCIED}}{\texttt{SCIED}} & \text{else} \end{cases}$ $\mathbf{Isc}_{t^+} = \begin{cases} \mathbf{Isc}_{t^-} & \text{if} \quad \texttt{SCEF} = 0[x][x] \\ \frac{O^{rf}(\texttt{SCMO}, t) - \texttt{SCIED}}{\texttt{SCIED}} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
CE	0.0	STF_AD_PAM()

7.2. **LAM: Linear Amortizer.**

LAM: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
IED		Same as PAM
PR	$t^{PR} = S(s, \texttt{PRCL}, T^{MD}, F)$	with $s = \begin{cases} \varnothing & \text{if} \quad \texttt{PRANX} = \varnothing \land \texttt{PRCL} = \varnothing \\ \texttt{IED} + \texttt{PRCL} & \text{else if} \quad \texttt{PRANX} = \varnothing \\ \texttt{PRANX} & \text{else} \end{cases}$
MD	$t^{MD} = \mathbf{Md}_{t_0}$	
PP		Same as PAM
PY		Same as PAM
FP		Same as PAM
PRD		Same as PAM
TD		Same as PAM
IP		Same as PAM
IPCI		Same as PAM
IPCB	$\vec{t}^{IPCB} = \begin{cases} \varnothing & \text{if} \quad \texttt{IPCB} \neq \text{'NTL'} \\ S(s, \texttt{IPCBCL}, T^{MD}) & \text{else} \end{cases}$	with $s = \begin{cases} \varnothing & \text{if} \quad \texttt{IPCBANX} = \varnothing \land \texttt{IPCBCL} = \varnothing \\ \texttt{IED} + \texttt{IPCBCL} & \text{else if} \quad \texttt{IPCBANX} = \varnothing \\ \texttt{IPCBANX} & \text{else} \end{cases}$
RR		Same as PAM
RRF		Same as PAM
SC		Same as PAM
CE		Same as PAM

LAM: State Variables Initialization

State	Initialization per t_0	Comments
Md	$\mathbf{Md}_{t_0} = \begin{cases} \texttt{MD} & \text{if} \quad \texttt{MD} \neq \varnothing \\ t^- + ceil(\frac{\texttt{NT}}{\texttt{PRNXT}})\texttt{PRCL} \end{cases}$	where $t^- = \begin{cases} \texttt{PRANX} & \text{if} \quad \texttt{PRANX} \neq \varnothing \land \texttt{PRANX} \geq t_0 \\ \texttt{IED} + \texttt{PRCL} & \text{else if} \quad \texttt{IED} + \texttt{PRCL} \geq t_0 \\ \sup t \in \vec{t}^{PR} \mid t < t_0 & \text{else} \end{cases}$
Nt		Same as PAM
Ipnr		Same as PAM
Ipac		Same as PAM
Feac		Same as PAM
Nsc		Same as PAM
Isc		Same as PAM

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State	Initialization per t_0	Comments
Prf		Same as PAM
Sd		Same as PAM
Prnxt	$\mathbf{Prnxt}_{t_0} = \begin{cases} \texttt{PRNXT} & \text{if} \quad \texttt{PRNXT} \neq \varnothing \\ \texttt{NT}(ceil(\frac{Y(s,T^{MD})}{Y(s,s+\texttt{PRCL})}))^{-1} & \text{else} \end{cases}$	with $s = \begin{cases} \texttt{PRANX} & \text{if} \quad \texttt{PRANX} \neq \varnothing \wedge \texttt{PRANX} > t_0 \\ \texttt{IED}+\texttt{PRCL} & \text{else if} \quad \texttt{PRANX} = \varnothing \wedge \texttt{IED}+\texttt{PRCL} > t_0 \\ t^- & \text{else} \end{cases}$ and where $t^- = \sup t \in \vec{t}^{PR} \mid t < t_0$
Ipcb	$\mathbf{Ipcb}_{t_0} = \begin{cases} 0.0 & \text{if} \quad t_0 < \texttt{IED} \\ R(\texttt{CNTRL})\texttt{NT} & \text{else if} \quad \texttt{IPCB} = \text{'NT'} \\ R(\texttt{CNTRL})\texttt{IPCBA} & \text{else} \end{cases}$	

LAM: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	STF_AD_PAM()
IED	POF_IED_PAM()	$\mathbf{Nt}_{t^+} = R(\texttt{CNTRL})\texttt{NT}$ $\mathbf{Ipnr}_{t^+} = \texttt{IPNR}$ $\mathbf{Ipac}_{t^+} = \begin{cases} \texttt{IPAC} & \text{if} \quad \texttt{IPAC} \neq \varnothing \\ y\mathbf{Nt}_{t^+}\mathbf{Ipnr}_{t^+} & \text{if} \quad \texttt{IPANX} \neq \varnothing \wedge \texttt{IPANX} < t \\ 0.0 & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ $\mathbf{Ipcb}_{t^+} = \begin{cases} R(\texttt{CNTRL})\texttt{NT} & \text{if} \quad \texttt{IPCB} = \text{'NT'} \\ R(\texttt{CNTRL})\texttt{IPCBA} & \text{else} \end{cases}$
PR	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})\mathbf{Nsc}_{t^-}\mathbf{Prnxt}_{t^-}$	$\mathbf{Nt}_{t^+} = \mathbf{Nt}_{t^-} - R(\texttt{CNTRL})\mathbf{Prnxt}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Ipcb}_{t^+} = \begin{cases} \mathbf{Ipcb}_{t^-} & \text{if} \quad \texttt{IPCB} \neq \text{'NT'} \\ \mathbf{Nt}_{t^+} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
MD	POF_MD_PAM()	$\mathbf{Nt}_{t^+} = 0.0$ $\mathbf{Ipac}_{t^+} = 0.0$ $\mathbf{Feac}_{t^+} = 0.0$ $\mathbf{Ipcb}_{t^+} = 0.0$ $\mathbf{Sd}_{t^+} = t$
PP	POF_PP_PAM()	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Nt}_{t^+} = \mathbf{Nt}_{t^-} - f(O^{ev}(\texttt{CID}, \texttt{PP}, t))$ $\mathbf{Ipcb}_{t^+} = \begin{cases} \mathbf{Ipcb}_{t^-} & \text{if} \quad \texttt{IPCB} \neq \text{'NT'} \\ \mathbf{Nt}_{t^+} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$

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Event	Payoff Function	State Transition Function
PY	POF_PY_PAM()	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
FP	POF_FP_PAM()	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-}$ $\mathbf{Feac}_{t^+} = 0.0$ $\mathbf{Sd}_{t^+} = t$
PRD	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})(-1)(\texttt{PPRD} + \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-})$	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
TD	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})(\texttt{PTD} + \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-})$	STF_TD_PAM()
IP	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)\mathbf{Isc}_{t^-}(\mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-})$	STF_IP_PAM()
IPCI	POF_IPCI_PAM()	$\mathbf{Nt}_{t^+} = \mathbf{Nt}_{t^-} + \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-}$ $\mathbf{Ipac}_{t^+} = 0.0$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Ipcb}_{t^+} = \begin{cases} \mathbf{Ipcb}_{t^-} & \text{if } \texttt{IPCB} \neq \text{'NT'} \\ \mathbf{Nt}_{t^+} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
IPCB	0.0	$\mathbf{Ipcb}_{t^+} = \mathbf{Nt}_{t^-}$ $\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
RR	POF_RR_PAM()	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Ipnr}_{t^+} = \min(\max(\mathbf{Ipnr}_{t^-} + \Delta r, \texttt{RRLF}), \texttt{RRLC})$ $\mathbf{Sd}_{t^+} = t$ with $\Delta r = \min(\max(O^{rf}(\texttt{RRMO},t)\texttt{RRMLT} + \texttt{RRSP} - \mathbf{Ipnr}_{t^-}, \texttt{RRPF}), \texttt{RRPC})$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$

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Event	Payoff Function	State Transition Function
RRF	POF_RRF_PAM()	$\mathbf{Ipac}_{t+} = \mathbf{Ipac}_{t-} + Y(\mathbf{Sd}_{t-}, t)\mathbf{Ipnr}_{t-}\mathbf{Ipcb}_{t-}$ $\mathbf{Feac}_{t+} = \begin{cases} \mathbf{Feac}_{t-} + Y(\mathbf{Sd}_{t-}, t)\mathbf{Nt}_{t-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})} R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Ipnr}_{t+} = \texttt{RRNXT}$ $\mathbf{Sd}_{t+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
SC	POF_SC_PAM()	$\mathbf{Ipac}_{t+} = \mathbf{Ipac}_{t-} + Y(\mathbf{Sd}_{t-}, t)\mathbf{Ipnr}_{t-}\mathbf{Ipcb}_{t-}$ $\mathbf{Feac}_{t+} = \begin{cases} \mathbf{Feac}_{t-} + Y(\mathbf{Sd}_{t-}, t)\mathbf{Nt}_{t-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})} R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Nsc}_{t+} = \begin{cases} \mathbf{Nsc}_{t-} & \text{if } \texttt{SCEF} = [x]0[x] \\ \frac{O^{rf}(\texttt{SCMO},t) - \texttt{SCIED}}{\texttt{SCIED}} & \text{else} \end{cases}$ $\mathbf{Isc}_{t+} = \begin{cases} \mathbf{Isc}_{t-} & \text{if } \texttt{SCEF} = 0[x][x] \\ \frac{O^{rf}(\texttt{SCMO},t) - \texttt{SCIED}}{\texttt{SCIED}} & \text{else} \end{cases}$ $\mathbf{Sd}_{t+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
CE	POF_CE_PAM()	STF_AD_PAM()

7.3. **LAX: Exotic Linear Amortizer.**

LAX: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
IED		Same as PAM
PR	$\vec{t}^{PR} = \begin{cases} \{t_1, t_2, ..., t_i, ...\} & \text{if } \texttt{ARPRCL} = \varnothing \\ s_1 \cup s_2 \cup ... \cup s_i \cup ... & \text{else} \end{cases}$ with $s_i = S(\texttt{ARPRANX}_i, \vec{C}_i, \texttt{ARPRANX}_{i+1}),\ i \in \{1, 2, ..., \mid \texttt{ARINCDEC} \mid \} \mid \texttt{ARINCDEC}_i = \text{'DEC'}$	with $\vec{C} = \begin{cases} \texttt{ARPRCL} & \text{if } \mid \texttt{ARPRCL} \mid = \mid \texttt{ARPRANX} \mid \\ \{c_1, c_2, ..., c_n\} & \text{else} \end{cases}$ where $n = \mid \texttt{ARPRANX} \mid, c_k = \texttt{ARPRCL}_1 \forall k$
MD		Same as PAM
PI	$\vec{t}^{PI} = \begin{cases} \{t_1, t_2, ..., t_i, ...\} & \text{if } \texttt{ARPRCL} = \varnothing \\ s_1 \cup s_2 \cup ... \cup s_i \cup ... & \text{else} \end{cases}$ with $s_i = S(\texttt{ARPRANX}_i, \vec{C}_i, \texttt{ARPRANX}_{i+1}),\ i \in \{1, 2, ..., \mid \texttt{ARINCDEC} \mid \} \mid \texttt{ARINCDEC}_i = \text{'INC'}$	with $\vec{C} = \begin{cases} \texttt{ARPRCL} & \text{if } \mid \texttt{ARPRCL} \mid = \mid \texttt{ARPRANX} \mid \\ \{c_1, c_2, ..., c_n\} & \text{else} \end{cases}$ where $n = \mid \texttt{ARPRANX} \mid, c_k = \texttt{ARPRCL}_1 \forall k$
PRF	$\vec{t}^{PRF} = \texttt{ARPRANX}$	
PP		Same as PAM
PY		Same as PAM
FP		Same as PAM
PRD		Same as PAM
TD		Same as PAM
IP	$\vec{t}^{IP} = \vec{S}(\texttt{ARIPANX}, \texttt{ARIPCL}, \mathbf{Md}_{t_0})$	
IPCI		Same as PAM
IPCB		Same as LAM
RR	$\vec{t}^{RR} = \begin{cases} \{t_1, t_2, ..., t_i, ...\} & \text{if } \texttt{ARRRCL} = \varnothing \\ s_1 \cup s_2 \cup ... \cup s_i \cup ... & \text{else} \end{cases}$ with $s_i = S(\texttt{ARRRANX}_i, \vec{C}_i, \texttt{ARRRANX}_{i+1}),\ i \in \{1, 2, ..., \mid \texttt{ARFIXVAR} \mid \} \mid \texttt{ARFIXVAR}_i = \text{'V'}$	with $\vec{C} = \begin{cases} \texttt{ARRRCL} & \text{if } \mid \texttt{ARRRCL} \mid = \mid \texttt{ARRRANX} \mid \\ \{c_1, c_2, ..., c_n\} & \text{else} \end{cases}$ where $n = \mid \texttt{ARRRANX} \mid, c_k = \texttt{ARRRCL}_1 \forall k$

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Event	**Schedule**	**Comments**
RRF	$\vec{t}^{RRF} = \begin{cases} \{t_1, t_2, ..., t_i, ...\} & \text{if} \quad \texttt{ARRRCL} = \varnothing \\ s_1 \cup s_2 \cup ... \cup s_i \cup ... & \text{else} \end{cases}$ with $s_i = S(\texttt{ARRRANX}_i, \vec{C}_i, \texttt{ARRRANX}_{i+1}),\ i \in \{1, 2, ..., \mid \texttt{ARFIXVAR} \mid\} \mid \texttt{ARFIXVAR}_i = \text{'F'}$	with $\vec{C} = \begin{cases} \texttt{ARRRCL} & \text{if} \quad \mid \texttt{ARRRCL} \mid = \mid \texttt{ARRRANX} \mid \\ \{c_1, c_2, ..., c_n\} & \text{else} \end{cases}$ where $n = \mid \texttt{ARRRANX} \mid, c_k = \texttt{ARRRCL}_1 \forall k$
SC		Same as PAM
CE		Same as PAM

LAX: State Variables Initialization

State	**Initialization per t_0**	**Comments**
Md	$\mathbf{Md}_{t_0} = \begin{cases} \texttt{MD} & \text{if} \quad \texttt{MD} \neq \varnothing \\ \inf t > t_0 \mid N(t) = 0 & \text{else} \end{cases}$ with $N(t) = \texttt{NT} + \sum_{i=1}^{n(t)} (-1)^k \texttt{ARPRNXT}_i \mid s_i \mid$	where $n(t) = \begin{cases} \sup k \in \mathbb{N} \mid \texttt{ARPRANX}_k < t & \text{if} \quad t < max(\texttt{ARPRANX}) \\ \mid \texttt{ARPRANX} \mid & \text{else} \end{cases}$ $k = \begin{cases} 0 & \text{if} \quad \texttt{ARINCDEC}_i = \text{'INC'} \\ 1 & \text{else} \end{cases}$ $s_i = \begin{cases} \{\texttt{ARPRANX}_i\} & \text{if} \quad \texttt{ARPRCL} = \varnothing \\ S(\texttt{ARPRANX}_i, \vec{C}_i, T_i) & \text{else} \end{cases}$ $T_i = \begin{cases} \texttt{ARPRANX}_{i+1} & \text{if} \quad i < \mid \texttt{ARPRANX} \mid \\ t & \text{else} \end{cases}$ $\vec{C} = \begin{cases} \texttt{ARRRCL} & \text{if} \quad \mid \texttt{ARRRCL} \mid = \mid \texttt{ARRRANX} \mid \\ \{c_1, c_2, ..., c_n\} & \text{else} \end{cases}$
Nt		Same as PAM
Ipnr		Same as PAM
Ipac		Same as PAM
Feac		Same as PAM
Nsc		Same as PAM
Isc		Same as PAM
Prf		Same as PAM
Sd		Same as PAM
Prnxt	$\mathbf{Prnxt}_{t_0} = \begin{cases} 0.0 & \text{if} \quad t_0 \geq \texttt{ARPRANX}_1 \\ \texttt{ARPRNXT}_i & \text{else} \end{cases}$	where $i = \sup k \in \mathbb{N} \mid \texttt{ARPRANX}_k < t_0$
Ipcb		Same as LAM

LAX: State Transition Functions and Payoff Functions

Event	**Payoff Function**	**State Transition Function**
AD	POF_AD_PAM()	STF_AD_PAM()
IED	POF_IED_PAM()	STF_IED_LAM()
PR	POF_PR_LAM()	STF_PR_LAM()
MD	POF_MD_PAM()	STF_MD_LAM()

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Event	Payoff Function	State Transition Function
PI	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})(-1)\mathbf{Nsc}_{t^-}\mathbf{Prnxt}_{t^-}$	$\mathbf{Nt}_{t^+} = \mathbf{Nt}_{t^-} + R(\texttt{CNTRL})\mathbf{Prnxt}_{t^-}$ $\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Ipcb}_{t^+} = \begin{cases} \mathbf{Ipcb}_{t^-} & \text{if } \texttt{IPCB} \neq \text{'NT'} \\ \mathbf{Nt}_{t^+} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
PRF	0.0	$\mathbf{Prnxt}_{t^+} = \texttt{ARPRNXT}_i$ $\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Ipcb}_{t^+} = \begin{cases} \mathbf{Ipcb}_{t^-} & \text{if } \texttt{IPCB} \neq \text{'NT'} \\ \mathbf{Nt}_{t^+} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $i = \sup k \in \mathbb{N} \mid \texttt{ARPRANX}_k = t$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
PP	POF_PP_PAM()	STF_PP_LAM()
PY	POF_PY_PAM()	STF_PY_LAM()
FP	POF_FP_PAM()	STF_FP_LAM()
PRD	POF_PRD_LAM()	STF_PRD_LAM()
TD	POF_TD_LAM()	STF_TD_PAM()
IP	POF_IP_LAM()	STF_IP_PAM()
IPCI	POF_IPCI_PAM()	STF_IPCI_LAM()
IPCB	POF_IPCB_LAM()	STF_IPCB_LAM()
RR	POF_RR_PAM()	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Ipnr}_{t^+} = \min(\max(\mathbf{Ipnr}_{t^-} + \Delta r, \texttt{RRLF}), \texttt{RRLC})$ $\mathbf{Sd}_{t^+} = t$ with $\Delta r = \min(\max(O^{rf}(\texttt{RRMO}, t)\texttt{RRMLT} + \texttt{ARRATE}_i - \mathbf{Ipnr}_{t^-}, \texttt{RRPF}), \texttt{RRPC})$ $i = \sup k \in \mathbb{N} \mid \texttt{ARPRANX}_k = t$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
RRF	POF_RRF_PAM()	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Ipnr}_{t^+} = \texttt{ARRATE}_i$ $\mathbf{Sd}_{t^+} = t$ with $i = \sup k \in \mathbb{N} \mid \texttt{ARRRANX}_k = t$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
SC	POF_SC_PAM()	STF_SC_LAM()
CE	POF_CE_PAM()	STF_AD_PAM()

7.4. **NAM: Negative Amortizer.**

NAM: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
IED		Same as PAM
PR		Same as LAM
MD		Same as PAM
PP		Same as PAM
PY		Same as PAM
FP		Same as PAM
PRD		Same as PAM
TD		Same as PAM
IP	$\vec{t}^{IP} = (\vec{u}, \vec{v})$ where $\vec{u} = \begin{cases} \varnothing & \text{if} \quad \texttt{IPANX} = \varnothing \wedge \texttt{IPCL} = \varnothing \\ \varnothing & \text{if} \quad \texttt{IPCED} \neq \varnothing \wedge \texttt{IPCED} \geq \texttt{T} \\ S(r, \texttt{IPCL}, T) & \text{else} \end{cases}$ $\vec{v} = S(s, \texttt{PRCL}, T^{MD})$	with $r = \begin{cases} \texttt{IPCED} & \text{if} \quad \texttt{IPCED} \neq \varnothing \\ \texttt{IPANX} & \text{else if} \quad \texttt{IPANX} \neq \varnothing \\ \texttt{IED} + \texttt{IPCL} & \text{else if} \quad \texttt{IPCL} \neq \varnothing \\ \varnothing & \text{else} \end{cases}$ $T = s - \texttt{PRCL}$ $s = \begin{cases} \texttt{IED} + \texttt{PRCL} & \text{if} \quad \texttt{PRANX} = \varnothing \\ \texttt{PRANX} & \text{else} \end{cases}$
IPCI		Same as PAM
IPCB		Same as LAM
RR		Same as PAM
RRF		Same as PAM
SC		Same as PAM
CE		Same as PAM

NAM: State Variables Initialization

State	Initialization per t_0	Comments
Md	$\mathbf{Md}_{t_0} = \begin{cases} \texttt{MD} & \text{if} \quad \texttt{MD} \neq \varnothing \\ t^- + n\texttt{PRCL} & \text{else} \end{cases}$ with $n = ceil(\frac{\texttt{NT}}{\texttt{PRNXT} - \texttt{NT}Y(t^-, t^- + \texttt{PRCL})\texttt{IPNR}})$	where $t^- = \begin{cases} \texttt{PRANX} & \text{if} \quad \texttt{PRANX} \neq \varnothing \wedge \texttt{PRANX} \geq t_0 \\ \texttt{IED} + \texttt{PRCL} & \text{else if} \quad \texttt{IED} + \texttt{PRCL} \geq t_0 \\ \sup t \in t^{PR} \mid t < t_0 & \text{else} \end{cases}$
Nt		Same as PAM
Ipnr		Same as PAM
Ipac		Same as PAM
Feac		Same as PAM
Nsc		Same as PAM
Isc		Same as PAM
Prf		Same as PAM
Sd		Same as PAM
Prnxt	$\mathbf{Prnxt}_{t_0} = R(\texttt{CNTRL})\texttt{PRNXT}$	
Ipcb		Same as LAM

NAM: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	STF_AD_PAM()
IED	POF_IED_PAM()	STF_IED_LAM()

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Event	Payoff Function	State Transition Function
PR	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)\mathbf{Nsc}_{t^-}(\mathbf{Prnxt}_{t^-} - \mathbf{Ipac}_{t^-} - Y(\mathbf{Sd}_{t^-},t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-})$	$\mathbf{Nt}_{t^+} = \mathbf{Nt}_{t^-} - (\mathbf{Prnxt}_{t^-} - \mathbf{Ipac}_{t^+})$ $\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Ipcb}_{t^+} = \begin{cases} \mathbf{Ipcb}_{t^-} & \text{if } \texttt{IPCB} \neq \text{'NT'} \\ \mathbf{Nt}_{t^+} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
MD	POF_MD_PAM()	STF_MD_LAM()
PP	POF_PP_PAM()	STF_PP_LAM()
PY	POF_PY_PAM()	STF_PY_LAM()
FP	POF_FP_PAM()	STF_FP_LAM()
PRD	POF_PRD_LAM()	STF_PRD_LAM()
TD	POF_TD_LAM()	STF_TD_PAM()
IP	POF_IP_LAM()	STF_IP_PAM()
IPCI	POF_IPCI_PAM()	STF_IPCI_LAM()
IPCB	POF_IPCB_LAM()	STF_IPCB_LAM()
RR	POF_RR_PAM()	STF_RR_LAM()
RRF	POF_RRF_PAM()	STF_RRF_LAM()
SC	POF_SC_PAM()	STF_SC_LAM()
CE	POF_CE_PAM()	STF_AD_PAM()

7.5. ANN: Annuity.

ANN: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
IED		Same as PAM
PR		Same as LAM
MD		Same as PAM
PP		Same as PAM
PY		Same as PAM
FP		Same as PAM
PRD		Same as PAM
TD		Same as PAM
IP		Same as NAM
IPCI		Same as PAM
IPCB		Same as LAM
RR		Same as PAM
RRF		Same as PAM
SC		Same as PAM
CE		Same as PAM

ANN: State Variables Initialization

State	Initialization per t_0	Comments
Md		Same as NAM
Nt		Same as PAM
Ipnr		Same as PAM
Ipac		Same as PAM
Feac		Same as PAM
Nsc		Same as PAM
Isc		Same as PAM
Prf		Same as PAM

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State	Initialization per t_0	Comments
Sd		Same as PAM
Prnxt	$\mathbf{Prnxt}_{t_0} = \begin{cases} R(\texttt{CNTRL})\texttt{PRNXT} & \text{if} \quad \texttt{PRNXT} \neq \varnothing \\ (\texttt{NT} + \mathbf{Ipac}_{t_0})\frac{todo}{todo} & \text{else} \end{cases}$	where $n = \lvert\vec{t}\rvert$ with $\lvert a\rvert$ indicating the cardinality of set a
Ipcb		Same as LAM

ANN: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	STF_AD_PAM()
IED	POF_IED_PAM()	STF_IED_LAM()
PR	POF_PR_NAM()	STF_PR_NAM()
MD	POF_MD_PAM()	STF_MD_LAM()
PP	POF_PP_PAM()	STF_PP_LAM()
PY	POF_PY_PAM()	STF_PY_LAM()
FP	POF_FP_PAM()	STF_FP_LAM()
PRD	POF_PRD_LAM()	STF_PRD_LAM()
TD	POF_TD_LAM()	STF_TD_PAM()
IP	POF_IP_LAM()	STF_IP_PAM()
IPCI	POF_IPCI_PAM()	STF_IPCI_LAM()
IPCB	POF_IPCB_LAM()	STF_IPCB_LAM()
RR	POF_RR_PAM()	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Ipnr}_{t^+} = \min(\max(\mathbf{Ipnr}_{t^-} + \Delta r, \texttt{RRLF}), \texttt{RRLC})$ $\mathbf{Prnxt}_{t^+} = A(t, \mathbf{Md}_{t^+}, \mathbf{Nt}_{t^+}, \mathbf{Ipac}_{t^+}, \mathbf{Ipnr}_{t^+})$ $\mathbf{Sd}_{t^+} = t$ with $\Delta r = \min(\max(O^{rf}(\texttt{RRMO}, t)\texttt{RRMLT} + \texttt{RRSP} - \mathbf{Ipnr}_{t^-}, \texttt{RRPF}), \texttt{RRPC})$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
RRF	POF_RRF_PAM()	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Ipcb}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Ipnr}_{t^+} = \texttt{RRNXT}$ $\mathbf{Prnxt}_{t^+} = A(t, \mathbf{Md}_{t^+}, \mathbf{Nt}_{t^+}, \mathbf{Ipac}_{t^+}, \mathbf{Ipnr}_{t^+})$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
SC	POF_SC_PAM()	STF_SC_LAM()
CE	POF_CE_PAM()	STF_AD_PAM()

7.6. **CLM: Call Money.**

CLM: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
IED		Same as PAM
PR		Same as PAM
FP		Same as PAM
IP	$t^{IP} = \mathbf{Md}_{t_0}$	

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Event	Schedule	Comments
IPCI	$\vec{t}^{IPCI} = \begin{cases} \varnothing & \text{if} \quad \texttt{IPNR} = \varnothing \\ S(s, \texttt{IPCL}, \mathbf{Md}_{t_0}) & \text{else} \end{cases}$	where $s = \begin{cases} \texttt{IPANX} & \text{if} \quad \texttt{IPANX} \neq \varnothing \\ \texttt{IED} + \texttt{IPCL} & \text{else} \end{cases}$
RR		Same as PAM
RRF		Same as PAM
CE		Same as PAM

CLM: State Variables Initialization

State	Initialization per t_0	Comments
Md	$\mathbf{Md}_{t_0} = \begin{cases} \texttt{MD} & \text{if} \quad \texttt{MD} \neq \varnothing \\ s & \text{else if} \quad O^{ev}(\texttt{CID}, t_0) \neq \{\} \\ t^{max} & \text{else} \end{cases}$	where $s = \sup t \in \tau(O^{ev}(\texttt{CID}, t_0))$
Nt		Same as PAM
Ipnr		Same as PAM
Ipac		Same as PAM
Feac		Same as PAM
Prf		Same as PAM
Sd		Same as PAM

CLM: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	STF_AD_PAM()
IED	$X^{\texttt{CURS}}_{\texttt{CUR}}(t) R(\texttt{CNTRL})(-1)\texttt{NT}$	STF_IED_PAM()
PR	POF_PR_PAM()	STF_PR_PAM()
FP	POF_FP_PAM()	STF_FP_PAM()
IP	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)(\mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Nt}_{t^-})$	$\mathbf{Ipac}_{t^+} = 0.0$ $\mathbf{Sd}_{t^+} = t$
IPCI	POF_IPCI_PAM()	STF_IPCI_PAM()
RR	POF_RR_PAM()	STF_RR_PAM()
RRF	POF_RRF_PAM()	STF_RRF_PAM()
CE	POF_CE_PAM()	STF_AD_PAM()

7.7. UMP: Undefined Maturity Profile.

UMP: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
IED		Same as PAM
PR	$\vec{t}^{PR} = O^{ev}(\texttt{CID}, i, t_0)$	with $i \in \{\texttt{PR}, \texttt{PI}\}$
FP		Same as PAM
IPCI	$\vec{t}^{IPCI} = \begin{cases} \varnothing & \text{if} \quad \texttt{IPNR} = \varnothing \\ S(s, \texttt{IPCL}, \mathbf{Md}_{t_0}) & \text{else} \end{cases}$	where $s = \begin{cases} \texttt{IPANX} & \text{if} \quad \texttt{IPANX} \neq \varnothing \\ \texttt{IED} + \texttt{IPCL} & \text{else} \end{cases}$
RR		Same as PAM
RRF		Same as PAM
CE		Same as PAM

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State	Initialization per t_0	Comments

UMP: State Variables Initialization

State	Initialization per t_0	Comments
Md	$\mathbf{Md}_{t_0} = \begin{cases} s & \text{if} \quad O^{ev}(\texttt{CID}, t_0) \neq \{\} \\ t^{max} & \text{else} \end{cases}$	where $s = \sup t, t \in \tau(O^{ev}(\texttt{CID}, i, t_0))$ $i \in \{\texttt{PR}, \texttt{PI}\}$
Nt		Same as PAM
Ipnr		Same as PAM
Ipac		Same as PAM
Feac		Same as PAM
Prf		Same as PAM
Sd		Same as PAM

UMP: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	STF_AD_PAM()
IED	POF_IED_CLM()	STF_IED_PAM()
PR	$f(e_t^{PR})$	$\mathbf{Ipac}_{t^+} = \mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Nt}_{t^-}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^-,t)}{Y(t^-,t^+)}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Nt}_{t^+} = \mathbf{Nt}_{t^-} - f(e_t^{PR})$ $\mathbf{Sd}_{t^+} = t$
FP	POF_FP_PAM()	STF_FP_PAM()
IPCI	POF_IPCI_PAM()	STF_IPCI_PAM()
RR	POF_RR_PAM()	STF_RR_PAM()
RRF	POF_RRF_PAM()	STF_RRF_PAM()
CE	POF_CE_PAM()	STF_AD_PAM()

7.8. **CSH: Cash.**

CSH: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM

CSH: State Variables Initialization

State	Initialization per t_0	Comments
Nt	$\mathbf{Nt}_{t_0} = R(\texttt{CNTRL})\texttt{NT}$	
Sd		Same as PAM

CSH: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Sd}_{t^+} = t$

7.9. **STK: Stock.**

STK: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as PAM
TD		Same as PAM
$\text{DV}^{(fix)}$	$t^{DV^{(fix)}} = \begin{cases} \varnothing & \text{if} \quad \texttt{DVNP} = \varnothing \\ \texttt{DVANX} & \text{else} \end{cases}$	

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Event	Schedule	Comments
DV	$\vec{t}^{DV} = \begin{cases} \varnothing & \text{if } \texttt{DVANX} = \varnothing \wedge \texttt{DVCL} = \varnothing \\ S(s, \texttt{DVCL}, t^{max}) & \text{else} \end{cases}$	where $s = \begin{cases} \texttt{DVANX} & \text{if } \texttt{DVNP} = \varnothing \\ \texttt{DVANX} + \texttt{DVCL} & \text{else} \end{cases}$
CE		Same as PAM

STK: State Variables Initialization

State	Initialization per t_0	Comments
Prf		Same as PAM
Sd		Same as PAM

STK: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Sd}_{t^+} = t$
PRD	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})(-1)\texttt{PPRD}$	$\mathbf{Sd}_{t^+} = t$
TD	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})\texttt{PTD}$	$\mathbf{Sd}_{t^+} = t$
$\text{DV}^{(fix)}$	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})\texttt{DVNP}$	$\mathbf{Sd}_{t^+} = t$
DV	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})O^{rf}(concat(\texttt{CID}, \text{"-DV"}), t)$ with $concat(x, y, z)$ indicates the string concatenation function	$\mathbf{Sd}_{t^+} = t$
CE	POF_CE_PAM()	STF_AD_STK()

7.10. **COM: Commodity.**

COM: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as STK
TD		Same as STK

COM: State Variables Initialization

State	Initialization per t_0	Comments
Sd		Same as STK

COM: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	STF_AD_STK()
PRD	POF_PRD_STK()	STF_PRD_STK()
TD	POF_PRD_STK()	STF_PRD_STK()

7.11. **FXOUT: Foreign Exchange Outright.**

FXOUT: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as PAM
TD		Same as PAM
STD	$t^{STD} = \begin{cases} \varnothing & \text{if } \texttt{DS} = \text{'D'} \\ \mathbf{Md}_{t_0} & \text{else} \end{cases}$	
$\text{STD}^{(1)}$	$t^{STD} = \begin{cases} \varnothing & \text{if } \texttt{DS} = \text{'S'} \\ \mathbf{Md}_{t_0} & \text{else} \end{cases}$	
$\text{STD}^{(2)}$	$t^{STD} = \begin{cases} \varnothing & \text{if } \texttt{DS} = \text{'S'} \\ \mathbf{Md}_{t_0} & \text{else} \end{cases}$	

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Event	**Schedule**	**Comments**
CE		Same as PAM

FXOUT: State Variables Initialization

State	**Initialization per t_0**	**Comments**
Md	$\mathbf{Md}_{t_0} = \begin{cases} \texttt{MD} & \text{if} \quad \texttt{STD} = \varnothing \\ \texttt{STD} & \text{else} \end{cases}$	
Prf		Same as PAM
Sd		Same as PAM

FXOUT: State Transition Functions and Payoff Functions

Event	**Payoff Function**	**State Transition Function**
AD	POF_AD_PAM()	STF_AD_STK()
PRD	POF_PRD_STK()	STF_PRD_STK()
TD	POF_TD_STK()	STF_TD_STK()
STD	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})(\texttt{NT} - O^{rf}(i, \mathbf{Md}_t)\texttt{NT2})$ where $i = concat(\texttt{CUR2}, "/", \texttt{CUR})$ and $concat(x, y, z)$ indicates the string concatenation function	$\mathbf{Sd}_{t^+} = t$
STD$^{(1)}$	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})\texttt{NT}$	$\mathbf{Sd}_{t^+} = t$
STD$^{(2)}$	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})(-1)\texttt{NT2}$	$\mathbf{Sd}_{t^+} = t$
CE	POF_CE_PAM()	STF_CD_STK()

7.12. **SWPPV: Plain Vanilla Interest Rate Swap.**

SWPPV: Contract Schedule

Event	**Schedule**	**Comments**
AD		Same as PAM
PRD		Same as PAM
TD		Same as PAM
IED		Same as PAM
PR		Same as PAM
IP	$\vec{t}^{IP} = \begin{cases} \varnothing & \text{if} \quad \texttt{DS} = \text{'D'} \\ \mathbf{Md}_{t_0} & \text{else if} \texttt{IPCL} = \varnothing \\ S(s, \texttt{IPCL}, \mathbf{Md}_{t_0}) & \text{else} \end{cases}$	where $s = \begin{cases} \texttt{IPANX} & \text{if} \quad \texttt{IPANX} \neq \varnothing \\ \texttt{IED} + \texttt{IPCL} & \text{else} \end{cases}$
IP$^{(fix)}$	$\vec{t}^{IP(fix)} = \begin{cases} \varnothing & \text{if} \quad \texttt{DS} = \text{'S'} \\ \mathbf{Md}_{t_0} & \text{else if} \texttt{IPCL} = \varnothing \\ S(s, \texttt{IPCL}, \mathbf{Md}_{t_0}) & \text{else} \end{cases}$	
IP$^{(var)}$	$\vec{t}^{IP(var)} = \begin{cases} \varnothing & \text{if} \quad \texttt{DS} = \text{'S'} \\ \mathbf{Md}_{t_0} & \text{else if} \texttt{IPCL} = \varnothing \\ S(s, \texttt{IPCL}, \mathbf{Md}_{t_0}) & \text{else} \end{cases}$	
RR	$\vec{t}^{RR} = S(s, \texttt{RRCL}, \mathbf{Md}_{t_0})$	where $s = \begin{cases} \texttt{RRANX} & \text{if} \quad \texttt{RRANX} \neq \varnothing \\ \texttt{IED} + \texttt{RRCL} & \text{else} \end{cases}$
CE		Same as PAM

SWPPV: State Variables Initialization

State	**Initialization per t_0**	**Comments**
Md		Same as PAM
Nt		Same as PAM

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State	Initialization per t_0	Comments
Ipnr	$\mathbf{Ipnr}_{t_0} = \begin{cases} 0.0 & \text{if} \quad \texttt{IED} > t_0 \\ \texttt{IPNR2} & \text{else} \end{cases}$	
Ipac	$\mathbf{Ipac}_{t_0} = \begin{cases} \texttt{IPAC} & \text{if} \quad \texttt{IPAC} \neq \varnothing \\ Y(t^-, t_0)\mathbf{Nt}_{t_0}(\texttt{IPNR} - \mathbf{Ipnr}_{t_0}) & \text{else} \end{cases}$	with $t^- = \sup t, t \in t^{IP}, t < t_0$
Ipac1	$\mathbf{Ipac1}_{t_0} = Y(t^-, t_0)\mathbf{Nt}_{t_0}\texttt{IPNR}$	with $t^- = \sup t, t \in t^{IP}, t < t_0$
Ipac2	$\mathbf{Ipac2}_{t_0} = Y(t^-, t_0)\mathbf{Nt}_{t_0}\mathbf{Ipnr}_{t_0}$	with $t^- = \sup t, t \in t^{IP}, t < t_0$
Prf		Same as PAM
Sd		Same as PAM

SWPPV: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Ipac}_{t^+} = Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t_0}(\texttt{IPNR} - \mathbf{Ipnr}_{t_0})$ $\mathbf{Ipac1}_{t^+} = Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t_0}\texttt{IPNR}$ $\mathbf{Ipac2}_{t^+} = Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t_0}\mathbf{Ipnr}_{t_0}$ $\mathbf{Sd}_{t^+} = t$
IED	0.0	$\mathbf{Nt}_{t^+} = R(\texttt{CNTRL})\texttt{NT}$ $\mathbf{Ipac}_{t^+} = 0.0$ $\mathbf{Ipac1}_{t^+} = 0.0$ $\mathbf{Ipac2}_{t^+} = 0.0$ $\mathbf{Ipnr}_{t^+} = \texttt{IPNR2}$ $\mathbf{Sd}_{t^+} = t$
PR	0.0	$\mathbf{Nt}_{t^+} = 0.0$ $\mathbf{Ipnr}_{t^+} = 0.0$ $\mathbf{Sd}_{t^+} = t$
PRD	POF_PRD_STK()	$\mathbf{Ipac}_{t^+} = Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t_0}(\texttt{IPNR} - \mathbf{Ipnr}_{t_0})$ $\mathbf{Ipac1}_{t^+} = Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t_0}\texttt{IPNR}$ $\mathbf{Ipac2}_{t^+} = Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t_0}\mathbf{Ipnr}_{t_0}$ $\mathbf{Sd}_{t^+} = t$
TD	POF_TD_STK()	$\mathbf{Nt}_{t^+} = 0.0$ $\mathbf{Ipac}_{t^+} = 0.0$ $\mathbf{Ipac1}_{t^+} = 0.0$ $\mathbf{Ipac2}_{t^+} = 0.0$ $\mathbf{Ipnr}_{t^+} = 0.0$ $\mathbf{Sd}_{t^+} = t$
IP	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})(\mathbf{Ipac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)(\texttt{IPNR} - \mathbf{Ipnr}_{t^-})\mathbf{Nt}_{t^-})$	$\mathbf{Ipac}_{t^+} = 0.0$ $\mathbf{Sd}_{t^+} = t$
IP$^{(fix)}$	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})(\mathbf{Ipac1}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\texttt{IPNR}\mathbf{Nt}_{t^-})$	$\mathbf{Ipac1}_{t^+} = 0.0$ $\mathbf{Sd}_{t^+} = t$
IP$^{(var)}$	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})(\mathbf{Ipac2}_{t^-} - Y(\mathbf{Sd}_{t^-}, t)\mathbf{Ipnr}_{t^-}\mathbf{Nt}_{t^-})$	$\mathbf{Ipac2}_{t^+} = 0.0$ $\mathbf{Sd}_{t^+} = t$
RR	POF_RR_PAM()	$\mathbf{Ipac}_{t^+} = Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t_0}(\texttt{IPNR} - \mathbf{Ipnr}_{t_0})$ $\mathbf{Ipac1}_{t^+} = Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t_0}\texttt{IPNR}$ $\mathbf{Ipac2}_{t^+} = Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t_0}\mathbf{Ipnr}_{t_0}$ $\mathbf{Ipnr}_{t^+} = \texttt{RRMLT}O^{rf}(\texttt{RRMO}, t) + \texttt{RRSP}$ $\mathbf{Sd}_{t^+} = t$
CE	POF_CE_PAM()	STF_AD_SWPPV()

7.13. **SWAPS: Swap.**

SWAPS: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as PAM
TD		Same as PAM
k	$\{e_t^k\} = \begin{cases} \{e_t^{k,1}\} \cup \{e_s^{l,2}\} & \text{if } \texttt{DS} = \text{'D'} \\ \{e_t^{k,1}\} + \{e_s^{l,2}\} & \text{else} \end{cases}$ with $\{e_t^{k,1}\} + \{e_s^{l,2}\} = U \cup V$ and $U = \{e_t^{k,1}\} \triangle \{e_s^{l,2}\}$ $V = \{x_\tau^m + y_\tau^m\}$ where for any two events $x_t^k \in \{e_t^{k,1}\}$, $y_s^l \in \{e_s^{l,2}\}$ we have $x_t^k = y_s^l \iff t = s \wedge k = l$, $\triangle$ is the *distinct union*-operator, and $x_t^k + y_s^l = z_\tau^m$ with $\tau = t = s$, $m = k = l \in \{\text{IED,IP,PR}\}$ indicates that any two congruent events of type IED, IP, or PR are *merged* into a new *aggregate* event (see payoff and state transition function below).	with $\{e_t^{k,1}\} = U^{ev}(\texttt{CTST}_{FirstLeg}^{Contract}, t_0 \mid \{\texttt{CNTRL} = r^{(1)}\})$ $\{e_s^{l,2}\} = U^{ev}(\texttt{CTST}_{SecondLeg}^{Contract}, t_0 \mid \{\texttt{CNTRL} = r^{(2)}\})$ $r^{(1)} = \begin{cases} RPA & \text{if } \texttt{CNTRL} = RFL \\ RPL & \text{else} \end{cases}$ $r^{(2)} = \begin{cases} RPL & \text{if } \texttt{CNTRL} = RFL \\ RPA & \text{else} \end{cases}$
CE		Same as PAM

SWAPS: State Variables Initialization

State	Initialization per t_0	Comments
Md	$\mathbf{Md}_{t_0} = \max(U^{sv}(\texttt{CTST}_{FirstLeg}^{Contract}, t_0, \mathbf{Tmd}), U^{sv}(\texttt{CTST}_{SecondLeg}^{Contract}, t_0, \mathbf{Tmd}))$	
Ipac	$\mathbf{Ipac}_{t_0} = U^{sv}(\texttt{CTST}_{FirstLeg}^{Contract}, t_0, \mathbf{Ipac} \mid \{\texttt{CNTRL} = r^{(1)}\}) + U^{sv}(\texttt{CTST}_{SecondLeg}^{Contract}, t_0, \mathbf{Ipac} \mid \{\texttt{CNTRL} = r^{(2)}\})$	with $r^{(1)} = \begin{cases} RPA & \text{if } \texttt{CNTRL} = RFL \\ RPL & \text{else} \end{cases}$ $r^{(2)} = \begin{cases} RPL & \text{if } \texttt{CNTRL} = RFL \\ RPA & \text{else} \end{cases}$
Prf		Same as PAM
Sd		Same as PAM

SWAPS: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Ipac}_{t^+} = U^{sv}(\texttt{CTST}_{FirstLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = r^{(1)}\}) + U^{sv}(\texttt{CTST}_{SecondLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = r^{(2)}\})$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if } \texttt{FEB} = \text{'N'} \\ \frac{Y(t^-,t)}{Y(t^-,t^+)}\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$
PRD	$X_{\texttt{CUR}}^{\texttt{CURS}}(t)((-1)\texttt{PPRD} + U^{sv}(\texttt{CTST}_{FirstLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = r^{(1)}\}) + U^{sv}(\texttt{CTST}_{SecondLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = r^{(2)}\}))$	$\mathbf{Ipac}_{t^+} = U^{sv}(\texttt{CTST}_{FirstLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = r^{(1)}\}) + U^{sv}(\texttt{CTST}_{SecondLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = r^{(2)}\})$ $\mathbf{Sd}_{t^+} = t$
TD	$X_{\texttt{CUR}}^{\texttt{CURS}}(t)(\texttt{PTD} + U^{sv}(\texttt{CTST}_{FirstLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = r^{(1)}\}) + U^{sv}(\texttt{CTST}_{SecondLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = r^{(2)}\}))$	$\mathbf{Ipac}_{t^+} = 0.0$ $\mathbf{Sd}_{t^+} = t$
z_τ^m	$f(x_\tau^m) + f(y_\tau^m)$	$\mathbf{Ipac}_{t^+} = U^{sv}(\texttt{CTST}_{FirstLeg}^{Contract}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = r^{(1)}\}) + U^{sv}(SecondLeg, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = r^{(2)}\})$ $\mathbf{Sd}_{t^+} = t$
CE	POF_CE_PAM()	STF_AD_SWAPS()

7.14. **CAPFL: Cap-Floor.**

CAPFL: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as PAM
TD		Same as PAM
k	$\{e^k_t\} = \{x^k_t + y^l_s\}$ for all events $x^k_t \in U^{ev}(\texttt{CTST}^{Contract}_{Underlying}, t_0 \mid \{\texttt{CNTRL} = RPA\})$, $y^l_s \in U^{ev}(\texttt{CTST}^{Contract}_{Underlying}, t_0 \mid \{\texttt{CNTRL} = RPA, \texttt{RRLC} = \texttt{RRLC}, \texttt{RRLF} = \texttt{RRLF}\})$ with $t = s \wedge k = l = IP$. That is, any two congruent events of the child-contract schedule, evaluated once without $\texttt{RRLC}, \texttt{RRLF}$ defined and once with the attributes defined, which are of type IP are *merged* into a new *aggregate* event (see payoff and state transition function below).	
CE		Same as PAM

CAPFL: State Variables Initialization

State	Initialization per t_0	Comments
Md	$\mathbf{Md}_{t_0} = \max(U^{sv}(\texttt{CTST}^{Contract}_{Underlying}, t_0, \mathbf{Tmd} \mid \{\texttt{CNTRL} = RPA\}), U^{sv}(\texttt{CTST}^{Contract}_{Underlying}, t_0, \mathbf{Tmd} \mid \{\texttt{CNTRL} = RPA, \texttt{RRLC} = \texttt{RRLC}, \texttt{RRLF} = \texttt{RRLF}\}))$	
Ipac	$\mathbf{Ipac}_{t_0} = R(\texttt{CNTRL})abs(U^{sv}(\texttt{CTST}^{Contract}_{Underlying}, t_0, \mathbf{Ipac} \mid \{\texttt{CNTRL} = r^{(1)}\}) - U^{sv}(\texttt{CTST}^{Contract}_{Underlying}, t_0, \mathbf{Ipac} \mid \{\texttt{CNTRL} = RPA, \texttt{RRLC} = \texttt{RRLC}, \texttt{RRLF} = \texttt{RRLF}\}))$	
Prf		Same as PAM
Sd		Same as PAM

CAPFL: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Ipac}_{t+} = R(\texttt{CNTRL})abs(U^{sv}(\texttt{CTST}^{Contract}_{Underlying}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = RPA\}) - U^{sv}(\texttt{CTST}^{Contract}_{Underlying}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = RPA, \texttt{RRLC} = \texttt{RRLC}, \texttt{RRLF} = \texttt{RRLF}\}))$ $\mathbf{Sd}_{t+} = t$
PRD	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)((-1)\texttt{PPRD} + R(\texttt{CNTRL})abs(U^{sv}(\texttt{CTST}^{Contract}_{Underlying}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = RPA\}) - U^{sv}(\texttt{CTST}^{Contract}_{Underlying}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = RPA, \texttt{RRLC} = \texttt{RRLC}, \texttt{RRLF} = \texttt{RRLF}\})))$	$\mathbf{Ipac}_{t+} = R(\texttt{CNTRL})abs(U^{sv}(\texttt{CTST}^{Contract}_{Underlying}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = RPA\}) - U^{sv}(\texttt{CTST}^{Contract}_{Underlying}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = RPA, \texttt{RRLC} = \texttt{RRLC}, \texttt{RRLF} = \texttt{RRLF}\}))$ $\mathbf{Sd}_{t+} = t$
TD	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)(\texttt{PTD} + \mathrm{R}(\texttt{CNTRL})\mathrm{abs}(U^{sv}(\texttt{CTST}^{Contract}_{Underlying}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = RPA\}) - U^{sv}(\texttt{CTST}^{Contract}_{Underlying}, t, \mathbf{Ipac} \mid \{\texttt{CNTRL} = RPA, \texttt{RRLC} = \texttt{RRLC}, \texttt{RRLF} = \texttt{RRLF}\})))$	$\mathbf{Ipac}_{t+} = 0.0$ $\mathbf{Sd}_{t+} = t$
z^m_τ	$R(\texttt{CNTRL})abs(f(x^m_\tau) - f(y^m_\tau))$ where $abs(u)$ defines that the absolute value of u is taken.	$\mathbf{Ipac}_{t+} = 0.0$ $\mathbf{Sd}_{t+} = t$
CE	POF_CE_PAM()	STF_AD_CAPFL()

7.15. **OPTNS: Option.**

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Event	Schedule	Comments

OPTNS: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as PAM
TD		Same as PAM
MD	$t^{MD} = \mathtt{MD}$	
XD	$t^{XD} = \begin{cases} \mathbf{Xd}_{t_0} & \text{if} \quad \mathbf{Xd}_{t_0} \neq \varnothing \\ \inf \tau(e^{XD}) & \text{else if} \quad \tau(e^{XD}) \neq \varnothing \wedge \mathtt{OPXT} \neq' E' \\ \varnothing & \text{else} \end{cases}$	with $e^{XD} = e \in O^{ev}(i, \mathtt{XD}, t_0) \vert \tau(e) \leq \mathbf{Md}_{t_0}$ and $i = \mathtt{CTST}^{Contract}_{CoveredContract}(i)$
STD	$t^{STD} = \begin{cases} \varnothing & \text{if} \quad \mathtt{XD} = \varnothing \\ \mathtt{XD} + \mathtt{STD} & \text{else} \end{cases}$	
CE		Same as PAM

OPTNS: State Variables Initialization

State	Initialization per t_0	Comments
Xd	$\mathbf{Xd}_{t_0} = \mathtt{XD}$	
Xa	$\mathbf{Xa}_{t_0} = \mathtt{XA}$	
Prf		Same as PAM
Sd		Same as PAM

OPTNS: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Sd}_{t^+} = t$
PRD	$X^{\mathtt{CURS}}_{\mathtt{CUR}}(t)(-1)\mathtt{PPRD}$	STF_PRD_STK()
TD	$X^{\mathtt{CURS}}_{\mathtt{CUR}}(t)\mathtt{PTD}$	STF_TD_STK()
MD	0.0	$\mathbf{Xd}_{t^+} = \begin{cases} \mathbf{Xd}_{t^-} & \text{if} \quad \mathbf{Xd}_{t^-} \neq \varnothing \\ \varnothing & \text{else if} \quad x = 0.0 \\ t & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $x = \begin{cases} \max(S_t - \mathtt{OPS1}, 0) & \text{if} \quad \mathtt{OPTP} = \text{'C'} \\ \max(\mathtt{OPS1} - S_t, 0) & \text{else if} \quad \mathtt{OPTP} = \text{'P'} \\ \max(S_t - \mathtt{OPS1}, 0) & \text{else} \\ \quad + \max(\mathtt{OPS2} - S_t, 0) & \end{cases}$ $S_t = O^{rf}(U^{ca}(\mathtt{CTST}^{Contract}_{Underlying}, MOC), t)$
XD	0.0	$\mathbf{Xa}_{t^+} = \begin{cases} \max(S_t - \mathtt{OPS1}, 0) & \text{if} \quad \mathtt{OPTP} = \text{'C'} \\ \max(\mathtt{OPS1} - S_t, 0) & \text{else if} \quad \mathtt{OPTP} = \text{'P'} \\ \max(S_t - \mathtt{OPS1}, 0) & \text{else} \\ \quad + \max(\mathtt{OPS2} - S_t, 0) & \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $S_t = O^{rf}(U^{ca}(\mathtt{CTST}^{Contract}_{Underlying}, MOC), t)$
STD	$X^{\mathtt{CURS}}_{\mathtt{CUR}}(t)R(\mathtt{CNTRL})\mathbf{Xa}_{t^-}$	$\mathbf{Xa}_{t^+} = 0.0$ $\mathbf{Sd}_{t^+} = t$
CE	POF_CE_PAM()	STF_AD_OPTNS()

7.16. FUTUR: Future.

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Event	Schedule	Comments

FUTUR: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as PAM
TD		Same as PAM
MD		Same as OPTNS
XD		Same as OPTNS
STD		Same as OPTNS
CE		Same as PAM

FUTUR: State Variables Initialization

State	Initialization per t_0	Comments
Xd		Same as OPTNS
Xa		Same as OPTNS
Prf		Same as PAM
Sd		Same as PAM

FUTUR: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	STF_AD_OPTNS()
PRD	POF_PRD_OPTNS()	STF_PRD_STK()
TD	POF_TD_OPTNS()	STF_TD_STK()
MD	0.0	$\mathbf{Xd}_{t^+} = \begin{cases} \varnothing & \text{if} \quad x = 0.0 \\ t & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $x = S_t - \texttt{PFUT}$ $S_t = O^{rf}(U^{ca}(\texttt{CTST}^{Contract}_{Underlying}, MOC), t)$
XD	0.0	$\mathbf{Xa}_{t^+} = S_t - \texttt{PFUT}$ $\mathbf{Sd}_{t^+} = t$ with $S_t = O^{rf}(U^{ca}(\texttt{CTST}^{Contract}_{Underlying}, MOC), t)$
STD	POF_STD_OPTNS()	STF_STD_OPTNS()
CE	POF_CE_PAM()	STF_AD_FUTUR()

7.17. **CEG: Credit Enhancement Guarantee.**

CEG: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
PRD		Same as PAM
TD		Same as PAM
FP	$\vec{t}^{FP} = \begin{cases} \varnothing & \text{if} \quad \texttt{FER} = \varnothing \vee \texttt{FER} = 0 \\ S(s, \texttt{FECL}, T) & \text{else} \end{cases}$	with $s = \begin{cases} \varnothing & \text{if} \quad \texttt{FEANX} = \varnothing \wedge \texttt{FECL} = \varnothing \\ \texttt{PRD} + \texttt{FECL} & \text{else if} \quad \texttt{FEANX} = \varnothing \\ \texttt{FEANX} & \text{else} \end{cases}$ $T = \begin{cases} \texttt{MD} & \text{if} \quad t^{XD} = \varnothing \\ t^{XD} & \text{else} \end{cases}$
MD	$t^{MD} = \begin{cases} \mathbf{Md}_{t_0} & \text{if} \quad t^{XD} = \varnothing \\ \varnothing & \text{else} \end{cases}$	

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Event	Schedule	Comments
XD	$t^{XD} = \begin{cases} \mathbf{Xd}_{t_0} & \text{if} \quad \mathbf{Xd}_{t_0} \neq \varnothing \\ \inf \tau(e^{CE}) & \text{else if} \quad \tau(e^{CE}) \neq \varnothing \\ \varnothing & \text{else} \end{cases}$	with $e^{CE} = e \in O^{ev}(i, \texttt{CE}, t_0) \mid \mathbf{Prf}_{\tau(e)+} == \texttt{CET} \wedge \tau(e) \leq \mathbf{Md}_{t_0}$ and $i = \texttt{CTST}^{Contract}_{CoveredContract}(i)$
STD		Same as OPTNS
CE		Same as PAM

CEG: State Variables Initialization

State	Initialization per t_0	Comments
Md	$\mathbf{Md}_{t_0} = \begin{cases} \texttt{MD} & \text{if} \quad \texttt{MD} \neq \varnothing \\ \max(\tau(\{e^k_t\})) & \text{else} \end{cases}$	with $\{e^k_t\} = \{e^{k,1}_t\} \cup \{e^{k,2}_t\} \cup ... \cup \{e^{k,n}_t\}$ $\{e^{k,i}_t\} = U^{ev}(\texttt{CTST}^{Contract}_{CoveredContract}(i), t_0 \mid \{ \})n = \mid \texttt{CTST}^{Contract}_{CoveredContract} \mid$
Nt	$\mathbf{Nt}_{t_0} = \begin{cases} 0.0 & \text{if} \quad t_0 \geq \mathbf{Md}_{t_0} \\ \texttt{CECV}R(\texttt{CNTRL})\texttt{NT} & \text{else if} \quad \texttt{NT} \neq \varnothing \\ \texttt{CECV}R(\texttt{CNTRL}) \sum_{i=1}^{\lvert I \rvert} n_i & \text{else} \end{cases}$	with $n_i = \begin{cases} U^{sv}(I(i), t_0, \mathbf{Nt} \mid \{x\}) & \text{if} \quad \texttt{CEGE} = NO \\ U^{sv}(I(i), t_0, \mathbf{Nt} \mid \{x\}) & \text{else if} \quad \texttt{CEGE} = NI \\ \quad +U^{sv}(I(i), t_0, \mathbf{Ipac} \mid \{x\}) & \\ O^{rf}(U^{ca}(I(i), MOC), t_0) & \text{else} \end{cases}$ where $I = \texttt{CTST}^{Contract}_{CoveredContract}$
Feac	$\mathbf{Feac}_{t_0} = \begin{cases} 0.0 & \text{if} \quad \texttt{FER} = \varnothing \\ \texttt{FEAC} & \text{else if} \quad \texttt{FEAC} \neq \varnothing \\ \mathbf{Nt}_{t_0} Y(t^-, t_0)\texttt{FER} & \text{else if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t_0)}{Y(t^{FP-}, t^{FP+})} R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$	with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$
Xd		Same as OPTNS
Xa		Same as OPTNS
Prf		Same as PAM
Sd		Same as PAM

CEG: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-}, t)\mathbf{Nt}_{t^-}\texttt{FER} & \text{if} \quad \texttt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-}, t)}{Y(t^{FP-}, t^{FP+})} R(\texttt{CNTRL})\texttt{FER} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$

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Event	Payoff Function	State Transition Function
PRD	POF_PRD_STK()	$\mathbf{Nt}_{t^+} = \begin{cases} \mathbf{Nt}_{t^-} & \text{if } \mathtt{NT} \neq \varnothing \\ \mathtt{CECV}\sum_{i=1}^{\lvert I\rvert} n_i & \text{else} \end{cases}$ $\mathbf{Feac}_{t^+} = \mathbf{Feac}_{t_0}$ $\mathbf{Sd}_{t^+} = t$ with $n_i = \begin{cases} U^{sv}(I(i),t,\mathbf{Nt} \mid \{\}) & \text{if } \mathtt{CEGE} = NO \\ U^{sv}(I(i),t,\mathbf{Nt} \mid \{\}) & \text{else if } \mathtt{CEGE} = NI \\ \quad +U^{sv}(I(i),t,\mathbf{Ipac} \mid \{\}) & \\ O^{rf}(U^{ca}(I(i),MOC),t) & \text{else} \end{cases}$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$ where $I = \mathtt{CTST}^{Contract}_{CoveredContract}$
FP	$X^{\mathtt{CURS}}_{\mathtt{CUR}}(t)R(\mathtt{CNTRL})\mathtt{FER}$ if $\mathtt{FEB} = A$ $X^{\mathtt{CURS}}_{\mathtt{CUR}}(t)\left(\mathbf{Feac}_{t^-} + \mathbf{Nt}_{t^-}Y(t^-,t)\mathtt{FER}\right)$ else	$\mathbf{Nt}_{t^+} = \begin{cases} \mathbf{Nt}_{t^-} & \text{if } \mathtt{NT} \neq \varnothing \\ \mathtt{CECV}\sum_{i=1}^{\lvert I\rvert} n_i & \text{else} \end{cases}$ $\mathbf{Feac}_{t^+} = 0.0$ $\mathbf{Sd}_{t^+} = t$ with $n_i = \begin{cases} U^{sv}(I(i),t,\mathbf{Nt} \mid \{\}) & \text{if } \mathtt{CEGE} = NO \\ U^{sv}(I(i),t,\mathbf{Nt} \mid \{\}) & \text{else if } \mathtt{CEGE} = NI \\ \quad +U^{sv}(I(i),t,\mathbf{Ipac} \mid \{\}) & \\ O^{rf}(U^{ca}(I(i),MOC),t) & \text{else} \end{cases}$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$ where $I = \mathtt{CTST}^{Contract}_{CoveredContract}$
MD	0.0	$\mathbf{Nt}_{t^+} = 0.0$ $\mathbf{Sd}_{t^+} = t$
XD	POF_XD_OPTNS()	$\mathbf{Nt}_{t^+} = \begin{cases} \mathbf{Nt}_{t^-} & \text{else if } \mathtt{NT} \neq \varnothing \\ \mathtt{CECV}\sum_{i=1}^{\lvert I\rvert} n_i & \text{else} \end{cases}$ $\mathbf{Feac}_{t^+} = \begin{cases} \mathbf{Feac}_{t^-} + Y(\mathbf{Sd}_{t^-},t)\mathbf{Nt}_{t^-}\mathtt{FER} & \text{if } \mathtt{FEB} = \text{'N'} \\ \frac{Y(t^{FP-},t)}{Y(t^{FP-},t^{FP+})}R(\mathtt{CNTRL})\mathtt{FER} & \text{else} \end{cases}$ $\mathbf{Xa}_{t^+} = \mathtt{CECV}\mathbf{Nt}_{t^-}$ $\mathbf{Sd}_{t^+} = t$ with $n_i = \begin{cases} U^{sv}(I(i),t,\mathbf{Nt} \mid \{\}) & \text{if } \mathtt{CEGE} = NO \\ U^{sv}(I(i),t,\mathbf{Nt} \mid \{\}) & \text{else if } \mathtt{CEGE} = NI \\ \quad +U^{sv}(I(i),t,\mathbf{Ipac} \mid \{\}) & \\ O^{rf}(U^{ca}(I(i),MOC),t) & \text{else} \end{cases}$ $t^{FP-} = \sup t \in \vec{t}^{FP} \mid t < t_0$ $t^{FP+} = \inf t \in \vec{t}^{FP} \mid t > t_0$ where $I = \mathtt{CTST}^{Contract}_{CoveredContract}$
STD	$X^{\mathtt{CURS}}_{\mathtt{CUR}}(t)\left(\mathbf{Xa}_{t^-} + \mathbf{Feac}_{t^-}\right)$	$\mathbf{Nt}_{t^+} = 0.0$ $\mathbf{Feac}_{t^+} = 0.0$ $\mathbf{Sd}_{t^+} = t$
CE	POF_CE_PAM()	STF_AD_CEG()

7.18. **CEC: Credit Enhancement Collateral.**

CEC: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
XD		Same as CEG
STD		Same as CEG
MD		Same as CEG

CEC: State Variables Initialization

State	Initialization per t_0	Comments
Md	$\mathbf{Md}_{t_0} = \begin{cases} \texttt{MD} & \text{if } \texttt{MD} \neq \varnothing \\ \max(\tau(\{e_t^k\})) & \text{else} \end{cases}$	with $\{e_t^k\} = \{e_t^{k,1}\} \cup \{e_t^{k,2}\} \cup ... \cup \{e_t^{k,n}\}$ $\{e_t^{k,i}\} = U^{ev}(\texttt{CTST}^{Contract}_{CoveredContract}(i), t_0 \mid \{ \})n =\mid \texttt{CTST}^{Contract}_{CoveredContract} \mid$
Nt	$\mathbf{Nt}_{t_0} = \begin{cases} 0.0 & \text{if } t_0 \geq \mathbf{Md}_{t_0} \\ \min\left(\sum_{j=1}^{\|J\|} v_j, \texttt{CECV} \sum_{i=1}^{\|I\|} n_i\right) & \text{else} \end{cases}$	with $n_i = \begin{cases} U^{sv}(I(i), t_0, \mathbf{Nt} \mid \{\}) & \text{if } \texttt{CEGE} = NO \\ U^{sv}(I(i), t_0, \mathbf{Nt} \mid \{\}) & \text{else if } \texttt{CEGE} = NI \\ \quad +U^{sv}(I(i), t_0, \mathbf{Ipac} \mid \{\}) & \\ O^{rf}(U^{ca}(I(i), MOC), t_0) & \text{else} \end{cases}$ $v_j = O^{rf}(U^{ca}(J(j), MOC), t_0)$ $I = \texttt{CTST}^{Contract}_{CoveredContract}$ $J = \texttt{CTST}^{Contract}_{CoveringContract}$
Sd		Same as PAM

CEC: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Nt}_{t^+} = \begin{cases} 0.0 & \text{if } t \geq \mathbf{Md}_{t_0} \\ \min\left(\sum_{j=1}^{\|J\|} v_j, \texttt{CECV} \sum_{i=1}^{\|I\|} n_i\right) & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$ with $n_i = \begin{cases} U^{sv}(I(i), t_0, \mathbf{Nt} \mid \{\}) & \text{if } \texttt{CEGE} = NO \\ U^{sv}(I(i), t_0, \mathbf{Nt} \mid \{\}) & \text{else if } \texttt{CEGE} = NI \\ \quad +U^{sv}(I(i), t_0, \mathbf{Ipac} \mid \{\}) & \\ O^{rf}(U^{ca}(I(i), MOC), t_0) & \text{else} \end{cases}$ $v_j = O^{rf}(U^{ca}(J(j), MOC), t_0)$ $I = \texttt{CTST}^{Contract}_{CoveredContract}$ $J = \texttt{CTST}^{Contract}_{CoveringContract}$
XD	POF_XD_OPTNS()	$\mathbf{Nt}_{t^+} = \min\left(\sum_{j=1}^{\|J\|} v_j, \texttt{CECV} \sum_{i=1}^{\|I\|} n_i\right)$ $\mathbf{Sd}_{t^+} = t$ with $n_i = \begin{cases} U^{sv}(I(i), t_0, \mathbf{Nt} \mid \{\}) & \text{if } \texttt{CEGE} = NO \\ U^{sv}(I(i), t_0, \mathbf{Nt} \mid \{\}) & \text{else if } \texttt{CEGE} = NI \\ \quad +U^{sv}(I(i), t_0, \mathbf{Ipac} \mid \{\}) & \\ O^{rf}(U^{ca}(I(i), MOC), t_0) & \text{else} \end{cases}$ $v_j = O^{rf}(U^{ca}(J(j), MOC), t_0)$ $I = \texttt{CTST}^{Contract}_{CoveredContract}$ $J = \texttt{CTST}^{Contract}_{CoveringContract}$

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Event	Payoff Function	State Transition Function
STD	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)\mathbf{Nt}_{t-}$	$\mathbf{Nt}_{t+}=0.0$ $\mathbf{Sd}_{t+}=t$
MD	0.0	$\mathbf{Nt}_{t+}=0.0$ $\mathbf{Sd}_{t+}=t$

7.19. **CERTF: Certificate.**

CERTF: Contract Schedule

Event	Schedule	Comments
AD		Same as PAM
ID	$t^{ID}=\texttt{ID}$	
MD	$t^{MD}=\begin{cases}\varnothing & \text{if} \quad \texttt{MD}=\varnothing\\ \texttt{MD} & \text{else}\end{cases}$	
RD	$t^{RD}=\begin{cases}\varnothing & \text{if} \quad \texttt{RDANX}=\varnothing \wedge \texttt{RDCL}=\varnothing \wedge \texttt{MD}=\varnothing\\ S(s,\texttt{RDCL},\texttt{MD}) & \text{else}\end{cases}$	where $s=\begin{cases}\texttt{RDANX} & \text{if} \quad \texttt{RDANX}\neq\varnothing\\ \texttt{ID}+\texttt{RDCL} & \text{else}\end{cases}$
XO	$t^{XO}=O^{ev}(\texttt{CID},\texttt{XO},t)$	
TD	$t^{TD}=O^{ev}(\texttt{CID},\texttt{TD},t)$	
XD	$t^{XD}=\begin{cases}\varnothing & \text{if} \quad t^{RD}=\varnothing\\ t^{RD}+\texttt{XP} & \text{else}\end{cases}$	
STD	$t^{STD}=\begin{cases}\varnothing & \text{if} \quad t^{RD}=\varnothing\\ t^{RD}+\texttt{STP} & \text{else}\end{cases}$	
CE		Same as PAM

CERTF: State Variables Initialization

State	Initialization per t_0	Comments
Xo	$\mathbf{Xo}_{t_0}=\texttt{XO}$	
Xq	$\mathbf{Xq}_{t_0}=\texttt{XQ}$	
Xa	$\mathbf{Xa}_{t_0}=\texttt{XA}$	
Td	$\mathbf{Td}_{t_0}=\texttt{TD}$	
Prf		Same as PAM
Sd		Same as PAM

CERTF: State Transition Functions and Payoff Functions

Event	Payoff Function	State Transition Function
AD	POF_AD_PAM()	$\mathbf{Sd}_{t+}=t$
ID	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)(-1)R(\texttt{CNTRL})\texttt{QT}\times\texttt{NP}$	$\mathbf{Qt}_{t+}=\texttt{QT}$ $\mathbf{Sd}_{t+}=t$
MD	0.0	$\mathbf{Xo}_{t+}=\mathbf{Qt}_t$ $\mathbf{Md}_{t+}=t$ $\mathbf{Sd}_{t+}=t$
RD	0.0	$\mathbf{Xa}_{t+}=f^{RA}(O^{rf}(\texttt{CTST}^{MOC}_{Underlying},t),t)$ $\mathbf{Sd}_{t+}=t$ with f^{RA} the contract Redemption Amount formula.
XO	0.0	$\mathbf{Xo}_{t+}=min(\mathbf{Qt}_t,\mathbf{Xo}_t+ev^{QT})$ $\mathbf{Sd}_{t+}=t$ with ev^{QT} the exercised quantity as per XO event.
TD	0.0	$\mathbf{Xo}_{t+}=\mathbf{Qt}_t$ $\mathbf{Td}_{t+}=t$ $\mathbf{Sd}_{t+}=t$

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Event	Payoff Function	State Transition Function
XD	0.0	$\mathbf{Xq}_{t^+} = \mathbf{Xo}_t$ $\mathbf{Xo}_{t^+} = 0$ $\mathbf{Sd}_{t^+} = t$
STD	$X^{\texttt{CURS}}_{\texttt{CUR}}(t)R(\texttt{CNTRL})\mathbf{Xq}_t\mathbf{Xa}_t$	$\mathbf{Qt}_{t^+} = \mathbf{Qt}_t - \mathbf{Xq}_t$ $\mathbf{Xq}_{t^+} = 0.0$ $\mathbf{Xa}_{t^+} = 0.0$ $\mathbf{Prf}_{t^+} = \begin{cases} \mathbf{Prf}_t & \text{if } \mathbf{Md}_t = \varnothing \wedge \mathbf{Td}_t = \varnothing \\ \text{matured} & \text{else if } \mathbf{Md}_t \neq \varnothing \\ \text{terminated} & \text{else} \end{cases}$ $\mathbf{Sd}_{t^+} = t$
CE	POF_CE_PAM()	STF_AD_OPTNS()

