

Appendix D Lists of Model Variables, Parameters, and Full Model Formulation

Table A.1: List of Variables

Variable	Definition	Unit
k_j^{TES}	Power rating for TES in building j	KW
g_{jt}^{TES}	Energy discharge from TES in building to serve load j at time t	kWh
g_{jt}^{HP-TES}	Heat pump output to TES in building j at time t	kWh
g_{jt}^{HP-L}	Heat pump output to serve load in building j at time t	kWh
d_{jt}^{HP}	Purchased electricity to power heat pump/resistance heater in building j at time t	MWh
x_{jt}^{TES}	State of charge for TES in building j at time t	kWh
v_{jt}^{TES}	Whether TES is discharging (1) or not (0) in building j at time t	Binary
v_{jt}^{HP-TES}	Whether heat pump is charging TES (1) or not (0) in building j at time t	Binary

Table A.2: List of Parameters and Sets

Parameter/Set	Definition	Unit
<i>Parameters:</i>		
r	Interest rate = 0.07	–
l	Storage lifetime	years
f^r	Capital recovery factor	–
f^d	Discharging efficiency	–
f^c	Charging efficiency	–
$(x_A, y_A), (x_B, y_B)$	Cut-off points for piece-wise function of TES power rating and SOC	(%, kW)
e_j^{TES}	Energy capacity for TES in building j	KWh
h^{TES}	TES hours	hours
p_{jt}^R	Residential electricity price at location of building j at time t	\$/kWh
T_t^O	Outdoor temperature at location of building j at time t	°C
k^{HP}	Heat pump capacity	KWh
COP_{jt}	Heat pump Coefficient of performance in building j at time t	°C
d_{jt}	Heating load in building j at time t	KWh
p_{jt}^R	Residential electricity rate in building j at time t	KWh
<i>Sets:</i>		
\mathbb{J}	Set of buildings, index j	–
\mathbb{T}	Set of hours in a typical year, index $t = \{1, 2, 3, \dots, 8760\}$	–
\mathbb{Z}_2	Set of binary numbers, $\mathbb{Z}_2 = \{0, 1\}$	–
\mathbb{Z}_1	Set of real numbers ≥ 1 , $\mathbb{Z}_1 = \{x \in \mathbb{Z}_1 : x \geq 1\}$	–

$$\min_{\substack{g_{jt}^{TES}, g_{jt}^{HP-TES}, g_{jt}^{HP-L} \\ d_{jt}^{HP}, x_{jt}^{TES}, v_{jt}^{TES}, v_{jt}^{HP-TES}}} TC_j = \sum_t p_{jt}^R d_{jt}^{HP} \quad (D.1)$$

s.t.

$$TES \text{ constraints: } 0 \leq g_{jt}^{TES} \leq k_j^{TES}, \quad \forall j \in \mathbb{J}, \forall t \in \mathbb{T} \quad (D.2)$$

$$0 \leq g_{jt}^{TES} \leq x_{jt}^{TES}, \quad \forall j \in \mathbb{J}, \forall t \in \mathbb{T} \quad (D.3)$$

$$x_{jt}^{TES} \leq e_j^{TES}, \quad \forall j \in \mathbb{J}, \forall t \in \mathbb{T} \quad (D.4)$$

$$x_{jt}^{TES} = x_{j(t-1)}^{TES} + f_j^c g_{jt}^{HP-TES} - f_j^d g_{jt}^{TES}, \quad \forall j \in \mathbb{S}, \forall t > 1 \in \mathbb{T} \quad (D.5)$$

$$Piece\text{-}wise \text{ constraints: } k_{jt}^{TES} = \frac{y_B - y_A}{x_B - x_A} \frac{x_{jt}^{TES}}{e_j^{TES}} + y_A - \frac{y_B - y_A}{x_B - x_A} x_A, \quad \forall j \in \mathbb{J}, \forall t \in \mathbb{T} \quad (D.6)$$

$$\text{for } x_A \leq x_{jt}^{TES} \leq x_B$$

$$Heat \text{ pump constraints: } g_{jt}^{HP-L} \geq 0, \quad \forall j \in \mathbb{J} \quad (D.7)$$

$$0 \leq g_{jt}^{HP-TES} \leq k_j^{TES}, \quad \forall j \in \mathbb{J}, \forall t \in \mathbb{T} \quad (D.8)$$

$$g_{jt}^{HP-L} + g_{jt}^{HP-TES} \leq k_j^{HP}, \quad \forall j \in \mathbb{J}, \forall t \in \mathbb{T} \quad (D.9)$$

$$COP_{jt} d_{jt}^{HP} \geq g_{jt}^{HP-L} + g_{jt}^{HP-TES}, \quad \forall j \in \mathbb{J}, \forall t \in \mathbb{T} \quad (D.10)$$

$$Market \text{ Clearing: } g_{jt}^{HP-L} + g_{jt}^{TES} \geq d_{jt}, \quad \forall j \in \mathbb{J} \quad (D.11)$$

$$\min_{\substack{k_{jt}^{TES}, g_{jt}^{TES}, g_{jt}^{HP-TES}, g_{jt}^{HP-L} \\ d_{jt}^{HP}, x_{jt}^{TES}, v_{jt}^{TES}, v_{jt}}} TC_j = \sum_t p_{jt}^R d_{jt}^{HP} \quad (D.12)$$

s.t.

$$\text{TES constraints: } 0 \leq g_{jt}^{TES} \leq k_j^{TES}, \quad \forall j \in \mathbb{J}, \forall t \in \mathbb{T} \quad (D.13)$$

$$0 \leq g_{jt}^{TES} \leq x_{jt}^{TES}, \quad \forall j \in \mathbb{J}, \forall t \in \mathbb{T} \quad (D.14)$$

$$x_{jt}^{TES} \leq e_j^{TES}, \quad \forall j \in \mathbb{J}, \forall t \in \mathbb{T} \quad (D.15)$$

$$x_{jt}^{TES} = x_{j(t-1)}^{TES} + f^c g_{jt}^{HP-TES} - f^d g_{jt}^{TES}, \quad \forall j \in \mathbb{S}, \forall t > 1 \in \mathbb{T} \quad (D.16)$$

$$v_{jt}^{TES} + v_{jt}^{HP-TES} \leq 1, \quad \forall j \in \mathbb{S}, \forall t \in \mathbb{T}$$

$$\text{Piece-wise constraints: } k_{jt}^{TES} = \frac{y_B - y_A}{x_B - x_A} \frac{x_{jt}^{TES}}{e_j^{TES}} + y_A - \frac{y_B - y_A}{x_B - x_A} x_A, \quad \forall j \in \mathbb{J}, \forall t \in \mathbb{T} \quad (D.17)$$

$$\text{for } x_A \leq x_{jt}^{TES} \leq x_B$$

$$\text{Heat pump constraints: } g_{jt}^{HP-L} \geq 0, \quad \forall j \in \mathbb{J} \quad (D.18)$$

$$0 \leq g_{jt}^{HP-TES} \leq k_{jt}^{TES}, \quad \forall j \in \mathbb{J}, \forall t \in \mathbb{T} \quad (D.19)$$

$$g_{jt}^{HP-L} + g_{jt}^{HP-TES} \leq k_j^{HP}, \quad \forall j \in \mathbb{J}, \forall t \in \mathbb{T} \quad (D.20)$$

$$COP_{jt} d_{jt}^{HP} \geq g_{jt}^{HP-L} + g_{jt}^{HP-TES}, \quad \forall j \in \mathbb{J}, \forall t \in \mathbb{T} \quad (D.21)$$

$$\text{Big M constraints: } g_{jt}^{HP-TES} \leq M v_{jt}, \quad \forall j \in \mathbb{J} \quad (D.22)$$

$$v_{jt} \leq M g_{jt}^{HP-TES}, \quad \forall j \in \mathbb{J} \quad (D.23)$$

$$g_{jt}^{TES} \leq M (1 - v_{jt}), \quad \forall j \in \mathbb{J}, \forall t \in \mathbb{T} \quad (D.24)$$

$$(1 - v_{jt}) \leq M g_{jt}^{TES}, \quad \forall j \in \mathbb{J}, \forall t \in \mathbb{T} \quad (D.25)$$

$$\text{Market Clearing: } g_{jt}^{HP-L} + g_{jt}^{TES} \geq d_{jt}, \quad \forall j \in \mathbb{J} \quad (D.26)$$