CS201: Data Structures and Algorithms **Autumn 2020**

Assignment 1

IIT Dharwad *Computer Science and Engineering*

Notes

- There are two questions in this assignment.
- Total marks: 6+4 = 10.
- The submission is allowed only through Moodle.
- Submit a single pdf file.
- Please prepare your answer using any editor.
- Please do not scan and submit hand-written answers: There will be a penalty of 10% if there is any hand-written content.
- The name of our submission file must be <roll-no>.pdf and it must be a pdf file, where the roll no. is your roll no.
- Violation of naming policy attracts 10% penalty.
- Last date for submission without penalty: 23 Oct 2020 (Friday)
- Last date for submission with 25% penalty: 30 Oct 2020 (Friday)

1 Question 1

1.1 Description

You are given with two functions on n: $f(n)$ and $g(n)$. You need to figure out how $f(n)$ and $g(n)$ are related with respect to the three asymptotic notations O, Ω, and Θ. That is, you have to answer (with proof) the following questions.

(i) Is $f(n) = O(g(n))$?

(ii) Is $f(n) = \Omega(g(n))$?

(iii) Is $f(n) = \Theta(g(n))$?

(iv) Is $g(n) = O(f(n))$?

(v) Is $g(n) = \Omega(f(n))$?

(vi) Is $g(n) = \Theta(f(n))$?

Let us recall the following definitions of the asymptotic notations (read Section 3.1 in the textbook).

Definition 1 $O(q(n)) = \{p(n) :$ *there exists positive constants* c, *and* n_0 *such that* $0 \leq p(n) \leq c \cdot q(n), \forall n \geq n_0\}$

Definition 2 $\Omega(q(n)) = \{p(n) :$ *there exists positive constants* c, *and* n_0 *such that* $0 \leq c \cdot g(n) \leq p(n), \forall n \geq n_0\}$

Definition 3 $\Theta(q(n)) = \{p(n) :$ *there exists positive constants* c_1, c_2, *and* n_0 *such that* $0 \leq c_1 \cdot g(n) \leq p(n) \leq c_2 \cdot g(n), \forall n \geq n_0\}$

Let us also recall the following results related to the asymptotic notations (see Section 3.1 in the textbook).

Theorem 1 $p(n) = O(q(n))$ *if and only if* $q(n) = \Omega(p(n))$.

Theorem 2 $p(n) = \Theta(q(n))$ *if and only if* $p(n) = O(q(n))$ *and* $p(n) = \Omega(q(n))$.

Theorem 3 $p(n) = \Theta(q(n))$ *if and only if* $q(n) = \Theta(p(n))$.

1.2 An Example

Let us solve an example. Let $f(n) = n$ and $g(n) = n^2$.

(i) Is $f(n) = O(g(n))$?

Binary Answer: Yes

Proof: Let $c = 1$ and $n_0 = 1$. Then, $0 \leq n \leq 1 \cdot n^2, \forall n \geq 1$. Therefore, as per Definition 1, $n = O(n^2)$.

(ii) Is $f(n) = \Omega(g(n))$?

Binary Answer: No

Proof: We will prove that there exist no positive constants c and n_0 such that $c \cdot n^2 \leq n, \forall n \geq n_0$. Then the answer follows from Definition 2. For a contradiction, assume that there exist such constants c and n_0. Then $0 \leq c \cdot n^2 \leq n$, $\forall n \geq n_0$. Then, $n^2 \leq n/c, \forall n \geq n_0$. But $n^2 \leq n/c$ is wrong for all $n > 1/c$, which is a contradiction.

(iii) Is $f(n) = \Theta(g(n))$?

Binary Answer: No

Proof: Follows from the answer to (ii) and Theorem 2.

(iv) Is $g(n) = O(f(n))$?

Binary Answer: No

Proof: Follows from the answer to (ii) and Theorem 1.

(v) Is $g(n) = \Omega(f(n))$?

Binary Answer: Yes

Proof: Follows from the answer to (i) and Theorem 1.

(vi) Is $g(n) = \Theta(f(n))$?

Binary Answer: No

Proof: Follows from the answer to (iii) and Theorem 3.

1.3 Instructions

- Each student will receive a different pair of functions $f(n)$ and $g(n)$. The following are the functions from which you will receive the functions $f(n)$ and $g(n)$. Every log is base 2.
  - $h_1(n) = \sqrt{2}^{\log n}$
  - $h_2(n) = (3/2)^n$
  - $h_3(n) = n^3$
  - $h_4(n) = \log^2 n$
  - $h_5(n) = 2^{2^n}$
  - $h_6(n) = n^{1/\log n}$
  - $h_7(n) = \log\log n$
  - $h_8(n) = n \cdot 2^n$
  - $h_9(n) = n^{\log\log n}$
  - $h_{10}(n) = (\log n)^{\log n}$
  - $h_{11}(n) = 4^{\log n}$
  - $h_{12}(n) = \sqrt{\log n}$
  - $h_{13}(n) = 2^{\sqrt{2\log n}}$
  - $h_{14}(n) = n\log n$
- Check the row corresponding to your roll no. in Appendix A to see which among these functions are your $f(n)$ and $g(n)$.
- As explained in the description, there are six parts for the question. Each part carries 1 mark and the total marks for this question is 6.
- In each part, your answer should have two sub-parts (as shown in the example) - a binary answer (Yes/No) and a proof. The binary answer (Yes/No) carries 0.25 marks and the proof carries 0.75 marks.
- In your proofs, you can refer to the definitions and theorems mentioned in the description (as done in the example). If you want to refer to any other definition or results mentioned in the textbook, you have to write it explicitly.

2 Question 2

2.1 Description

You will be given with a pre-order traversal of a binary search tree (BST). There are two tasks for you:

(i) Draw the BST corresponds to the given pre-order traversal.

(ii) Write the post-order traversal corresponds to the BST represented by the given pre-order traversal.

2.2 Instructions

- Each student will receive the pre-order traversal of a different BST.
- Check the row corresponding to your roll no. in Appendix B to see the pre-order traversal assigned to you.
- This question carries 4 marks - each of the two parts carries 2 marks.

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