MTL704: Problems Sheet-2

1. Find the number of iterations required to minimize $\phi(x) = (x-2)^2$ with an error in the minimum point $x^\star$ not more than 0.042 by Golden Section method. Perform two complete iterations of the method.

2. Use Golden Section (illustrate 3 iterations) rule to find, within 10%, the value of x in $[0,1]$ that maximize
$f(x) = \min\{\ x,\ 2x^2,\ 2-2x-x^2\ \}$.

3. Consider the function $f(x)$ over $[0,4]$ as follows
$$f(x) = \begin{cases} 2-x, & 0 \le x \le 1 \\ x^2, & 1 < x \le 2 \\ x+2, & 2 < x \le 4 \end{cases}$$
Is $f(x)$ unimodal in $[0,4]$? Perform three iterations of Golden Section rule and three iterations of Fibonacci Search technique and compare the two output thereon.

4. Use the Fibonacci search technique (illustrate 2 complete iterations) to minimize $f(x) = 3x^2 - \exp(x)$ over $[0,1]$ with an error in optimal solution not more than 0.05.

5. Perform three complete iterations of Golden Section rule and Fibonacci method to minimize the function $\phi(x)$ on $[-1,1]$, where $\phi(x) = \max\left\{x^2, \dfrac{1-x}{2}\right\}$. Compare the ratios $\dfrac{I_4}{I_1}$ as obtained by the two methods. Does the numerical results agree with the theoretical estimations?

6. Consider the function $f(x) = x^4 - 14x^3 + 60x^2 - 70x$. If Fibonacci search technique is used in $[0,2]$, then how many iterations are needed to find the minimum x^* within the range 0.3? Also obtain the point of local minima of $f(x)$ (perform 2 complete iterations).

7. Let the sequence $< r_k >$ satisfying $r_{k+1} = \frac{1}{r_k} - 1,\ r_k \ge \frac{1}{2},\ k = 1, 2, \ldots$. Prove that for $k \ge 2$,
$$r_k = -\left(\frac{F_{k-2} - F_{k-1}r_1}{F_{k-3} - F_{k-2}r_1}\right).$$

8. Let $F_0, F_1, F_2, \ldots$ be the Fibonacci sequence. Show that, for each $k \ge 2$,
$$F_{k-2}F_{k+1} - F_{k-1}F_k \;=\; (-1)^k.$$
(Hint: use induction principle).

9. Consider the iterative process $x_{k+1} = \dfrac{1}{2}\left(x_k + \dfrac{a}{x_k}\right)$, $a > 0$.
Assuming the process converges then find the point to which it converges? What is the order of convergence of this algorithm?

10. Discuss the convergence and the order of convergence for the following methods
    - (a) $x_k = r^k,\ \ 0 < r < 1;$
    - (b) $x_k = r^{2^k},\ \ 0 < r < 1;$
    - (c) $x_k = 1/k^k.$

11. Develop a derivative free quadratic interpolation scheme to minimize a unimodular function $f(x)$ in the interval of uncertainty $[a,b]$. Perform 3 iterations of the proposed scheme to minimize $f(x) = (x-1)(x-2)(x-3)$ in $[1,3]$. Verify your result by using basic calculus rules.

12. Solve the problem minimize $f(x) = (x-1)(x-2)(x-3)$ in $[1,3]$. Perform 3 iterations of Fibonacci method. Compare your result with the result of previous question. What can you infer?

13. A convex quadratic function $h(x)$ assumes the values h_1, h_2, and h_3 at $x = x_1, x_2$, and x_3, respectively, where $x_1 = x_2\delta$ and $x_3 = x_2 + \delta$. Show that the minimum of the function is given by
$$h_2 - \frac{(h_1 - h_3)^2}{8(h_1 - 2h_2 + h_3)}.$$

14. Use Hooke and Jeeves' method to minimize $f(x) = 3x_1^2 - 2x_1x_2 + x_2^2 + 4x_1 + 3x_2$. Take $(0,0)^T$ as the initial base point, $\lambda_1 = \lambda_2 = 1$ as initial step lengths. Take 0.25 as the threshold in step length reduction in both directions as stopping criterion.

15. Use the method of Nelder and Mead to minimize $f(x_1, x_2) = 4(x_1 - 5)^2 + 6(x_2 - 6)^2$ The initial simplex has the following three vertices A(8, 9), B(10, 11), C(8, 11) Carry out 2 complete iterations.