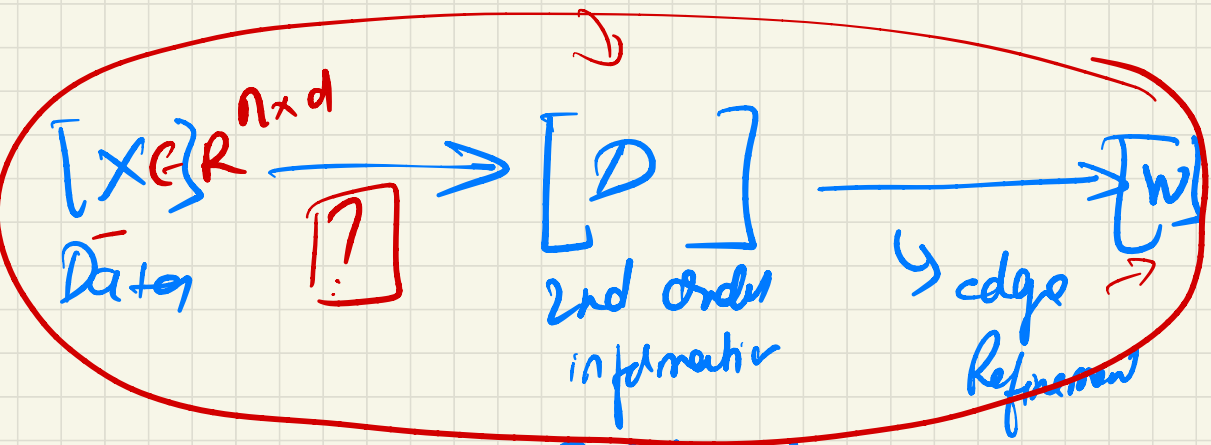
Lecture - 2

24/02/2022

Graph learning from Data $X \rightarrow$



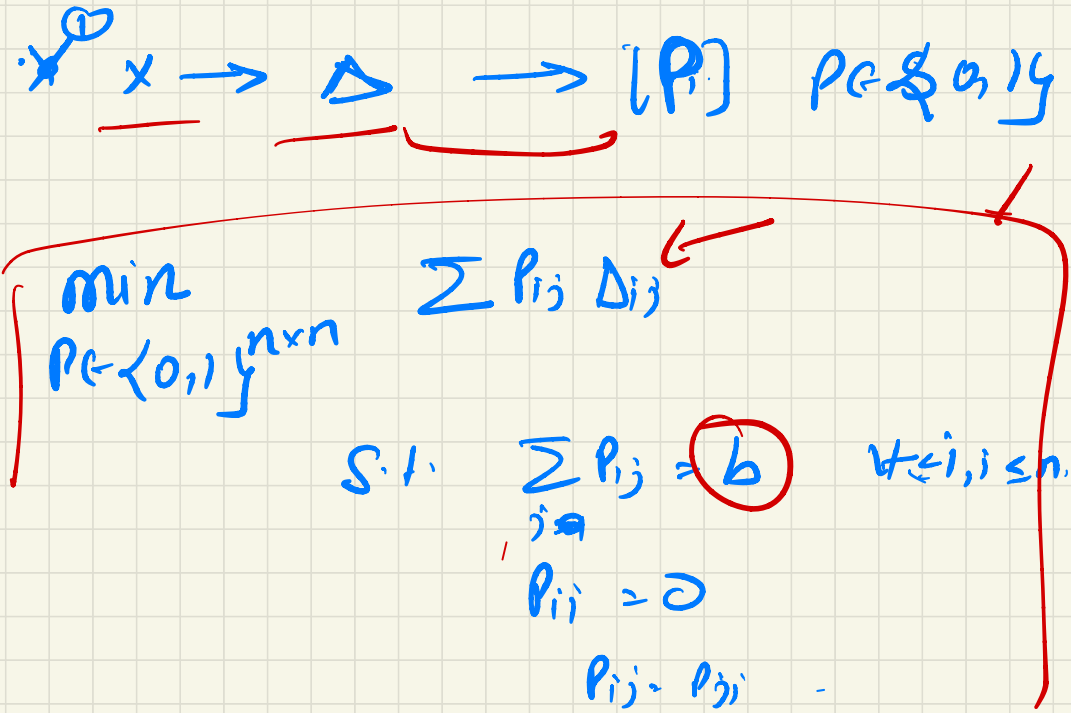
=> Euclidean distance

=> Sample covariance

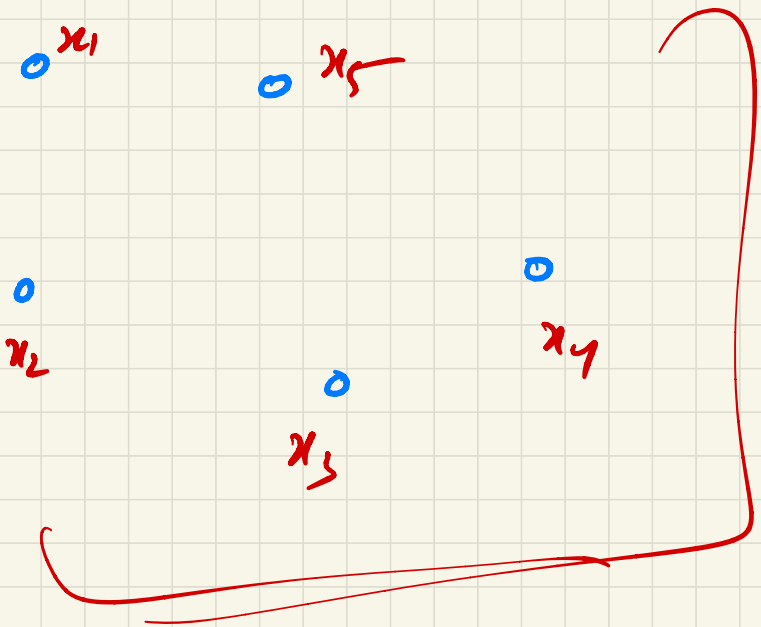
=> kernel

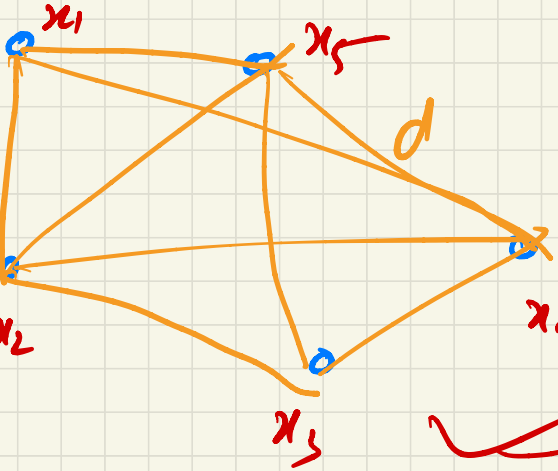
=> RBF

eg



δi


x_1
x_5
x_2
x_4
x_3



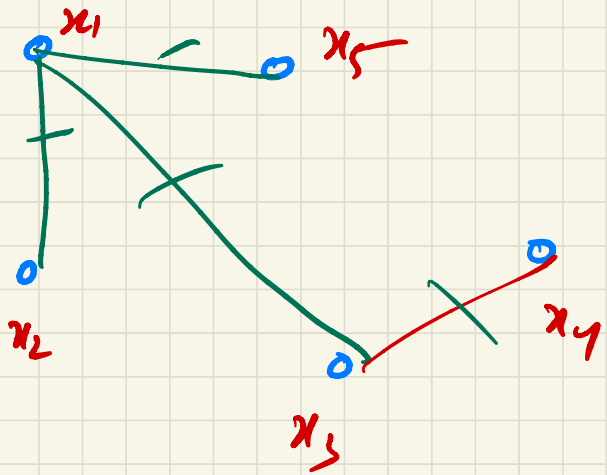
d

(D)

$\min \sum p_{ij} \Delta_{ij}$

$p_{ij} \in \{0,1\}^{mn}$

p

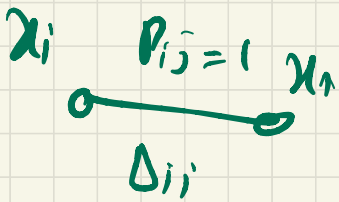


- **B.S.L**

$p_{ij} = 0 \quad w_{ij} = 0$

$p_{ij} = 1 \quad w_{ij} = 1$

- Gaussian kernel



$$w_{ij} = p_{ij} \exp\left(\frac{-\|x_i - x_j\|}{2\sigma^2}\right)$$

$$= p_{ij} \exp\left(-\frac{\Delta_{ij}}{2\sigma^2}\right)$$

- Graph learning problem

① [X] → [C] ← Conv

② [C] —(X)→ [W] ←

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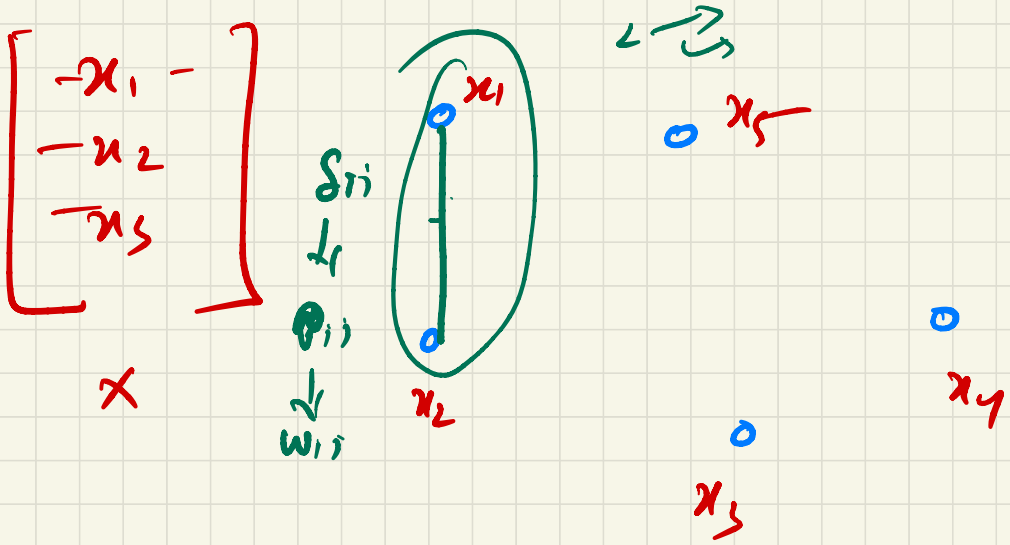
② [X] → [W] ?

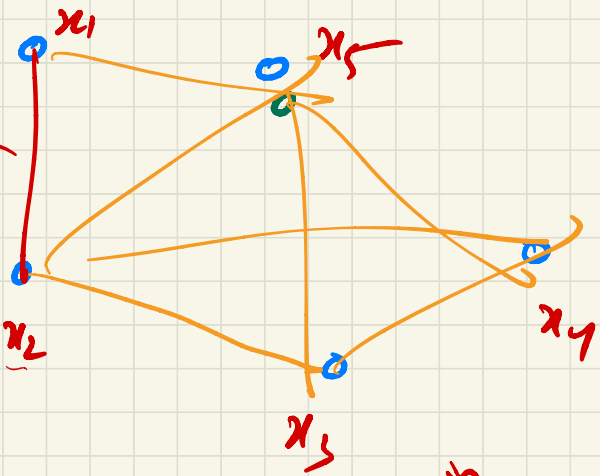
[X] →



Model

1. Linear Combination Model
2. Smooth Gifford Model
   - Diffusion Model
3. Probabilistic Graphical Model

(1)



$$x_2 = \sum_{j \neq 2} \alpha_{2j} x_j$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$x_1 = \sum_{j \neq 1} \alpha_{1j} x_j$$

$$\vdots$$

$$x_n = \sum_{j \neq n} \alpha_{nj} x_j$$

$$x_i = \sum_{j \neq n} \alpha_{ij} x_j \quad (1)$$

$$Mx = \lambda x$$

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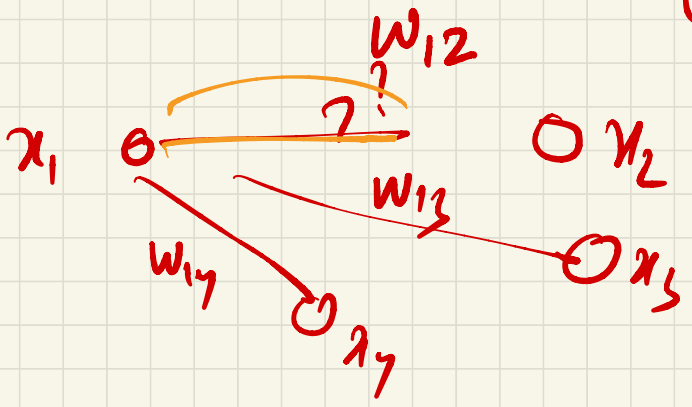
$$\underset{y}{A}\ \underset{\uparrow}{x} = \underset{\downarrow}{b} \qquad \min \|Ax - b\|_F^2$$

$$a_1 x_1 + a_2 x_2 + \cdots = b$$

$$\alpha_{ij} = w_{ij}$$

$$W = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ \vdots & & & \\ w_{n1} & \cdots & & w_{nn} \end{bmatrix}$$

$$\Rightarrow \min_{\alpha}$$



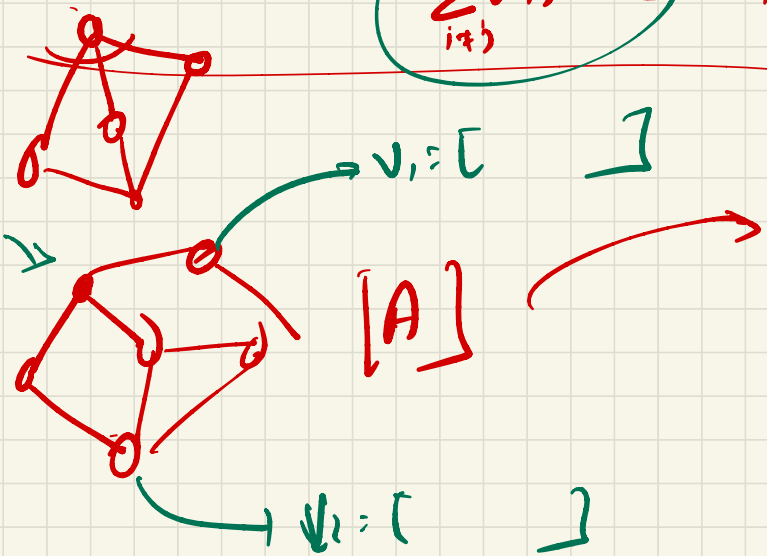
$$\min_{w_1 = [w_{11} \cdots w_{1n}]} \|x_1 - \sum_{j \neq 1} w_{1j} x_j\|^2$$

$$\min_{W}$$

$$\min_{W \in \mathbb{R}^{n \times n}} \sum_{i=1}^{n} \| x_i - \sum_{i \neq j} w_{ij} x_j \|^2$$

$$w_{ij} = w_{ji}, \quad w_{ii} \geq 0$$

$$\sum_{i \neq j} w_{ij} = b \quad , \forall i = 1 \cdots n$$



$$f(w) = \sum_{i=1}^{n} \left\| x_i - \sum_{j \neq i} w_{ij} x_j \right\|^2$$

$$\min \quad f(w)$$

$$s.t \quad w \in S$$

Model

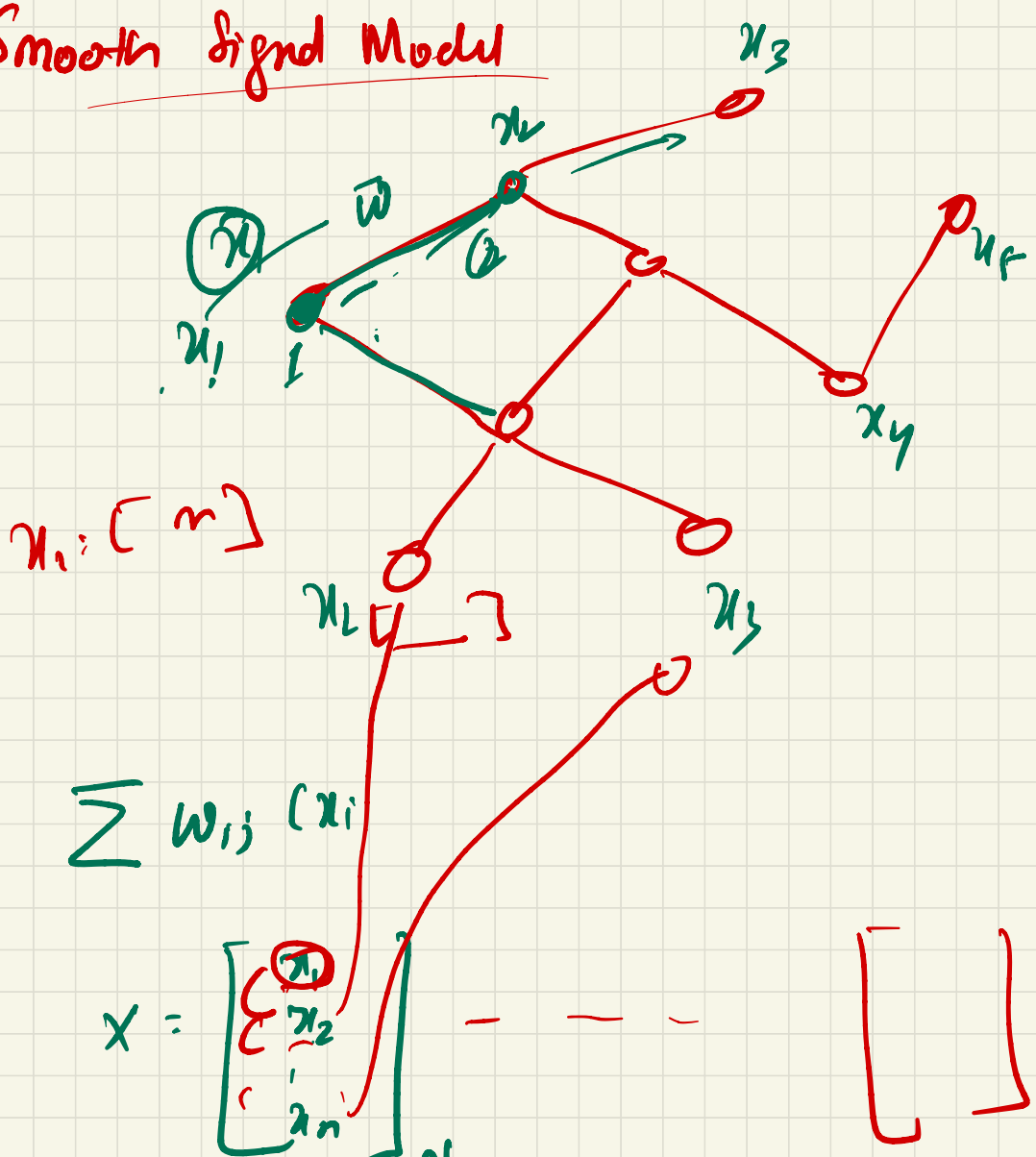
$$\Downarrow$$

$f(w)$

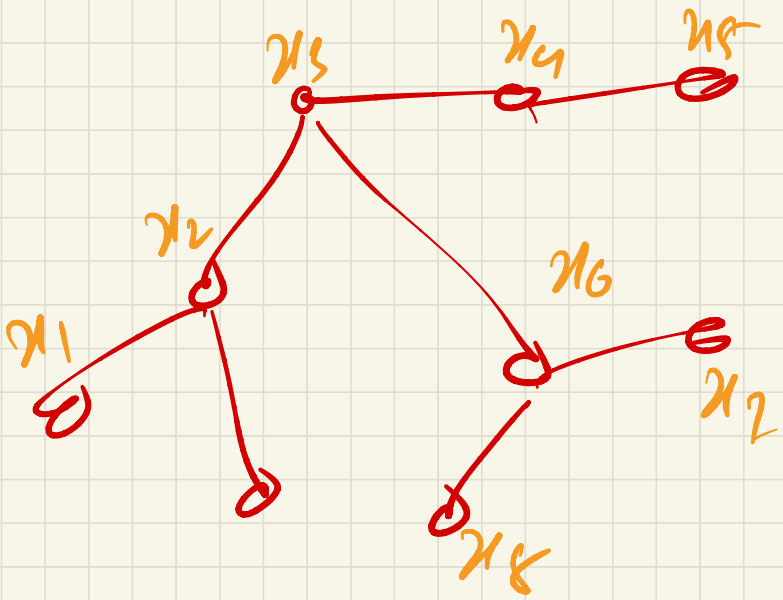
$$\min_{w} \quad f(w) + \lambda h(w)$$

$$s.t \quad w \in S$$

Smooth Signal Model



$X = [\ ]$



$W,\ L$

$L = D - W$

$X^T L X$

$= X^T (D - W) X$

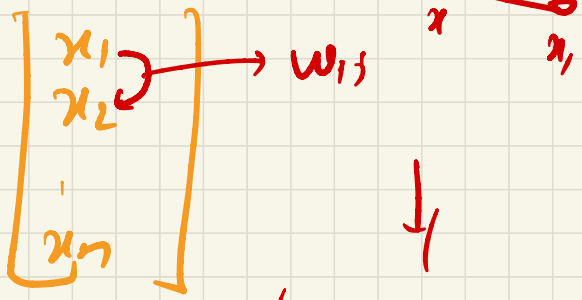
$= X^T D X - X^T W X$

$X^T \begin{bmatrix} \sum_{j \neq 1} w_{1j} & & \\ & \ddots & \\ & & \sum_{n \neq i} w_{ni} \end{bmatrix} X - X^T \begin{bmatrix} w_{11} & \cdots \\ \vdots & \ddots \end{bmatrix} X$

$= \sum w_{ij} (x_i - x_j)^2$

X is given $\underline{x} \in R^N$: Smoothness

W?



$$\min_L \; x^T L x$$

$$\min_W \sum w_{ij} (x_i - x_j)^2$$

?