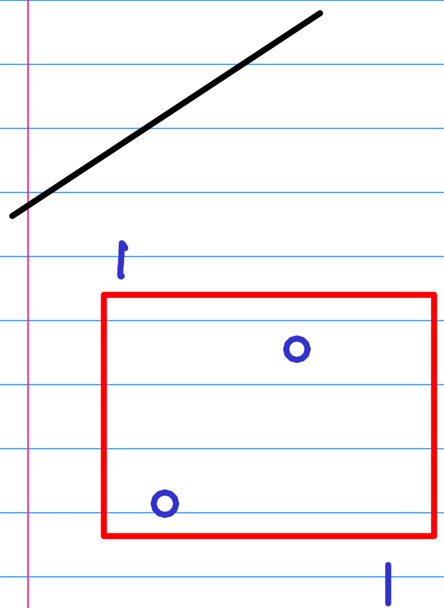
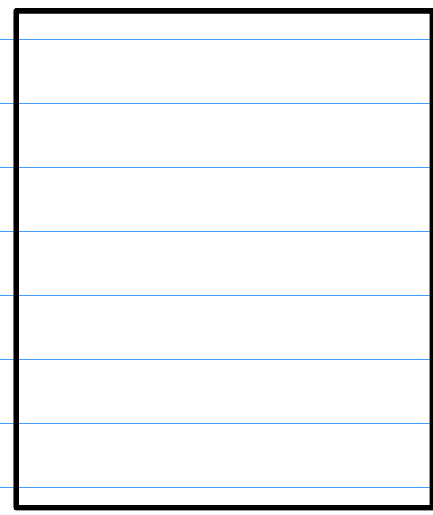




$$w_1 x_1 + w_2 x_2 + \dots \quad w_n x_n + b = 0$$

$$\|W\|_0 \leq k$$

$$\|W\|_p = \left(|w_1|^p + |w_2|^p + \dots \; |w_n|^p\right)^{\frac{1}{p}}$$

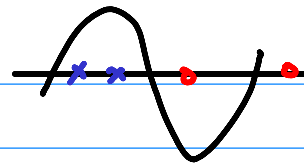
$$\|W\|_0 = \lim_{p \to 0} \left(|w_1|^p + |w_2|^p + \dots \; |w_n|^p\right)^{\frac{1}{p}}$$

M : samples $\qquad 6\Delta_f(2M)$

n : dimension

d

polynomials: 2^M



$\sin(kx)$

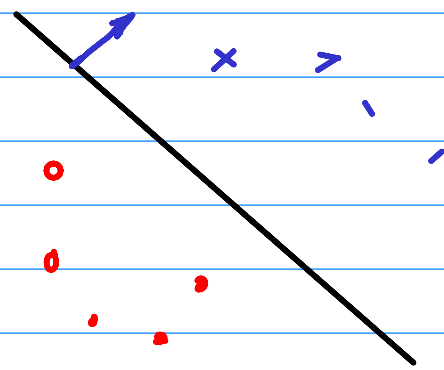
$n+1$

$\geq |\hat{u}^T \hat{x}^i|$

$$\text{Min } \frac{R}{d} \equiv \min_{\hat{u}} \frac{\max_{i=1,2,..M} \|\hat{u}\| \|\hat{x}^i\|}{\min_{i=1,2,..,M} \|\hat{u}^T \hat{x}^i\|}$$

$$d \geq d_{min} \quad \min_{u,v} k \frac{\max_{i=1,2..M} |u^T x^i + v|}{\min_{i=1,2,..,M} |u^T x^i + v|}$$

Linearly separable dataset (x^i, y_i), $i=1,2,..M$



$y_i \in \{-1, 1\}$

$y_i = 1 \quad : \quad u^T x^i + v \geq 1$

$y_i = -1 \quad : \quad u^T x^i + v \leq -1$

$$|u^T x^i + v| = y_i(u^T x^i + v)$$

$$\text{Min } \frac{R}{d} \equiv \min_{u,v} \frac{\max_{i=1,2,..M} y_i(u^T x^i + v)}{\min_{i=1,2,..,M} y_i(u^T x^i + v)}$$

$$\mathrm{Min}\ \frac{R}{d} \equiv \underset{u,v}{\mathrm{Min}} \frac{\underset{i=1,2,\ldots M}{\mathrm{Max}}\ y_i(u^T x^i + v)}{\underset{i=1,2,\ldots,M}{\mathrm{Min}}\ y_i(u^T x^i + v)}$$

$$\mathrm{Min}\ \frac{g}{l}$$ LFPP

$$g \geq y_i(u^T x^i + v),\ i = 1, 2, \ldots, M$$

$$l \leq y_i(u^T x^i + v),\ i = 1, 2, \ldots, M$$

$$l \neq 0$$

$$\cancel{y_i(u^T x^i + v)} \geq 1$$

Charnes - Cooper transformation (Num. Opt. S. Chandra)

$$p \equiv \frac{1}{l} \qquad l \neq 0.$$

$$\mathrm{Min}\ \frac{g}{l} \equiv g \cdot p$$

$$g \geq y_i(u^T x^i + v),\ i = 1, 2, \ldots, M$$

$$pg \geq y_i\left[(pu)^T x^i + (pv)\right]$$

$$l \leq y_i(u^T x^i + v),\ i = 1, 2, \ldots, M$$

$$p \cdot l \leq y_i\left[(pu)^T x^i + (pv)\right]$$

$$\text{Min } \frac{g}{l} \equiv g \cdot p$$

$$g \geq y_i\,(u^T x^i + v), \quad i = 1, 2, \dots, M$$

$$pg \geq y_i\,[(pu)^T x^i + (pv)]$$

$$l \leq y_i\,(u^T x^i + v), \quad i = 1, 2, \dots, M$$

$$p \cdot l \leq y_i\,[(pu)^T x^i + (pv)]$$

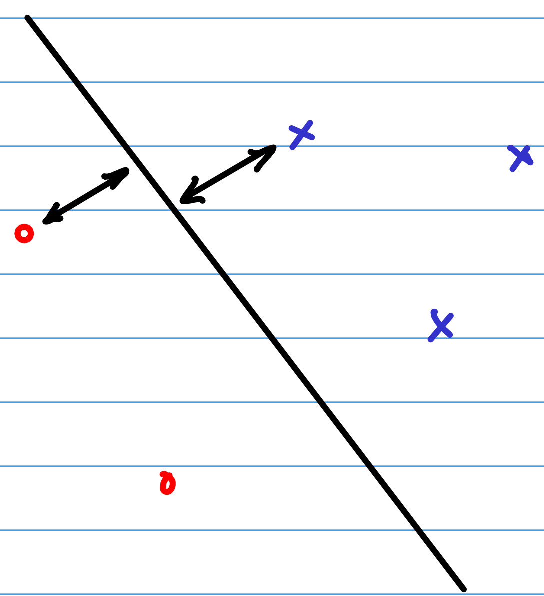
$$h \equiv g \cdot p, \qquad (pu) \equiv w, \qquad (pv) \equiv b$$

$$\text{Min } h$$

s.t.

$$h \geq y_i\,[w^T x^i + b], \quad i = 1, 2, \dots, M$$

$$y_i\,(w^T x^i + b) \geq 1, \quad i = 1, 2, \dots, M$$



soft margin MCM

$h = \frac{R}{d}$

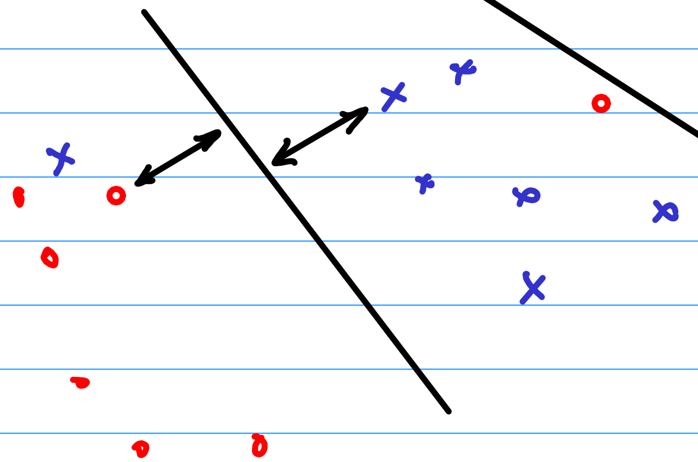
$$\text{Min } h + C \sum_{i=1}^{M} q_i$$

st.

$$h \geq y_i \left[W^T x^i + b \right] \quad , i = 1, 2, \dots, M$$

$$y_i (W^T x^i + b) + q_i \geq 1 \quad , i = 1, 2, \dots, M$$

$$q_i \geq 0$$



What is the dual? Does it serve any purpose?

$$\underset{\in \mathbb{R}^n}{x} \longrightarrow \phi \longrightarrow \phi(x) \in \mathbb{R}^d$$

$d >> n$.

Cover's theorem

$$W^T \phi(x) + b = 0$$

$$\text{Min } h + C \sum q_i$$

st

$$h \geq y_i \left[W^T \phi(x^i) + b \right]$$

$$y_i \left[W^T \phi(x^i) + b \right] + q_i \geq 1$$

$$q_i \geq 0$$