Q3
a)

$$V_\pi(s) = \underset{\pi}{\mathbb{E}}\Big[\sum \gamma^i r(s_{t+i}, a_{t+i}, s_{t+i+1}) \Big| s_t = s \Big]$$

$a_i \sim \pi(\cdot|s_i)$ ← this runs from $i=0$ to ∞

$$= \underset{\pi}{\mathbb{E}}\Big[r(s_t, a_t, s_{t+1}) + \gamma . r(s_{t+1}, a_{t+1}, s_{t+2}) + \underbrace{\sum_{i=2}^{\infty} \gamma^i r(s_{t+i}, a_{t+i}, s_{t+i+1})}_{} \Big| s_t = s \Big]$$

$$\underset{\pi}{\mathbb{E}}\Big[\gamma^2 \sum_{i=0}^{\infty} \gamma^i r(s_{t+2+i}, a_{t+2+i}, a_{t+2+i+1}) \Big| s_{t+2} = s_{t+2}\Big]$$

$$= \gamma^2 \underset{\pi}{\mathbb{E}}\Big[\sum_{i=0}^{\infty} \gamma^i r(s_{t+2+i}, a_{t+2+i}, a_{t+2+(i+1)}) \Big| s_{t+2}\Big]$$

$$= \gamma^2 V_\pi(s_{t+2}).$$

$$\therefore V_\pi(s) = \mathbb{E}\Big[r(s_t, a_t, s_{t+1}) + \gamma r(s_{t+1}, a_{t+1}, s_{t+2}) + \gamma^2 V_\pi(s_{t+2}) \Big| s_t = s \Big]$$