Math 21
Module 1: Limits and Continuity

Nile Jocson
<atraphaxia@gmail.com>

February 19, 2025

Contents

1 Limit of a Function: An Intuitive Approach **3**
1.1 3
1.2 4
1.2.1 4
1.2.2 5
1.2.3 6
1.2.4 7
1.2.5 8
1.2.6 9
1.2.7 10
1.2.8 11
1.2.9 12
1.2.10 13
1.3 14
1.3.1 14
1.3.2 15

2 One-Sided Limits **16**
2.1 16
2.1.1 16
2.1.2 17
2.1.3 18
2.1.4 19
2.1.5 20
2.1.6 21
2.1.7 22
2.1.8 23

2 One-Sided Limits

2.1

Evaluate the following limits.

2.1.1

$$\lim_{x \to 3^-} \sqrt{7x^2 - 12x + 5}$$

$= \lim_{x\to 3^-} \sqrt{7x^2 - 7x - 5x + 5}$	Factor.
$= \lim_{x\to 3^-} \sqrt{(7x-5)(x-1)}$	
$= \sqrt{(7(3)-5)(3-1)}$	Since $\sqrt{(7x-5)(x-1)} \not\to 0$ as $x \to 3^-$, eliminate the limit.
$= \sqrt{(16)(2)}$	
$= 4\sqrt{2}$	Final answer.
	■

2.1.2

$$\lim_{x \to \frac{3}{2}} \sqrt{4x^2 - 12x + 9}$$

$= \lim_{x \to \frac{3}{2}} \sqrt{4x^2 - 6x - 6x + 9}$	Factor.
$= \lim_{x \to \frac{3}{2}} \sqrt{(2x-3)^2}$	
$= 2(\frac{3}{2}) - 3$	
$= 0$	Final answer.
	$\blacksquare$

2.1.3

$$\lim_{x \to \frac{2}{3}^+} \lfloor 3x + 1 \rfloor$$

$= \lim_{x \to \frac{2}{3}^+} \lfloor 3^+ \rfloor$	
$= 3$	Final answer.
	$\blacksquare$

2.1.4

$$\lim_{x \to \frac{2}{3}^-} \lfloor 3x + 1 \rfloor$$

$= \lim_{x \to \frac{2}{3}^-} \lfloor 3^- \rfloor$	
$= 2$	Final answer.
	■

2.1.5

$$\lim_{x \to \frac{2}{3}} \lfloor 3x + 1 \rfloor$$

DNE	Final answer. If one-sided limits aren't equal, the limit doesn't exist. ■

2.1.6

$$\lim_{x\to 1} \frac{|4x| - |x-5|}{1-x}$$

$= \lim_{x\to 1} \frac{4x-\lvert x-5\rvert}{1-x}$	$\lvert 4x\rvert = 4x, x \geq 0$
$= \lim_{x\to 1} \frac{4x-(-x+5)}{1-x}$	$\lvert x-5\rvert = -x+5, x < 5$
$= \lim_{x\to 1} \frac{5x-5}{1-x}$	
$= \lim_{x\to 1} \frac{-5\cancel{(1-x)}}{\cancel{1-x}}$	
$= -5$	Final answer.
	■

2.1.7

$$\lim_{t\to -\frac{1}{2}} \frac{2t+1}{|2t^2-3t-2|}$$

$\lim_{t\to -\frac{1}{2}^-} \frac{2t+1}{\lvert 2t^2-3t-2\rvert}$	Evaluate negative one-sided limit.
$= \lim_{t\to -\frac{1}{2}^-} \frac{2t+1}{2t^2-3t-2}$	$\lvert 2t^2-3t-2\rvert = 2t^2-3t-2, x < -\frac{1}{2}$
$= \lim_{t\to -\frac{1}{2}^-} \frac{2t+1}{2t^2-4t+t-2}$	Factor.
$= \lim_{t\to -\frac{1}{2}^-} \frac{\cancel{2t+1}}{\cancel{(2t+1)}(t-2)}$	
$= \lim_{t\to -\frac{1}{2}^-} \frac{1}{t-2}$	
$= \frac{1}{-\frac{1}{2}-2}$	
$= -\frac{2}{5}$	
$\lim_{t\to -\frac{1}{2}^+} \frac{2t+1}{\lvert 2t^2-3t-2\rvert}$	Evaluate positive one-sided limit.
$= \lim_{t\to -\frac{1}{2}^-} \frac{2t+1}{-(2t^2-3t-2)}$	$\lvert 2t^2-3t-2\rvert = -(2t^2-3t-2), x \geq -\frac{1}{2}$
$= \lim_{t\to -\frac{1}{2}^-} -\frac{2t+1}{2t^2-3t-2}$	
$= \lim_{t\to -\frac{1}{2}^-} -\frac{2t+1}{2t^2-4t+t-2}$	Factor.
$= \lim_{t\to -\frac{1}{2}^-} -\frac{\cancel{2t+1}}{\cancel{(2t+1)}(t-2)}$	
$= \lim_{t\to -\frac{1}{2}^-} -\frac{1}{t-2}$	
$= -\frac{1}{-\frac{1}{2}-2}$	
$= \frac{2}{5}$	
DNE	Final answer. If one-sided limits aren't equal, the limit doesn't exist.
	■

2.1.8

$$\lim x \to 1^- \frac{x\lfloor x \rfloor - 1}{\lfloor x \rfloor - 1}$$

$= \lim_{x\to 1^-} \frac{1\lfloor 1^- \rfloor - 1}{\lfloor 1^- \rfloor - 1}$	
$= \lim_{x\to 1^-} \frac{1(0)-1}{(0)-1}$	
$= 1$	Final answer.
	■