

Math 21
Module 1: Limits and Continuity

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Contents

1	Limit of a Function: An Intuitive Approach	3
1.1	3
1.2	4
1.2.1	4
1.2.2	5
1.2.3	6
1.2.4	7
1.2.5	8
1.2.6	9
1.2.7	10
1.2.8	11
1.2.9	12
1.2.10	13
1.3	14
1.3.1	14
1.3.2	15
2	One-Sided Limits	16
2.1	16
2.1.1	16
2.1.2	17
2.1.3	18
2.1.4	19
2.1.5	20
2.1.6	21
2.1.7	22
2.1.8	23
3	Limits Involving Infinity	24
3.1	24
3.1.1	24
3.1.2	25
3.1.3	26
3.1.4	27
3.1.7	28
3.1.8	29
3.1.9	30

3 Limits Involving Infinity

3.1

Evaluate the following limits.

3.1.1

$$\lim_{x \rightarrow \frac{1}{3}^-} \frac{2}{1 - 3x}$$

$$= \lim_{x \rightarrow \frac{1}{3}^-} \frac{2}{0^+}$$

$$= +\infty$$

Final answer.



3.1.2

$$\lim_{t \rightarrow 0^+} \left(\frac{1}{t\sqrt{t+3}} - \frac{1}{t} \right)$$

Indeterminate, type $\infty - \infty$

$$= \lim_{t \rightarrow 0^+} \left(\frac{1}{t\sqrt{t+3}} - \frac{\sqrt{t+3}}{t\sqrt{t+3}} \right)$$

$$\text{lcm} = \sqrt{t+3}$$

$$= \lim_{t \rightarrow 0^+} \frac{1 - \sqrt{t+3}}{t\sqrt{t+3}}$$

$$= \lim_{t \rightarrow 0^+} \frac{1 - \sqrt{3}}{0^+}$$

$$= -\infty$$

Final answer. $1 - \sqrt{3}$ is negative.



3.1.3

$$\lim_{x \rightarrow 2^-} \frac{x-2}{2 - \sqrt{4x-x^2}}$$

Indeterminate, type $\frac{0}{0}$

$$= \lim_{x \rightarrow 2^-} \frac{(x-2)(2+\sqrt{4x-x^2})}{x^2-4x+4}$$

Rationalize.

$$= \lim_{x \rightarrow 2^-} \frac{\cancel{(x-2)}(2+\sqrt{4x-x^2})}{\cancel{(x-2)}(x-2)}$$

Factor.

$$= \lim_{x \rightarrow 2^-} \frac{2+\sqrt{4x-x^2}}{x-2}$$

$$= \lim_{x \rightarrow 2^-} \frac{4}{0^-}$$

$$= -\infty$$

Final answer.



3.1.4

$$\lim_{x \rightarrow -2^-} \left(\frac{4}{(x+2)^2(2-x)} - \frac{1}{(x+2)^2} - \frac{1}{x} \right)$$

Indeterminate, type $\infty - \infty$

$$= \lim_{x \rightarrow -2^-} \left(\frac{4x}{x(x+2)^2(2-x)} - \frac{x(2-x)}{x(x+2)^2(2-x)} - \frac{(x+2)^2(2-x)}{x(x+2)^2(2-x)} \right) \quad \text{lcm} = x(x+2)^2(2-x)$$

$$= \lim_{x \rightarrow -2^-} \frac{4x - x(2-x) - (x+2)^2(2-x)}{x(x+2)^2(2-x)}$$

$$= \lim_{x \rightarrow -2^-} \frac{x^2 + 2x - (x+2)^2(2-x)}{x(x+2)^2(2-x)}$$

$$= \lim_{x \rightarrow -2^-} \frac{x(x+2) - (x+2)^2(2-x)}{x(x+2)^2(2-x)}$$

$$= \lim_{x \rightarrow -2^-} \frac{\cancel{(x+2)}(x - (x+2)(2-x))}{x(x+2)\cancel{(2-x)}}$$

$$= \lim_{x \rightarrow -2^-} \frac{x - (x+2)(2-x)}{x(x+2)(2-x)}$$

$$= \lim_{x \rightarrow -2^-} \frac{-2 - (-2+2)(2+2)}{-2(-2+2)(2+2)}$$

$$= \lim_{x \rightarrow -2^-} \frac{-2}{-2(0^-)(4)}$$

$$= \lim_{x \rightarrow -2^-} \frac{-2}{0^+}$$

$$= -\infty$$

Final answer.



3.1.7

$$\lim_{x \rightarrow +\infty} \frac{2x^3 - 6x + 5}{4 + 7x - 6x^3}$$

Indeterminate, type $\frac{\infty}{\infty}$

$$= \lim_{x \rightarrow +\infty} \frac{2x^3 - 6x + 5}{4 + 7x - 6x^3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}}$$

Divide by highest denominator power.

$$= \lim_{x \rightarrow +\infty} \frac{2 - \frac{6}{x^2} + \frac{5}{x^3}}{\frac{4}{x^3} + \frac{7}{x^2} - 6}$$

$$= \lim_{x \rightarrow +\infty} \frac{2}{-6}$$

$$= -\frac{1}{3}$$

Final answer.



3.1.8

$$\lim_{z \rightarrow +\infty} \frac{4z^3 + 5}{1 - 2z + 3z^2}$$

Indeterminate, type $\frac{\infty}{\infty}$

$$= \lim_{z \rightarrow +\infty} \frac{4z^3 + 5}{1 - 2z + 3z^2} \cdot \frac{\frac{1}{z^2}}{\frac{1}{z^2}}$$

Divide by highest denominator power.

$$= \lim_{z \rightarrow +\infty} \frac{4z + \frac{5}{z^2}}{\frac{1}{z^2} - \frac{2}{z} + 3}$$

$$= \lim_{z \rightarrow +\infty} \frac{\infty}{3}$$

$$= +\infty$$

Final answer.



3.1.9

$$\lim_{x \rightarrow +\infty} \frac{3 - x^2}{\sqrt{4x^2 + 1} + x^2}$$

Indeterminate, type $\frac{\infty}{\infty}$

$$= \lim_{x \rightarrow +\infty} \frac{3 - x^2}{\sqrt{4x^2 + 1} + x^2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

Divide by highest denominator power.

$$= \lim_{x \rightarrow +\infty} \frac{\frac{3}{x^2} - 1}{\sqrt{\frac{4}{x^2} + \frac{1}{x^4}} + 1}$$

$$= \lim_{x \rightarrow +\infty} \frac{-1}{1}$$

$$= -1$$

Final answer.

