Math 21
Module 1: Limits and Continuity

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Contents

3 Limits Involving Infinity

3.1

Evaluate the following limits.

3.1.1

$$\lim_{x \to \frac{1}{3}^-} \frac{2}{1-3x}$$

$= \lim_{x \to \frac{1}{3}^-} \frac{2}{0^+}$
$= +\infty$ Final answer. ■

3.1.2

$$\lim_{t\to 0^+}\left(\frac{1}{t\sqrt{t+3}}-\frac{1}{t}\right)$$

Indeterminate, type $\infty-\infty$	
$=\lim_{t\to 0^+}(\frac{1}{t\sqrt{t+3}}-\frac{\sqrt{t+3}}{t\sqrt{t+3}})$	lcm $=\sqrt{t+3}$
$=\lim_{t\to 0^+}\frac{1-\sqrt{t+3}}{t\sqrt{t+3}}$	
$=\lim_{t\to 0^+}\frac{1-\sqrt{3}}{0^+}$	
$=-\infty$	Final answer. $1-\sqrt{3}$ is negative.
	■

3.1.3

$$\lim_{x\to 2^-} \frac{x-2}{2-\sqrt{4x-x^2}}$$

Indeterminate, type $\frac{0}{0}$	
$= \lim_{x\to 2^-} \frac{(x-2)(2+\sqrt{4x-x^2})}{x^2-4x+4}$	Rationalize.
$= \lim_{x\to 2^-} \frac{\cancel{(x-2)}(2+\sqrt{4x-x^2})}{\cancel{(x-2)}(x-2)}$	Factor.
$= \lim_{x\to 2^-} \frac{2+\sqrt{4x-x^2}}{x-2}$	
$= \lim_{x\to 2^-} \frac{4}{0^-}$	
$= -\infty$	Final answer.
	■

3.1.4

$$\lim_{x\to -2^-}\left(\frac{4}{(x+2)^2(2-x)}-\frac{1}{(x+2)^2}-\frac{1}{x}\right)$$

Indeterminate, type $\infty-\infty$

$= \lim_{x\to -2^-}\left(\frac{4x}{x(x+2)^2(2-x)}-\frac{x(2-x)}{x(x+2)^2(2-x)}-\frac{(x+2)^2(2-x)}{x(x+2)^2(2-x)}\right)$ lcm $= x(x+2)^2(2-x)$

$= \lim_{x\to -2^-}\frac{4x-x(2-x)-(x+2)^2(2-x)}{x(x+2)^2(2-x)}$

$= \lim_{x\to -2^-}\frac{x^2+2x-(x+2)^2(2-x)}{x(x+2)^2(2-x)}$

$= \lim_{x\to -2^-}\frac{x(x+2)-(x+2)^2(2-x)}{x(x+2)^2(2-x)}$

$= \lim_{x\to -2^-}\frac{\cancel{(x+2)}(x-(x+2)(2-x))}{x(x+2)\cancel{^2}(2-x)}$

$= \lim_{x\to -2^-}\frac{x-(x+2)(2-x)}{x(x+2)(2-x)}$

$= \lim_{x\to -2^-}\frac{-2-(-2+2)(2+2)}{-2(-2+2)(2+2)}$

$= \lim_{x\to -2^-}\frac{-2}{-2(0^-)(4)}$

$= \lim_{x\to -2^-}\frac{-2}{0^+}$

$= -\infty$ Final answer.

■

3.1.7

$$\lim_{x\to+\infty}\frac{2x^3-6x+5}{4+7x-6x^3}$$

Indeterminate, type $\frac{\infty}{\infty}$	
$=\lim_{x\to+\infty}\frac{2x^3-6x+5}{4+7x-6x^3}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}}$	Divide by highest denominator power.
$=\lim_{x\to+\infty}\frac{2-\frac{6}{x^2}+\frac{5}{x^3}}{\frac{4}{x^3}+\frac{7}{x^2}-6}$	
$=\lim_{x\to+\infty}\frac{2}{-6}$	
$=-\frac{1}{3}$	Final answer.
■	

3.1.8

$$\lim_{z\to+\infty} \frac{4z^3+5}{1-2z+3z^2}$$

Indeterminate, type $\frac{\infty}{\infty}$	
$= \lim_{z\to+\infty} \frac{4z^3+5}{1-2z+3z^2} \cdot \frac{\frac{1}{z^2}}{\frac{1}{z^2}}$	Divide by highest denominator power.
$= \lim_{z\to+\infty} \frac{4z+\frac{5}{z^2}}{\frac{1}{z^2}-\frac{2}{z}+3}$	
$= \lim_{z\to+\infty} \frac{\infty}{3}$	
$= +\infty$	Final answer.
■	

3.1.9

$$\lim_{x\to+\infty} \frac{3-x^2}{\sqrt{4x^2+1}+x^2}$$

Indeterminate, type $\frac{\infty}{\infty}$	
$= \lim_{x\to+\infty} \frac{3-x^2}{\sqrt{4x^2+1}+x^2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{\sqrt{x^4}}}$	Divide by highest denominator power.
$= \lim_{x\to+\infty} \frac{\frac{3}{x^2}-1}{\sqrt{\frac{4}{x^2}+\frac{1}{x^4}}+1}$	
$= \lim_{x\to+\infty} \frac{-1}{1}$	
$= -1$	Final answer.
	■