Math 21
Module 1: Limits and Continuity

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Contents

1.2

Evaluate the following limits.

1.2.1

$$\lim_{x \to -1} x(x-2)(x+2)$$

$= -1(-1-2)(-1+2)$	$\lim_{x \to a} f(x) = f(a)$ if a is in the domain of f.
$= -1(-3)(1)$	
$= 3$	Final answer.
	■

1.2.2

$$\lim_{x\to -1} \frac{3x^2+2x-1}{x^3+1}$$

Indeterminate, type $\frac{0}{0}$.	
$= \lim_{x\to -1} \frac{3x^2+3x-x-1}{x^3+1}$	Factor by grouping.
$= \lim_{x\to -1} \frac{(3x-1)(x+1)}{x^3+1}$	
$= \lim_{x\to -1} \frac{(3x-1)\cancel{(x+1)}}{\cancel{(x+1)}(x^2-x+1)}$	Factor using sum of two cubes.
$= \frac{(3(-1)-1)}{(-1)^2-(-1)+1}$	
$= -\frac{4}{3}$	Final answer.
	■

1.2.3

$$\lim_{t\to 18} \frac{\sqrt{t-2}-4}{t-18}$$

Indeterminate, type $\frac{0}{0}$.

$= \lim_{t\to 18} \frac{\sqrt{t-2}-4}{t-18} \cdot \frac{\sqrt{t-2}+4}{\sqrt{t-2}+4}$ Rationalize.

$= \lim_{t\to 18} \frac{t-2-16}{(t-18)(\sqrt{t-2}+4)}$

$= \lim_{t\to 18} \frac{\cancel{t-18}}{\cancel{(t-18)}(\sqrt{t-2}+4)}$

$= \lim_{t\to 18} \frac{1}{\sqrt{t-2}+4}$

$= \frac{1}{\sqrt{18-2}+4}$

$= \frac{1}{8}$ Final answer.

■

1.2.4

$$\lim_{s\to 3}\frac{2s^2-7s+3}{s^2-4s+3}$$

Indeterminate, type $\frac{0}{0}$.

$= \lim_{s\to 3} \frac{2s^2-6s-s+3}{(s-3)(s-1)}$ Factor by grouping.

$= \lim_{s\to 3} \frac{(2s-1)\cancel{(s-3)}}{\cancel{(s-3)}(s-1)}$

$= \lim_{s\to 3} \frac{2s-1}{s-1}$

$= \frac{2(3)-1}{3-1}$

$= \frac{5}{2}$ Final answer.

■

1.2.5

$$\lim_{x\to 4} \frac{2x^2 - 13x + 20}{x^3 - 64}$$

Indeterminate, type $\frac{0}{0}$.

$= \lim_{x\to 4} \frac{2x^2-5x-8x+20}{x^3-64}$ Factor by grouping.

$= \lim_{x\to 4} \frac{(x-4)(2x-5)}{x^3-64}$

$= \lim_{x\to 4} \frac{\cancel{(x-4)}(2x-5)}{\cancel{(x-4)}(x^2+4x+16)}$ Factor using difference of two cubes.

$= \lim_{x\to 4} \frac{2x-5}{x^2+4x+16}$

$= \frac{2(4)-5}{4^2+4(4)+16}$

$= \frac{3}{48}$

$= \frac{1}{16}$ Final answer.

■

1.2.6

$$\lim_{z\to 2}\left(\frac{2z-z^2}{z^2-4}\right)^3$$

Indeterminate, type $\frac{0}{0}$.

$= \lim_{z\to 2}\left(\frac{-z\cancel{(z-2)}}{\cancel{(z-2)}(z+2)}\right)^3$ Factor.

$= \lim_{z\to 2}\left(\frac{-z}{z+2}\right)^3$

$= \left(\frac{-2}{2+2}\right)^3$

$= \left(-\frac{1}{2}\right)^3$

$= -\frac{1}{8}$

■

1.2.7

$$\lim_{x \to -1} \frac{\sqrt{x^2+3}-2}{x^2-1}$$

Indeterminate, type $\frac{0}{0}$.	
$= \lim_{x\to -1} \frac{\sqrt{x^2+3}-2}{x^2-1} \cdot \frac{\sqrt{x^2+3}+2}{\sqrt{x^2+3}+2}$	Rationalize.
$= \lim_{x\to -1} \frac{x^2+3-4}{(x^2-1)(\sqrt{x^2+3}+2)}$	
$= \lim_{x\to -1} \frac{\cancel{x^2-1}}{\cancel{(x^2-1)}(\sqrt{x^2+3}+2)}$	
$= \lim_{x\to -1} \frac{1}{\sqrt{x^2+3}+2}$	
$= \frac{1}{\sqrt{(-1)^2+3}+2}$	
$= \frac{1}{4}$	Final answer.
	■

1.2.8

$$\lim_{x\to 2}\frac{\sqrt{2x}-\sqrt{6-x}}{4-x^2}$$

Indeterminate, type $\frac{0}{0}$.

$= \lim_{x\to 2} \frac{\sqrt{2x}-\sqrt{6-x}}{4-x^2} \cdot \frac{\sqrt{2x}+\sqrt{6-x}}{\sqrt{2x}+\sqrt{6-x}}$ Rationalize.

$= \lim_{x\to 2} \frac{2x-(6-x)}{(4-x^2)(\sqrt{2x}+\sqrt{6-x})}$

$= \lim_{x\to 2} \frac{3x-6}{(4-x^2)(\sqrt{2x}+\sqrt{6-x})}$

$= \lim_{x\to 2} \frac{-3\cancel{(2-x)}}{\cancel{(2-x)}(2+x)(\sqrt{2x}+\sqrt{6-x})}$ Factor.

$= \lim_{x\to 2} -\frac{3}{(2+x)(\sqrt{2x}+\sqrt{6-x})}$

$= -\frac{3}{(2+2)(\sqrt{2(2)}+\sqrt{6-2})}$

$= -\frac{3}{16}$

■

1.2.9

$$\lim_{p \to 1} \frac{p^3 - 1}{\sqrt{2p-1} - 1}$$

Indeterminate, type $\frac{0}{0}$.

$= \lim_{p\to 1} \frac{p^3-1}{\sqrt{2p-1}-1} \cdot \frac{\sqrt{2p-1}+1}{\sqrt{2p-1}+1}$ Rationalize.

$= \lim_{p\to 1} \frac{(p^3-1)(\sqrt{2p-1}+1)}{2p-1-1}$

$= \lim_{p\to 1} \frac{(p^3-1)(\sqrt{2p-1}+1)}{2p-2}$

$= \lim_{p\to 1} \frac{\cancel{(p-1)}(p^2+p+1)(\sqrt{2p-1}+1)}{2\cancel{(p-1)}}$ Factor.

$= \lim_{p\to 1} \frac{(p^2+p+1)(\sqrt{2p-1}+1)}{2}$

$= \frac{(1^2+1+1)(\sqrt{2(1)-1}+1)}{2}$

$= \frac{6}{2}$

$= 3$ Final answer.

■

1.2.10

$$\lim_{x\to -3} \frac{6x+2x^2}{-1-\sqrt[3]{2x+5}}$$

Indeterminate, type $\frac{0}{0}$.

$= \lim_{x\to -3} \frac{6x+2x^2}{-(1+\sqrt[3]{2x+5})}$

$= \lim_{x\to -3} \frac{6x+2x^2}{-(1+\sqrt[3]{2x+5})} \cdot \frac{1-\sqrt[3]{2x+5}+(\sqrt[3]{2x+5})^2}{1-\sqrt[3]{2x+5}+(\sqrt[3]{2x+5})^2}$ Rationalize using sum of two cubes.

$= \lim_{x\to -3} \frac{(6x+2x^2)(1-\sqrt[3]{2x+5}+(\sqrt[3]{2x+5})^2)}{-(1+2x+5)}$

$= \lim_{x\to -3} \frac{(6x+2x^2)(1-\sqrt[3]{2x+5}+(\sqrt[3]{2x+5})^2)}{-2x-6}$

$= \lim_{x\to -3} \frac{-x\cancel{(-2x-6)}(1-\sqrt[3]{2x+5}+(\sqrt[3]{2x+5})^2)}{\cancel{-2x-6}}$ Factor.

$= \lim_{x\to -3} -x(1-\sqrt[3]{2x+5}+(\sqrt[3]{2x+5})^2)$

$= -(-3)(1-\sqrt[3]{2(-3)+5}+(\sqrt[3]{2(-3)+5})^2)$

$= 3(1-\sqrt[3]{-1}+(\sqrt[3]{-1})^2)$

$= 3(1+1+1)$

$= 9$ Final answer.

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