Computational Physics 10/23/2020

Midterm Friday

1. False. It is not always better to choose a smaller Δt. One example is for numerical integration where change in the function is small over a large interval. In that case choosing small Δt will increase the run time considerabply. Also, very small Δt gives significant round-off error in adaptative integration

2. In non-adaptive numerical method Δt is constant in the simulation whereas, in adaptative numerical method, Δt is adjusted depending on the function.

One disadvantage of adaptative numerical methods is that round off error becomes very significant at very small time steps. For this reason it may fail to adapt a time step of desired accuracy.

3b) Comparing the figures from 3b and 3a we see that the y axis of power spectrum decreases as frequency increases and after a certain value of frequency it again increases. Now, an ideal low pass filter allows only low frequency signal below a cutoff to pass, attenuating high frequency signals. Thus, it can be argued that the averaging method is an example of a low pass filter (though not an ideal one). In other words, without averaging, the power spectrum had ~~same~~ similar y axis values, which decreased for ~~high~~ increasing frequencies ~~after~~ within an interval, which is expected from a low pass filter.

- Without averaging, the signal had rapid changes in magnitude which implies it had high frequency components. Averaging smoothed the signal thus lowering ~~the~~ some high frequency components. So, the name low-pass filter is appropriate for the averaging method.

4.a) In order for the sum to converge, $\|I - A\| < 1$.
⇒ A can not be null matrix and all its eigen values have to be +ve

$A \rightarrow n \times n$ matrix

$(I-A) \rightarrow n^2$ elements each $\rightarrow O(n^2)$ operations

$(I-A)^2 \rightarrow n \times n$ elements

$(I-A)^2_{ij} = \sum_{k=1}^{n} (I-A)_{ik} (I-A)_{kj} \rightarrow O(n)$

$\Rightarrow (I-A)^2 \longrightarrow O(n^2 \times n) = O(n^3)$

$(I-A)^3 = (I-A)^2 \cdot (I-A) = O(n^3) \cdot O(n^3)$

$= O(n^6) = O\{(n^3)^{3-1}\}$

$(I-A)^k = O[(n^3)^{k-1}]$

$\frac{(I-A)^k}{n} = O(n^2)$

$-\sum_{k=1}^{n} \frac{(I-A)^k}{k} = O(m \times n^2)$ (truncating series at m)

$\therefore$ Complexity $= O\left(mn^2 + \sum_k (n^3)^{k-1} + n^2\right)$

$= O\left(\cancel{m(n} \; O\left(n^2(m+1) + \sum_k (n^{3k} - n^3)\right)\right)$

$= O\left(n^2(m+1) + \sum_{k=1}^{m} n^{3k} - n^3 m\right)$

$$\sum_{k=1}^{m} n^{3k} = n^{3k} \left(\frac{n^{3m} - 1}{n^{3} - 1} \right.$$

$m = 200$

$\therefore$ <u>complexity</u> : $O\left(201 n^{2} + \frac{n^{603} - 1}{n^{3} - 1} - n^{3} 20 \right.$

for large n

$$\Rightarrow O\left(201 n^{2} + \frac{n^{603}}{n^{3}} - 201 n^{3} \right)$$

$$\simeq O\left(201 n^{2} (1 - n) + n^{301} \right)$$

5b) At $t=0$, $\vec{r}_1 = 0$, $\vec{r}_2 = -\vec{r}_3$, $\vec{V}_2 = -\vec{V}_3$

at $t=0$, $\ddot{\vec{r}}_1 = \dot{\vec{v}}_1 = -\dfrac{-\vec{r}_2}{|-\vec{r}_2|^3} - \dfrac{\vec{r}_2}{|r_2|^3}$

$= 0$

also $\because$ at $t=0$, $\vec{r}_1 = 0$, $\dot{\vec{r}}_1 = 0$.

$\therefore$ the star 1 will remain stationary at all time.

$\vec{r}_1 = d\vec{r}_1$.

~~if at $t=0$, $\vec{r}_1 = dr$.~~ $\vec{v}_1 = \dfrac{d\vec{r}_1}{dt}$

$\dot{\vec{r}}_1 = v_1$

$\dot{\vec{v}}_1 = -\dfrac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3} - \dfrac{\vec{r}_1 + \vec{r}_2}{|\vec{r}_1 + \vec{r}_2|^3}$

$\cancel{\dot{\vec{v}}_1 = \left[0 \quad \dfrac{1}{|\vec{r}_1 - \vec{r}_2|^3} \right] \quad 1 \quad \dot{\vec{v}}_1 = \dfrac{d\vec{r}_1 - \vec{r}_2}{|d\vec{r}_1 - \vec{r}_2|^3} -}$

Is this solution is not stable if $\vec{r}_1$ is purturbed

a) $U(R_1, R_2, R_3) = -GM^2\left(\frac{1}{|R_1 - R_2|} + \frac{1}{|R_2 - R_3|} + \frac{1}{|R_1 - R_3|}\right)$

$\cancel{\vec{R_1} - \vec{R_2} = \vec{r_1}}$

$$\vec{F}_{12} = -\nabla U_{12} = -\nabla\left(\frac{-GM^2}{|R_1 - R_2|}\right)$$

$$\vec{R_1} - \vec{R_2} = \vec{r}_{12}.$$

$$\vec{F}_{12} = -\frac{GM^2}{r_{12}^2}\hat{r}_{12} = -\frac{GM^2}{r_{12}^3}\vec{r}_{12}$$

$$\vec{F}_{13} = -\frac{GM^2}{r_{13}^3}\vec{r}_{13}.$$

$$\cancel{M}\frac{d^2\vec{r_1}}{dt^2} = -GM^{\cancel{2}}\left[\frac{\vec{r}_{12}}{r_{12}^3} + \frac{\vec{r}_{13}}{r_{13}^3}\right]$$

$$\frac{d^2\vec{r_1}}{dt^2} = -GM\left[\frac{\vec{r}_{12}}{r_{12}^3} + \frac{\vec{r}_{13}}{r_{13}^3}\right]$$

$$\frac{1}{GM}\frac{d^2}{dt^2} = \frac{d^2}{d\tau^2} \quad \text{or,} \quad \frac{1}{\sqrt{GM}}\frac{d}{dt} = \frac{d}{d\tau}$$

$$\tau = \sqrt{GM}\,t$$

$$\ddot{\vec{r}}_1 = -\left(\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3} + \frac{\vec{r}_1 - \vec{r}_3}{|\vec{r}_1 - \vec{r}_3|^3}\right), \ldots$$

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$$\frac{dr}{d\tau} = \frac{1}{\sqrt{GM}}\frac{dr}{dt} = \frac{1}{\sqrt{GM}} 1\,Km/s$$

$$= \cancel{\frac{1}{\sqrt{N\,Kg^{-2}m^2M}} \times 10^3 m/s}$$

$$GM = \frac{\cancel{G} \times 4\pi^2 \times (1\,AU)^3}{\cancel{G} \times (1\,yr)^2}$$

$$G = 6.672\,59 \times 10^{-11}\,Nm^2/Kg^2$$

$$1\,M = 2 \times 10^{30}\,kg,\ G$$

~~Can be done~~

$$\sqrt{GM} = \sqrt{1.334518 \times 10^{20} Nm^2/kg}$$

$$= 1.15 \times 10^{10}\,N^{-1/2}\,m/kg$$

$$= 1.15 \times 10^{10} \sqrt{\frac{Kg\,m/s^2}{Kg}}\,m$$

$$V = \frac{10^3}{1.15 \times 10^{10}} \frac{m\,s^{-1}}{m^{1+\frac{1}{2}}\,.\,s^{-1}}$$

$6.674 \times 10^{-11}\ m^3\, kg^{-1}\, s^{-2}$

$13.248 \times 10^{-11+30} = 1.325 \times 10^{20}$

$[G] = N\, kg^{-2}\, m^2 = kg\, m\, s^{-2}\, kg^{-2}\, m^2 = kg^{-1}\, m^3\, s^{-2}$

$[M] = kg$

$[GM] = m^3\, s^{-2}$

$1 - 3/2$

$= \frac{2-3}{2} = -1/2$

$\sqrt{[GM]} = m^{3/2}\, s^{-1}$

$\frac{dr}{dt} = 1\ km/s$

$\frac{1}{\sqrt{GM}} \simeq \frac{1}{1.15 \times 10^{10}} \times \frac{1\ km/s}{m^{3/2}/s} = \frac{1}{\sqrt{m}}$

$= \frac{10^{3-10}}{1.15} \simeq 8.69 \times 10^{-8}$ units.

$\frac{d^2R}{dt^2} = -GM(\ \ )$

5d) The orbit is no longer stable and periodic

5e) The trajectory is different. This is due to ringing effect