Spatiotemporal forecasting of plant populations and a proposal to partition forecast uncertainty

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FORECASTING

Skillfull forecasts are in high demand because the world is changing fast.

1. Responses to global climate change
2. Species invasions
3. Emergence and spread of infectious diseases

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1. Responses to global climate change
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ROAD MAP

1. Plant population forecasts over large spatial extents
2. A proposal for partitioning forecast uncertainty

§ 1

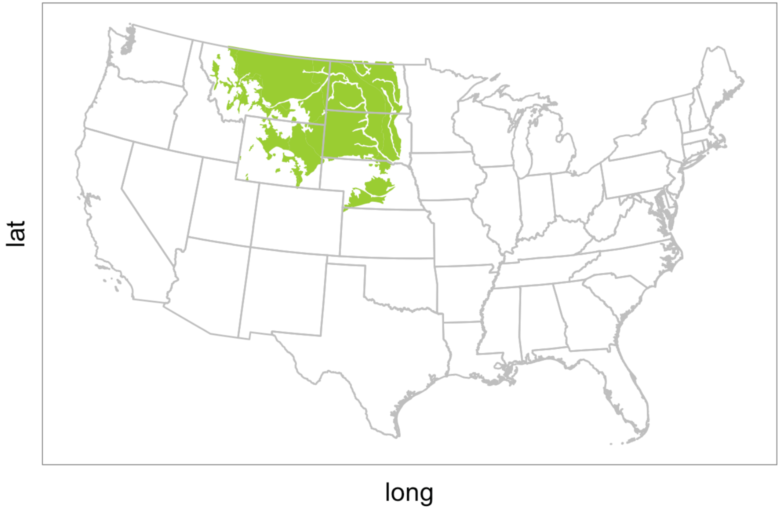
Plant population forecasts

COLLABORATORS

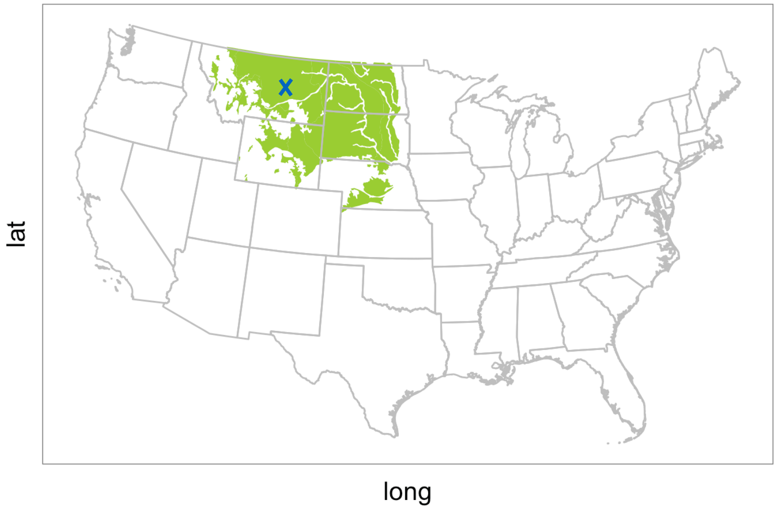
Peter Adler (USU)

Mevin Hooten (CSU)

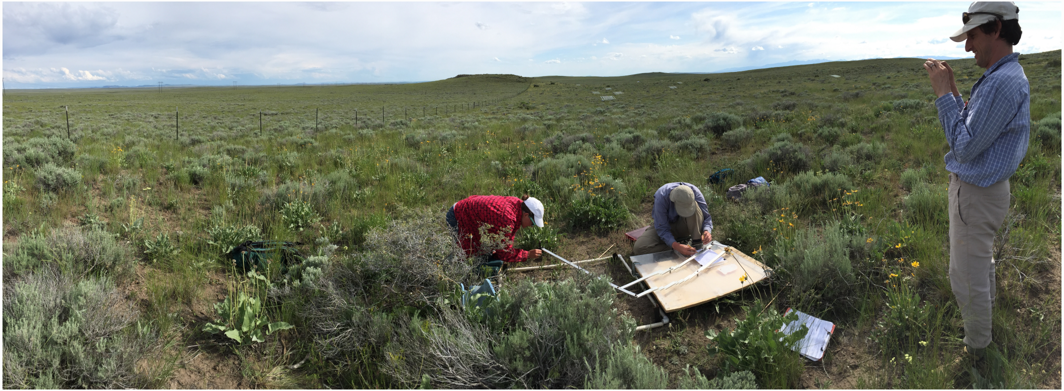
WHAT LAND MANAGERS WANT



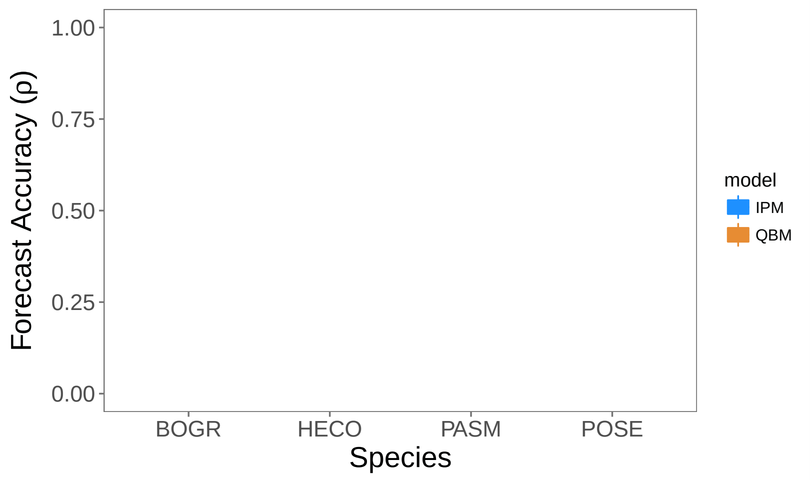
WHAT LAND MANAGERS GET



REALLY HARD WORK

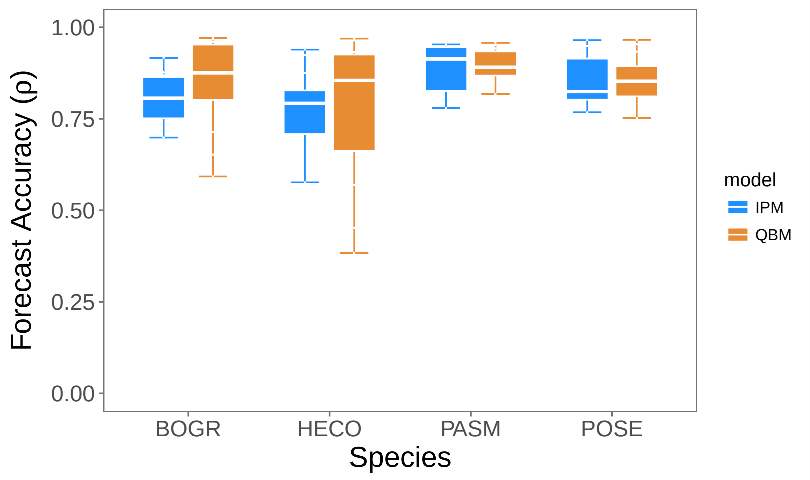


DO WE NEED DEMOGRAPHIC DATA?



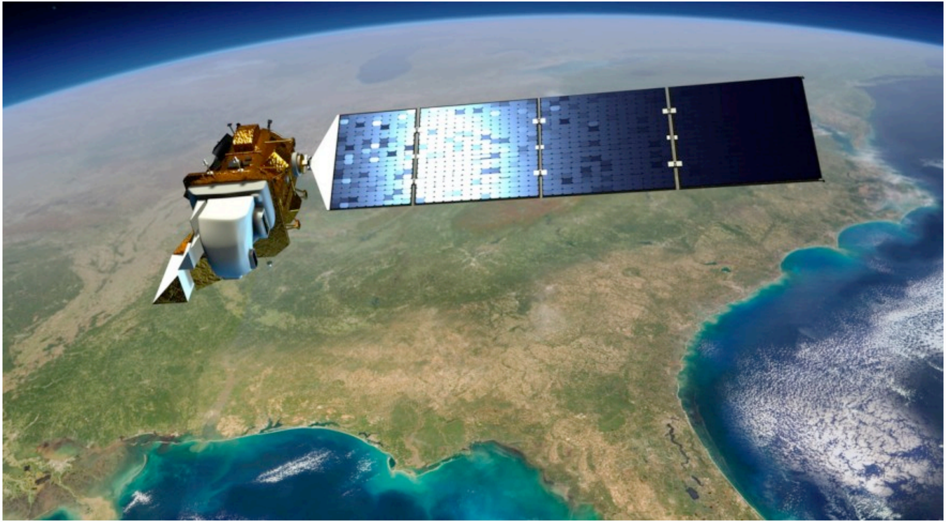
Tredennick et al., 2017, *Methods in Ecol. Evol.*

FREE FROM THE TYRANNY OF DEMOGRAPHIC DATA!



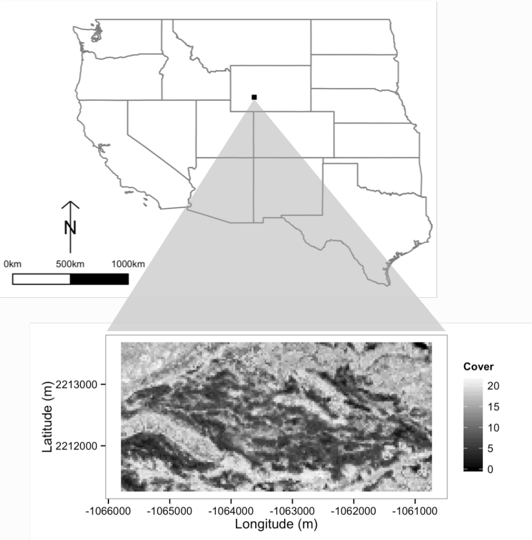
Tredennick et al., 2017, *Methods in Ecol. Evol.*

LET’S USE SATELLITES

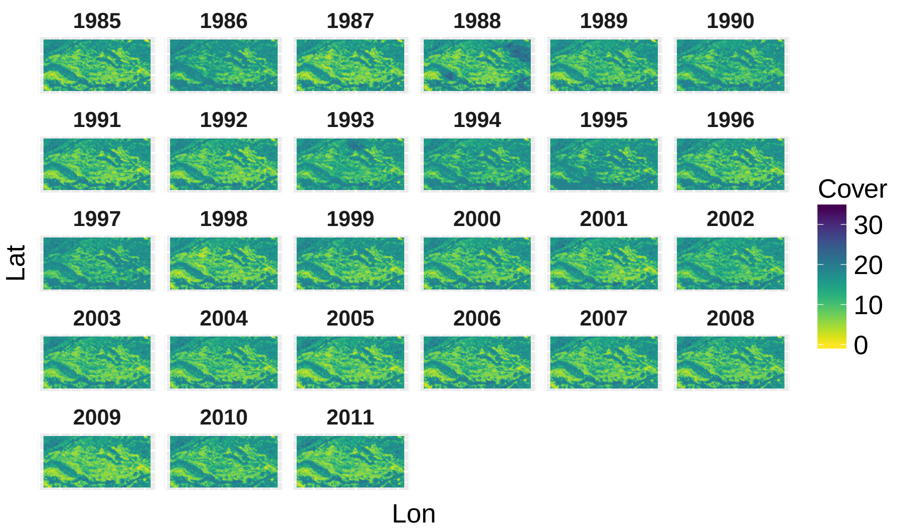


SAGEBRUSH SEA IN WYOMING

STUDY AREA


N
0km
500km
1000km
Cover
20
15
10
5
0
2213000
2212000
Latitude (m)
-1066000
-1065000
-1064000
-1063000
-1062000
-1061000
Longitude (m)

LANDSAT TIME SERIES

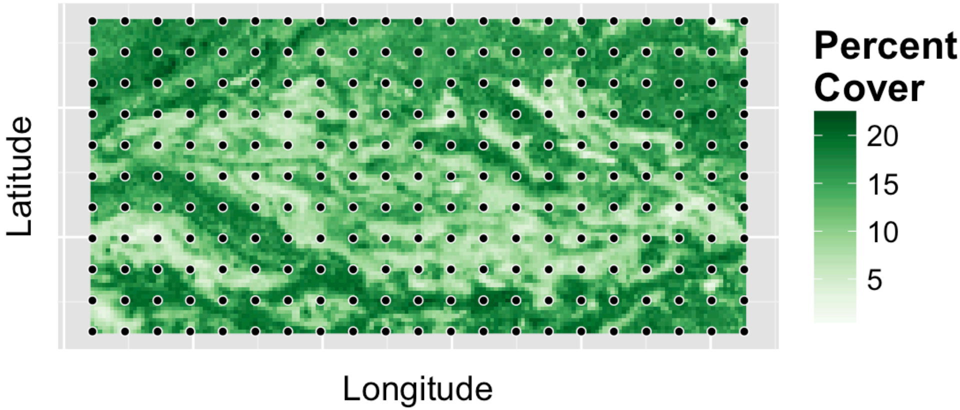


DYNAMIC COVER MODEL

$$y_{i,t} \sim \text{Poisson}(\mu_{i,t})$$

$$\log(\mu_{i,t}) = \underbrace{\beta_{0,t} + \beta_1 y_{i,t-1}}_{\text{temporal + dens. dep}} + \underbrace{\mathbf{x}_t' \gamma}_{\text{climate}} + \underbrace{\eta_i}_{\text{spatial}}$$

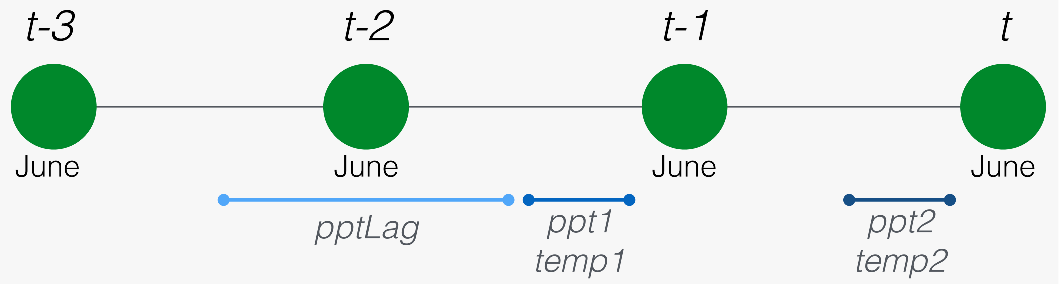
DIMENSION REDUCTION FOR SPATIAL EFFECT



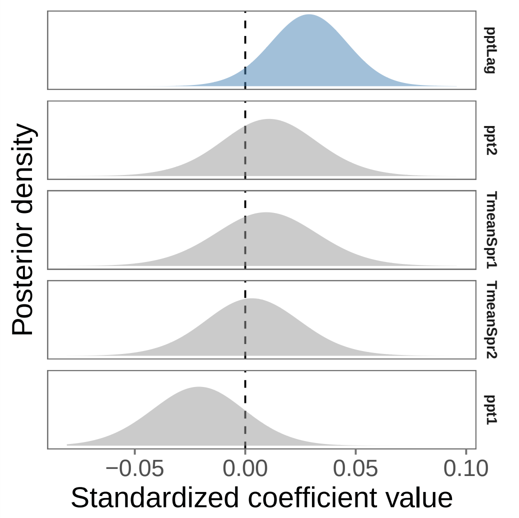
DYNAMIC COVER MODEL

$$
\begin{aligned}
y_{i,t} &\sim \text{Poisson}(\mu_{i,t}) \\
\log(\mu_{i,t}) &= \underbrace{\beta_{0,t} + \beta_1 y_{i,t-1}}_{\text{temporal + dens. dep}} + \underbrace{\mathbf{x}_t'\gamma}_{\text{climate}} + \underbrace{\eta_i}_{\text{spatial}} \\
\eta &\approx \mathbf{K}\alpha, \\
\mathbf{K} &= \mathbf{w}_{s,m} / \sum_{s=1}^{S} \mathbf{w}_{s,m} \\
\mathbf{w}_{s,m} &= \exp\left(-d_{s,m}/\sigma\right) \\
\alpha_m &\sim \text{Normal}(0, \sigma_\eta^2)
\end{aligned}
$$

CLIMATE COVARIATES

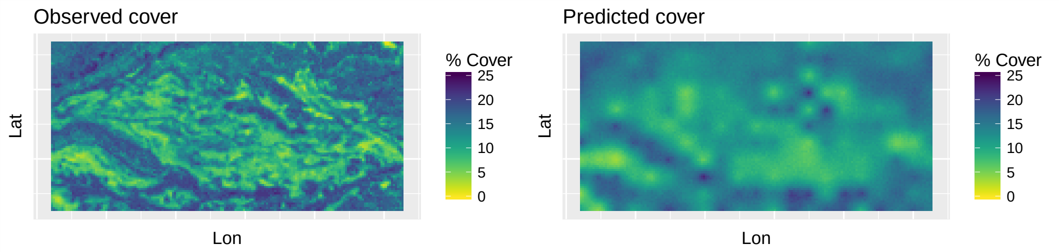


CLIMATE EFFECTS



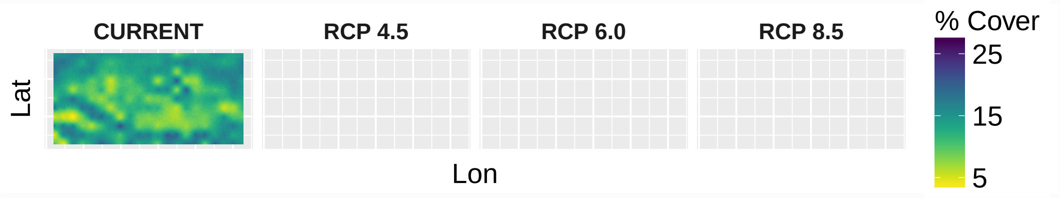
MODEL PERFORMANCE

In-sample RMSE ≈ 4%



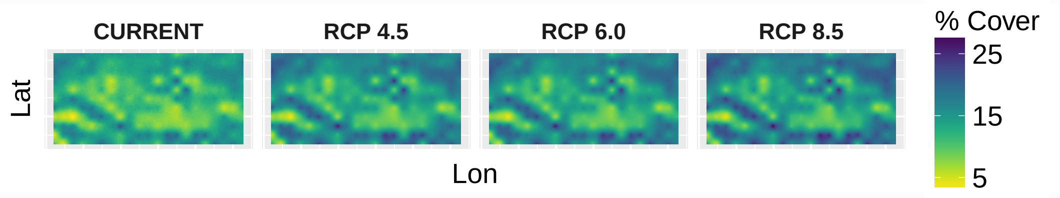
Tredennick et al., 2016, *Ecosphere*

FORECASTS UNDER CLIMATE CHANGE: SPATIAL



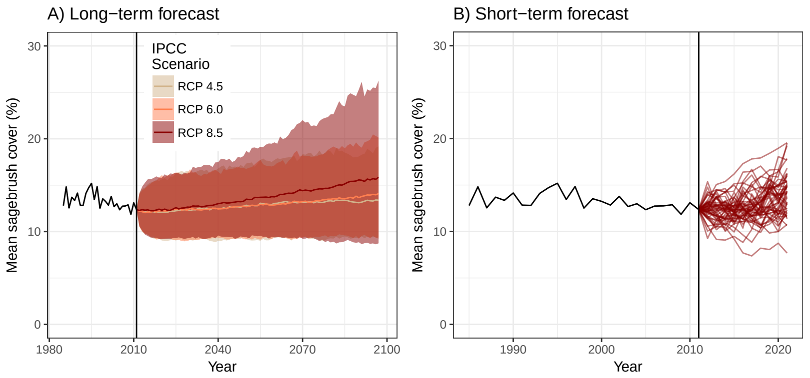
Tredennick et al., 2016, *Ecosphere*

FORECASTS UNDER CLIMATE CHANGE: SPATIAL



Tredennick et al., 2016, *Ecosphere*

FORECASTS UNDER CLIMATE CHANGE: TEMPORAL



Tredennick et al., 2016, *Ecosphere*

§ 2

Partitioning forecast uncertainty

FORECAST UNCERTAINTY, TO A FIRST APPROXIMATION

Forecast of state z at $t+1$ from function q: $q = f\big(z_t, x_t, \theta, \varepsilon_{t+1}\big)$

$$\mathrm{var}[y_{t+1}] \approx \underbrace{\left(\frac{\delta q}{\delta y}\right)^2}_{\text{stability}} \underbrace{\mathrm{var}[y_t]}_{\text{IC uncert.}} + \underbrace{\left(\frac{\delta f}{\delta x}\right)^2}_{\text{driver sens.}} \underbrace{\mathrm{var}[x_t]}_{\text{driver uncert.}} + \underbrace{\left(\frac{\delta f}{\delta \theta}\right)^2}_{\text{param sens.}} \underbrace{\mathrm{var}[\theta]}_{\text{param. uncert.}} + \underbrace{\mathrm{var}[\varepsilon_{t+1}]}_{\text{process error}}$$

Dietze, 2017, *Ecological Applications*

ERROR PROPAGATION

For some function q: $q = f(x_1, x_2, \ldots, x_n)$

$$\begin{aligned}\sigma_q^2 &= \left(\frac{\delta q}{\delta x_1}\sigma_{x_1}\right)^2 + \left(\frac{\delta q}{\delta x_2}\sigma_{x_2}\right)^2 + \cdots + \left(\frac{\delta q}{\delta x_n}\sigma_{x_n}\right)^2 \\ &= \sum_{i=1}^{n}\left(\frac{\delta q}{\delta x_i}\sigma_{x_i}\right)^2\end{aligned}$$

Ku, 1966, *J. Res. National Bureau of Standards - C*

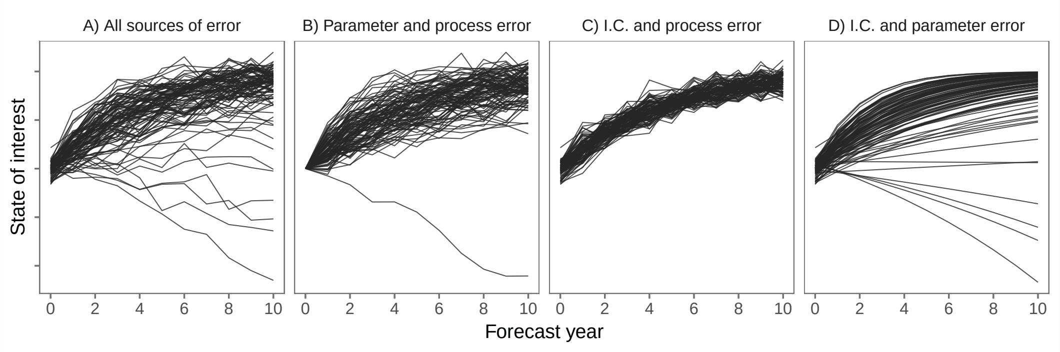
$$\sigma_q^2 = \underbrace{\sum_{i=1}^{n} \left(\frac{\delta q}{\delta x_i} \sigma_{x_i} \right)^2}_{\text{variances}} + \underbrace{\sum_{i=1}^{n} \sum_{j(j \neq i)}^{n} 2\sigma_{ij} \left(\frac{\delta q}{\delta x_i} \right) \left(\frac{\delta q}{\delta x_j} \right)}_{\text{covariances}}$$

Ku, 1966, *J. Res. National Bureau of Standards - C*

CAN WE IGNORE COVARIANCES?

$$
\begin{aligned}
z_{t+1} &= z_t\beta_0 + x_t\beta_1 + \varepsilon_{t+1}, \\
z_{t=1} &\sim \text{Normal}\left(z_0, \sigma^2_{\text{init.}}\right), && \text{initial conditions uncertainty} \\
\boldsymbol{\beta} &\sim \text{MVN}\left(0, \sigma^2_{\text{param.}}\mathbf{I}\right), && \text{parameter uncertainty} \\
\varepsilon_t &\sim \text{Normal}\left(0, \sigma^2_{\text{proc.}}\right) && \text{process uncertainty}
\end{aligned}
$$

INTERACTION (COVARIANCES) CANNOT BE IGNORED



AN *INVERSE* ERROR PROPOGATION PROBLEM

$$\mathrm{var}[y_{t+1}] \approx \underbrace{\left(\frac{\delta q}{\delta y}\right)^2}_{\text{stability}} \underbrace{\mathrm{var}[y_t]}_{\text{IC uncert.}} + \underbrace{\left(\frac{\delta f}{\delta x}\right)^2}_{\text{driver sens.}} \underbrace{\mathrm{var}[x_t]}_{\text{driver uncert.}} + \underbrace{\left(\frac{\delta f}{\delta \theta}\right)^2}_{\text{param sens.}} \underbrace{\mathrm{var}[\theta]}_{\text{param. uncert.}} + \underbrace{\mathrm{var}[\varepsilon_{t+1}]}_{\text{process error}}$$

HIERARCHICAL BAYESIAN MODELS PROPAGATE UNCERTAINTY FOR US

Data Model: $y_t \sim [y_t \mid z_t, \sigma^2_{\mathrm{o}}], \qquad t = 1, \ldots, T,$

Process Models: $z_t \sim [z_t \mid \mu_t, \sigma^2_{\mathrm{p}}],$

$\mu_t = g(z_{t-1}, \mathbf{x}'_t, \theta), \qquad t = 2, \ldots, T,$

Parameter Models: $\varphi \sim [\theta, \sigma^2_{\mathrm{p}}, \sigma^2_{\mathrm{o}}, z_{t=1}]$

Berliner, 1996, *in* Maximum entropy and bayesian methods

THE FORECAST DISTRIBUTION

$$[z_{T+1}|y_1,\ldots,y_T] = \int\int\ldots\int \left[z_{T+1}|z_T,\mathbf{x}_T,\theta,\sigma^2_{\mathrm{p}}\right]$$
$$\times\left[z_1,\ldots,z_{T+1},\theta,\sigma^2_{\mathrm{p}}|y_1,\ldots,y_T\right] d\theta d\sigma^2_{\mathrm{p}} dz_1\ldots dz_T d\mathbf{x}_1\ldots d\mathbf{x}_T$$

Hobbs and Hooten, 2015, Bayesian Models: A Statistical Primer for Ecologists

THE FORECAST DISTRIBUTION, VIA MCMC

We have:

- $k = 1, \ldots, K$ MCMC iterations
- $j = 1, \ldots, J$ realizations of the covariate, resampled to match K
- Forecasts at times $T + q, \ldots, T + Q$

$$z_{T+q}^{(k)} \sim \left[z_{T+q} \middle| g\left(z_{T+q-1}^{(k)}, \mathbf{x}_{T+q}^{(j(k))}, \theta^{(k)}\right), \sigma_{\mathrm{p}}^{2(k)} \right]$$

Ignore initial conditions uncertainty

$$z_T^{(*)} = E(z_T | y_1, \ldots, y_T) \approx \frac{\sum_{k=1}^{K} z_T^{(k)}}{K}$$

$$z_{T+q} \sim \begin{cases} \left[z_{T+q} | g\left(z_{T+q-1}^{(k)}, \mathbf{x}_T^{(j(k))}, \theta^{(k)}\right), \sigma_{\mathbf{p}}^{2(k)} \right], & q > 1 \\ \left[z_{T+q} | g\left(z_T^{(*)}, \mathbf{x}_T^{(j(k))}, \theta^{(k)}\right), \sigma_{\mathbf{p}}^{2(k)} \right], & q = 1. \end{cases}$$

POST HOC PARTITIONING FROM MCMC SAMPLES

k	z_T	θ_1	$\cdots$
1	$z_T^{(1)}$	$\theta_1^{(1)}$	$\cdots$
2	$z_T^{(2)}$	$\theta_1^{(2)}$	$\cdots$
3	$z_T^{(3)}$	$\theta_1^{(3)}$	$\cdots$
4	$z_T^{(4)}$	$\theta_1^{(4)}$	$\cdots$
5	$z_T^{(5)}$	$\theta_1^{(5)}$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
K	$z_T^{(K)}$	$\theta_1^{(K)}$	$\cdots$

POST HOC PARTITIONING FROM MCMC SAMPLES

k	z_T	θ_1	$\cdots$
1	$z_T^{(*)}$	$\theta_1^{(1)}$	$\cdots$
2	$z_T^{(*)}$	$\theta_1^{(2)}$	$\cdots$
3	$z_T^{(*)}$	$\theta_1^{(3)}$	$\cdots$
4	$z_T^{(*)}$	$\theta_1^{(4)}$	$\cdots$
5	$z_T^{(*)}$	$\theta_1^{(5)}$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
K	$z_T^{(*)}$	$\theta_1^{(K)}$	$\cdots$

POST HOC PARTITIONING FROM MCMC SAMPLES

$$\mathbf{Z}^{(I)} = \mathbf{Z}^{(I,\overline{PA},\overline{D},\overline{PS})}$$

$$\mathbf{Z}^{(I)} \approx \left[z_{T+q} \mid g\left(z_{T+q-1}^{(k)}, \mathbf{x}_T^{(*)}, \theta^{(*)}\right), 0 \right]$$

POST HOC PARTITIONING FROM MCMC SAMPLES

$$\mathbf{z}^{(l)} = \mathbf{z}^{(l,\overline{PA},\overline{D},\overline{PS})}$$

$$\mathbf{z}^{(l)} \approx \left[z_{T+q} \mid g\left(z_{T+q-1}^{(k)}, \mathbf{x}_T^{(*)}, \theta^{(*)}\right), 0 \right]$$

POST HOC PARTITIONING FROM MCMC SAMPLES

$$\mathbf{z}^{(l)} = \mathbf{z}^{(l,\overline{PA},\overline{D},\overline{PS})}$$

$$\mathbf{z}^{(l)} \approx \left[z_{T+q} \mid g(z_{T+q-1}^{(k)}, \mathbf{x}_T^{(*)}, \theta^{(*)}), 0 \right]$$

$$V^{(l)} = \mathrm{var}(\mathbf{z}^{(l)})$$

POST HOC PARTITIONING FROM MCMC SAMPLES

Source of Uncertainty	Notation
Initial conditions	$V^{(I)} = V^{(I,\overline{PA},\overline{D},\overline{PS})}$
Parameter uncertainty	$V^{(PA)} = V^{(\bar{I},PA,\overline{D},\overline{PS})}$
Driver uncertainty	$V^{(D)} = V^{(\bar{I},\overline{PA},D,\overline{PS})}$
Process uncertainty	$V^{(PS)} = V^{(\bar{I},\overline{PA},\overline{D},PS)}$

PARTITION FORECAST UNCERTAINTY: ANOVA

$$V_{T+q}^{(F)} = V_{T+q}^{(I)} + V_{T+q}^{(PA)} + V_{T+q}^{(D)} + V_{T+q}^{(PS)}$$

PARTITION FORECAST UNCERTAINTY: ANOVA

$$
\begin{aligned}
V_{T+q}^{(F)} = V_{T+q}^{(I)} + V_{T+q}^{(PA)} + V_{T+q}^{(D)} + V_{T+q}^{(PS)} \\
&+ \varepsilon_{T+q}^{(I,PA)} + \varepsilon_{T+q}^{(I,D)} + \varepsilon_{T+q}^{(I,PS)} + \varepsilon_{T+q}^{(PA,PS)} + \varepsilon_{T+q}^{(PA,D)} + \varepsilon_{T+q}^{(D,PS)} \\
&+ \varepsilon_{T+q}^{(I,PA,D)} + \varepsilon_{T+q}^{(I,PA,PS)} + \varepsilon_{T+q}^{(I,D,PS)} + \varepsilon_{T+q}^{(PA,D,PS)} \\
&+ \varepsilon_{T+q}^{(I,PA,D,PS)}
\end{aligned}
$$

PARTITION FORECAST UNCERTAINTY: ANOVA

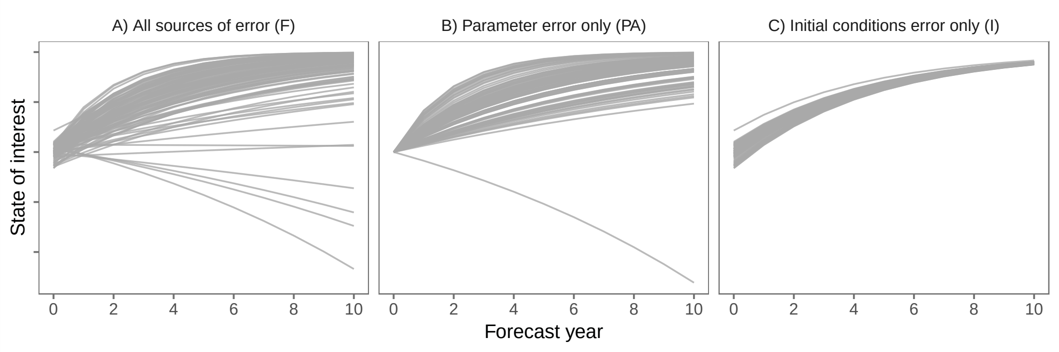
Example where forecast is influenced by initial conditions (I) and parameter uncertainty (PA):

$$V_{T+q}^{(F)} = V_{T+q}^{(I)} + V_{T+q}^{(PA)} + \varepsilon_{T+q}^{(I,PA)}$$

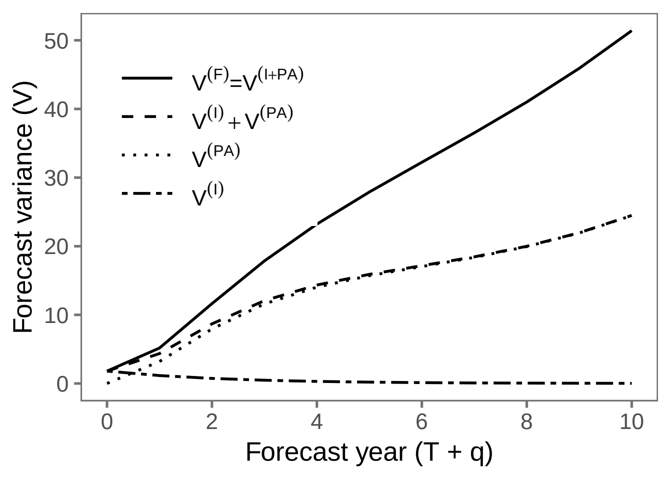
so,

$$\varepsilon_{T+q}^{(I,PA)} = V_{T+q}^{(F)} - \left[V_{T+q}^{(I)} + V_{T+q}^{(PA)} \right]$$

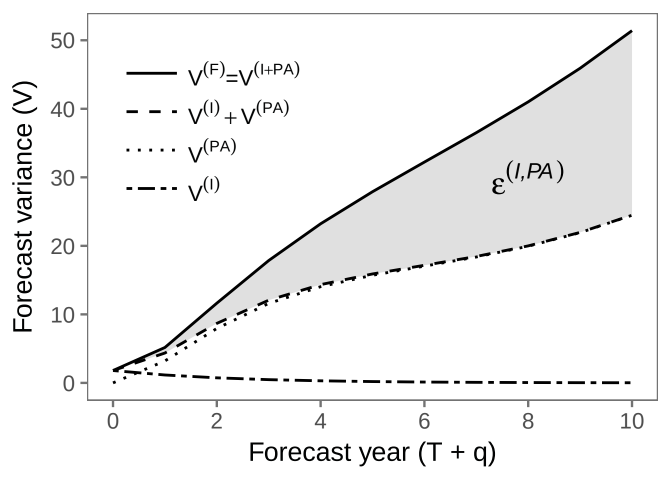
RETURN TO EXAMPLE OF AR(1) PROCESS



PARTITIONED FORECAST VARIANCE OVER TIME



PARTITIONED FORECAST VARIANCE OVER TIME



§ 3

Conclusions

CONCLUSIONS

1. Partition uncertainty to advance ecological forecasting – how do we get better?
2. Partition uncertainty to advance scientific progress – what don’t we know?
3. Hierarchical Bayesian models ideally suited for partitioning uncertainty because they allow us to fully specify the inclusion of uncertainty.
4. Proof of concept – formal treatment in the works.