

# **Spatiotemporal forecasting of plant populations and a proposal to partition forecast uncertainty**

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## FORECASTING

Skillfull forecasts are in high demand because the world is changing fast.

1. Responses to global climate change
2. Species invasions
3. Emergence and spread of infectious diseases



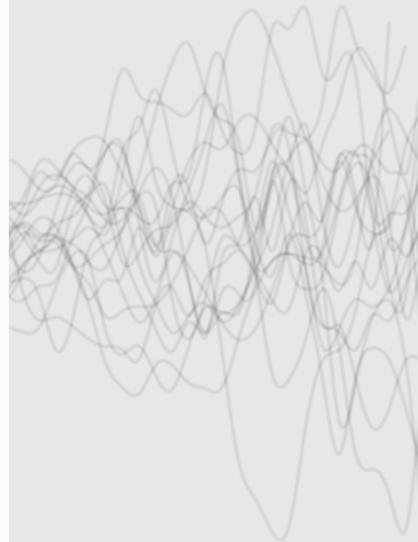
## FORECASTING

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## Plant population forecasts





## COLLABORATORS

Peter Adler (USU)



Mevin Hooten (CSU)



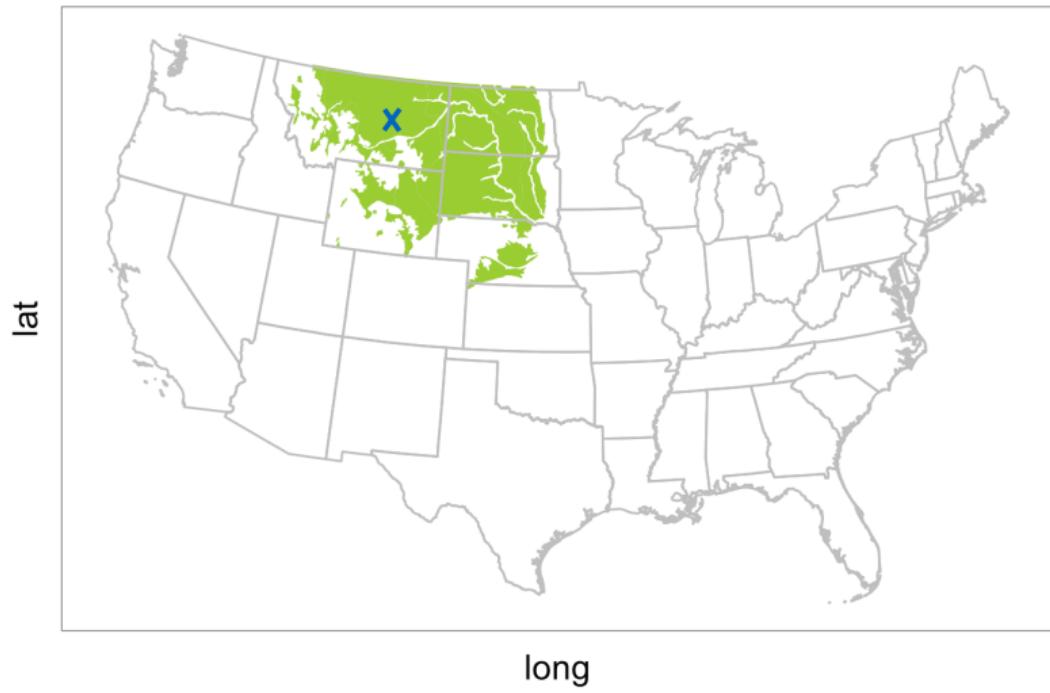


## WHAT LAND MANAGERS WANT





## WHAT LAND MANAGERS GET



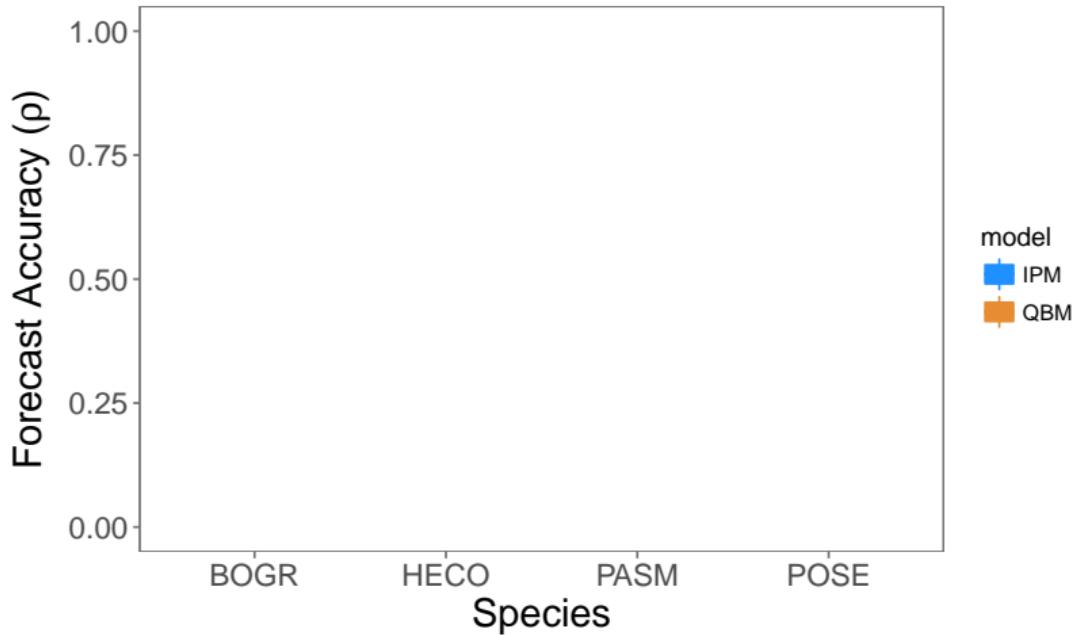


# REALLY HARD WORK



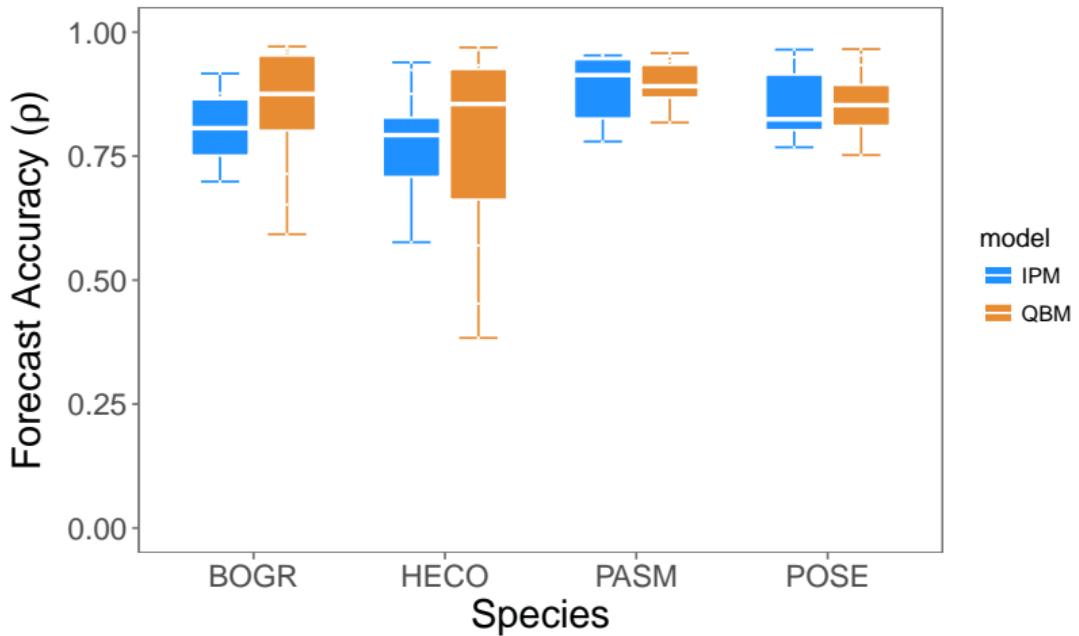


## DO WE NEED DEMOGRAPHIC DATA?



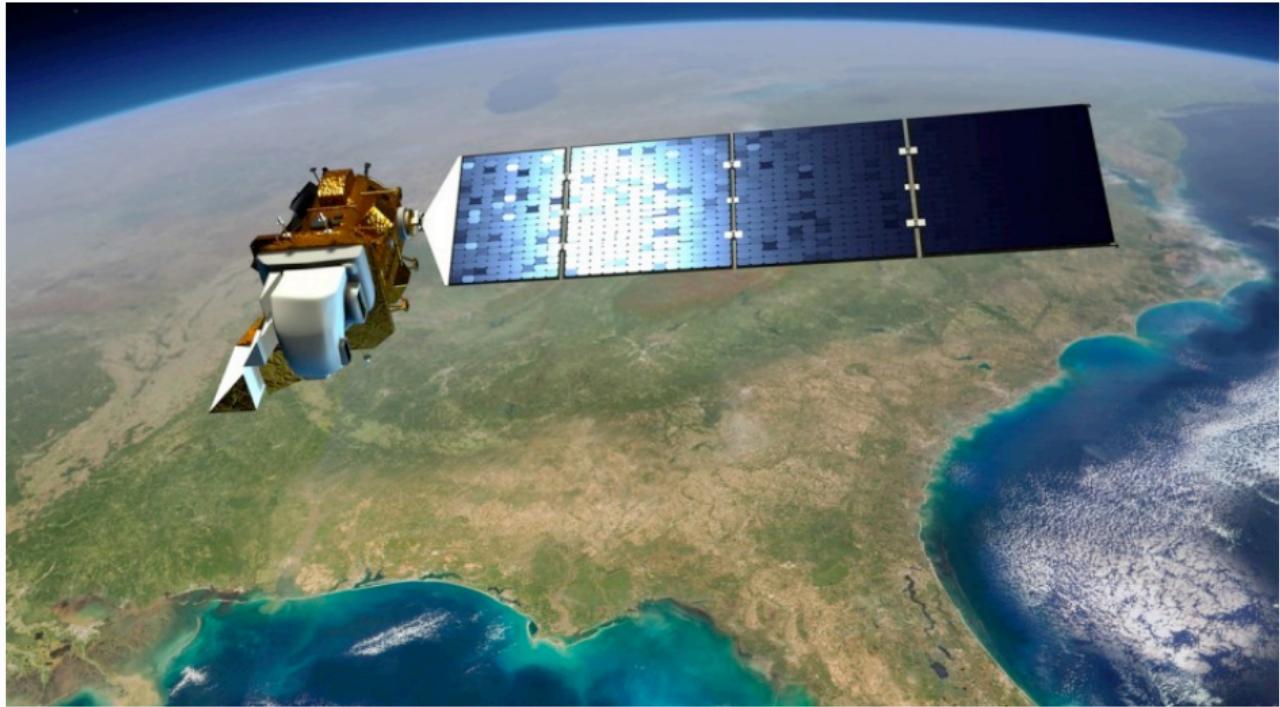


FREE FROM THE TYRANNY OF DEMOGRAPHIC DATA!





LET'S USE SATELLITES



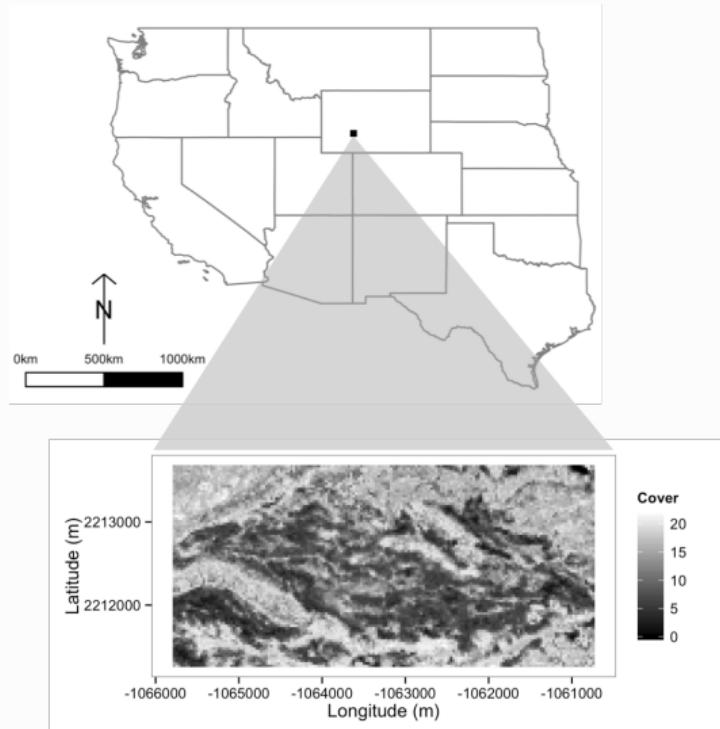


## SAGEBRUSH SEA IN WYOMING



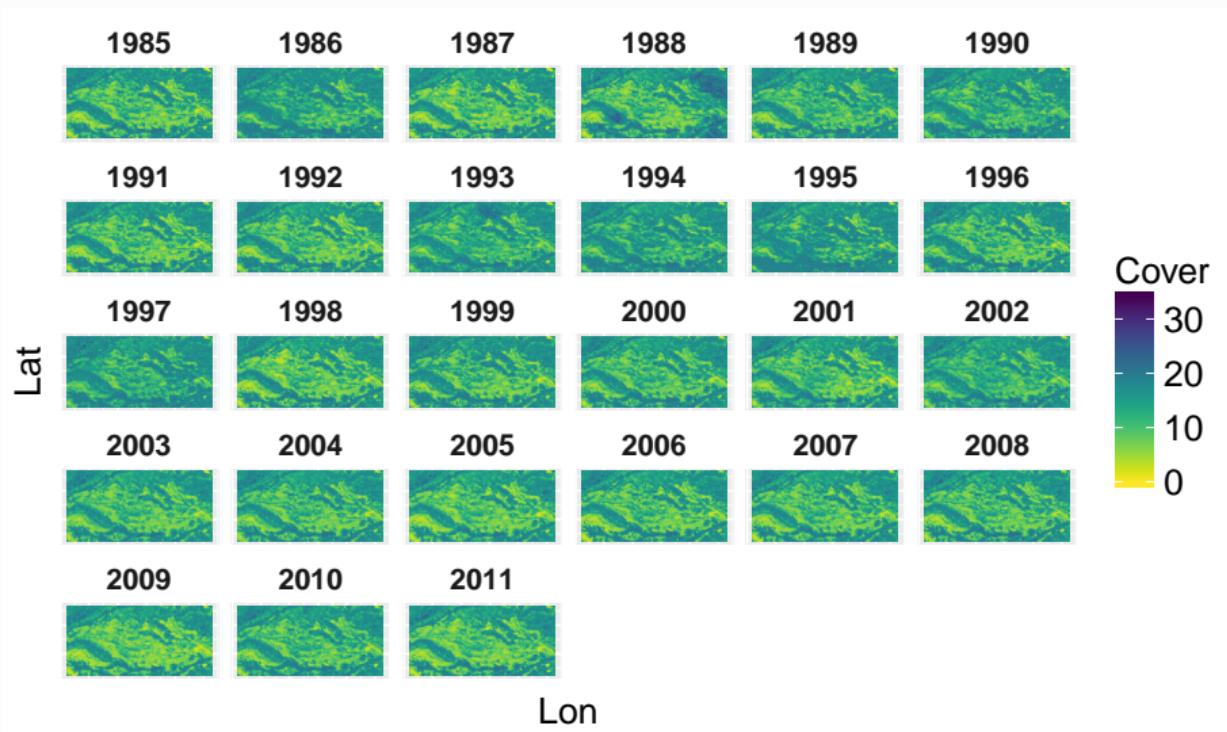


## STUDY AREA





## LANDSAT TIME SERIES





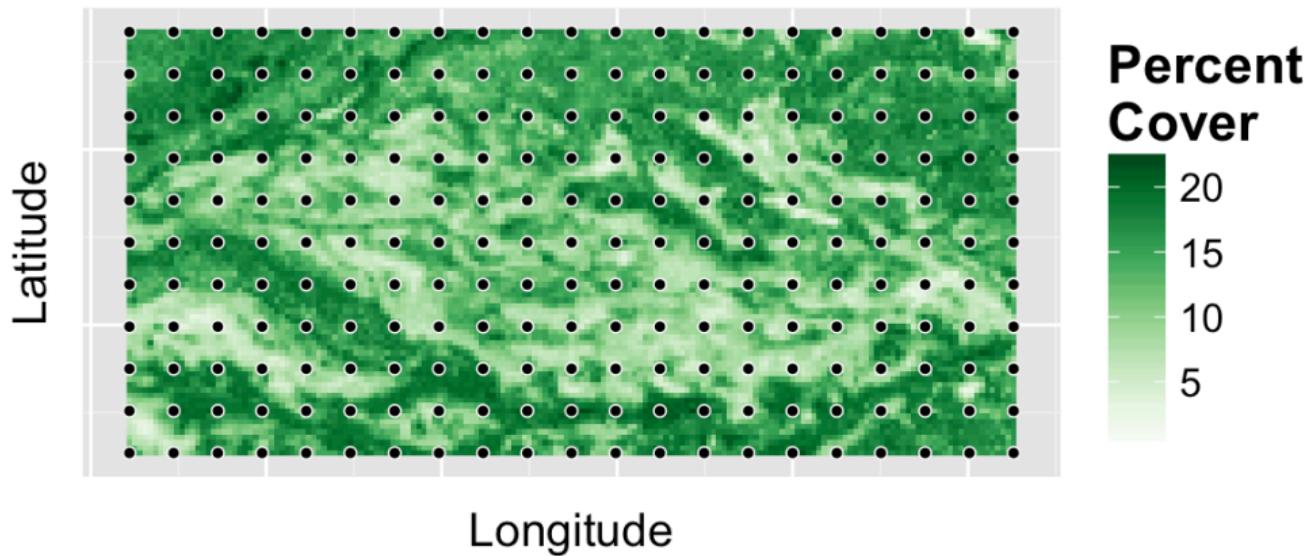
## DYNAMIC COVER MODEL

$$y_{i,t} \sim \text{Poisson}(\mu_{i,t})$$

$$\log(\mu_{i,t}) = \underbrace{\beta_{0,t} + \beta_1 y_{i,t-1}}_{\text{temporal + dens. dep}} + \underbrace{\mathbf{x}'_t \gamma}_{\text{climate}} + \underbrace{\eta_i}_{\text{spatial}}$$



## DIMENSION REDUCTION FOR SPATIAL EFFECT





## DYNAMIC COVER MODEL

$$y_{i,t} \sim \text{Poisson}(\mu_{i,t})$$

$$\log(\mu_{i,t}) = \underbrace{\beta_{0,t} + \beta_1 y_{i,t-1}}_{\text{temporal + dens. dep.}} + \underbrace{\mathbf{x}'_t \gamma}_{\text{climate}} + \underbrace{\eta_i}_{\text{spatial}}$$

$$\eta \approx \mathbf{K}\alpha,$$

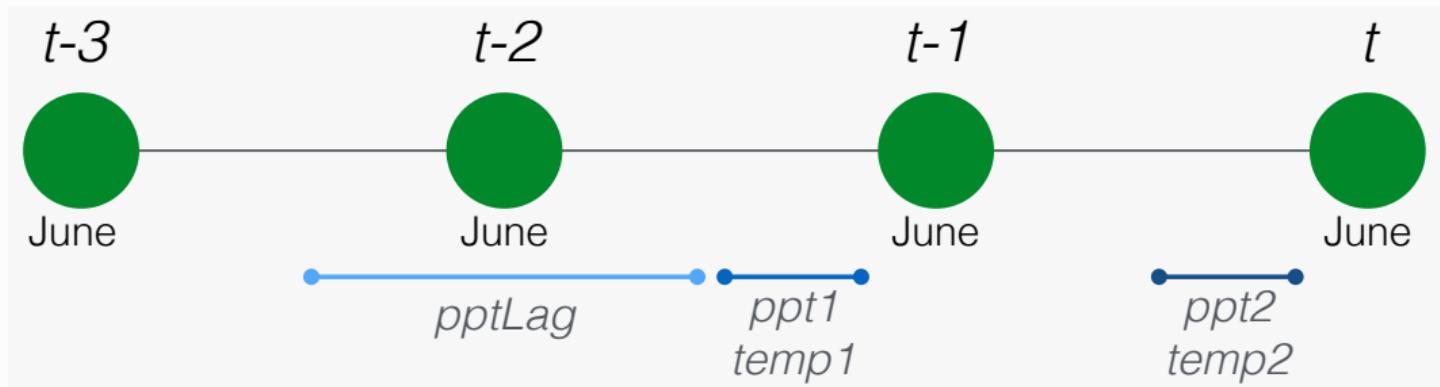
$$\mathbf{K} = \mathbf{w}_{s,m} / \sum_{s=1}^S \mathbf{w}_{s,m}$$

$$\mathbf{w}_{s,m} = \exp(-d_{s,m}/\sigma)$$

$$\alpha_m \sim \text{Normal}(0, \sigma_\eta^2)$$

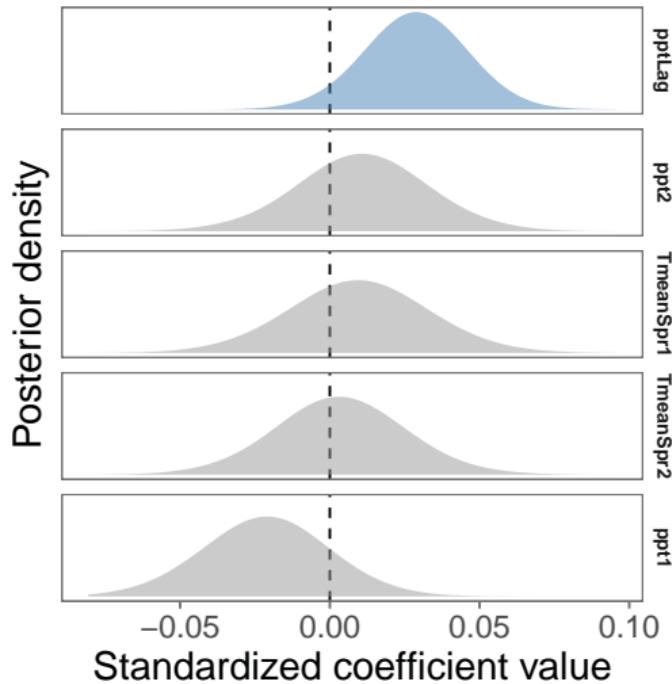


## CLIMATE COVARIATES





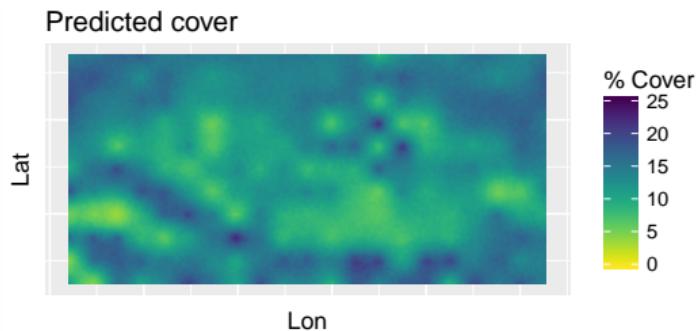
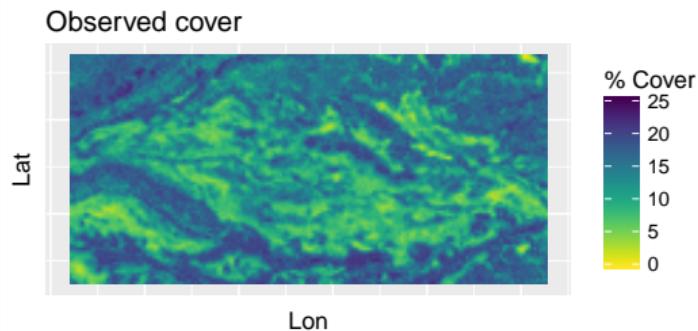
## CLIMATE EFFECTS





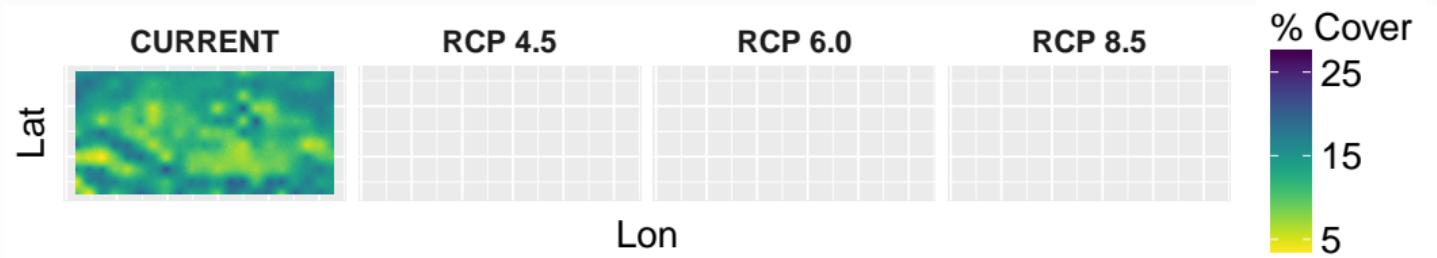
## MODEL PERFORMANCE

In-sample RMSE  $\approx 4\%$



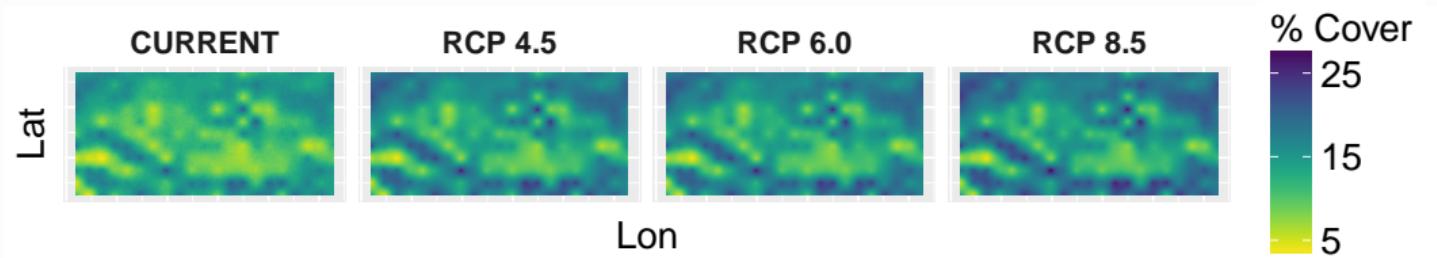


## FORECASTS UNDER CLIMATE CHANGE: SPATIAL



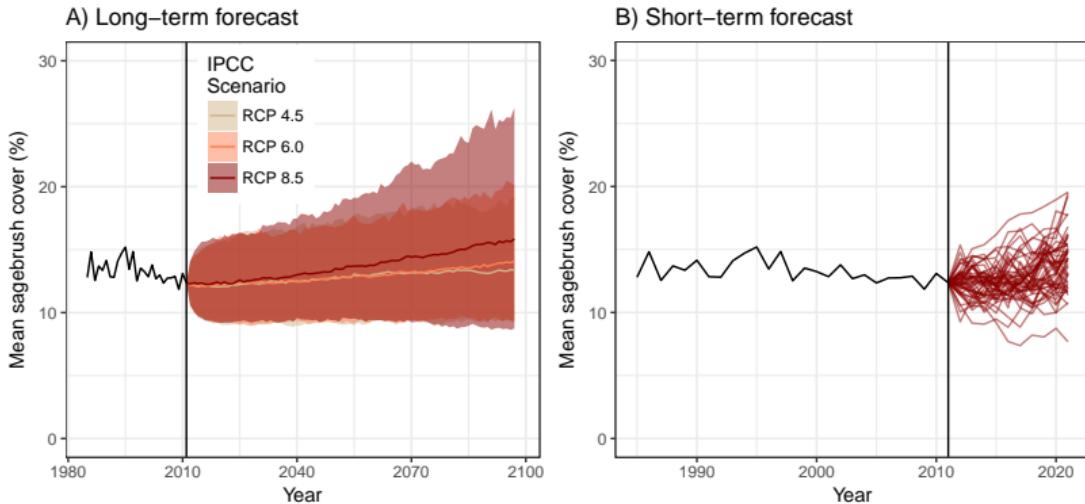


## FORECASTS UNDER CLIMATE CHANGE: SPATIAL



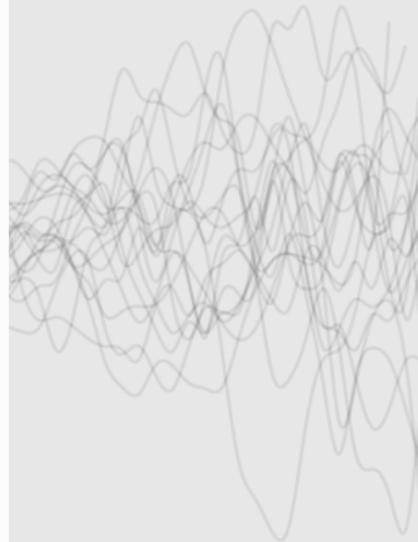


## FORECASTS UNDER CLIMATE CHANGE: TEMPORAL



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## Partitioning forecast uncertainty





## FORECAST UNCERTAINTY, TO A FIRST APPROXIMATION

Forecast of state  $z$  at  $t + 1$  from function  $q$ :  $q = f(z_t, x_t, \theta, \varepsilon_{t+1})$

$$\text{var}[y_{t+1}] \approx \underbrace{\left( \frac{\delta q}{\delta y} \right)^2}_{\text{stability}} \underbrace{\text{var}[y_t]}_{\text{IC uncert.}} + \underbrace{\left( \frac{\delta f}{\delta x} \right)^2}_{\text{driver sens.}} \underbrace{\text{var}[x_t]}_{\text{driver uncert.}} + \underbrace{\left( \frac{\delta f}{\delta \theta} \right)^2}_{\text{param sens.}} \underbrace{\text{var}[\theta]}_{\text{param. uncert.}} + \underbrace{\text{var}[\varepsilon_{t+1}]}_{\text{process error}}$$



## ERROR PROPAGATION

For some function  $q$ :  $q = f(x_1, x_2, \dots, x_n)$

$$\begin{aligned}\sigma_q^2 &= \left( \frac{\delta q}{\delta x_1} \sigma_{x_1} \right)^2 + \left( \frac{\delta q}{\delta x_2} \sigma_{x_2} \right)^2 + \cdots + \left( \frac{\delta q}{\delta x_n} \sigma_{x_n} \right)^2 \\ &= \sum_{i=1}^n \left( \frac{\delta q}{\delta x_i} \sigma_{x_i} \right)^2\end{aligned}$$



## ERROR PROPAGATION

$$\sigma_q^2 = \underbrace{\sum_{i=1}^n \left( \frac{\delta q}{\delta x_i} \sigma_{x_i} \right)^2}_{\text{variances}} + \underbrace{\sum_{i=1}^n \sum_{j(j \neq i)}^n 2\sigma_{ij} \left( \frac{\delta q}{\delta x_i} \right) \left( \frac{\delta q}{\delta x_j} \right)}_{\text{covariances}}$$



## CAN WE IGNORE COVARIANCES?

$$z_{t+1} = z_t \beta_0 + x_t \beta_1 + \varepsilon_{t+1},$$

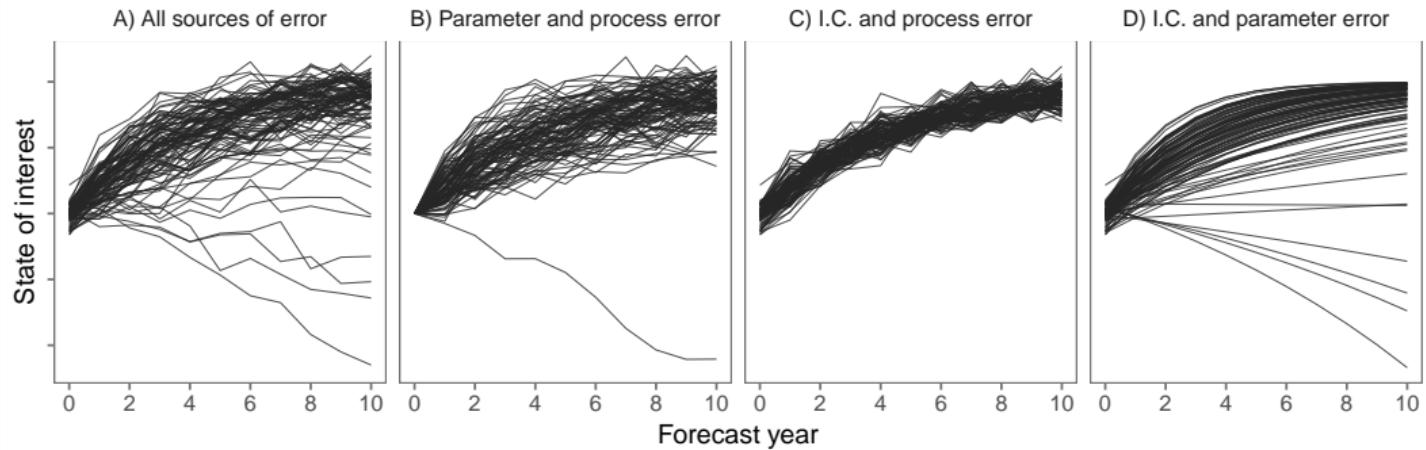
$$z_{t=1} \sim \text{Normal}(z_0, \sigma_{\text{init.}}^2), \quad \text{initial conditions uncertainty}$$

$$\boldsymbol{\beta} \sim \text{MVN}(0, \sigma_{\text{param.}}^2 \mathbf{I}), \quad \text{parameter uncertainty}$$

$$\varepsilon_t \sim \text{Normal}(0, \sigma_{\text{proc.}}^2) \quad \text{process uncertainty}$$



## INTERACTION (COVARIANCES) CANNOT BE IGNORED





## AN INVERSE ERROR PROPOGATION PROBLEM

$$\text{var}[y_{t+1}] \approx \underbrace{\left(\frac{\delta q}{\delta y}\right)^2 \text{var}[y_t]}_{\text{stability}} + \underbrace{\left(\frac{\delta f}{\delta x}\right)^2 \text{var}[x_t]}_{\text{IC uncert.}} + \underbrace{\left(\frac{\delta f}{\delta \theta}\right)^2 \text{var}[\theta]}_{\text{driver sens. driver uncert.}} + \underbrace{\text{var}[\varepsilon_{t+1}]}_{\text{param sens. param. uncert.}} + \underbrace{\text{var}[\varepsilon_{t+1}]}_{\text{process error}}$$



## HIERARCHICAL BAYESIAN MODELS PROPAGATE UNCERTAINTY FOR US

**Data Model:**  $y_t \sim [y_t | z_t, \sigma_o^2], \quad t = 1, \dots, T,$

**Process Models:**  $z_t \sim [z_t | \mu_t, \sigma_p^2],$

$\mu_t = g(z_{t-1}, \mathbf{x}'_t, \theta), \quad t = 2, \dots, T,$

**Parameter Models:**  $\varphi \sim [\theta, \sigma_p^2, \sigma_o^2, z_{t=1}]$



## THE FORECAST DISTRIBUTION

$$\begin{aligned} [z_{T+1}|y_1, \dots, y_T] &= \int \int \dots \int [z_{T+1}|z_T, \mathbf{x}_T, \theta, \sigma_p^2] \\ &\times [z_1, \dots, z_{T+1}, \theta, \sigma_p^2|y_1, \dots, y_T] d\theta d\sigma_p^2 dz_1 \dots dz_T d\mathbf{x}_1 \dots d\mathbf{x}_T \end{aligned}$$



## THE FORECAST DISTRIBUTION, VIA MCMC

We have:

- $k = 1, \dots, K$  MCMC iterations
- $j = 1, \dots, J$  realizations of the covariate, resampled to match  $K$
- Forecasts at times  $T + q, \dots, T + Q$

$$z_{T+q}^{(k)} \sim \left[ z_{T+q} | g(z_{T+q-1}^{(k)}, \mathbf{x}_{T+q}^{(j(k))}, \theta^{(k)}), \sigma_p^{2(k)} \right]$$



## POST HOC PARTITIONING FROM MCMC SAMPLES

**Ignore initial conditions uncertainty**

$$z_T^{(*)} = E(z_T | y_1, \dots, y_T) \approx \frac{\sum_{k=1}^K z_T^{(k)}}{K}$$

$$z_{T+q} \sim \begin{cases} \left[ z_{T+q} | g(z_{T+q-1}^{(k)}, \mathbf{x}_T^{(j(k))}, \theta^{(k)}), \sigma_p^{2(k)} \right], & q > 1 \\ \left[ z_{T+q} | g(z_T^{(*)}, \mathbf{x}_T^{(j(k))}, \theta^{(k)}), \sigma_p^{2(k)} \right], & q = 1. \end{cases}$$



## POST HOC PARTITIONING FROM MCMC SAMPLES

$$\mathbf{z}^{(l)} = \mathbf{z}^{(l, \overline{PA}, \overline{D}, \overline{PS})}$$

$$\mathbf{z}^{(l)} \approx \left[ z_{T+q} \mid g(z_{T+q-1}^{(k)}, \mathbf{x}_T^{(*)}, \theta^{(*)}), 0 \right]$$



## POST HOC PARTITIONING FROM MCMC SAMPLES

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## POST HOC PARTITIONING FROM MCMC SAMPLES

$$\mathbf{z}^{(l)} = \mathbf{z}^{(l, \overline{PA}, \overline{D}, \overline{PS})}$$

$$\mathbf{z}^{(l)} \approx \left[ z_{T+q} \mid g(z_{T+q-1}^{(k)}, \mathbf{x}_T^{(*)}, \boldsymbol{\theta}^{(*)}), 0 \right]$$

$$V^{(l)} = \text{var}(\mathbf{z}^{(l)})$$



## POST HOC PARTITIONING FROM MCMC SAMPLES

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Source of Uncertainty	Notation
Initial conditions	$V^{(I)} = V^{(I, \bar{PA}, \bar{D}, \bar{PS})}$
Parameter uncertainty	$V^{(PA)} = V^{(\bar{I}, PA, \bar{D}, \bar{PS})}$
Driver uncertainty	$V^{(D)} = V^{(\bar{I}, \bar{PA}, D, \bar{PS})}$
Process uncertainty	$V^{(PS)} = V^{(\bar{I}, \bar{PA}, \bar{D}, PS)}$

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## PARTITION FORECAST UNCERTAINTY: ANOVA

$$V_{T+q}^{(F)} = V_{T+q}^{(I)} + V_{T+q}^{(PA)} + V_{T+q}^{(D)} + V_{T+q}^{(PS)}$$



## PARTITION FORECAST UNCERTAINTY: ANOVA

$$\begin{aligned} V_{T+q}^{(F)} = & V_{T+q}^{(I)} + V_{T+q}^{(PA)} + V_{T+q}^{(D)} + V_{T+q}^{(PS)} \\ & + \varepsilon_{T+q}^{(I,PA)} + \varepsilon_{T+q}^{(I,D)} + \varepsilon_{T+q}^{(I,PS)} + \varepsilon_{T+q}^{(PA,PS)} + \varepsilon_{T+q}^{(PA,D)} + \varepsilon_{T+q}^{(D,PS)} \\ & + \varepsilon_{T+q}^{(I,PA,D)} + \varepsilon_{T+q}^{(I,PA,PS)} + \varepsilon_{T+q}^{(I,D,PS)} + \varepsilon_{T+q}^{(PA,D,PS)} \\ & + \varepsilon_{T+q}^{(I,PA,D,PS)} \end{aligned}$$



## PARTITION FORECAST UNCERTAINTY: ANOVA

Example where forecast is influenced by initial conditions ( $I$ ) and parameter uncertainty ( $PA$ ):

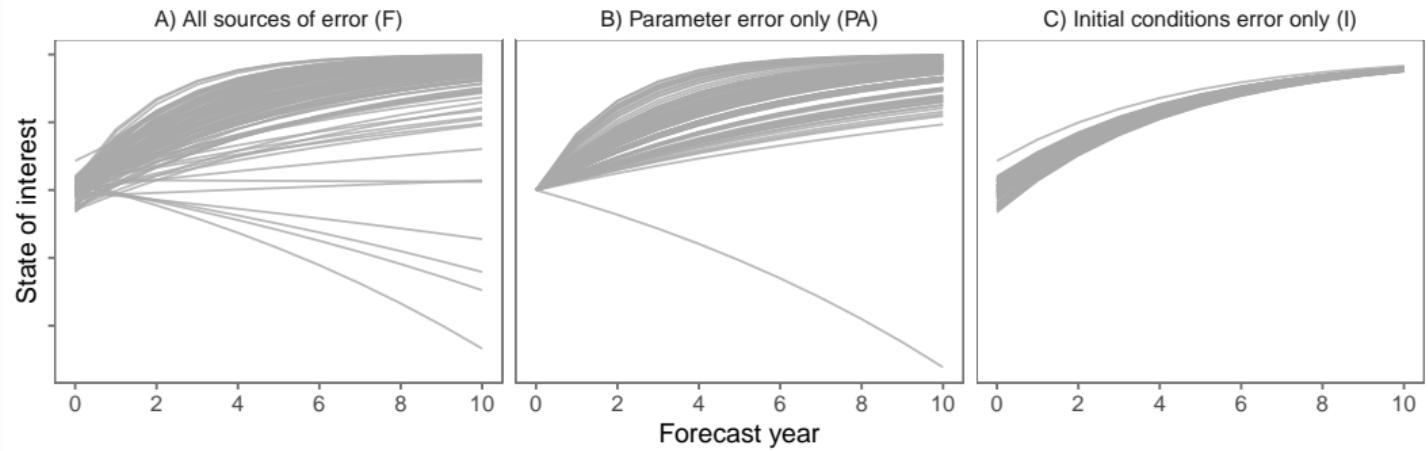
$$V_{T+q}^{(F)} = V_{T+q}^{(I)} + V_{T+q}^{(PA)} + \varepsilon_{T+q}^{(I,PA)}$$

so,

$$\varepsilon_{T+q}^{(I,PA)} = V_{T+q}^{(F)} - \left[ V_{T+q}^{(I)} + V_{T+q}^{(PA)} \right]$$

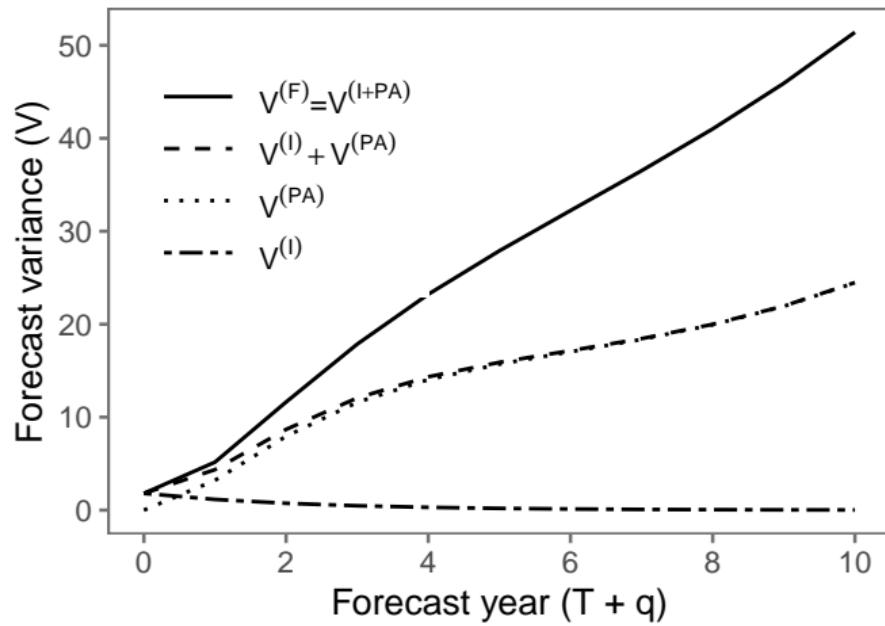


## RETURN TO EXAMPLE OF AR(1) PROCESS



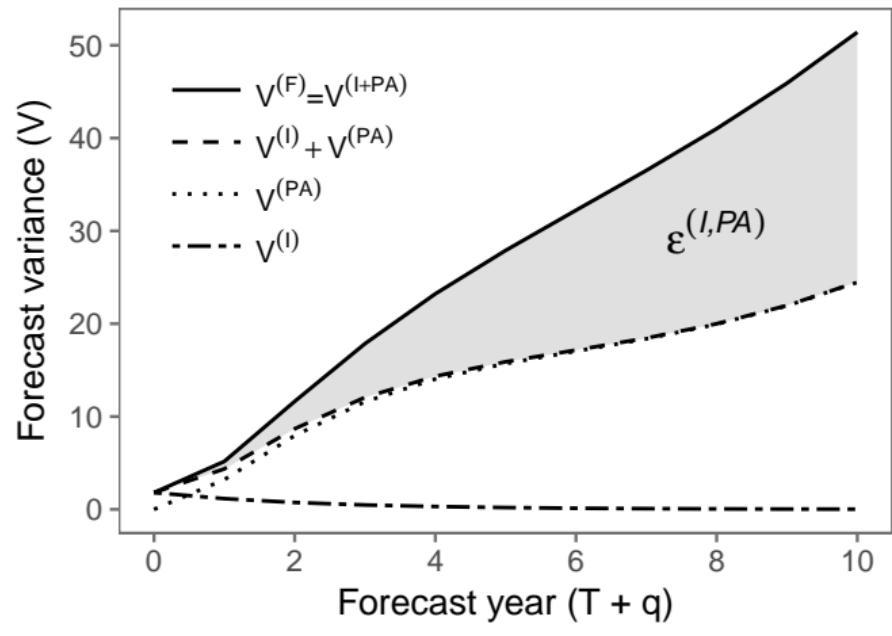


## PARTITIONED FORECAST VARIANCE OVER TIME



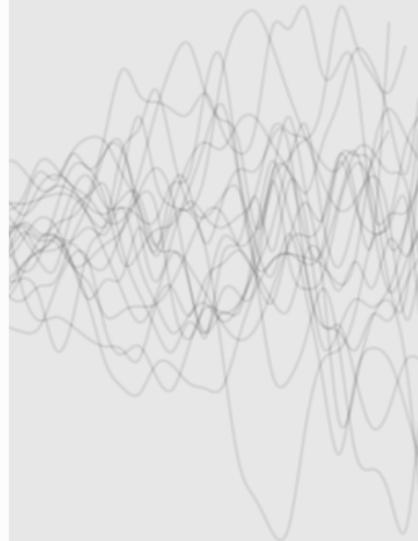


## PARTITIONED FORECAST VARIANCE OVER TIME



§ 3

## Conclusions





## CONCLUSIONS

1. Partition uncertainty to **advance ecological forecasting** – how do we get better?
2. Partition uncertainty to **advance scientific progress** – what don't we know?
3. Hierarchical Bayesian models ideally suited for partitioning uncertainty because they allow us to fully specify the inclusion of uncertainty.
4. Proof of concept – formal treatment in the works.

