Spatiotemporal forecasting of plant populations and a proposal to partition forecast uncertainty

Andrew Tredennick
University of Georgia

October 12, 2018

COLLABORATORS

Peter Adler (USU)

Mevin Hooten (CSU)

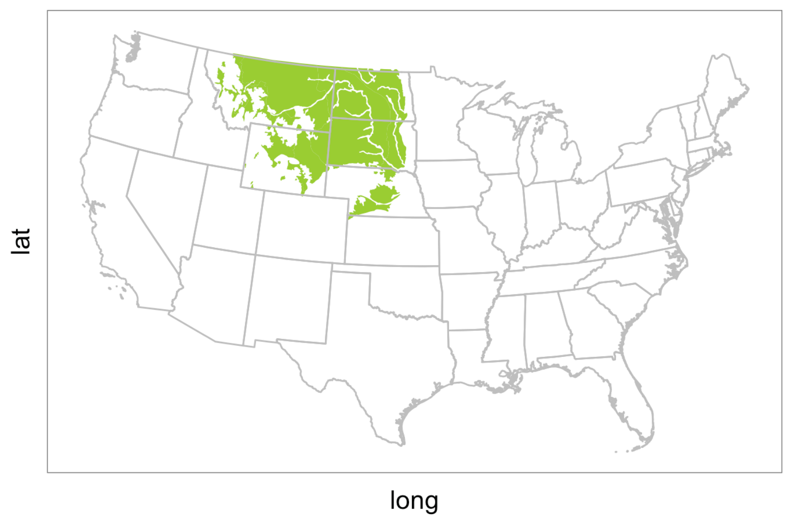
ROAD MAP

1. Plant population forecasts over large spatial extents
2. A proposal for partitioning forecast uncertainty

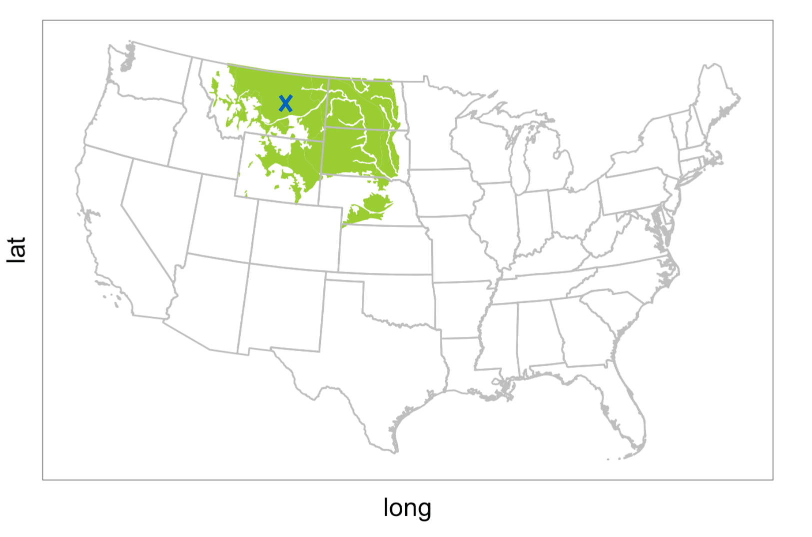
§ 1

Plant population forecasts

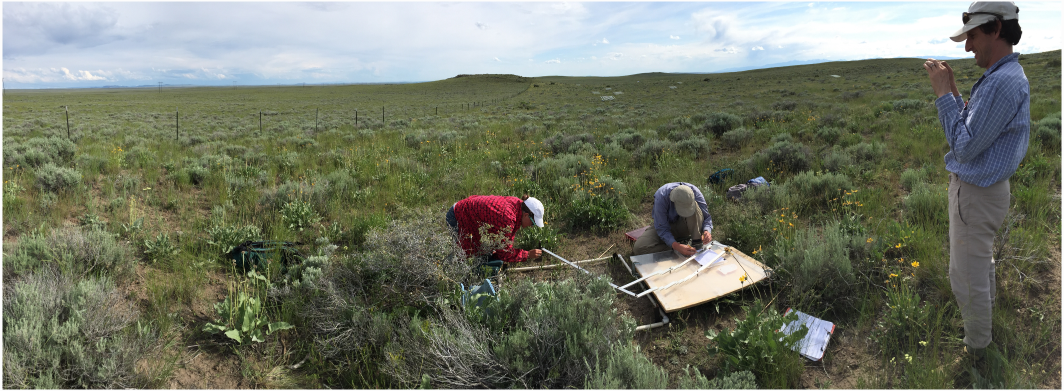
WHAT LAND MANAGERS WANT



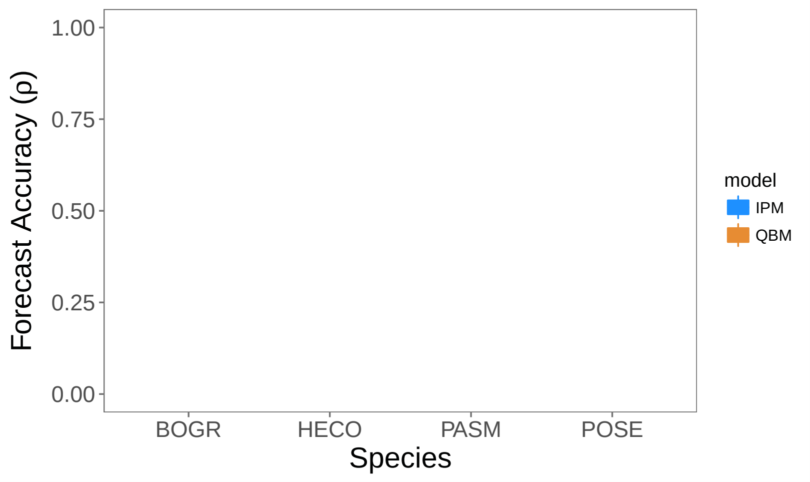
WHAT LAND MANAGERS GET



REALLY HARD WORK

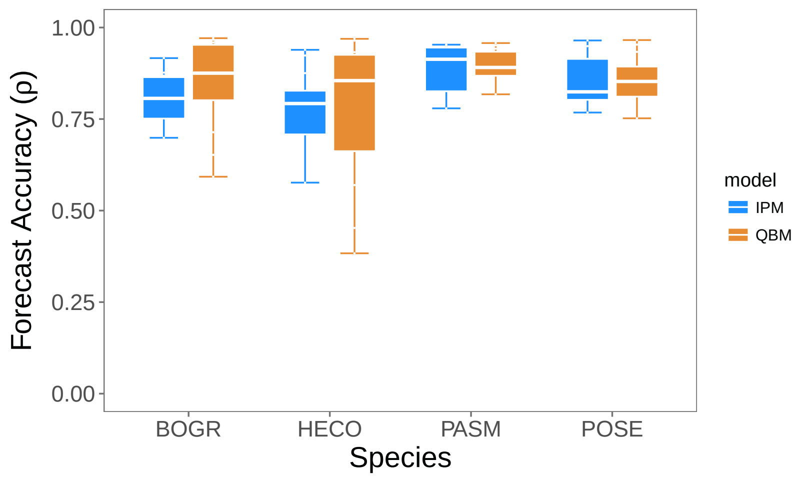


DO WE NEED DEMOGRAPHIC DATA?



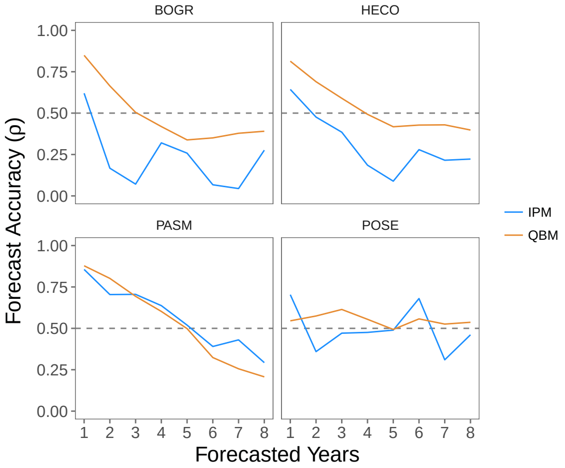
Tredennick et al., 2017, *Methods in Ecol. Evol.*

FREE FROM THE TYRANNY OF DEMOGRAPHIC DATA!



Tredennick et al., 2017, *Methods in Ecol. Evol.*

FORECAST HORIZONS

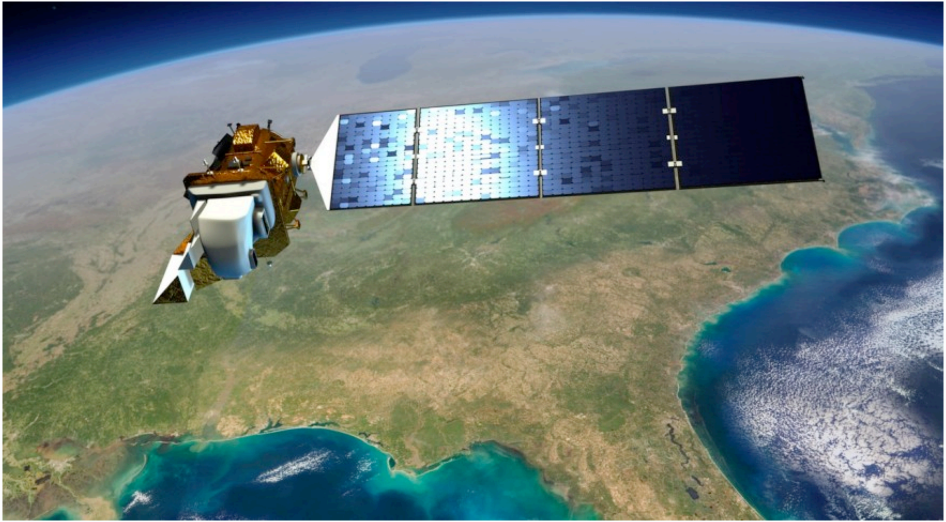


Tredennick et al., 2017, *Methods in Ecol. Evol.*

DO WE NEED DEMOGRAPHIC DATA?

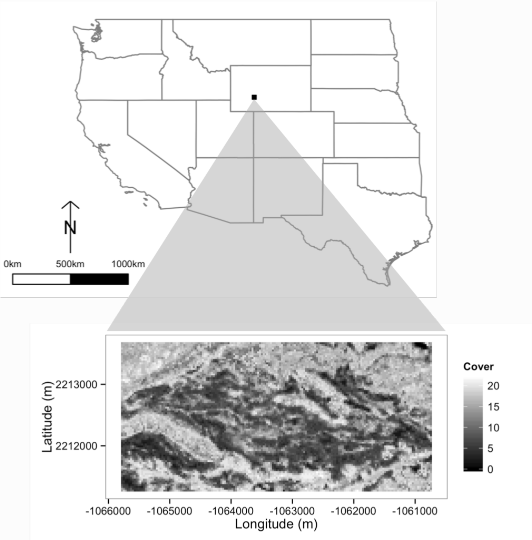
Maybe not.

LET’S USE SATELLITES



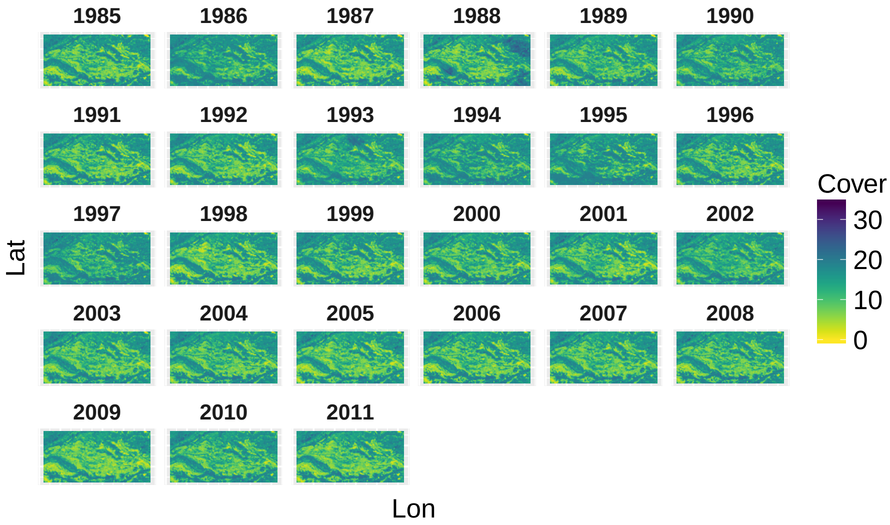
SAGEBRUSH SEA IN WYOMING

STUDY AREA



0km
500km
1000km
N
Cover
20
15
10
5
0
Latitude (m)
2213000
2212000
-1066000
-1065000
-1064000
-1063000
-1062000
-1061000
Longitude (m)

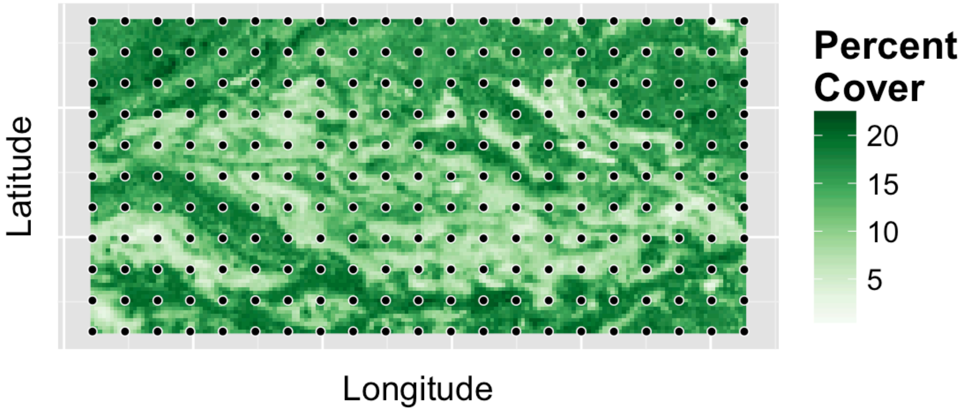
LANDSAT TIME SERIES



DYNAMIC COVER MODEL

$$
\begin{aligned}
y_{i,t} &\sim \text{Poisson}(\mu_{i,t}) \\
\log(\mu_{i,t}) &= \underbrace{\beta_{0,t} + \beta_1 y_{i,t-1}}_{\text{temporal + dens. dep}} + \underbrace{\mathbf{x}_t' \gamma}_{\text{climate}} + \underbrace{\eta_i}_{\text{spatial}}
\end{aligned}
$$

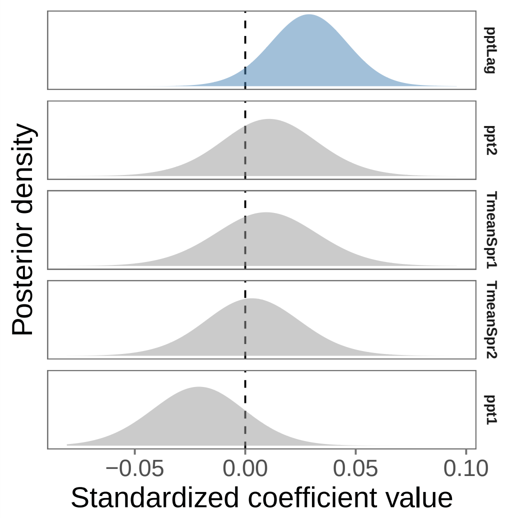
DIMENSION REDUCTION FOR SPATIAL EFFECT



DYNAMIC COVER MODEL

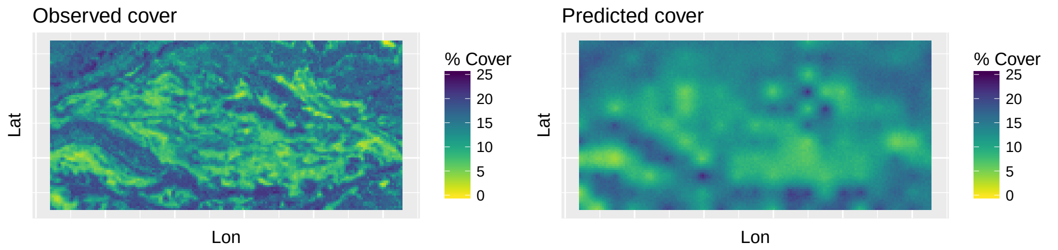
$$
\begin{aligned}
y_{i,t} &\sim \text{Poisson}(\mu_{i,t}) \\
\log(\mu_{i,t}) &= \underbrace{\beta_{0,t} + \beta_1 y_{i,t-1}}_{\text{temporal + dens. dep}} + \underbrace{\mathbf{x}_t'\gamma}_{\text{climate}} + \underbrace{\eta_i}_{\text{spatial}} \\
\eta &\approx \mathbf{K}\alpha, \\
\mathbf{K} &= \mathbf{w}_{s,m} \Big/ \sum_{s=1}^{S} \mathbf{w}_{s,m} \\
\mathbf{w}_{s,m} &= \exp\left(-d_{s,m}/\sigma\right) \\
\alpha_m &\sim \text{Normal}(0, \sigma_\eta^2)
\end{aligned}
$$

CLIMATE EFFECTS



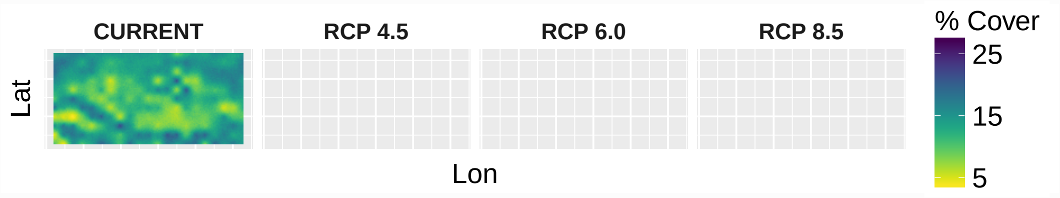
MODEL PERFORMANCE

In-sample RMSE ≈ 4%



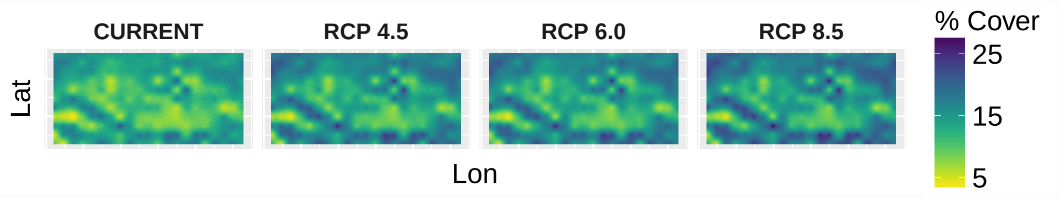
Tredennick et al., 2016, *Ecosphere*

FORECASTS UNDER CLIMATE CHANGE: SPATIAL



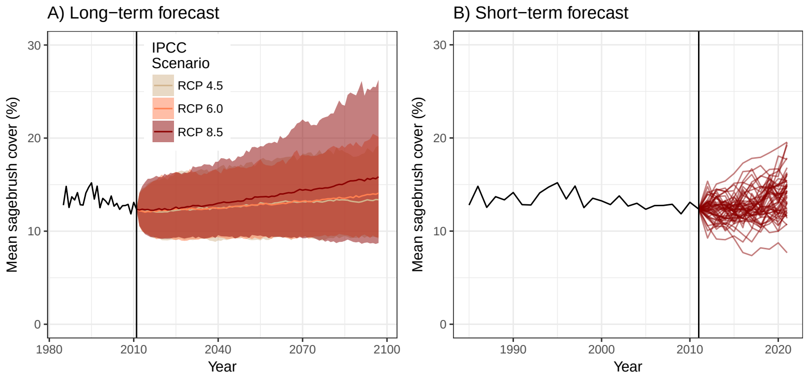
Tredennick et al., 2016, *Ecosphere*

FORECASTS UNDER CLIMATE CHANGE: SPATIAL



Tredennick et al., 2016, *Ecosphere*

FORECASTS UNDER CLIMATE CHANGE: TEMPORAL



Tredennick et al., 2016, *Ecosphere*

§ 2

Partitioning forecast uncertainty

FORECAST UNCERTAINTY, TO A FIRST APPROXIMATION

Forecast of state z at $t+1$ from function q: $q = f(z_t, x_t, \theta, \varepsilon_{t+1})$

$$Var[y_{t+1}] \approx \underbrace{\left(\frac{\delta q}{\delta y}\right)^2}_{\text{stability}} \underbrace{Var[y_t]}_{\text{IC uncert.}} + \underbrace{\left(\frac{\delta f}{\delta x}\right)^2}_{\text{driver sens.}} \underbrace{Var[x_t]}_{\text{driver uncert.}} + \underbrace{\left(\frac{\delta f}{\delta \theta}\right)^2}_{\text{param sens.}} \underbrace{Var[\theta]}_{\text{param. uncert.}} + \underbrace{Var[\varepsilon_{t+1}]}_{\text{process error}}$$

Dietze, 2017, *Ecological Applications*

AN *INVERSE* ERROR PROPAGATION PROBLEM

For some function q: $q = f(x_1, x_2, \ldots, x_n)$

$$\sigma_q^2 = \left(\frac{\delta q}{\delta x_1}\sigma_{x_1}\right)^2 + \left(\frac{\delta q}{\delta x_2}\sigma_{x_2}\right)^2 + \cdots + \left(\frac{\delta q}{\delta x_n}\sigma_{x_n}\right)^2$$
$$= \sum_{i=1}^{n}\left(\frac{\delta q}{\delta x_i}\sigma_{x_i}\right)^2$$

Ku, 1966, *J. Res. National Bureau of Standards - C*

AN *INVERSE* ERROR PROPAGATION PROBLEM

$$\sigma_q^2 = \underbrace{\sum_{i=1}^{n} \left(\frac{\delta q}{\delta x_i} \sigma_{x_i} \right)^2}_{\text{variances}} + \underbrace{\sum_{i=1}^{n} \sum_{j(j \neq i)}^{n} 2\sigma_{ij} \left(\frac{\delta q}{\delta x_i} \right) \left(\frac{\delta q}{\delta x_j} \right)}_{\text{covariances}}$$

Ku, 1966, *J. Res. National Bureau of Standards - C*

INTERACTION (COVARIANCES) CANNOT BE IGNORED

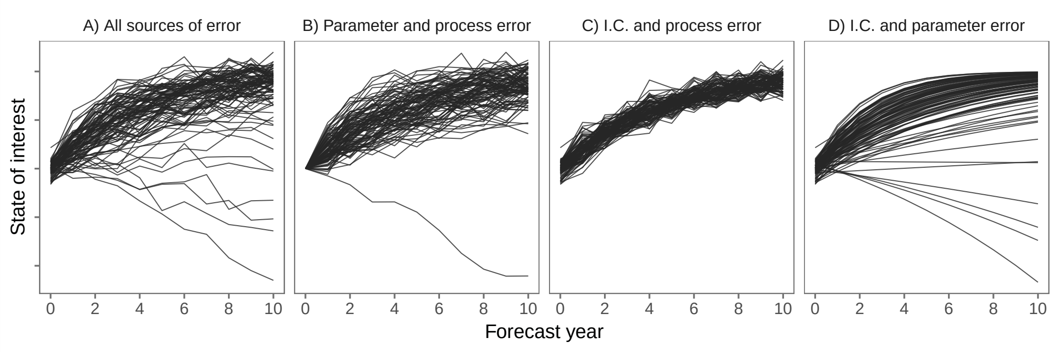
$$z_{t+1} = z_t\beta_0 + x_t\beta_1 + \varepsilon_{t+1},$$

$$z_{t=1} \sim \text{Normal}\left(z_0, \sigma^2_{\text{init.}}\right),$$ initial conditions uncertainty

$$\boldsymbol{\beta} \sim \text{MVN}\left(0, \sigma^2_{\text{param.}}\mathbf{I}\right),$$ parameter uncertainty

$$\varepsilon_t \sim \text{Normal}\left(0, \sigma^2_{\text{proc.}}\right)$$ process uncertainty

INTERACTION (COVARIANCES) CANNOT BE IGNORED



HIERARCHICAL BAYESIAN MODELS PROPAGATE UNCERTAINTY FOR US

Data Model: $y_t \sim \left[y_t \mid z_t, \sigma^2_{\mathrm{o}}\right], \qquad t = 1, \ldots, T,$

Process Models: $z_t \sim \left[z_t \mid \mu_t, \sigma^2_{\mathrm{p}}\right],$

$\mu_t = g\left(z_{t-1}, \mathbf{x}'_t, \theta\right), \qquad t = 2, \ldots, T,$

Parameter Models: $\varphi \sim \left[\theta, \sigma^2_{\mathrm{p}}, \sigma^2_{\mathrm{o}}, z_{t=1}\right]$

Berliner, 1996, *in* Maximum entropy and bayesian methods

$$
\begin{aligned}
[z_{T+1}|y_1,\ldots,y_T] = \int\int\ldots\int &\left[z_{T+1}|z_T,\mathbf{x}_T,\theta,\sigma_{\mathrm{p}}^2\right] \\
\times &\left[z_1,\ldots,z_{T+1},\theta,\sigma_{\mathrm{p}}^2|y_1,\ldots,y_T\right] d\theta d\sigma_{\mathrm{p}}^2 dz_1\ldots dz_T
\end{aligned}
$$

Hobbs and Hooten, 2015, Bayesian Models: A Statistical Primer for Ecologists

THE FORECAST DISTRIBUTION, VIA MCMC

We have:

- $k = 1, \ldots, K$ MCMC iterations
- $j = 1, \ldots, J$ realizations of the covariate, resampled to match K
- Forecasts at times $T + q, \ldots, T + Q$

$$z_{T+q}^{(k)} \sim \left[z_{T+q} \middle| g\left(z_{T+q-1}^{(k)}, \mathbf{x}_{T+q}^{(j(k))}, \theta^{(k)} \right), \sigma_{\mathrm{p}}^{2(k)} \right]$$

Ignore initial conditions uncertainty

$$z_T^{(*)} = E\left(z_T \middle| y_1, \ldots, y_T\right) \approx \frac{\sum_{k=1}^{K} z_T^{(k)}}{K}$$

$$z_{T+q} \sim \begin{cases} \left[z_{T+q} \middle| g\left(z_{T+q-1}^{(k)}, \mathbf{x}_T^{(j(k))}, \theta^{(k)}\right), \sigma_{\mathbf{p}}^{2(k)}\right], & q > 1 \\ \left[z_{T+q} \middle| g\left(z_T^{(*)}, \mathbf{x}_T^{(j(k))}, \theta^{(k)}\right), \sigma_{\mathbf{p}}^{2(k)}\right], & q = 1. \end{cases}$$

POST HOC PARTITIONING FROM MCMC SAMPLES

k	z_T	θ_1	$\cdots$
1	$z_T^{(*)}$	$\theta_1^{(1)}$	$\cdots$
2	$z_T^{(*)}$	$\theta_1^{(2)}$	$\cdots$
3	$z_T^{(*)}$	$\theta_1^{(3)}$	$\cdots$
4	$z_T^{(*)}$	$\theta_1^{(4)}$	$\cdots$
5	$z_T^{(*)}$	$\theta_1^{(5)}$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
K	$z_T^{(*)}$	$\theta_1^{(K)}$	$\cdots$

POST HOC PARTITIONING FROM MCMC SAMPLES

$$\mathbf{z}^{(I)} = \mathbf{z}^{(I,\overline{PA},\overline{D},\overline{PS})}$$

$$\mathbf{z}^{(I)} \approx \left[z_{T+q} \mid g\left(z_{T+q-1}^{(k)}, \mathbf{x}_T^{(*)}, \theta^{(*)}\right), 0 \right]$$

POST HOC PARTITIONING FROM MCMC SAMPLES

$$\mathbf{z}^{(l)} = \mathbf{z}^{(l,\overline{PA},\overline{D},\overline{PS})}$$

$$\mathbf{z}^{(l)} \approx \left[z_{T+q} \mid g\left(z_{T+q-1}^{(k)}, \mathbf{x}_T^{(*)}, \theta^{(*)}\right), 0 \right]$$

POST HOC PARTITIONING FROM MCMC SAMPLES

$$\mathbf{Z}^{(l)} = \mathbf{Z}^{(l,\overline{PA},\overline{D},\overline{PS})}$$

$$\mathbf{Z}^{(l)} \approx \left[z_{T+q} \mid g(z_{T+q-1}^{(k)}, \mathbf{x}_T^{(*)}, \theta^{(*)}), 0\right]$$

$$V^{(l)} = \mathrm{var}(\mathbf{Z}^{(l)})$$

POST HOC PARTITIONING FROM MCMC SAMPLES

Source of Uncertainty	Notation
Initial conditions	$V^{(I)} = V^{(I,\overline{PA},\overline{D},\overline{PS})}$
Parameter uncertainty	$V^{(PA)} = V^{(\bar{I},PA,\overline{D},\overline{PS})}$
Driver uncertainty	$V^{(D)} = V^{(\bar{I},\overline{PA},D,\overline{PS})}$
Process uncertainty	$V^{(PS)} = V^{(\bar{I},\overline{PA},\overline{D},PS)}$

PARTITION FORECAST UNCERTAINTY: ANOVA

$$V^{(F)}_{T+q} = V^{(I)}_{T+q} + V^{(PA)}_{T+q} + V^{(D)}_{T+q} + V^{(PS)}_{T+q}$$

PARTITION FORECAST UNCERTAINTY: ANOVA

$$\begin{aligned}
V^{(F)}_{T+q} = V^{(I)}_{T+q} + V^{(PA)}_{T+q} + V^{(D)}_{T+q} + V^{(PS)}_{T+q} \\
+ \varepsilon^{(I,PA)}_{T+q} + \varepsilon^{(I,D)}_{T+q} + \varepsilon^{(I,PS)}_{T+q} + \varepsilon^{(PA,PS)}_{T+q} + \varepsilon^{(PA,D)}_{T+q} + \varepsilon^{(D,PS)}_{T+q} \\
+ \varepsilon^{(I,PA,D)}_{T+q} + \varepsilon^{(I,PA,PS)}_{T+q} + \varepsilon^{(I,D,PS)}_{T+q} + \varepsilon^{(PA,D,PS)}_{T+q} \\
+ \varepsilon^{(I,PA,D,PS)}_{T+q}
\end{aligned}$$

PARTITION FORECAST UNCERTAINTY: ANOVA

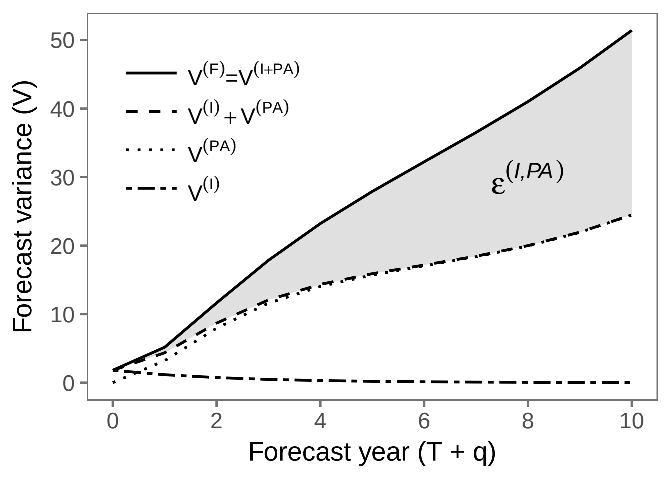
Example where forecast is influenced by initial conditions (I) and parameter uncertainty (PA):

$$V_{T+q}^{(F)} = V_{T+q}^{(I)} + V_{T+q}^{(PA)} + \varepsilon_{T+q}^{(I,PA)}$$

so,

$$\varepsilon_{T+q}^{(I,PA)} = V_{T+q}^{(F)} - \left[V_{T+q}^{(I)} + V_{T+q}^{(PA)}\right]$$

RETURN TO EXAMPLE OF AR(1) PROCESS



§ 3

Concluding thoughts

CONCLUSIONS

1. Partition uncertainty to advance ecological forecasting – how do we get better?
2. Partition uncertainty to advance scientific progress – what don’t we know?
3. Hierarchical Bayesian models ideally suited for partitioning uncertainty because they allow us to fully specify the inclusion of uncertainty.