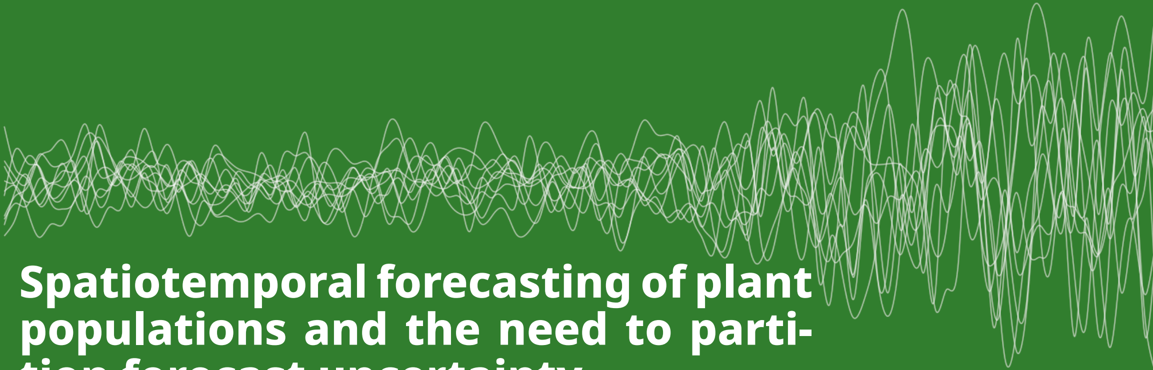


Spatiotemporal forecasting of plant populations and the need to partition forecast uncertainty

Andrew Tredennick

University of Georgia

October 12, 2018

COLLABORATORS

Peter Adler (USU)

Mevin Hooten (CSU)

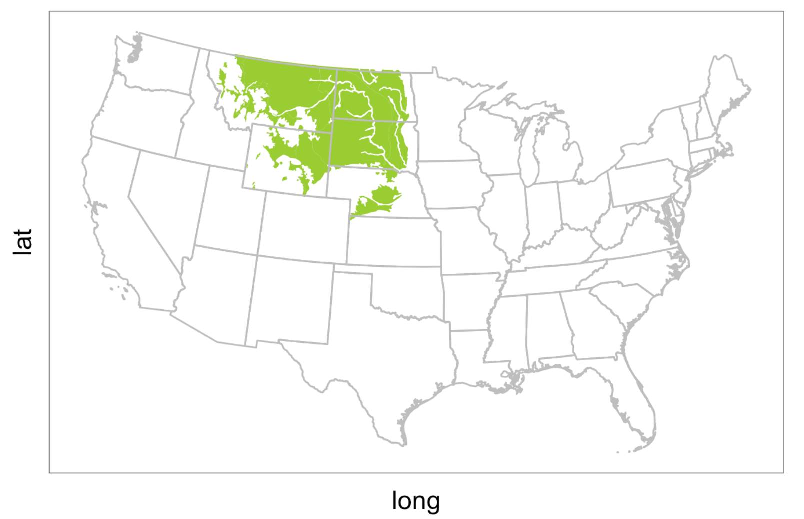
ROAD MAP

1. Plant population forecasts over large spatial extents
2. A plea (and a proposal) to partitioning forecast uncertainty

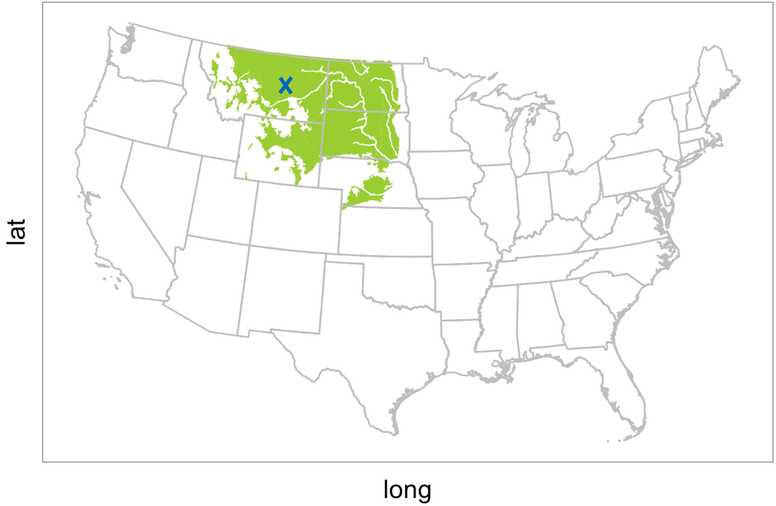
§ 1

Plant population forecasts

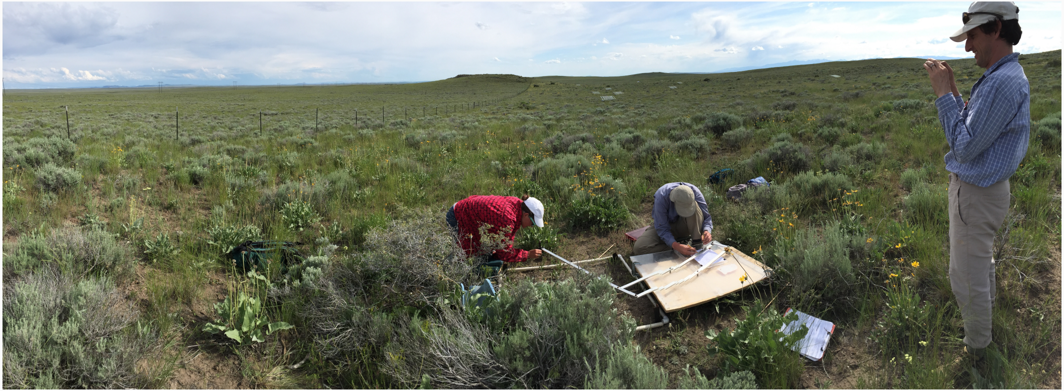
WHAT LAND MANAGERS WANT



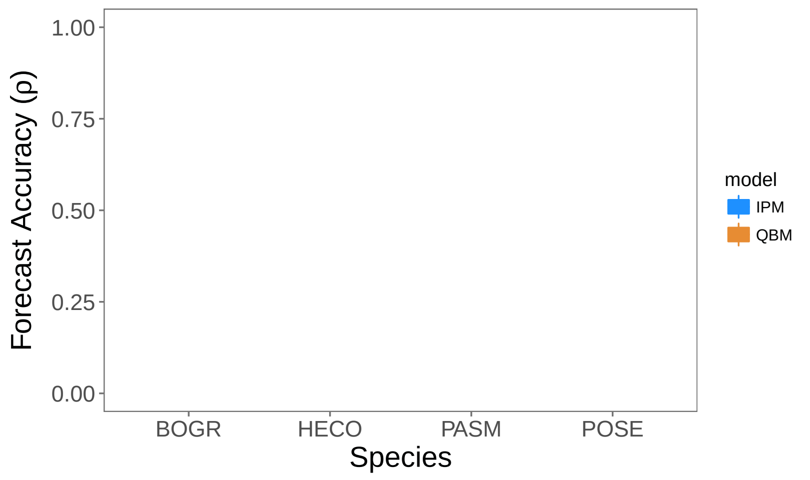
WHAT LAND MANAGERS GET



REALLY HARD WORK

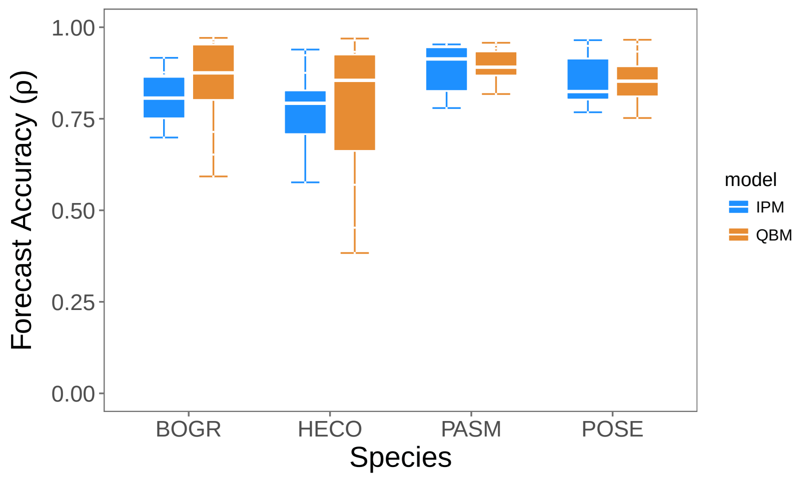


FREE FROM THE TYRANNY OF DEMOGRAPHIC DATA!



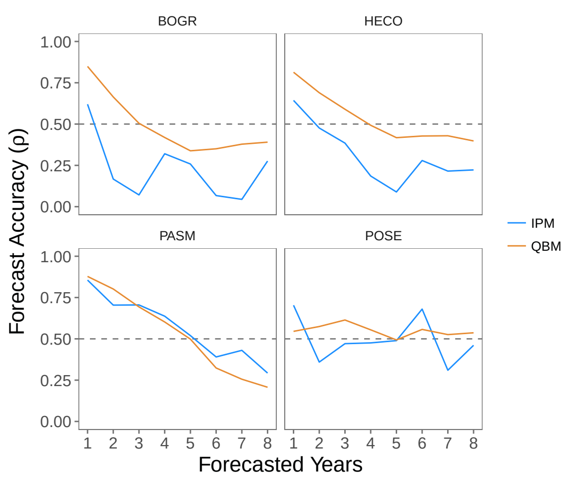
Tredennick et al., 2017, *Methods in Ecol. Evol.*

FREE FROM THE TYRANNY OF DEMOGRAPHIC DATA!



Tredennick et al., 2017, *Methods in Ecol. Evol.*

FORECAST HORIZONS

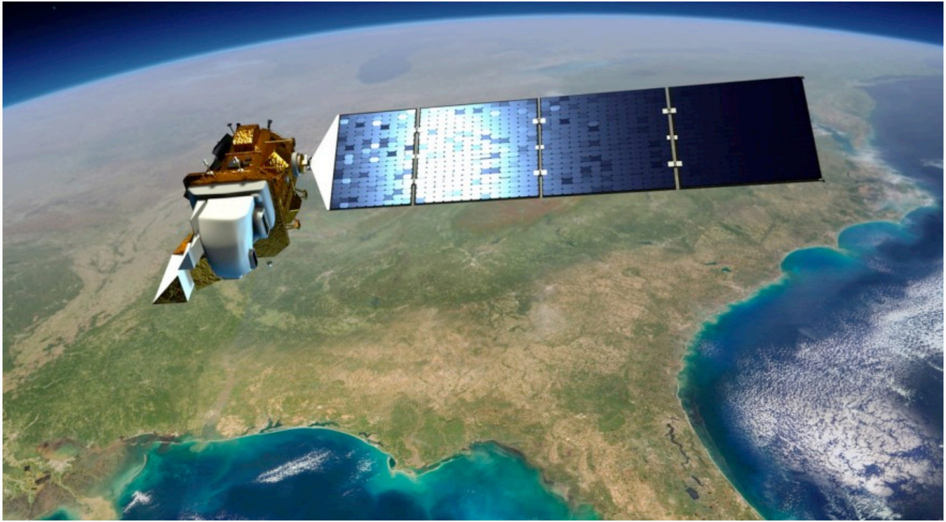


Tredennick et al., 2017, *Methods in Ecol. Evol.*

DO WE NEED DEMOGRAPHIC DATA?

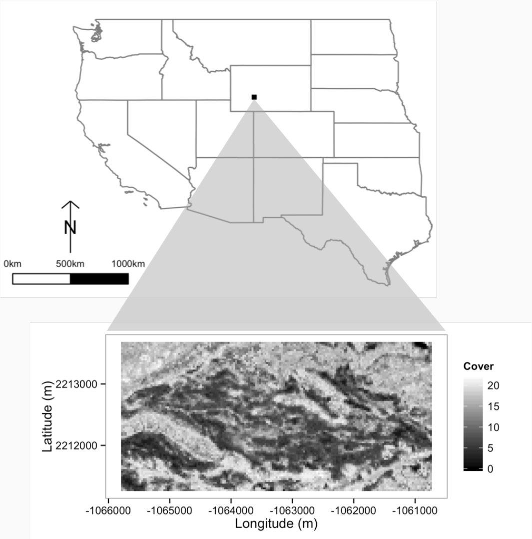
Maybe not.

LET’S USE SATELLITES



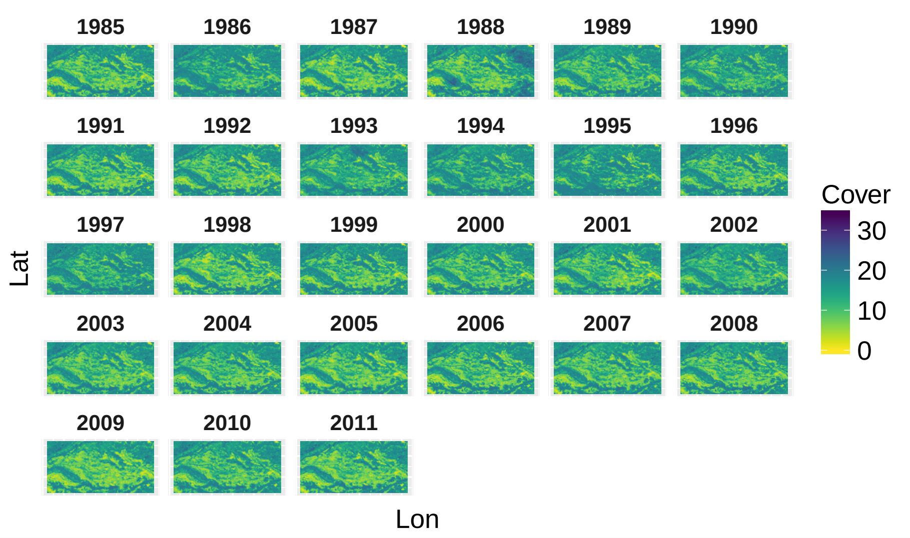
SAGEBRUSH SEA IN WYOMING

STUDY AREA



N
0km
500km
1000km
Cover
20
15
10
5
0
Latitude (m)
2213000
2212000
-1066000
-1065000
-1064000
-1063000
-1062000
-1061000
Longitude (m)

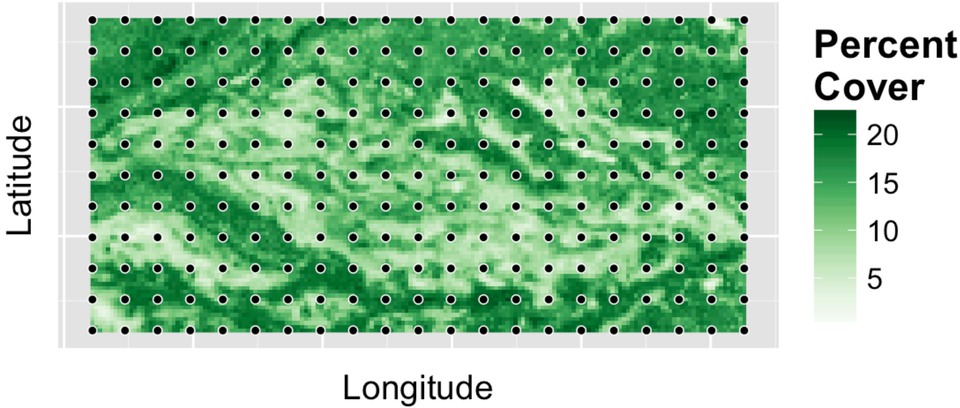
LANDSAT TIME SERIES



DYNAMIC COVER MODEL

$$
\begin{aligned}
y_{i,t} &\sim \text{Poisson}(\mu_{i,t}) \\
\log(\mu_{i,t}) &= \underbrace{\beta_{0,t} + \beta_1 y_{i,t-1}}_{\text{temporal + dens. dep}} + \underbrace{\mathbf{x}_t' \gamma}_{\text{climate}} + \underbrace{\eta_i}_{\text{spatial}}
\end{aligned}
$$

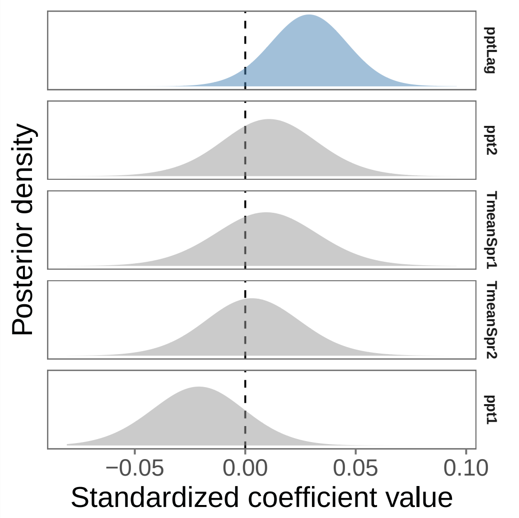
DIMENSION REDUCTION FOR SPATIAL EFFECT



DYNAMIC COVER MODEL

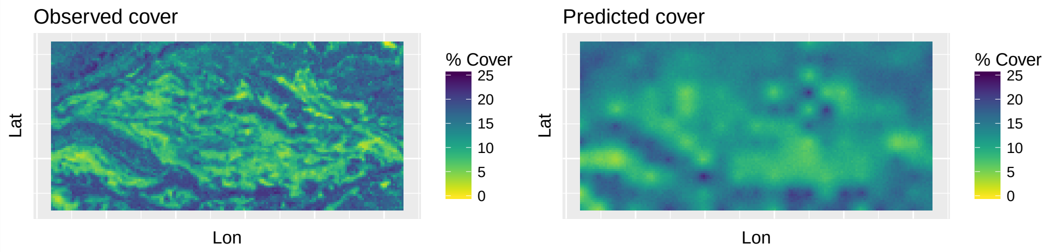
$$
\begin{aligned}
y_{i,t} &\sim \text{Poisson}(\mu_{i,t}) \\
\log(\mu_{i,t}) &= \underbrace{\beta_{0,t} + \beta_1 y_{i,t-1}}_{\text{temporal + dens. dep}} + \underbrace{\mathbf{x}_t'\gamma}_{\text{climate}} + \underbrace{\eta_i}_{\text{spatial}} \\
\eta &\approx \mathbf{K}\alpha, \\
\mathbf{K} &= \mathbf{w}_{s,m} / \sum_{s=1}^{S} \mathbf{w}_{s,m} \\
\mathbf{w}_{s,m} &= \exp\left(-d_{s,m}/\sigma\right) \\
\alpha_m &\sim \text{Normal}(0, \sigma_\eta^2)
\end{aligned}
$$

CLIMATE EFFECTS



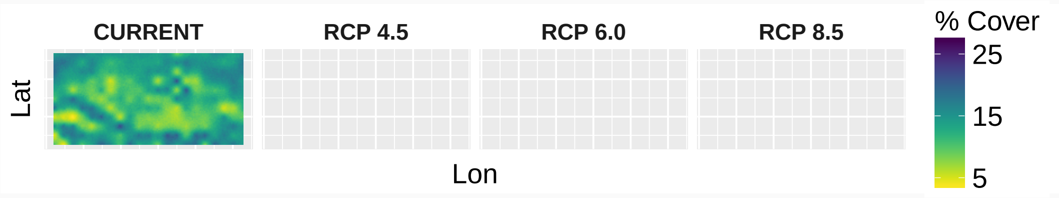
MODEL PERFORMANCE

RMSE $\approx$ 4%



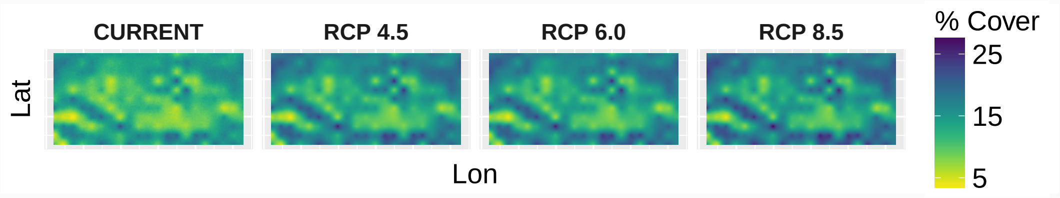
Tredennick et al., 2016, *Ecosphere*

FORECASTS UNDER CLIMATE CHANGE: SPATIAL



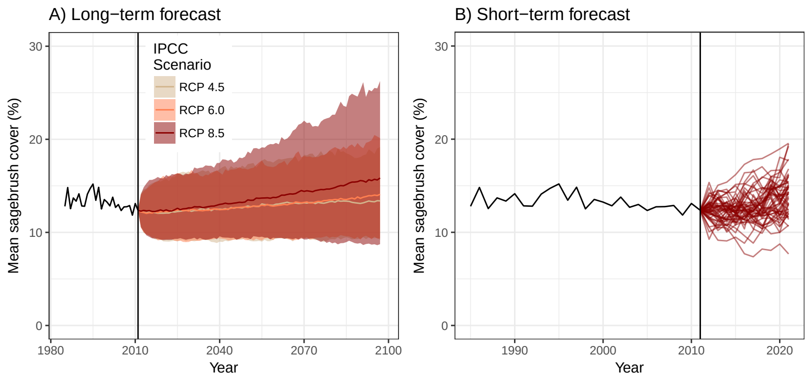
Tredennick et al., 2016, *Ecosphere*

FORECASTS UNDER CLIMATE CHANGE: SPATIAL



Tredennick et al., 2016, *Ecosphere*

FORECASTS UNDER CLIMATE CHANGE: TEMPORAL



Tredennick et al., 2016, *Ecosphere*

§ 2

Partitioning forecast uncertainty

AN (*INVERSE*) ERROR PROPAGATION PROBLEM

For some function q: $q = f(x_1, x_2, \ldots, x_n)$

$$
\begin{aligned}
\sigma_q^2 &= \left(\frac{\delta q}{\delta x_1}\sigma_{x_1}\right)^2 + \left(\frac{\delta q}{\delta x_2}\sigma_{x_2}\right)^2 + \cdots + \left(\frac{\delta q}{\delta x_n}\sigma_{x_n}\right)^2 \\
&= \sum_{i=1}^{n}\left(\frac{\delta q}{\delta x_i}\sigma_{x_i}\right)^2
\end{aligned}
$$

Ku, 1966, *J. Res. National Bureau of Standards - C*

AN (*INVERSE*) ERROR PROPAGATION PROBLEM

$$\sigma_q^2 = \underbrace{\sum_{i=1}^{n} \left(\frac{\delta q}{\delta x_i} \sigma_{x_i} \right)^2}_{\text{variances}} + \underbrace{\sum_{i=1}^{n} \sum_{j(j \neq i)}^{n} 2\sigma_{ij} \left(\frac{\delta q}{\delta x_i} \right) \left(\frac{\delta q}{\delta x_j} \right)}_{\text{covariances}}$$

Ku, 1966, *J. Res. National Bureau of Standards - C*

RECAST AS FORECAST UNCERTAINTY

Forecast of state z at $t+1$ from function q: $q = f(z_t, x_t, \theta, \varepsilon_{t+1})$

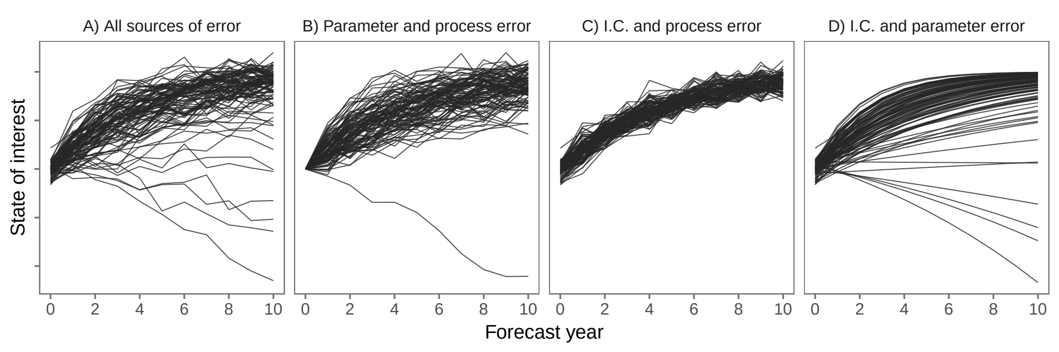
$$Var[y_{t+1}] \approx \underbrace{\left(\frac{\delta q}{\delta y}\right)^2}_{\text{stability}} \underbrace{Var[y_t]}_{\text{IC uncert.}} + \underbrace{\left(\frac{\delta f}{\delta x}\right)^2}_{\text{driver sens.}} \underbrace{Var[x_t]}_{\text{driver uncert.}} + \underbrace{\left(\frac{\delta f}{\delta \theta}\right)^2}_{\text{param sens.}} \underbrace{Var[\theta]}_{\text{param. uncert.}} + \underbrace{Var[\varepsilon_{t+1}]}_{\text{process error}}$$

Dietze, 2017, *Ecological Applications*

INTERACTION (COVARIANCES) CANNOT BE IGNORED

$$
\begin{aligned}
z_{t+1} &= z_t\beta_0 + x_t\beta_1 + \varepsilon_{t+1}, \\
z_{t=1} &\sim \text{Normal}\left(z_0, \sigma^2_{\text{init.}}\right), && \text{initial conditions uncertainty} \\
\boldsymbol{\beta} &\sim \text{MVN}\left(0, \Sigma\mathbf{I}\right), && \text{parameter uncertainty} \\
\varepsilon_t &\sim \text{Normal}\left(0, \sigma^2_{\text{proc.}}\right) && \text{process uncertainty}
\end{aligned}
$$

INTERACTION (COVARIANCES) CANNOT BE IGNORED



HIERARCHICAL BAYESIAN MODELS PROPAGATE UNCERTAINTY FOR US

Data Model: $y_t \sim \left[y_t \mid z_t, \sigma_\text{o}^2\right], \qquad t = 1, \ldots, T,$

Process Models: $z_t \sim \left[z_t \mid \mu_t, \sigma_\text{p}^2\right],$

$\mu_t = g\left(z_{t-1}, \mathbf{x}_t', \theta\right), \qquad t = 2, \ldots, T,$

Parameter Models: $\varphi \sim \left[\theta, \sigma_\text{p}^2, \sigma_\text{o}^2, z_{t=1}\right],$

Berliner, 1996, *in* Maximum entropy and bayesian methods

THE FORECAST DISTRIBUTION

$$
\begin{aligned}
\left[z_{T+1}\middle|y_1, \ldots, y_T\right] = \int\int \ldots \int \left[z_{T+1}\middle|z_T, \mathbf{x}_T, \theta, \sigma_{\mathrm{p}}^2\right] \\
\times \left[\boldsymbol{z_1, \ldots, z_{T+1}}, \theta, \sigma_{\mathrm{p}}^{\mathbf{2}}\middle|\boldsymbol{y_1, \ldots, y_T}\right] \boldsymbol{d}\theta\boldsymbol{d}\sigma_{\mathrm{p}}^{\mathbf{2}}\boldsymbol{dz_1 \ldots dz_T}.
\end{aligned} \tag{1}
$$

Hobbs and Hooten, 2015, Bayesian Models: A Statistical Primer for Ecologists

THE FORECAST DISTRIBUTION, VIA MCMC

We have: - $k = 1, \ldots, K$ MCMC iterations - $j = 1, \ldots, J$ realizations of the covariate, resampled to match K - Forecasts at times $T + q, \ldots, T + Q$

$$z_{T+q}^{(k)} \sim \left[z_{T+q} | g\left(z_{T+q-1}^{(k)}, \mathbf{x}_{T+q}^{(j(k))}, \theta^{(k)} \right), \sigma_{\mathrm{p}}^{2(k)} \right]$$