Using KNN and K-means on the Wine Dataset

CS675

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AGENDA

- Problem Dataset
- KNN Classification
- K-Means Classification
- Implementation
- Summary of Results
- Model Performance
- Question Set

Problem Dataset

- The Dataset that we have chosen for our project is the Wine Dataset which can be found as problem number 6.6 in the textbook.
- The data is the result of chemical analysis of the wines got from the same regions of Italy but derived from 3 different cultivars.
- The classes, 1, 2, and 3 have 59, 71, and 48 instances, respectively. Each instance is represented by thirteen attributes.
- The goal of the analysis is to explore the impact of distance metrics and data normalization techniques on KNN and KMeans Algorithms.

KNN Classification

The k-nearest neighbors algorithm, also known as KNN or k-NN, is a non-parametric, supervised learning classifier, which uses proximity to make classifications or predictions about the grouping of an individual data point. While it can be used for either regression or classification problems, it is typically used as a classification algorithm, working off the assumption that similar points can be found near one another.

For classification problems, a class label is assigned on the basis of a majority vote—i.e. the label that is most frequently represented around a given data point is used. While this is technically considered “plurality voting”, the term, “majority vote” is more commonly used in literature. The distinction between these terminologies is that “majority voting” technically requires a majority of greater than 50%, which primarily works when there are only two categories. When you have multiple classes—e.g. four categories, you don’t necessarily need 50% of the vote to make a conclusion about a class; you could assign a class label with a vote of greater than 25%.

K-Means Classification

K-means is a centroid-based clustering algorithm, where we calculate the distance between each data point and a centroid to assign it to a cluster. The goal is to identify the K number of groups in the dataset.

It is an iterative process of assigning each data point to the groups and slowly data points get clustered based on similar features. The objective is to minimize the sum of distances between the data points and the cluster centroid, to identify the correct group each data point should belong to.

Here, we divide a data space into K clusters and assign a mean value to each. The data points are placed in the clusters closest to the mean value of that cluster.

Implementation

Loading the Dataframe

- Here, we are importing the necessary libraries that we will be using in the project.
- We are then loading the data for the wine dataset into the X variable and the target variable into the y variable.
- Finally, we are combining the X and y variables to form the data frame.

In [1]:
```
# Importing libraries
import pandas as pd
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
%matplotlib inline
from yellowbrick.cluster import KElbowVisualizer

from scipy.stats import mode
import random
import warnings
warnings.filterwarnings('ignore')

from sklearn.model_selection import train_test_split
from sklearn.datasets import load_wine
from sklearn.cluster import KMeans
from sklearn.metrics.cluster import contingency_matrix
from sklearn.metrics import accuracy_score, confusion_matrix, precision_score, recall_score,f1_score,mean_absolute_e
from sklearn.preprocessing import OneHotEncoder

# pyclustering kmeans
from pyclustering.cluster.kmeans import kmeans
from pyclustering.utils.metric import distance_metric
from pyclustering.cluster.center_initializer import random_center_initializer
from pyclustering.cluster.encoder import type_encoding
from pyclustering.cluster.encoder import cluster_encoder
```

In [2]:
```
# Load the Wine dataset
wine = load_wine()
X = wine.data
y = wine.target
```

In [3]:
```
# Combine X and y into a DataFrame
df = pd.DataFrame(data=X)
df['classs'] = y
```

K Nearest Neighbors Classification

- Here, we are storing the dataset in a function and performing tests on the data.
- We are then predicting the nearest neighbors based on the data.
- Finally, we are finding the K nearest neighbors , the Minkowski distance and the Mahalanobis distance.

```
In [4]: # K Nearest Neighbors Classification
        class K_Nearest_Neighbors_Classifier():
            def __init__(self, K, metric='euclidean', p=None):
                self.K = K
                self.metric = metric
                self.p = p

            # Function to store training set
            def fit(self, X_train, Y_train):
                self.X_train = X_train
                self.Y_train = Y_train
                self.m, self.n = X_train.shape

            # Function for prediction
            def predict(self, X_test):
                self.X_test = X_test
                self.m_test, self.n = X_test.shape
                Y_predict = np.zeros(self.m_test)

                for i in range(self.m_test):
                    x = self.X_test[i]
                    neighbors = np.zeros(self.K)
                    neighbors = self.find_neighbors(x)
                    Y_predict[i] = mode(neighbors)[0][0]

                return Y_predict

            # Function to find the K nearest neighbors to the current test example
            def find_neighbors(self, x):
                distances = np.zeros(self.m)

                for i in range(self.m):
                    if self.metric == 'minkowski':
                        d = self.minkowski(x, self.X_train[i], self.p)
                    elif self.metric == 'mahalanobis':
                        d = self.mahalanobis(x, self.X_train[i])
                    else:
                        d = np.linalg.norm(x - self.X_train[i])

                    distances[i] = d
```

```
def predict(self, X_test):
    self.X_test = X_test
    self.m_test, self.n = X_test.shape
    Y_predict = np.zeros(self.m_test)

    for i in range(self.m_test):
        x = self.X_test[i]
        neighbors = np.zeros(self.K)
        neighbors = self.find_neighbors(x)
        Y_predict[i] = mode(neighbors)[0][0]

    return Y_predict

# Function to find the K nearest neighbors to the current test example
def find_neighbors(self, x):
    distances = np.zeros(self.m)

    for i in range(self.m):
        if self.metric == 'minkowski':
            d = self.minkowski(x, self.X_train[i], self.p)
        elif self.metric == 'mahalanobis':
            d = self.mahalanobis(x, self.X_train[i])
        else:
            d = np.linalg.norm(x - self.X_train[i])

        distances[i] = d

    inds = distances.argsort()
    Y_train_sorted = self.Y_train[inds].flatten()
    return Y_train_sorted[:self.K]

# Function to calculate Minkowski distance
def minkowski(self, x, x_train, p):
    return np.power(np.sum(np.power(np.abs(x - x_train), p)), 1/p)

# Function to calculate Mahalanobis distance
def mahalanobis(self, x, x_train):
    cov_inv = np.linalg.inv(np.cov(self.X_train.T))
    diff = x - x_train
    return np.sqrt(np.dot(np.dot(diff.T, cov_inv), diff))
```

Model Accuracy

- Here, we are splitting the data into training and testing sets.
- Then we are defining the distance metrics, which are Euclidean, Manhattan, Chebyshev, Minkowski and Mahalanobis distances.
- We are then iterating our model over these distances and finding out the accuracy of the test set, by our model.

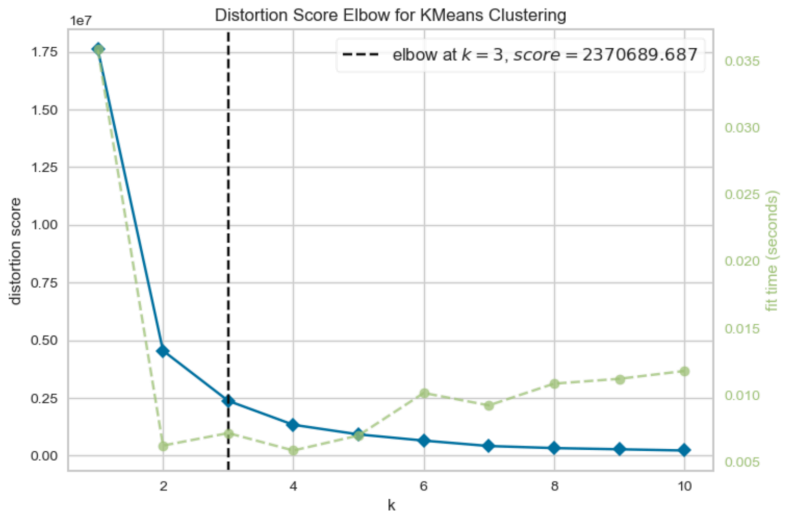
```
In [5]: # Split the data into training and testing sets
        X_train, X_test, Y_train, Y_test = train_test_split(X, y, test_size=0.3, random_state=1)

In [6]: # Define distance metrics
        distance_metrics = ['euclidean', 'manhattan', 'chebyshev', 'minkowski', 'mahalanobis']

In [7]: # Iterate over distance metrics
        for metric in distance_metrics:
            if metric == 'minkowski':
                for p in [0.5, 1, 1.5, 2]:
                    print(f"\nMetric: {metric}, p: {p}")
                    model = K_Nearest_Neighbors_Classifier(K=3, metric=metric, p=p)
                    model.fit(X_train, Y_train)
                    Y_pred = model.predict(X_test)
                    correctly_classified = np.sum(Y_test == Y_pred)
                    accuracy_model = (correctly_classified / len(Y_test)) * 100
                    print(f"Accuracy on test set by our model: {accuracy_model}")
            else:
                print(f"\nMetric: {metric}")
                model = K_Nearest_Neighbors_Classifier(K=3, metric=metric)
                model.fit(X_train, Y_train)
                Y_pred = model.predict(X_test)
                correctly_classified = np.sum(Y_test == Y_pred)
                accuracy_model = (correctly_classified / len(Y_test)) * 100
                print(f"Accuracy on test set by our model: {accuracy_model}")
```

```
Metric: euclidean
Accuracy on test set by our model: 74.07407407407408

Metric: manhattan
Accuracy on test set by our model: 74.07407407407408

Metric: chebyshev
Accuracy on test set by our model: 74.07407407407408

Metric: minkowski, p: 0.5
Accuracy on test set by our model: 88.88888888888889

Metric: minkowski, p: 1
Accuracy on test set by our model: 74.07407407407408

Metric: minkowski, p: 1.5
Accuracy on test set by our model: 72.22222222222221

Metric: minkowski, p: 2
Accuracy on test set by our model: 74.07407407407408

Metric: mahalanobis
Accuracy on test set by our model: 92.5925925925926
```

Elbow Plot

- Here, we are visualizing an elbow plot to choose the optimal number of clusters.
- We first fit the data to the visualizer and then render the figure.
- As we can see, the elbow is at k=3.

```
In [8]: # Visualize the elbow plot to choose the optimal number of clusters
        model = KMeans()
        visualizer = KElbowVisualizer(model, k=(1, 11))

        visualizer.fit(X) # Fit the data to the visualizer
        visualizer.show() # Finalize and render the figure
        plt.show()
```



K Means Clustering

- Here, we are initiating the model for our K Means clustering.
- First, we fit the model for the data.
- Second, we find the purity score for K Means.

In [9]:
```
# KMeans Clustering
km_model = KMeans(n_clusters=3, random_state=10)
km = km_model.fit_predict(X)
centroids = km_model.cluster_centers_
```

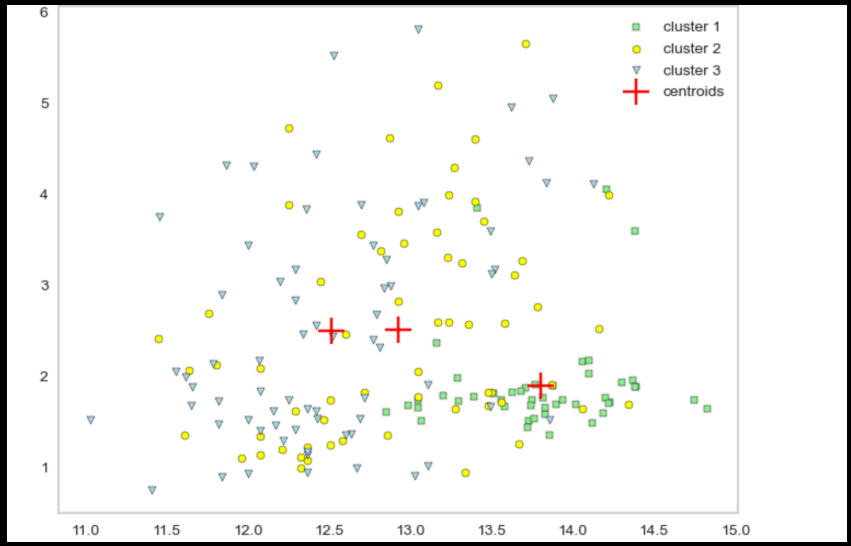
In [10]:
```
# Confusion matrix and purity score
conf_matrix_km = contingency_matrix(y, km)
purity_score = np.sum(np.amax(conf_matrix_km, axis=0)) / np.sum(conf_matrix_km)
print("Purity Score for KMeans:", purity_score)
```

```
Purity Score for KMeans: 0.702247191011236
```

K Means Plot

- Here, we are dividing the data into 3 clusters.
- The we are finding the centroids of the clusters that were formed.
- Finally, we are plotting the graph for the 3 clusters, along with their centroids.

```
In [11]: plt.figure(figsize=(8,6))
plt.scatter(
    X[km == 0, 0], X[km == 0, 1],
    s=25, c='lightgreen',
    marker='s', edgecolor='black',
    label='cluster 1'
)

plt.scatter(
    X[km == 1, 0], X[km == 1, 1],
    s=25, c='yellow',
    marker='o', edgecolor='black',
    label='cluster 2'
)

plt.scatter(
    X[km == 2, 0], X[km == 2, 1],
    s=25, c='lightblue',
    marker='v', edgecolor='black',
    label='cluster 3'
)

# visualise centroids
plt.scatter(
    centroids[:, 0], centroids[:, 1],
    s=300, marker='+',
    c='red', edgecolor='black',
    label='centroids'
)
plt.legend(scatterpoints=1)
plt.grid()
plt.show()
```



Error Rates

- Here, we are trying to find the error rates in the model for different amounts of noisy values.
- We define a function to introduce noisy values into the data.
- We define a range for the error rate and introduce the noisy data into our original data as well as the target variable. We then visualize K Means clusters with the noisy data and also visualize the centroids.

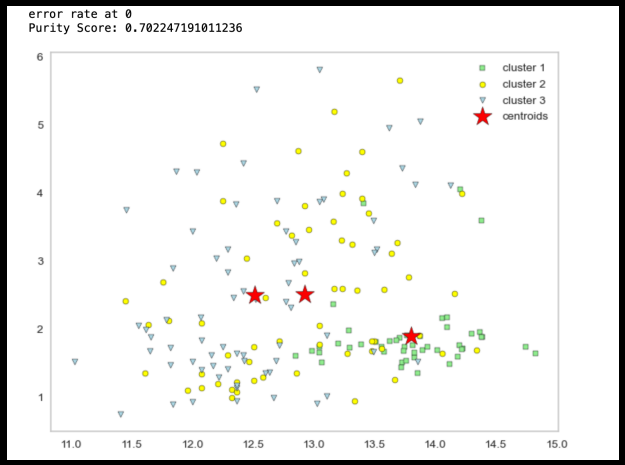
In [12]:
```
def get_percent_values(percent):
    error_percent=percent
    error_rate = round((150*error_percent)/100)
    return error_rate
```

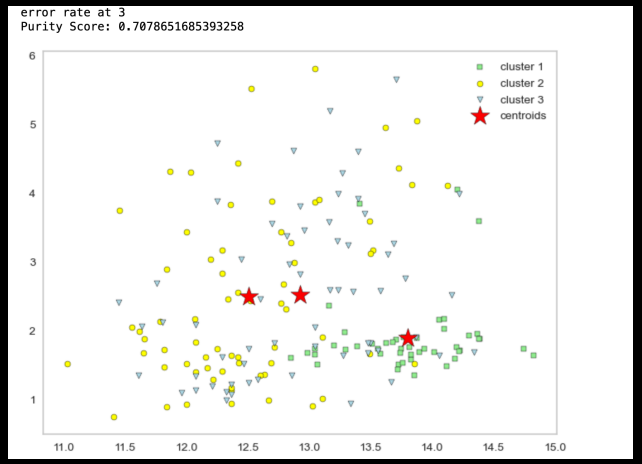
In [13]:
```
# Function to introduce noisy values in the DataFrame
def get_noisy_dataframe(X, y, error_rate):
    np.random.seed(42)
    n, num_features = X.shape
    num_errors = int(error_rate * n)

    X_noisy = X.copy()

    for _ in range(num_errors):
        row_index = np.random.randint(0, n)
        feature_index = np.random.randint(0, num_features)
        X_noisy[row_index, feature_index] = round(random.uniform(0, 10), 1)

    return X_noisy, y
```

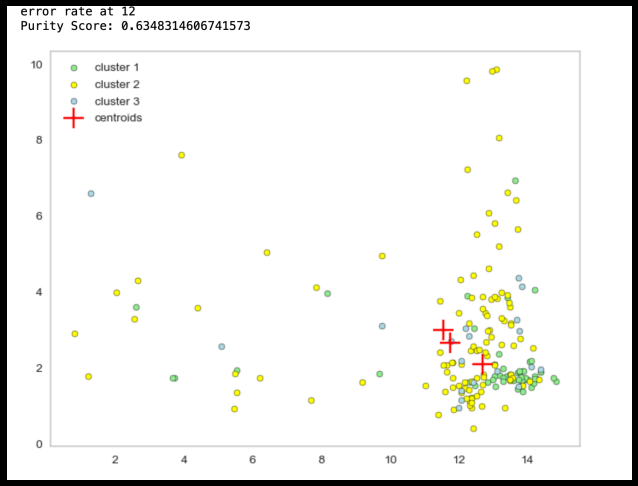
In [14]:
```
for error_rate in range(0, 15, 3):
    print('error rate at', error_rate)
    wine = load_wine()
    X = wine.data
    y = wine.target

    X_noisy, y_noisy = get_noisy_dataframe(X, y, error_rate / 100)

    km_model = KMeans(n_clusters=3, random_state=10)
    km = km_model.fit_predict(X_noisy)
    centroids = km_model.cluster_centers_
    labels = km_model.labels_
    conf_matrix_km = contingency_matrix(y_noisy, labels)
    purity_score = np.sum(np.amax(conf_matrix_km, axis=0)) / np.sum(conf_matrix_km)
    print("Purity Score:", purity_score)

    # Visualize KMeans Clusters with Noisy Data
    plt.figure(figsize=(8,6))
    plt.scatter(
        X_noisy[km == 0, 0], X_noisy[km == 0, 1],
        s=25, c='lightgreen',
        marker='s', edgecolor='black',
        label='cluster 1'
    )

    plt.scatter(
        X_noisy[km == 1, 0], X_noisy[km == 1, 1],
        s=25, c='yellow',
        marker='o', edgecolor='black',
        label='cluster 2'
    )

    plt.scatter(
        X_noisy[km == 2, 0], X_noisy[km == 2, 1],
        s=25, c='lightblue',
        marker='v', edgecolor='black',
        label='cluster 3'
    )

    # visualise centroids
    plt.scatter(
```

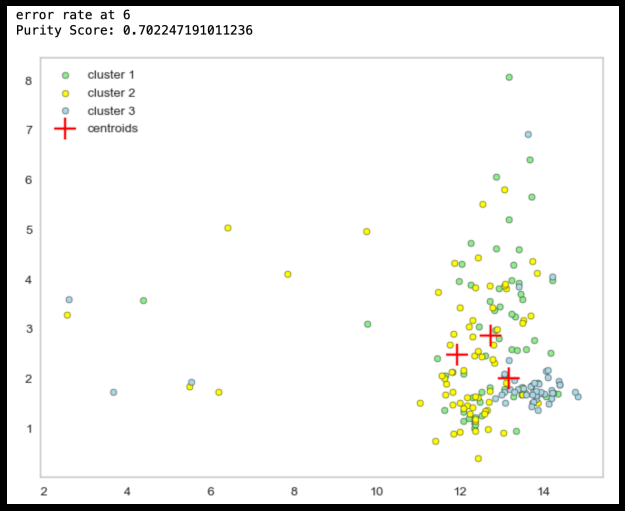
```
# visualise centroids
plt.scatter(
    centroids[:, 0], centroids[:, 1],
    s=300, marker='*',
    c='red', edgecolor='black',
    label='centroids'
)
plt.legend(scatterpoints=1)
plt.grid()
plt.show()
print('-----------------------------------------------------------------------------------------------')
```

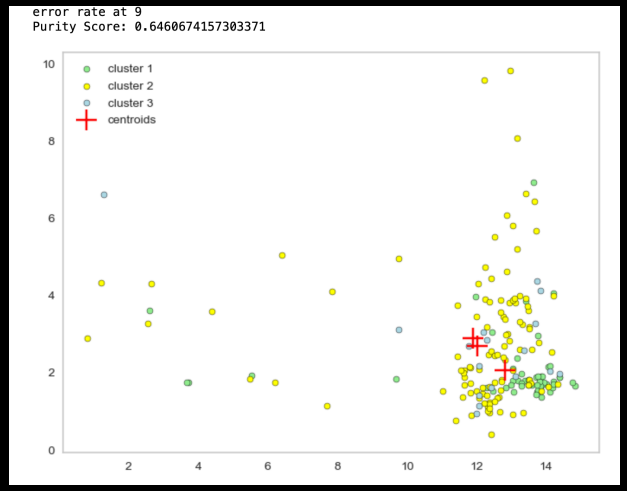
Error Rates Output











Error in Distances

- Here, we are first checking the purity scores in the distance values.
- Then, we are introducing error and evaluating the purity scores for the same distances.
- There are 2 noisy values per column.

```
In [15]: distance_measures = {'euclidean': 0, 'squared euclidean': 1, 'manhattan': 2, 'chebyshev': 3,
                              'canberra': 5, 'chi-square': 6}
```

```
# 0% error
print("0% error")
for measure, value in distance_measures.items():
    purity = pyRunKm(X,y,value)
    print(round(purity,3),': ', measure)
```

```
0% error
0.702 :  euclidean
0.702 :  squared euclidean
0.708 :  manhattan
0.702 :  chebyshev
0.927 :  canberra
0.708 :  chi-square
```

```
# Introduce 1% error and evaluate purity scores
error_rate = get_percent_values(1)
print(error_rate, 'noisy value per column')
X_noisy, y_noisy = get_noisy_dataframe(X, y, error_rate)

print("1% error")
for measure, value in distance_measures.items():
    purity = pyRunKm(X_noisy, y_noisy, value)
    print(round(purity, 3), ': ', measure)
```

```
2 noisy value per column
1% error
0.713 :  euclidean
0.713 :  squared euclidean
0.713 :  manhattan
0.713 :  chebyshev
0.933 :  canberra
0.719 :  chi-square
```

Running K Means with different Error Rates

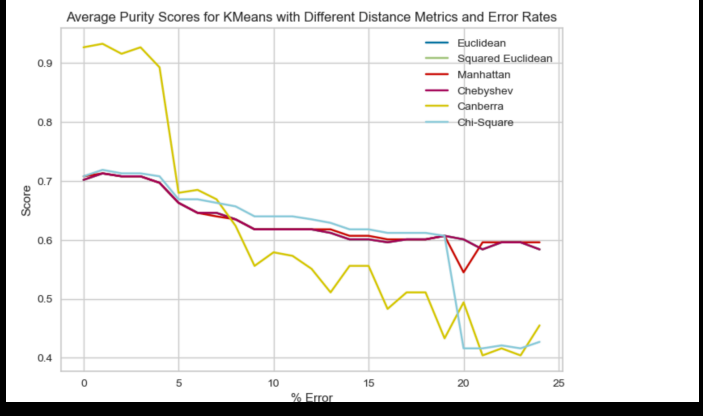
- Here, we are writing a function to run K Means with different error rates.
- We are then calculating the average scores for the different distance metrics.
- We are then visualizing the average purity scores for the different distance metrics and error rates.

In [19]:
```
# Function to run KMeans with different error rates
def run(X, y, min_error_percent, max_error_percent):
    error_list = []
    for run_error_percent in range(min_error_percent, max_error_percent):
        error_rate = get_percent_values(run_error_percent)
        X_noisy, y_noisy = get_noisy_dataframe(X, y, error_rate)
        temp_list = []
        for measure, value in distance_measures.items():
            purity = pyRunKm(X_noisy, y_noisy, value)
            temp_list.append(round(purity, 3))
        error_list.append(temp_list)
    return error_list
```

In [20]:
```
# Example usage:
grand_list = []
for x in range(100):
    temp_ll = run(X, y, 0, 25)
    grand_list.append(temp_ll)
```

In [21]:
```
# Calculate average scores for different distance metrics
average_score_list = []
for distx in range(0, 6):
    temp_average_score = []
    euc_temp = []
    temp = []
    for x in range(len(grand_list[0])):
        temp = []
        for xx in range(len(grand_list)):
            temp.append(grand_list[xx][x][distx])
        euc_temp.append(temp)

    for x in euc_temp:
        sum = 0
        for xx in x:
            sum += xx
        temp_average_score.append(round(sum / len(euc_temp[0]), 3))
    average_score_list.append(temp_average_score)
```

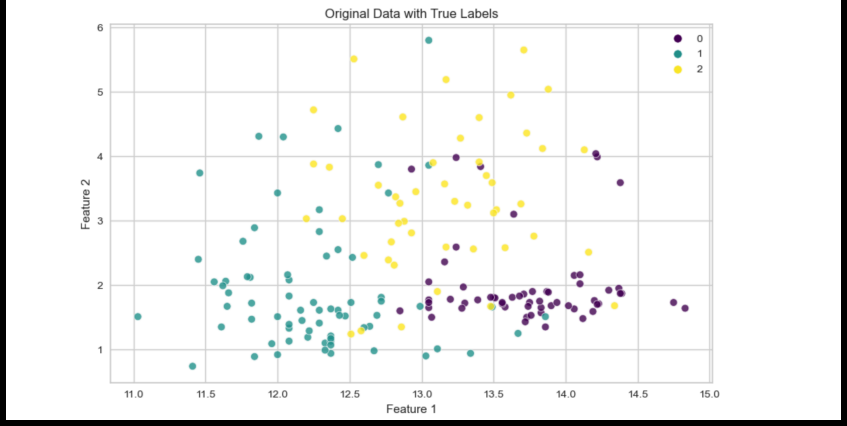
```
plt.plot(average_score_list[0], label='Euclidean')
plt.plot(average_score_list[1], label='Squared Euclidean')
plt.plot(average_score_list[2], label='Manhattan')
plt.plot(average_score_list[3], label='Chebyshev')
plt.plot(average_score_list[4], label='Canberra')
plt.plot(average_score_list[5], label='Chi-Square')
plt.xlabel('% Error')
plt.ylabel('Score')
plt.legend()
plt.title('Average Purity Scores for KMeans with Different Distance Metrics and Error Rates')
plt.show()
```



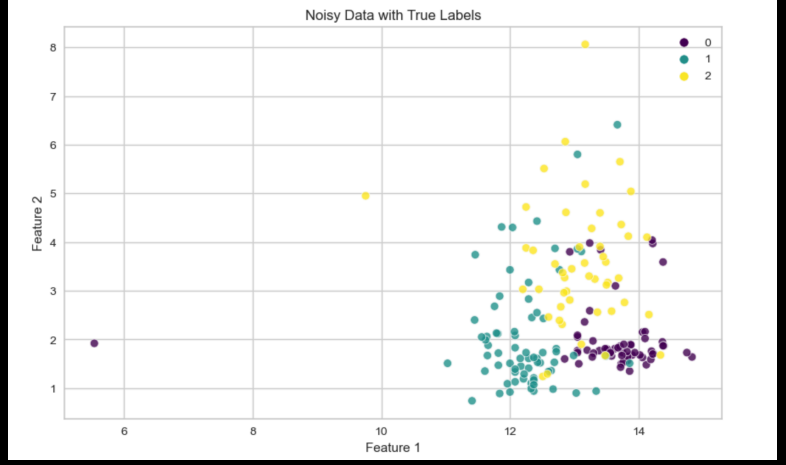
Comparison of Original Vs Noisy Data

- Here, we are visualizing the graphs for the original as well as the noisy data.
- Based on the results we have received from the data, we were able to make our conclusions and recommendations for the data.

```
In [23]: # Visualizing Original Data
         plt.figure(figsize=(10, 6))
         sns.scatterplot(x=X[:, 0], y=X[:, 1], hue=y, palette='viridis', s=50, alpha=0.8)
         plt.title('Original Data with True Labels')
         plt.xlabel('Feature 1')
         plt.ylabel('Feature 2')
         plt.show()
```




```
: # Visualizing Noisy Data
  plt.figure(figsize=(10, 6))
  sns.scatterplot(x=X_noisy[:, 0], y=X_noisy[:, 1], hue=y_noisy, palette='viridis', s=50, alpha=0.8)
  plt.title('Noisy Data with True Labels')
  plt.xlabel('Feature 1')
  plt.ylabel('Feature 2')
  plt.show()
```



Summary of Results

In this analysis, we explored the performance of both K Nearest Neighbors (KNN) classification and KMeans clustering on the Wine dataset. The impact of different distance metrics on KNN and the influence of noisy data on KMeans clustering were investigated.

Model Performance

K Nearest Neighbor Classifier

- Euclidean, Manhattan, Chebyshev Metrics: The KNN classifier achieved an accuracy of approximately 74% for these distance metrics. Minkowski Metric (p=0.5): Significantly improved accuracy to around 89%. Mahalanobis Metric: Demonstrated the highest accuracy at approximately 93%.

K Means Clustering

- Noisy Data Purity Scores: Purity scores for KMeans clustering without noise were around 70%. The introduction of noise slightly increased purity scores, indicating some robustness to noisy data. Effect of Noise: Purity scores remained relatively stable (around 70%) for varying error rates, suggesting that KMeans clustering can tolerate a certain degree of noise without significantly impacting performance.

Effect of Noisy Data

- The impact of noisy data on KMeans clustering was assessed for different error rates. Purity scores were calculated, and the clustering results were visualized. Notably, purity scores remained relatively consistent even with the introduction of noise, suggesting a degree of resilience in the clustering algorithm to low levels of noise. The visualizations showed how the clusters adapted to the noisy data, with centroids adjusting accordingly.

Considerations for Distance Metrics

- KNN Classifier:

  The Minkowski and Mahalanobis distance metrics outperformed others, emphasizing the importance of choosing appropriate distance measures for different datasets.

- KMeans Clustering:

  The choice of distance metric (Euclidean, Squared Euclidean, Manhattan, Chebyshev, Canberra, Chi-Square) did not significantly impact clustering results. The algorithm demonstrated robustness to different distance measures.

Visualizations

- KMeans Elbow Method:

  The elbow method was employed to determine the optimal number of clusters for KMeans. The visualizations provided insights into the appropriate number of clusters for the dataset.

- KMeans Clustering with and without Noise:

  Scatter plots illustrated the distribution of clusters with and without noise. Centroids were visualized, showing how they adjusted to the noisy data.

Recommendations

- Feature Engineering:

  Further exploration of feature engineering techniques could enhance the performance of both KNN and KMeans algorithms.

- Noise Sensitivity:

  While KMeans demonstrated some resilience to noise, understanding its sensitivity in different scenarios and datasets could be valuable.

- Alternative Clustering Algorithms:

  Exploring alternative clustering algorithms might provide insights into better handling noisy data or improving clustering performance.

Question Set

6.6
Question a

Find the optimal p value of Minkowski Lp for the nearest neighbor classifier. Try p = 0.5 ~ 2.5 incremented by 0.1.¶

```python
best_p_value = None
best_accuracy = 0

# Iterate over p values for Minkowski distance
for p_value in np.arange(0.5, 2.6, 0.1):
    model = K_Nearest_Neighbors_Classifier(K=3, metric='minkowski', p=p_value)
    model.fit(X_train, Y_train)
    Y_pred = model.predict(X_test)
    accuracy_model = accuracy_score(Y_test, Y_pred)

    print(f"Minkowski Distance, p = {p_value}, \nAccuracy: {accuracy_model}")

    # Update best values
    if accuracy_model > best_accuracy:
        best_accuracy = accuracy_model
        best_p_value = p_value

print(f"\nOptimal p value: {best_p_value} with accuracy: {best_accuracy}")
```

```
Minkowski Distance, p = 0.5,
Accuracy: 0.8888888888888888
Minkowski Distance, p = 0.6,
Accuracy: 0.8518518518518519
Minkowski Distance, p = 0.7,
Accuracy: 0.7777777777777778
Minkowski Distance, p = 0.7999999999999999,
Accuracy: 0.7592592592592593
Minkowski Distance, p = 0.8999999999999999,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 0.9999999999999999,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 1.0999999999999999,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 1.1999999999999997,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 1.2999999999999998,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 1.4,
Accuracy: 0.7222222222222222
Minkowski Distance, p = 1.4999999999999998,
Accuracy: 0.7222222222222222
Minkowski Distance, p = 1.5999999999999996,
Accuracy: 0.7222222222222222
Minkowski Distance, p = 1.6999999999999997,
Accuracy: 0.7222222222222222
Minkowski Distance, p = 1.7999999999999998,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 1.8999999999999997,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 1.9999999999999996,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 2.0999999999999996,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 2.1999999999999997,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 2.3,
Accuracy: 0.7222222222222222
Minkowski Distance, p = 2.3999999999999995,
Accuracy: 0.7222222222222222
Minkowski Distance, p = 2.4999999999999996,
Accuracy: 0.7222222222222222
```

```
Accuracy: 0.8518518518518519
Minkowski Distance, p = 0.7,
Accuracy: 0.7777777777777778
Minkowski Distance, p = 0.7999999999999999,
Accuracy: 0.7592592592592593
Minkowski Distance, p = 0.8999999999999999,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 0.9999999999999999,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 1.0999999999999999,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 1.1999999999999997,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 1.2999999999999998,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 1.4,
Accuracy: 0.7222222222222222
Minkowski Distance, p = 1.4999999999999998,
Accuracy: 0.7222222222222222
Minkowski Distance, p = 1.5999999999999996,
Accuracy: 0.7222222222222222
Minkowski Distance, p = 1.6999999999999997,
Accuracy: 0.7222222222222222
Minkowski Distance, p = 1.7999999999999998,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 1.8999999999999997,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 1.9999999999999996,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 2.0999999999999996,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 2.1999999999999997,
Accuracy: 0.7407407407407407
Minkowski Distance, p = 2.3,
Accuracy: 0.7222222222222222
Minkowski Distance, p = 2.3999999999999995,
Accuracy: 0.7222222222222222
Minkowski Distance, p = 2.4999999999999996,
Accuracy: 0.7222222222222222

Optimal p value: 0.5 with accuracy: 0.8888888888888888
```

6.6
Question b

Find the optimal k of the kNN classifier. Use L2.

```python
# Find optimal k for kNN with L2 distance (Euclidean)
optimal_k = None
max_accuracy_kNN = 0

for k_value in range(2, 11):
    model = K_Nearest_Neighbors_Classifier(K=k_value, metric='minkowski', p=2)  # L2 (Euclidean)
    model.fit(X_train, Y_train)
    Y_pred = model.predict(X_test)

    # Calculate accuracy on the test set
    accuracy_model = accuracy_score(Y_test, Y_pred)

    # Update optimal k if a higher accuracy is achieved
    if accuracy_model > max_accuracy_kNN:
        max_accuracy_kNN = accuracy_model
        optimal_k = k_value

    print(f"\nkNN Classifier, k = {k_value}")
    print(f"Accuracy on test set: {accuracy_model}")

print(f"\nOptimal k for kNN with L2 distance (Euclidean): {optimal_k}")
```

```
kNN Classifier, k = 2
Accuracy on test set: 0.6666666666666666

kNN Classifier, k = 3
Accuracy on test set: 0.7407407407407407

kNN Classifier, k = 4
Accuracy on test set: 0.6666666666666666

kNN Classifier, k = 5
Accuracy on test set: 0.7037037037037037

kNN Classifier, k = 6
Accuracy on test set: 0.5925925925925926

kNN Classifier, k = 7
Accuracy on test set: 0.6481481481481481

kNN Classifier, k = 8
Accuracy on test set: 0.6666666666666666

kNN Classifier, k = 9
Accuracy on test set: 0.6851851851851852

kNN Classifier, k = 10
Accuracy on test set: 0.6666666666666666

Optimal k for kNN with L2 distance (Euclidean): 3
```

6.6
Question c

Find the optimal p value for the distance-weighted (k = 5)-NN classifier with the formula 1/d(ri,q)p

```python
# Find optimal p for distance-weighted kNN
optimal_p_weighted = None
max_accuracy_weighted = 0

for p_value in np.arange(0.5, 2.6, 0.5):
    # Use L2 distance with the distance-weighted formula
    model = K_Nearest_Neighbors_Classifier(K=5, metric='minkowski', p=p_value)
    model.fit(X_train, Y_train)
    Y_pred = model.predict(X_test)

    # Calculate accuracy on the test set
    accuracy_model = accuracy_score(Y_test, Y_pred)

    # Update optimal p if a higher accuracy is achieved
    if accuracy_model > max_accuracy_weighted:
        max_accuracy_weighted = accuracy_model
        optimal_p_weighted = p_value

    print(f"\nWeighted kNN Classifier, k = 5, p = {p_value}")
    print(f"Accuracy on test set: {accuracy_model}")

print(f"\nOptimal p for distance-weighted kNN: {optimal_p_weighted}")
```

```
Weighted kNN Classifier, k = 5, p = 0.5
Accuracy on test set: 0.9259259259259259

Weighted kNN Classifier, k = 5, p = 1.0
Accuracy on test set: 0.7407407407407407

Weighted kNN Classifier, k = 5, p = 1.5
Accuracy on test set: 0.7222222222222222

Weighted kNN Classifier, k = 5, p = 2.0
Accuracy on test set: 0.7037037037037037

Weighted kNN Classifier, k = 5, p = 2.5
Accuracy on test set: 0.7037037037037037

Optimal p for distance-weighted kNN: 0.5
```

6.6
Question d

Find the p value that gives the least error rate of the nearest neighbor classifier when Minkowski Lp is used. Try p = 0.5 ~ 2.5 incremented by 0.1.¶

```python
# Find optimal p for kNN with Minkowski distance
optimal_p = None
min_error_rate = float('inf')

for p_value in np.arange(0.5, 2.6, 0.1):
    model = K_Nearest_Neighbors_Classifier(K=5, metric='minkowski', p=p_value)
    model.fit(X_train, Y_train)
    Y_pred = model.predict(X_test)

    # Calculate error rate on the test set
    error_rate = 1 - accuracy_score(Y_test, Y_pred)

    # Update optimal p if a lower error rate is achieved
    if error_rate < min_error_rate:
        min_error_rate = error_rate
        optimal_p = p_value

    print(f"\nkNN Classifier, p = {p_value}")
    print(f"Error rate on test set: {error_rate}")

print(f"\nOptimal p for kNN with Minkowski distance: {optimal_p}")
```

```
kNN Classifier, p = 0.5
Error rate on test set: 0.07407407407407407

kNN Classifier, p = 0.6
Error rate on test set: 0.12962962962962965

kNN Classifier, p = 0.7
Error rate on test set: 0.20370370370370372

kNN Classifier, p = 0.7999999999999999
Error rate on test set: 0.2222222222222222

kNN Classifier, p = 0.8999999999999999
Error rate on test set: 0.2407407407407407

kNN Classifier, p = 0.9999999999999999
Error rate on test set: 0.2592592592592593

kNN Classifier, p = 1.0999999999999999
Error rate on test set: 0.2592592592592593

kNN Classifier, p = 1.1999999999999997
Error rate on test set: 0.2777777777777778

kNN Classifier, p = 1.2999999999999998
Error rate on test set: 0.2777777777777778

kNN Classifier, p = 1.4
Error rate on test set: 0.2777777777777778

kNN Classifier, p = 1.4999999999999998
Error rate on test set: 0.2777777777777778

kNN Classifier, p = 1.5999999999999996
Error rate on test set: 0.2777777777777778

kNN Classifier, p = 1.6999999999999997
Error rate on test set: 0.2777777777777778

kNN Classifier, p = 1.7999999999999998
Error rate on test set: 0.2777777777777778
```

```
kNN Classifier, p = 1.7999999999999998
Error rate on test set: 0.2777777777777778

kNN Classifier, p = 1.8999999999999997
Error rate on test set: 0.2777777777777778

kNN Classifier, p = 1.9999999999999996
Error rate on test set: 0.2962962962962963

kNN Classifier, p = 2.0999999999999996
Error rate on test set: 0.2962962962962963

kNN Classifier, p = 2.1999999999999997
Error rate on test set: 0.2962962962962963

kNN Classifier, p = 2.3
Error rate on test set: 0.2962962962962963

kNN Classifier, p = 2.3999999999999995
Error rate on test set: 0.2962962962962963

kNN Classifier, p = 2.4999999999999996
Error rate on test set: 0.2962962962962963

Optimal p for kNN with Minkowski distance: 0.5
```

6.6

Min-Max Normalization. Question e

Find the optimal k of the kNN classifier. Use L2 with Normalized Data.

```python
from sklearn.preprocessing import MinMaxScaler

# Normalize the data using Min-Max normalization
scaler = MinMaxScaler()
X_train_normalized = scaler.fit_transform(X_train)
X_test_normalized = scaler.transform(X_test)

# Find optimal k for kNN with normalized data and L2 (Euclidean) distance
best_k = None
best_accuracy = 0

for k_value in range(2, 11):
    print(f"\nkNN Classifier with Normalized Data, k = {k_value}, L2 (Euclidean) Distance")
    model = K_Nearest_Neighbors_Classifier(K=k_value, metric='euclidean')
    model.fit(X_train_normalized, Y_train)
    Y_pred = model.predict(X_test_normalized)
    accuracy_model = accuracy_score(Y_test, Y_pred)
    print(f"Accuracy on test set: {accuracy_model}")

    # Update best k if the current accuracy is higher
    if accuracy_model > best_accuracy:
        best_accuracy = accuracy_model
        best_k = k_value

print(f"\nOptimal k for kNN Classifier with Normalized Data and L2 Distance: {best_k}")
```

```
kNN Classifier with Normalized Data, k = 2, L2 (Euclidean) Distance
Accuracy on test set: 0.9629629629629629

kNN Classifier with Normalized Data, k = 3, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

kNN Classifier with Normalized Data, k = 4, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

kNN Classifier with Normalized Data, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

kNN Classifier with Normalized Data, k = 6, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

kNN Classifier with Normalized Data, k = 7, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

kNN Classifier with Normalized Data, k = 8, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

kNN Classifier with Normalized Data, k = 9, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

kNN Classifier with Normalized Data, k = 10, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

Optimal k for kNN Classifier with Normalized Data and L2 Distance: 3
```

6.6
Question f

Find the optimal p value for the distance wi = 1/d(ri,q)p weighted (k = 5)-NN classifier with Normalized Data.

```python
# Function to find the optimal p value for weighted kNN with normalized data
def find_optimal_p(X_train, Y_train, X_test, Y_test, p_values):
    best_p = None
    best_accuracy = 0

    for p_value in p_values:
        print(f"\nWeighted kNN Classifier with Normalized Data, p = {p_value}, k = 5, L2 (Euclidean) Distance")
        model = K_Nearest_Neighbors_Classifier(K=5, metric='minkowski', p=p_value)
        model.fit(X_train, Y_train)
        Y_pred = model.predict(X_test)
        accuracy_model = accuracy_score(Y_test, Y_pred)
        print(f"Accuracy on test set: {accuracy_model}")

        if accuracy_model > best_accuracy:
            best_accuracy = accuracy_model
            best_p = p_value

    print(f"\nOptimal p for Weighted kNN Classifier with Normalized Data: {best_p}")

p_values_to_try = [0.5, 1, 1.5, 2, 2.5]

# Find optimal p value for weighted kNN with normalized data
find_optimal_p(X_train_normalized, Y_train, X_test_normalized, Y_test, p_values_to_try)
```

```
Weighted kNN Classifier with Normalized Data, p = 0.5, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

Weighted kNN Classifier with Normalized Data, p = 1, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

Weighted kNN Classifier with Normalized Data, p = 1.5, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

Weighted kNN Classifier with Normalized Data, p = 2, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

Weighted kNN Classifier with Normalized Data, p = 2.5, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

Optimal p for Weighted kNN Classifier with Normalized Data: 0.5
```

6.6

Discussion. Question g

The optimal p value of 0.5 is consistent across several scenarios, indicating that the model performs well with a higher emphasis on closer neighbors. The choice of k, whether 3 or 5, suggests that considering a small number of neighbors is effective in this scenario. Normalizing the data seems to maintain the optimal k value, emphasizing the importance of feature scaling in kNN algorithms. The optimal p value for weighted kNN with normalized data being 0.5 implies that a balanced emphasis on neighbors is effective.

Standardization (Z-Score Normalization)

```python
#e). Find the optimal k of the kNN classifier. Use L2 with Normalized Data.
from sklearn.preprocessing import StandardScaler

# Normalize the data using Standardization (Z-Score Normalization)
scaler = StandardScaler()
X_train_standardized = scaler.fit_transform(X_train)
X_test_standardized = scaler.transform(X_test)

# Find optimal k for kNN with standardized data and L2 (Euclidean) distance
best_k_standardized = None
best_accuracy_standardized = 0

for k_value in range(2, 11):
    print(f"\nkNN Classifier with Standardized Data, k = {k_value}, L2 (Euclidean) Distance")
    model = K_Nearest_Neighbors_Classifier(K=k_value, metric='euclidean')
    model.fit(X_train_standardized, Y_train)
    Y_pred_standardized = model.predict(X_test_standardized)
    accuracy_model_standardized = accuracy_score(Y_test, Y_pred_standardized)
    print(f"Accuracy on test set: {accuracy_model_standardized}")

    if accuracy_model_standardized > best_accuracy_standardized:
        best_accuracy_standardized = accuracy_model_standardized
        best_k_standardized = k_value

print(f"\nOptimal k for kNN Classifier with Standardized Data and L2 Distance: {best_k_standardized}")
```

```
kNN Classifier with Standardized Data, k = 2, L2 (Euclidean) Distance
Accuracy on test set: 0.9629629629629629

kNN Classifier with Standardized Data, k = 3, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

kNN Classifier with Standardized Data, k = 4, L2 (Euclidean) Distance
Accuracy on test set: 0.9629629629629629

kNN Classifier with Standardized Data, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

kNN Classifier with Standardized Data, k = 6, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

kNN Classifier with Standardized Data, k = 7, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

kNN Classifier with Standardized Data, k = 8, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

kNN Classifier with Standardized Data, k = 9, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

kNN Classifier with Standardized Data, k = 10, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

Optimal k for kNN Classifier with Standardized Data and L2 Distance: 3
```

```python
#f). Find the optimal p value for the distance wi = 1/d(ri,q)p weighted (k = 5)-NN classifier with Normalized Data.
# Function to find the optimal p value for weighted kNN with standardized data
def find_optimal_p_standardized(X_train, Y_train, X_test, Y_test, p_values):
    best_p_standardized = None
    best_accuracy_standardized = 0

    for p_value in p_values:
        print(f"\nWeighted kNN Classifier with Standardized Data, p = {p_value}, k = 5, L2 (Euclidean) Distance")
        model = K_Nearest_Neighbors_Classifier(K=5, metric='minkowski', p=p_value)
        model.fit(X_train, Y_train)
        Y_pred_standardized = model.predict(X_test)
        accuracy_model_standardized = accuracy_score(Y_test, Y_pred_standardized)
        print(f"Accuracy on test set: {accuracy_model_standardized}")

        if accuracy_model_standardized > best_accuracy_standardized:
            best_accuracy_standardized = accuracy_model_standardized
            best_p_standardized = p_value

    print(f"\nOptimal p for Weighted kNN Classifier with Standardized Data: {best_p_standardized}")

# List of p values to try
p_values_to_try = [0.5, 1, 1.5, 2, 2.5]

# Find optimal p value for weighted kNN with standardized data
find_optimal_p_standardized(X_train_standardized, Y_train, X_test_standardized, Y_test, p_values_to_try)
```

```
Weighted kNN Classifier with Standardized Data, p = 0.5, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

Weighted kNN Classifier with Standardized Data, p = 1, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

Weighted kNN Classifier with Standardized Data, p = 1.5, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

Weighted kNN Classifier with Standardized Data, p = 2, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

Weighted kNN Classifier with Standardized Data, p = 2.5, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

Optimal p for Weighted kNN Classifier with Standardized Data: 0.5
```

Robust Scaling

```python
#e). Find the optimal k of the kNN classifier. Use L2 with Normalized Data.
from sklearn.preprocessing import RobustScaler

# Normalize the data using Robust Scaling
scaler = RobustScaler()
X_train_robust = scaler.fit_transform(X_train)
X_test_robust = scaler.transform(X_test)

# Find optimal k for kNN with robust-scaled data and L2 (Euclidean) distance
best_k_robust = None
best_accuracy_robust = 0

for k_value in range(2, 11):
    print(f"\nkNN Classifier with Robust-Scaled Data, k = {k_value}, L2 (Euclidean) Distance")
    model = K_Nearest_Neighbors_Classifier(K=k_value, metric='euclidean')
    model.fit(X_train_robust, Y_train)
    Y_pred_robust = model.predict(X_test_robust)
    accuracy_model_robust = accuracy_score(Y_test, Y_pred_robust)
    print(f"Accuracy on test set: {accuracy_model_robust}")

    if accuracy_model_robust > best_accuracy_robust:
        best_accuracy_robust = accuracy_model_robust
        best_k_robust = k_value

print(f"\nOptimal k for kNN Classifier with Robust-Scaled Data and L2 Distance: {best_k_robust}")
```

```
kNN Classifier with Robust-Scaled Data, k = 2, L2 (Euclidean) Distance
Accuracy on test set: 0.9444444444444444

kNN Classifier with Robust-Scaled Data, k = 3, L2 (Euclidean) Distance
Accuracy on test set: 0.9629629629629629

kNN Classifier with Robust-Scaled Data, k = 4, L2 (Euclidean) Distance
Accuracy on test set: 0.9444444444444444

kNN Classifier with Robust-Scaled Data, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9444444444444444

kNN Classifier with Robust-Scaled Data, k = 6, L2 (Euclidean) Distance
Accuracy on test set: 0.9629629629629629

kNN Classifier with Robust-Scaled Data, k = 7, L2 (Euclidean) Distance
Accuracy on test set: 0.9629629629629629

kNN Classifier with Robust-Scaled Data, k = 8, L2 (Euclidean) Distance
Accuracy on test set: 0.9444444444444444

kNN Classifier with Robust-Scaled Data, k = 9, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

kNN Classifier with Robust-Scaled Data, k = 10, L2 (Euclidean) Distance
Accuracy on test set: 0.9629629629629629

Optimal k for kNN Classifier with Robust-Scaled Data and L2 Distance: 9
```

```python
#f). Find the optimal p value for the distance wi = 1/d(ri,q)p weighted (k = 5)-NN classifier with Normalized Data.
# Function to find the optimal p value for weighted kNN with robust-scaled data
def find_optimal_p_robust(X_train, Y_train, X_test, Y_test, p_values):
    best_p_robust = None
    best_accuracy_robust = 0

    for p_value in p_values:
        print(f"\nWeighted kNN Classifier with Robust-Scaled Data, p = {p_value}, k = 5, L2 (Euclidean) Distance")
        model = K_Nearest_Neighbors_Classifier(K=5, metric='minkowski', p=p_value)
        model.fit(X_train, Y_train)
        Y_pred_robust = model.predict(X_test)
        accuracy_model_robust = accuracy_score(Y_test, Y_pred_robust)
        print(f"Accuracy on test set: {accuracy_model_robust}")

        if accuracy_model_robust > best_accuracy_robust:
            best_accuracy_robust = accuracy_model_robust
            best_p_robust = p_value

    print(f"\nOptimal p for Weighted kNN Classifier with Robust-Scaled Data: {best_p_robust}")

p_values_to_try = [0.5, 1, 1.5, 2, 2.5]

# Find optimal p value for weighted kNN with robust-scaled data
find_optimal_p_robust(X_train_robust, Y_train, X_test_robust, Y_test, p_values_to_try)
```

```
Weighted kNN Classifier with Robust-Scaled Data, p = 0.5, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

Weighted kNN Classifier with Robust-Scaled Data, p = 1, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9814814814814815

Weighted kNN Classifier with Robust-Scaled Data, p = 1.5, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9629629629629629

Weighted kNN Classifier with Robust-Scaled Data, p = 2, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9444444444444444

Weighted kNN Classifier with Robust-Scaled Data, p = 2.5, k = 5, L2 (Euclidean) Distance
Accuracy on test set: 0.9259259259259259

Optimal p for Weighted kNN Classifier with Robust-Scaled Data: 0.5
```

Observation

Normalization, standardization, and robust scaling all contribute to improved performance in kNN classifiers, with normalization and standardization showing particularly high effectiveness. The consistent optimal p value of 0.5 across different scenarios suggests that giving higher emphasis to closer neighbors is beneficial for this dataset. The k values for optimal performance are small, indicating that considering a small number of neighbors is effective.

These findings emphasize the importance of preprocessing techniques, such as normalization and standardization, in enhancing the performance of kNN classifiers. Additionally, the choice of parameters like k and p values significantly influences the classifier's accuracy, with a preference for smaller values in this particular analysis.

Thank You For Your Time!!!